Supersymmetric QCD correction to top-quark production at the Tevatron

Zack Sullivan
Department of Physics, University of Illinois
1110 West Green Street, Urbana, IL 61801
Email: z-sulli@uiuc.edu

ABSTRACT

We calculate the supersymmetric QCD correction to top-quark production at the Fermilab Tevatron, allowing for arbitrary left-right mixing of the squarks. We find that the corrections are significant for several combinations of gluino and squark masses, and may reach 44%, when $m_{\widetilde{g}} = 200$ GeV.

I. INTRODUCTION

The discovery of the top quark [1] provides an unique opportunity to search for effects beyond the Standard Model. Supersymmetry is a promising candidate for this new physics. The top quark mass $m_t = 176.8 \pm 6$ GeV has been measured to 3.5%, and the cross section has been measured to $\approx 25\%$ [2]. With the copious production of top quarks in Run II of the Fermilab Tevatron and future upgrades, the cross section will be measured to within 6% with 10 fb^{-1} of data [3]. Comparison of the theoretical cross section to that measured will test the Standard Model and may indicate the presence of SUSY. Currently, only lower bounds on the masses of the superpartners have been set. Barring discovery, direct searches for SUSY will eliminate a small range of parameter space, since these searches depend strongly on the modelling of the decays of the supersymmetric particles. In contrast, some effects of virtual SUSY may be generalized, thus extending the reach of experiment. If virtual SUSY effects are found to be large enough, an indirect search may provide the first sign of supersymmetry. In this paper we calculate the supersymmetric QCD correction to $t\bar{t}$ production at the Fermilab Tevatron.

Direct searches for SUSY are generally performed separately for top squarks, the light quark superpartners, and gluinos. This is motivated by minimal supergravity models which argue that all scalar particles acquire a mass on the order of the SUSY breaking scale [4, 5]. A heavy top quark loop dominates the running of the masses to low energies, forcing the mass of the two top squarks below that of the rest of the scalars. Additionally, mixing of the left-right weak eigenstates of the top squarks may result in the top squark $\widetilde{t}_1$ becoming the lightest squark [6]. It has been argued that a light top squark $m_{\widetilde{t}_1} < 100$ GeV may explain the deviations from the Standard Model values for $\Gamma(Z \to b\bar{b})$ and $\alpha_s(m_Z^2)$ [7]. Limits on the mass of the gluino have reached $m_{\widetilde{g}} > 154$ GeV [8] when the light quark superpartners are assumed to be heavier. The lightest top squark has a limit of $m_{\widetilde{t}_1} > 55$ GeV [9]. Current experimental limits are extracted assuming specific values of the SUSY parameters, and may be relaxed [10]. Other regions of parameter space have been eliminated [11, 12], but these limits are generally model dependent. Exhaustive direct searches will reach 300 GeV for gluinos and 100 GeV for top squarks with 10 fb^{-1} of data at the Tevatron [3]. Until the advent of the CERN Large Hadron Collider, the presence of heavier SUSY particles will only be suggested by their effects on Standard Model processes.

The NLO QCD cross section for $t\bar{t}$ production with resummed gluon emission at a $\sqrt{S} = 2$ TeV $p\bar{p}$ collider has been calculated [13]. The dominant mechanism of top-quark production at the Tevatron is $q\bar{q}$ annihilation. It is expected that the dominant SUSY contribution to top production will be in the form of QCD corrections to this process. We consider the SUSY corrections to the cross section as a correction to the dominant process as shown in Fig. 1. The calculation of the SUSY corrections to top-quark production is different from typical SUSY calculations in that the number of assumptions about supersymmetry neccesary to predict phenomenologically interesting results is minimal. It is assumed that R-parity is conserved so that the interaction terms in the Lagrangian are the simple supersymmetrization of the Standard Model interactions. No assumptions about the mechanism of SUSY breaking or of unification are required. In a strong-interaction process, the correction depends only on the observed masses of the gluino and squarks, and the mixing angle that relates the squark mass eigenstates to their in-

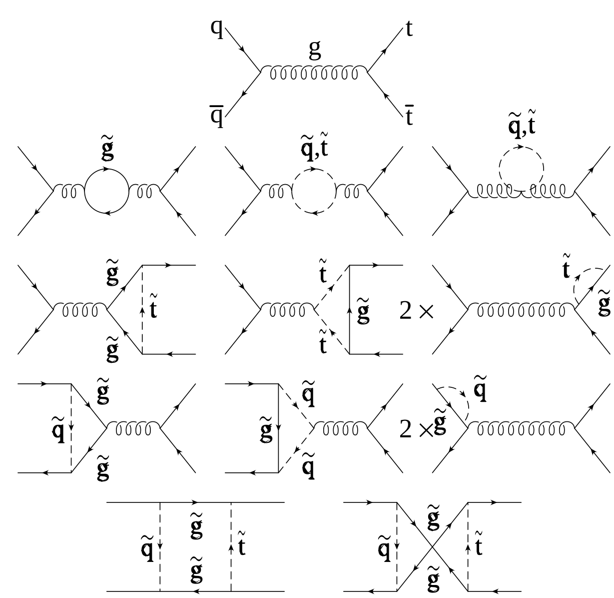

Figure 1: Feynman diagrams for the one-loop SUSY QCD correction to top quark production at the Tevatron. The first row contains the tree-level diagram. The second row contains the vacuum polarization corrections to the gluon propagator due to squarks and gluinos. The third row contains the final state vertex corrections and wave-function renormalization diagrams. The fourth row contains the initial state vertex corrections and wave-function renormalization diagrams. The last row contains the box and crossed-box diagrams.

teraction eigenstates. The assumption of universal scalar masses is common to experimental extractions of mass limits on the superpartners, but there is no compelling theoretical basis for believing this to be the case [5, 10]. For the purpose of this calculation, and in order to cover the greatest range of models, we treat top squarks $\widetilde{t}$ separately from the light-quark superpartners $\widetilde{q}$. We present analytic and numerical results for degenerate squark masses, and for the case where the top squarks are light compared to the light-quark superpartners, the 'heavy squarks'. Results for $m_{\widetilde{t}}, m_{\widetilde{q}} > 50$ GeV, and $m_{\widetilde{g}} > 150$ GeV are presented.

The SUSY QCD corrections to top production in e^+e^- annihilation has been studied in Ref. [14]. The corrections in $p\bar{p}$ annihilation were calculated by Li *et al.* [15] for the case of degenerate squark mass. Numerical results were obtained for two gluino masses and a small range of squark masses. That paper also provided analytic results for their calculation. However, that work neglected the contribution of the vacuum polarization and the crossed-box diagram, which arises because the gluino is a Majorana particle. These contributions are found to be numerically significant. We provide a complete calculation of the SUSY correction to the cross section, and discuss the phenomenological significance of the result.

This paper is organized as follows. In Section 2, we present the analytic form of the $O(\alpha_s)$ SUSY QCD corrections to the $p\bar{p} \to t\bar{t}$ cross section. In Section 3A, we present numerical results for the correction to the $p\bar{p} \to t\bar{t}$ cross section. In Section 3B, we show $t\bar{t}$ invariant mass distributions for several choices of gluino mass. In Section 4, conclusions are drawn. A more detailed presentation of this calculation will be presented in a subsequent paper [16].

II. ANALYTIC SUPERSYMMETRIC QCD CORRECTION

The one loop supersymmetric QCD contribution to the $q\bar{q} \to t\bar{t}$ cross section at leading order in α_s is attributed to the cross term in the matrix element between the tree level diagram and the one loop diagrams presented in Fig. 1. The general form of the vertex corrections, consistent with current conservation, is

$$\begin{aligned} i\bar{u}(p_3)\Gamma^{\mu A}v(p_4) &= -i\,g_s\Bigg[\bar{u}(p_3)T^A\gamma^\mu v(p_4) \\ &- \frac{\alpha_s}{4\pi}\,\bar{u}(p_3)T^A\left[V\gamma^\mu + S(p_3^\mu - p_4^\mu)\right. \\ &\left. + A(\gamma^\mu q^2 - 2m_t q^\mu)\gamma_5\right]v(p_4)\Bigg] , \end{aligned}$$

where p_3 and p_4 are the outgoing momenta of the top and anti-top, $q = p_3 + p_4$, T^A is a $SU(3)$ generator, and V, S, and A are the vector, scalar, and anapole form factors respectively. The analytic forms of V, S, A, the gluon vacuum polarization Π, and the corrections due to the box and cross-box diagrams, B and C, will be given in Ref. [16]. The anapole term A does not contribute to the total cross section at this order in the expansion. It is used in Ref. [16], however, in determining the parity-violating left-right asymmetry due to the squark mixing. The Dirac algebra and loop integrals were evaluated using dimensional regularization. The analytic cross section was derived in the $\overline{MS}$ renormalization scheme. The Feynman rules for the SUSY verticies were derived from Ref. [4] for the physically-relevant mass eigenstates of the squarks rather than the interaction eigenstates. Mixing of the squarks is therefore explicit and parameterized by mixing angles $\theta_{\widetilde{t}}$ and $\theta_{\widetilde{q}}$ for the stops and light quark superpartners respectively.

$$\begin{pmatrix} \widetilde{t}_1 \\ \widetilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\widetilde{t}} & \sin\theta_{\widetilde{t}} \\ -\sin\theta_{\widetilde{t}} & \cos\theta_{\widetilde{t}} \end{pmatrix} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix}$$

$$\begin{pmatrix} \widetilde{q}_1 \\ \widetilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\widetilde{q}} & \sin\theta_{\widetilde{q}} \\ -\sin\theta_{\widetilde{q}} & \cos\theta_{\widetilde{q}} \end{pmatrix} \begin{pmatrix} \widetilde{q}_L \\ \widetilde{q}_R \end{pmatrix}$$

The spin- and color-averaged parton-level differential cross section is given by

$$\frac{d\widehat{\sigma}}{dz} = \frac{\beta}{32\pi\widehat{s}}|M|^2 ,$$

where z is the cosine of the angle between the incoming quark and the top quark, $\beta = \sqrt{1 - 4m_t^2/\widehat{s}}$, and $\sqrt{\widehat{s}}$ is the parton center-of-momentum energy. The Born matrix element squared is given by

$$|M_0|^2 = \frac{32\pi^2{\alpha_s}^2}{9}[2 - \beta^2(1 - z^2)] .$$

Integrating over $-1 \le z \le 1$ readily yields the Born-level cross section

$$\widehat{\sigma}_0 = \frac{4\pi\alpha_s^2\beta}{9\widehat{s}}(1 - \beta^2/3) .$$

The correction arises from the cross term in the square of the amplitude. This correction will be the sum of the terms:

$$\begin{aligned} 2\mathrm{Re}[M_0^\dagger M_\Pi] &= -\frac{\alpha_s^2}{2\pi}|M_0|^2\mathrm{Re}[\Pi(\widehat{s}) - \Pi(0)] \\ 2\mathrm{Re}[M_0^\dagger M_V] &= -\frac{\alpha_s^2}{2\pi}|M_0|^2\mathrm{Re}[V] \\ 2\mathrm{Re}[M_0^\dagger M_S] &= \frac{32\pi\beta^2\alpha_s^3 m_t}{9}(1 - z^2)\mathrm{Re}[S] \\ 2\mathrm{Re}[M_0^\dagger M_B] &= \frac{32\pi\alpha_s^3}{9\widehat{s}}\mathrm{Re}\left[\frac{7}{3}B + \frac{2}{3}C\right] . \end{aligned}$$

The integration over phase space is trivial except for the box and cross-box matrix elements, B and C, which depend implicitly on z. The relative sign between the box and cross-box terms should be noted. The color factor asscociated with C is $-2/3$. However, there is a non-trivial relative sign difference between the two diagrams that arises from the proper ordering of the gluino fields in the amplitude. The net result is that the two contributions constructively interfere.

The total cross section for top production in $p\bar{p}$ annihilation is obtained by convolving the parton cross section for annihilation into a $t\bar{t}$ final state with the parton distribution functions of the proton and antiproton. The integral may be parameterized as

$$\sigma = \int_{4m_t^2/S}^{1} d\tau\, \widehat{\sigma}(\tau S) \times \int_{\ln(\tau)/2}^{-\ln(\tau)/2} d\eta\, P(\sqrt{\widehat{s}}, \sqrt{\tau} e^{\eta})\, \bar{P}(\sqrt{\widehat{s}}, \sqrt{\tau} e^{-\eta}) \ ,$$

where $\sqrt{S} = 2$ TeV, $\tau = \widehat{s}/S$ and $P(\sqrt{\widehat{s}}, \sqrt{\tau} e^{\eta})$, $\bar{P}(\sqrt{\widehat{s}}, \sqrt{\tau} e^{-\eta})$ are the proton and antiproton parton distribution functions (PDF's).

In the following section, numerical results are presented for a top quark of mass $m_t = 175$ GeV. Analytic expressions were reduced to scalar n-point integrals [17] and evaluated with the aid of the code FF [18] in order to ensure numerical stability. For those cases that FF does not handle, the analytic solutions to the integrals were substituted. The integrals were evaluated using both the MRS(A′) [19] and CTEQ3M [20] PDF's. The coupling α_s was taken to be as defined in the PDF's in order to be consistent. Nearly identical results were obtained using both sets, therefore, only the results obtained using the MRS(A′) PDF's are presented.

III. NUMERICAL RESULTS

A. $t\bar{t}$ Cross Section

The correction to the $p\bar{p} \to t\bar{t}$ cross section is shown in Fig. 2 as a function of gluino mass for a wide range of degenerate squark masses $m_{\widetilde{Q}}$, where $m_{\widetilde{Q}} = m_{\widetilde{q}} = m_{\widetilde{t}}$. As expected from decoupling, the magnitude of the correction decreases as the squark mass increases. For squarks of mass 50 GeV, the correction ranges from -11.8% for a gluino of 150 GeV to $+44\%$ for a gluino of 200 GeV. The correction changes sign as $m_{\widetilde{g}}$ approaches m_t. The correction changes rapidly as the threshold for gluino production moves through the top-quark threshold. It is noteworthy that the corrections are nearly independent of gluino mass when $m_{\widetilde{g}} > 400$ GeV. In this region, the corrections are entirely dominated by the squark vacuum terms and, to a lesser extent, the box terms.

In Figure 3 the correction to the total cross section is shown as a function of degenerate squark mass $m_{\widetilde{Q}} = m_{\widetilde{q}} = m_{\widetilde{t}}$, for several gluino masses. Once $m_{\widetilde{Q}} > 400$ GeV, the correction becomes small and the squarks effectively decouple. In this region, the correction is dominated by the gluino vacuum terms. In Figs. 2,3 there is a large jump in the cross section when $m_t^2 = m_{\widetilde{t}}^2 + m_{\widetilde{g}}^2$. When $m_{\widetilde{g}} = 150$ GeV, the correction jumps from $+6.5\%$ to -9.3% around $m_{\widetilde{t}} = 90.1$ GeV. This corresponds to a jump in the real part of the final state vertex correction [21], when the anomalous threshold crosses the real threshold for superpartner production in the complex s-plane [22].

The largest corrections occur when $m_{\widetilde{g}} = 200$ GeV. Therefore, this mass is used in Fig. 4 to show the correction as a function of heavy squark mass $m_{\widetilde{q}}$, for a variety of top squark masses. This figure demonstrates that the correction is mostly influenced by the mass of the top squark. The correction is 21%

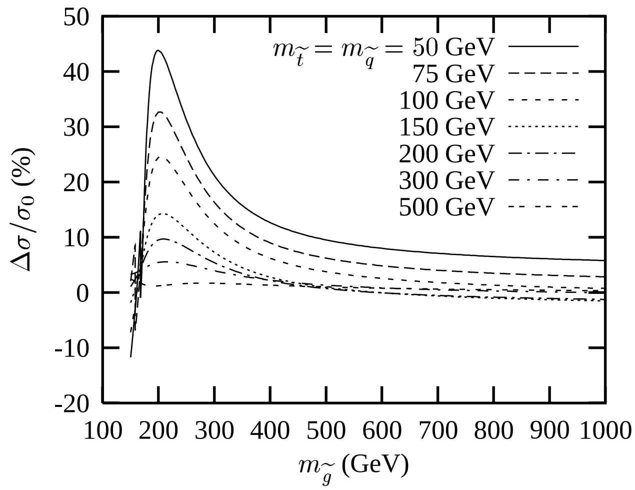

Figure 2: Change in the cross section for $p\bar{p} \to t\bar{t}$, as a function of gluino mass $m_{\widetilde{g}}$, for $m_t = 175$ GeV. Curves of constant degenerate squark mass $m_{\widetilde{q}} = m_{\widetilde{t}}$ are shown.

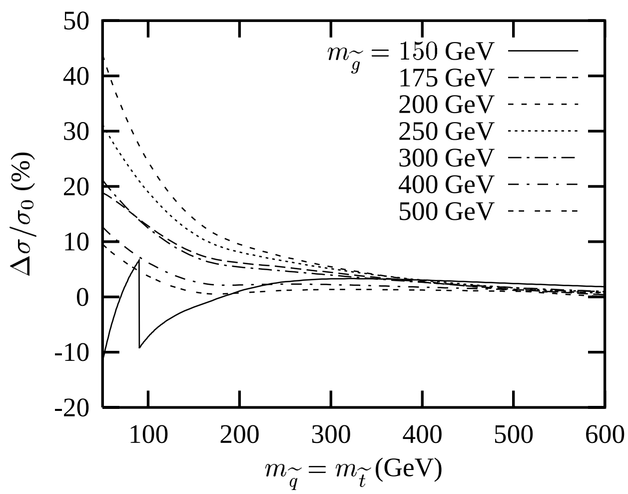

Figure 3: Change in the cross section for $p\bar{p} \to t\bar{t}$, as a function of degenerate squark mass $m_{\widetilde{q}} = m_{\widetilde{t}}$, for $m_t = 175$ GeV. Curves of constant gluino mass $m_{\widetilde{g}}$ are shown.

for $m_{\widetilde{t}} = 50$ GeV, and $m_{\widetilde{q}} = 300$ GeV; whereas the correction is 16% for $m_{\widetilde{t}} = 300$ GeV, and $m_{\widetilde{q}} = 50$ GeV. Even if the heavy squarks decouple, the correction remains significant as long as $m_{\widetilde{t}} < 150$ GeV. The magnitude of the correction will be reduced if the masses of the stops are different and they mix. The numerical effects of this mixing will be presented in Ref. [16].

B. $t\bar{t}$ Invariant Mass Distributions

Since total cross section measurements are difficult to normalize, it is advantageous to look for deviations from the lineshapes predicted by the Standard Model. A sampling of the invariant mass of $t\bar{t}$ events provides another avenue to search for supersymmetry. In Figure 5 we show the total differential cross section as a function of $t\bar{t}$ invariant mass $M_{t\bar{t}}$, for gluinos

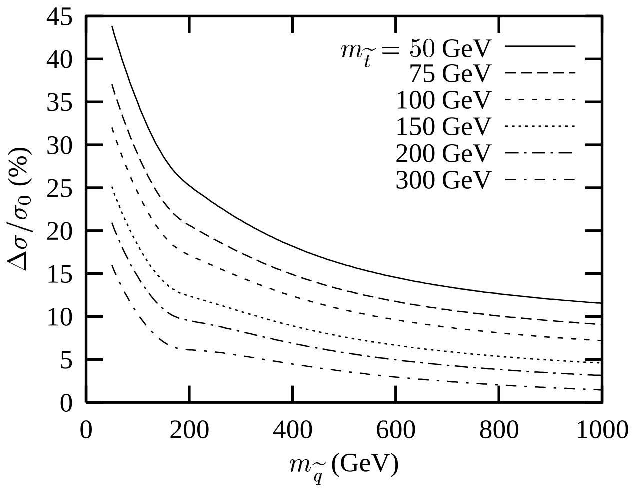

Figure 4: Change in the cross section for $p\bar{p} \to t\bar{t}$, as a function of heavy squark mass $m_{\tilde{q}}$, for $m_t = 175$ GeV, and $m_{\tilde{g}} = 200$ GeV. Curves of constant top squark mass $m_{\tilde{t}}$ are shown.

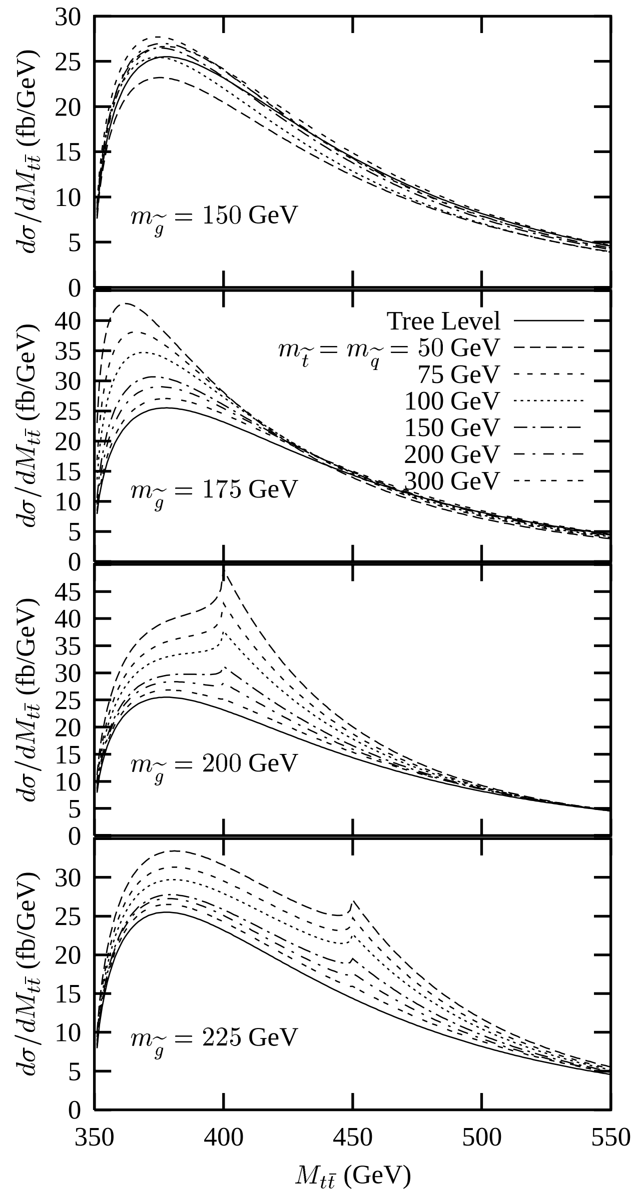

Figure 5: Differential cross section for $p\bar{p} \to t\bar{t}$, as a function of $t\bar{t}$ invariant mass $M_{t\bar{t}}$, for $m_t = 175$ GeV. Figures are shown for $m_{\tilde{g}} = 150$, 175, 200, and 225 GeV. Curves of constant degenerate squark mass $m_{\tilde{q}} = m_{\tilde{t}}$ are shown.

of mass $m_{\tilde{g}} = 150$, 175, 200, and 225 GeV. Several choices of degenerate squark mass $m_{\tilde{q}} = m_{\tilde{t}}$, are presented. By looking for an excess in the invariant mass distribution, a gluino of mass between 175 GeV and 225 GeV may be observable.

There are two types of enhancement to the cross section that appear in Fig. 5. If $m_{\tilde{g}} \approx m_t$, the maximum of the invariant mass distribution is shifted toward the common threshold. This would also produce a steeper top-quark threshold region in the data. A singularity at the threshold for gluino pair production causes a cusp at $2m_{\tilde{g}}$. The largest cusp occurs when $m_{\tilde{g}} = 200$ GeV, and $m_{\tilde{Q}} = 50$ GeV. The amplitude of the cusp is 112% of the Standard Model differential cross section at this point. Despite the large normalization, the cusp will sit on a large contiuum background. If we assume purely statistical errors, this cusp will appear at the 3σ level with 3 fb^{-1} of integrated luminosity. For $m_{\tilde{g}} \approx 200$ GeV, the corrections are most apparent for $m_{\tilde{Q}} < 150$ GeV. If $m_{\tilde{g}} > 225$ GeV, then even with light squarks, the correction will be difficult to observe.

IV. CONCLUSIONS

The supersymmetric QCD correction to the top quark cross section, as measured at the Tevatron, has been calculated. We present analytic results for a minimal supersymmetric model that depends only on the masses of the superpartners and their mixing. We obtain numerical results for the total correction for all masses $m_{\tilde{g}} > 150$ GeV, $m_{\tilde{q}} > 50$ GeV and $m_{\tilde{t}} > 50$ GeV. Corrections are found to be as large as $+44\%$ for a gluino of mass $m_{\tilde{g}} = 200$ GeV, when $m_{\tilde{q}} = m_{\tilde{t}} = 50$ GeV. If light top squarks $m_{\tilde{t}} < 150$ GeV exist, then the correction should be observable with 10 fb^{-1} at the Tevatron for $m_{\tilde{g}} < 400$ GeV, even if the heavy squarks decouple. If all of the squarks remain light, then the correction is significant even if the gluinos decouple.

Should the gluino mass turn out to be near the current experimental limits, a gluino-pair threshold may be found near the top-quark production threshold. The advantage of looking for a cusp in the $t\bar{t}$ invariant mass distribution, is that the dependence on the modelling of SUSY decays is eliminated. When invariant mass resolutions and smearings are accounted for, this search will be very challenging. It is reasonable to expect that at least 10 fb^{-1} of integrated luminosity would be required to find a cusp for the best case of $m_{\tilde{g}} \approx 200$ GeV, and $m_{\tilde{Q}} < 150$ GeV. Despite the difficulties in detection, virtual SUSY thresholds are common in quark production [23]. A full detector-based analysis of these threshold regions would help determine the experimental significance of our results.

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