ANALYSIS OF EMITTANCE GROWTH AND COMPLEX IMPEDANCE FOR COHERENT SYNCHROTRON RADIATION SHIELDED BY TWO PARALLEL PLATE

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Abstract

Self-interaction of electron bunch in circular motion is numerically studied. It has been shown that the self-interaction is expressed in terms of Liénard-Wiechert potential formula and can be calculated with three-dimensional particle tracking. The relation between shielded radiation force and complex impedance of beam pipe is also discussed.

1 INTRODUCTION

In resent years several studies have shown that emittance growth due to self field of electrons traveling through circular motion may become a severe problem in the design of X-ray free-electron lasers, where electron-beam of extremely low emittance generated by photo-cathode RF-gun is accelerated in a linear accelerator without dilution of emittance and compressed by magnetic bunch compressors to achieve high peak current. The self field in circular motion is called as coherent synchrotron radiation (CSR) force or noninertial space charge force.

Coherent synchrotron radiation has been studied since 1940s when high energy accelerators were innovated [1]. The early studies were conducted mainly to estimate the amount of energy loss caused by radiation, because the analysis of energy loss was relevant to the construction of a high energy accelerator. In the present days CSR is an important issue regarding to emittance growth in a bunch compressor for X-ray FELs.

There have been several studies on this subject: one-dimensional analysis of CSR force on short electron bunches [2], transient analysis including finite length of magnets [3], and CSR in the motion of small period such as undulator [4]. As for numerical studies, it has been shown that Liénard-Wiechert potential together with particle tracking can be applied to the calculation of CSR and noninertial space charge force [5]. Emittance growth by shielded CSR has also been discussed [6] [7] [8].

In the present study, self-interaction of microbunch in circular motion is numerically studied with a three-dimensional particle tracking code. The relation between emittance growth due to shielded CSR force and complex impedance of beam pipe is also introduced.

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2 ENERGY CHANGE AND EMITTANCE GROWTH IN A CIRCULAR PATH

When a relativistic electron bunch moves along a straight path, filed generated by each electron is propagating with speed of light and hugging the source electron, then it does not much affect on the motion of other electrons. Self-interaction of the bunch, therefore, can be neglected for a relativistic energy.

In a circular path, however, field generated by tail of bunch can overtake head of bunch as shown in figure 1, and this self-interaction may affects the electrons motion. Electric field applied on single electron can be obtained by integrating contribution from whole of the bunch which is expressed in Liénard-Wiechert potential formula. In one-dimensional model, electron bunch can be expressed with line charge along the circular path. For line charge of Gaussian distribution, averaged energy loss of an electron in the bunch due to the self-interaction is given by

$$\Delta E_{ave} = \frac{0.34\; qe\; L_d}{\rho^{2/3}\sigma^{4/3}} \quad , \tag{1}$$

where q and σ are charge and characteristic length of the bunch, L_d and ρ are path length and radius of the circular motion. Total energy loss of the bunch is consistent with the power of coherent synchrotron radiation emitted from the bunch, then the self-interaction is often called as coherent synchrotron radiation force.

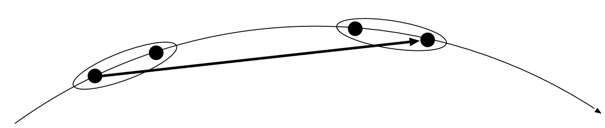

Figure 1: Schematic view of noninertial space charge field.

Estimation of emittance growth is more complicated than energy loss calculation. Although emittance growth can be discussed in terms of additional energy spread in the circular motion and expressed with a simple formula [9], it is limited to ideal situation. Therefore, numerical simulation is required for practical design analysis of bunch compressors.

The emittance growth due to self-interaction in circular motion can be numerically calculated with particle tracking method. Radiative and static field of relativistic electrons is obtained in the form of retarded potential, which is reduced to Liénard-Wiechert potential in case of small line charge of density λ traveling through circular trajectory of radius R and azimuthal electric field in the static frame is expressed as [5]:

$$E_\theta = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{r_{ret} - \vec{r}_{ret}\cdot\vec{u}_{ret}/c} \left(\frac{1}{\gamma^2} - \frac{x}{\rho}\beta^2 + \frac{r}{\rho}\beta^2\left[1-\cos(\zeta')\right]\right)\Bigg|_{\zeta_r}^{\zeta_f}, \quad (2)$$

where the first term is usual space charge force proportional to γ^{-2} which is often neglected for high energy electron beam, the second term is centrifugal force which does not appear in one-dimensional analysis, and the third term represents CSR force. The retarded position of source particle, ζ', is obtained from geometrical consideration. A three-dimensional particle tracking code **JPP** [10] has been modified to calculate the self-interaction in circular motion, where the numerical routine for the self-field is based on Eq.(2).

We consider an electron beam transport through two-dipole dog-leg as shown in figure 2. Parameters are chosen as: radius of circular path $R = 2.292m$, path length in each magnet $L_d = 20cm$, energy of electrons $E = 400MeV$. The electron bunch is assumed to be Gaussian shape in longitudinal direction , $\sigma_z = 0.4mm$, uniform in transverse direction, $r_b = 1mm$, and $q = 1nC$. Figure 3 shows calculated longitudinal phase plot after the dog-leg, where negative s means head of bunch. The results of one-dimensional theory is also plotted as a solid line. Emittance growth in the dog-leg is 0.44 mm-mrad, while the growth is 0.002 mm-mrad without the centrifugal force and CSR force.

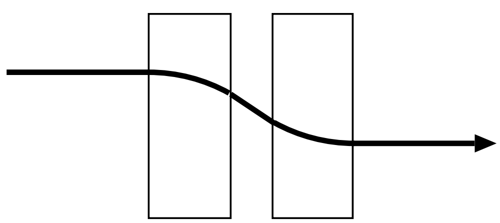

Figure 2: Two-dipole dog-leg consisting of two rectangular magnets.

3 EMITTANCE GROWTH BY SHIELDED RADIATION FORCE

In this section shielded CSR force is calculated by **JPP** code and compared with the results of impedance analysis.

The energy loss of electron bunch by CSR force in a metallic beam pipe, can be expressed as

$$\left(\frac{dW}{dt}\right)_{CSR} = -\left(qc\beta/\rho\right)^2 \sum_{n=-\infty}^{\infty} |f_n|^2\,\mathrm{Re}[Z_n] = -(qc\beta/2\pi\rho)^2\,\mathcal{R}, \quad (3)$$

where q is charge of the bunch, Z_n is complex impedance

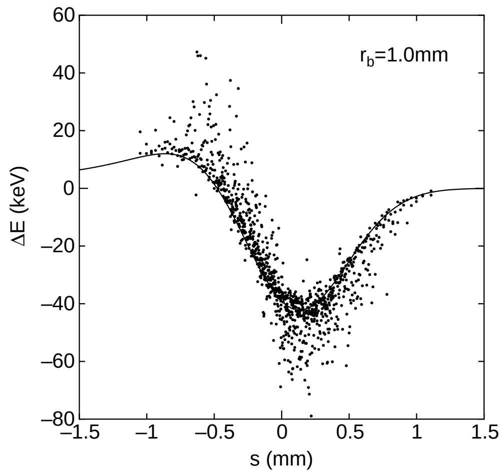

Figure 3: Longitudinal phase plot of electron bunch after the dog-leg.

of the beam pipe, $\mathcal{R}$ is radiation resistance and f_n is a longitudinal Fourier component of electron bunch defined as

$$f_n = \frac{1}{2\pi}\int_0^{2\pi} \exp(-in\theta)\lambda(\theta)d\theta, \quad (4)$$

where $\lambda(\theta)$ is longitudinal charge distribution of the bunch. For infinite two metallic plates placed parallel to the orbit plane, the impedance can be expressed as [1]

$$\frac{Z_n}{n} = \frac{2\pi^2 Z_0}{\beta}\frac{\rho}{h}\sum_{j=1,3,\ldots}\left[\beta^2 J_n'\left(J_n' + iY_n'\right) + \frac{(j\pi\rho/h)^2}{(n\beta)^2 - (j\pi\rho/h)^2} J_n\left(J_n + iY_n\right)\right], \quad (5)$$

where h is the distance between two plates, the argument of Bessel functions is $\sqrt{(n\beta)^2 - (j\pi\rho/h)^2}$ and we assume perfect conductivity of the plates.

The effect of conducting plates on the self field in a particle tracking code can be completely simulated by image charge method. Although infinite number of image charges are required to fulfill the boundary condition on the two parallel plates, we can truncate the number of image charges without degradation of accuracy in the simulation, regarding the field dependency on the distance between source and observer. The number of image charges in the following calculations is carefully determined not to introduce numerical error arising from this truncation. In the calculation of retarded position, a tentative solution with "pencil beam" approximation is refined through iterative algorithm, the approximation is, however, not appropriate for image particles far from central axis of the trajectory. A special approximation, therefore, is introduced to obtain a tentative retarded position for image particles [10].

We consider a two-dipole dog-leg having a beam line surrounded by infinite two parallel plates to study the shielding effect on coherent radiation and emittance

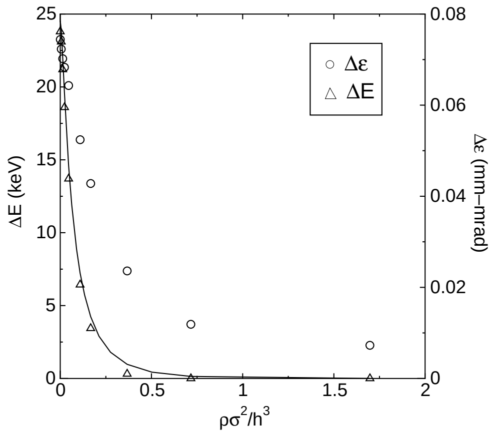

Figure 4: Averaged energy loss of an electron and emittance growth as a function of dimensionless shielding parameter: energy loss (△) and emittance growth (∘) calculated by **JPP** code, energy loss calculated from impedance analysis (solid line).

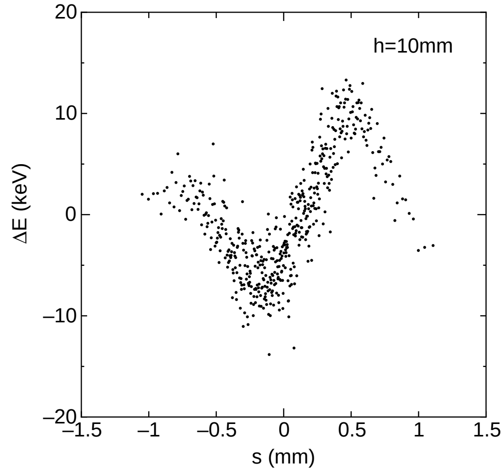

Figure 5: Longitudinal phase plot of electron bunch after the dog-leg.

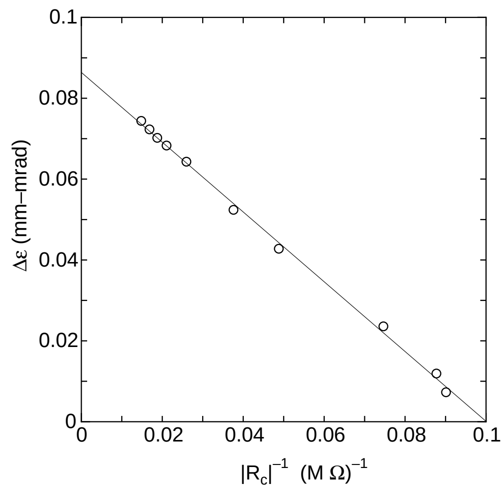

Figure 6: Obtained emittance growth by **JPP** code as a function of inverse of magnitude of complex radiation resistance.

growth. The energy loss of electrons and normalized emittance growth are calculated with various distance between two plates. Figure 4 shows the calculated energy loss and emittance growth as a function of dimensionless shielding parameter $\rho\sigma^2/h^3$, where the simulation parameters are chosen as: $E = 400MeV$, $r_b = 0.1mm$, $q = 1nC$, $\sigma_z = 0.4mm$, $\rho = 2.292m$, $L_d = 0.2m$.

It can be seen that obtained energy loss by **JPP** code agrees with the result of impedance analysis, where energy loss can be neglected for large shielding parameter as predicted before [11]. It seems that the reduction of emittance growth by shielding of metallic walls is not as drastic as the energy loss. While the energy loss due to coherent radiation for $h = 10mm$ ($\rho\sigma^2/h^3 = 0.37$) is less than 1keV, 1/30 of energy loss without shielding, the emittance growth is about 1/3 of open duct. This different property of shielding effect between energy loss and emittance growth can be explained by longitudinal phase plot after the dog-leg as shown in figure 5.

It shows that relatively large energy modulation still exists, while total energy loss of the electron bunch is almost zero. This energy modulation arisen in the dog-leg results in emittance growth. The estimation of emittance growth, therefore, requires amount of energy modulation of the electron bunch. It means that we should consider the imaginary part of impedance in the emittance analysis as well as the real part which is directly related with total CSR power. We introduce a concept of complex radiation resistance to include imaginary part of the impedance:

$$\mathcal{R}_c = (2\pi)^2 \sum_{n=-\infty}^{\infty} |f_n|^2 \left(\mathrm{Re}[Z_n] + i\ \mathrm{Im}[Z_n]\right). \quad (6)$$

Figure 6 shows obtained emittance growth by **JPP** code as a function of inverse of magnitude of complex radiation resistance calculated from Eq.(6). It seems that the emittance growth of an electron bunch due to shielded coherent synchrotron radiation can be expressed as a function of complex radiation resistance.

4 SUMMARY

In the present study we have applied three-dimensional particle tracking together with Liénard-Wiechert potential to the calculation of self-interaction of microbunch in circular motion, which should be considered in the design of bunch compressors for X-ray FELs. Averaged energy loss of an electron due to CSR force is calculated with the simulation code and found to be consistent with one-dimensional analytical results. In the case of shielded CSR force, the obtained energy loss agrees with the result of impedance analysis. It is also found that emittance growth of an electron bunch caused by shielded CSR force can be expressed as a function of complex radiation resistance of beam pipe.

5 REFERENCES

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