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Steps Toward a Formalism for the Formulary Model

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Introduction

In a previous memo Hoffman [1969] described a system for controlling the access to a large data base in an interactive environment. Simply stated, the system consists of a set of procedures (called a formulary) which interact with the user and control his access to the data base. In this memo we present some mathematical notation which we hope will be of use in discussing the problems of data base access in general and formularies in particular.

Definitions and Notation

Let p_i represent a formulary and q_j represent a data item. Assume for convenience that the q_j's represent atomic (indivisible) quanta of data. That is, the q's stand for the smallest pieces of data which may be accessed individually. Any set of data items not possessing this atomic property may be simply transformed into a (larger) set with this property. Assume for tractability that there is a finite set of m formularies ($1 \leq i \leq m$) and a finite set of n data items ($1 \leq j \leq n$).

Define a pseudo length function on formularies and data items.

$|p_i|$ = the number of bits required to represent formulary p_i.

$|q_j|$ = the number of bits required to represent data item q_j.

Let M_p be the total number of bits needed to represent formularies, M_q be the total number of bits needed to represent data items, and M be the total number of bits of memory (core or rotating) available.

$$M_p = \sum_{i=1}^{m} |p_i| \qquad M_q = \sum_{j=1}^{n} |q_j|$$

Clearly we are constrained to have:

$$M \geq M_p + M_q$$

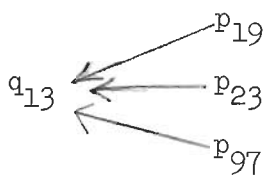
The storage efficiency η of a formulary system can be defined as:

$$\eta = \frac{M_q}{M_p + M_q}$$

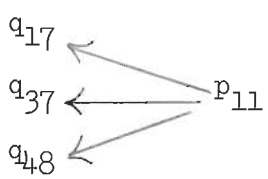
η is a measure of how much of the storage used by the system is actually used for storing useful data.

We note that M_p/m is the average number of bits per formulary and M_q/n is the average number of bits per data item. For each data item q_j there are some number of formularies which allow some level of access to it. Note that the association between data items and formularies can be:

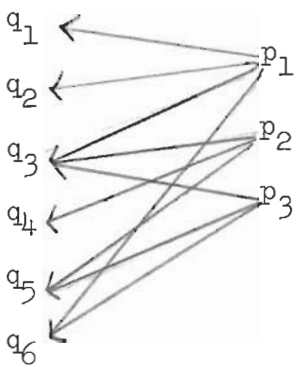
a) one-many



b) many-one



c) more complex



Define a predicate on data items and formularies:

PERMIT (q,p) is true iff formulary p allows access to data item q.

The value of the predicate is effectively computable since we are dealing with finite sets of formularies and data items and it is possible to try to access each data item with each formulary.

Using this predicate we can define some set relations on formularies and data:

Let S_j be the index set of formularies which allow access to q_j.

$$S_j = \{ x \mid \text{PERMIT}(q_j, p_x) \}$$

Let T_i be the index set of data items accessed by p_i.

$$T_i = \{ y \mid \text{PERMIT}(q_y, p_i) \}$$

The set operations of intersection and union have the following interpretations:

$S_a \cup S_b$ is the index set of formularies which access q_a or q_b (or both).

$S_a \cap S_b$ is the index set of formularies which access both q_a and q_b.

$T_a \cup T_b$ is the index set of data items accessed by p_a or p_b (or both).

$T_a \cap T_b$ is the index set of data items accessed by both p_a and p_b.

In example c) above:

$S_1 = \{1\}$ $\quad T_1 = \{1, 2, 3, 6\}$

$S_2 = \{1\}$ $\quad T_2 = \{3, 4, 5\}$

$S_3 = \{1, 2, 3\}$ $\quad T_3 = \{3, 5, 6\}$

$S_4 = \{2\}$

$S_5 = \{2, 3\}$

$S_6 = \{1, 3\}$

The <u>protective overhead</u> V_j for a data item q_j is defined to be the number of bits of formulary needed to control access to q_j.

$$V_j = \sum_{i \epsilon S_j} |p_i|$$

Now

$$M_p \leq \sum_{j=1}^{n} \sum_{i \epsilon S_j} |p_i| = \sum_{j=1}^{n} V_j$$

with equality attained iff

$$(\forall x \forall y \quad (T_x \cap T_y = \emptyset)) \wedge (\forall z \exists w \quad (\text{PERMIT}(q_w, p_z)))$$

(i.e., no two formularies control a common data item and there are no formularies which do not control at least one data item).

Similarly we may define the <u>data space</u> U_i attributed to a formulary p_i as the number of bits of data accessed by p_i.

$$U_i = \sum_{j \epsilon T_i} |q_j|$$

The analogous relation for M_q is:

$$M_q \leq \sum_{i=1}^{m} \sum_{j \epsilon T_i} |q_j| = \sum_{i=1}^{m} U_i$$

with equality iff

$$(\forall x \forall y \ (S_x \cap S_y = \emptyset)) \wedge (\forall w \exists z \ (\text{PERMIT}(q_w, p_z)))$$

(each data item is controlled by only one formulary and there are no inaccessible data items).

Application to a Common Problem

One problem which is often studied in conjunction with systems which allow multiple concurrent access to a data base is the problem of simultaneous updating. This situation arises when two users of the data base attempt to change the value of a particular data item at the same time. In a previous memo [Hoffman 1969], techniques were outlined which would solve the problem by allowing a user to LOCK a data item and make it inaccessible to other users. Here we will present a formal condition as to when simultaneous updating can occur.

Consider an interactive environment with some number of formularies actively being invoked by various users to access a common data base. Define

$$R_t = \{x \mid p_x \text{ is active at time } t\}$$

A sufficient (overly restrictive) condition to preclude simultaneous updating of any data item at a particular time t is:

$$(\forall x \forall y \ (((x \epsilon R_t) \wedge (y \epsilon R_t)) \supset ((x \neq y) \supset (T_x \cap T_y = \emptyset))))$$

Data on a Practical System

The quantities defined above can be used to describe memory usage in systems which use the formulary model. In a system which uses memory efficiently we would expect to have $M_p \ll M_q$. Inefficient memory usage would occur when $M_p \geq Mq$.

In the Stanford Health Service system the following data has been observed:

M_p	$\sim$	8.0×10^4	bits	$m = 3$
M_q		6.4×10^6	bits	$n \sim 10^4$
M_p/m		2.66×10^4	bits	
M_q/n	$\sim$	640	bits	

$$\eta = \frac{6.4 \times 10^6}{6.48 \times 10^6} = 0.988$$

Reference

Hoffman, Lance J. (1969). "Formularies -- Program Controlled Privacy in Large Data Bases", CGTM No. 60, Stanford Linear Accelerator Center, Computation Group, Stanford, California, February 1969