

# USING 3-D PERSPECTIVES AS A VISUALIZATION TOOL FOR PHASE SPACE DATA

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## *Abstract*

Two-dimensional projections of six-dimensional phase space data are routinely used in the analysis of accelerator beam dynamics phenomena. Plots of the distribution of particle coordinates and momenta in a given phase plane (e.g.  $x, x'$ ), or for the coordinates of beam cross sections (e.g.  $x, y$ ), are among the most commonly used projections. Computer-based visualizations of higher-dimensional projections of phase space data offer the possibility of providing improved insight into complex beam dynamics phenomena. This paper illustrates one of these types of visualizations that use interactive three-dimensional perspective displays.

## 1 INTRODUCTION

There are a number of new tools available that focus on improving the productivity of scientists and engineers involved in the study of particle beams and accelerator beamlines. Increasingly powerful multi-particle simulation codes are being developed that offer unprecedented capability in modeling the beams in accelerator systems. Ryne and collaborators [1] are running three dimensional simulations in parallel computing environments that use as many as 100 million macro-particles, a number which is approaching the actual number of particles that occur within the bunches of real beams. Efficient ways to interpret and assimilate the detailed information contained in the particle distributions produced by these simulations should prove useful.

A prototype interactive three-dimensional (3-D) perspective display tool has been developed to explore its utility. The prototype tool was designed to work with any optics code that generates 6-D phase space distributions, but the work described here uses distributions created with the Particle Beam Optics Laboratory (PBO Lab™). The PBO Lab provides an intelligent graphic user interface that works with several accelerator modeling and simulation programs [2,3].

## 2 EXAMPLE BEAMLINER PROBLEM

To illustrate how the use of 3-D visualization can aid in understanding and designing particle optics beamlines an example problem will be used. The beamline is a four-cell, second-order achromatic bend of the type that has been frequently described in the literature and often used as a building block for many accelerator designs [4,5]. The four cells are identical and are composed of a sector bend, two quadrupoles and two sextupoles, interspersed with drifts. Each cell of the achromat has transverse phase advances of  $90^\circ$  and is sometimes referred to as a quarter-

wave transformer. Figure 1 illustrates an iconic representation of the example beamline.

A scalable version of the example achromatic bend has been set up with the PBO Lab using formulas that incorporate all of the dependent relationships between the different elements. A description of the construction of the example has been published previously [6]. The system has twelve (12) independent parameters, all defined for the first cell. Eight (8) are parameters related to the size of the beamline: the first drift length, the effective lengths and apertures for the first quadrupole, sextupole and dipole magnets, and the bend angle of the first dipole magnet. The remaining four (4) parameters are magnetic field strengths: two quadrupole pole tip fields and two sextupole pole tip fields. Note that the dipole field strength is not an independent parameter in this example. It is computed by one of the expert system rules of the PBO Lab [6], from the specified bend angle and orbit length of the dipole.

For the results presented here we use the same parameters for the achromat that are described in reference [6]. That design is for a 100 MeV electron beam. Each sector dipole has a  $10^\circ$  bend angle, so that the total bend for the 4-cell achromat is  $40^\circ$ . The length of the first drift is 5 cm, and the lengths of the first sextupole and quadrupole are 10 cm. The central trajectory length through the first dipole is also 10 cm. The drift between the first sextupole and quadrupole is set to be equal to the first drift length. The drift between the first quadrupole and dipole is set with a formula that assures that the length of the first half of the first cell is equal to ten (10) times the first drift length. All other lengths in the first cell are specified by algebraic expressions so that the second half of the cell is a mirror image of the first half cell. The first cell thus has a length of 1 meter in this example. This length may easily be adjusted, while still preserving the cell symmetry, by changing the lengths of the first few elements. The second cell of the achromat is completely defined in terms of the parameters of the first cell by algebraic expressions. The third and fourth cells are copies of the second cell. The TRANSPORT Module of the PBO Lab is used in two steps, to initially fit the quadrupole strengths to achieve the desired first-order optical conditions, and then to determine the sextupole strengths to eliminate the second-order aberrations. The third-order TRANSPORT version [7] was used for these fits. The results of this fitting procedure are given in reference [6].

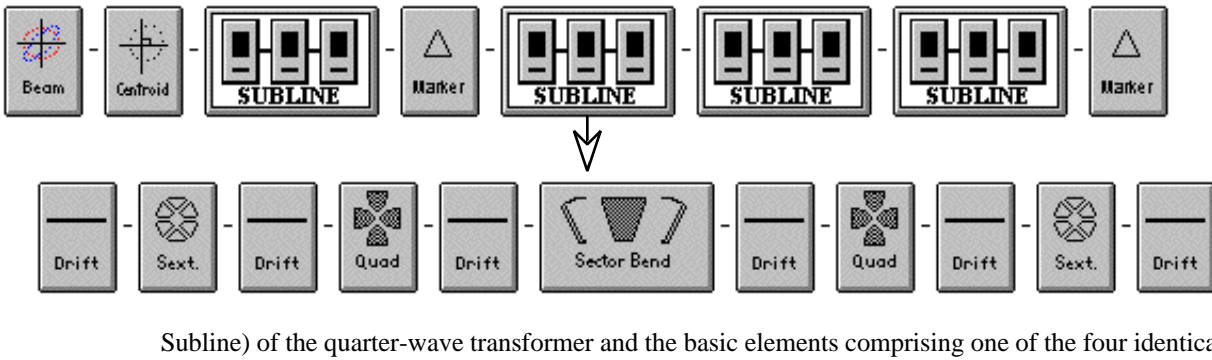


Figure 1. Icon image (from a PBO Lab Document Window) illustrating the four-cell layout (each cell represented by a

Subline) of the quarter-wave transformer and the basic elements comprising one of the four identical cells.

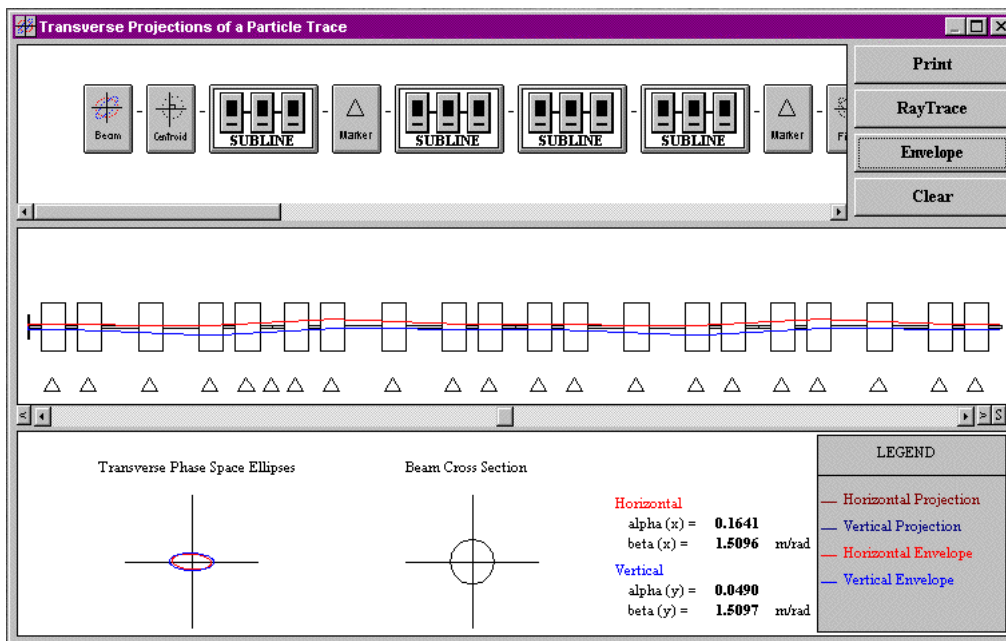


Figure 2. Four-cell achromat layout (top pane), side view scaled image of the achromat with horizontal (x) and vertical (y) envelopes overlain (center pane), and beam ellipses half way through the achromat (lower pane). The window illustrated is from the PBO Lab Trajectories Module.

Figure 2 shows a scaled layout of the example beamline, together with the first-order envelopes, and the transverse phase space ellipses and beam cross section at the mid-point of the achromat.

### 3 2-D VIEWS OF 6-D DATA

The TURTLE Module of the PBO Lab was used to generate two-dimensional (2-D) views of a beam propagating through the example beamline. TURTLE is a multi-particle ray tracing program [8] that models beams with a full six-dimensional phase space representation. For this work, an initial beam of 1000 macro-particles was used with semi-axes parameters as given in Table 1. The initial beam bunch is nearly spherical, with an aspect ratio (length/radius) of 1.25, but the correlation parameters are not symmetric. The transverse divergences are small and different values of the momentum spread,  $\sigma_p$ , were used.

Table 1. Semi-axes parameters for the initial beam.

Semi-Axis	Value	Units
x	4.000	mm
x'	0.101	mrاد
r12	0.14	-
y	4.000	mm
y'	0.101	mrاد
r34	0.16	-
z	5.000	mm
	3-27	%
r56	-0.09	-

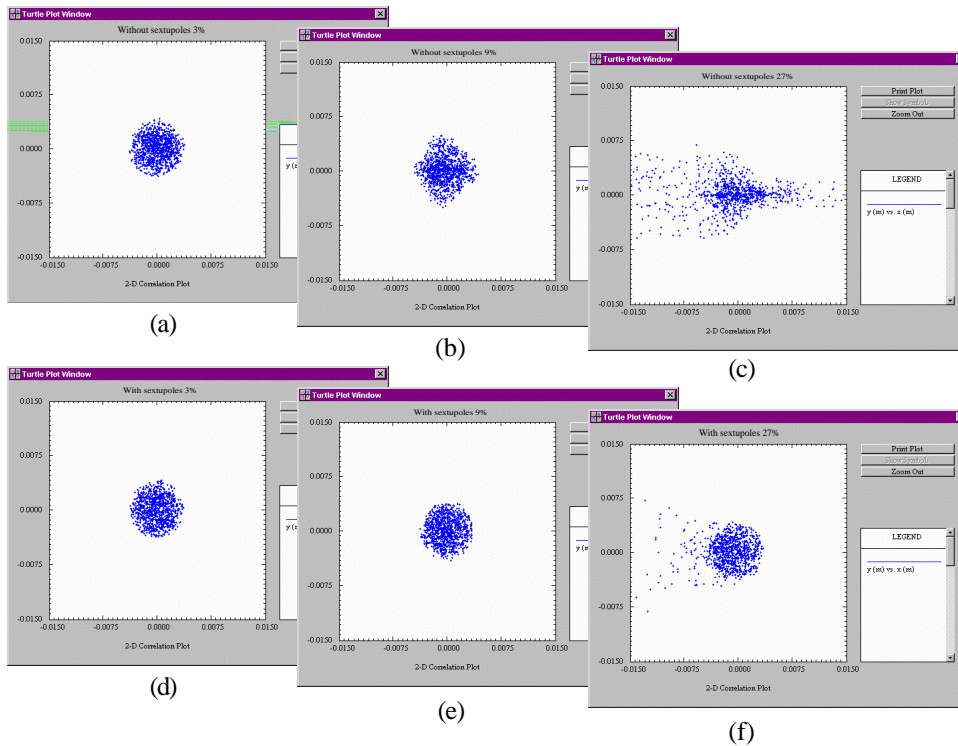


Figure 3. Two-dimensional scatter plots of the beam cross section at the end of the achromatic bend, without sextupole correctors [top (a)-(c)] and with sextupole correctors [bottom (d)-(f)]. In both cases, results are shown for three different values of the initial momentum spread : 3%, 9% and 27%, increasing from left to right. Window images are from the PBO Lab TURTLE Module.

Figure 3 illustrates the results of six different TURTLE simulations of the achromat performance. For the first-order design (without sextupole correction), the cylindrical symmetry of the initial beam is preserved by the achromat for a momentum spread of 3%, but aberrations are apparent at  $\approx 9\%$ , and for  $\approx 27\%$  the beam cross section projection is highly distorted. For the second-order achromat (with the sextupole correction), the cylindrical symmetry remains largely intact throughout the range of momentum spreads illustrated, although for  $\approx 27\%$  some scattering of particles in the bend plane is apparent. For a precision spot imaging system, the results suggest that the first-order design is adequate for  $\approx 3\%$ , that a second-order achromat will be required for  $\approx 9\%$ , and the second-order achromat may be adequate for  $\approx 27\%$  with aperture scraping, if the particle loss can be tolerated. What do these beams look like in 3-D?

#### 4 3-D PERSPECTIVE VIEWS

A 3-D perspective display tool has been developed on a Macintosh computer in order to visualize particle distributions in three dimensions. The tool uses data provided by a 6-column data file of phase space coordinate and momentum values. A simple dialog-type window is used to select the distribution file and the three axes for plotting. Figure 4 illustrates the window.

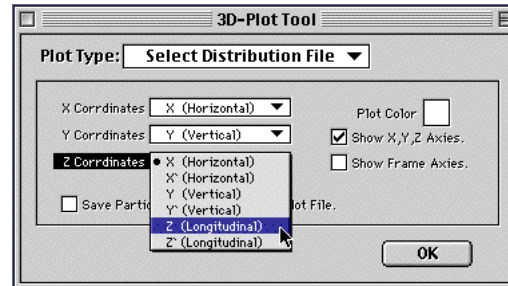


Figure 4. Selection of axes for 3-D visualization tool.

Other settings, such as the degree of the perspective and the size of the data points, can be adjusted interactively from menu selections after the data is displayed. Keyboard commands are used to zoom in and out, to perform controlled rotations about specific axes, and for other tasks. Mouse-based interactive features include the ability to “grab” the beam bunch and rotate it about any axis in order view the distribution from any direction. A variety of other interactive features are also available.

The capability to interactively change the viewing direction is one of the key features that can be used to gain insight into the physical phenomena occurring in the transport of the beam. Figure 5 illustrates a sequence of “snapshots” taken by rotating the viewpoint.

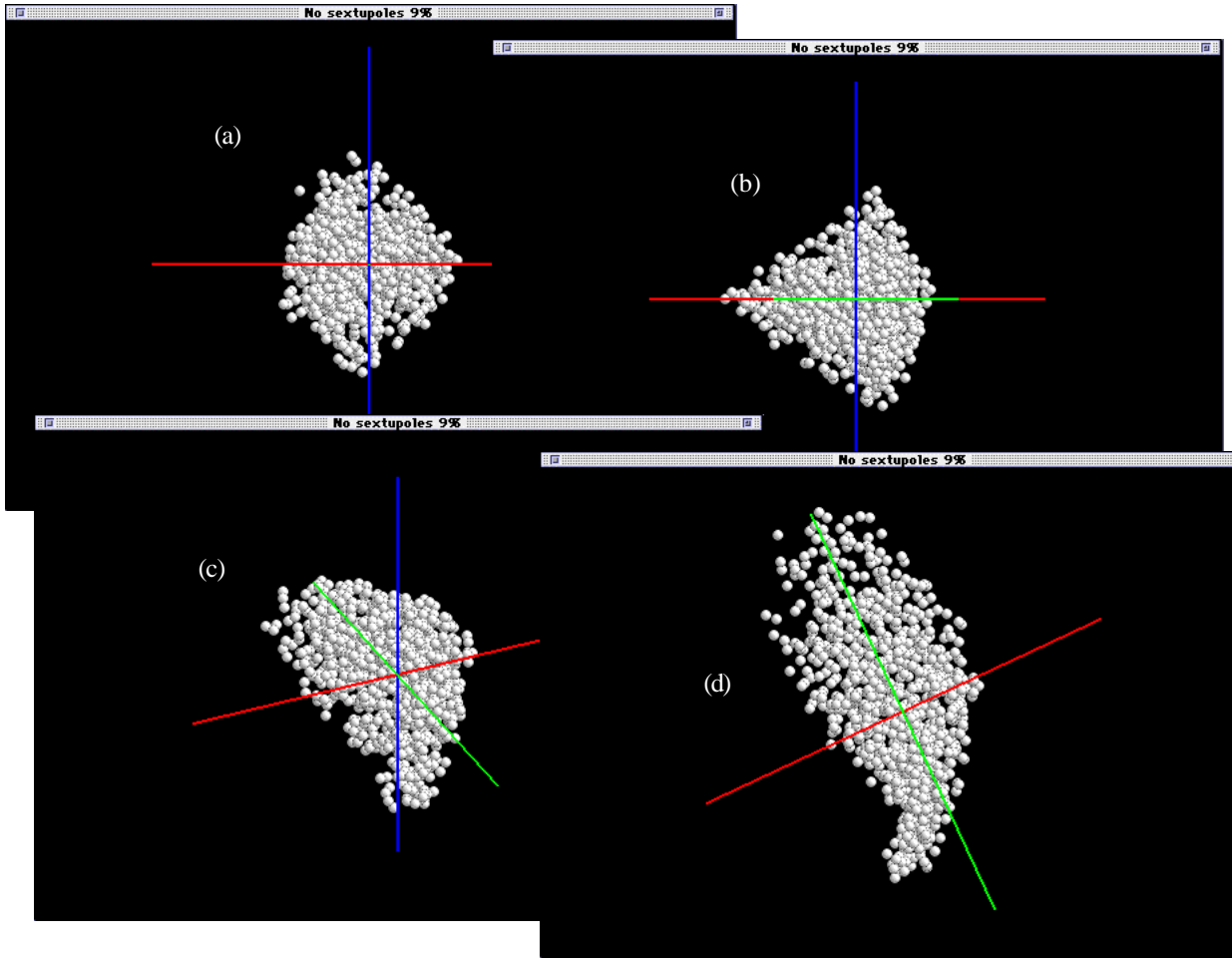


Figure 5. Snapshots of the rotation of the three-dimensional (x,y,z) perspective view of the particle distribution at the exit of the uncorrected achromat. The initial, nearly spherical, beam has a 9% momentum spread. The x-y view in (a), corresponding to Figure 4(b), has been rotated about the y (vertical) axis by approximately  $45^\circ$  in (b), rotated about the new horizontal axis by  $45^\circ$  in (c), and then by  $90^\circ$  in (d). Fanning in different directions at the (longitudinal) ends of the beam, as well as a wrapping of the beam around the y-axis, are apparent.

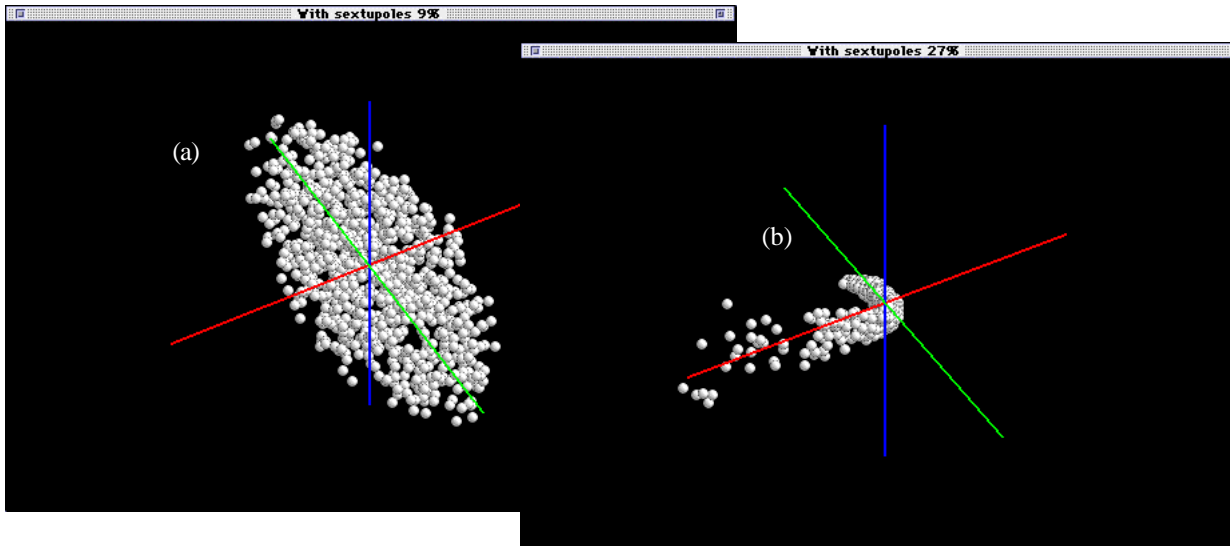


Figure 6. Snapshots of exit distributions of the corrected achromat, for initial beams with momentum spreads of (a) 9% and (b) 27%. The view orientations correspond to that of Figure 5(c), but are not to the same scale.

Figures 5 and 6 illustrate selected views of the three dimensional spatial distribution of the exit beam, for the uncorrected and corrected beamline, respectively. The uncorrected beam illustrated in Figure 5 fans out in different directions: the beam spreads in the horizontal ( $x$ - $z$ ) or bend plane at one (longitudinal) end, but in the vertical ( $y$ - $z$ ) plane at the other end. The beam also tends to wrap around the vertical axis. This same beam (9% momentum spread) is a well-formed 3-D ellipsoid when corrected, as shown in Figure 6(a).

The caption to Figure 5 states that the fanning and wrapping are “apparent” for the uncorrected achromat, but in fact, these effects are difficult to see from a set of snapshots. However, the effects are immediately recognizable by a user interactively rotating the beam bunch on the computer screen; the human eye and brain quickly process the image as a three dimensional object being rotated in space.

At larger momentum spread (27%), the fanning and wrapping become more distorted for the uncorrected beamline (not shown). As Figure 6(b) shows, the wrapping is still apparent for the corrected achromat, while the fanning is effectively confined to one end of the beam. Figure 6(b) also demonstrates that the scattered particles appearing in Figure 4(f) are confined to a narrow fan in the vertical plane at one end of the beam bunch.

## 5 SUMMARY

A prototype interactive tool for visualizing three-dimensional scatter plots has been developed. The tool can be used for displaying any three of the coordinate or momentum variables of a 6-D phase space distribution in a 3-D perspective view. The 3-D display can be interactively manipulated to study the distribution: the image can be rotated about any axis, the perspective can be changed, zoom in and out is supported, and the sizes of the particles and axes can be adjusted. The tool is easy to use and has proven useful in examining the nature of the non-linear structure of beams transported through a classical second-order achromat. The tool has also been used with PARMILA-generated distributions to study the free energy relaxation of non-equipartitioned beams [9] and should prove useful for exploring a variety of other physical phenomena associated with particle beams.

## ACKNOWLEDGMENT

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## REFERENCES

- [1] S Habib, J Qiang, R. Ryne, K. Ko, Z. Li, B. McCandless, W. Mi, C. Ng, M. Saporov, V. Srinivas, Y. Sun, and X. Zhan, “The Department of Energy Grand Challenge in Computational Accelerator Physics,” to be published in the proceedings of XIXth International Linear Accelerator Conference (Chicago, 1998).
- [2] G. H. Gillespie, B. W. Hill, N. A. Brown, H. Martono and D. C. Carey, “The Particle Beam Optics Interactive Computer Laboratory,” AIP Conference Proceedings 391, 264-269 (1996).
- [3] G. H. Gillespie, B. W. Hill, H. Martono, J. M. Moore, N. A. Brown, and M. C. Lampel, “The Particle Beam Optics Interactive Computer Laboratory for Personal Computers and Workstations,” Proceedings of the 1997 IEEE Particle Accelerator Conference, 2562-2564 (1998).
- [4] K. L. Brown, “A Second-Order Magnetic Optical Achromat,” IEEE Trans. Nuc. Sci. NS-26, 3490-3492 (1979).
- [5] K. L. Brown and R. V. Servranckx, “Applications of the Second-Order Achromat Concept to the Design of Particle Accelerators,” IEEE Trans. Nuc. Sci. NS-32, 2288-2290 (1985).
- [6] G. H. Gillespie, B. W. Hill, H. Martono, J. M. Moore, M. C. Lampel and N. A. Brown, “Using the Particle Beam Optics Laboratory (PBO Lab™) for Beamline Design and Analysis,” to be published in proceedings of the 15th International Conference on Cyclotrons and Their Applications (Caen, France, 15-19 June 1998), 4 pages (1998).
- [7] D. C. Carey, K. L. Brown and F. Rothacker, “Third-Order TRANSPORT- A Computer Program for Designing Charged Particle Beam Transport Systems,” Stanford Linear Accelerator Center Report No. SLAC-R-95-462, 295 pages (1995).
- [8] D. C. Carey, “TURTLE (Trace Unlimited Rays Through Lumped Elements) A Computer Program for Simulating Charged Particle Beam Transport Systems,” Fermi National Accelerator Laboratory Report No. NAL-64, 45 pages (1978).
- [9] N. A. Brown, G. H. Gillespie and B. W. Hill, “Free Energy Relaxation in a Drift-Tube Linac,” Proceedings of 1997 IEEE Particle Accelerator Conference, 1894-1896 (1998).
- [10] *PBO Lab™* is available from AccelSoft Inc., see [www.ghga.com/accelsoft](http://www.ghga.com/accelsoft).