Scattering Amplitudes from LHC to LIGO and Beyond

Lance Dixon
“Quantum Universe” Colloquium
DESY and University of Hamburg
23 June 2020
“The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time.”
- S. Chandrasekhar (1979)

Event Horizon Telescope (2019)
Where most black holes have been “seen”

LIGO, Hanford, WA; also LIGO, Livingston, LA; VIRGO, near Pisa
Binary inspiral

![Binary inspiral image](image-url)
Some events (many more on tape)
LHC, Geneva: Probing matter at the shortest distances
A few events, out of trillions

CMS  $H \rightarrow \gamma\gamma$

$H \rightarrow \mu^+\mu^-\mu^+\mu^-$

$pp \rightarrow 2 \text{ jets}$

13 TeV  $m_{jj} = 8$ TeV

6 jets

$tttt$
At the heart of the LHC

- The most copious events at short distances (large momentum transfer $p_T$) are from quantum chromodynamics (QCD), also known as the strong interactions.
- Force carried by gluons ($g$), a nonlinear cousin of the photon ($\gamma$)

\[
\gamma \overset{\text{= 0}}{\longrightarrow} \quad g \overset{\text{= } g_s = \sqrt{\alpha_s}}{\longrightarrow}
\]
Typical LHC event

hadronic jets

nonlinearities small, perturbatively calculable

confining

Particle Data Group

F. Krauss
Typical LIGO Event

- Nonlinearities small, perturbatively calculable
- NS-NS mergers @ LIGO/VIRGO/...
or BH’s @ LISA
  → many, many cycles in perturbative regime
  → phase of orbit can be measured very precisely
- Numerical relativity
- Quasi-normal modes
Scattering amplitudes

• “The most perfect microscopic structures in the universe”
• Where the “rubber meets the road” between quantum field theory and experiment
• Amplitudes for quantum chromodynamics (QCD) dominate collisions at LHC
• Smooth functions of the external kinematic variables and essentially nothing else
Both shrouded in mystery

• Black holes often shrouded in gas and dust (which makes it possible to “see” them electromagnetically)

• Quarks and gluons scattering in QCD are shrouded by confinement: bound within the initial protons, and emerge as collimated jets of hadrons in the final state.
QCD Factorization & Parton Model

At short distances, quarks and gluons (partons) in proton are almost free. Sampled “one at a time”

Parton distribution functions (from experiment)

Short-distance cross section \( \hat{\sigma}(\alpha_s, \mu_F, \mu_R) \)

predictable using perturbative QCD

The “femto-universe”

size = factorization scale \( \mu_F \)

(“arbitrary”)

“typical” infrared safe final state

\( p \)

\( \bar{q} \)

\( q \)

\( g \)

\( Z \)

\( e^+ \)

\( e^- \)
Leading-order (LO) predictions qualitative: poor convergence of expansion in $\alpha_s(\mu)$.
Uncertainty bands from varying $\mu_R = \mu_F = \mu$.

Example: Higgs gluon fusion cross section vs. LHC CM energy $\sqrt{s}$.

LO $\rightarrow$ NNNLO $\rightarrow$ factor of 2.7 increase!
Short-distance cross sections built out of scattering amplitudes, S-matrix elements

\[ \sigma = f \]
Black hole scattering vs. inspiral

- Related by “analytic continuation around $r = \infty$”
- Accomplish with effective Hamiltonian, e.g. Cheung, Rothstein, Solon, 1808.02489
- Or more directly in terms of trajectories Kälin, Porto, 1910.03008, 1911.09130
Spinless black hole example

- **Scattering** depends on both relative velocity $v$ and strength of potential $G_N M_1 M_2 / r \equiv G / (r_{Schw})$ (deviation from Minkowski metric)
- In **bound state**, locked together by **virial theorem**:
  - Kinetic energy $\sim$ potential energy
    $$ v^2 \sim G $$
- Common parameter controls perturbative **post-Newtonian** approximation relevant for inspiral accuracy
- But in **scattering** one can compute separate orders in $v^2$ (or $p^2$) and $G$
  $$ H^{(0)}(r^2, p^2) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + \frac{G}{r} c_1^{(0)}(p^2) + \left(\frac{G}{r}\right)^2 c_2^{(0)}(p^2) + \mathcal{O}(G^3) $$
- Powers of $G$ alone referred to as **post-Minkowskian**
Double expansion of spinless conservative Hamiltonian

\[
\begin{array}{cccccc}
& 0\text{PN} & 1\text{PN} & 2\text{PN} & 3\text{PN} & 4\text{PN} & 5\text{PN} \\
1\text{PM} & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots) G & \boxed{1 + v^2} & \boxed{1 + v^2 + v^4} & \boxed{1 + v^2 + v^4 + v^6} & \boxed{1 + v^2 + v^4 + v^6 + v^8} & \boxed{1 + v^2 + v^4 + v^6 + v^8 + \cdots} G \\
2\text{PM} & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots) G^2 & \boxed{1 + v^2} & \boxed{1 + v^2 + v^4} & \boxed{1 + v^2 + v^4 + v^6} & \boxed{1 + v^2 + v^4 + v^6 + v^8} & \boxed{1 + v^2 + v^4 + v^6 + v^8 + \cdots} G^2 \\
3\text{PM} & (1 + v^2 + v^4 + v^6 + v^8 + \cdots) G^3 & \boxed{1 + v^2} & \boxed{1 + v^2 + v^4} & \boxed{1 + v^2 + v^4 + v^6} & \boxed{1 + v^2 + v^4 + v^6 + v^8} & \boxed{1 + v^2 + v^4 + v^6 + v^8 + \cdots} G^3 \\
4\text{PM} & (1 + v^2 + v^4 + v^6 + \cdots) G^4 & \boxed{1 + v^2} & \boxed{1 + v^2 + v^4} & \boxed{1 + v^2 + v^4 + v^6} & \boxed{1 + v^2 + v^4 + v^6 + v^8} & \boxed{1 + v^2 + v^4 + v^6 + v^8 + \cdots} G^4 \\
5\text{PM} & \boxed{1 + v^2 + v^4 + \cdots} G^5 & \boxed{1 + v^2} & \boxed{1 + v^2 + v^4} & \boxed{1 + v^2 + v^4 + v^6} & \boxed{1 + v^2 + v^4 + v^6 + v^8} & \boxed{1 + v^2 + v^4 + v^6 + v^8 + \cdots} G^5 \\
\text{almost} & \text{almost} & \text{almost} & \text{almost} & \text{almost} & \text{almost} \\
6\text{PM} & \boxed{1 + v^2 + \cdots} G^6 & \boxed{1 + v^2} & \boxed{1 + v^2 + v^4} & \boxed{1 + v^2 + v^4 + v^6} & \boxed{1 + v^2 + v^4 + v^6 + v^8} & \boxed{1 + v^2 + v^4 + v^6 + v^8 + \cdots} G^6 \\
\end{array}
\]

Many contributed to these advances, for PN notably T. Damour and collaborators.
QCD at LHC:
LO = Trees

LO cross section uses only Feynman diagrams with no closed loops – tree diagrams.
Here’s a very simple one:

$\hat{\sigma}(0)_{\text{LO}}$

Although there are many kinds of trees, some harder than others, “textbook” methods often suffice.
NLO needs 1 loop first quantum corrections
Challenging in QCD if many legs – depends on many variables

\[ \bar{q}q \to W + n \text{ gluons} \]

\[ = + 256,264 \text{ more} \]
“NLO QCD revolution”

Gavin Salam (2012)

2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al]
2011: NLO $WWjj$ [Rocket: Melia et al]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al]
2011: first automation [MadNLO: Hirschi et al]
2011: first automation [Helac NLO: Bevilacqua et al]
2011: first automation [GoSam: Cullen et al]
2011: $e^+e^- \rightarrow 7j$ [Becker et al, leading colour]

[unitarity]
[unitarity]
[unitarity]
[unitarity]
[unitarity + feyn.diags]
[unitarity]
[feyn.diags(+unitarity)]
[numerical loops]
NLO $pp \rightarrow Z + 1, 2, 3, 4$ jets vs. ATLAS 2011 data

ATLAS 1304.7098

BH+S: 1108.2229

L. Dixon  Amplitudes, LHC, LIGO & Beyond

DESY & U. Hamburg  23.6.2020
Revolution made possible by new perspective on particle scattering:
“On-shell” Methods, or
Granularity vs. Fluidity
QCD Tree Amplitudes

Many tree-level helicity amplitudes vanish or are very simple. Much simpler than individual Feynman diagrams!

\[
\begin{align*}
\text{right-handed} & : \ h = +1 \\
\text{left-handed} & : \ h = -1
\end{align*}
\]

\[
\begin{array}{c}
\text{Analyticity/unitarity makes it possible to recycle this} \\
\text{simplicity into loop amplitudes}
\end{array}
\]

\[
\begin{align*}
A_n + A_n & = 0 \\
A_n & = \frac{\langle i j \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \cdots \langle n 1 \rangle} \\
& \text{Parke-Taylor formula (1986)}
\end{align*}
\]
Recycling “Fluid” Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

Picture leads directly to BCFW (on-shell) recursion relations:
Reconstruct amplitude from poles in complex plane, where intermediate particle is on shell, and amplitude factorizes into product of 2 simpler amplitudes with fewer external legs

Britto, Cachazo, Feng, Witten, hep-th/0501052

Trees recycled into trees
All Gluon Tree Amplitudes Built From:

\[
\begin{align*}
3^+ &\rightarrow \quad 1^- \\
2^- &\quad = \quad \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle}
\end{align*}
\]

In contrast to Feynman vertices, it’s on-shell, completely physical

- On-shell recursion $\rightarrow$ very compact \textit{analytic} formulae, fast \textit{numerical} implementation.
- Can do same sort of thing at \textit{loop level}.
Branch cut information →
Generalized Unitarity (One-loop Fluidity)

**Ordinary unitarity:**
put 2 particles on shell, with real momenta

**Generalized unitarity:**
put 3 or 4 particles on shell, complex momenta

Trees recycled into loops!
What does all this have to do with black holes?

Orbiting black holes obey General Relativity:

\[ S_{EH} = \int d^4x \sqrt{g} \ R \]

Gluons scatter according to Yang-Mills theory

\[ S_{YM} = \int d^4x \ F_{\mu\nu}^a F^{\mu\nu,a} \]
Both theories contain massless particles

- Graviton is a traceless symmetric tensor → helicity ±2

- Gluon helicity ±1
  like photon
Gedanken calculation

- Graviton scattering amplitudes $\mathcal{M}_n$
- Could compute Feynman rules from
  \[
  S_{EH} = -\frac{2}{\kappa^2} \int d^4x \sqrt{g} \ R(g_{\mu\nu})
  \]
  by expanding metric around Minkowski space,
  \[
  g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \kappa = \sqrt{32\pi G_N}
  \]
- But action contains inverse metric, leading to incredibly complicated
  Feynman vertices of arbitrary multiplicity
- While Yang-Mills $\to$ 3- and 4-gluon vertices only
- Feynman rules not the way to go
“On-shell” methods

• Ignore Lagrangians and Feynman rules, construct on-shell scattering amplitudes using causality and analyticity
• Basic on-shell amplitude, exposing theory’s nonlinearity, is 3-point amplitude
• For all massless particles, complex momenta needed to realize it nonsingularly:
  \[ s_{ab} \equiv (k_a + k_b)^2 = k_c^2 = 0 \]

→ All 3 vectors parallel, if \( k_{a,b,c} \) all real
3-point amplitudes (cont.)

- Complex 3-point kinematics: no free parameters, like scattering angles
- 3-point amplitudes completely dictated by
  1. External helicities
  2. Overall energy scaling (dimension of coupling)
- For YM, $g_s$ dimensionless and helicity $\pm 1$, find:
  \[ A_3^{YM}(1^+, 2^+, 3^+) = A_3^{YM}(1^-, 2^-, 3^-) = 0 \]
  \[ A_3^{YM}(1^-, 2^-, 3^+) = g_s \frac{<12>^3}{<23><31>} f_{a_1 a_2 a_3} \]
- $<ab>$ are inner products of Weyl spinors, would be $\sqrt{S_{ab}}$ if momenta were real
3-point graviton amplitude

• For gravity, \( \kappa \propto \frac{1}{M_{\text{Planck}}} \) has dimension \(-1\) and graviton helicity \( \pm 2 \), together dictate:

\[
\mathcal{M}_3^{\text{grav}} (1^{++}, 2^{++}, 3^{++}) = \mathcal{M}_3^{\text{grav}} (1^{--}, 2^{--}, 3^{--}) = 0
\]

\[
\mathcal{M}_3^{\text{grav}} (1^{--}, 2^{--}, 3^{++}) = \frac{\kappa}{2} \left[ \frac{<12>^3}{<23><31>} \right]^2 \propto [\mathcal{A}_3^{\text{YM}}]^2
\]

Note: \(<ab> \propto \text{energy}. 1 \text{ more } <ab> \text{ compensates } \kappa\)

• In summary: \(1+1=2\), Lorentz symmetry, and dimension of 3-point couplings

\[
\Rightarrow \boxed{\text{Gravity} = \text{YM}^2}
\]
Beyond 3-point tree

- **BCFW recursion relations** work for gravity as well as gauge theory.
- Uniquely determine $n$-point tree amplitude $\mathcal{M}_n$, given $\mathcal{M}_3$. (Higher-point Feynman vertices all linked to 3-point vertex by gauge symmetry.)
- Suggests that in general $\mathcal{M}_n \sim A_n \otimes A'_n$
- Relations actually known much earlier from string theory (KLT)
Other theories lurking in background

Open superstring \[\xrightarrow{\text{KLT relations}}\] Closed superstring

\[\xrightarrow{\text{low energy limit}}\]

N=4 super Yang-Mills theory \[\xrightarrow{\text{KLT relations}}\] N=8 supergravity

\[\xrightarrow{\text{closed subsector}}\]

Yang-Mills theory \[\xrightarrow{}\] gravity
KLT relations

Kawai, Lewellen, Tye (1985)

1-dimensional string sweeps out a 2-dimensional world-sheet
open → with boundary (disk)  closed → no boundary (sphere)

\[ A_n^{\text{open}} \sim \int dx_a \, f(x_b, k_b) \]

\[ M_n^{\text{closed}} \sim \iint dz_a d\bar{z}_a \, |f(z_b, k_b)|^2 \]

deform integral contours, take low energy limit, ignore couplings and color factors

\[
M_4^{\text{tree}}(1, 2, 3, 4) = -i s_{12} A_4^{\text{tree}}(1, 2, 3, 4) A_4^{\text{tree}}(1, 2, 4, 3), \\
M_5^{\text{tree}}(1, 2, 3, 4, 5) = i s_{12} s_{34} A_5^{\text{tree}}(1, 2, 3, 4, 5) A_5^{\text{tree}}(2, 1, 4, 3, 5) + i s_{13} s_{24} A_5^{\text{tree}}(1, 3, 2, 4, 5) A_5^{\text{tree}}(3, 1, 4, 2, 5), \\
\ldots
\]
KLT relations (cont.)

• Machine for building any graviton tree amplitude from simpler gauge theory ones.

• Can also get massless external scalars, by dimensional reduction or supersymmetry, make them massive by giving them momentum in a 5th dimension (Kaluza-Klein):

  \[ k^\mu = (E, \vec{k}, k_5) \]

  \[ k^2 = 0 = E^2 - \vec{k}^2 - k_5^2 \quad \rightarrow \quad \text{mass} = k_5 \]

  • Massive scalar ~ spinless black hole!
Quantum supergravity

- **Unitarity** \( \rightarrow \) construct loop amplitudes from tree amplitudes
- **KLT relations** \( \rightarrow \) (super)gravity trees from (super)YM trees
- Three-loop example:

\[
\begin{array}{c}
\text{N=8 supergravity} \\
\end{array}
\]

\[
\begin{array}{c}
\text{N=4 SYM} \\
\end{array}
\]

\( \rightarrow \) Determine at what loop (if ever?) various supergravities have ultraviolet divergences

\( \rightarrow \) For example, find that N=8 SUGRA is finite through 5 loops!

Z. Bern et al., 9802162, 0702112, 0808.4112, 0905.2326, 1201.5366, 1708.06807, 1804.09311

- These calculations would be impossible using Feynman diagrams

L. Dixon Amplitudes, LHC, LIGO & Beyond

DESY & U. Hamburg 23.6.2020
Another approach – the double copy


- Can always write a gauge theory amplitude in terms of cubic graphs:

\[ A_m^{(L)} = i^L g^{m-2+2L} \sum_{i \in \Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \prod_{\alpha_i} \frac{n_i C_i}{p_{\alpha_i}^2} \]

\( n_i \) are not unique, can be shifted around. Remarkably, can (usually) be chosen to obey a set of 3-term identities.
Double copy (cont.)

- Color factors $C_i$ for these triplets of graphs, obey a Jacobi identity ($f \cdots f \cdots = f \cdots f \cdots + f \cdots f \cdots$)

\[ C_i = C_j + C_k \]

Also require
\[ n_i = n_j + n_k \]

for every such triplet.
Double copy (cont.)

- Then the gravity amplitude is the literal square of the YM amplitude:

\[
\mathcal{M}_m^{(L)} = \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i \in \Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \prod_{\alpha_i} \frac{(n_i)^2}{p_{\alpha_i}^2}
\]

- Machine for building graviton loop amplitudes from simpler gauge theory ones
Loops contain classical pieces

- Especially if particles move slowly, lots of time for multiple exchanges of virtual gravitons, to build up smooth classical trajectory.
But not all loop diagrams are classical

• E.g. black holes don’t annihilate in this regime

• graviton vacuum is pure quantum

• Black holes never get very far off shell
Classical restrictions compatible with on-shell methods

3PM computation
Bern, Cheung, Roiban, Shen, Solon, 1901.04424, 1908.01493

Figure 12: The independent generalized cuts needed at two loops for the classical potential. The remaining contributing cuts are given by simple relabeling of external legs. Here the straight lines represent on-shell scalars and the wiggly lines correspond to on-shell gravitons or gluons.
One of first contributions of “amplitudes” to LIGO physics

\[
\begin{align*}
\text{0PN} & : 1956 & & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots) G \\
\text{1PN} & : 1960 & & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots) G^2 \\
\text{2PN} & : 1972 - 1985 & & (1 + v^2 + v^4 + v^6 + v^8 + \cdots) G^3 \\
\text{3PN} & : 1987 & & (1 + v^2 + v^4 + v^6 + v^8 + \cdots) G^4 \\
\text{4PN} & : 2001 & & (1 + v^2 + v^4 + v^6 + \cdots) G^5 \\
\text{5PN} & : 2014 & & (1 + v^2 + \cdots) G^6 \\
\text{6PN} & : 2019 & & \text{almost} \\
\end{align*}
\]

Many contributed to these advances, for PN notably T. Damour and collaborators
Spin and tidal effects also computable within similar framework

Bern, Luna, Roiban, Shen, Zeng, 2005.03071
Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory

Cheung, Solon, 2006.06665

Tidal Effects in the Post-Minkowskian Expansion

operator(s) encoding multipole moments of neutron star

analogous to $H F_{\mu \nu} F^{\mu \nu} + \ldots$

operator(s) encoding couplings of gluons to Higgs boson at LHC
Effective one-body approach

Inspired by properties of bound states in QFT.

Interpolates information from various sources, including PN and PM expansions, test particle limit $m_1 \ll m_2$, and numerical relativity results.

Provides accurate gravitational wave templates very close in, faster than NR, allowing many combinations of initial masses and spins.
Adding more PN and PM orders improves EOB performance, but it also depends strongly on matching scheme, PS vs. $\bar{PS}$.
Conclusions

- At the heart of the LHC is the “square root” of the fundamental nonlinearity of General Relativity

\[ h^2 = \left[ \begin{array}{c} g \\ g \end{array} \right]^2 \]

- One can combine this information with on-shell methods, originally developed to improve QCD predictions for LHC, and study supergravities to high quantum loop order

- Similar methods just now beginning to bear fruit for the classical problem of binary inspiral, relevant for LIGO, VIRGO, and future gravitational wave detectors!

- Bright future ahead for higher precision waveforms, thanks to modern understanding of particle scattering in gauge theory and gravity.
Thanks for your attention!
Extra slides
NNLO QCD revolution still in progress

All $2 \rightarrow 1$ or $2 \rightarrow 2$ processes (still, in 2020), except for one $2 \rightarrow 3$ process, $\gamma \gamma \gamma$

Chawdry, Czakon, Mitov, Poncelet, 1911.00479

G. Salam, LHCP 2016

explosion of calculations in past 18 months

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New Physics Example: Supersymmetry

- Symmetry between fermions (matter) and bosons (forces)
- Very elegant, solves hierarchy problem
- Lightest supersymmetric particle (LSP) can be dark matter
- Cornucopia of new elementary particles at LHC.
Classic SUSY dark matter signature

Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

→ Missing transverse energy
MET + 4 jets
Not background free: happens in Standard Model too

MET + 4 jets from $pp \rightarrow Z + 4 \text{ jets}$,
$Z \rightarrow \nu \nu$

Neutrinos escape detector.
Irreducible background.
Plus there are many reducible backgrounds from $W + \text{ jets, } tt + \text{ jets, ...}$

Precision theory (typically NLO) can help with this, usually when embedded in parton shower Monte Carlos
From searches to measurements

• No convincing evidence for SUSY, or any other direct production of new particles.
• Also look for deviations in rates for Standard Model processes, especially involving the brand-new Higgs boson.
• Measurements are hard, take a while to perform.
• More precise theory typically needed.
Precision theory, from NLO to NNLO and even NNNLO required here!

**ATLAS** Preliminary
Run 1,2 $\sqrt{s} = 7,8,13$ TeV

even 3 electroweak vector bosons seen
Many Automated Programs for One-Loop QCD

**Blackhat:** Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…

+ Sherpa $\rightarrow$ NLO $W,Z + 3,4,5$ jets pure QCD 4 jets

**CutTools:**
- Ossola, Papadopolous, Pittau, 0711.3596
- Binoth+OPP, 0804.0350
- Ossola, Papadopolous, Pittau, 0711.3596
- Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621

**MadLoop:**
- Binoth+OPP, 0804.0350
- Bevilacqua et al, 1110.1499

**HELAC-NLO:**
- Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621

**Rocket:**
- Giele, Zanderighi, 0805.2152
- Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762

**SAMURAI $\rightarrow$ GoSAM:**
- Mastrolia, Ossola, Reiter, Tramontano, 1006.0710,…

**NGluon:**
- Badger, Biedermann, Uwer, 1011.2900,…

**OpenLoops:**
- Cascioli, Maierhofer, Pozzorini, 1111.5206,…
On to two loops

- State-of-art currently stuck at $2 \to 2$
  - with a couple of $2 \to 3$ exceptions
- Why? In part because 2 loop multiscale integrals are typically very hard
- All 1 loop integrals with external legs in $D=4$ are reducible to scalar box integrals + simpler

$\to$ combinations of

+ simpler

\[
\text{Li}_2(x) = - \int_0^x \frac{dt}{t} \ln(1 - t)
\]

First 2 loop $2 \rightarrow 3$ amplitudes

- All massless partons (or photons) in large $N_c$ (planar) limit for QCD gauge group $SU(3) \rightarrow SU(N_c)$:

$$gg \rightarrow ggg, \; qg \rightarrow qgg, \; q\bar{q} \rightarrow q\bar{q}g, \ldots$$

Gehrmann, Henn, Lo Presti, 1511.05409;
Badger, Brønnum-Hansen, Hartanto, Peraro, 1712.02229, 1811.11699;
Abreu, Dormans, Febres Cordero, Ita, Page, Zeng, Sotnikov, 1712.03946, 1812.04586, 1904.00945

- More work needed to compute NNLO cross section for $pp \rightarrow 3$ jets

- And $q\bar{q} \rightarrow \gamma\gamma\gamma$

  – already with NNLO cross section for $pp \rightarrow \gamma\gamma\gamma$!

Chawdhry, Czakon, Mitov, Poncelet, 1911.00479
The “QCD for LHC” revolution

- Many important hadron collider processes have been computed at NLO and NNLO in the past decade (even $2 \rightarrow 1$ at NNNLO), well beyond what was previously thought possible.
- Required a new understanding of scattering amplitudes, at a formal level, as well as efficient, stable implementation.
- Many people contributed to this progress.
- Parallel progress in understanding supersymmetric gauge & gravity theories.
- Revolution far from over; e.g. NNLO $2 \rightarrow 3+$ awaits!