CALCULATION OF IMPEDANCE FOR MULTIPLE WAVEGUIDE JUNCTION USING SCATTERING MATRIX FORMULATION

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Abstract

A method of computing the electromagnetic characteristics of a complex 3D cavity consisting of series of waveguides with arbitrary cross section is derived. The scattering matrix formulation is used for the simulation. For calculations of modes in waveguide with arbitrary cross sections a finite element code SLANS is applied. The computer code based on the method is used to calculate scattering parameters, dispersion characteristics of periodical structures, resonances, longitudinal and transverse impedances. The method was originally developed for simulations of long range wake fields in accelerator structures and for calculations of RF windows. Advantages of the technology make possible to simulate long open cavities such as accelerating structures and complex vacuum chambers. The method and some results are presented.

1 INTRODUCTION

Design of modern accelerators requires extensive simulation of an influence of accelerator components on the beam. This influence can be characterised by such parameters as resonant frequency, resonant shunt impedance, coupling impedance. A lot of well established methods and computer codes were developed to obtain the parameters of RF cavities, periodical structures, and obstacles in a beam pipe. In general the methods can be divided into three parts. This are grid and finite element based methods, then mode-matching based and equivalent circuit based methods. Up to now, calculation of complex and 3D structures by the first methods is memory-space and computer time consuming problem. Equivalent circuit method is a very effective one but limited mostly in use for structures with simple coupling such as single-mode waveguides or lower passbands of accelerating structure. Mode-matching method has several advantages that make it useful for the simulations. The method permits to simulate complex 3D structures an efficient way and make qualitative analysis of the results simpler then the other methods. It can be used also for non-relativistic particles with curved trajectories. Results of simulation by grid or finite element codes can be naturally incorporated into the method. The method allows to calculate a structure partially as well. An example of such calculation is a matching of accelerator structure couplers. During the matching one needs to calculate scattering matrix of the structure once and then recalculate only couplers.

The present paper discusses a mode-matching based method. The method uses a scattering matrix technology to determine coupling impedance, scattering parameters, resonant frequency, quality factor, and fields in a RF cavity, as well as periodical structure parameters. The code using the method was developed. The aim of the code was simulation of 3D structures such as RF windows and accelerating structures with 3D couplers. Some results were presented in [1]. The interactive code was written on object oriented language C++ and compiled for PC. The similar approach was developed by S.A.Heifets and S.A.Kheifets [2], and by Ursula Van Reinen[3] for calculation of rotationally symmetrical detuned accelerator structure.

2 SCATTERING MATRIX FORMULATION

Theory of mode-matching and multimode scattering matrixes is well known in electrical engineering and will be described briefly. Main steps of the calculation are: to build multimode scattering matrix for each waveguide junction, then to apply the scattering matrix technology to resolve characteristics of the whole structure, and then to post-process the resulted fields. The derivation begins by assuming an expansion of the transverse fields $E_\perp$ and $H_\perp$ in terms of the eigenmodes e_l in the waveguide as:

$$E_\perp = \sum_{l=1}^{M} (A_l + B_l) e_l \; , \qquad \textbf{(1)}$$

$$H_\perp = \sum_{l=1}^{M} Y_l (A_l - B_l) e_l \times \vec{z} \; . \qquad \textbf{(2)}$$

where ***A*** is a modal amplitude of the incident wave and ***B*** is the amplitude of the reflected wave (Fig.1), and ***Y*** is the characteristic wave admittance of the mode. Normalization of the eigenmodes was chosen so that the modes were orthogonal, that is,

$$\int e_l \cdot e_m ds = \delta_{lm} \; , \qquad \textbf{(3)}$$

where δ is Kronecker delta function. Longitudinal electric field is the sum over E (TM) modes :

$$E_z = \sum_{l=1}^{Me} i \frac{(k_l^e)^2}{\gamma_l^e} (A_l^e - B_l^e) ez_l \; , \qquad \textbf{(4)}$$

where k_l^e is cut-off wave value of the mode, γ_l^e is propagation value for the particular frequency, and ez_l is normalized as:

$$(k_l^e)^2 \int ez_l \cdot ez_l ds = 1. \qquad \textbf{(5)}$$

Derivation of the eingenmodes can be done analytically only for simple geometries such as circle or rectangle. So, for arbitrary cross section another method has to be applied. The method should be able to calculate 2D flat scalar function that represents z - component of the field in a waveguide and correspondent cut-off frequency. In this particular case a finite element code SLANS [4] was used. After derivation of eigenmodes, applying continuity of fields in common aperture area yields the relation between incident and reflected waves i.e. scattering S matrix. Let us assume that right waveguide has smaller cross-section than the left one (Fig.1).

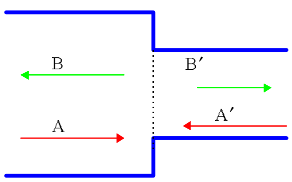

Figure 1: Modes in scattering matrix formulation

At first, we need to build two coupling matrices with the elements:

$$\eta_{ab}^{left} = \int e_a^{left} \cdot e_b^{right} ds, \qquad \textbf{(6)}$$

where a = 1..Mleft, b = 1..Mrigth; The values of Mleft and Mrigth are governed by the relative convergence phenomena and strongly depend on the geometry.

$$\eta_{ab}^{right} = \delta_{ab}, \qquad \textbf{(7)}$$

where a, b =1..Mrigth, integration over common cross-section, e_a^{left} is the a-th eigenmode of the left waveguide and e_b^{right} is b-th eigenmode of the right waveguide. The integration can be provided either analytically, for simple cross-section, or numerically, for arbitrary ones. Next step is derivation of admittance matrixes:

$$Y_{ml}^{left} = \sum_{a=1}^{Mleft} Y_a^{left} \eta_{am}^{left} \eta_{al}^{right}, \qquad \textbf{(8)}$$

$$Y_{ml}^{right} = \sum_{b=1}^{Mright} Y_b^{right} \delta_{ml}, \qquad \textbf{(9)}$$

where m, l = 1..Mrigt. Equation for the full admittance matrix is $Y = Y^{left} + Y^{right}$. Scattering matrix of the junction is:

$$S = \begin{pmatrix} (2Y^{-1}Y^{left})^T \eta^{left} - I & (2Y^{-1}Y^{left})^T \eta^{right} \\ (2Y^{-1}Y^{right})^T \eta^{left} & (2Y^{-1}Y^{right})^T \eta^{right} - I \end{pmatrix}. \qquad \textbf{(10)}$$

There are several advantages of application of scattering matrices instead of transfer or ABCD matrices[5]. The first one is that on every step of the calculation we have parameters that can be easily measured by a network analyser. Another one is that by the method we can avoid numerical instabilities that appear for frequencies far above cut-off. Next steps in the calculation depend on required results. There are two methods implemented in the discussed code. One is cascading and other sparse matrix LU technology. Cascading leads to effective calculation of dense matrices, and it is used for calculation of scattering parameters and periodical structures. For the impedance calculation a sparse matrix technology [5] was used.

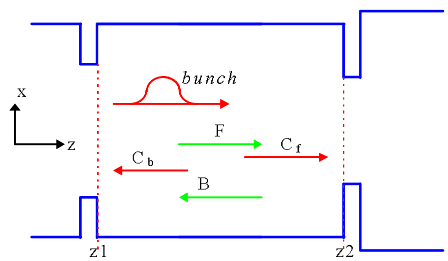

Figure 2: Excitation of a waveguide between two obstacles.

3 COUPLING IMPEDANCE

Intensity of beams in an accelerator is limited mostly by the coupling beam impedance. Therefore study and minimization of the impedance are importants part of accelerator design. The impedance can be obtained analytically for rather simple geometries only, so numerical methods are applied. Application of time domain methods for 3D or long structures is limited by computing time. The use of cavity codes for the calculation of the impedance is limited by high-Q structures and can be carefully applied for frequencies above beam-pipe cut-off. At the same time properties of the scattering matrix approach allow one to calculate frequency dependent impedance for 3D long structures, frequencies below and above cut-off, and to simulate non-relativistic particles. Computation time for the method does not depend on the length of the structure. Let us consider a wavegude between two obstacles as shown on Fig.2.

The beam excites waveguide modes. The well known formula for the amplitude of the excited mode is [6]:

$$C_{b,f} = \frac{1}{N_s}\int_{z1}^{z2} \vec{j}_\omega \vec{E}_{b,f}\, dv, \; N_s = 2\int \vec{E}\times\vec{H} ds, \quad \textbf{(11)}$$

here indexes *b, f* denote the modes propagating along forward *(+z)* and backward *(-z)* direction, $\vec{j}_\omega$ - beam current density, *dv* - integration over space from *z1* to *z2*, *ds* - integration over waveguide cross-section, *E* and *H* - eigenmode fields. Using (11) amplitude of modes in the waveguide for *z1* coordinate we have:

$$B = (I - S^{right}S^{left})^{-1}(S^{right}S^{left}C_b + S^{right}C_f^{z1}),$$
$$F = S^{left}(B + C_b), \quad \textbf{(12)}$$

where S^{right} , S^{left} are scattering matrices that resulted from cascading to the right and left from *z1*, C_b is a vector of modes induced towards *(-z)* direction, and $\left(C_f^{z1}\right)_m = \left(C_f\right)_m e^{i\gamma(z1-z2)}$ is a vector of amplitudes excited forward but converted to *z1* coordinate. Longitudinal impedance for frequency ω is given by:

$$Z_l(\omega) = \int_{-\infty}^{+\infty} Ez_s(\omega) e^{-i\cdot k\cdot z} dz, \quad \textbf{(13)}$$

where $k = 2\pi\omega/\beta c$, βc - beam velocity.
Longitudinal electric field is:

$$Ez_s(\omega) = Ez_{\text{mod}}(\omega) + Ez_{excit}(\omega) + E_{bunch}(\omega),$$

where $Ez_{mod}(\omega)$ follows from (4), $E_{bunch} = j_\omega / i\omega$ and similar to the (4):

$$Ez_{excit}(z) = \sum_{l=1}^{Me} i\frac{(k_l^e)^2}{\gamma_l^e}(C(z)_f - C(z)_b)ez_l \;. \quad \textbf{(14)}$$

The integration (13) is performed along a witness bunch trajectory. The impedance for structures with arbitrary cross-section has rather complex *(x,y)* dependence in the transverse plane[7]. Mapping of the impedance is used in the code. The *Ez* field is integrated along five parallel trajectories with coordinates: (x_w,y_w), (x_w+dx,y_w), (x_w,y_w+dy), (x_w-dx,y_w), (x_w,y_w-dy). The transverse impedance for ultra-relativistic particles is obtained by taking the transverse derivative of $Z_l(\omega)$. Direct integration of the transverse forces can be used for non-relativistic particles. Sum of $Z_l(\omega)$ over all waveguides, that form the whole structure, gives coupling impedance for the structure.

4 THE USE OF CASCADING FOR IMPEDANCE CALCULATION

New technology of cascading of impedances was developed. The cascading is not restricted to “waveguide to waveguide” junction and then can be applied for arbitrary 3D geometries. The technology is based on separate calculation of parts of a structure and then combining them. The result of the cascading includes the complete impedance as well as total scattering matrix. The parameters of a part of the structure are a frequency-dependent matrix related to mode-mode (scattering matrix), bunch-mode, mode-bunch, bunch-bunch interaction. This matrix can be obtained by any code, capable to calculate current-field and mode-mode dependencies. Finite-element or grid methods can be used for the purpose. The size of the matrix depends on quantity of waveguide modes and bunch trajectories and rarely exceeds *100×100*. Because of the small matrix size the process of cascading requires modest computing time. The cascading is particularly effective for simulation of structures consisting of several identical parts such as periodical accelerating structures or vacuum chambers.

Let us suppose (for simplicity) that the structure consists of two identical parts with scattering matrix:

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}. \quad \textbf{(15)}$$

Vectors of amplitudes of modes excited by the bunch and values of field integrals caused by particular mode are:

$$Vbm = \begin{bmatrix} If \\ Ib \end{bmatrix}, \; Vmb = \begin{bmatrix} Ff \\ Fb \end{bmatrix}.$$

Then, according to (12) amplitudes between the parts of the structure are:

$$B = (I - S_{11}S_{11})^{-1}(S_{11}S_{11}Ib\cdot e^{i\cdot k\cdot z} + S_{11}If),$$
$$F = S_{11}(B + Ib\cdot e^{i\cdot k\cdot z}). \quad \textbf{(16)}$$

Using (16) we can obtain equation for complete impedance *Zc*:

$$Zc = (B + Ib\cdot e^{i\cdot k\cdot z})\cdot Fb + (F + If)\cdot Ff\cdot e^{-i\cdot k\cdot z} + 2Zp,$$

where *z* is length of the part, $k = 2\pi\omega/\beta c$, *Zp* is impedance of the part.

5 EXAMPLES

5.1 Mode trapping above beam-pipe cut-off

The code was used for calculation of modes trapped in a beam pipe above its cut-off frequency. Narrow-band high - Q impedance is responsible for coupling between first bunch and successive bunches. The standard way to describe the impedance above beam-pipe cut-off is low-Q resonances, assuming the loading by radiation into the beam pipes. But simulation shows that low-Q resonances can trap fields with Q ≈ 10^4. Geometry with rotational symmetry was used for the simulation. Longitudinal impedance for monopole modes, resonant frequency and Q-value were calculated assuming perfectly conducting walls, so that Q-values are caused only by radiation into beam-pipes. Geometry consisting of two identical pillboxes was simulated (Fig.3). Dimensions of the pillboxes are *a = 1cm, b = 2cm, g = 1cm.* The pillboxes are connected by a long beam pipe with a = *1cm,* length L = *40cm,* and loaded by the same radius beam pipe with infinite length. At first, longitudinal impedance for the one separate pillbox was calculated for frequencies above beam-pipe cut-off. As we can see on the Fig.4 there are no high-Q resonances appeared on the impedance curve.

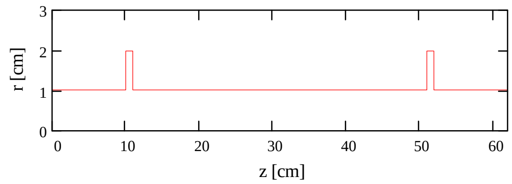

Figure: 3 Geometry of two pillboxes coupled through beam pipe.

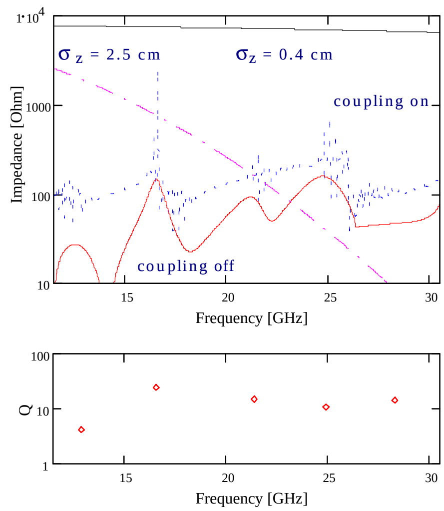

Figure 4: Upper picture: Comparison of longitudinal impedance for two pillboxes above beam-pipe cut-off frequency. Solid line - no coupling of the pillboxes through the beam-pipe. Dot line - coupled pillboxes. Dot-dash line - shape of bunch spectra [a.u.] for σ_z= *2.5 cm*. Upper curve - shape of bunch spectra [a.u.] for σ_z= *0.4 cm*. Lower picture: Q - value for resonances in the one separate pillbox loaded by the beam-pipes.

Then, frequencies and Q-values were calculated. The largest Q does not exceed 50 (lower picture on Fig.4). After that, the two coupled pillboxes were calculated. The results show sharp resonances located near low-Q resonances of the one pillbox. Additional curves on the Fig.4 present shape of beam spectra with r.m.s. z-length *0.4cm* and *2.5cm* to emphasise the effect of the impedance on short bunches. Blow-up of frequency range near the second resonance of the one pillbox is shown on Fig.5. As we can see on the lower picture of Fig.5 the loaded Q-value of the two-pillbox open cavity reaches the value of 10^4. And no specific behaviour appears near the second cut-off frequency of *26.36GHz*.

The narrow resonances can cause coupled bunch microwave instability. Hence, during the vacuum chamber design, one should pay attention to modes trapped by low-Q "chokes" above beam pipe cut-off frequency.

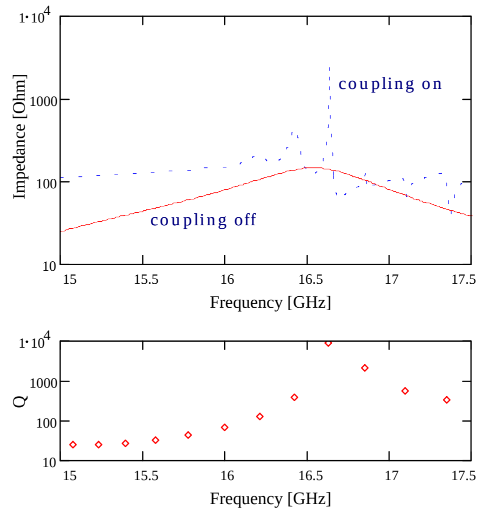

Figure 5: Upper picture: Comparison of longitudinal impedances for two pillboxes above beam-pipe cut-off frequency. Blow-up of curves from Fig.4. Solid line - no coupling of the pillboxes through the beam-pipe. Dot line - coupled pillboxes. Lower picture: Q - value for the resonator consisting of two coupled pillboxes loaded by the beam-pipes.

5.2 Transverse field caused by a shifted cell

Next example concerns accelerating structure design. An accelerating structure for modern projects of linear colliders is a disk-loaded waveguide[8][9]. Parameters of the structure is determined, between others, by requirements on the emittance preservation along the linac. Transverse forces inside a structure cause a dilution of the transverse emittance. The forces can be generated both by bunches (wake fields) and by accelerating field. Transverse force created by shifting of a cell of an accelerating structure was estimated. The shifted cell brakes cylindrical symmetry of the structure so that the high gradient accelerating field generates transverse field. Dimensions of the waveguide are: disk radius a = *4.5mm*, disk width t = *2mm*, cell radius b = *10.778mm*, period D = *8.7474mm*. That dimensions represent middle cell for X-band (*11.424 GHz*) detuned accelerator structure for JLC linear collider [8]. First step was matching of a disk-loaded waveguide to simulate a travelling wave. 7-cell stack was excited by monopole mode. Radius b of the end-cells and radiuses of the outer discs were changed to obtain reflection coefficient less than *0.02.* Matched dimensions was b = *10.986mm, a*= *5.99mm.* Then the middle cell was shifted as shown on Fig.6. Electric field was integrated along 5 trajectories to obtain map of the resulted fields. Fitting value V_{sh} from linear fitting of the results is equal to *8.3 V/mm/sqrt(W)*. Then we can estimate transverse voltage $V_{\perp}$ for full accelerator structure using following parameters: number of cells *N=150*, input power *P=50 MW*, average shift of cells *dy=1μm*. Using the equation: $V_{\perp} = dyV_{sh}\sqrt{N}\sqrt{P}$, we can obtain $V_{\perp} = 720V$.

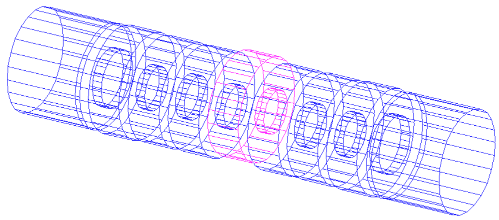

Figure 6: Geometry of the matched disk-loaded waveguide with shifted middle cell (*dy=1mm*).

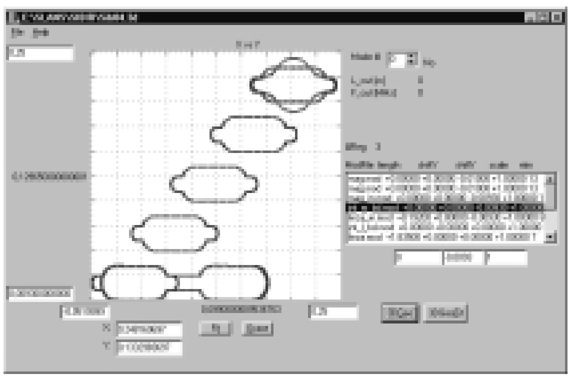

Figure 7: Dump of the preprocessor window. Outer part of the geometry is shown. The geometry was used for calculation of the longitudinal impedance of the vacuum chamber.

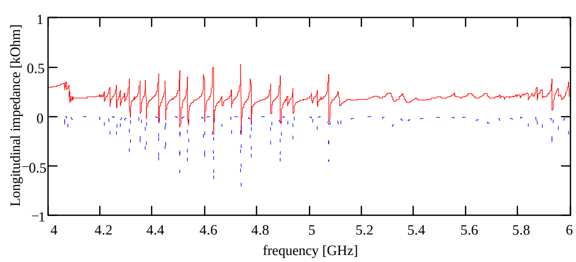

Figure 8: Longitudinal impedance of the vacuum chamber. Solid line - *Im(Z),* dashed line - *Re(Z).*

5.3 Impedance of a vacuum chamber

Synchrotron radiation source "Siberia-2" was designed and produced by Budker INP. The longitudinal coupling impedance was calculated for vacuum chamber located inside a quadrupole lens. The vacuum chamber has length of *183.5cm* and cross-section about *5.2cm×10.5cm*. The cross-section is fitted to the poles of the lens. Both sides of the chamber are connected to dipole magnet's chamber by insertion that is used for adjusting of the cross-sections. The geometry of the junction is shown on Fig.7. Resulted impedance we can see on Fig.8. The chamber has a reach spectrum above *4GHz* of the modes trapped inside, in spite of that the lowest cut-off mode has frequency about *2 GHz.*

6 CONCLUSION

The interactive code was developed for simulation of 3D accelerator components. The code uses scattering matrix method. Performance of the code allows to calculate coupling impedance for complicate structures with arbitrary cross sections. Using the code a technology of cascading of impedances was tested. The cascading can be used in combination with a finite element or grid methods. The cascading is a universal method and it can serve as powerful tool for simulations of complex 3D structures. The code was applied for open cavity consisting of two pillboxes loaded by beam pipe. The calculations show that fields can be trapped by low Q resonances above the beam-pipe cut-off frequency. Transverse voltage caused by shift of a cell in a disk-loaded waveguide was obtained. Calculation of a part of the "Siberia-2" vacuum chamber shows sharp resonances far above the first cut-off frequency.

7 REFERENCES

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