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Errata and Addenda for An Introduction to the Curves and Surfaces of Computer-Aided Design

by
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Since the book *An Introduction to the Curves and Surfaces of Computer-Aided Design* was first published in April of 1991, a few errors and omissions, mostly minor, have been found. In addition, a number of additional results have been found that fit very nicely into the book. This document will describe all of these items.

Individual changes are separated by horizontal rulings. The page number of the modification and the position on the page is given. If someone else initially pointed out the problem to me, that contribution is acknowledged.

Page ix: *The following paragraph should be added after the fourth paragraph:*

The index is rather extensive; it contains concepts as well as key words and these items are cross referenced. For example, the index contains the entry

reason for using column vectors 6

but it also contains

column vectors, reason for using 6

and

vectors, reason for using column 6.

Page x: *The following entries should be added after the entry defining the length of a vector:*

$\mathbf{A} \cdot \mathbf{B}$	The dot product of the vectors **A** and **B**.
$\mathbf{A} \times \mathbf{B}$	The cross product of the vectors **A** and **B**.

Page 15: *For compatibility with the addition on page 16, Equation (1.29) should be changed to the equivalent form:*

$$\mathbf{A} = \begin{pmatrix} 1-\left(u_2^2+u_3^2\right)(1-\cos\theta) & -u_3\sin\theta + u_1u_2(1-\cos\theta) & u_2\sin\theta + u_1u_3(1-\cos\theta) \\ u_3\sin\theta + u_1u_2(1-\cos\theta) & 1-\left(u_1^2+u_3^2\right)(1-\cos\theta) & -u_1\sin\theta + u_2u_3(1-\cos\theta) \\ -u_2\sin\theta + u_1u_3(1-\cos\theta) & u_1\sin\theta + u_2u_3(1-\cos\theta) & 1-\left(u_1^2+u_2^2\right)(1-\cos\theta) \end{pmatrix}. \tag{1.29}$$

Page 16: *The following paragraphs should be added at the end of Section 1.2.4:*

Notice that Equation (1.29) can be put into the form

$$\begin{aligned} \mathbf{A} &= \mathbf{I} + \sin\theta \begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{pmatrix} \\ &\quad + (1-\cos\theta)\begin{pmatrix} -(u_2^2+u_3^2) & u_1u_2 & u_1u_3 \\ u_1u_2 & -(u_1^2+u_3^2) & u_2u_3 \\ u_1u_3 & u_2u_3 & -(u_1^2+u_2^2) \end{pmatrix} \\ &= \mathbf{I} + \sin\theta\mathbf{H} + (1-\cos\theta)\mathbf{H}^2 \end{aligned} \tag{1.29a}$$

where

$$\mathbf{H} = \begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{pmatrix}.$$

Equation (1.29a) can be manipulated in many ways. If we let $\mathbf{X} = (\,x \quad y \quad z\,)^T$ and $\mathbf{X}' = (\,x' \quad y' \quad z'\,)^T$, then the transformation given by Equation (1.29) can be put into the equivalent form

$$\mathbf{X}' = \mathbf{X} + \sin\theta\,(\mathbf{U}\times\mathbf{X}) + (1-\cos\theta)\,(\mathbf{U}\times(\mathbf{U}\times\mathbf{X}))\,.$$

If only a few points need to be rotated, this is a very efficient way to process them; if a large number of points are to be transformed, it is best to calculate the matrix **A** and use that to rotate the points.

We will now use Equation (1.29a) to obtain a very interesting but not particularly useful result related to the exponential of a matrix. If **B** is a square matrix, the exponential may be defined by

$$e^{\mathbf{B}} = \mathbf{I} + \frac{\mathbf{B}}{1!} + \frac{\mathbf{B}^2}{2!} + \frac{\mathbf{B}^3}{3!} + \cdots.$$

It can be shown that this series always converges. Now compute the powers of **H** to get $\mathbf{H}^3 = -\mathbf{H}$, $\mathbf{H}^4 = -\mathbf{H}^2$, and $\mathbf{H}^5 = \mathbf{H}$. Thus the powers of **H** cycle through $\mathbf{H}$, $\mathbf{H}^2$, $-\mathbf{H}$, and $-\mathbf{H}^2$. Therefore

$$\begin{aligned}
e^{\theta\mathbf{H}} &= \mathbf{I} + \frac{\theta\mathbf{H}}{1!} + \frac{\theta^2\mathbf{H}^2}{2!} - \frac{\theta^3\mathbf{H}}{3!} - \frac{\theta^4\mathbf{H}^2}{4!} + \cdots \\
&= \mathbf{I} + \left(\frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)\mathbf{H} + \left(\frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \frac{\theta^6}{6!} - \cdots\right)\mathbf{H}^2 \\
&= \mathbf{I} + \left(\frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)\mathbf{H} + \left(1 - \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right)\right)\mathbf{H}^2 \\
&= \mathbf{I} + \sin\theta\mathbf{H} + (1-\cos\theta)\mathbf{H}^2
\end{aligned}$$

and we have

$$\mathbf{A} = e^{\theta\mathbf{H}}. \qquad (1.29b)$$

Equation (1.29b) is a generalization of the equation

$$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} = e^{\theta\mathbf{K}}$$

where

$$\mathbf{K} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}.$$

This result may be proven using the infinite series or it may be obtained by noticing that the correspondence

$$a + ib \longleftrightarrow a\mathbf{I} + b\mathbf{K}$$

is an isomorphism between the complex numbers and matrices of the form

$$\begin{pmatrix}a & -b\\ b & a\end{pmatrix}$$

with **K** playing the role of i. Multiplication of a complex number by $e^{i\theta} = \cos\theta + i\sin\theta$ rotates the number counterclockwise about the origin through an angle of θ. Because of the isomorphism, multiplication of a vector emanating from the origin by $e^{\theta\mathbf{K}}$ must have the same effect.

Page 25: *The following paragraph should be added at the end of Section 1.4.1:*

When stereo views of an object are required, two separate transformations, a left eye transformation and a right eye transformation, can be produced. The proper procedure is to use two eye points but define the projection plane in exactly the same manner for both transformations. It is usually best to have the object being viewed near the projection plane or slightly behind it.

Page 36: *The second paragraph on the page should be:*

The first thing to notice is that proving that Equations (1.73) have a unique solution is equivalent to proving that the homogeneous problem, that is, the problem with $\beta_0 = \cdots = \beta_n = 0$, only has $a_0 = \cdots = a_n = 0$ as a solution. This fact is easy to see if we consider the determinant of the α_{ij}'s in Equation (1.73). If the determinant is nonzero, Cramer's rule shows that the system has a unique solution and that the solution is $a_0 = \cdots = a_n = 0$. If, on the other hand, the determinant of the α_{ij}'s is zero, then the homogeneous system has infinitely many solutions because the equations are not independent. In that case the nonhomogeneous problem does not have a solution or has infinitely many solutions.

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Page 54: *The following paragraph should be added after the second paragraph in Section 1.7.*

We earlier stated that Equation (1.29b) was interesting but not very useful. The reason it is not useful is that the computation of the exponential of a matrix can pose serious difficulties. These problems are studied by Moler and Van Loan in their article, "Nineteen dubious ways to compute the exponential of a matrix" [MOL78].

Page 98: *Figure (3.1) should have the word* "Non-Uniform" *changed to* "Nonuniform" *to be consistent with the text.*

Page 103: *Figure (3.3) should have the word* "Non-Uniform" *changed to* "Nonuniform" *to be consistent with the text.*

Page 135: *Figure (4.2) should have the word* "Non-Uniform" *changed to* "Nonuniform" *to be consistent with the text.*

Page 136: *The third line from the top of the page should be:*

Figure (4.4) illustrates the manipulation of the a_i values. The natural value of

Page 140: *The following paragraphs should be added after the second paragraph in Section 4.5.*

This method is related to the Catmull-Rom spline described by Catmull and Rom in "A class of local interpolating splines" in *Computer Aided Geometric Design* [BAR74]. That article displays the matrix in Equation (4.8). Their methodology is potentially more general than the methods used in this chapter.

Bessel's method is also related to the work of Akima. He has published two reports "A new method of interpolation and smooth curve fitting based on local procedures" [AKI70] and "A method of univariate interpolation that has the accuracy of a third-degree polynomial" [AKI91]. Akima is especially interested in maintaining monotonic interpolation curves through monotonic points.

Page 174: *Figures (6.3) and (6.4) should have the word* "Non-Uniform" *changed to* "Nonuniform" *to be consistent with the text.*

Page 243: *Figure (9.3) should have the word* "Non-Uniform" *changed to* "Nonuniform" *to be consistent with the text.*

Page 246: *Figure (9.6) should have the word* "Non-Uniform" *changed to* "Nonuniform" *to be consistent with the text.*

Page 248: *A new section should be added after Section 9.3. The new section is:*

9.3a. Higher Order B-splines in Matrix Form

In principal, the matrix form of the B-spline can be produced for any degree. The desired result is to express each entry in the matrix as a simple combination of terms whose denominators will reduce to the factorial function in the uniform case. When this is done, the result is a formulation that substantially reduces the computation when many points on a curve are to be evaluated. The method clearly displays all of the common factors in the calculation. In practice, the algebra becomes very difficult after the third degree; discovering the form of the k_i values in the following equations is not straightforward.

However, with the aid of a symbolic algebraic manipulator, the fourth and fifth degree B-splines have been put into this form. Even with such a manipulator, it is unlikely that these results will ever be extended to higher degree.

The fourth degree B-spline is given by:

$$\mathbf{P}(t) = (\mathbf{P}_0 \quad \mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3 \quad \mathbf{P}_4) \begin{pmatrix} k_1 & -4k_1 & 6k_1 & -4k_1 & k_1 \\ -(k_1+k_4+k_8+k_9) & 4(k_1+k_4+k_8) & -6(k_1+k_4-k_6) & 4(k_1-k_3-k_6-k_7) & 1-(k_1+k_2-k_5) \\ (k_4+k_8+k_9+k_{13}+k_{14}+k_{15}) & -4(k_4+k_8+k_{13}) & 6(k_4-k_6-k_{12}) & 4(k_3+k_6+k_7-k_{11}) & (k_2-k_5-k_{10}) \\ -(k_{13}+k_{14}+k_{15}+k_{16}) & 4k_{13} & 6k_{12} & 4k_{11} & k_{10} \\ k_{16} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^4 \\ t^3 \\ t^2 \\ t \\ 1 \end{pmatrix}$$

where

$$k_1 = \frac{d_4^3}{(d_1+d_2+d_3+d_4)(d_2+d_3+d_4)(d_3+d_4)},$$

$$k_2 = \frac{d_2^2 d_3 + d_2^2 d_4 + 3d_2 d_3^2 + 3d_2 d_3 d_4 + 2d_3^3 + 3d_3^2 d_4}{(d_2+d_3+d_4+d_5)(d_2+d_3+d_4)(d_3+d_4)},$$

$$k_3 = \frac{d_2 d_4^2 + d_3 d_4^2}{(d_2+d_3+d_4+d_5)(d_2+d_3+d_4)(d_3+d_4)},$$

$$k_4 = \frac{d_4^3}{(d_2+d_3+d_4+d_5)(d_2+d_3+d_4)(d_3+d_4)},$$

$$k_5 = \frac{d_3^3}{(d_2+d_3+d_4+d_5)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_6 = \frac{d_3 d_4^2}{(d_2+d_3+d_4+d_5)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_7 = \frac{d_3 d_4 d_5}{(d_2+d_3+d_4+d_5)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_8 = \frac{d_4^3}{(d_2+d_3+d_4+d_5)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_9 = \frac{d_4^3}{(d_2+d_3+d_4+d_5)(d_3+d_4+d_5)(d_4+d_5)},$$

$$k_{10} = \frac{d_3^3}{(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_{11} = \frac{d_3^2 d_4}{(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_{12} = \frac{d_3 d_4^2}{(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_{13} = \frac{d_4^3}{(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_3+d_4)},$$

$$k_{14} = \frac{d_4^3}{(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_4+d_5)},$$

$$k_{15} = \frac{d_4^3}{(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{16} = \frac{d_4^3}{(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)}.$$

The fifth degree B-spline is given by:

$$\mathbf{P}(t) = (\mathbf{P}_0 \quad \mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3 \quad \mathbf{P}_4 \quad \mathbf{P}_5)$$

$$\begin{pmatrix}
-k_1 & 5k_1 & -10k_1 & 10k_1 & -5k_1 & k_1 \\
(k_1+k_3+k_6+k_{11}+k_{12}) & -5(k_1+k_3+k_6+k_{11}) & 10(k_1+k_3+k_6-k_9) & -10(k_1+k_3-k_5-k_9-k_{10}) & 5(k_1+k_3+k_6-k_8) & 1-(k_1+k_2-k_4+k_7) \\
-(k_3+k_6+k_{11}+k_{12}+k_{16}+k_{21}+k_{22}+k_{27}+k_{28}+k_{29}) & 5(k_3+k_6+k_{11}+k_{16}+k_{21}+k_{27}) & -10(k_3+k_6-k_9+k_{16}-k_{19}-k_{26}) & 10(k_3-k_5-k_9-k_{10}-k_{15}-k_{19}-k_{20}+k_{25}) & -5(k_3+k_6-k_8+k_{14}-k_{18}-k_{24}) & (k_2-k_4+k_7-k_{13}+k_{17}+k_{23}) \\
(k_{16}+k_{21}+k_{22}+k_{27}+k_{28}+k_{29}+k_{34}+k_{35}+k_{36}+k_{37}) & -5(k_{16}+k_{21}+k_{27}+k_{34}) & 10(k_{16}-k_{19}-k_{26}-k_{33}) & 10(k_{15}+k_{19}+k_{20}-k_{25}-k_{32}) & 5(k_{14}-k_{18}-k_{24}-k_{31}) & (k_{13}-k_{17}-k_{23}-k_{30}) \\
-(k_{34}+k_{35}+k_{36}+k_{37}+k_{38}) & 5k_{34} & 10k_{33} & 10k_{32} & 5k_{31} & k_{30} \\
k_{38} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix} t^5 \\ t^4 \\ t^3 \\ t^2 \\ t \\ 1 \end{pmatrix}$$

where

$$k_1 = \frac{d_5^4}{(d_1 + d_2 + d_3 + d_4 + d_5)(d_2 + d_3 + d_4 + d_5)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$\begin{aligned} k_2 =& (d_2^2 d_3 d_4 + d_2^2 d_3 d_5 + d_2^2 d_4^2 + 2d_2^2 d_4 d_5 + d_2^2 d_5^2 + 3d_2 d_3^2 d_4 + 3d_2 d_3^2 d_5 \\ & + 7d_2 d_3 d_4^2 + 10 d_2 d_3 d_4 d_5 + 3d_2 d_3 d_5^2 + 4d_2 d_4^3 + 8d_2 d_4^2 d_5 + 4d_2 d_4 d_5^2 \\ & + 2d_3^3 d_4 + 2d_3^3 d_5 + 7d_3^2 d_4^2 + 10 d_3^2 d_4 d_5 + 3d_3^2 d_5^2 + 8d_3 d_4^3 + 16 d_3 d_4^2 d_5 \\ & + 8d_3 d_4 d_5^2 + 3d_4^4 + 8d_4^3 d_5 + 6d_4^2 d_5^2)/ \\ & (d_2 + d_3 + d_4 + d_5 + d_6)(d_2 + d_3 + d_4 + d_5)(d_3 + d_4 + d_5)(d_4 + d_5), \end{aligned}$$

$$k_3 = \frac{d_5^4}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_2 + d_3 + d_4 + d_5)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$k_4 = \frac{d_3^3 d_4 + d_3^3 d_5 + 4d_3^2 d_4^2 + 4d_3^2 d_4 d_5 + 6d_3 d_4^3 + 6d_3 d_4^2 d_5 + 3d_4^4 + 4d_4^3 d_5}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$k_5 = \frac{d_3 d_5^3 + d_4 d_5^3}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$k_6 = \frac{d_5^4}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$k_7 = \frac{d_4^4}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_4 + d_5 + d_6)(d_4 + d_5)},$$

$$k_8 = \frac{3d_4 d_5^3 + 3d_4 d_5^2 d_6 + d_4 d_5 d_6^2 + 2d_5^4 + 3d_5^3 d_6 + d_5^2 d_6^2}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_4 + d_5 + d_6)(d_4 + d_5)},$$

$$k_9 = \frac{d_4 d_5^3}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_4 + d_5 + d_6)(d_4 + d_5)},$$

$$k_{10} = \frac{d_4 d_5^2 d_6}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_4 + d_5 + d_6)(d_4 + d_5)},$$

$$k_{11} = \frac{d_5^4}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_4 + d_5 + d_6)(d_4 + d_5)},$$

$$k_{12} = \frac{d_5^4}{(d_2 + d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5 + d_6)(d_4 + d_5 + d_6)(d_5 + d_6)},$$

$$k_{13} = \frac{d_3^3 d_4 + d_3^3 d_5 + 4d_3^2 d_4^2 + 4d_3^2 d_4 d_5 + 6d_3 d_4^3 + 6d_3 d_4^2 d_5 + 3d_4^4 + 4d_4^3 d_5}{(d_3 + d_4 + d_5 + d_6 + d_7)(d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$k_{14} = \frac{d_3^2 d_4 d_5 + d_3^2 d_5^2 + 3d_3 d_4^2 d_5 + 3d_3 d_4 d_5^2 + 2d_4^3 d_5 + 3d_4^2 d_5^2}{(d_3 + d_4 + d_5 + d_6 + d_7)(d_3 + d_4 + d_5 + d_6)(d_3 + d_4 + d_5)(d_4 + d_5)},$$

$$k_{15} = \frac{d_3 d_5^3 + d_4 d_5^3}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_4+d_5)},$$

$$k_{16} = \frac{d_5^4}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_3+d_4+d_5)(d_4+d_5)},$$

$$k_{17} = \frac{d_4^4}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{18} = \frac{d_4^3 d_5}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{19} = \frac{d_4 d_5^3}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{20} = \frac{d_4 d_5^2 d_6}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{21} = \frac{d_5^4}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{22} = \frac{d_5^4}{(d_3+d_4+d_5+d_6+d_7)(d_3+d_4+d_5+d_6)(d_4+d_5+d_6)(d_5+d_6)},$$

$$k_{23} = \frac{d_4^4}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{24} = \frac{d_4^3 d_5}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{25} = \frac{d_4^2 d_5^2}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{26} = \frac{d_4 d_5^3}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{27} = \frac{d_5^4}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{28} = \frac{d_5^4}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_5+d_6)},$$

$$k_{29} = \frac{d_5^4}{(d_3+d_4+d_5+d_6+d_7)(d_4+d_5+d_6+d_7)(d_5+d_6+d_7)(d_5+d_6)},$$

$$k_{30} = \frac{d_4^4}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{31} = \frac{d_4^3 d_5}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{32} = \frac{d_4^2 d_5^2}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{33} = \frac{d_4 d_5^3}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{34} = \frac{d_5^4}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_4+d_5)},$$

$$k_{35} = \frac{d_5^4}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_4+d_5+d_6)(d_5+d_6)},$$

$$k_{36} = \frac{d_5^4}{(d_4+d_5+d_6+d_7+d_8)(d_4+d_5+d_6+d_7)(d_5+d_6+d_7)(d_5+d_6)},$$

$$k_{37} = \frac{d_5^4}{(d_4+d_5+d_6+d_7+d_8)(d_5+d_6+d_7+d_8)(d_5+d_6+d_7)(d_5+d_6)},$$

$$k_{38} = \frac{d_5^4}{(d_5+d_6+d_7+d_8+d_9)(d_5+d_6+d_7+d_8)(d_5+d_6+d_7)(d_5+d_6)}.$$

Although the computation for this fifth degree B-spline can be made efficient if all of the common factors are taken into consideration, the formulation has clearly lost the simplicity of the lower degrees.

Page 254: *The figure caption should be:*

Figure 10.1. Examples of rational Bézier curves

Page 257: *The following paragraph should be added after the first full paragraph on the page:*

Actually, the preceding results in this section would be almost obvious if we had spent more time investigating homogeneous coordinates. A three-dimensions to two-dimensions projective transformation can be viewed as an affine transformation from the $(x \quad y \quad z)^T$ system to the $(x' \quad y' \quad w)^T$ system followed by a projection through the origin onto the $w = 1$ plane. Both of these operations preserve the interpolation and derivative matching characteristics of the curve.

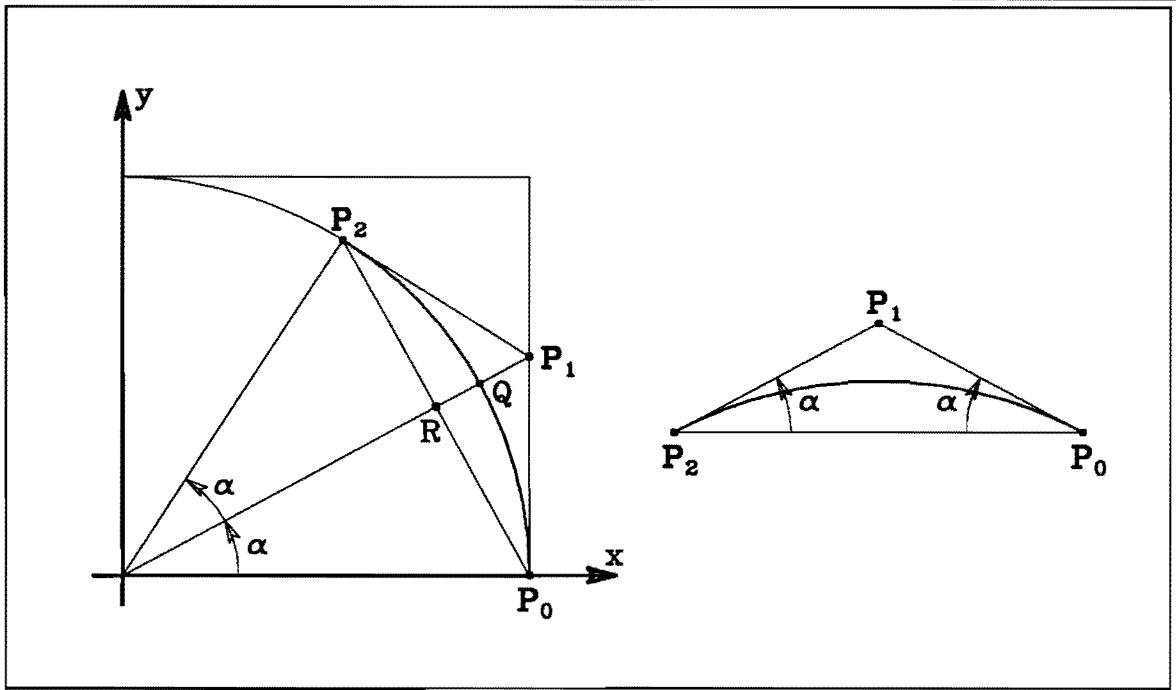

Figure 10.5. Constructing an arc of a circle from a rational Bézier curve

Pages 261 through 263: *Section 10.3.1 determines a Bézier curve that exactly matches a quarter circle. The problem of matching a general arc of a circle is actually simpler so Sections 10.3.1 and 10.3.2 should be replaced by:*

10.3.1. An Arc of a Circle as a Rational Bézier Curve

Let $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$ be the Bézier control points for the arc of a unit circle in the first quadrant as shown in the left part of Figure (10.5). The angle α is also shown in that figure. The points $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$ are therefore given by

$$\mathbf{P}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{P}_1 = \begin{pmatrix} 1 \\ \tan\alpha \end{pmatrix}, \text{ and } \mathbf{P}_2 = \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha - \sin^2\alpha \\ 2\cos\alpha\sin\alpha \end{pmatrix}.$$

The points $\mathbf{Q}$ and $\mathbf{R}$ are also given by

$$\mathbf{Q} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \text{ and } \mathbf{R} = \frac{1}{2}(\mathbf{P}_0 + \mathbf{P}_2) = \begin{pmatrix} \dfrac{1 + \cos^2\alpha - \sin^2\alpha}{2} \\ \cos\alpha\sin\alpha \end{pmatrix}.$$

Using the y component of Equation (10.18), we get

$$\cos\alpha\sin\alpha = \left(\frac{(1-t)^2}{(1-t)^2 + t^2}\right) 0 + \left(\frac{t^2}{(1-t)^2 + t^2}\right) 2\cos\alpha\sin\alpha.$$

This immediately reduces to

$$\frac{1}{2} = \frac{t^2}{(1-t)^2 + t^2}$$

or $t = 1/2$. The x component will give the same result but is not quite as simple to work with. Equation (10.19) then gives $\beta = 1$. The y component of Equation (10.20) becomes

$$\sin\alpha = \frac{1}{w_1+1}\cos\alpha\sin\alpha + \frac{w_1}{w_1+1}\frac{\sin\alpha}{\cos\alpha}$$

which simplifies to

$$w_1 = \cos\alpha. \tag{10.22}$$

Thus, with w_1 given by Equation (10.22), we obtain the circular arc.

Although we derived this result for the part of the unit circle starting at the x axis, Equation (10.21) is independent of its coordinate system and is invariant under a magnification. It will therefore always generate an arc of a circle provided $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$ form an isosceles triangle and w_1 is given by Equation (10.22). As shown in the right half of Figure (10.5), the angle α is easily determined from the control points.

Notice that a quarter circle is produced when $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$ form an equilateral right triangle and

$$w_1 = \frac{\sqrt{2}}{2}.$$

10.3.2. A Full Circle as a Rational B-spline

We shall now derive a rational B-spline representation for a full circle. The full circle will be made up of four quarter circles. Since a Bézier curve is a special case of a B-spline, we can quickly convert Equation (10.21) into a rational B-spline. We have already verified, in Section 9.4, that the matrix of the quadratic B-spline reduces to the quadratic Bézier matrix if we choose $d_1 = d_3 = 0$ in Equations (9.26) and (9.27). Thus Equation (10.5) will represent the quarter circle in the first quadrant if we let $\mathbf{B}$ be the nonuniform B-spline matrix and set

$$(d_1, d_2, d_3) = (0, 1, 0),$$
$$(w_0, w_1, w_2) = \left(1, \sqrt{2}/2, 1\right),$$

and let $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$ be the points shown in Figure (10.6).

The extension of the B-spline into the second quadrant will be controlled by $\mathbf{P}_2$, $\mathbf{P}_3$, and $\mathbf{P}_4$. The associated d_i's, that is, d_3, d_4, and d_5, must have values of 0, 1, and 0, respectively. There is, however, a slight problem with the formulation of the quadratic B-spline as we have given it up to now. If we just concatenate the points $\mathbf{P}_0$, ..., $\mathbf{P}_4$ together, we will get a B-spline controlled by $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$. The associated d_i's will be 1, 0, and 1. In that case, the d_2 interval in Figure (9.2)

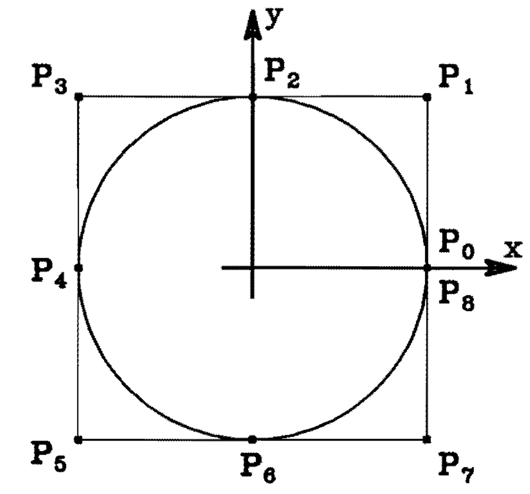

Figure 10.6. Constructing a full circle from rational B-spline curves

collapses to zero and it is clear that we should just skip this interval since all of the points evaluated in this interval will be identical. If this constraint is imposed, then the B-spline for the full circle is controlled by the points $\mathbf{P}_0, \ldots, \mathbf{P}_8$ as shown in Figure (10.6) and

$$(d_1, \ldots, d_9) = (0,1,0,1,0,1,0,1,0),$$
$$(w_0, \ldots, w_8) = \left(1, \sqrt{2}/2, 1, \sqrt{2}/2, 1, \sqrt{2}/2, 1, \sqrt{2}/2, 1\right).$$

Page 278: *The ninth line in the program should be:*

```
C *     CALL DRRBEZ(NPTS,PNTS,NWTS,WGTS)                                  *
```

Pages 289 through 296: *The following items should be added to the references:*

[AKI70] Hiroshi Akima, "A new method of interpolation and smooth curve fitting based on local procedures," *Journal of the Association for Computing Machinery*, Volume 17, Number 4, pages 589–602, October 1970.

See Chapter 4.

[AKI91] Hiroshi Akima, "A method of univariate interpolation that has the accuracy of a third-degree polynomial," *ACM Transactions on Mathematical Software*, Volume 17, Number 3, pages 341–366, September 1991.

See Chapter 4.

[BAR74] Robert E. Barnhill and Richard F. Riesenfeld (editors), *Computer Aided Geometric Design*, Academic Press, New York, 1974.

An additional item has been cited in this reference.

Edwin Catmull and Raphael Rom, "A class of local interpolating splines," pages 317–326.

See Chapter 4.

[MOL78] Cleve Moler and Charles Van Loan, "Nineteen dubious ways to compute the exponential of a matrix," *SIAM Review*, Volume 20, Number 4, pages 801–836, October 1978.

See Chapter 1.