1 Introduction

The most important goal of the Large Hadron Collider is to discover new elementary particles. We understand many things about elementary particle physics and the subnuclear forces. However, there are important things that we do not know. One of these mysteries is the origin of the masses of the quarks, leptons, and $W$ and $Z$ bosons. Another is the nature of dark matter, the neutral, very weakly interacting substance that makes up 80% of the mass in the universe [1]. In quantum physics, every force is generated by an associated particle. So, for every mystery in the physics of very small distances, there is a particle responsible, and we must find it.

If we are looking at the LHC for the particle that makes up dark matter, we must be looking for a particle that is stable, neutral, and weakly interacting. Searches for the dark matter particle in sensitive underground detectors tell us that its probability to interact with ordinary matter is infinitesimal—in fact, less than $10^{-20}$ of the interaction probability for protons. So, if dark matter particles are produced at the LHC, the detectors set up there will not be able to observe them. However, most theories of dark matter include more particles than just this one. Typically, there are heavier particles that are produced in pairs at the LHC and then decay to the dark matter particle with observable decay products—quarks, gluons, or leptons. One of the most popular models to explain both dark matter and mass generation is supersymmetry. In supersymmetry, the dark matter particle is a matter particle related to the photon. This particle has a very small probability of being produced directly at the LHC. However, the corresponding partners of quarks and gluons, which are heavier particles that couple to the strong interactions, should be produced with a relatively larger rate. These particles would then decay to the photon partner plus additional quarks and gluons.

Supersymmetry is one of a large class of model in which dark matter particles are made at the LHC by heavy particle pair-production and decay. In all of these models, events with dark matter particles have the general form shown in Fig. 1.
Figure 1: A typical event at the LHC that would produce particles of dark matter. (a.) The process of heavy particle production and decay. In the figure, \(N\) represents a dark matter particle, \(Q\) represents a new heavy particle, and \(q\) represents a quark or antiquark. This is the simplest case. Depending on the model, the primary particle \(Q\) will decay to the dark matter particle by emission of a quark, a gluon, or multiple quarks, gluons, and leptons. (b.) The resulting configuration of high-energy particles, visible in a detector. The proton-proton collisions also creates many lower-energy tracks not shown in this drawing.

Visible particles are present, but these apparently have unbalanced momenta. The momentum is balanced by the unobservable dark matter particles.

When quarks and gluons are produced by ordinary strong interactions, all final particles are visible (except for the occasional neutrino). Thus you might think that it would be easy to distinguish events with new particles and dark matter production from typical events created by the well-understood strong interactions. This would be true if strong interaction events and dark matter events appeared at the same rate. A relatively small number of proton-proton collisions at the LHC contain scatterings of quarks and gluons through the strong interactions with very high energy transfer, leading to particles with more than 200 GeV of energy. The probability of such an event is about 1 in 10,000. However, our models predict that the heavy particle production that leads to dark matter happens in about one out of every \(10^{11}\) proton-
proton collisions, a rate smaller by a factor of 10 million. To discover such events, we need to be sure that ordinary quark-gluon events cannot be confused or mismeasured by our detectors to mimic the signal of large unbalanced momentum, even for 1 event in 10 million. This presents a very difficult challenge to the experiments.

Because the problem of discriminating new particle events from ordinary quark-gluon processes is so difficult, it is essential to have a systematic approach. Both of the major LHC experiments ATLAS and CMS have detailed computer simulations that include the locations of every bolt and baffle in the apparatus. Such simulations can be trusted to compute the signal that will be registered by a given event at the few-percent level. However, these simulations cannot be trusted to compute the probabilities of unusual detector responses at the level of 1 in 10 million.

A systematic approach to this question can be built in one of two ways. First, we could enumerate the standard reactions that are most likely to create signals of unbalanced momentum, measure the rates for each, and measure the factors that might lead these reactions to general false signals of dark matter. Both ATLAS and CMS have devoted much effort and energy to this program. It will be useful to list the standard reactions that are the most dangerous. The first is ordinary quark-gluon scattering. I will refer to this below as “QCD”. If the detector makes an error in measuring the energy of the scattered quarks or gluons, the event may appear to have missing momentum. As explained above, the rate for these events at the LHC is $10^7$ times larger than the rate for dark matter production. The second is the production of a $W$ or $Z$ boson together with quarks and gluons. The $W$ boson decays about $\frac{1}{3}$ of the time to a lepton and an invisible neutrino. If the lepton is emitted into a region of the detector where it would not be identified, the $W$ is invisible. The $Z$ boson has a probability of 20% to decay to a pair of neutrinos. This decay is completely invisible. If the production of a $W$ or a $Z$ boson is accompanied by the radiation of two gluons or quarks, that event could easily be mistaken for dark matter production. The rate for these events at the LHC is $10^4$ times larger than the rate for dark matter production. Finally, there is a substantial probability to create the top quark, the heaviest type of quark, at the LHC. Top quarks are usually produced in pairs. In a top quark pair production event, the two heavy quarks decay about 10% of the time to two neutrinos, two leptons (which might miss being observed) and two quarks. The rate for these events at the LHC is 100 times larger than the rate for dark matter production. Physicists in the two experiments have measured the rates for each type of event—QCD, W/Z, and top—and assessed for each the probability of extra quark emission, quark energy mismeasurement, and other effects that would allow these events to look like dark matter events. For QCD events, this probability is very small; for top events, there is a higher probability multiplying a lower intrinsic rate.
The other way to understand whether we can trust an observation of unbalanced momentum is to invent a variable that separates standard but mismeasured events from genuinely new physics processes. Observing the distribution of events in this variable can help us evaluate whether a signal of unbalanced momentum comes from standard or new processes. The CMS experiment has made effective use of two such variables. One of them will be refered to here as the ”scissors” variable. This variable was invented by Lisa Randall and David Tucker-Smith [2]. The second is the “razor” variable, invented by Chris Rogan [3]. Randall and Tucker-Smith are well known theoretical physicists; Randall is a very well-known Harvard professor. Rogan is a Caltech graduate student who is one of the 3000 members of the CMS collaboration. But a clever idea will be noticed no matter who you are.

2 Scissors

Randall and Tucker-Smith consider events of the type shown in Fig. 1(b), with two clusters of high-energy particles emitted. The clusters are called “jets”. Each jet originates from the emission of a quark or gluon. This particle then radiates other quarks and gluons and eventually forms a cluster of strongly-interacting particles. For each jet, let $\vec{p}_T$ be the component of the momentum vector of the jet in the direction perpendicular to the axis of the incoming proton beams [4]. This quantity is called the “transverse momentum”. I will denote the length of this vector as $p_T$. It is difficult to measure the momentum of reaction products that emerge at very small angles to the proton directions, so it is difficult to check that the component of momentum along the beam direction balances completely. Thus, the clearest sign of the presence of invisible particles is the imbalance of transverse momentum. Let jet 1 be the jet with the higher value of $p_T$ and jet 2 be the jet with the lower value of $p_T$. In a quark-gluon scattering event, the two jets would have equal and opposite vectors $\vec{p}_T$. The mass of the two-jet system is given by

$$m_{jj} = \sqrt{E_{jj}^2 - |\vec{p}_{jj}|^2} = \sqrt{(E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2}.$$ 

However, since we are most interested in transverse momentum imbalance, it is useful to define a mass estimator based on transverse information.

$$m_T = \sqrt{(p_{T1} + p_{T2})^2 - |\vec{p}_{T1} + \vec{p}_{T2}|^2}.$$ 

The quantity $m_T$ is called the “transverse mass”. It can be shown that $m_T$ is always less than the two-jet mass $m_{jj}$.

Randall and Tucker-Smith then define the scissors variable

$$\alpha_T = \frac{p_{T2}}{m_T}.$$ 


For 2-jet QCD events in which the transverse momenta $\vec{p}_T^1$ and $\vec{p}_T^2$ are equal and opposite, $\alpha_T = \frac{1}{2}$. Events with $\alpha_T > \frac{1}{2}$ signal actual invisible transverse momentum. Events with large values of $\alpha_T$ resemble a scissors, as we see in Fig. 1(b).

The variable $\alpha_T$ has two properties that assist this identification. First, assume that we have a 2-jet QCD event with both quarks having $p_T = P$, but in which the energy of one quark is measured correctly but the energy of the other quark is measured low by a factor $f$. There is missing transverse momentum

$$\vec{p}_T^1 + \vec{p}_T^2 = (1 - f)\vec{p}_T^1 .$$

In this situation, the value of $p_T^2$ would be measured incorrectly as $fP$, and the transverse mass would be measured incorrectly as $\sqrt{4fP^2}$. Then we would find

$$\alpha_T = \sqrt{f} \cdot \frac{1}{2} ,$$

so that these events would not appear in the signal region $\alpha_T > \frac{1}{2}$. Second, assume that we have a 3-jet event, resulting, for example, from a quark-quark scattering event with emission of an extra gluon. Then we have three transverse vectors, with $\vec{p}_T^1 + \vec{p}_T^2 + \vec{p}_T^3 = 0$. If the jet with transverse momentum $\vec{p}_T^3$ is not included in the event reconstruction, the event will register with unbalanced transverse momentum. Let $P = p_T^1 + p_T^2 + p_T^3$, and

$$x_1 = \frac{2p_T^1}{P} , \quad x_2 = \frac{2p_T^2}{P} , \quad x_3 = \frac{2p_T^3}{P} .$$

Then

$$x_1 + x_2 + x_3 = 2$$

and, for the combination of jets 1 and 2,

$$m_T^2(12) = 4P^2(1 - x_3) .$$

Then

$$\alpha_T = \frac{1}{2} \frac{x_2}{\sqrt{1 - x_3}} .$$

Typically in QCD events, the direction of jet 3 will be close to that of jet 2. In this case, $x_2 + x_3 \approx 1$ to balance momentum, and then the expression for $\alpha_T$ is close to $\frac{1}{2} \sqrt{x_2} < \frac{1}{2}$. Only if there is a distinct missed jet at a large angle can we have $\alpha_T > \frac{1}{2}$, and QCD events of this type are relatively improbable. Thus, the scissors variable very effectively removes QCD events from the sample.

The use of the scissors variable in practice is shown in a set of figures from the paper of the CMS collaboration that applies this variable to searches for supersymmetry [5, 6]. Fig. 2(a) shows the effectiveness of the scissors variable in removing
Figure 2: Use of the scissors variable in a search for dark matter production, from [5]:

(a.) Comparison of physics simulation and data for the distribution of 2-jet events in the scissors variable $\alpha_T$. The expectation for QCD events is shown as a green histogram. The expectation for $W/Z$ and top events is shown as a blue histogram. The red and orange histograms are the expectations for two supersymmetry/dark matter models. The data is shown in black. Notice that hundreds of thousands of events are observed for $\alpha_T < 0.5$, but only a few events survive in the region $\alpha_T > 0.55$. (b.) Fraction of jet events that survive into the regions $\alpha_T > 0.51$ (red) and $\alpha_T > 0.55$ (black), as measured in data and evaluated from a physics simulation. The variable $H_T$ measures the total energy deposited in the event. The blue stars check the second result by counting jet events in the $W$ boson production with jets, using events in which the muon from $W$ decay is observed.

2-jet QCD events. Hundreds of thousands of very energetic 2-jet events are observed in the data at values of $\alpha_T$ less than $\frac{1}{2}$. At $\alpha_T > 0.55$, only a few events remain, and, according to the simulation, most of these events arise from the more complex $W/Z$ and top production processes. Fig. 2(b) shows the measurement of $R_{\alpha_T}$, the fraction of 2- and 3-jet events from the total sample that pass the requirements $\alpha_T > 0.51$ and $\alpha_T > 0.55$. The figure shows that this fraction is a very smooth function of the variable $H_T$ that measures the total energy deposition in the detector. By measuring $R_{\alpha_T}$ for events with low energy deposition, we can confidently extrapolate the value to the sample of events with large energy deposition, where the events from supersymmetry and dark matter production are expected to lie. If there is an excess of events in the region of large energy deposition, we might be able to claim that we have discovered
supersymmetry. Unfortunately, no such excess shows up in this analysis. The value found for $R_{\alpha_T}$ agrees with the simulation, but this really does not matter, as long as the data suffices to determine the value that represents the standard processes.

3 Razor

Rogan’s razor variable is based on a further observation on the kinematic differences between heavy particle production and standard processes. The proton is not a truly elementary particle. It is described, rather, as a bound state of quarks and gluons. To produce a pair of heavy particles in proton-proton collisions, we annihilate a particle in one proton with its antiparticle in the other proton (a quark with an antiquark, or a gluon with a gluon) and materialize the energy released as a pair of particles of a new type. Each quark, antiquark, or gluon in the proton carries a fraction of the proton’s total energy. If the new particles are heavy, the probability that the annihilating particles carry enough energy is small. This implies that the annihilations should occur with not much energy to spare. This, in turn, means that the pair of new particles should be produced almost at rest, with as little extra kinetic energy as possible. The razor variable takes advantage of this to separate new particle production from standard reactions, for which the energy requirements are much less stringent.

In a typical annihilation, the initial particle coming out of one proton will have higher energy than the particle from the other proton. So, the pair of new particles that is produced will have nonzero momentum along the beam direction. We can remove this momentum by making a boost along the beam direction. Ideally, the two heavy particles are produced just at rest in this frame. We can define some kinematic variables that make use of this idealized kinematics.

Look back at Fig. 1(a). If the heavy particles $Q$ and $\bar{Q}$ have equal mass (as particle and antiparticle would) and are at rest in some frame, and if both decay to a quark or antiquark and a dark matter particle $N$, and if the energies are so high that we can ignore the masses of the quarks, the quark and antiquark would have the same magnitude of momentum in this frame. So, if we have a 2-jet event, it is useful to define a frame obtained by boosting along the beam direction such that the energies of the two jets are equal in that frame. Call this the “R-frame”. Let the momenta of the two jets in the lab frame be $\vec{p}_1$ and $\vec{p}_2$. For approximately zero-mass particles, the energies of the jets are $E_1 = |\vec{p}_1|$, $E_2 = |\vec{p}_2|$ [4]. Let the beam direction be the $\hat{z}$ axis. Then it is a simple exercise in relativistic kinematics to show that the boost
from the lab frame to the R-frame is given by

$$v_R = \frac{E_1 - E_2}{p_1^z - p_2^z},$$

and that the jet momenta in this frame are given by

$$P_R = |\vec{p}_1|_R = |\vec{p}_2|_R = \sqrt{\frac{(E_1 p_1^z - E_2 p_2^z)^2}{(p_1^z - p_2^z)^2 - (E_1 - E_2)^2}}.$$

For the idealized situation I have described, in which each heavy particle at rest in the R-frame decays to a massless quark and a dark matter particle with mass $m_N$, the value of $P_R$ would be

$$P_R = \frac{m_Q^2 - m_N^2}{2m_Q}$$

More generally, the variable $P_R$ defined from the event momenta would vary from event to event with a distribution peaked near this value.

If a heavy particle decays at rest, the decay products will have an roughly equal probability to going out in any direction. For the standard QCD, W/Z, and top processes, however, it is easiest to create 2-jet events in which these jets have relatively low values of the transverse momentum. Events with high transverse momentum require hard scattering, which has a probability that decreases sharply with the momentum that is exchanged. It thus makes sense to define another variable that estimates the transverse momentum in the R-frame and compare its value to that of $P_R$.

In the idealized kinematics, in the R-frame, not only are the quark and anti-quark momenta $p_1$ and $p_2$ equal, but also the two dark matter particle momenta $k_1$ and $k_2$ are equal. In fact, if the frame in which the $Q$ and $\bar{Q}$ are produced has no transverse momentum with respect to the beam direction, as we have been assuming so far, then

$$\vec{p}_1 + \vec{k}_1 = 0 \quad \vec{p}_2 + \vec{k}_2 = 0$$

in the R-frame. Rogan relaxes the last assumption slightly. The total momentum imbalance in the event $\vec{P}$ should equal $(\vec{k}_1 + \vec{k}_2)$. This quantity, or, at least, is transverse component, is separately measureable. Rogan estimates the momenta $\vec{k}_1$ and $\vec{k}_2$ by assigning each to be equal to $\frac{1}{2} \vec{P}$.

The equation in the previous paragraph implies that, in the R-frame,

$$P_R = \frac{1}{2}(p_1 + k_1) = \frac{1}{2}(p_2 + k_2)$$

We could also compute $P_R$ from the mass of each two-vector system. If we would like to concentrate on transverse information only, we could compute the transverse
mass of each two-vector system

\[ M_T = \sqrt{(p_{T1} + k_{T1})^2 - (\vec{p}_{T1} + \vec{k}_{T1})^2} \]

and similarly for the \( \vec{Q} \). Here \( \vec{p}_{T1} \) is the 2-dimensional vector of transverse components of \( \vec{p}_1 \), as in our discussion of the scissors variable, and \( p_{T1} \) is the magnitude of this vector. This transverse mass variable can also be evaluated as

\[ M_T = \sqrt{2p_{T1}k_{T1} - 2\vec{p}_{T1} \cdot \vec{k}_{T1}}. \]

When the vectors \( \vec{p}_1 \) and \( \vec{k}_1 \) are oriented in the transverse direction, \( M_T = 2P_R \); more generally, this transverse mass will take values up to \( P_R \). Rogan combines the two values of \( M_T \) and estimates \( \vec{k}_1 \) and \( \vec{k}_2 \) as half of the measured momentum imbalance. This produces, finally, the formula

\[ P_{TR} = \frac{1}{2} \sqrt{\frac{1}{2} P_T(p_{T1} + p_{T2}) - \frac{1}{2} \vec{p}_T \cdot (\vec{p}_{T1} + \vec{p}_{T2})}. \]

Since transverse components of momentum are unchanged by boosts along the beam axis, this expression makes no reference to the R-frame and can be evaluated in the laboratory frame.

Rogan then defines the “razor” variable

\[ R = \frac{P_{TR}}{P_R}. \]

The variable \( R \) should be less than 1. In simulations of heavy particle production, it has a distribution with a peak near \( R = \frac{1}{2} \). Then heavy particle events are peaked both in the variable \( P_R \), near the value given above, and in \( R \), near the value \( \frac{1}{2} \).

The standard reactions have a completely different behavior in these two variables. First, the distributions for these reactions are highly suppressed if the value of \( R \) is not small. Rogan shows in his paper, using arguments similar to those given above for \( \alpha_T \), that if apparent missing momentum is generated by jet energy mismeasurement or by missing detection of additional jets, then these events will almost always have \( R < 0.4 \). Second, in the region where \( R \) is large, the events from standard processes peak in \( P_R \) at low values. For the W/Z and top processes, these peaks are near half the W, Z, or top quark masses. The distribution in \( P_R \) falls off exponentially above the peak.

These features of the distributions of events in \( R \) and \( P_R \) are illustrated with simulation data in Fig. 3. For the signal, the variables \( R \) and \( P_R \) are almost uncorrelated. Each provides information that discriminates new particle production from standard processes.
Figure 3: Simulation of the distribution of events in the variables $R$ and $M_R = 2P_R$ for standard and new physics processes, from [7]: top left: QCD; top right: W; bottom left: top; bottom right: an illustrative supersymmetry model.
Figure 4: Use of the razor variable in a search for dark matter production, from [7]: left: Distribution of events recorded by the CMS experiment in events with an isolated electron, unbalanced momentum, and jets, in the variable $M_R = 2P_R$, in the region $R > 0.45$. The data is compared to a physics simulation of the standard W/Z and top processes. right: The same distribution for events with an isolated muon, unbalanced momentum, and jets.

Thus, a possible strategy for supersymmetry discovery is the following: Restrict attention to events with large $R$. Measure the distribution in $P_R$ at values below 100 GeV. Using the exponential decay law to extrapolate, predict the number of standard events at high values of $P_R$. If there is an excess, this might indicate production of new heavy particles. With enough data, we should see the peaking both in $R$ and in $P_R$.

This strategy for controlling the rates of the standard processes seems to work very well in practice. Fig. 4 shows the distributions in $P_R$ measured by the CMS experiment for events with $R > 0.45$ with identified leptons that signal that these events come from $W$, $Z$, or top production. The variable used in the plots is $M_R = 2P_R$, where $P_R$ is measured as explained above. For $M_R > 250$ GeV or $P_R > 125$ GeV, the distributions indeed fall exponentially. This allows the number of standard events in the supersymmetry search region to be estimated directly from the data. The plots also show the expectations from physics simulations of the W/Z and top processes. The data is in good agreement with these simulations. This gives confidence that the method makes sense, but that agreement is not actually necessary in searching for supersymmetry.

As in our discussion of the scissors variable, no significant excess of events is found in the region where supersymmetry is to be expected. So far, CMS obtains only limits.
on the dark matter models, and no discovery yet.

4 Conclusion

The scissors and razor variables provide clever methods to confront the problem of the large rates for standard reactions at the LHC. Using these variables, we are able to cut these rates down to size and allow processes with new particles, which occur at much smaller rates, to become visible. These methods complement more direct approaches to this problem that have been carried out by both the ATLAS and CMS experiments. As of this moment, neither experiment has seen an excess of events over the standard expectation that could be interpreted as the discovery of dark matter. The current data excludes possible primary particles $Q$ with masses below about 500 GeV.

However, the LHC program is just beginning. The experiments are confident that, before the summer of this year, they will accumulate 20 times more data than was used for the analyses reviewed in this paper. By the end of 2012, we hope for 200 times more data than is present in these samples. Each increase in data by a factor of 10 gives, roughly, sensitivity to supersymmetry production and to other possible new particles of mass about a factor 2 higher. It is likely that the dark matter particle and its partners are out there, and perhaps we do not have long to wait to find them.

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References


[4] In this note, I will measure energy, momentum, and mass in energy units. The unit of 1 GeV is approximately the rest energy of the proton, $0.937 \text{ GeV} = m_p c^2$. I will convert masses to GeV units by multiplying by $c^2$. I will convert momenta to GeV units by multiplying by $c$.


[6] The complete set of CMS results on searches for supersymmetry can be found at https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUS