Probing Quartic Couplings Through Three Gauge Boson Production at an e^+e^- Linear Collider *

S. Dawson[a], A. Likhoded[b], G. Valencia[c], and O. Yushchenko[d]
[a] *Brookhaven National Laboratory, Upton, NY 11973*
[b] *International Institute of Theoretical and Applied Physics, Ames, IA. 50011*
[c] *Iowa State University, Ames, IA 50011*
[d] *Institute for High Energy Physics, Protvino, 142284, Russia*

ABSTRACT

We explore the capability of a 500 or 1000 GeV e^+e^- linear collider to measure anomalous quartic gauge boson couplings. In the framework of a non-linear effective Lagrangian with a custodial $SU(2)$ symmetry, there are only two next-to-leading order operators which contribute to quartic, but not to two- and three- gauge boson interactions. The limits on the coefficients of these operators from present and future e^+e^- colliders are compared with those available from other sources.

I. INTRODUCTION

The non-Abelian structure of the Standard Model of electroweak interactions gives precise predictions for the three- and four- gauge boson self-interactions. Precision tests of these interactions can thus be used to search for new physics beyond the Standard Model. There exist numerous studies of the capabilities of various machines such as the Tevatron and LEP II to measure anomalous three-gauge boson couplings [2]. However, only very indirect limits exist on the four-gauge boson couplings and the prospects for direct measurements have not been explored thoroughly in the literature.

The direct study of four-gauge boson couplings requires the production of at least three gauge bosons or the observation of vector-boson scattering processes. For this reason they are harder to probe directly than their three-gauge boson counterparts and higher energy machines are needed. The sensitivity of the four-gauge boson couplings to new physics is illustrated by the fact that the lowest order couplings in the standard model (without the Higgs boson) lead to gauge-boson scattering amplitudes that violate unitarity at an energy scale around $1.8\ TeV$ [1]. In the standard model this bad high energy behavior is corrected by the exchange of a Higgs boson. In this way, gauge-boson scattering amplitudes constitute a real probe of the symmetry breaking sector of the model. Thus we expect to learn something about the mechanism of electroweak symmetry breaking by studying the four gauge boson couplings.

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Here, we consider three-gauge boson production in e^+e^- collisions in the processes, $e^+e^- \to ZZZ$ and $e^+e^- \to W^+W^-Z$. Three Z production has a very small cross section in the Standard Model, making it an exceptionally good place to search for new physics. The rate for W^+W^-Z production has a different dependance on the quartic couplings and so by combining results from the two reactions, significant limits on the quartic coupling constants can be obtained. These processes have previously been considered by Belanger and Boudjema [3]. We extend their analysis by carefully considering the details of the two reactions. In particular we study the possibility of improving the bounds that can be placed on quartic couplings by imposing different kinematic cuts. We find that unlike the case of triple-gauge boson couplings, it will be very difficult to improve the sensitivity to quartic couplings by means other than increasing the energy of the machine.

II. PRELIMINARIES

We consider a picture in which there is no Higgs boson at low energy and the new physics responsible for the electroweak symmetry breaking occurs at some high scale, $\Lambda < 4\pi v \sim 3\ TeV$. In this case, the physics at low energy can be written in terms of an effective Lagrangian describing the interactions of the $SU(2)_L \times U(1)_Y$ gauge fields with the Goldstone bosons which become the longitudinal components of the $W^\pm$ and Z gauge bosons, $\omega^\pm$ and z. The minimal Lagrangian which describes the interactions of the $SU(2)_L \times U(1)_Y$ gauge bosons with the Goldstone bosons is,

$$\mathcal{L}_2 = \frac{v^2}{4} Tr\left[D^\mu \Sigma^\dagger D_\mu \Sigma\right] + \text{Kinetic Energy Terms} \tag{1}$$

and has been discussed in detail in Refs. [4, 5]. This non-renormalizable Lagrangian is to be interpreted as an effective field theory, valid below the scale Λ and yields the gauge boson self interactions which we use in this calculation [5]. These interactions are identical to those of the Standard Model when the Higgs mass is taken to be very large.

In this scenario, the effects of new physics are described in terms of a derivative expansion in powers of s/Λ^2. At $\mathcal{O}(s/\Lambda^2)$ there are 13 possible new interactions. We will assume a custodial $SU(2)_C$ symmetry along with CP conservation which restricts the number of operators at this order to 5. With these assumptions there are only two new interactions which contribute to four- gauge boson vertices, but not to two- and three- gauge boson vertices,

$$\mathcal{L}_4 = \frac{v^2}{\Lambda^2}\left\{ L_1 Tr\left(D_\mu \Sigma D^\mu \Sigma^\dagger\right)^2 + L_2\left(D_\mu \Sigma D^\nu \Sigma^\dagger\right)^2 \right\} \quad . \tag{2}$$

It is these truly quartic interactions which we study. With the normalization we have chosen, L_1 and L_2 are naturally of $\mathcal{O}(1)$. The Feynman rules for the four- gauge boson vertices resulting from this Lagrangian are given in the appendix of Ref. [5].

At present, the only experimental limits on L_1 and L_2 are indirect limits from precision measurements at LEP. These limits arise from loop effects and depend logarithmically on a cutoff, which we take to be $1\ TeV$. Implicit in the limits obtained from the LEP results is the assumption that there are no cancellations between the effects of different operators. Assuming only L_1 and L_2 are non-zero, the 95% confidence level bounds are,[1]

$$-28\left(\frac{\Lambda}{2\ TeV}\right)^2 < L_1 + \frac{5}{2}L_2 < 19\left(\frac{\Lambda}{2\ TeV}\right)^2 \quad . \tag{3}$$

The LHC will also be able to limit four- gauge boson interactions through vector boson scattering, $pp \rightarrow VVX$.[6, 8] Since the three- gauge boson interactions also contribute to these processes, it is again necessary to assume that there are no cancellations between the contributions of the different operators. Bounding one coefficient at a time, Ref. [8] found that the LHC will be able to obtain limits of roughly,

$$L_1, L_2 < \mathcal{O}(1)\left(\frac{\Lambda}{2\ TeV}\right)^2 \quad . \tag{4}$$

III. $e^+e^- \rightarrow ZZZ$

In an e^+e^- collider, there is no $4\ Z$ coupling and $3\ Z$ production can be computed at lowest order in the energy expansion using the Lagrangian of Eq. 1. (This is equivalent to the Standard Model with the Higgs graphs removed or the Higgs boson taken very massive.) We find [2]

$$\sigma_{\rm SM}(e^+e^- \rightarrow ZZZ) = \begin{cases} .8\ fb & \text{at } \sqrt{s} = 500\ GeV \\ .7\ fb & \text{at } \sqrt{s} = 1\ TeV \end{cases} \quad . \tag{5}$$

For an integrated luminosity of $50\ fb^{-1}$ at $\sqrt{s} = 500\ GeV$, this yields only 400 events. When we include final state branching ratios and detector efficiencies, it is apparent that this process will be statistics limited.

To explore new physics beyond the lowest order in the energy expansion, we include the interactions of Eq. 2. It is important to note that there are no contributions from 3 gauge boson vertices to this process at this order in the energy expansion, so $e^+e^- \rightarrow ZZZ$ is an unambiguous test of the quartic $ZZZZ$ vertex and gives limits on the couplings L_1 and L_2 without the assumption of naturalness. It is this feature which makes the $3Z$ production so useful for exploring new physics.[3]

We include in our calculation an error on the luminosity measurement of $.1\%$ and a realistic efficiency for the ZZZ final state reconstruction of

$$\epsilon_{ZZZ} = 15\% \tag{6}$$

as estimated for anticipated NLC detectors [9]. In Fig. 1, we demonstrate the effects of non-zero L_1 and L_2 and plot $d\sigma/dM_{ZZ}$ (where M_{ZZ} is the invariant mass of any pair of Z's). We see that the effects are rather small, of order a few percent. This is unfortunately the same with all the distributions we have considered. We also tried to implement some kinematic cuts to enhance the sensitivity to L_1 and L_2. The most effective values of the cuts can be obtained by studying the statistical significance which is defined as,

$$S \equiv \frac{\mid \sigma^{\rm NEW} - \sigma^{\rm SM} \mid}{\sqrt{\sigma^{\rm SM}}}\sqrt{\mathcal{L}} \quad , \tag{7}$$

where $\mathcal{L}$ is the integrated luminosity. In Fig. 2 we show the dependance of the statistical significance S on the chosen M_{ZZ} cut. The presence of the clear peak at $M_{ZZ}^{cut} \simeq 240\ GeV$ implies that this is the optimal value of the cut for enhancing the sensitivity to the new physics effects. In Fig. 3, we show the

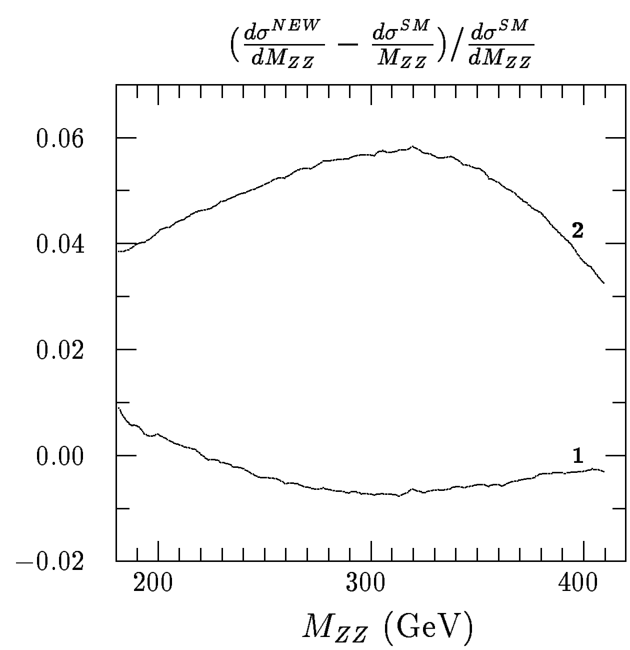

Figure 1: $d\sigma/dM_{ZZ}$ for $e^+e^- \rightarrow ZZZ$ at $\sqrt{s} = 500\ GeV$. The line labelled 1 (2) has $L_1 + L_2 = 3\ (-3)$. This figure assumes $\Lambda = 2\ TeV$. $\sigma^{\rm NEW}$ is the cross section with the interactions of Eqs. 1 and 2, while $\sigma^{\rm SM}$ includes only the interactions of Eq. 1.

[1] We have updated the limits of Ref. [5].

[2] These numbers agree with Ref. [7] for $M_H = 1\ TeV$.

[3] The e^+e^-Z vertex is renormalized by a factor sensitive to the coupling L_{10}.[5]. However, L_{10} is severely restricted by LEP measurements (since it contributes to the gauge boson two-point functions) and so the inclusion of this operator would not change our results.

allowed regions (at 95% confidence level) of L_1 and L_2 which are obtainable with and without the cut on M_{ZZ}. We see that even though Fig. 2 demonstrates the usefulness of the kinematic cut, the bound of Fig. 3 which results from utilizing the entire kinematic region is more stringent than that obtained by imposing the cut. This is because imposing kinematic cuts leads to a reduction in the cross section and consequently to an increase in the statistical error. Hence, with such a small cross section as in $3\ Z$ production, the cuts are ineffective for all distributions which we have considered.

The bounds at $\sqrt{s}=1\ TeV$ are considerably more stringent than those at lower energy.[4] The region which can be probed depends linearly on L_1+L_2 and for $\sqrt{s}=500\ GeV$, we obtain roughly,

$$\mid L_1+L_2\mid < 10\left(\frac{\Lambda}{2\ TeV}\right)^2 \quad . \tag{8}$$

This is only a slight improvement over the LEP limits of Eq. 3, but is a factor of 4 improvement of the limits obtained in Ref. [3].

The dependance on L_1+L_2 can easily be understood by computing the rate for $e^+e^-\to Zzz$ using the electroweak equivalence theorem. At high energy, this is the dominant process contributing to $3\ Z$ production. In this case, there is a single s-channel diagram for $e^+e^-\to Z^*\to Zzz$ and the $ZZzz$ vertex depends only on the combination L_1+L_2. From Ref. [8], the

[4] We have taken $\mathcal{L}=50\ fb^{-1},\ 100\ fb^{-1}$ for $\sqrt{s}=500\ GeV,\ 1\ TeV$.

amplitude for $Z^\mu(q_1)Z^\nu(q_2)\to z(p)z(p')$ is

$$\begin{aligned}\mathcal{A}(ZZ\to zz) \quad = \quad & \frac{-g^2}{2\pi^2v^2}\frac{L_1+L_2}{\cos^2\theta_W}\left[\frac{s}{2}g^{\mu\nu}+p^\mu p'^\nu+p^\nu p'^\mu\right] \\ & +\frac{g^2}{\pi^2v^2\cos^2\theta_W}\left[-\frac{s}{2}g^{\mu\nu}+q_2^\mu q_1^\nu\right] \\ & \cdot(1-4\sin^2\theta_W\cos^2\theta_W) \\ & +\frac{g^2}{16\pi^2v^2}\left[g^{\mu\nu}\frac{s}{2}\log\left(\frac{s}{\mu^2}\right)\right. \\ & \left.+p^\nu p'^\mu\log\left(\frac{-u}{\mu^2}\right)+p^\mu p'^\nu\log\left(\frac{-t}{\mu^2}\right)\right],\end{aligned} \tag{9}$$

where $s=2p\cdot p'$, $t=(q_1-p)^2$, $u=(q_1-p')^2$, and μ is an arbitrary renormalization scale. It is straightforward to turn this amplitude into a cross section for $e^+e^-\to Zzz$ and the results yield good agreement with those shown in Fig. 3. The first line in Eq. 9 is the contribution from the $\mathcal{O}(\frac{s}{\Lambda^2})$ Lagrangian of Eq. 2, while the others are the one- loop corrections obtained using the lowest order Lagrangian of Eq. 1. Both sets of terms are formally of the same order in the energy expansion and must be included for a consistent analysis. Numerically we find, however, that for the range of values of L_1+L_2 being probed at possible next linear colliders, the loop corrections to the amplitude are extremely small when compared to the contribution from the L_1+L_2 term. This gives us confidence that the results shown in Fig. 3 will not be significantly altered by the inclusion of electroweak radiative corrections in the analysis.

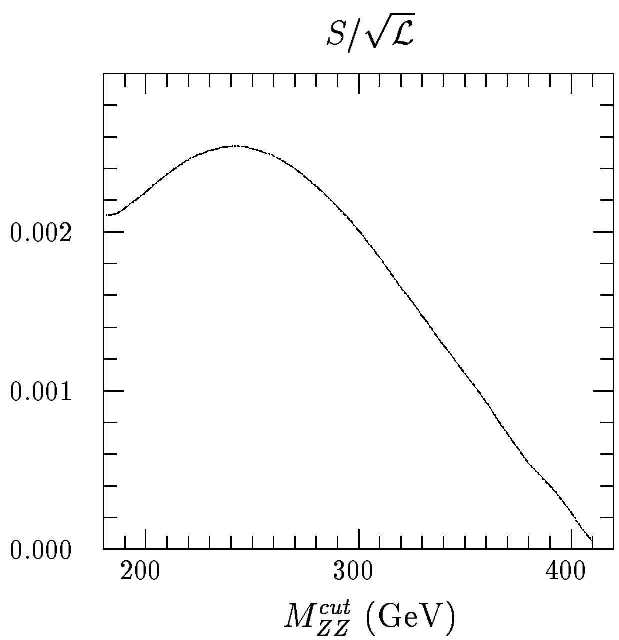

Figure 2: Statistical significance for the signal using a cut on M_{ZZ} at $\sqrt{s}=500\ GeV$ from $e^+e^-\to ZZZ$. This figure assumes $L_1+L_2=3$ and $\Lambda=2\ TeV$.

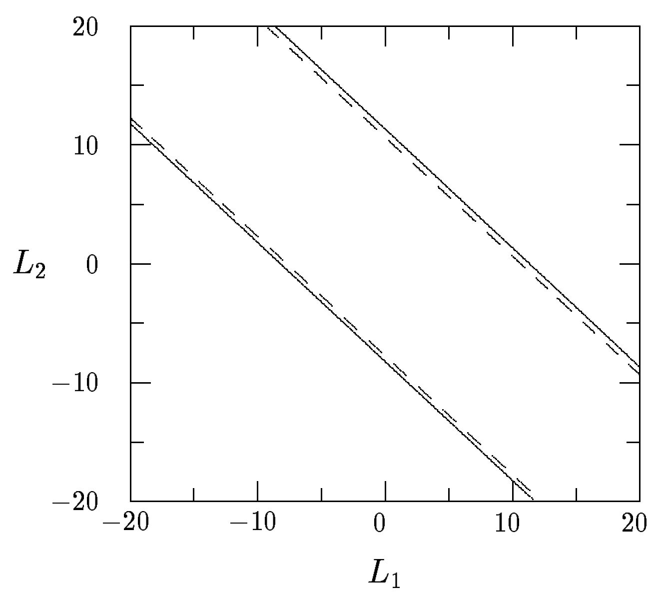

Figure 3: 95% confidence level bounds on L_1 and L_2 from $e^+e^-\to ZZZ$ at $\sqrt{s}=500\ GeV$. The solid (dashed) lines show the bound with (without) the cut on M_{ZZ}. This figure assumes $\Lambda=2\ TeV$.

IV. $e^+e^- \to W^+W^-Z$

The process $e^+e^- \to W^+W^-Z$ has a larger cross section than that for $e^+e^- \to ZZZ$,[5]

$$\sigma(e^+e^- \to W^+W^-Z)_{SM} = \begin{cases} 43\ fb & \text{at } \sqrt{s} = 500\ GeV \\ 70\ fb & \text{at } \sqrt{s} = 1\ TeV \end{cases} \quad . \tag{10}$$

We see that the production of W^+W^-Z depends on the three-gauge boson couplings as well as the quartic couplings. Hence to obtain limits on the quartic couplings from this process requires that we assume that there are no cancellations between the various terms, an assumption which was not necessary in the $3\ Z$ case. We assume that the three- gauge boson vertices have their Standard Model forms and consider only the effects of non-zero L_1 and L_2.

To model detector efficiencies, we again use an efficiency for the WWZ reconstruction modeled on potential NLC detectors [10] and take

$$\epsilon_{WWZ} = 12\% \quad . \tag{11}$$

As in the process $e^+e^- \to ZZZ$, the kinematic cuts are ineffective for extracting the effects of L_1 and L_2, which are again rather small. Some insight can be gained by writing the cross section (in fb) at $\sqrt{s} = 500\ GeV$ as,

$$\begin{aligned} \sigma(e^+e^- \to W^+W^-Z) &= 43 - .022L_1 + .049L_2 \\ &\quad -.001L_1L_2 + .006L_1^2 + .003L_2^2 \end{aligned} \tag{12}$$

The smallness of the coefficients of the L_i makes it apparent that the effects of the new physics are extremely small.

In Fig. 4, we show the 95% confidence level bounds on L_1 and L_2 resulting from combining the ZZZ and the W^+W^-Z process. We also compare with the bounds previously obtained from LEPI.

[5] These results agree with Ref. [7] for $M_H = 1\ TeV$.

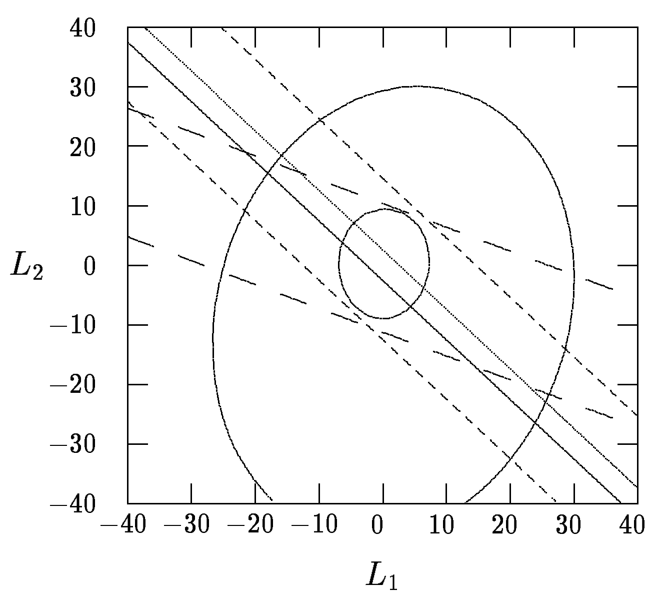

Figure 4: $95\%\ CL$ bounds on L_1 and L_2. The short- dashed (solid) bands are from $e^+e^- \to ZZZ$ at $\sqrt{s} = 500\ GeV$ ($1\ TeV$) and the large (small) ellipse are from $e^+e^- \to W^+W^-Z$ at $\sqrt{s} = 500\ GeV$ ($1\ TeV$). The long dashed band is the limit from LEPI. This figure assumes $\Lambda = 2\ TeV$.

V. CONCLUSIONS

By combining the results from $e^+e^- \to ZZZ$ and $e^+e^- \to W^+W^-Z$, we see that an NLC will be sensitive to a small region in the L_1, L_2 plane. This limit will be a considerable improvement over the present indirect limit from precision measurements at LEP. In addition, the limit from $e^+e^- \to ZZZ$ is cleaner theoretically than other limits since there is no dependance on three- gauge boson couplings.

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