Questioning Quantum Mechanics?

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SASS Talk
January 25th, 2012
Quantum Mechanics is presently the framework for physical law.

What if this isn’t the case?
What if:

Fundamental Theory → Some Limit → (almost) Effective Field Theory

NOT Quantum Mechanics → (almost) Quantum Mechanics

What deviations from quantum mechanics are possible?

What are their observable consequences?
What Is Quantum Mechanics?

- A state $|\psi\rangle$ is a ray in a Hilbert space
- Probabilities: $|\langle \varphi | \psi \rangle|^2$
- Measured quantities are eigenvalues of Hermitian operators
- Time evolution described by Schrödinger equation:

$$i \frac{d}{dt} |\psi\rangle = H |\psi\rangle$$

Linear
Possible generalization: linearity

- Nonlinearity is common in everyday phenomena; why not in quantum systems as well?

- To what extent can we test for non-linearity in a model-independent manner?

- Can we preserve logical consistency in a non-linear quantum theory?
Outline

- Quantum Mechanics from non-linear dynamics?
  - Concrete example: Weinberg’s framework.

- Observable consequences of a fundamentally new character?
  - Yes; state-dependent phases, nonlocality. parallel universe communication

- Can a nonlinear theory be logically consistent?
  - Remove nonlocality? (Polchinski, Kibble)
  - Fundamental challenges

A Nonlinear Extension: Weinberg’s Framework
Making QM Nonlinear

\[ i \frac{d\psi_k}{dt} = H_{kl} \psi_l \]
Making QM Nonlinear

\[ i \frac{d\psi_k}{dt} = \frac{\partial}{\partial \psi_k^*} (\psi_j^* H_{j\ell} \psi_\ell) \]

real, bilinear function
Making QM Nonlinear

\[ i \frac{d\psi_k}{dt} = \frac{\partial}{\partial \psi_k^*} \left( h(\psi_j^*, \psi_l) \right) \]

real, NON-bilinear function
One Further Requirement: Homogeneity

- Require that any nonlinear observable $a$ satisfy:

$$a(Z^*\psi^*, Z\psi) = Z^*Za(\psi^*, \psi)$$

- Why?
  - It makes it easy to keep wavefunction’s normalization arbitrary

Achievements

- Constant in time:
  - Wavefunction’s normalization, Hamiltonian

- Can find many nonlinear Hamiltonians that respect Galilean invariance
  - Requires taking the standard linear generators for momentum, angular momentum, etc.

- Solutions with harmonic time dependence (stationary states)
What are the observable consequences of Weinberg nonlinearities?
N-atom coherent system, neglecting all but two closely split states

\[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\xrightarrow{\text{Rabi flop}}
\begin{bmatrix}
\sqrt{1-a} \\
\sqrt{a}
\end{bmatrix}
\xrightarrow{\text{Free evolution}}
\begin{bmatrix}
\sqrt{1-a} \\
\sqrt{ae^{i\delta}}
\end{bmatrix}
\]

If the free evolution is non-linear a la Weinberg, $\delta$ depends on $a$, a feature that is qualitatively distinct from any linear quantum mechanical evolution.
Experiment by Bollinger et al*

- Used a hyperfine nuclear transition of $^9\text{Be}^+$

- Model non-linearity: $2\varepsilon a^2$

- Set a limit:

  $$\varepsilon \leq 8 \pm 16 \cdot 10^{-21} \text{eV}$$

Nonlinearity and Nonlocality

\[ \psi \otimes \phi = \alpha_{II}(\psi \otimes \phi) = ? ? ? \]
Non-locality

- How can we combine separated systems?

- Weinberg proposes a form that permits separable solutions

- However, this form is basis-dependent

- Polchinski has proven that it permits instantaneous communication
Polchinski and Kibble

Can we get logical consistency?
Remove nonlocality: Polchinski’s proposal*

- Basis-independent solution: reduced density matrices

- Eliminates nonlocality, but causes communication between wavefunction branches
  - Binary messages of arbitrary length can be exchanged between the same observer in parallel worlds(!)

- In practice, wavefunction-branch coupling might be so complex that effects would be unobservable

Wavefunction Branch Communication

Observer + two-state system

Observe spin

Nonlinear time evolution

Observe spin again--outcome depends on choice of A or B!

Choose action A or B

“Many Worlds” interpretation seems natural—But is this consistent?

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Kibble’s Nonlinear QFT*

- Start in Schrödinger picture, introduce nonlinearity by allowing the Hamiltonian to depend on the wavefunction:

\[ i \frac{d}{dt} \psi = H_\psi \psi \]

- Preserve locality: write Hamiltonian as an integral over a density:

\[ H_\psi = \int \left[ \frac{1}{2} \pi^2 + \frac{1}{2} \left( \nabla \phi \right)^2 + \frac{m}{2} \phi^2 + \frac{g}{4!} \phi^4 + \alpha + \beta \langle \phi^2 \rangle_\psi + \frac{\lambda}{2} \langle \phi^2 \rangle_\psi \phi^2 \right] d^3 x \]

Physical Consequences \((g=0)\)

\[
\omega^2 = \left| \vec{k} \right|^2 + m^2 + \lambda \langle \phi^2 (\vec{x}) \rangle \psi
\]

- Effects will be strongest for localized wavefunctions
- Wavepackets will show anomalous dispersion
- Scattering can occur even in the absence of interactions
  - No “free” theory, really
Generic Difficulties for NLQM

- Perturbation theory difficult
  - Can’t get to a Heisenberg or interaction picture easily

- Consistent probabilistic interpretation challenging
  - Scattering not a well-posed question

- Basic logical consistency is uncertain
Conclusions
Questioning Quantum Mechanics?

- Can Quantum Mechanics from non-linear dynamics?
  - Concrete example: Weinberg’s framework.

- Are there observable consequences of a fundamentally new character?
  - Yes; state-dependent phases, nonlocality, parallel universe communication

- Can a nonlinear theory be logically consistent?
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The End! Thanks for listening!

Schrodinger's cat survived the experiment

Another consequence of non-linear quantum mechanics?

but his new abilities were entirely unexpected
Details of New Phase

\[ n \equiv |\psi_1|^2 + |\psi_2|^2 \quad \quad a \equiv \frac{|\psi_2|^2}{n} \quad \text{(definitions)} \]

Model Hamiltonian function:

\[ h(\psi^*, \psi) = n[(1 - a)E_1 + aE_2 + 2\varepsilon a^2] \]

Phase difference between the two components after a time \( t \):

\[ \delta = t[E_1 - E_2 - 4\varepsilon a] \]
Nonlinearity and Nonlocality

\[ \psi_k \quad \phi_l \]

\[ a_{II} = |\phi_1|^4 \]

\[ a_{II} = \sum_{k=1}^{2} |\Psi_{k1}|^4 \]

\[ |\Psi\rangle = |\psi_1\rangle \otimes |\phi_1\rangle \]

\[ |\Psi\rangle = \left( \sqrt{1 - \varepsilon^2} |\psi_1\rangle + \varepsilon |\psi_2\rangle \right) \otimes |\phi_1\rangle \]

\[ a_{II} = 1 \]

\[ a_{II} = 1 - 2\varepsilon^2 + 2\varepsilon^4 \]