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# RESOLVING THE LEFT-RIGHT AMBIGUITY BY PRINCIPAL COMPONENT ANALYSIS

Henk Wind
Computation Research Group
Stanford Linear Accelerator Center
Stanford, California 94305

## ABSTRACT

From the drift time of electrons in a drift chamber, one can get an estimate of the distance from a track to the sense wire. However, the point of closest approach is not known from the drift time. Since the direction of the track is usually known, the lack of information is called the left-right ambiguity. To resolve this ambiguity, we propose to generate and use linear constraints between drift times. In simulated tracks, we thus assign a plus or a minus sign to the drift time, depending on whether the track is left or right from the sense wire. Knowing these constraints then allows us to resolve the ambiguity in actual observations.

The difficulty of doing pattern recognition resides in the number of degrees of freedom, M. We thus typically consider a group of M observed coordinates and check if there is an M + 1 -th coordinate consistent with the M under consideration.[1]

We have considered the case where there are only three degrees of freedom for the tracks which we want to recognize. This is inspired by the Mark II experiment.[2] In fact, most tracks have a vertex so close to the origin that we found that most simulated tracks can be resolved by assuming only two degrees of freedom. However, the price we have to pay for this is that we not only need to find the most likely left-right assignment, but also need to know its reliability. This is done by comparing the "best" assignment with the "best but one". If the difference is less than a set value, we have to reject this combination.

This is, however, a procedure which we would recommend more generally in pattern recognition; i.e., first make a fast but sloppy test which can deal, at least, with part of the cases considered. If, indeed, we have a method of judging the reliability of this analysis, we can thus economize on the number of cases to be treated with a more sophisticated but slower test.

For our numerical experiment, we generated tracks originating from a point of a distance r = RAN6(-1) from the origin. RAN6(-1) gives a random number with a rectangular distribution between zero and one. The initial direction was taken as $\varphi = 2\pi$*RAN6(-1), the track's radius as R = 750.+250./RAN6(-1), and the charge randomly to be positive or negative. This corresponds with the model proposed in reference two. We then computed the closed distance to a sense wire in three cylindrical drift chambers with radii 413.6, 620.4 and 827.2. All measurements are in millimeters.

We have a special reason for selecting these particular detectors even though on the Mark II there are many more detectors. Firstly, to reduce the number of degrees of freedom, we want to consider truly cylindrical detectors only, as opposed to those having "slanted" wires to measure also an axial component. Secondly, we choose these particular detectors because the number of evenly distributed sense wires in these are 144, 216 and 144, respectively. For these, there is consequently a periodicity of 5°. We will now show why this periodicity is so important. The magnetic field causes the drift time not to be simply proportional with the smallest distance between the track and the sense wire.[2] Moreover, this time-distance relation also depends on the angle between the track and the detector's cylinder. Suppose for a moment that we would use the drift times in a particular drift chamber without being restricted to one particular sense wire. Then, if we would plot the intersection of a track and this cylinder versus the corresponding drift time, we would see two types of "kinks" (discontinuity in the derivative). One is due to another sense wire taking over and another kink is due to left-right equivalence for short distances to the sense wire. This latter kink can be avoided by assuming that the drift time has a minus sign when the track passes on the left-hand side. In doing so, the first "kink", however, now becomes a true discontinuity. Such a function would be unfit for parameterization.[3] For this reason, we consider a different constraint for each possible wire combination. Another advantage of separating various wire combinations is that the earlier mentioned dependence on incident angle can then be implemented in the drift times. If we were to use spacial coordinates for our constraints, we would not be restricted to one particular wire combination, but to get from the observed drift times to spacial coordinates we would need to know an estimate of the incidence angle which is, however, uncertain if we have not resolved the left-right ambiguity.

By taking into account the drift times of one specific combination of wires, the angular dependence is already built-in.

As a result of the periodicity of 5°, we found that when generating 5000 random tracks only 30 essentially different wire combinations occurred in 4613 of these cases. For each of these tracks, we computed the drift time (with a sign) from the shortest distance to a sense wire, the incidence angle and the known time-distance relations.[2] The latter relation was parameterized with a double Chebyshev expansion for rapid use. We then derived, using the Principal Component Analysis technique,[3] 30 linear relations of the drift times.

We then generate a new track with, say, unknown signs of the drift times. One could then resolve the left-right ambiguity by looping over all eight possible signs of the "observed" drift times.

However, we applied some refinements. One, as mentioned earlier, is that when considering all possible signs and accepting the one which satisfies best the linear constraint, we also consider the second best. If the latter is not much worse, then we decide that we cannot resolve the ambiguity for sure and reject this track. The second refinement is the application of the trick[1] mentioned earlier: "if there are M degrees of freedom, one wants to check if there is an M + 1-th coordinate consistent with the M under consideration". In our present case M=2, so if we have provisionally accepted two drift times with certain signs, we know what the third drift time would need to be to satisfy the constraint. So we compare the absolute value of the latter with the drift time observed. This way, we only have four (rather than eight) possibilities to consider. A minor disadvantage of this is that the rejection test is weaker since the second best out of four possibilities is likely to be worse than the second best out of eight possibilities. In particular, it is possible that although the signs of the first two coordinates can be unambiguously determined, the third drift time happens to be so small that its sign is not determinable nor relevant. We have, therefore, implemented a third refinement in the testing program, i.e., to recognize tracks whose left-right ambiguity resolution are "almost right". Meaning, wrong but irrelevant. Supposedly, once we have acquired more information on this track (from other detectors) we can resolve this unimportant ambiguity for the precise quantitative analysis.

As mentioned before, we derived 30 constraint equations using about 5000 randomly generated reference tracks. The constraint equations[3] are of the type

$$W_{1j}(t_1-\bar{t}_{ij}) + W_{2j}(t_2-\bar{t}_{2j}) + W_{3j}(t_3-\bar{t}_{3j}) \approx 0 \qquad j = 1,\ldots 30 \qquad (1)$$

$$\text{with } W_{1j}^2 + W_{2j}^2 + W_{3j}^2 = 1$$

where $t_i$ are the observed drift times with the appropriate sign and $\bar{t}_{ij}$ is the mean drift time (again with appropriate sign) of the i-th detector and the j-th wire combination. $W_{ij}$ are the three components of the eigenvector corresponding to the smallest eigenvalue of the dispersion matrices of $t_{ij}$.[3]

For testing, we then generated another 10 000 random tracks. Of these, 784 were of the type (i.e., wire combination) for which we had insufficient (less than 50) reference tracks. Of 4893, the left-right ambiguity was correctly found (in four tests). Of the remaining, there were 4219 rejected because two left-right ambiguities were too close together. Of these latter, 3435 would have been classified rightly had they not been rejected and of the other 784, which had they not been rejected would have been wrong, there were 257 "almost right". All of the remaining 104 that were accepted, but wrong, were "almost right". No truly wrong assignment was made. The criterion for "almost right" was that the track passed to within .8mm of the sense wire.

The above results depend, of course, on the thresholds set. It is possible to reject fewer but have an occasional bad one slip through.

If we would include errors of measurement, we would probably need to take observations from four detectors before a reliable test would be possible. However, what we wanted to illustrate was that one could make a very rapid test and have the number of possible "patterns" considerably reduced. For example, if 50% is classified correctly then a factor 2 is economized on a much more complicated test. The crucial criterion in the fast test is not only to find the right pattern but in how far this is unique.

REFERENCES

1 H. Wind, CERN/DHR 79-1, 1211 Geneve 23, Switzerland

2 R.H. Schindler, Thesis, December 1979, Stanford University, Stanford, California

3 H. Wind. Function Parameterization in CERN 72-21, Appendix 3 (for Principal Component Analysis) and page 60-62, 77-79, for convergence.