Prospects for the Precision Measurement of α_s

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ABSTRACT

The prospects for the measurement of the strong coupling constant $\alpha_{\overline{\rm MS}}(M_Z)$ to a relative uncertainty of $1\,\%$ are discussed. Particular emphasis is placed on the implications relating to future High Energy Physics facilities.

I. INTRODUCTION

Quantum Chromodynamics (QCD), the theory of the strong interaction, has a single free parameter, the strong coupling α_S. The coupling depends on the renormalization scheme and the energy scale, Q. Once $\alpha_S(Q)$ is determined from an experimentally measured process, any other process mediated by the strong interaction can be calculated to arbitrary accuracy, at least in principle. Most determinations of α_S are based on perturbative QCD, where it is conventional to evaluate the coupling in the $\overline{\rm MS}$ scheme, which is only defined in perturbation theory. Furthermore, it is also customary to choose the Z^0 mass, M_Z, as the reference scale. We shall adhere to these conventions and quote, for the most part, $\alpha_{\overline{\rm MS}}(M_Z)$ in our discussions.

A precise measurement of α_S is motivated by a number of considerations:

1. The couplings of the electroweak theory, $\alpha_{\rm em}$ and $\sin^2\theta_W$, have been determined with a precision of about $0.1\,\%$. In contrast, the strong coupling is presently known only to about $5\,\%$. It is pertinent to improve the accuracy with which the strong coupling has been measured in order to place it on a more equal basis with respect to the other interactions. For example, the current accuracy of α_S measurements is one of the main limitations on Standard Model electroweak tests at LEP and SLC [1]. In addition, attempts to constrain Grand Unified models, from the convergence of the standard model SU(3), SU(2) and U(1) couplings at a Grand Unification Scale, are similarly limited by the accuracy with which α_S has been measured.

2. QCD with its one parameter, α_S, must account for the rich phenomenology which is attributed to the strong interaction, including perturbative and nonperturbative phenomena. A fundamental test then of QCD is the determination of α_S from experimental measurements which probe complementary processes. This test is only meaningful if the values of α_S being compared have been measured with similar, good accuracy.

3. The QCD β-function (which is known to three loops in the $\overline{\rm MS}$ scheme) determines the evolution of the coupling. Accurate measurements of α_S over a wide range of momenta provide an additional fundamental test of the theory. Tests of the QCD β-function constrain physics beyond the standard model, in particular models with additional colored particles. Measurements of the energy dependence of observables in a single experiment, such as jet variables at e^+e^- or $p\overline{p}$ colliders, can also test the QCD β-function.

The last two reasons given above for an accurate measurement of α_S emphasize that it is not sufficient to determine α_S using a single method, but that precise measurements are necessary using different processes and widely different Q^2 values.

For the presentation here, we consider the prospect to measure $\alpha_{\overline{\rm MS}}(M_Z)$ with $1\,\%$ accuracy. We attempt to identify those methods which offer the greatest potential for such precision.

Figure 1 [3] presents a summary of the most accurate measurements of α_S which are currently available. Measurements performed at Q^2 scales different from M_Z^2 have been evolved to Q^2=M_Z^2 using the three loop QCD β-function (in the $\overline{\rm MS}$ scheme). All determinations of α_S receive contributions from theoretical systematic errors. These are, in many cases, the dominant sources of uncertainty. In general, they are difficult to estimate. In determinations based on perturbative QCD, sources of such errors are the truncation of the perturbative series and nonperturbative effects (such as hadronization). Most of the perturbative calculations have been carried out to next-to-leading order (NLO), and, in a few cases, to next-to-next-to-leading order (NNLO). The sources of theoretical uncertainty in determinations based on lattice QCD are discussed in section III. In a few cases, α_S results are limited by experimental uncertainties.

Theoretical uncertainties are in general not gaussian-distributed and are estimated from a variety of different methods. Consequently, the correlations between different α_S measurements are difficult to estimate. Given this difficulty, there is not a unique procedure to define a world average for the results shown in Figure 1. A number of proposals for world averages

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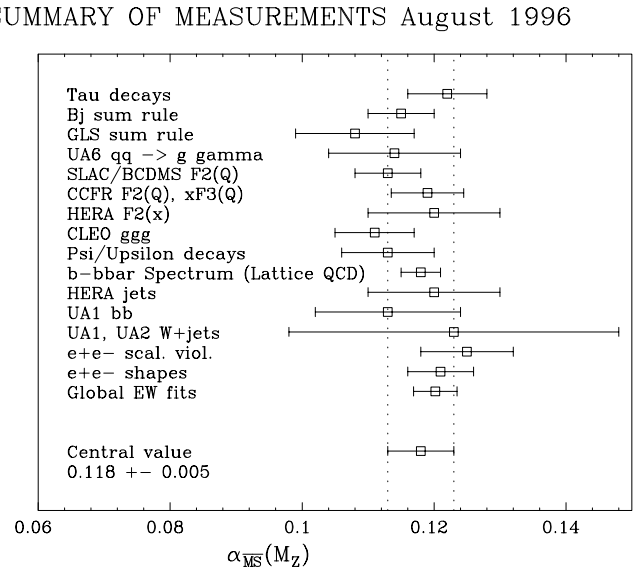

Figure 1: Summary of current accurate measurements of α_S, by technique. The α_S measurements are based on perturbative QCD, except where otherwise noted.

exist [2, 3, 4]. We shall use [3]

$$\alpha_{\overline{\rm MS}}(M_Z) = 0.118 \pm 0.005 \qquad (1)$$

as our nominal value for the world average.

In conducting this study, we considered a wide range of approaches to the measurement of $\alpha_{\overline{\rm MS}}(M_Z)$, eventually identifying four methods which exhibit the potential to yield results with about $1\,\%$ precision. The existence of perturbative calculations to at least next-to-next-to-leading order is a prerequisite for $1\,\%$ accuracy. A number of such calculations are in progress [5], and we shall assume that they will be available for the experimental measurements in question.

The four methods are: (1) the Q^2 evolution of the parity violating structure function xF_3, (2) the Gross-Llewellyn-Smith sum rule, (3) spin averaged splittings in the Υ and ψ systems, and (4) hadronic observables in e^+e^- annihilations. Items (1) and (2) in the above list are measured in deep inelastic neutrino scattering experiments. Item (3) is based on lattice QCD, all other methods use perturbative QCD. In addition to these four approaches, we found that two other methods, the Q^2 evolution of the parity non-violating structure function F_2 at high x, and the jet E_T spectrum in high energy proton-(anti)proton collisions, offer the possibility to determine α_S with good accuracy in regions of Q^2 which are complementary to those of the other measurements. The ultimate accuracy with which α_S can be determined using these last two techniques is uncertain at present, however. Therefore we do not include them in our final list of techniques which might yield an α_S result with $1\,\%$ precision.

The remainder of this report is devoted to a presentation of the various methods we considered for an α_S measurement, with a particular emphasis on the implications for future High Energy Physics facilities.

II. DEEP INELASTIC SCATTERING

Measurements of nucleon structure functions from the deep inelastic scattering (DIS) of a lepton on a nuclear target have yielded some of the most precise results for the strong coupling constant α_S. The field of nucleon structure functions remains very active, with major experiments in operation at CERN, DESY, Fermilab and SLAC. New structure function data, extending the measurements to previously unmeasured regions of the kinematic variables x and Q^2 and utilizing polarized targets and probes, have recently become available. These programs are expected to continue for at least the rest of this decade.

In this section, we assess which of these new data have the potential to yield an α_S measurement with $1\,\%$ accuracy. We do not review the formalism of structure functions or provide more than an indication of the methods used to determine α_S from them. References to existing literature with such information are given below where deemed appropriate.

A. DIS Nucleon Structure Functions

The basic kinematic variables of DIS are the Q^2 of the interaction, given by the difference in 4-momentum squared between the outgoing and incoming leptons, and the Feynman variable x defined by x=$Q^2/(2M(E\text{-}E'))$, where M is the mass of the target nucleon, with E and E' the energies of the initial- and final-state leptons, respectively, as measured in the laboratory frame. In the quark–parton model, x is interpreted to be the fraction of the nucleon's energy carried by the struck parton. Experiments in DIS measure the energy and scattering angle of the final-state lepton and/or recoiling hadronic system. The lepton probes are either electrically charged (electron e and muon μ probes) or neutral (neutrino ν or antineutrino $\overline{\nu}$ probes). The dominant mechanism for charged lepton scattering is single photon exchange in the t-channel between the lepton and nucleon system, while that for ν or $\overline{\nu}$ scattering is single $W^{\pm}$ exchange. For Q^2 values which approach or exceed M_W, $W^{\pm}$ and Z^0 exchange become important for charged lepton scattering. Nonperturbative QCD corrections to single-parton scattering contribute higher twist terms to the cross sections, which scale like $(1/Q^2)^{\rm n}$ (n=1,2,3$\cdots$) and are important at low Q^2 (typically Q^2<4-5 GeV^2).

Of the many structure functions necessary to describe DIS cross sections in their most general form, only three are candidates for a precise measurement of α_S: the structure functions $F_2(x,Q^2)$, $F_3(x,Q^2)$ and $g_1(x,Q^2)$. F_2 is measured from the neutral current cross section for unpolarized charged leptons to scatter from unpolarized targets and from the sum of the charged current cross sections for neutrinos ν and antineutrinos $\overline{\nu}$ to scatter from unpolarized targets. F_3 is measured from the difference between the charged current ν and $\overline{\nu}$ cross sections for scattering from unpolarized targets. g_1 is measured from the asymmetry in the cross sections for longitudinally polarized charged leptons to scatter from polarized targets if the beam and target polarizations are parallel, compared to the case that they are antiparallel, and from the corresponding asymmetry for targets which are polarized perpendicular to the beam polarization

directions [6]. The techniques that have been used to determine α_S using F_2, F_3 and g_1 are mentioned in the following section.

Structure functions can be resolved into color singlet and color non-singlet components. In QCD, the singlet and non-singlet terms evolve differently with Q^2 [7]. The singlet component receives a contribution from gluon splitting into $q\overline{q}$ pairs. As a consequence, the Q^2 evolution of the color singlet term depends not only on the running coupling constant $\alpha_S(Q^2)$ but also on the probability for gluon splitting, given by the gluon distribution function $g(x,Q^2)$. This dependence on $g(x,Q^2)$ is not important if x is larger than about 0.25 because the probability for gluon splitting at large x is small. Gluon splitting does not contribute to the non-singlet component of the structure functions: the Q^2 evolution of this term depends on $\alpha_S(Q^2)$ only, irrespective of the x range. Depending on the nature of the target (e.g. deuterium d_2 or hydrogen h_2), F_2 is either a pure singlet or a mixture of a singlet and a non-singlet, whereas F_3 is always a pure non-singlet.

B. Methods used to Determine α_S from DIS Structure Functions

The following methods have been used to determine α_S using the F_2, F_3 and g_1 structure functions:

1. the Q^2 evolution of F_2 at high x values, measured in charged lepton scattering ($4\,\%$) [8];
2. the same method as given in item 1, except at low x values ($11\,\%$) [9];
3. the Q^2 evolution of F_3 multiplied by the kinematic variable x, i.e. the evolution of xF_3 ($5\,\%$) [10];
4. the Gross, Llewellyn-Smith (GLS) sum rule, based on F_3 at fixed Q^2, integrated over all x values ($7\,\%$) [11];
5. the Bjorken sum rule, based on the difference between the g_1 structure functions of protons and neutrons at fixed Q^2, integrated over all x values ($4\,\%$) [12];
6. the shape of F_2 from charged lepton scattering in the limit of very large Q^2 and very small x values ($9\,\%$) [13].

The reference given after each item refers to the most precise result available for the method. This precision itself, $\Delta\alpha_{\overline{\rm MS}}(M_Z)/\alpha_{\overline{\rm MS}}(M_Z)$, is given by the number in parentheses, where $\alpha_{\overline{\rm MS}}(M_Z)$ is the value of α_S after it has been evolved to the scale of the Z^0 mass and $\Delta\alpha_{\overline{\rm MS}}(M_Z)$ is the corresponding uncertainty including statistical and systematic terms.

In addition to the methods listed above, DIS experiments have measured α_S using one technique which is not based on structure functions: the measurement of α_S using jet rates [14]. This method is very similar to the one based on event shapes from e^+e^- annihilations and has similar sources of systematic uncertainty. The present accuracy of the result for $\alpha_{\overline{\rm MS}}(M_Z)$ from DIS jet rates is about $8\,\%$. It is not likely that this method will yield a result for $\alpha_{\overline{\rm MS}}(M_Z)$ with precision better than about $5\,\%$ unless a next-to-next-to leading order QCD calculation becomes available. The overall situation for this measurement is similar to that for jet rates from e^+e^- collisions and we will not discuss it further.

From the above list, it is seen that the most precise DIS results for α_S are obtained from the Q^2 evolution of F_2 at high x (item 1), the Q^2 evolution of xF_3 (item 3), the Bjorken sum rule (item 5), and – with somewhat less precision at present – the GLS sum rule (item 4). It is of note that all three structure functions F_2, F_3 and g_1 contribute at least one measurement with 4-5 % accuracy, illustrating the strength of the complementarity offered by the unpolarized charged lepton, neutrino, and polarized charged lepton programs. The results utilizing the Q^2 evolution of F_2 at low x (item 2) and the shape of F_2 (item 6) are less accurate. Method 2 is unlikely to provide a precise result for α_S in the future, since the Q^2 evolution of the singlet component at small x depends on the gluon distribution function $g(x,Q^2)$, as mentioned above: this situation will not change for future data sets. If data are collected using different nuclear targets so that the singlet and non-singlet components of F_2 can be separated, the evolution of the non-singlet component of F_2 at relatively small x values could still be a viable method for an accurate α_S result: this was not found to be the case in [9], however, which included such an analysis. It is more difficult to assess the future status of the α_S result based on method 6 since this method has only recently been proposed. This method is based on the asymptotic behavior of the QCD resumed prediction for F_2 at large Q^2 and small x and has been applied to HERA data. The dominant uncertainty arises from the ambiguity in the choice of the renormalization and factorization scales [13]. This suggests that a reduction in the uncertainty of $\alpha_{\overline{\rm MS}}(M_Z)$ below the $5\,\%$ level will require the inclusion of subleading terms, the prospects for which are unknown. Further theoretical understanding of this method will probably be required before it can be used to accurately measure α_S. We will not consider this method further. The remaining discussion on the prospects for a precise α_S measurement from DIS therefore concentrates on items 1, 3, 4 and 5 in the above list.

C. Future Prospects for a Precise α_S Measurement from DIS

The future facilities which we consider for the purpose of evaluating the potential for a $1\,\%$ measurement of $\alpha_{\overline{\rm MS}}(M_Z)$ from DIS are the following:

1. HERA with a luminosity upgrade, able to deliver data samples of about 150 pb^{-1} per year, yielding a total data sample for the HERA experiments of 500-1000 pb^{-1};
2. an electron–hadron collider utilizing the LHC, referred to as "LEP$\times$LHC" in the following;
3. a ν ($\overline{\nu}$) beam from the Tevatron with upgraded luminosity, i.e. Tevatron "Run 2" and TeV33, available for fixed target experiments; it should be emphasized that the prospective fixed target neutrino facility under consideration here would make use of the *full energy* Tevatron beam;
4. a ν ($\overline{\nu}$) beam from the LHC, available for fixed target experiments;

5. future experiments measuring the polarized structure function g_1.

The facilities listed above have often been presented as natural extensions to the HERA, Tevatron, and LHC programs, with the possible exception of a fixed target facility at the LHC. It is not clear whether it is feasible to incorporate a fixed target neutrino facility into the LHC program.

We next discuss each of items 1, 3, 4 and 5 from section II.B in the context of these future facilities.

Q^2 Evolution of F_2 at High x

A measurement of α_S from the Q^2 evolution of F_2 is best performed using charged lepton scattering on unpolarized targets (for a review of this method, see [15]). The relevant future facilities for this measurement are an upgraded HERA and LEP$\times$LHC.

An α_S measurement based on the evolution of F_2 is not possible using the current HERA data sample because of the scarcity of data at x values above about 0.20. HERA has kinematic access to x values up to about 0.50 for Q^2 values greater than 10^3 GeV^2, however. For the purposes of an α_S measurement, HERA data at $Q^2 \sim 10^4$ GeV^2 and $x \sim 0.50$ are interesting because the large x value ensures suppression of the contribution from the gluon distribution function, while the Q^2 value is similar to that in Z^0 decays: this offers the opportunity for a direct comparison of the DIS results with those from e^+e^- Z^0 experiments. The region $Q^2 \sim 10^4$ GeV^2, $x \sim 0.50$ is near the kinematic limit of HERA, making it likely that data samples of about 1000 pb^{-1} will be necessary for an accurate α_S measurement based on the evolution of F_2. Furthermore, weak effects due to Z^0 exchange contribute to the neutral current cross sections for such Q^2 values. It will be necessary to combine electron–proton and positron-proton data in order to correct for the weak interference terms, leading to additional possibilities for systematic error. It has been estimated [16] that an uncertainty on $\alpha_{\overline{\rm MS}}(M_Z)$ of about 0.002 might ultimately be achieved from measurements of the evolution of F_2 at HERA, implying a precision of 1.5-2.0 %. Such a precision may require a combination of HERA and fixed target results for F_2, however [16].

Another possibility which has been envisioned is to operate HERA with electron–deuteron collisions. Comparison of the electron–proton and electron–deuteron data would allow the singlet component of F_2 to be extracted. A recent study [16] implies that this method could provide an improvement of about 25 % in the uncertainty of α_S, relative to what can be achieved using the electron–proton data alone.

The comments made above emphasize the relevance of considering electron–proton, positron–proton and electron–deuteron options for LEP$\times$LHC. A detailed estimate of the α_S precision achievable using LEP$\times$LHC has not yet been made. Assuming that there is not a great difference between the systematic sources of uncertainty at HERA and LEP$\times$LHC, it may be presumed that an α_S measurement with a precision on the order of 2 % is also possible at this latter facility. We note that LEP$\times$LHC offers the possibility for an accurate determination of α_S in the Q^2 range of 2-3$\cdot 10^5$ GeV^2, i.e. the same Q^2 range as a 500 GeV e^+e^- collider.

We therefore conclude that an α_S result with a precision of about 2 % is a possibility for HERA at a Q^2 value of about 10^4 GeV^2. Extrapolating to LEP$\times$LHC, a measurement of similar accuracy may be possible at a Q^2 value of about 2-3$\cdot 10^5$ GeV^2.

Q^2 Evolution of xF_3

The Q^2 evolution of xF_3 offers an advantageous method to measure α_S because it is independent of the gluon distribution function $g(x,Q^2)$ over the entire x range, as noted above. Measurements of xF_3 are best obtained using the difference between the ν and $\overline{\nu}$ cross sections for scattering on unpolarized targets [15]. These measurements require a fixed target program in order to collect adequate collision statistics. There is an active experiment at Fermilab (the NuTeV Collaboration), which is expected to improve the precision on $\alpha_{\overline{\rm MS}}(M_Z)$ to about 2.5 % within the next few years, using this method [17]. This is likely to become one of the world's most precise measurements of α_S and to remain so for some time. The uncertainties on the NuTeV result are roughly evenly divided between statistical and systematic sources, with the systematic uncertainty dominated by imprecise knowledge of the neutrino beam flux and of the calorimeter energy scale.

To improve the precision of the α_S result from this technique yet further, higher statistics from tagged neutrino beams will be necessary (tagged neutrino beams allow an event-by-event determination of the incident neutrino energy, as well as *apriori* knowledge of whether the interaction was caused by a neutrino or an antineutrino). The future facilities which could potentially provide beams for a precise xF_3 measurement of α_S are therefore the primary Tevatron beam with an upgraded luminosity, such as TeV33, and the LHC. Given the good result for α_S which is anticipated from the ongoing experiment, mentioned above, and given the improvement in accuracy expected from higher statistics and the introduction of event-by-event neutrino tagging, it is plausible that this method can provide an α_S measurement with 1 % precision. A study of the precision attainable at a LHC fixed target experiment has not yet been performed, however.

In conclusion, the method based on the Q^2 evolution of xF_3 is a strong candidate to provide a 1 % measurement of $\alpha_{\overline{\rm MS}}(M_Z)$, assuming that fixed target programs with tagged neutrino beams are available at either TeV33 or the LHC. We note that the necessary matrix elements are already available at NLO [5]. Since the β-function is known to three loops, all that is needed for a full NNLO analysis of α_S using the xF_3 method are the splitting functions calculated at NNLO. It is reasonable to expect that this result will become available and that the theoretical uncertainty will be below 1%.

GLS Sum Rule

The situation regarding the Gross, Llewellyn-Smith (GLS) sum rule [18] is similar to that discussed above for the Q^2 evolution of xF_3 since both methods rely on the F_3 structure func-

tion measured in neutrino fixed target experiments. The GLS sum rule is based on the integral

$$\int_0^1 \left[x F_3(x, Q^2) \right] \frac{\mathrm{d}x}{x} \ , \tag{2}$$

the QCD prediction for which has been calculated to $\mathcal{O}({\alpha_S}^3)$ (the next-to-next-to leading order in α_S). This is one of the few quantities calculated to such a high order in QCD perturbation theory. The integral (2) is evaluated experimentally using fairly low values of Q^2, which allows small values of x. (The small x region is particularly important because of the $1/x$ weighting in (2).) The Q^2 value for present experiments [10] is about 3 GeV^2. Because of the low Q^2 value, higher twist corrections are important. Furthermore, it is necessary to extrapolate into the unmeasured region at low x.

Like the result for α_S based on the evolution of xF_3, the current precision of the α_S measurement from the GLS sum rule is partially statistics-limited. There are several sources of experimental systematic uncertainty which are relevant for the GLS result and which are not relevant for the xF_3 evolution result, however: the measurement of the absolute cross sections for both the ν and $\overline{\nu}$ beams, the extrapolation into the low x region, and higher twist corrections. The NuTeV experiment expects to attain a precision of about $3\,\%$ for α_S using the GLS sum rule. At TeV33 and the LHC, higher statistics, larger Q^2 values (reducing the uncertainty from higher twists) and better low x reach (reducing the extrapolation uncertainty) should yield smaller statistical and systematic errors, making a measurement of $\alpha_{\overline{\mathrm{MS}}}(M_Z)$ with a precision of $1.5\,\%$ a possibility.

In conclusion, the GLS sum rule provides an independent method to determine α_S from a neutrino fixed target experiment, using the structure function F_3. Systematic uncertainties should be reduced at the higher Q^2 values offered by TeV33 or the LHC, relative to the current experiments, making an α_S measurement with a precision of about $1.5\,\%$ feasible.

Bjorken Sum Rule

Lastly, we consider the determination of α_S using the Bjorken sum rule [19]. The Bjorken sum rule is based on the quantity

$$\int_0^1 \left[g_1^p(x, Q^2) - g_1^n(x, Q^2) \right] \mathrm{d}x \ , \tag{3}$$

where g_1^p and g_1^n are the g_1 structure functions for proton and neutron targets, respectively. The Bjorken sum rule method for determining α_S differs from the others discussed here in that it is based on polarized cross sections. The method resembles the GLS sum rule technique, however, because it is based on an integral over x of a structure function measured at fixed Q^2, utilizes low Q^2 measurements (for current experiments, the Q^2 value is about 2 GeV^2), requires extrapolation into the unmeasured x regions, and has a QCD prediction available at the next-to-next-to leading order.

Like the method based on the shape of F_2 (item 6 in section II.B), it is somewhat difficult to assess the future status of the α_S result obtainable from the Bjorken sum rule because it is only recently that this method has been used to determine α_S. The current result (about $4\,\%$ accuracy [12]) is quite precise by current standards, however. Given that additional polarized structure function data are currently being collected at CERN, DESY and SLAC, and that additional experiments are being planned, it can be anticipated that a reduction in the uncertainty in α_S from this method will be possible. Many sources of systematic uncertainty (higher twists, extrapolation into the unmeasured x region, measurement of the absolute cross sections) are common between this method and the GLS one. The Bjorken sum rule measurement is complicated by its reliance on polarized targets and probes, however, and thus has sources of systematic uncertainty which are not present for the GLS measurement. Therefore, we presume that the ultimate accuracy for $\alpha_{\overline{\mathrm{MS}}}(M_Z)$ achievable from the Bjorken sum rule for experiments currently running or being planned lies between the current precision ($4\,\%$) and that which we estimate will be achievable from the GLS sum rule ($1.5\,\%$). Thus, an estimate of about $2.5\,\%$ precision seems justifiable.

Although no study has been done at this point, we wish to emphasize that some of the systematics which degrade the accuracy of the Bjorken sum rule measurement of α_S, including those due to higher twist and the low x extrapolation, may improve with a high statistics, high energy beam. Such a beam would be available if a high energy e^+e^- collider were constructed with longitudinal polarization and a fixed-target capability. In this way, it is plausible that the Bjorken sum rule measurement could be more accurate than the estimate given above.

Table I: The estimated precision for $\alpha_{\overline{\mathrm{MS}}}(M_Z)$ attainable at future DIS experiments.

Method	*Precision*	*Facility*
Q^2 evolution of F_2 at high x	$2\,\%$	HERA, LEP$\times$LHC
Q^2 evolution of F_3	$1\,\%$	TeV33 fixed target, LHC fixed target
GLS sum rule	$1.5\,\%$	TeV33 fixed target, LHC fixed target
Bjorken sum rule	$2.5\,\%$	Future polarized DIS experiments

D. Conclusion for a Precise α_S Result from DIS

Table I summarizes our estimates of the precision which might be attainable for $\alpha_{\overline{\mathrm{MS}}}(M_Z)$ from DIS experiments at future colliders. These estimates are mostly based on extrapolations from current experiments rather than on detailed studies of future facilities. The best prospect for a $1\,\%$ measurement of $\alpha_{\overline{\mathrm{MS}}}(M_Z)$ from a DIS experiment is from a fixed target neutrino facility at a hadron collider with high flux, tagged ν and $\overline{\nu}$ beams. The most promising measurement technique is the observation of the Q^2 evolution of xF_3.

We again emphasize, however, the importance of accurate α_S measurements at widely different Q^2 values. The DIS results

based on xF_3 offer the possibility for a precise measurement of α_S in the Q^2 range from about 5 to 20 GeV^2. Those based on the high x region of F_2 offer the possibility for an accurate measurement at a much larger Q^2 value, up to about 10^5 GeV^2 for LEP$\times$LHC.

III. THE HADRON SPECTRUM

Lattice QCD is, so far, the only systematic, first principles approach to nonperturbative QCD. The experimentally observed hadron spectrum, like the high-energy observables discussed in the other sections, provide us with information on the free parameter of QCD, α_S. Determinations of α_S from the experimentally observed hadron spectrum, based on lattice QCD, are thus complementary to determinations which are based on perturbative QCD.

While an introduction to lattice QCD is beyond the scope of this report (see [20] for pedagogical introductions and reviews), we shall, in the following, outline the strategy for determinations of α_S based on lattice QCD. In general, determinations of α_S can be divided into three steps:

The first step is always an experimental measurement. In α_s determinations based on perturbative QCD this might be a cross section or (ratio of) decay rates. In determinations based on lattice QCD this is usually a hadron mass or mass splitting, for example the mass of the ρ meson, or a better choice, spin-averaged splittings in the charmonium and bottomonium systems. In "lattice language" this step is often referred to as "setting the scale" (see section III.A).

The second step involves a choice of renormalization scheme. In perturbative QCD the standard choice is the $\overline{\rm MS}$ scheme. With lattice QCD a nonperturbative scheme may be chosen, and there are many candidates. In order to compare with perturbative QCD, any such scheme should be accessible to perturbative calculations (without excessive effort).

Finally, the third step is an assessment of the experimental and theoretical errors associated with the strong coupling determination. This is of course the most important (and sometimes also the most controversial) step as it allows us to distinguish and weight different determinations. The experimental errors on hadron masses are negligibly small in lattice determinations of α_s at this point. The theoretical errors that are part of α_s determinations based on perturbative QCD include higher order terms in the truncated perturbative series and the associated dependence on the renormalization scale, and hadronization or other generic nonperturbative effects. In lattice QCD the theoretical errors include (but are not limited to) discretization errors (due to the finite lattice spacing, $a \neq 0$), finite volume effects, and errors associated with the partial or total omission of sea quarks.

The consideration of systematic uncertainties should guide us towards quantities where these uncertainties are controlled, for a reliable determination of α_s. As has been argued by Lepage [21], quarkonia are among the easiest systems to study with lattice QCD, since systematic errors can be analyzed easily with potential models if not by brute force.

Finite-volume errors are much easier to control for quarkonia than for light hadrons, since quarkonia are smaller. Lattice-spacing errors, on the other hand, can be larger for quarkonia and need to be considered. This error can be controlled by studying the lattice spacing dependence of physical quantities (in physical units). The lattice spacing is reduced (while keeping the physical volume of the lattice fixed) until the error is under sufficient control. The source of the lattice spacing dependence are the discretizations used in the lattice lagrangian (or action). Thus, an alternative to reducing the lattice spacing in order to control this systematic error is the use of better discretizations. This procedure is generally referred to as improving the action. For quarkonia, the size of lattice-spacing errors in a numerical simulation can be *anticipated* by calculating expectation values of the corresponding operators using potential-model wave functions. They are therefore ideal systems to test and establish improvement techniques.

A lot of the work of phenomenological relevance is done in what is generally referred to as the "quenched" (and sometimes as the "valence") approximation. In this approximation gluons are not allowed to split into quark - anti-quark pairs (sea quarks). This introduces a systematic error into the calculation. However, for quarkonia, a number of calculations now exist which partially include the effect of sea quarks, thereby significantly reducing this systematic error. This is further discussed in sections III.A and III.C.

A. Determination of the Lattice Spacing and the Quarkonium Spectrum

The experimental input to the strong coupling determination is a mass or mass splitting, from which by comparison with the corresponding lattice quantity the lattice spacing, a, is determined in physical units. For this purpose, one should identify quantities that are insensitive to lattice errors. In quarkonia, spin-averaged splittings are good candidates. The experimentally observed 1P-1S and 2S-1S splittings depend only mildly on the quark mass (for masses between m_b and m_c). Figure 2 shows the observed mass dependence of the 1P-1S splitting in a lattice QCD calculation. The comparison between results from different lattice actions illustrates that higher-order lattice-spacing errors for these splittings are small [22, 23].

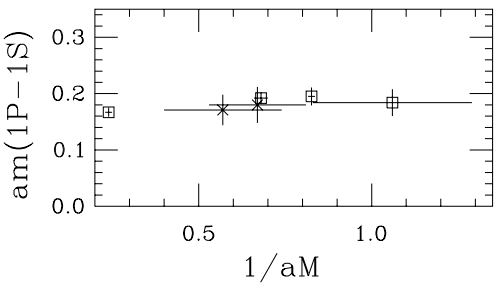

Figure 2: The 1P-1S splitting as a function of the 1S mass (statistical errors only) from Ref. [23]; □: $\mathcal{O}(a^2)$ errors; ×: $\mathcal{O}(a)$ errors.

Two different formulations for fermions have been used in lattice calculations of the quarkonia spectra. In the non-relativistic limit the QCD action can be written as an expansion in powers of v^2 (or $1/m$), where v is the velocity of the heavy quark inside the bound state [24]; Henceforth, this approach shall be referred to as NRQCD. Lepage and collaborators [25] have adapted this formalism to the lattice regulator. Several groups have performed numerical calculations of quarkonia in this approach. In Refs. [26, 27] the NRQCD action is used to calculate the $b\bar{b}$ and $c\bar{c}$ spectra, including terms up to $\mathcal{O}(mv^4)$ and $\mathcal{O}(a^2)$. In addition to calculations in the quenched approximation, this group is also using gauge configurations that include two flavors of sea quarks with mass $m_q \sim \frac{1}{2}m_s$ to calculate the $b\bar{b}$ spectrum [22, 28]. The leading order NRQCD action is used in Ref. [29] for a calculation of the $b\bar{b}$ spectrum in the quenched approximation.

The Fermilab group [30] developed a generalization of previous approaches, which encompasses the non-relativistic limit for heavy quarks as well as Wilson's relativistic action for light quarks. Lattice-spacing errors are analyzed for quarks with arbitrary mass. Ref. [23] uses this approach to calculate the $b\bar{b}$ and $c\bar{c}$ spectra in the quenched approximation. The authors considered the effect of reducing lattice-spacing errors from $\mathcal{O}(a)$ to $\mathcal{O}(a^2)$. The SCRI collaboration [31] is also using this approach for a calculation of the $b\bar{b}$ spectrum using the same gauge configurations as the NRQCD collaboration with $n_f = 2$ and an improved fermion action (with $\mathcal{O}(a^2)$ errors).

All but one group use gauge configurations generated with the Wilson action, leaving $\mathcal{O}(a^2)$ lattice-spacing errors in the results. The lattice spacings, in this case, are in the range $a \simeq 0.05 - 0.2$ fm. Ref. [32] uses an improved gauge action together with a non-relativistic quark action improved to the same order (but without spin-dependent terms) on coarse ($a \simeq 0.4 - 0.24$ fm) lattices. The results for the $b\bar{b}$ and $c\bar{c}$ spectra from all groups are summarized in Figures 3 and 4.

The agreement between the experimentally-observed spectrum and lattice QCD calculations is impressive. As indicated in the preceding paragraphs, the lattice artifacts are different for all groups. Figures 3 and 4 therefore emphasize the level of control over systematic errors.

Results with two flavors of degenerate sea quarks have now become available from a number of groups [22, 33, 34, 28], with lattice-spacing and finite-volume errors similar to the quenched calculations, significantly reducing this systematic error. However, several systematic effects associated with the inclusion of sea quarks still need to be studied further. They include the dependence of the quarkonium spectrum on the number of flavors of sea quarks, and the sea-quark action (staggered vs. Wilson). The inclusion of sea quarks with realistic light-quark masses is very difficult and can, at present, only be done by extrapolation from $m_q \simeq 0.3 - 0.5m_s$ to $m_{u,d}$. However, the dependence of the quarkonium splittings on the sea quark masses can be analyzed with chiral perturbation theory [35] to guide the extrapolation.

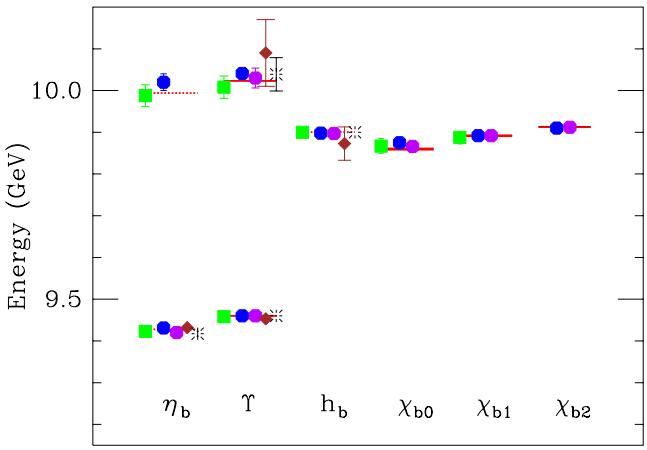

Figure 3: A comparison of lattice QCD results for the $b\bar{b}$ spectrum (statistical errors only). –: Experiment; □: FNAL [23]; ○: NRQCD ($n_f = 0$) [26]; •: NRQCD ($n_f = 2$) [22]; ◇: UK(NR)QCD [29]; ∗: SCRI [31].

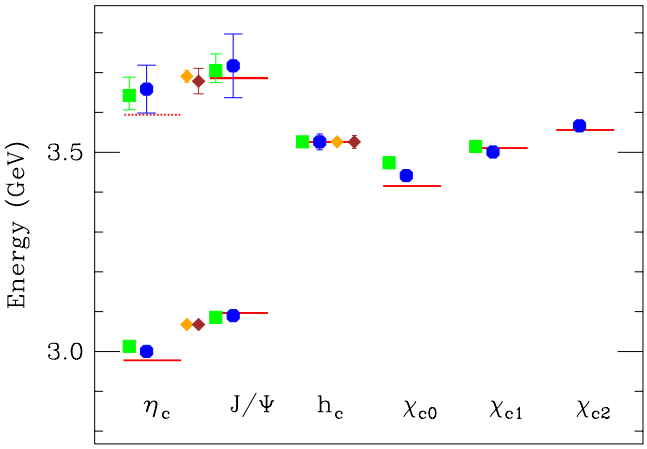

Figure 4: A comparison of lattice QCD results for the $c\bar{c}$ spectrum (statistical errors only). –: Experiment; □: FNAL [23]; ○: NRQCD ($n_f = 0$) [27]; ◇: ADHLM [32].

B. Definition of a Renormalized Coupling

Within the framework of lattice QCD the conversion from the bare to a renormalized coupling can, in principle, be made nonperturbatively. In the definition of a renormalized coupling, systematic uncertainties should be controllable, and at short distances, its (perturbative) relation to other conventional definitions calculable. For example, the renormalized coupling, α_V, can be defined from the nonperturbatively computed heavy-quark potential [36]. An elegant approach has been developed in Ref. [37], where a renormalized coupling is defined nonperturbatively through the Schrödinger functional. The authors compute the evolution of the coupling nonperturbatively using a finite size scaling technique, which allows them to vary the momentum scales by an order of magnitude. The same technique has also been applied to the renormalized coupling defined from twisted Polyakov loops [38]. The numerical calculations include only gluons at the moment. However, the inclusion of fermions is possible. Once such simulations become available they should yield very accurate information on α_S and its evolution. A renormalized coupling can also be defined from the three-gluon vertex, suitably evaluated on the lattice [39].

An alternative is to define a renormalized coupling through short distance lattice quantities, like small Wilson loops or Creutz ratios which can be calculated perturbatively and by numerical simulation. For example, the coupling defined from the plaquette (the smallest Wilson loop on the lattice), $\alpha_\Box = -3\ln\langle \mathrm{Tr}\, U_\Box\rangle/4\pi$, can be expressed as [40]:

$$\alpha_\Box = \alpha_P(q)[1-(1.19+0.07\,n_f)\alpha_P(q)] \qquad (4)$$

at $q = 3.41/a$, close to the ultraviolet cut-off. The coupling α_P is chosen such that it equals α_V at one-loop:

$$\alpha_P(q) = \alpha_V(q) + \mathcal{O}(\alpha_V^3)\ . \qquad (5)$$

α_P is related to the more commonly used $\overline{\mathrm{MS}}$ coupling by

$$\alpha_{\overline{\mathrm{MS}}}(Q) = \alpha_P(e^{5/6}Q)\left(1+\tfrac{2}{\pi}\alpha_P + c_2(n_f)\alpha_P^2 + \ldots\right)\ . \qquad (6)$$

The size of higher-order corrections associated with the above defined coupling constants can be tested by comparing perturbative predictions for short-distance lattice quantities with nonperturbative results [40]. The comparison of the nonperturbatively calculated coupling of Ref. [37] with the perturbative predictions for this coupling using Eq. (4) is an additional consistency test.

The relation of α_P to $\alpha_{\overline{\mathrm{MS}}}$,Eq. (6), has recently been calculated to two loops [41, 42] in the quenched approximation (no sea quarks, $n_f = 0$):

$$c_2(n_f = 0) = 0.96\ . \qquad (7)$$

This term shifts $\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z)$ by $+0.002$. Because of the unknown n_f dependence in the two-loop term, c_2, the perturbative uncertainty is still ± 0.002 (at M_Z). The extension of the two-loop calculation to $n_f \neq 0$ will reduce this uncertainty to well below $1\,\%$ for $\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z)$.

C. Sea Quark Effects

Calculations that properly include all sea-quark effects do not yet exist. If we want to make contact with the "real world", these effects have to be estimated phenomenologically or extrapolated away.

The phenomenological correction necessary to account for the sea-quark effects omitted in calculations of quarkonia that use the quenched approximation gives rise to the dominant systematic error in this calculation [43, 44]. By demanding that, say, the spin-averaged 1P-1S splitting calculated on the lattice reproduce the experimentally observed one (which sets the lattice spacing, a^{-1}, in physical units), the effective coupling of the quenched potential is in effect matched to the coupling of the effective three flavor potential at the typical momentum scale of the quarkonium states in question. The difference in the evolution of the zero flavor and 3,4 flavor couplings from the effective low-energy scale to the ultraviolet cut-off, where α_S is determined, is the perturbative estimate of the correction.

For comparison with other determinations of α_S, the $\overline{\mathrm{MS}}$ coupling can be evolved to the Z^0 mass scale. An average of Refs. [43, 44] yields for α_S from calculations in the quenched approximation:

$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.110 \pm 0.006\ . \qquad (8)$$

The phenomenological correction described in the previous paragraph has been tested from first principles in Ref. [33]. The 2-loop evolution of $n_f = 0$ and $n_f = 2$ $\overline{\mathrm{MS}}$ couplings – extracted from calculations of the $c\bar{c}$ spectrum using the Wilson action in the quenched approximation and with two flavors of sea quarks respectively – to the low-energy scale gives consistent results. After correcting the two flavor result to $n_f = 3$ in the same manner as before and evolving $\alpha_{\overline{\mathrm{MS}}}$ to the Z^0 mass, they find [33]

$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.111 \pm 0.005 \qquad (9)$$

in good agreement with the previous result in Eq. (8). The total error is now dominated by the rather large statistical errors and the perturbative uncertainty.

The most accurate result to date comes from the NRQCD collaboration [22, 28]. They use results for α_S from the $b\bar{b}$ spectrum with 0 and two flavors of sea quarks to extrapolate the inverse coupling to the physical three flavor case directly at the ultraviolet momentum, $q = 3.41/a$. They obtain a result consistent with the old procedure. Recently, they have begun to study the dependence of α_S on the masses of the sea quarks. Their preliminary result is:

$$\alpha^{(3)}_P(8.2\,\mathrm{GeV}) = 0.195 \pm 0.003 \pm 0.001 \pm 0.004\ . \qquad (10)$$

The first error is statistics, the second error an estimate of residual cut-off effects and the third (dominant) error is due to the quark mass dependence. The conversion to $\overline{\mathrm{MS}}$ (including the 2-loop term in Eq. (6) and evolution to the Z^0 mass then gives:

$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.118 \pm 0.003\ , \qquad (11)$$

where the error now also includes the perturbative uncertainty from eq. (6). A similar analysis is performed in Ref. [34] on the same gauge configurations but using the Wilson action for a calculation of the $c\bar{c}$ spectrum. The result for the coupling is consistent with Refs. [22, 33].

The preliminary calculation of the SCRI collaboration [31] ($n_f = 2$) can be combined with the result of Ref. [23]. Using the same analysis as in Ref. [22] gives [28]

$$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.116 \pm 0.003 \quad , \tag{12}$$

nicely consistent with Eq. (11). Clearly, more work is needed to confirm the results of Eqs. (11) and (12), especially in calculations with heavy quark actions based on Ref. [30]. In particular, the systematic errors associated with the inclusion of sea quarks into the simulation have to be checked, as outlined in section III.A.

D. Conclusions

Phenomenological corrections are a necessary evil that enter most coupling constant determinations. In contrast, lattice QCD calculations with control over all sources of systematic error can, at least in principle, yield truly first-principles determinations of α_S from the experimentally observed hadron spectrum.

At present, determinations of α_S from the experimentally measured quarkonia spectra using lattice QCD are comparable in reliability and accuracy to other determinations based on perturbative QCD from high energy experiments (see Figure 1). The phenomenological corrections for the most important sources of systematic errors in lattice QCD calculations of quarkonia have already been replaced by first principles calculations. This has led to a significant increase in the accuracy of α_S determinations from quarkonia.

Still lacking for a first-principles result is the proper inclusion of sea quarks. A difficult problem in this context is the inclusion of sea quarks with physical light quark masses. At present, this can only be achieved by extrapolation (from $m_q \simeq 0.3 - 0.5 m_s$ to $m_{u,d}$). Given sufficient numerical results on the light quark mass dependence, chiral perturbation theory can be used for the extrapolation [35]. These calculations can most likely be performed with currently available computational resources leading to first-principles results for the quarkonia spectra. They should, in turn, yield determinations of $\alpha_{\overline{\rm MS}}(M_Z)$ with a total uncertainty below $1\,\%$.

IV. ELECTRON-POSITRON ANNIHILATION

The measurement of α_S via hadronic observables in e^+e^- annihilation is a mature subject. Prospects for the accurate measurement of α_S in high-energy e^+e^- annihilation have been under discussion for some time [45]. To assess the potential for an α_S measurement from this method, it is useful to examine an experimental analysis in detail, to assess those areas in which the uncertainties might be reduced in the future. For this purpose, we choose a recent comprehensive study of α_S published by the SLD Collaboration [46]. Similar studies have been published by the LEP experiments [47]. The SLD result

$$\alpha_{\overline{\rm MS}}(M_Z) = 0.1200 \pm 0.0025 \pm 0.0078 \tag{13}$$

was derived from the consideration of 15 different infrared-safe hadronic observables, including various event shape parameters, jet rates derived with several different jet finding schemes, and energy-energy correlations. The ± 0.0025 experimental error received contributions of ± 0.0009 from event statistics, and ± 0.0024 from detector-related uncertainties. The ± 0.0078 theoretical uncertainty resulted from contributions of ± 0.0024 from uncertainties in the hadronization process, and ± 0.0074 from missing higher orders in the perturbative calculation of the 15 observables. Currently, all 15 observables have been calculated to next-to-leading order in α_S. In addition, for six of the 15 observables, the leading and sub-leading logarithms have been resummed and combined with the next-to-leading order calculations.

This breakdown of the uncertainty provides a basis for estimating the accuracy of a similar measurement of α_S at an electron–positron collider of cms energy $\sqrt{s} = 500$ GeV, such as the proposed NLC. Statistically, the SLD measurement was performed with the sample of 37,000 hadronic ($e^+e^- \to q\overline{q}$) events remaining in the 1993 SLD event sample after the application of hadronic event selection cuts. At a design luminosity of 5×10^{33} cm^{-2}sec^{-1}, with a Born-level cross section of 3.1 pb, an NLC detector would collect approximately 150,000 $e^+e^- \to q\overline{q}$ events in a "Snowmass" year of 10^7 seconds. The effects of initial state radiation and beamstrahlung, and inefficiencies introduced by event selection (to be discussed below), reduce this to approximately 25,000 $e^+e^- \to q\overline{q}$ events per year at $\sqrt{s} \simeq 500$ GeV, adequate for a statistical precision of $\pm 1\,\%$ on the value of α_S at that scale. A well designed NLC Detector calorimeter should permit a substantial reduction in the $\pm 2\,\%$ detector uncertainty.

The determination of α_S involves the comparison of the hadronic observables with parton-level perturbative calculations which depend upon α_S. The relationship between the parton-level calculations and the final state observables is thus obscured by the fragmentation process. This introduces a correction, and corresponding uncertainty, which must be applied to the extracted value of α_S. It is generally expected [48] that effects which alter the relation between the perturbative parton-level calculations of observables, and the actual hadron-level observables, scale as an inverse power of the momentum transfer Q, so that for some observable O,

$$\delta O \equiv O - O_{\rm pert} \quad \sim \quad \frac{a}{Q^n} \ . \tag{14}$$

Typically, n equals 1 or 2. On the other hand, the perturbative evolution of α_S scales roughly as $(\ln Q)^{-1}$. Thus, one expects the relative uncertainty on α_S due to fragmentation effects to scale as $\ln Q/Q$. As a result, the $\sim 10\,\%$ correction applied to the value of α_S extracted from hadronic observables at the Z^0 pole is expected to reduce to a $\sim 2\,\%$ correction at $\sqrt{s} = 500$ GeV, with an uncertainty of $1\,\%$ or less.

In addition to the fragmentation, the relationship between the perturbative parton-level calculation and the measured observables is compromised by missing higher orders in the perturbative expansion. In the SLD analysis, this uncertainty was estimated to be $\Delta\alpha_{\overline{\rm MS}}(M_Z) = \pm 0.0074$ by varying the renormalization scale of the perturbative calculation over a range permitted by consistency with the hadronic data, and observing the corresponding variation in the extracted value of α_S. Current perturbative calculations are done to order α_S^2; thus, uncertainties due to missing higher orders should scale as α_S^3, leading to an uncertainty of $\pm$0.003–0.004 at $\sqrt{s} = 500$ GeV. Evolving this back to the benchmark scale Q^2=M_Z^2 using the three-loop QCD β-function yields an uncertainty of $\pm$0.005–0.006, or 4–5 % relative, on the value of $\alpha_{\overline{\rm MS}}(M_Z)$ extracted from hadronic observables at the NLC. Should next-to-next-to-leading order perturbative calculations become available, it should thus be possible to approach the target uncertainty of $\pm 1\,\%$.

A final issue associated with the measurement of α_S in e^+e^- annihilation at large cms energy is that of identifying a clean sample of $e^+e^- \to q\overline{q}$ ($q \neq t$) events. At $\sqrt{s} = 500$ GeV, without event selection cuts, $q\overline{q}$ ($q \neq t$) production ($\sigma_{Born} = 3.1$ pb) has a substantially smaller cross section than W^+W^- production ($\sigma_{Born} = 7.0$ pb), as well as significant backgrounds from Z^0Z^0 ($\sigma_{Born} = 0.4$ pb) and $t\overline{t}$ ($\sigma_{Born} = 0.3$ pb) production. A study performed by the European Linear Collider QCD Working Group [49] identified a set of kinematic cuts which select an $83\,\%$ pure sample of $e^+e^- \to q\overline{q}$ ($q \neq t$) events. However, these cuts substantially impacted the hadronic distributions of the remaining $q\overline{q}$ events, leading to large ($\sim 20\,\%$) corrections to the extracted value of α_S. To this end, in preparation for the Snowmass Workshop, a Monte Carlo study [50] was undertaken in which one of the European Working Group kinematic cuts – the requirement that at least one hemisphere have a reconstructed invariant mass of less than $13\,\%$ of the visible energy in the event – was removed. Instead, events were used only if they were produced with the right-handed electron beam (to remove W^+W^- background), and if they gave no indication of the presence of B hadrons in the vertex detector (to eliminate $t\overline{t}$ background). For an electron beam polarization of $P_e = 80\,\%$ ($P_e = 90\,\%$), this yielded an $82\,\%$ ($87\,\%$) pure "Snowmass" sample of $e^+e^- \to q\overline{q}$ ($q \neq t$) events. A comparison of 3-jet rates between a pure Monte Carlo sample of $e^+e^- \to q\overline{q}$ ($q \neq t$) events, and the sample identified by the Snowmass cuts, indicated that corrections due to the Snowmass event selection are substantially less than $5\,\%$. Thus, with these cuts, the uncertainty on α_S due to the event selection process is expected to be well within the target of $\pm 1\,\%$. It should be noted that electron beams with $80\,\%$ polarization, and bunch populations exceeding that required for the operation of the NLC, are already in use at the SLAC Linear Collider, and that polarized running is part of the base-line proposal for the NLC [51].

As a final note, it has been pointed out [52] that the high luminosity of an e^+e^- linear collider, combined with the rise in the $e^+e^- \to q\overline{q}$ cross section with falling $\sqrt{s}$, may make it feasible to precisely constrain the evolution of α_S over a wide range of Q^2 in a single experiment. The execution of such a program would have an impact on the design of the high energy e^+e^- collider.

Thus, e^+e^- annihilation at high energy appears to be a promising avenue towards the measurement of $\alpha_{\overline{\rm MS}}(M_Z)$ to a relative uncertainty of $\pm 1\,\%$. Furthermore, the high momentum transfer scale associated with the measurement ($Q^2 \simeq s = (500\,{\rm GeV})^2$) makes this approach an important component of the program to constrain the possible anomalous running of α_S. For this accuracy to be achievable, next-to-next to leading order ($\mathcal{O}(\alpha_S^3)$) calculations of e^+e^- event shape observables will be required.

V. pp ($p\overline{p}$) COLLISIONS

The greatest potential to extend measurements of α_S to large values of the momentum transfer scale Q^2 resides with the pp ($p\overline{p}$) colliders. In adition, many approaches to the measurement of α_S in pp ($p\overline{p}$) collisions produce a range of values for α_S over a broad lever-arm in Q^2. For example, the inclusive jet E_T spectrum from the Tevatron extends out to almost 500 GeV (see Figure 5 [53]), providing sensitivity to α_S over a range in momentum transfer extending from 50 GeV to values nearly equivalent to that proposed for the next generation of electron–positron and electron–proton colliders. The LHC, currently scheduled to begin delivering beams in the middle of the next decade, will extend this reach to several TeV.

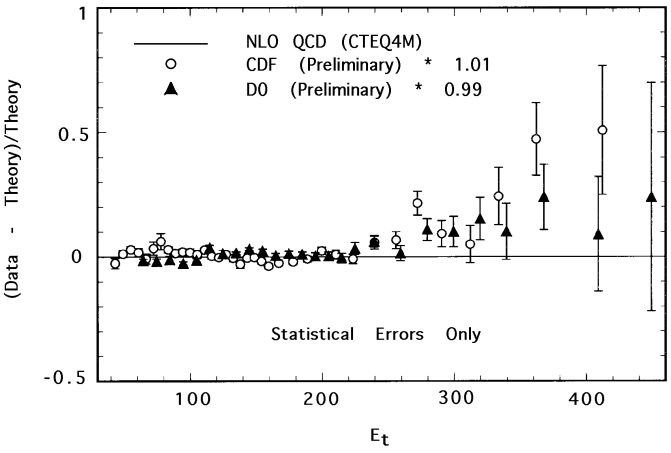

Figure 5: The preliminary CDF and D0 Run Ib data compared to NLO QCD using CTEQ4M parton distributions. Experimental points normalized as indicated. This figure is reproduced from Reference [53].

On the other hand, measurements of α_S from hadron colliders have not yet approached the level of accuracy achieved by the most accurate approaches. For example, a typical approach to constraining α_S in $p\overline{p}$ collisions is to study the ratio of W + 1 jet events to W + 0 jet events [54]. Experimental systematics, such as energy scale and resolution uncertainty, introduce large errors ($\pm \sim 0.015$) in the value of α_S extracted from this ratio. In addition, since gluons liberated from the nucleon sea can themselves form jets, the measurement is sensitive to the parton distributions used in calculating the W + 1 jet rate. For the most

recent measurement of this ratio [55], the D0 collaboration did not report a value for α_S, because of an inconsistency between the best fit value of α_S used in the parton distribution function and that used in the perturbative matrix element.

Current work on the measurement of α_S with $p\overline{p}$ data thus concentrates on developing approaches which remove or reduce the sensitivity of the method to variations in the parton distribution functions, and to experimental parameters. This work is still in its early stages, but a number of promising ideas are being pursued.

For example, a generalization of the W + jet method is the measurement of the jet cross section ratios

$$R_n^V = \frac{\sigma(V + (n+1)\text{jets})}{\sigma(V + n\text{jets})} \quad , \tag{15}$$

where $V = W^{\pm}$, Z^0 is a vector boson. The UA2 and D0 measurements involved R_0^V; for $n \neq 0$, however, the contribution from sea gluons, and thus the dependence on the parton distribution functions, largely cancels in the ratio. Another approach that is being pursued is the measurement of the $p_\perp$ spectrum of Z^0 production – for this measurement, there is no dependence on experimental errors such as hadronic energy scales and resolution, jet algorithm definitions, or hadronization, although the measurement still requires *apriori* knowledge of parton distribution functions, and only measures α_S at the single momentum transfer scale Q^2=M_Z^2. Finally, fits to the triple differential cross section [56]

$$\frac{d\sigma}{dE_T d\eta_1 d\eta_2} \propto \alpha_S^2 \{ f_{g_1}(x_1) f_{g_2}(x_2) A_{gg}(\eta^*) + f_{g_1}(x_1) f_2(x_2) A_{gq}(\eta^*) + f_2(x_1) f_2(x_2) A_{qq}(\eta^*) \}; \tag{16}$$

$$\eta^* = \frac{\eta_1 - \eta_2}{2}$$

$$x_{1,2} = \frac{2E_T}{\sqrt{s}} \left(e^{\pm\eta_1} + e^{\pm\eta_2} \right)$$

are being explored, which can simultaneously constrain α_S, the gluon distribution function $f_g(x)$, and the non-singlet quark distribution function $f_2(x)$, thus removing the uncertainty due to poorly constrained parton distribution functions.

All of these studies have only recently been started [57], inspired by the large data samples available with the completion of Tevatron Run I. Thus, it will be several years before the potential for the measurement of α_S at hadron colliders is fully understood. Finally, it should be noted that this method (like any other) requires that at least NNLO perturbative calculations be completed for a determination of $\alpha_{\overline{\rm MS}}(M_Z)$ with $1\,\%$ accuracy. However, most of the matrix elements needed here are identical to, or are limiting cases of, matrix elements necessary for NNLO calculations of hadronic observables in e^+e^- annihilation.

VI. CONCLUSIONS

Table II lists the methods we consider promising for accurate α_S determinations. The items listed above the double line are established methods for α_S measurements, and we can evaluate their potential for $1\,\%$ accuracy with reasonable confidence. The items listed below the double line are either expected to yield somewhat less accurate determinations of α_S, or they are less established methods which need further study to better evaluate their potential for α_S determinations with $1\,\%$ accuracy.

Traditionally, DIS at relatively low momentum transfer has produced precise determinations of α_S. In particular, measurements of α_S via the Q^2 evolution of xF_3 and the GLS sum rule are expected to each achieve experimental accuracies of 2-3 % in the upcoming run of the NuTeV Experiment, limited primarily by sample statistics, the uncertainty in the calorimeter energy scale, and the understanding of the composition of the incident neutrino beam. Thus a high flux tagged neutrino beamline derived from the *full energy* Tevatron primary beam, and eventually one of the LHC primary proton beams, is a strong candidate for a facility which will produce a $1\,\%$ measurement of α_S at low Q^2. At present, however, such a facility is not part of the future program of either laboratory. We also wish to mention the approach of measuring α_S via the Bjorken sum rule in polarized deep inelastic scattering. It is a relatively new method, but could yield a result as accurate as 2-3 %. Certain systematic limitations, such as the corrections for higher twist and the extrapolation of g_1 into the unmeasured region at low x, may be less problematic if polarized high energy NLC electron beams are available for fixed target physics. This issue is worthy of further study.

Lattice QCD calculations have matured considerably in the last few years. First principles calculations of the simplest hadronic systems, like quarkonia spectra, should be possible with current technology and computational resources. This implies the potential for very accurate determinations of α_S at relatively low Q^2 from experimental measurements of the hadron spectrum, with systematic uncertainties largely independent of all other approaches discussed here. Since the present experimental errors contribute much less than $1\,\%$, no future experimental facilities are required for a $1\,\%$ determination of $\alpha_{\overline{\rm MS}}(M_Z)$ from the hadron spectrum.

e^+e^- annihilation experiments at high center-of-mass energies are promising for accurate determinations of α_S from measurements of jet rates and other jet variables. Such experiments could be performed at an NLC collider with $E_{\rm cm} = 500$ GeV or higher.

ep scattering experiments at HERA which measure the structure function F_2 at high x over a wide range of Q^2 can potentially yield determinations of $\alpha_{\overline{\rm MS}}(M_Z)$ with about $2\,\%$ accuracy. It should be noted that a future LEP×LHC facility can potentially probe momentum transfers of $Q^2 \sim (500\,{\rm GeV})^2$.

Experiments at hadron colliders ($p\overline{p}$ or pp) have the highest potential energy reach. Accurate determinations of α_S will require either concurrent extractions of the parton distribution functions (PDFs) from the same experiment or prior knowledge of the PDFs (with error bars) over the range of x probed by the process under study. Feasibility studies are underway now for the Tevatron experiments, and it is expected that the potential for providing an accurate measurement of α_S in high energy hadron collisions will be understood within the next few years.

Table II: Summary of methods for potential $1\,\%$ determinations of $\alpha_{\overline{\rm MS}}(M_Z)$. The methods listed below the double line are either considered to yield somewhat less accurate determinations (F_2 at high x at HERA, Bjorken sum rule), or they have not yet been fully established ($p\overline{p}$ and pp collisions).

Process	*Approach*	*NNLO Calculation*	*Energy Scale*	*Facilities*
DIS	Q^2 evolution of F_3	Partial	2–15 GeV 2-45 GeV	TeV33 fixed target LHC fixed target
DIS	GLS sum rule	Available	few GeV $\sim$ 10 GeV	TeV33 fixed target LHC fixed target
Hadron spectrum	Spin-averaged $b\bar{b}$ and $c\bar{c}$ splittings	lattice QCD	few–10 GeV	none
e^+e^-	Hadronic observables	Partial	500 GeV	NLC
Polarized DIS	Bjorken sum rule	Available	few GeV $\sim$ 10 GeV	SLAC, DESY, HERA NLC fixed target
DIS	Q^2 evolution of F_2 at high x	Partial	few–100 GeV $\leq$ 500 GeV	HERA LEP$\times$LHC
$p\overline{p}$ pp	Jet properties	Partial	100–500 GeV $\leq$ few TeV	Tevatron LHC

The Q^2 reach of the LHC is substantially larger than that of any other accelerator mentioned here. Should it prove possible to accurately determine α_S in hadronic collisions, the construction of a higher energy collider would extend this reach even further.

In summary, the goal of measuring α_S to an accuracy of $1\,\%$, with a number of complementary approaches, and over a wide range of Q^2, seems feasible. The complete program will likely require a number of new facilities. At low Q^2, to approach a precision of $1\,\%$ in DIS experiments, it will most likely be necessary to establish a tagged neutrino beam facility utilizing the full energy Tevatron beam, or eventually one of the LHC proton beams. The determination of α_S from the hadron spectrum using lattice QCD is the only method without facility implications. At high Q^2, a measurement of α_S in e^+e^- annihilation fits quite naturally into the physics program for the proposed Next Linear Collider. The potential for complementary α_S determinations in $p\overline{p}$ collisions at the Tevatron (and pp collisions at the LHC) still needs further study, as do measurements of F_2 at high x at HERA or at a possible LEP$\times$LHC lepton-hadron collider.

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