Why Supersymmetry is Super

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Outline

• Introduce Quantum Field Theory
• Incorporating Supersymmetry in a Quantum Field Theory
• The MSSM
• Motivations for Supersymmetry in the “Real World”
Definition of a Quantum Field Theory
Quantum Mechanics and Relativity

\[ i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \]

- Not Lorentz invariant
  - Unequal number of time and space derivatives
  - Explicit mass in denominator
  - Potential usually depends explicitly on position
Quantum Mechanics and Relativity

• Different approach:
  Make a relativistic theory quantum mechanical!
• A Classical Field Theory is Lorentz invariant
• How to make it quantum mechanical:
  • Promote classical fields to operators!

\[
[p_i, x_j] = i\hbar\delta_{ij}
\]

\[
[\pi(\vec{x}, t), \phi(\vec{x}', t)] = i\hbar\delta(\vec{x} - \vec{x}')
\]
Quantum Mechanics and Relativity

• Moral:

Quantum Mechanics + Lorentz Invariance =

Quantum Field Theory
Quantum Field Theory

- Spin defines transformations under rotations and boosts
- Quantum Field Theory naturally explains spin
  - Not added ad hoc as in non-relativistic qm

\[ \begin{array}{ccc}
0 & \pi & 2\pi \\
Scalar & Scalar & Scalar \\
Fermion & Fermion & Fermion \\
\overrightarrow{Vector} & \overleftarrow{Vector} & \overrightarrow{Vector}
\end{array} \]
Example: Scalar (spin 0) theory

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 \]

- Kinetic energy
- Shear energy
- Mass energy
- Potential Energy

Euler-Lagrange Equation: \((\partial^2 + m^2)\phi = 0\)

Quantum Mechanical identification: \(\partial_\mu = i p_\mu\)

Einstein’s relation:
\[-p^2 + m^2 = -E^2 + |\vec{p}|^2 + m^2 = 0\]
Example: Fermion (spin 1/2) theory

\[ \mathcal{L} = i \bar{\psi} \sigma^\mu \partial_\mu \psi - m \bar{\psi} \psi \]

- Aligns spins
- Displaces fermions
- Mass
- Spins anti-aligned

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \]

“\( \psi^2 = \phi \)”
Supersymmetry
Familiar Quantum Operators

• Example: Momentum

\[ P_\mu |\text{boson}\rangle = p_\mu |\text{boson}\rangle \]

\[ P_\mu |\text{fermion}\rangle = p_\mu |\text{fermion}\rangle \]

\[ P_\mu \xrightarrow{2\pi \text{ rotation}} P_\mu \]

• Momentum is a vector (a boson)
• Momentum is Hermitian
• Momentum satisfies commutation relations
Unfamiliar Quantum Operators

\[ Q|\text{fermion}\rangle = |\text{boson}\rangle \]

\[ Q|\text{boson}\rangle = |\text{fermion}\rangle \]

\[ Q \quad \xrightarrow{2\pi\text{ rotation}} \quad 0 \]

- \( Q \) is a fermion! (spin 1/2)
- \( Q \) is not Hermitian!
  - Eigenstates of \( Q \) do not have well-defined spin
Unfamiliar Quantum Operators

• $Q$ satisfies anti-commutation relations:

$$\{Q, Q^\dagger\} = 4E \quad \left( E = P_0 = -i \frac{\partial}{\partial t} \right)$$

$$\{Q, Q\} = 0$$

$$\{Q^\dagger, Q^\dagger\} = 0$$

$$Q^\dagger = \mathcal{O} \neq Q$$

• Look familiar?
Unfamiliar Quantum Operators

• Rescale $Q$:

$$a = \frac{Q}{\sqrt{4E}}$$

$$a^\dagger = \frac{Q^\dagger}{\sqrt{4E}}$$

$$\{a, a^\dagger\} = 1$$

$$\{a, a\} = 0$$

$$\{a^\dagger, a^\dagger\} = 0$$

• Spin 1/2 or two-state raising and lowering operators!
Summary

• $Q$ satisfies the algebra:

\[
\{Q, Q^\dagger\} = 4E
\]

\[
\{Q, Q\} = 0
\]

\[
\{Q^\dagger, Q^\dagger\} = 0
\]

• $Q$ interpolates between two states:

\[
Q^\dagger |f\rangle = |b\rangle
\]

\[
Q |b\rangle = |f\rangle
\]

• $Q$ called the supercharge
• $f$ and $b$ are superpartners and form a supermultiplet
Simplest Supersymmetric Model

- Massless, noninteracting Wess-Zumino model:
  \[
  \mathcal{L}_{\text{W-Z}} = \partial_{\mu} \phi \partial^{\mu} \phi^{*} + i\bar{\psi}\sigma^{\mu} \partial_{\mu} \psi
  \]

- \(\phi\) is a complex scalar field:
  \[
  \phi = \phi_{1} + i\phi_{2}
  \]

- \(\psi\) is a spin 1/2 fermion field:
  \[
  \psi = \begin{pmatrix} a \\ b \end{pmatrix}
  \]
Wess-Zumino Model

- Transformations of fields under supersymmetry:
  \[ Q\phi = \psi \]
  \[ Q^\dagger \psi = i\sigma^\mu \partial_\mu \phi \]

- Leave the Wess-Zumino Lagrangian invariant:
  \[ L_{W-Z} = \partial_\mu \phi \partial^\mu \phi^* + i\bar{\psi}\sigma^\mu \partial_\mu \psi \]

- Wess-Zumino Lagrangian is *supersymmetric*!
What does Supersymmetry mean?

• Consider two electrons; one at rest, one with velocity $v$:
  
  $e \quad e \quad v$

• Lorentz transformations can change velocities
  • Doesn’t change laws of physics
  • Consequence: velocity is not a quantum number

• Consider a fermion and boson:
  
  $f \quad b$

• Supersymmetry doesn’t change laws of physics
  • Does change spin!
  • Consequence: spin is not a quantum number!
What does Supersymmetry mean? Summary

• Maps boson degrees of freedom to fermion degrees of freedom:

\[ \begin{align*}
\text{Bosons} & \leftrightarrow \text{Fermions} \\
Q, Q^\dagger & \\
\end{align*} \]

• Theory must have equal numbers of fermion and boson d.o.f.s
• Boson and fermion superpartners must have same interactions
  • Same mass, same charges
• Moral:

Supersymmetry:

Bosons \sim \text{Fermions}
The Minimal Supersymmetric Standard Model
Minimal Supersymmetric Standard Model

• “Minimal Supersymmetric”: The absolute minimum number of additional particles to make the Standard Model supersymmetric

• Can particles in the Standard Model be superpartners?
Minimal Supersymmetric Standard Model

• “Minimal Supersymmetric”: The absolute minimum number of additional particles to make the Standard Model supersymmetric

• Can particles in the Standard Model be superpartners?
  • No! No particles have same charges or mass and spin differing by 1/2
  • Need to double particle content!
Minimal Supersymmetric Standard Model

- Superpartners in the MSSM:
  - Scalar partner to a fermion = sfermion
    - Partner to electron = selectron
    - Partner to top quark = stop
  - Fermion partner to a boson = bosino
    - Partner to photon = photino
    - Partner to gluon = gluino

- I make no apology for the nomenclature; it is terribly silly
Minimal Supersymmetric Standard Model

• A (major) problem
  • Supersymmetry demands superpartners have same mass
    • We don’t observe a negatively charged, scalar with mass of 511 keV (the selectron)
  • Supersymmetry cannot be an exact symmetry of our world!
• Sparticle masses must be > 100 GeV to escape detection
• Many models predict masses in LHC range
  • Very exciting! (pending LHC problems)
Motivations for Supersymmetry in the Real World
Unification of Coupling Constants
Unification of Electricity and Magnetism

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} + \frac{\partial}{\partial t} \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{B} - \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \mathbf{E} = \mu_0 \mathbf{J} \]
Unification of Electricity and Magnetism

• Maxwell’s equations unify $E$ and $B$ fields
• Relates electric and magnetic couplings:

$$\mu_0 \varepsilon_0 = \frac{1}{c^2}$$

• Conversely, if couplings can be related, fields can be unified!
  • (At least a very strong indication of unification)
Unification in the Standard Model

• Three forces in Standard Model
  • Electromagnetism
  • Weak
  • Strong
• Three couplings in Standard Model relating strengths of forces
• If these couplings can be related, forces could be unified!
  • Could “explain” forces and their strengths
Unification in the Standard Model

• How to relate them?
  • Couplings depend on distance (energy)!
  • An electron polarizes the region of space around it
  • Dipoles screen bare electron charge
  • Closer to electron, charge looks larger!
Unification in the Standard Model

- Standard Model: dashed
  - No unification
- Standard Model + Supersymmetry: solid
  - Unification!
Hierarchy Problem
A Problem with Fundamental Scalars

• Or, why a massless fermion stays massless

\[ \frac{v = c}{\text{spin}} \]

Lorentz Transformation

\[ \frac{v = c}{\text{spin}} \]

• Recall massless fermion Lagrangian:

\[ \mathcal{L} = i \bar{\psi} \sigma^\mu \partial_\mu \psi \]
A Problem with Fundamental Scalars

- Mass term requires anti-aligned spins:

\[ \mathcal{L}_{\text{mass}} = -m \bar{\psi} \psi \]

- Quantum Mechanics respects Lorentz invariance
  - Velocity and spin are locked in place
  - Such a term can never be generated!
  - Massless fermion stays massless!
A Problem with Fundamental Scalars

- Consider a massless, fundamental scalar:

  \[ s \rightarrow \nu = c \quad | \quad \text{Lorentz Transformation} \quad | \quad s \rightarrow \nu = c \]

- Scalar has no intrinsic direction or vector
- Mass term of Lagrangian is not disallowed by any symmetries:

  \[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \]

- Anything that is not forbidden will happen in QM
  - Mass term is generated quantum mechanically!
- (Composite scalars do not suffer this problem as they are made of fermions and inherit chiral symmetry)
Standard Model Higgs Scalar Boson

• In the Standard Model, Higgs boson is responsible for mass
  • All particles have mass proportional to Higgs mass
• How does quantum mechanics affect Higgs mass?
  • Interactions with bosons: increases mass
  • Interactions with fermions: decreases mass
• In Standard Model, these contributions are unrelated
  • Quantum mechanic shifts can be (essentially) unbounded!
  • Obviously a bad thing
• Requiring supersymmetry:
  • Fermions and bosons have same interactions with Higgs
  • Contributions to Higgs mass exactly cancel!
  • A good thing
Standard Model Higgs mass

• Resolves the hierarchy problem:
  Supersymmetry resolves the hierarchy problem by demanding that fermion and boson contributions to the higgs mass exactly cancel.
Things I Didn’t Discuss

• Other solutions to the Hierarchy problem:
  • Extra Dimensions
    • Randall-Sundrum Models, etc.
  • Composite Higgs models
    • Technicolor, etc.
• None, there is no Hierarchy problem
  • Split Supersymmetry, etc.
Things I Didn’t Discuss

• Other connections of Supersymmetry
  • Adding more supersymmetries
    • Have only considered $N = 1$
    • Can add more supercharges to consider larger supermultiplets
  • Conformal theories; AdS/CFT
• Gravity + Supersymmetry = Supergravity
  • Could be a way to control the divergences that trouble a quantum theory of gravity
• String Theory
  • Requires supersymmetry
  • Currently the best quantum theory of gravity
An Explanation of Dark Matter
An Explanation of Dark Matter

• I won’t motivate the existence of dark matter here
• Necessary to explain the rotation curves of galaxies
• Necessary to explain the distribution of mass and the gravitational potential well in the Bullet Cluster

• There are many reasons that Standard Model particles could not be dark matter
  
  • Dark matter is dark, must be neutral and stable
  
  • Could possibly be neutrinos but are much too light to remain contained in galaxies
An Explanation of Dark Matter

• Supersymmetry “naturally” includes heavy, neutral Weakly interacting particles!
  • Called WIMPs
  • Constraints on cosmological parameters demand its mass be in the 100 GeV-1 TeV range