Update of Fit to Moments of Inclusive $B \to X_c \ell \bar{\nu}$ and $B \to X_s \gamma$ Decay Distributions for LP07

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We present an update of our fit to measured moments of inclusive distributions in $B \to X_c \ell \bar{\nu}$ and $B \to X_s \gamma$ decays to extract values for the CKM matrix element $|V_{cb}|$, the b- and c- quark masses, and higher order parameters that appear in the Heavy Quark Expansion. The fit is carried out using theoretical calculations in the kinetic scheme and includes moment measurements of the *BABAR*, Belle, CDF, CLEO and DELPHI collaborations for which correlation matrices have been published. We find $|V_{cb}| = (41.91 \pm 0.19_{\rm exp} \pm 0.28_{\rm HQE} \pm 0.59_{\Gamma_{\rm SL}}) \times 10^{-3}$ and $m_b = 4.613 \pm 0.022_{\rm exp} \pm 0.027_{\rm HQE}$ GeV where the errors are experimental and theoretical respectively. We also translate the heavy quark distribution function parameters m_b and μ_π^2 to the shape function scheme so that they can be used as input for the determination of $|V_{ub}|$.

I. INTRODUCTION

We present an update of our fit to measured moments of inclusive distributions in $B \to X_c \ell \bar{\nu}$ and $B \to X_s \gamma$ decays in the kinetic scheme [1]. The only difference with respect to [1] is the inclusion of the published hadron mass and lepton energy moment measurements by the Belle collaboration [2, 3]. When refering to the results presented in this note please cite Ref. [1].

II. EXPERIMENTAL INPUT

All results are based on the following set of moment measurements which are also summarised in Table I. Additional measurements for which correlation matrices are not available and thus cannot be used in the presented fit are listed in parentheses.

- *BABAR*
  Hadron mass [4] and lepton energy moments [5] from $B \to X_c \ell \bar{\nu}$ decays measured as a function of the minimum lepton energy $E_{\rm cut}$. The lepton moments used here differ slightly from those in the *BABAR* publication [5]. They have been updated by taking into account the recent improved measurements of the D_s and B branching fractions (*upper-vertex charm*) that impact the background subtraction. Moments of the photon energy spectrum in $B \to X_s \gamma$ decays as a function of the minimum photon energy $E_{\rm cut}$ from two independent analyses [6, 7].
- Belle
  Measurements of hadron mass and lepton energy moments as functions of the lower lepton energy [2, 3]. First and second moment of the photon energy spectrum as a function of the minimum photon energy $E_{\rm cut}$ [8, 9].
- CDF
  Hadron moment measurements with a minimum lepton energy of $E_{\rm cut} = 0.7$ GeV [10].
- CLEO
  Hadron moment measurements as a function of the minimum lepton energy [11]. First (and second) moment of the photon energy spectrum at $E_{\rm cut} = 2.0$ GeV [12]. (The measurement of lepton energy moments as a function of $E_{\rm cut}$ [13] is not given with the full covariance matrix and thus has not been included in the fit [14].)
- DELPHI
  Lepton energy and hadron mass moment measurements with no restriction on the lepton energy [15].

III. FIT TO MOMENTS

The fit to the moment measurements is carried out using HQE calculations in the kinetic scheme presented in Refs. [17, 18], including $E_{\rm cut}$ dependent perturbative corrections to the hadron moments [19–21].

As μ_G^2 and ρ_{LS}^3 are estimated from the $B^* - B$ mass splitting and heavy-quark sum rules, respectively, we impose Gaussian error constraints of $\mu_G^2 = 0.35 \pm 0.07\,{\rm GeV}^2$ and $\rho_{LS}^3 = -0.15 \pm 0.10\,{\rm GeV}^3$ on these parameters as advocated in Ref. [17].

The results of a combined fit of the HQEs to all moment measurements listed in Table I is shown in Table II.

In order to asses the consistency of the moment measurements from the two different decay processes, $B \to X_c \ell \bar{\nu}$ and $B \to X_s \gamma$, we also carry out separate fits to $B \to X_c \ell \bar{\nu}$ moments and to photon moments only. However, as the latter are not sensitive to all the heavy quark parameters, all but m_b and μ_π^2 are fixed to the result obtained from the combined fit.

The fit to $B \to X_c \ell \bar{\nu}$ moments only results in:

$$\begin{aligned}
|V_{cb}| &= 41.68 \pm 0.39_{\rm fit} \pm 0.58_{\Gamma_{SL}} \times 10^{-3}\,, \\
m_b &= 4.677 \pm 0.053_{\rm fit}\,{\rm GeV}\,, \\
m_c &= 1.285 \pm 0.078_{\rm fit}\,{\rm GeV}\,, \\
\mu_\pi^2 &= 0.387 \pm 0.039_{\rm fit}\,{\rm GeV}^2\,.
\end{aligned}$$

TABLE I: Summary of moment measurements used in the combined fit. n indicates the order of the (central) moment measurement of observable $\langle M_X^n \rangle_{E_{\rm cut}}$, $\langle E_\ell^n \rangle_{E_{\rm cut}}$ and $\langle E_\gamma^n \rangle_{E_{\rm cut}}$. $E_{\rm cut}$ indicates measurements with the corresponding minimum lepton momenta and photon energies in GeV.

Experiment	Hadron Moments $\langle M_X^n \rangle_{E_{\rm cut}}$	Lepton Moments $\langle E_\ell^n \rangle_{E_{\rm cut}}$	Photon Moments $\langle E_\gamma^n \rangle_{E_{\rm cut}}$
BABAR [4, 5] [6, 7]	n=2 $E_{\rm cut}$=0.9,1.0,1.1,1.2,1.3,1.4,1.5 n=4 $E_{\rm cut}$=0.9,1.0,1.1,1.2,1.3,1.4,1.5	n=0 $E_{\rm cut}$=0.6,1.2,1.5 n=1 $E_{\rm cut}$=0.6,0.8,1.0,1.2,1.5 n=2 $E_{\rm cut}$=0.6,1.0,1.5 n=3 $E_{\rm cut}$=0.8,1.2	n=1 $E_{\rm cut}$=1.9,2.0 [a] n=2 $E_{\rm cut}$=1.9 [a]
Belle [2, 3] [8, 9]	n=2 $E_{\rm cut}$=0.7,0.9,1.1,1.3 n=4 $E_{\rm cut}$=0.7,0.9,1.1,1.3	n=0 $E_{\rm cut}$=0.6,1.0,1.4 n=1 $E_{\rm cut}$=0.6,1.0,1.4 n=2 $E_{\rm cut}$=0.6,0.8 n=3 $E_{\rm cut}$=0.8,1.2	n=1 $E_{\rm cut}$=1.8,1.9 n=2 $E_{\rm cut}$=1.8,2.0
CDF [10]	n=2 $E_{\rm cut}$=0.7 n=4 $E_{\rm cut}$=0.7		
CLEO [11, 12]	n=2 $E_{\rm cut}$=1.0,1.5 n=4 $E_{\rm cut}$=1.0,1.5		n=1 $E_{\rm cut}$=2.0
DELPHI [15]	n=2 $E_{\rm cut}$=0.0 n=4 $E_{\rm cut}$=0.0 n=6 $E_{\rm cut}$=0.0	n=1 $E_{\rm cut}$=0.0 n=2 $E_{\rm cut}$=0.0 n=3 $E_{\rm cut}$=0.0	
HFAG [16]		n=0 $E_{\rm cut}$=0.6	

[a] A total of six photon moments from Refs. [6] and [7] are used.

From the fit to $B \to X_s\gamma$ moments only we obtain:

$$m_b = 4.56^{+0.08}_{-0.14}\,\mathrm{GeV}\,,$$
$$\mu_\pi^2 = 0.44^{+0.30}_{-0.17}\,\mathrm{GeV}^2\,.$$

A comparison of results from the combined fit with those obtained from fits to $B \to X_c\ell\bar{\nu}$ and $B \to X_s\gamma$ moments only can be found in Figure 1 where the $\Delta\chi^2 = 1$ contours for the fit results are shown in the $(m_b, |V_{cb}|)$ and (m_b, μ_π^2) planes. It can be seen that the inclusion of the photon energy moments adds additional sensitivity to the b-quark mass m_b.

In addition to the above we extract the difference in the quark masses as

$$m_b - m_c = 3.427 \pm 0.021\,\mathrm{GeV}\,.$$

Comparing the extracted values of the quark mass m_b with other determinations is often convenient in the commonly used $\overline{\rm MS}$ scheme. The translation between the kinetic and $\overline{\rm MS}$ masses to two loop accuracy and including the BLM part of the α_s^3 corrections was given in Ref. [22]. This leads to

$$\overline{m_b}(\overline{m_b}) = 4.22 \pm 0.04 \text{ GeV}$$

IV. TRANSLATION OF FIT RESULTS TO SHAPE FUNCTION SCHEME

We translate the results for m_b and μ_π^2 in the kinetic scheme to heavy quark distribution function parameters in other schemes so that they can be used for the extraction of $|V_{ub}|$. The translation is done by predicting the first and second moment of the photon energy spectrum above $E_{\rm cut} = 1.6\,\mathrm{GeV}$ based on the heavy quark parameters from Table II and using the calculations of Ref. [18].

The experimental and theoretical uncertainties in the fitted parameters as well as their correlations are propagated into the errors on the moments. The minimum photon energy of 1.6 GeV is chosen such as to be insensitive to the distribution function itself. At this threshold the local OPE calculation is applicable as the hardness $\mathcal{Q} = m_B - 2E_{\rm cut}$ of the process is sufficiently high such that cut-induced perturbative and non-perturbative corrections or biases are negligible. The predicted moments are given in Table III.

As the moments are physical observables which are scheme independent they can be used to extract the corresponding heavy quark distribution function parameters in other schemes. For this translation, grids for the first and second moments of the photon energy spectrum are generated as a function of the two parameters ($m_{b\ SF}$,$\mu^2_{\pi\ SF}$) for the Shape Function [23] scheme. A χ^2 is calculated for every set of parameters $\mu = (\langle E_\gamma \rangle(m_b, \mu_\pi^2), \langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle(m_b, \mu_\pi^2))$ as

$$\chi^2 = \sum_{i,j=1,2} (y_i - \mu_i)\, V_{ij}^{-1}\, (y_j - \mu_j) \text{ with } V_{ij} = \sigma_i\sigma_j\rho_{ij} \quad (1)$$

where the y_i are the predicted moments with their errors σ_i and ρ_{ij} is the correlation between them.

From the minimum value χ^2_{min} we obtain the central values for the parameters in the shape function scheme

TABLE II: Results for the combined fit to all moments with experimental and theoretical uncertainties. For $|V_{cb}|$ we add an additional theoretical error stemming from the uncertainty in the expansion for $\Gamma_{\rm SL}$ of 1.4%. Below the fit results the correlation matrix is shown.

$B \to X_c \ell \bar\nu$	OPE FIT RESULT: χ^2/N_{dof} =39.1/62							
$+ B \to X_s \gamma$	$\lvert V_{cb}\rvert$ $\times 10^{-3}$	m_b (GeV)	m_c (GeV)	μ_π^2 (GeV^2)	ρ_D^3 (GeV^3)	μ_G^2 (GeV^2)	ρ_{LS}^3 (GeV^3)	$BR_{c\ell\bar\nu}$ (%)
RESULT	41.91	4.613	1.187	0.408	0.191	0.261	-0.195	10.64
Δ exp	0.19	0.022	0.033	0.017	0.008	0.019	0.052	0.09
Δ HQE	0.28	0.027	0.040	0.031	0.019	0.035	0.068	0.07
Δ $\Gamma_{\rm SL}$	0.59							
$\lvert V_{cb}\rvert$	1.000	-0.450	-0.315	0.495	0.311	-0.275	0.070	0.674
m_b		1.000	0.962	-0.525	-0.225	-0.226	-0.211	0.121
m_c			1.000	-0.536	-0.310	-0.448	-0.100	0.152
μ_π^2				1.000	0.750	0.230	0.071	0.126
ρ_D^3					1.000	0.185	-0.507	0.123
μ_G^2						1.000	-0.034	-0.160
ρ_{LS}^3							1.000	-0.070
$BR_{c\ell\bar\nu}$								1.000

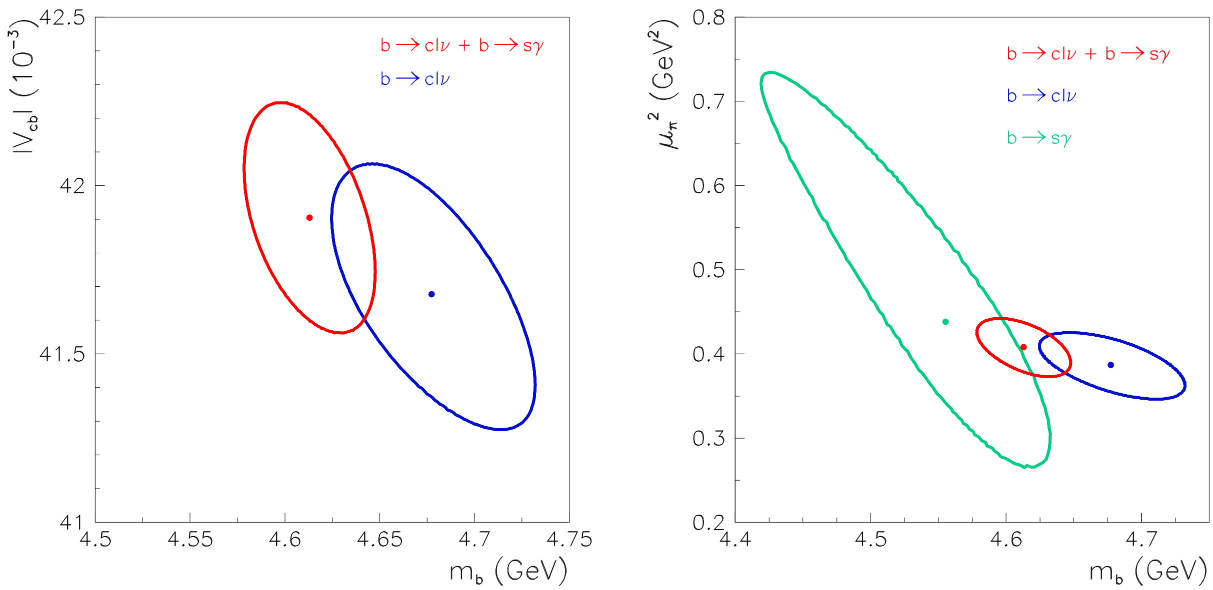

FIG. 1: Comparison of the different fit scenarios. Figure (a) shows the $\Delta\chi^2 = 1$ contour in the (m_b,μ_π^2) plane for the combined fit to all moments (red), the fit to hadron and lepton moments only (blue) and the fit to photon moments only (green). Figure (b) shows the results for the combined fit (red) and the fit to hadron and lepton moments only (blue) in the (m_b,$|V_{cb}|$) plane.

and determine the $\Delta\chi^2 = 1$ contour with respect to χ^2_{min}.

For the translation into the Shape-Function scheme [23, 24] we use a grid of moments obtained with a *Mathematica* notebook based on Ref. [25–28] that was provided to us by the authors. In this calculation the moments are determined from a spectrum that is obtained by convoluting a shape function with a perturbative kernel with next-to-leading order accuracy,

TABLE III: First and second moment of the photon spectrum predicted for $E_{\rm cut} = 1.6$ GeV on the basis of the fit results for the HQE parameters.

$E_{\rm cut}$(GeV)	$\langle E_\gamma\rangle$(GeV)	$\langle (E_\gamma - \langle E_\gamma\rangle)^2\rangle$(GeV^2)	ρ
1.6	2.297 $\pm$ 0.016	0.0431 $\pm$ 0.0030	-0.07

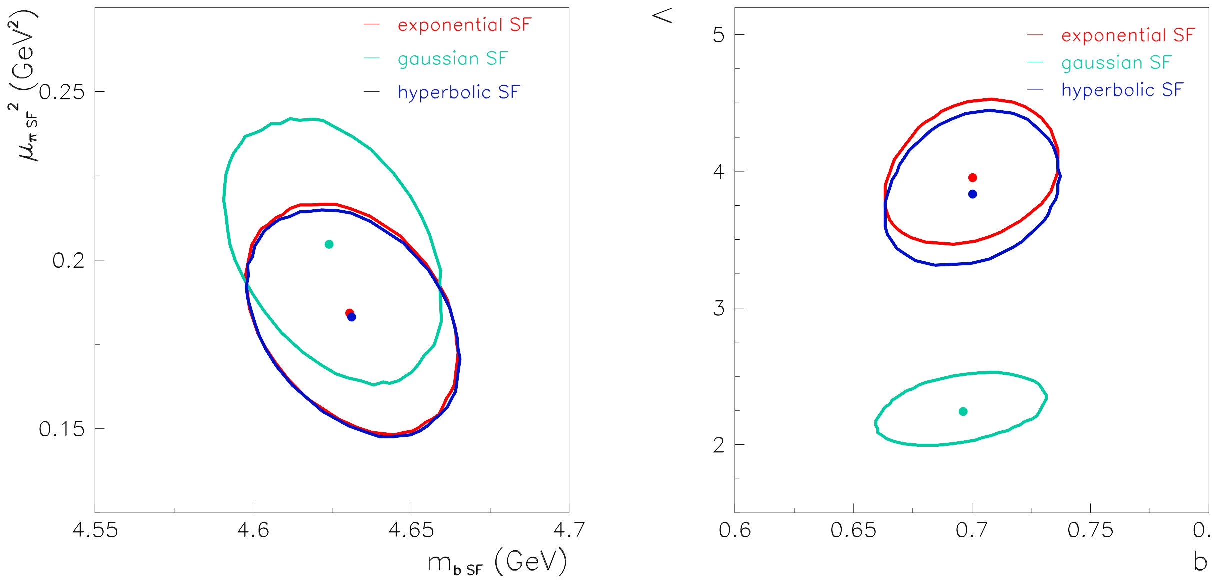

FIG. 2: Translation of fit results obtained in the kinetic scheme to the Shape Function scheme via predicted photon moments. Figure (a) shows the results for the shape function parameters in the $(m_{b\ SF}, \mu_{\pi\ SF}^2)$ plane using three different Ansätze for the Shape Function in the SF scheme. Figure (b) shows the corresponding results in the internal parameter space (b, Λ) (see text).

TABLE IV: Comparison of heavy quark distribution function parameters in the kinetic and Shape Function scheme together with their correlation ρ.

Shape Function Scheme		
Exponential SF		
$m_{b\ SF}$(GeV)	$\mu_{\pi\ SF}^2$(GeV^2)	ρ
4.631 ± 0.034	0.184 ± 0.036	-0.27
Hyperbolic SF		
$m_{b\ SF}$(GeV)	$\mu_{\pi\ SF}^2$(GeV^2)	ρ
4.631 ± 0.034	0.183 ± 0.036	-0.27
Gaussian SF		
$m_{b\ SF}$(GeV)	$\mu_{\pi\ SF}^2$(GeV^2)	ρ
4.624 ± 0.035	0.205 ± 0.041	-0.28

where we use three different Ansätze for the shape function (exponential, hyperbolic, and gaussian) given in Ref. [25]. This calculation is conceptually similar to the one for $B \to X_u \ell \bar{\nu}$ decays also presented in Ref. [25] which at present is used for the extraction of $|V_{ub}|$ by several experiments. It therefore allows for a consistent determination of the shape function parameters for both, $B \to X_s \gamma$ and $B \to X_u \ell \bar{\nu}$ decays. The numerical results for the shape function parameters are shown in Table IV and the $\Delta\chi^2 = 1$ contours are displayed in Figure 2 for the three different shape function Ansätze. In addition, the $\Delta\chi^2 = 1$ contours are shown in the internal parameter space (b, Λ). These are model parameters that depend on the used functional form.

As expected, the difference in the physical parameters $m_{b\ SF}$ and $\mu_{\pi\ SF}^2$ obtained with the three SF Ansätze is small as the translation between schemes was done using moments predicted at $E_{\rm cut} = 1.6\,{\rm GeV}$, where the sensitivity to shape function effects is expected to be small.

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