ANALYTIC MODEL OF HALO FORMATION

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Abstract

An analytical model for halo formation has been constructed based on a beam in a focusing channel which is "breathing" due to a mismatch with the channel. For a 2-D KV beam, an integral of motion can be obtained assuming the dominance of the parametric resonance (breathing mode frequency is twice the frequency of individual particles within the beam). These results correspond to the "peanut diagram" in particle phase space in all respects. The model is then extended to other 2-D distributions, as well as 3-D distributions involving both longitudinal and transverse breathing modes. Numerical simulations are then used to determine the behavior of the longitudinal and transverse halos which occur, and their dependence on the initial phase space distribution, the bunch charge and shape, and the amplitude of the mismatch.

1 INTRODUCTION

The need for high current in a variety of new accelerator applications has focused a great deal of attention on understanding the phenomenon of halo formation in ion beams, which can cause excessive radiation of the accelerator. This understanding requires both an analytical model which explains available observations as well as computer simulations to verify both the assumptions of the model and its predictions. Fortunately, the availability of high speed computing, whose application to this problem is one of the major foci of this conference, has allowed rapid progress in the understanding of halo formation in high current linear accelerators.

2 2-D MODEL

Early attention was devoted to the analytic study of 2-D round beams in a continuous focusing channel. In particular, the KV distribution [1], a hyperspherical shell in the 4-D phase space with the self-consistent [2] distribution

$$f(H) = N\delta(H_0 - H), \tag{1}$$

where

$$H = \frac{mv^2}{2} + \frac{kr^2}{2} + e\Phi_{sc}(\boldsymbol{r}), \tag{2}$$

had the useful features of a uniform charge density within the beam, and uniform density in the x and y phase space projections. Here H_0 and N are constants, k is the constant external focusing gradient, and $e\Phi_{sc}(\boldsymbol{r})$ is the potential energy at $\boldsymbol{r}$ due to space charge.

Use of the equation for the beam envelope [3] permitted the analytic description of a "breathing" beam, in which the charge density oscillated between too tight and too loose a match to the external focusing force. These oscillations provided a periodic force to the ion motion, which was simple harmonic as long as the ions remained inside the beam. But for ions which traveled beyond the beam boundary, the oscillations were non-linear. In this case the ion's non-linear motion in the presence of a periodic force allowed it to be trapped in the parametric resonance, where the breathing frequency was twice the ion oscillation frequency. The analytic model thus predicted the formatoin of a "halo" [4] for certain combinations of mismatch and tune depression. The numerical simulations using the "particle-core" model confirmed the validity of the models, and pointed as well to the existence of chaotic motion as the tune depression became more severe [5]-[9].

Subsequent work focused on the possible mechanism for particles escaping from the beam into the region of non-linear oscillation [10]. In addition, numerical simulations were run for other, more physical, self-consistent stationary distributions of the form

$$f(\boldsymbol{r}, \boldsymbol{v}) = N(H_0 - H)^n \tag{3}$$

with $n = 0, 1$ [11]. These simulations exhibited the same halo structure and phase space patterns seen for the KV distribution, but with somewhat different quantitative dependence on mismatch and tune depression. The localization of the halo radius to approximately the same value predicted by the KV distribution gave the linac designers confidence that a beam pipe wall could be placed far enough from the beam to avoid intercepting the halo particles.

3 3-D MODEL

Attention then shifted to short 3-D beam bunches of ellispoidal shape with c/a = length/width ratio in the range 2-4 [12, 13]. We continued our effort to study the self-consistent phase space stationary distributions of the form

$$f(\boldsymbol{r}, \boldsymbol{v}) = N(H_0 - H)^n \tag{4}$$

but this time, for $n = -1/2$, the differential equation for the charge density was linear and could be solve analytically [13]. In addition, for $c/a > 2$, the "breathing" modes could be approximately separated into transverse and longitudinal modes, each of which was capable of generating a halo. Thus the picture was of a beam bunch which, when mismatched accordingly, generated either a transverse or a longitudinal halo, or both. The signature of the longitudinal halo was the same as that of the transverse halo (a "peanut diagram" in the phase space projection). The transverse and longitudinal mismatch and tune depression parameter space was extensively explored with numerical simulations, as you will hear in subsequent talks. But a new concern surfaced: Would the longitudinal halo permit the loss of ions from the rf bucket? Unfortunately, the bucket "walls" cannot be moved far away without increasing the length and cost of the linac. You will also hear more about these studies in a non-linear rf bucket in a subsequent talk.

Other issues involving halo formation were looked at, including equipartitioned distributions which were rms matched but not self-consistent [14]. As you will hear, these involved a rapid initial phase space redistribution, leading to a relatively small change in the parameters and extent of the halo formation due to the mismatch. In addition, they also point to the presence of a transverse-longitudinal coupling which allows either kind of halo to develop from either a transverse or longitudinal mismatch [14].

4 SUMMARY

Analytic models have been developed to study halo development in both 2-D beams and 3-D beam bunches in a linac. These models suggest that the most likely explanation for the halos which have been observed and which are likely to be seen in future high current linacs involves the parametric resonance between the collective modes which describe "breathing" and the motion of individual ions. When these models are used in conjunction with multiparticle simulations involving millions of particles, which are now practical with supercomputers and parallel processing, one can have great confidence in the predictions for halo formation and emittance growth which are so crucial for the designs of high current acceleration of short beam bunches.

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