High Power Millimeter Wave and Terahertz Sources
Innovative design concepts and improved modeling techniques

Alysson Vrielink
SLAC Technology Innovation Directorate, RF Accelerator Research Division
Research Advisor: Prof. Sami Tantawi

Scientific Policy Committee Meeting, November 3rd, 2016
Wanted: compact, high average power THz

High bandwidth (BW) communications
- Japan to start 8K broadcasting in 2020
  - 300 GHz source still missing to achieve requisite data rates
- Europe to implement mm-wave wireless link from fiber networks to mobile

Medical Imaging and Spectroscopy
- Dynamic nuclear polarization NMR
  - 100 – 400 GHz sources needed
  - Currently using large gyrotrons requiring superconducting magnets (10-40 T)

Radar and Long Range Imaging
- High BW, mm-wave radar and imaging for brownout conditions and automotive applications
  - Searching for 60 – 300 GHz sources
A natural extension: Enabling compact accelerator technology

- Ultra-high resolution diagnostics and beam manipulation
- Radiofrequency Undulators
- Higher accelerating gradients

High Power Millimeter Wave and Terahertz Sources, SPC Meeting, November 3\textsuperscript{rd}, 2016
Why the lack of sources?
Mind the (THz) Gap!

• Power scaling laws result in a minimum: 0.1 – 10 THz

• Optical sources have demonstrated impressive peak powers

• Vacuum electronic devices (VEDs) are necessary for high average powers

Traditional VEDs are fundamentally limited

In a traditional VED (traveling wave tube amplifiers, klystrons):

→ Electron beam interacts with traveling EM waves in the **longitudinal** direction, resonant interaction results in large amplification of RF fields.

**Problem:** As $f \uparrow$, dimensions $\downarrow$

→ Cavity quality factor, $Q \propto f^{-\frac{1}{2}}$
→ Beam current, $I_{\text{beam}} \propto a^2 \propto f^{-2}$

**Result:** Efficiency, $P_{\text{out}}/(P_{\text{in}} + P_{\text{beam}})$, and output power are currently very low:

→ 3%, 80W at 230 GHz

High Power Millimeter Wave and Terahertz Sources, SPC Meeting, November 3rd, 2016
The Solution: Heavily overmoded, spherical cavities (dimensions $\gg \lambda$)

$\rightarrow$ Azimuthal interaction between rotating disc beam and traveling EM waves

$\rightarrow$ Disc sheet beam reduces space charge forces, allowing $\uparrow I_{beam}$

Inspiration: Whispering gallery modes

Optical

220 GHz
Novel Azimuthal Traveling Wave Structure

Cathode

Output Structure

Beam Envelope

Beam Dump

Beam Focusing Optics

Anode

Copper Shell

Radial Width

Beam Trajectory

Vacuum

$R = 50 \text{ mm}$
Total estimated efficiency for an 80 GHz structure: 60%
**Problem:**
Overmoded structures require a 10-100x denser mesh → very large simulations

**Solution:**
Currently developing a finite element (FEM) solver

$\rightarrow e^{im\phi}$ dependence included in FEM formulation $\Rightarrow$ 2D mesh

$\rightarrow$ Based on variation of Lagrangian with respect to $(\Phi, \vec{A})$
• Addressing the need for high efficiency, high average power, portable sources at 80-230 GHz

• Azimuthally traveling waves, radial sheet beam ⇒ we avoid the scaling laws for interaction efficiency and device size of traditional devices

• Currently completing the finite element code, and designing the RF electron gun and power coupling system

I would like to express my gratitude to:
• The organizing committee for the invitation to present
• My advisor, Prof. Sami Tantawi
• Filippos Toufexis, Emilio Nanni, Jeff Neilson, Matt Franzi, Zenghai Li, Massimo dal Forno, Valery Dolgashev, Ago Marinelli, Mamdouh Nasr and Tor Raubenheimer for many excellent discussions
## The Many Facets of THz @ SLAC

<table>
<thead>
<tr>
<th><strong>Single Cycle (Short Pulse)</strong></th>
<th><strong>Multi Cycle (Long Pulse)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Who:</strong> Matthias Hoffmann et al.</td>
<td><strong>Who:</strong> Emilio Nanni et al.</td>
</tr>
<tr>
<td><strong>What:</strong> &lt;1ps pulse, ~1MV/m peak fields for pump-probe</td>
<td><strong>What:</strong> High energy, &gt;1ns pulses using novel periodic poled LiNbO₃ OPA</td>
</tr>
<tr>
<td><strong>Why:</strong> Investigating phase transitions in superconductors, semiconductors</td>
<td><strong>Why:</strong> Electron acceleration, spectroscopy, imaging</td>
</tr>
</tbody>
</table>

### Pulsed

<table>
<thead>
<tr>
<th><strong>Who:</strong> Massimo Dal Forno et al</th>
<th><strong>Who:</strong> Sami Tantawi et al</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What:</strong> 100-200 GHz wakefields from GeV beam are used to accelerate witness bunch</td>
<td><strong>What:</strong> kW, CW 80 – 230 GHz vacuum electronic sources using azimuthal beam-wave interaction</td>
</tr>
<tr>
<td><strong>Why:</strong> Proof of principle THz accelerator, frequency scaling of RF breakdown</td>
<td><strong>Why:</strong> Compact accelerators, high BW communications, medical imaging and spectroscopy</td>
</tr>
</tbody>
</table>
57 GHz structure already tested, producing power

F. Toufexis, S. Tantawi

Photo of helicopter in brownout courtesy of Mex Merrill, available from [http://www.defenceimagery.mod.uk](http://www.defenceimagery.mod.uk)


Millimeter Wave and Terahertz
Current Applications and Sources

- Terahertz:
  - 100 GHz to 10THz
- Growing interest in high power THz sources
  - Compact accelerators and light sources
  - Ultra high bandwidth radar/communications
  - Non destructive testing and non-ionizing imaging

Image courtesy of Teraphysics (www.teraphysics.com)
Azimuthal Traveling Wave Structure

- Structure resonates only for spherical modes:

\[
E_r = \frac{\alpha n (n + 1) \sqrt{kr} e^{im\phi} [J_{n+\frac{1}{2}}(kr) + C1 Y_{n+\frac{1}{2}}(kr)]}{r^2 y} [P_n^m(\cos(\theta)) + C2 Q_n^m(\cos(\theta))]
\]

Traveling wave in \(\phi\)

- Interaction between electron beam and radial E field ensured by synchronizing phase velocity:

\[
\omega_{out} = m\omega_{in}
\]

Spherical Harmonics:

Image courtesy of Ellipsix Informatics (http://www.ellipsix.net/)
Lagrangian Formulation

\[ \mathcal{L} = \int \int \left[ \frac{\epsilon((\nabla \phi - \frac{\partial A}{\partial t})^2 - c^2(\nabla \times A)^2)}{2} - \rho \phi + A \cdot j \right] dvdt \]

 Converted to frequency domain: Steady state solution

\[ \mathcal{L} = \int \epsilon \left( (\nabla \phi_{\omega} - i \omega A_{\omega}) \cdot (-\nabla \phi_{\omega}^* + i \omega A_{\omega}^*) - c^2 (\nabla \times A_{\omega}) \cdot (\nabla \times A_{\omega}^*) \right) - \rho_{\omega} \phi_{\omega} + \rho_{\omega} \phi_{\omega}^* + A_{\omega}^* \cdot j_{\omega} + A_{\omega} \cdot j_{\omega}^* \, dv + \frac{1}{i \omega} \int (\nabla \phi_{\omega} - i \omega A_{\omega}) \cdot Y \cdot (-\nabla \phi_{\omega}^* + i \omega A_{\omega}^*) \, ds \] (2)
Lagrangian Formulation

\[ \mathcal{L} = \int \int \left[ \epsilon \left( \frac{(-\nabla \phi - \frac{\partial A}{\partial t})^2}{2} - c^2 (\nabla \times A)^2 \right) - \rho \phi + A \cdot j \right] dv dt \]

Converted to frequency domain:
Steady state solution

\[ \mathcal{L} = \int \epsilon \left( (-\nabla \phi_\omega - i \omega A_\omega) \cdot (-\nabla \phi_\omega^* + i \omega A_\omega^*) - c^2 (\nabla \times A_\omega) \cdot (\nabla \times A_\omega^*) \right) - \rho_\omega \phi_\omega^* - \rho_\omega^* \phi_\omega + \]

\[ A_\omega^* \cdot j_\omega + A_\omega \cdot j_\omega^* dv + \frac{1}{i \omega} \int (-\nabla \phi_\omega - i \omega A_\omega) \cdot Y \cdot (-\nabla \phi_\omega^* + i \omega A_\omega^*) ds \]  

Volume Integral Term, No Sources

Finite Element for Quasi Azimuthal Symmetry: Part 1
Lagrangian Formulation

\[ \mathcal{L} = \int \int \left[ \epsilon \left( \left( -\nabla \phi - \frac{\partial A}{\partial t} \right)^2 - c^2 (\nabla \times A)^2 \right) - \rho \phi + A \cdot j \right] \ dv \ dt \]

Converted to frequency domain: Steady state solution

\[ \mathcal{L} = \int \epsilon \left( \left( -\nabla \phi_\omega - i\omega A_\omega \right) \cdot \left( -\nabla \phi_\omega^* + i\omega A_\omega^* \right) - c^2 (\nabla \times A_\omega) \cdot (\nabla \times A_\omega^*) \right) - \rho_\omega \phi_\omega^* - \rho_\omega^* \phi_\omega \]

\[ A_\omega^* \cdot j_\omega + A_\omega \cdot j_\omega^* \ dv + \frac{1}{i\omega} \int \left( -\nabla \phi_\omega - i\omega A_\omega \right) \cdot Y \cdot \left( -\nabla \phi_\omega^* + i\omega A_\omega^* \right) \ ds \quad (2) \]

Source terms: Need to solve for fields in frequency domain, then solve for particle trajectory using numerical integrator in time domain, then convert back to frequency domain to solve for fields again. Iterative process.
Lagrangian Formulation

\[ \mathcal{L} = \int \int \left[ \frac{\epsilon((-\nabla \phi - \frac{\partial A}{\partial t})^2 - c^2(\nabla \times A)^2)}{2} - \rho \phi + A \cdot j \right] \, dv \, dt \]

Converted to frequency domain: Steady state solution

\[ \mathcal{L} = \int \epsilon \left( (-\nabla \phi_\omega - \text{i}\omega A_\omega) \cdot (-\nabla \phi^*_\omega + \text{i}\omega A^*_\omega) - c^2(\nabla \times A_\omega) \cdot (\nabla \times A^*_\omega) \right) - \rho_\omega \phi^*_\omega - \rho^*_\omega \phi_\omega + \]

\[ A^*_\omega \cdot j_\omega + A_\omega \cdot j^*_\omega \, dv + \frac{1}{\text{i}\omega} \int (-\nabla \phi_\omega - \text{i}\omega A_\omega) \cdot Y \cdot (-\nabla \phi^*_\omega + \text{i}\omega A^*_\omega) \, ds \]  \quad (2)

Imposed Boundary Condition: Y is a 3x3x3 tensor related to conductance
Substitution of the Azimuthal Dependance

\[ \vec{F} = (\Phi, \vec{A}) \propto \vec{F}(r, z)e^{im\theta} \]

\[ \mathcal{L}_V = 2\pi\epsilon \int \int \frac{1}{r} \left( (r^2\omega^2 - c^2m^2) |A_r(r, z)|^2 + (r^2\omega^2 - c^2m^2) |A_z(r, z)|^2 + (r^2\omega^2 \right. \\
- c^2 |A_\theta(r, z)|^2 - c^2r^2 |A_\theta^{(0,1)}(r, z)|^2 - c^2r^2 |A_\theta^{(1,0)}(r, z)|^2 + c^2 (-r^2) |A^{(0,1)}(r, z)|^2 \\
- c^2r^2 |A^{(1,0)}(r, z)|^2 + ic^2mrA_r(r, z)A_\theta(r, z)^* + ic^2mrA_r(r, z)A_\theta^{(1,0)}(r, z)^* \\
+ ic^2mrA_r(r, z)A_\theta^{(0,1)}(r, z)^* - ic^2mA_r(r, z)A_\theta(r, z) - ic^2mrA_r(r, z)^*A_\theta^{(0,1)}(r, z) - \\
ic^2mA_r(r, z)^*A_\theta^{(1,0)}(r, z) + c^2r^2A^{(0,1)}(r, z)A_z^{(1,0)}(r, z)^* + c^2r^2A_z^{(1,0)}(r, z)^*A_r^{(0,1)}(r, z)^* \\
- c^2rA_\theta(r, z)A^{(1,0)}(r, z)^* - c^2rA_\theta^{(1,0)}(r, z)^*A_\theta(r, z)^* + mr\omega\phi(r, z)A_\theta(r, z)^* \\
+ mr\omegaA_\theta(r, z)\phi(r, z)^* + ir^2\omegaA_r(r, z)\phi^{(1,0)}(r, z)^* + ir^2\omegaA_z(r, z)\phi^{(0,1)}(r, z)^* \\
- ir^2\omega\phi^{(0,1)}(r, z)A_z(r, z)^* \\
- ir^2\omega\phi^{(1,0)}(r, z)A_r(r, z)^* + m^2 |\phi(r, z)|^2 + r^2 |\phi^{(0,1)}(r, z)|^2 + r^2 |\phi^{(1,0)}(r, z)|^2 \right) \text{d}r \text{d}z \]
Substitution of the Azimuthal Dependance, m=0

\[ \mathcal{L}_V^{m=0} = 2\pi e \int \int - \left( c^2 r \left( |A_\theta^{(0,1)}(r, z)|^2 + |A_\theta^{(1,0)}(r, z)|^2 + |A_r^{(0,1)}(r, z)|^2 + |A_z^{(1,0)}(r, z)|^2 - A_z^{(1,0)}(r, z) A_r^{(0,1)}(r, z) A_\theta^{(1,0)}(r, z) \right)^* + \right. \\
\left. c^2 A_\theta^{(0,1)}(r, z) A_\theta^{(1,0)}(r, z) - i r \omega A_r^{(0,1)}(r, z) \phi^{(1,0)}(r, z) - i r \omega A_z^{(1,0)}(r, z) \phi^{(0,1)}(r, z) \right)^* + \right. \\
\left. i r \omega \phi^{(1,0)}(r, z) A_z^{(0,1)}(r, z) + i r \omega \phi^{(1,0)}(r, z) A_r^{(0,1)}(r, z) - r |\phi^{(0,1)}(r, z)|^2 - r |\phi^{(1,0)}(r, z)|^2 \right) dr dz \]

\[ \mathcal{L}_g = 2\pi \int \int 2r \left( Y_{rrn_r} \phi^{(1,0)}(r, z)^* \left( \phi^{(1,0)}(r, z) + i \omega A_r(r, z) \right) \right. \\
\left. + \omega Y_{rrn_r} A_r(r, z)^* \left( \omega A_r(r, z) - i \phi^{(1,0)}(r, z) \right) + Y_{zzz} n_z \left( \omega A_z(r, z) - i \phi^{(0,1)}(r, z) \right) \right. \\
\left. + i \phi^{(0,1)}(r, z)^* \right) \omega A_z(r, z)^* \right) \right) \right) ds \]

Takeaway: No coupling with A_{\theta}
Discretization: Finite Elements

Result: Quadratic Eigenvalue Problem, with $M$, $K$ and $C$ a function of the triangle coordinates. For example:

$$\frac{\pi c}{360 \text{Det } B} \left( 3 \text{Det } B^2 + 6 \text{Det } B(z_2 - z_3)(6r_1 + r_2 + r_3) + 4 \left( 39r_1^2 + 15r_1(r_2 + r_3) + 7 \left( r_2^2 + r_2r_3 + r_3^2 \right) \right) \left( r_2^2 - 2r_2r_3 + r_3^2 + (z_2 - z_3)^2 \right) \right)$$
Electrostatic FEM Solver for arbitrary m, m=0
Electrostatic FEM Solver for arbitrary m, m=10

Finite Element for Quasi Azimuthal Symmetry: Part 2
Progress:
- Mesher complete
- Linear algebra libraries compiled & linked against
- Basic electrostatic potential solver complete

To Do:
- Particle tracking & iterative space charge computation
- RF field solver
TE Modes: Pillbox Cavity

L=10mm, R1=0, R2=10mm
TE Modes: Pillbox Cavity