Dalitz Plots

Brian Lindquist
SLAC

SASS Talk

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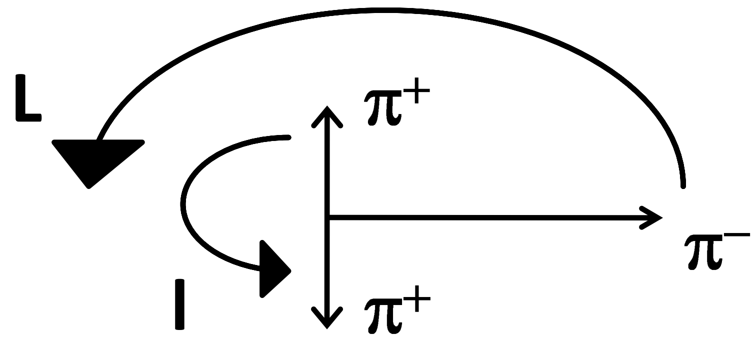
The Discoveries of the θ^+ and τ^+

- $\theta^+ \rightarrow \pi^+\pi^0$ discovered 1953 (Menon and O'Ceallaigh)
- Alternatively called the χ particle.
- m ~ 500 MeV/c^2
- Assuming parity conservation:
- $P(\theta) = P(\pi^+)P(\pi^0)(-1)^L = (-1)(-1)(-1)^L = (-1)^L$
- So, $J^P = 0^+, 1^-, 2^+, 3^-$, etc.
- In 1949, $\tau^+ \rightarrow \pi^+\pi^-\pi^+$ discovered using emulsions exposed to cosmic rays (Brown et. al.)
- Mass consistent with that of θ^+

J^P of τ^+

- What values are allowed for J^P for τ^+?
- Again, assume P conservation.
- $P(\tau^+) = P(\pi^+)P(\pi^-)P(\pi^+)\ (-1)^l(-1)^L\ = (-1)^l(-1)^{L+1}$



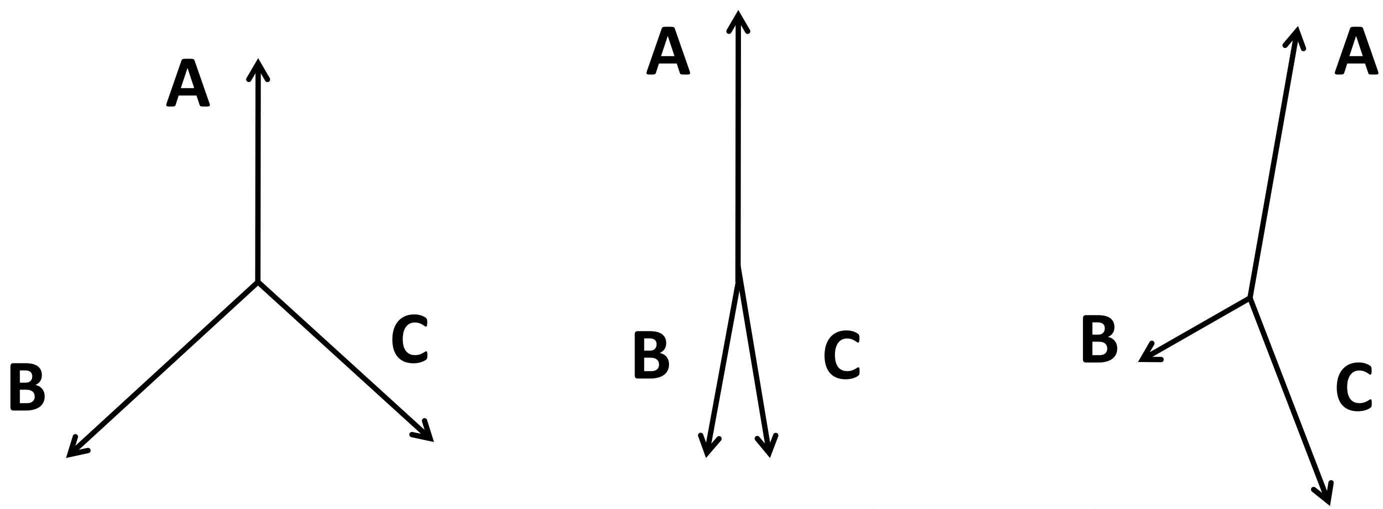
- By Bose symmetry, wavefunction symmetric under exchange of π^+'s, so l=even.
- $P(\tau^+) = -(-1)^L$

J^P of τ^+ (cont.)

- $\vec{J} = \vec{L} + \vec{l}$
- If J=0, then L=l=even, so only $J^P = 0^-$ allowed, not 0^+
- For J≠0, any value of J^P is possible – infinite number of L,l combinations allowed
- e.g.:
  - 1^-: (L=2,l=2), (L=4,l=4), (L=6,l=6), etc.
  - 1^+: (L=1,l=0), (L=1,l=2), (L=3,l=2), (L=3,l=4), etc.
  - 2^-: (L=0,l=2), (L=2,l=0), (L=2,l=2), (L=2,l=4), etc.
- To figure out J^P, we need to know what L,l contribute to decay!

Three-body decays

- For 2-body decays, M→AB, p_A and p_B are completely determined by E,p conservation.
- 3-body decays (M→ABC) have additional degrees of freedom.
- Different values of p_A, p_B, and p_C are possible, depending on decay configuration



A way to get more info about J^P!

- You might suspect that the configuration of a 3-body decay will depend upon the angular momentum between the 3 daughter particles – and you'd be right!
- Dalitz's idea:
  - Look at the frequency with which the different $\pi^+\pi^-\pi^+$ configurations occur
  - Use this information to figure out what values of L,l are present
  - Use that to infer J^P

How many degrees of freedom?

- For 3-body decay, $M \rightarrow ABC$ (where A,B, and C are spinless), we can specify final state with 3 four-vectors: $p^{\mu}{}_{A}$, $p^{\mu}{}_{B}$, $p^{\mu}{}_{C}$
- 12 parameters, but not all are independent/relevant
  - A,B,C all decay in the same plane, so can set $p_{i,z} = 0$. This removes 3 degrees of freedom (d.o.f.)
  - Remove 3 d.o.f. by $E_i = \sqrt{m_i^2 + p_i^2} \quad , i = A,B,C$
  - Remove 3 d.o.f. by $\vec{p}_M = \vec{p}_A + \vec{p}_B + \vec{p}_C$ and $E_M = E_A + E_B + E_C$
  - Can freely rotate entire system in x-y plane without effect. Removes 1 d.o.f.
- Only 2 degrees of freedom!

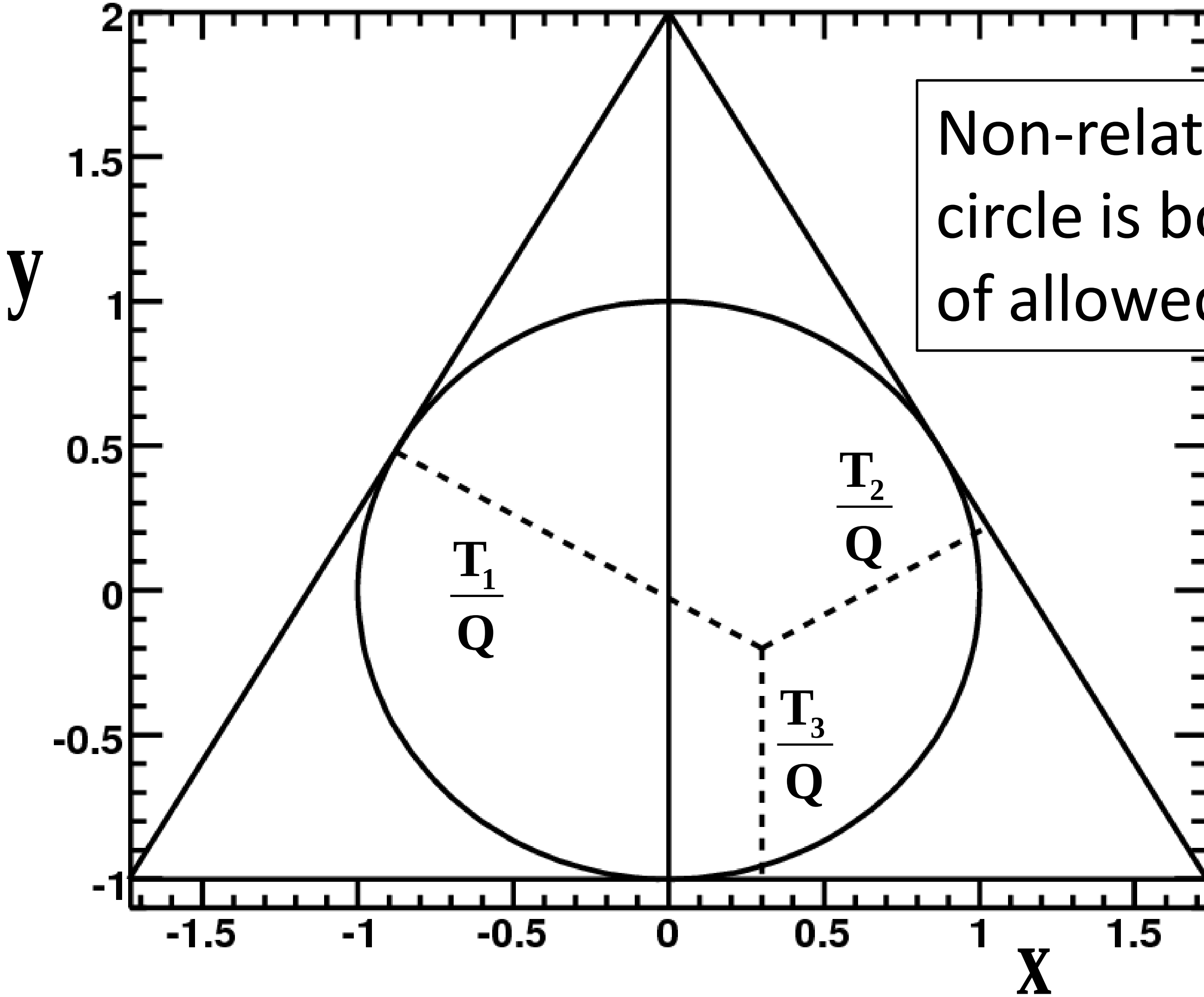
The Dalitz Plot

- So, we can describe the 3-body state with two variables (there are many choices of what variables to use).
- We can make a 2-D scatter plot, with one variable on the x-axis, and one on the y-axis.
- Dalitz chose these variables:

$$x = \frac{\sqrt{3}(T_1 - T_2)}{Q} \qquad y = \frac{(2T_3 - T_1 - T_2)}{Q}$$

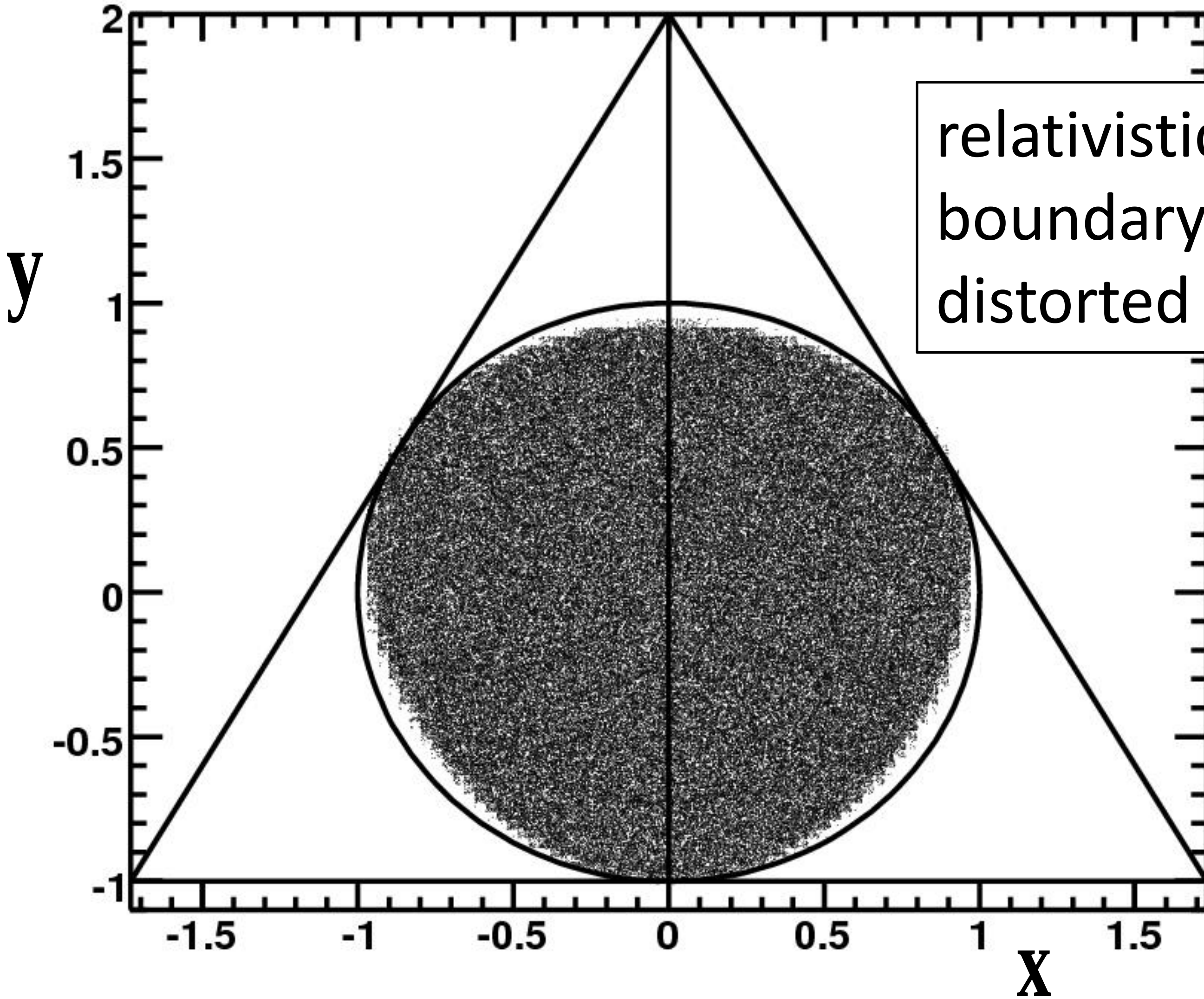
- T_1, T_2, T_3 are the kinetic energies of the pions
- Q is the energy released in the decay ($m_\tau - 3m_\pi$)

Dalitz Plot Geometry



Non-relativistically, circle is boundary of allowed events

Dalitz Plot Geometry

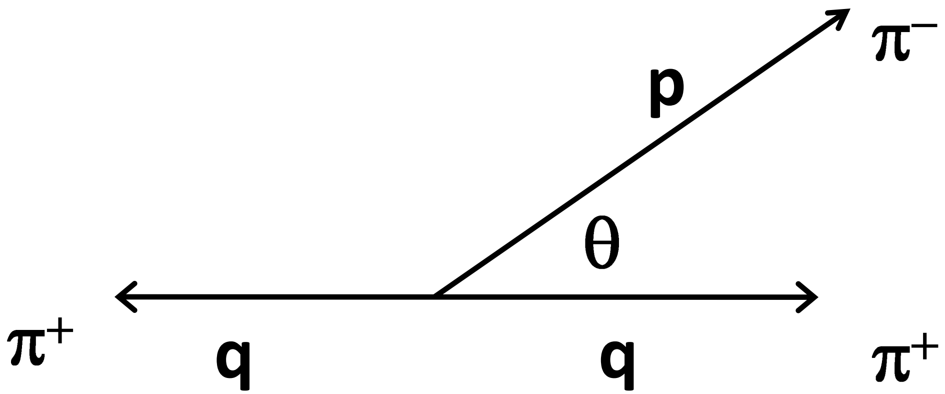


relativistically, boundary is slightly distorted

Calculating Density of Events

$$\rho(x,y) = \frac{1}{2J+1}\sum_{m_J}\left|A(m_J)\right|^2$$

$$A(m_J) = \sum_{L,l} f_{Ll}(p,q) C^{LlJ}_{0 m_J m_J} Y_l^{m_J}(\theta,\phi)$$



- This is still pretty complicated!

Approximation time!

- The τ has a small effective radius r, so larger angular momenta are suppressed (L = r x p)
- Assume $f_{Ll}(p,q)$ is slowly varying except for effect from centrifugal barrier:

$$f_{Ll}(p,q) \propto (pr)^L (qr)^l$$

- Since amplitudes fall off with angular momentum, only include the term with lowest (L+l).

Predicted Distributions

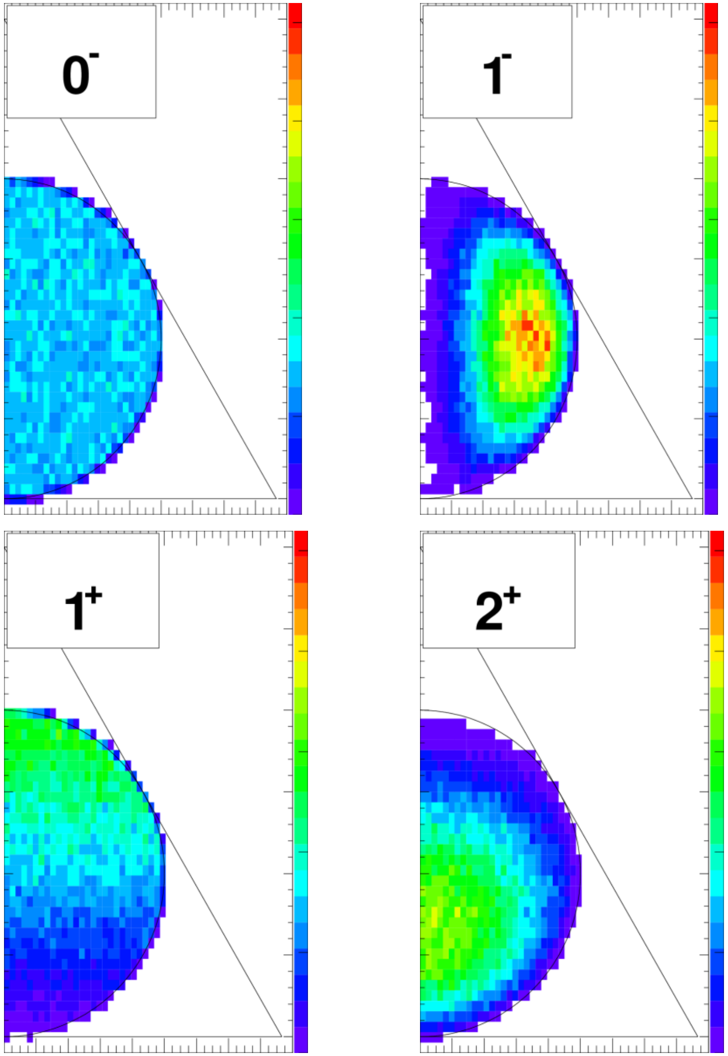
- P(x,y):

$0^-: \quad 1$

$1^+: \quad p^2$

$1^-: \quad p^4 q^4 \sin^2\theta \cos^2\theta$

$2^+: \quad p^2 q^4 \sin^2\theta$



Observed Distribution

- Early data inconclusive, but suggested even J, odd P.
- Increased data pointed strongly at even J, odd P, consistent with 0^-.
- But θ^+ can't be even J, odd P – why do two particles with seemingly identical masses have different J^P?
- The "tau-theta puzzle"

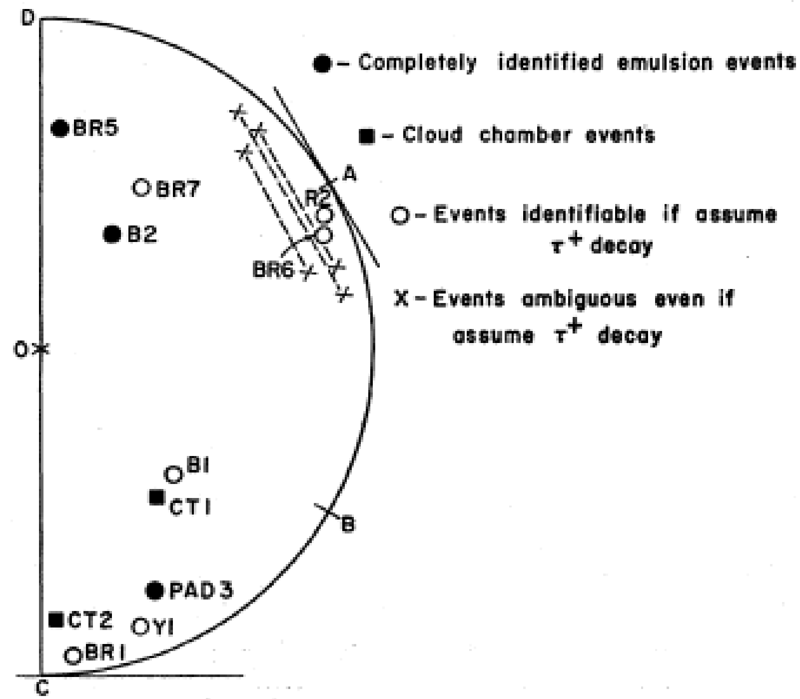


FIG. 3. The data on τ-meson decay events in which the signs of π-meson charges are established.

Dalitz, Phys. Rev. **94**, 1046-1051 (1954).

Solution to tau-theta puzzle

- This analysis assumed that P is conserved in τ and θ decays.
- In 1957, Wu et al reported P violation in Cobalt-60 beta decay.
- Tau-theta puzzle can now be solved!
- τ^+ and θ^+ are actually the same particle – now known as the K^+, which has $J^P=0^-$.

Intermission

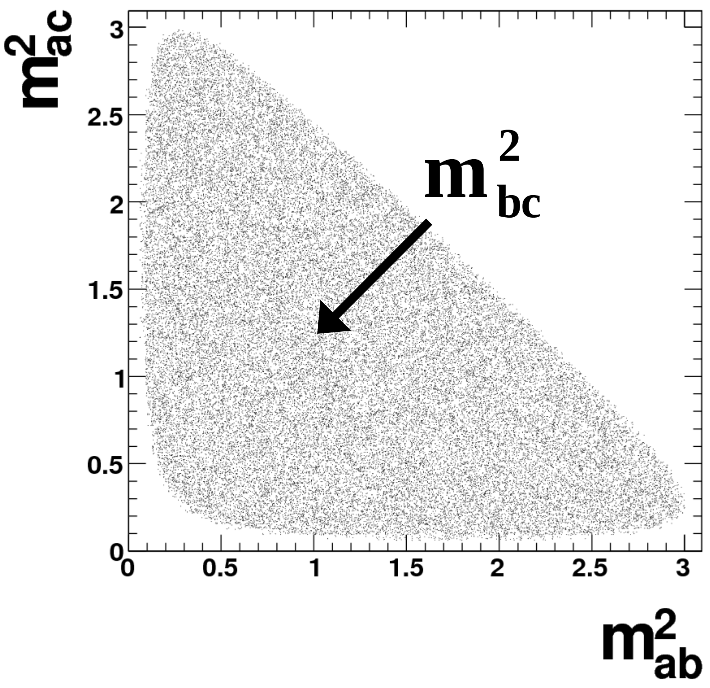
Dalitz plots in Heavy Meson Decays

- Dalitz plot analyses of decays of heavy mesons (D, B mesons) very popular in recent years.
- Relativistic – usually don't use the original Dalitz plot variables.
- Typically, for M$\rightarrow$abc, use

$$x = m_{ab}^2 \qquad y = m_{ac}^2$$

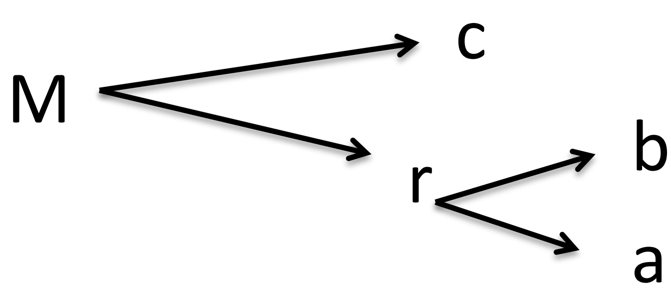
$$m_{ab}^2 = (p_a^\mu + p_b^\mu)^2$$

- "invariant mass" squared



Resonances

- Frequently, M will decay through intermediate particle, or "resonance," r.



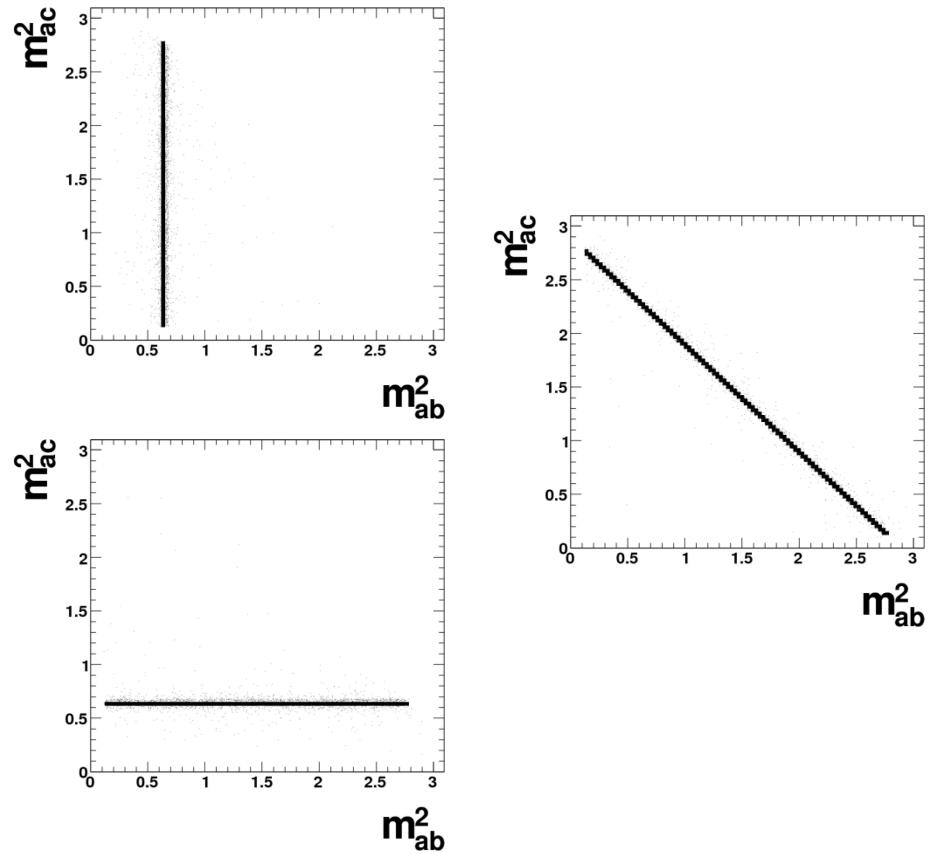
- r typically very short lived – can't observe directly
- But r can be studied by looking at Dalitz plot!

Resonances on Dalitz plot

- E and p conservation imply that if r→ab, then:

$$m_{ab}^2 = m_r^2$$

- Resonances show up as bands on Dalitz plot.



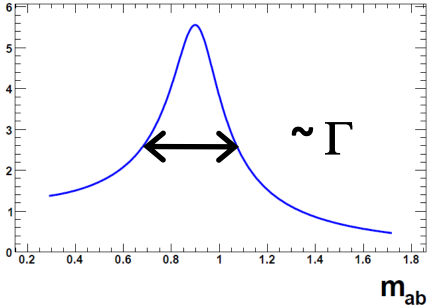
Resonance lifetimes

- Recall $(\Delta E)(\Delta t) \sim \hbar$
- Short-lived resonances have broad peak
- "Relativistic Breit-Wigner" amplitude

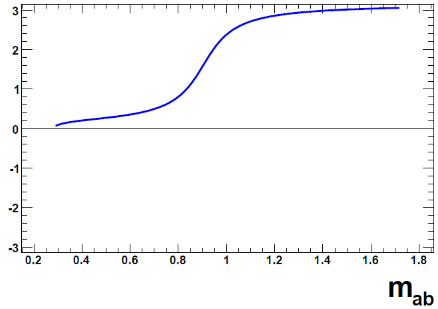
$$A_{RBW} \sim \frac{1}{m_r^2 - m_{ab}^2 - i m_r \Gamma} \qquad \Gamma = \frac{\hbar}{\tau}$$

- Width Γ inversely proportional to lifetime
- Plot of magnitude and phase of $A = |A| e^{i\phi}$

Magnitude



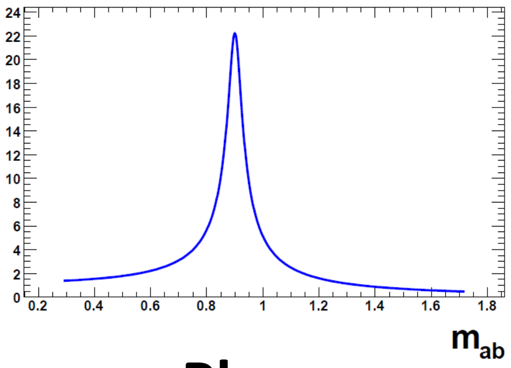
Phase



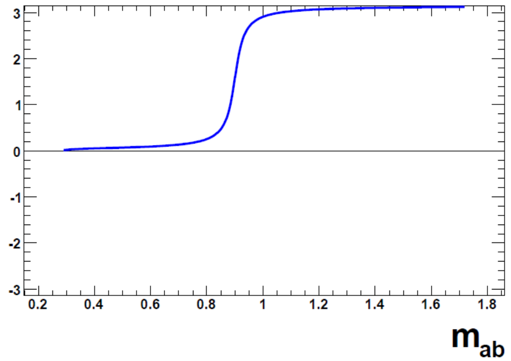
Resonance lifetimes (2)

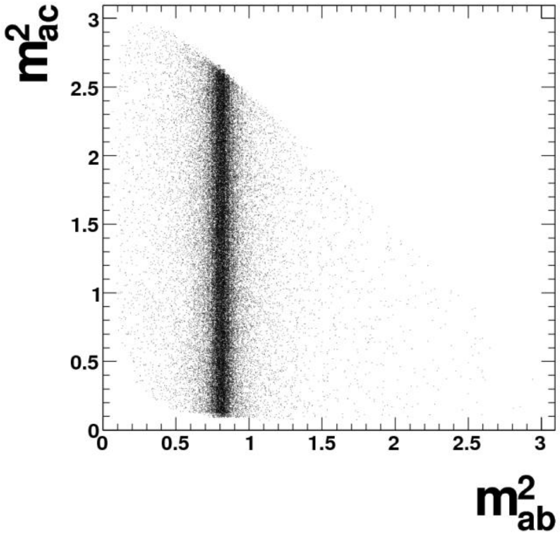
$$\rho(x, y) \sim |A|^2$$

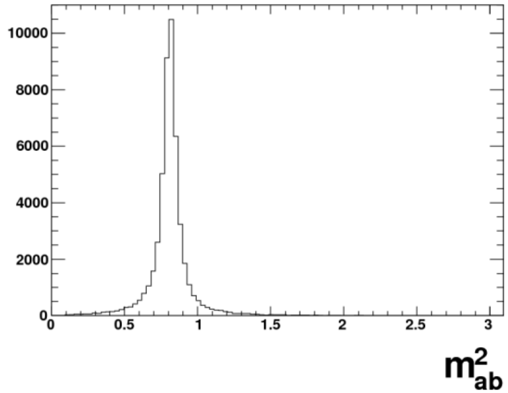
Magnitude



Phase







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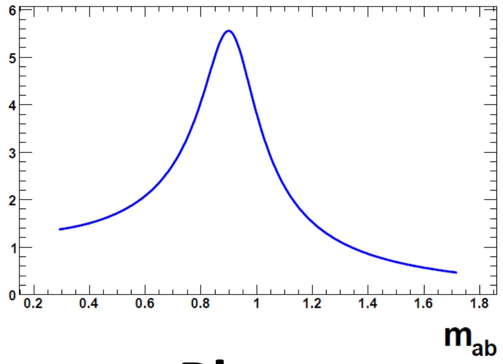
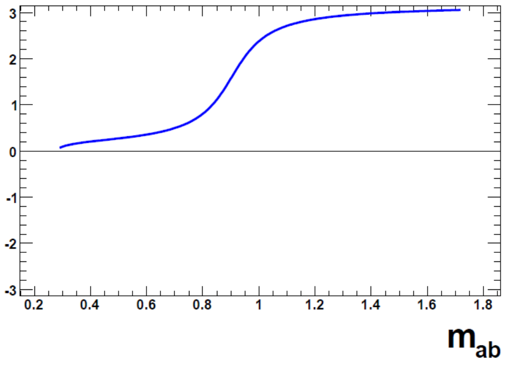
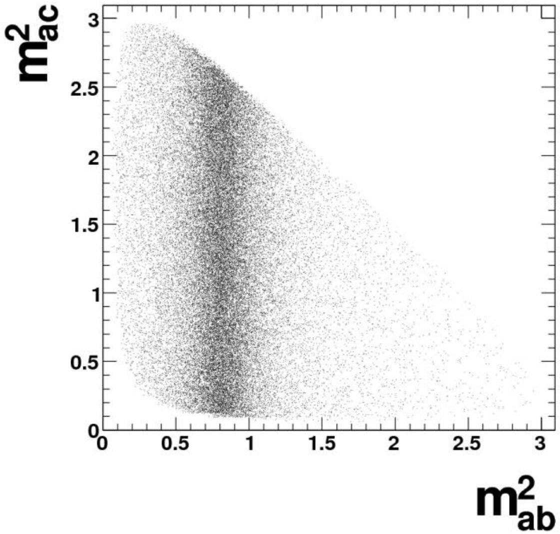
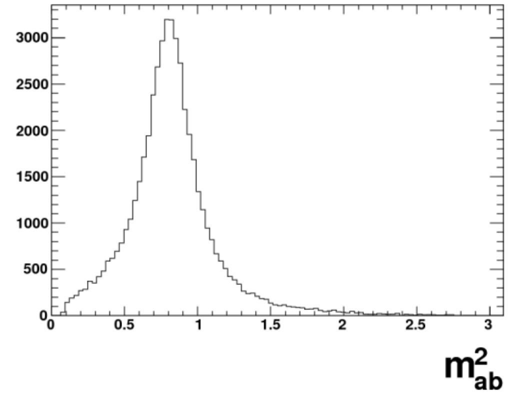


Phase





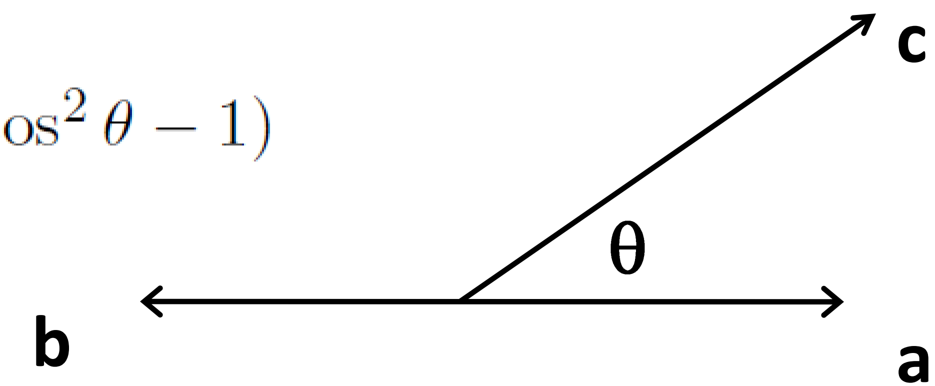


Resonance spins

- If the resonance has spin S, and M, a, b, and c are spin-0, then decay amplitude is proportional to Legendre polynomial:

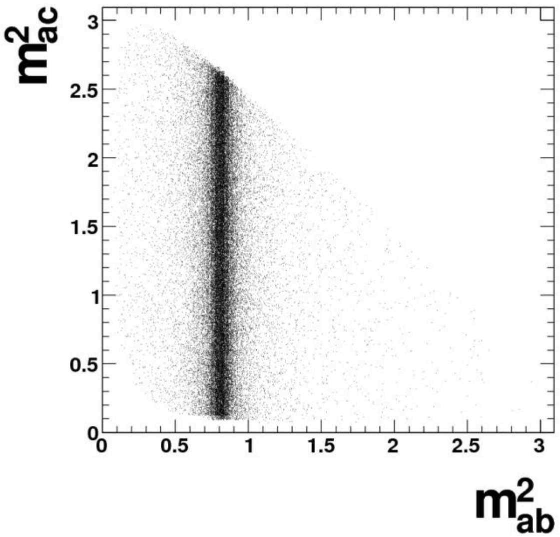
$$A \propto A_{RBW}(m_{ab}) P_S(\cos\theta)$$

$$P_0(\cos\theta) = 1$$
$$P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

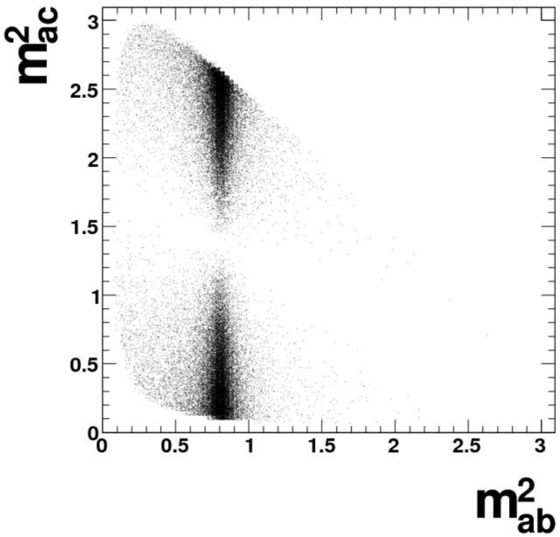


Spin on Dalitz Plot

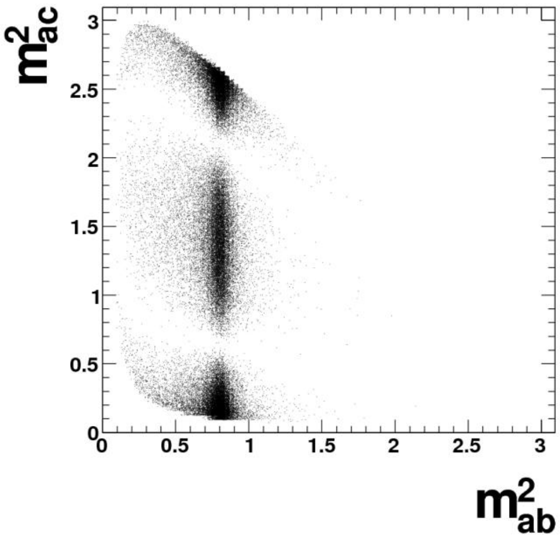
Spin-0



Spin-1

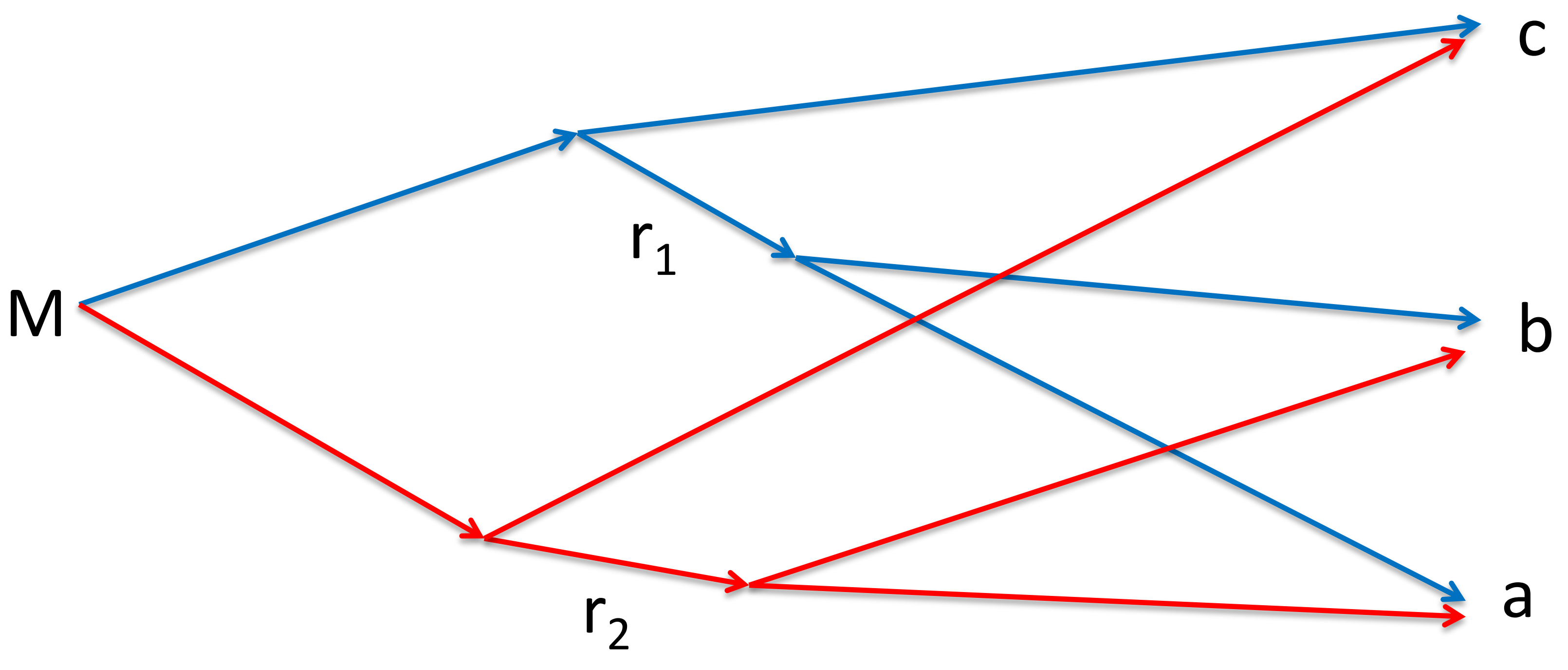


Spin-2



Multiple Resonances

- Typically, M can decay through multiple resonances
- Get interference like in Young's double slit experiment!



“Isobar Model”

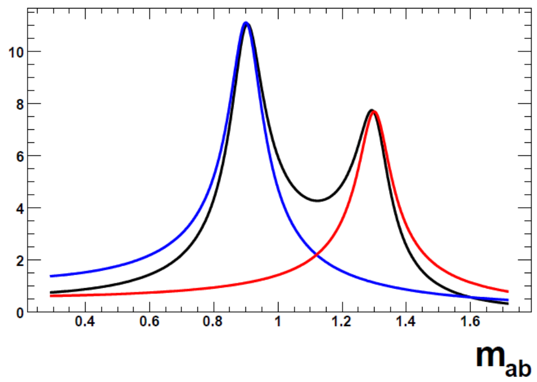
- Often, analysts will model the total decay amplitude as a sum of individual resonances, or “isobars”

$$A = \sum_k c_k e^{i\phi_k} A_k$$

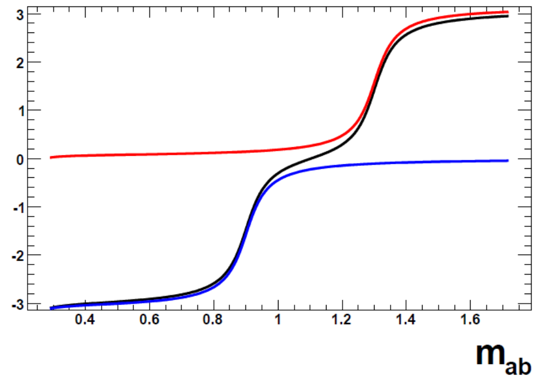
- $A_k = A_k(m_{ab}^2, m_{ac}^2)$ are the Dalitz-plot dependent amplitudes (e.g. relativistic Breit-Wigner, or may be a “nonresonant” term).
- c_k and ϕ_k are constants which can be measured in a maximum likelihood fit.
- Can measure the fractions and relative phases of different isobars

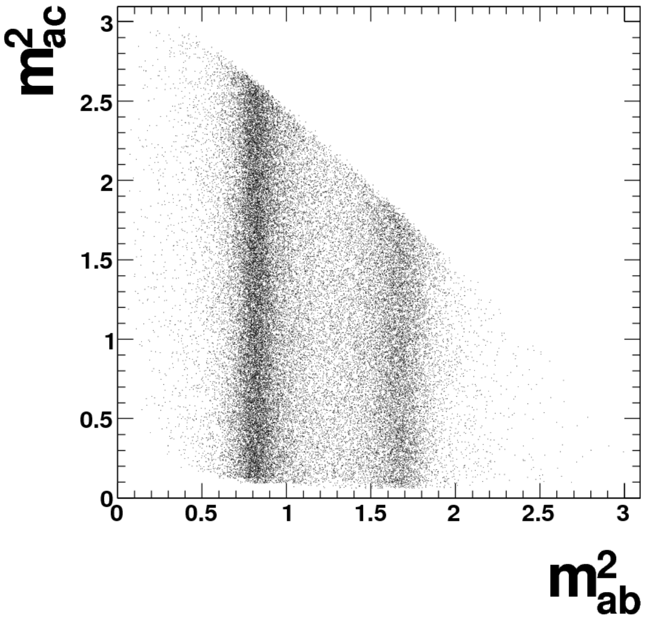
Constructive Interference

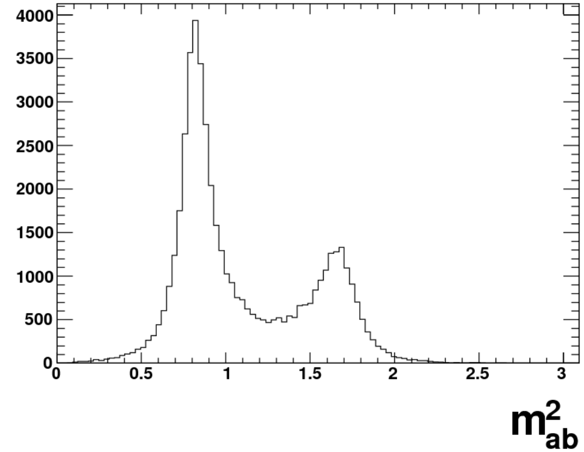
Magnitude



Phase

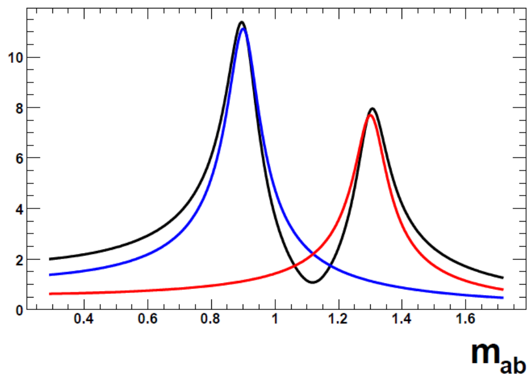




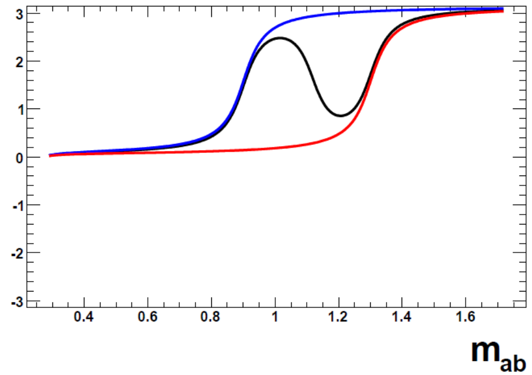


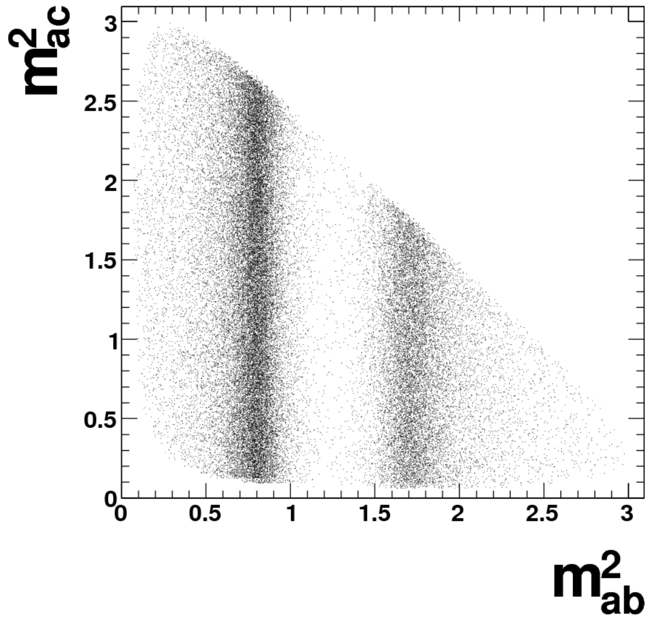
Destructive Interference

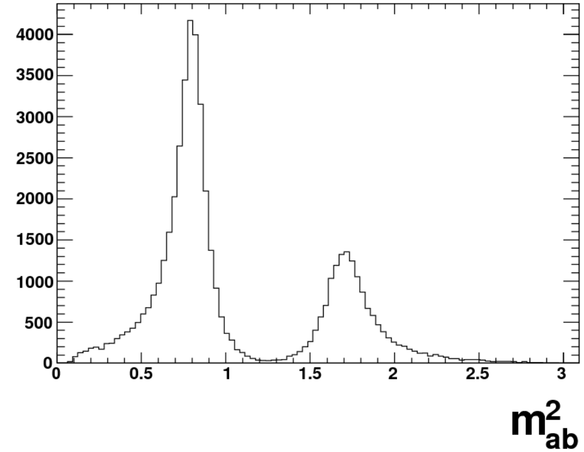
Magnitude



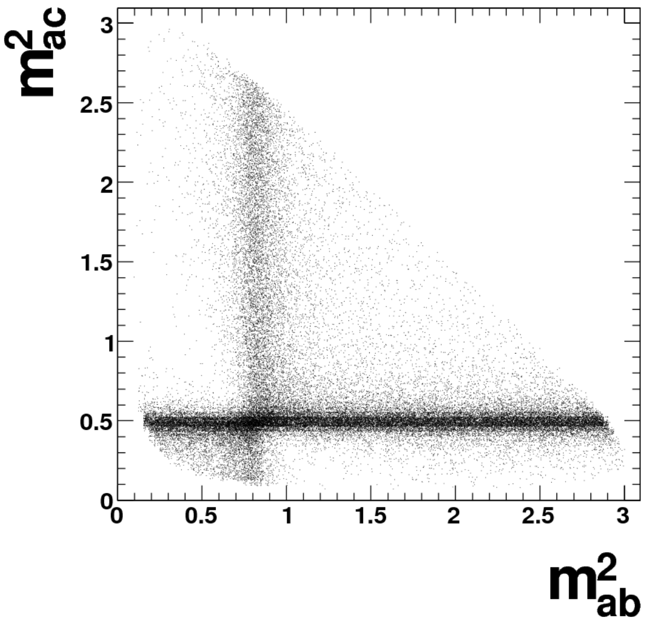
Phase

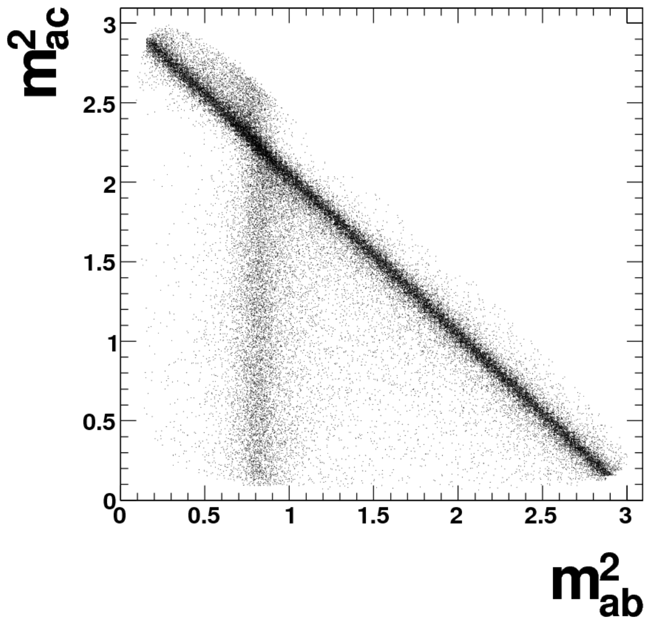






Cross-channel Interference





Pop Quiz!

- Look at this Dalitz plot from CLEO.
- How many resonances do you see?
- What are their spins?

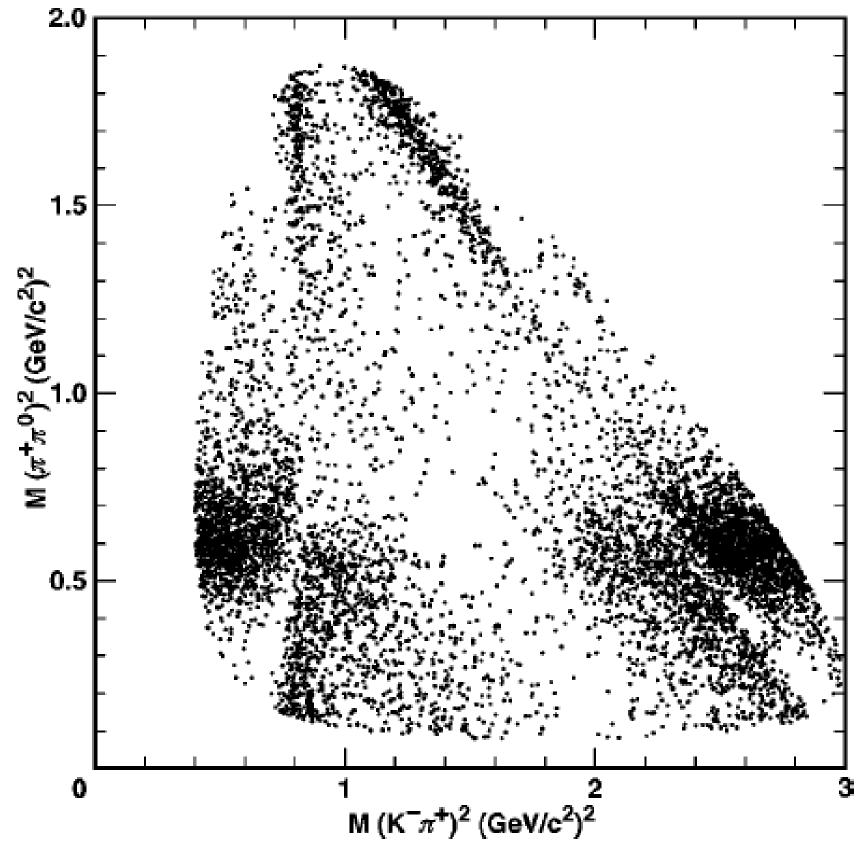


FIG. 3. The Dalitz distribution of all 7070 $D^0 \rightarrow K^- \pi^+ \pi^0$ candidates in our data sample shown in an unbinned scatter plot.

Kopp et al, Phys. Rev. D **63**, 092001 (2001).

A complicated Dalitz plot from Belle

Poluektov et al, Phys. Rev. D **73**, 112009 (2006).
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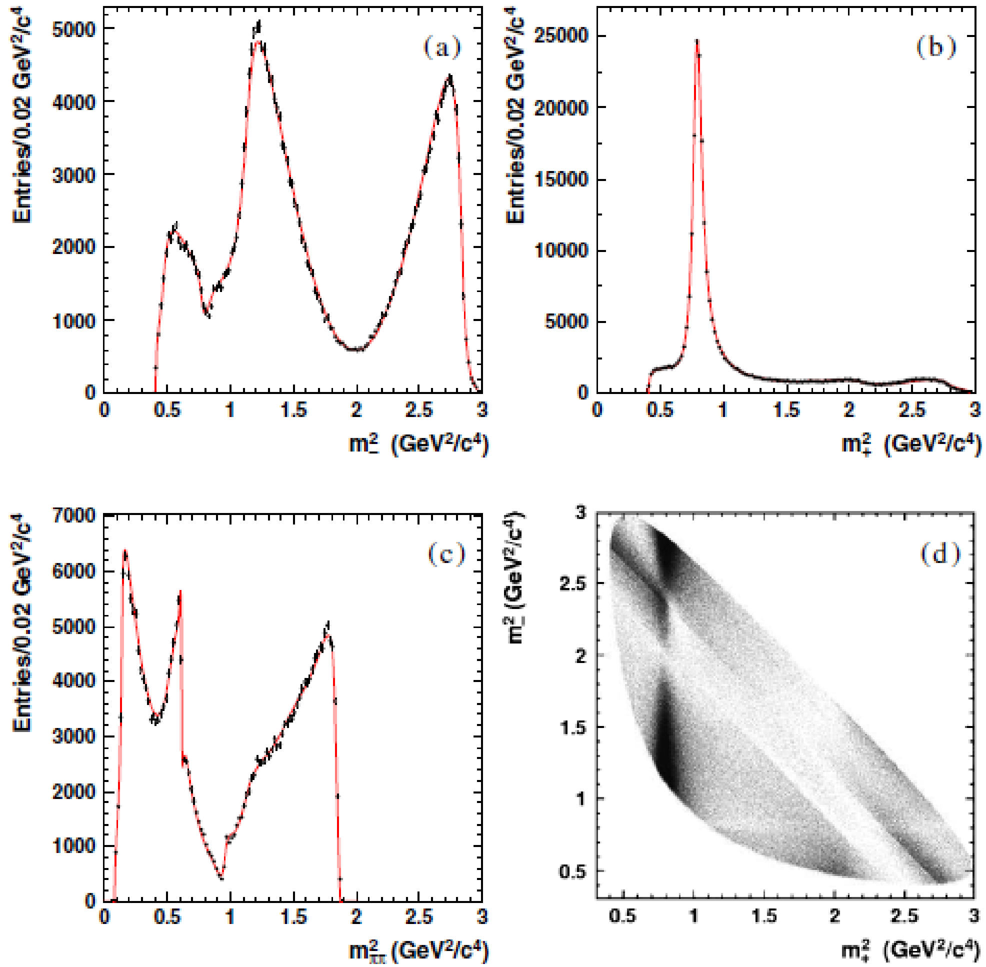

FIG. 2 (color online). (a) m_-^2, (b) m_+^2, (c) $m_{\pi\pi}^2$ and (d) Dalitz plot distribution for $D^{*-} \to \bar{D}^0 \pi_s^-$, $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$ decays from the $e^+e^- \to c\bar{c}$ continuum process. The points with error bars show the data; the smooth curve is the fit result.

Dalitz plot from Crystal Ball

- $p\bar{p}$ annihilation at rest.

Abele et al, Phys. Rev. D **57**, 3860 (1998).
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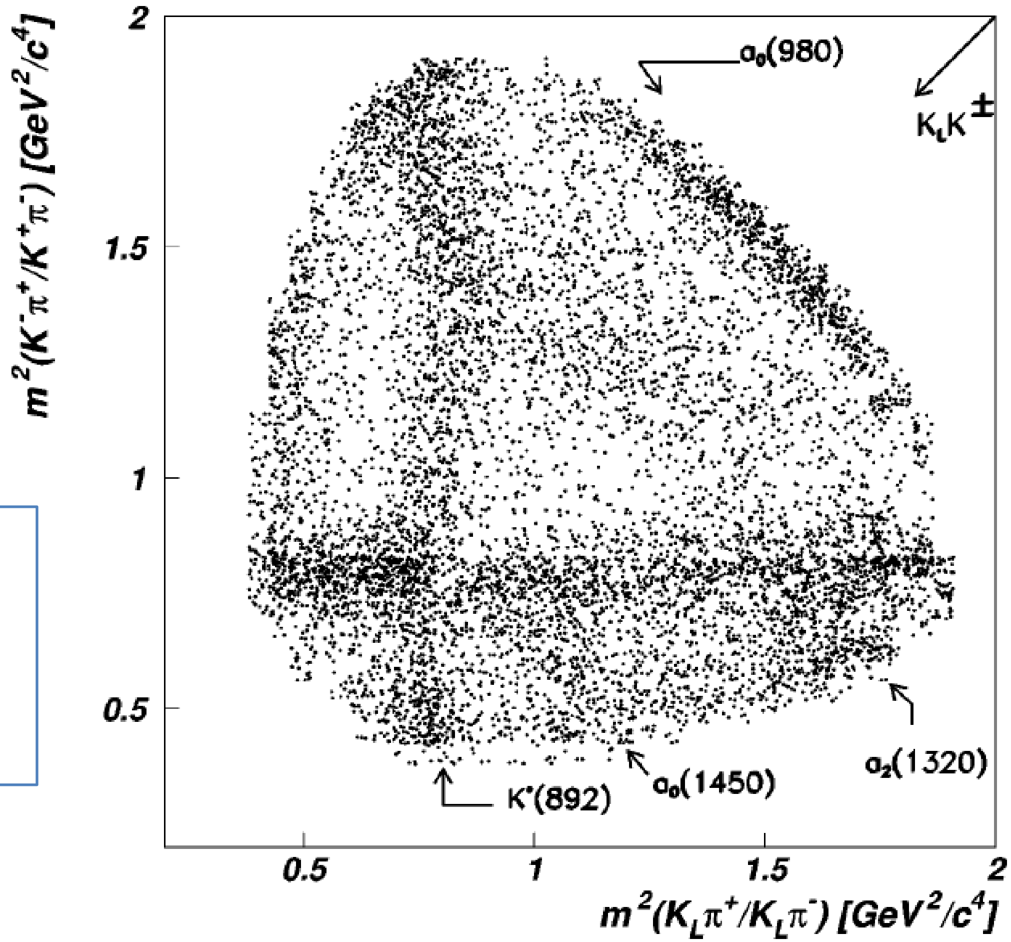

FIG. 4. Acceptance corrected and background subtracted $K_L K^{\pm}\pi^{\mp}$ Dalitz plot. The plot is divided into quadratic cells of size 0.045 $\times$0.045 GeV$^4/c^2$. The dots represent the event density.

Summary

- Dalitz plots are a powerful tool for studying three-body systems.
- The Dalitz plot was an important contributor to the tau-theta puzzle of the 1950's, which was eventually solved by the discovery of parity violation.
- Dalitz plots give information about particle masses, lifetimes, spins, and interference.

References

- Abele et al, "$p\bar{p}$ annihilation at rest into $K_L K^{+-} \pi^{-+}$," Phys. Rev. D **57**, 3860 (1998).
- Aitchison et al, "The scientific heritage of Richard Henry Dalitz, FRS (1925-2006)," arXiv:physics/0603219v1 [physics.hist-ph]
- Brown et al, "Observation with electron-sensitive plates exposed to cosmic radiation" Nature (1949).
- Dalitz, On the analysis of τ-meson data and the nature of the τ-meson, Phil. Mag. 44, 1068-1080 (1953).
- Dalitz, "Decay of τ Mesons of Known Charge," Phys. Rev. **94**, Num 4., 1046-1051 (1954).
- Dalitz, "Isotopic Changes in τ and θ Decay," Proc Phys Soc A 69, 527-540 (1956).
- Dalitz , "K Mesons and Hyperons their strong and weak interactions," Rep. Prog. Phys. 20, 163-303 (1957).
- E. Fabri, "A study of tau-meson decay", Il Nuovo Cimento Vol 11, 479-491 (1954).
- Franzinetti and Morpurgo, "The determination of the spin of the $K^+_{\pi 3}$", Il Nuovo Cimento Vol 6, Supplement 2, 641-659 (1957).
- Kopp et al, "Dalitz analysis of the decay $D^0 \rightarrow K^- \pi^+ \pi^0$," Phys. Rev. D **63**, 092001 (2001).
- K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010)
- Poluektov et al, "Measurement of ϕ_3 with a Dalitz plot analysis of $B^+ \rightarrow D^{(*)} K^{(*)+}$ decay," Phys. Rev. D **73**, 112009 (2006).
- Wu et al, "Experimental Test of Parity Conservation in Beta Decay." Phys. Rev. **105**, 1413 (1957).