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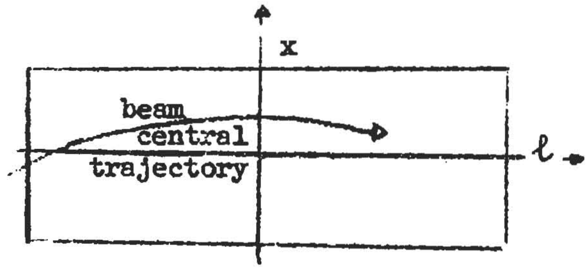
C.G.T.M. #8

Determination of Linear and Quadratic Non-Homogeneity Terms from Magnetic Measurements of the 3° Switchyard Bending Magnets

I. METHOD

Integral $Bd\ell$ data in the form:



$$u(x,B_o) \equiv \frac{\int_{-\infty}^{+\infty} B(x,\ell,B_o)d\ell}{\int_{-\infty}^{+\infty} B(0,\ell,B_o)d\ell} - 1$$

was measured and tabulated for values of x and B_o. The variables ℓ and x are the longitudinal and transverse co-ordinates respectively (see figure) and B_o is the nominal field strength at the magnetic center (as determined by a nuclear magnetic resonance measurement).

$u(x,B_o)$ was measured for a grid of values (x,B_o):

$$B_o = 4,\ 8,\ 12.5,\ 15.0 \text{ kg}$$

and x = -3.0 step 0.5 to 3.0 inches.

For each of the values of B_o, the <u>$u(x,B_o)$</u> vs. <u>x</u> data are (least-squares) fitted to the quadratic expansion of the magnetic field:

$$u = -n\left(\frac{x}{\rho}\right) + \beta\left(\frac{x}{\rho}\right)^2$$

where $\rho = \frac{L_{eff}}{\theta}$, $\theta = 3°$; $L_{eff} = 3$ meters, the bending angle and effective length respectively.

II. EXAMPLE

The following example data came from measurements on the 3° prototype magnet. $B_o = 12.5$ kg:

i	x_i(inches)	$u_i(x_i)$(per cent)
1	0	0
2	-0.5	-0.0004
3	-1.0	-0.003
4	-1.5	-0.0085
5	-2.0	-0.0169
6	-2.5	-0.0304
7	-3.0	-0.0476
8	-3-5/16	-0.0529
9	0.5	-0.0011
10	1.0	-0.0041
11	1.5	-0.0135
12	2.0	-0.0264
13	2.5	-0.0450
14	3.0	-0.0669
15	3+5/16	-0.0823

There are N = 15 data points. Values of n and β are chosen so as to minimize:

$$\chi^2 \equiv \sum_{i=1}^{N} \left[\frac{u_i}{100} + n\left(\frac{(0.0254)(3/57.29578)}{3.0}\right) x_i - \beta\left(\frac{(0.0254)(3/57.29578)}{3.0}\right)^2 x_i^2\right]^2$$

These values were: $n = 0.0732$

$\beta = 312.0$

III. THE TRANSPORT PARAMETER ϵ_1

The computer program TRANSPORT assumed parameter ϵ_1 which differs from β by a scale factor:

$$\epsilon_1 = \left(\frac{\text{1 unit of transverse or 'x' direction}}{\text{bending radius}}\right)^2 \beta$$

Thus ϵ_1 has the physical interpretation as the quadratic variation of field occurring at 1 transverse unit displacement from the magnetic axis. For switchyard magnets

$$\rho = \frac{3.0}{(3/57.29578)} = 57.29578 \text{ meters}$$

Since the transverse unit in TRANSPORT is usually in centimeters the above factor is:

$$\left(\frac{10^{-2}}{57.29578}\right)^2 = 3.046 \times 10^{-8}$$

so the value of ϵ_1 in the above example is

$$\epsilon_1 = 3.046 \times 10^{-8}\beta$$
$$= -0.95 \times 10^{-5}$$

This is the quadratic dropoff of field one cm from the magnetic axis on the smoothed curve.

IV. SUMMARY OF DATA ON THE PROTOTYPE MAGNET

B_0	n	β	ϵ_1	χ^2	$\sigma = (\chi^2/N)^{\frac{1}{2}}$
15.0	0.0796	-1390	$-4.22 \cdot 10^{-5}$	0.1745	0.0108
12.5	0.0732	-312	$-0.95 \cdot 10^{-5}$	0.0124	0.0028
8	0.0736	-167	$-0.51 \cdot 10^{-5}$	0.0031	0.0014
4	0.0255	-131	$-0.40 \cdot 10^{-5}$	0.0050	0.0018