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Domain Walls, Branes, and Fluxes in String Theory: New Ideas on the Cosmological Constant Problem, Moduli Stabilization, and Vacuum Connectedness

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A DISSERTATION
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Michael Brian Schulz
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I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Shamit Kachru, Principal Adviser

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Stephen H. Shenker

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Eva M. Silverstein

Approved for the University Committee on Graduate Studies:

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Physics is a collaborative endeavor. Occasionally, individuals working in isolation produce results of great interest, but it is more often the case that these results are obtained through the combined efforts of many. This is true both on the large scale—as evidenced by the 757+ works that together comprise our understanding of the AdS/CFT correspondence [1]—and also on the small scale: the majority of our favorite works have more than one author [2], for the basic reason that a few authors working together tend to accomplish more, and write clearer papers than the same authors working independently

For this reason, the work presented in this dissertation does not represent solely my own labor. It is the result of countless conversations, arguments, emails, phone calls, notes, drafts and revisions by myself and my collaborators Shamit Kachru, Xiao Liu, Eva Silverstein, and Sandip Trivedi. I feel that the work was truly a combined effort of all of the authors involved; it would be inaccurate to try to assign individual credit for any subset. In addition, we owe much to the help and enthusiasm of our colleagues.

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Contents

1. Introduction

1.1 An Appraisal

String theory is now in its 35th year of existence. While it would be inappropriate to refer the subject as middle-aged, it not unreasonable to perform the traditional introspection customary at this juncture, and look back on what the theory has accomplished. We can recognize and take pride in many crowning achievements. However, we should also begin to call it to task on a few questions that a quantum theory of gravity was thought to answer, but for which there has so far been little progress.

Our field has come a long way since its early days as a phenomenological theory of the strong interactions. The universal presence of a massless spin-2 particle in its spectrum, initially an obstacle to any realistic applications of the theory, guaranteed that string theory was a quantum theory of gravity. More importantly, it is a finite theory of gravity: string theory is divergence free order by order in perturbation theory. At low energies it reduces to point particle quantum field theory. In addition, it incorporates supersymmetry and naturally leads to grand unified theories. This was first demonstrated in the mid 1980s via four-dimensional, $\mathcal{N} = 1$ supersymmetric compactifications of the heterotic string.

In recent years, we have learned that string theory is a theory of more than just strings. It is also a theory of their magnetic cousins, the NS5-branes, and of topological defects of all dimension, called D-branes, on which the strings can end. Through a systematic study of nonperturbative objects such as these and an exploitation of exact consequences of the supersymmetry algebra, we have been able to count the microstates of black holes, thus providing the first microscopic basis for black hole thermodynamics. In perhaps the most famous success story, the study of D-branes has led to the first precise statement of holography involving a four-dimensional gauge theory. Very general arguments have long implied that on the one hand, gauge field theories with large gauge group are string theories, and on the other hand, that every gravitational theory in d dimensions contains redundant degrees of freedom and can be alternatively described by a non-gravitational theory in $(d-1)$ dimensions. We can now, at last, explicitly state this holographic correspondence in a large class of situations.

Despite its many successes, our current understanding of string theory has glaring shortcomings. We have had embarrassingly little to say, even qualitatively, about a few celebrated problems in which gravity and quantum field theory rub elbows, and an answer is specifically sought in a unified treatment of both. Examples include the cosmological constant problem, the hierarchy problem, and the problem of spacetime singularities. Related to this is the fact that string theory in a cosmological context remains almost completely unexplored. Clearly, to describe *our* universe, with the inflationary beginning and de Sitter endpoint currently favored by astrophysical observation, we will need to address this shortcoming.

Another expectation of string theory which has not yet seen fruition, is that it would have a predictive power that the standard model plus general relativity lacked. The standard model of particle physics has a number of arbitrary parameters. Its supersymmetric extensions have even more. In contrast, string theory is unique. There are no free parameters. Nevertheless, this uniqueness has been an aesthetic rather than practical advance, since the large space of *a priori* permissible free parameters has been replaced by an even larger space of possible vacuum states.

This begs the question of how the one particular vacuum in which we live is selected. However, there are really at least two problems. First, while some of the vacua are isolated, others occur in continuous families, parametrized by the vacuum expectation values of massless scalar fields, or moduli. These moduli are excluded both by precision tests of the equivalence principle and on cosmological grounds. Therefore, we need to understand the mechanisms that can ultimately give these fields masses, thus rendering them approximate rather than exact moduli. This leaves us with a number of isolated vacua and perhaps a few continuous spaces of unphysical vacua parametrized by moduli that cannot be lifted. Our second task is then to understand which of these vacua are in some sense connected, and to identify the dynamical mechanism that singles out one of these connected vacua.

In this dissertation, we will focus on three of the problems mentioned above: the cosmological constant problem, moduli stabilization, and vacuum connectedness. The remainder of the Introduction contains a short overview of each of these problems, together with a description of a specific string theoretic or string inspired approach toward a solution. The main ingredients are Dirichlet and Neveu-Schwarz p-branes, the corresponding $(p+2)$-form field strengths, and domain walls, which may or may not be composed of branes.[1] Due to the particular approaches employed, we will also have occasion to discuss singularities (in Sec. 1.2) and the hierarchy problem (in Sec. 1.3), but this discussion will be brief and secondary to the main discussion. The subsequent chapters form the nuts and bolts of the dissertation. There, the reader will find a complete exposition of the ideas outlined in the next three sections.

1.2 The Cosmological Constant Problem

As a starting point, consider the action

$$S_{\text{total}} = S_{\text{grav}} + S, \tag{1.2.1}$$

where

$$S_{\text{grav}} = \frac{1}{16\pi G_4} \int d^4x\sqrt{-g}(R - 2\Lambda_0) \tag{1.2.2}$$

is the four-dimensional Einstein-Hilbert action plus cosmological term. One can show that the gravitational field equations become

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda_0 g_{\mu\nu} = 8\pi G_4 T_{\mu\nu}. \tag{1.2.3}$$

Here Λ_0 is the bare cosmological constant, and $T_{\mu\nu}$ is the energy momentum tensor due to all the fields appearing in S. If we define

$$\rho_{\Lambda_0} = \frac{1}{8\pi G_4}\Lambda_0, \tag{1.2.4}$$

[1] In the context of SUSY changing bubbles (Sec. 1.4, Chap. 5) the domain walls are made of D5 and NS5 branes, but in the self-tuning approach to the cosmological constant problem (Sec. 1.2, Chaps. 2-3), the domain walls need not be made up of branes.

then Eq. (1.2.3) can be rewritten as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_4(T_{\mu\nu} - \rho_{\Lambda_0}g_{\mu\nu}). \tag{1.2.5}$$

The reason for the suggestive notation is that on symmetry grounds one generally has

$$\langle T_{\mu\nu}\rangle = -\rho_{\text{vac}}g_{\mu\nu}. \tag{1.2.6}$$

So, the bare cosmological constant has an effect identical to that of the vacuum energy density. In fact, it is customary to group these terms together into an effective cosmological constant,

$$\Lambda = \Lambda_0 + 8\pi G_4\rho_{\text{vac}}, \tag{1.2.7}$$

with

$$\rho_\Lambda = \rho_{\Lambda_0} + \rho_{\text{vac}}. \tag{1.2.8}$$

In field theory, we rarely talk about the vacuum energy density since it can be redefined away at will by shifting the definition of zero energy. However, this is no longer true once gravity is included. In this case, we need to retain all contributions to the vacuum energy density. These contributions include, first of all, a quantum zero-point energy density, which on dimensional grounds should go as

$$\rho_{\text{zero-pt}} \sim M_{\text{cutoff}}^4 \subset \rho_{\text{vac}}. \tag{1.2.9}$$

Here M_{cutoff} is the mass scale at which momentum integrals are regularized in the UV. If M_{cutoff} is the four-dimensional Planck mass M_p, then $\rho_{\text{zero-pt}} \sim {M_p}^4 = (10^{18}\ \text{GeV})^4$. In addition, if there are scalar fields, then the scalar potential also contributes to ρ_{vac} :

$$\rho_V = V\big(\langle\phi\rangle\big) \subset \rho_{\text{vac}}. \tag{1.2.10}$$

Note that ρ_V can change at phase transitions. For example, the Higgs boson has a potential

$$V(h) = \frac{\lambda h^4}{4} - \mu^2 h^2. \tag{1.2.11}$$

At temperatures above the scale of the electroweak phase transition, the Higgs field is in a disordered state with $\langle h\rangle = 0$. However, in the present epoch, $\langle h\rangle = v = \sqrt{2\mu}/\lambda$, and electroweak symmetry is broken. During the phase transition,

$$(\Delta\rho_{\text{vac}})_{\text{EW}} = \frac{\lambda v^4}{4} \sim (10^2\ \text{GeV})^4. \tag{1.2.12}$$

Similarly, one expects $(\Delta\rho_{\rm vac})_{\rm QCD} \sim (10^{-1}\ {\rm GeV})^4$ at the QCD phase transition and $(\Delta\rho_{\rm vac})_{\rm GUT} \sim (10^{16}\ {\rm GeV})^4$ if there is a GUT transition.

Gathering all contributions, we have

$$\rho_\Lambda = \rho_{\Lambda_0} + \rho_{\rm zero\text{-}pt} + (\Delta\rho_{\rm vac})_{\rm QCD} + (\Delta\rho_{\rm vac})_{\rm EW} + (\Delta\rho_{\rm vac})_{\rm GUT} + \cdots, \tag{1.2.13}$$

whereas from astrophysical observation

$$(\rho_\Lambda)_{\rm observed} \lesssim (10^{-12}\ {\rm GeV})^4. \tag{1.2.14}$$

Given the Planck scale estimate for $\rho_{\rm zero\text{-}pt}$, the disagreement between theory and experiment is whopping 120 orders of magnitude.

This celebrated discrepency is known as the *cosmological constant problem*. Even if our estimate for $\rho_{\rm zero\text{-}pt}$ is wrong,[2] the observed value of ρ_Λ is still far smaller than any of the other expected contrabutions to ρ_Λ, and there seems to be no good reason why a miraculous cancellation with ρ_{Λ_0} should take place. For a more extensive review of the cosmological constant problem, see [10] and the references contained therein. (The discussion presented above is very similar to §1.2 and §2.3 of [10]).

In most attempts to solve the cosmological constant problem, symmetry arguments are employed to justify certain cancellations. For example, in a theory with unbroken supersymmetry, $\rho_{\rm vac} = 0$ due to cancellations between bosons and fermions. If supersymmetry is broken at low energies at a scale $M_{\rm susy}$, then the prediction is $\rho_{\rm vac} \sim M_{\rm susy}^4$, which for $M_{\rm susy} \sim$ TeV, is still too large. However, gravity loops are supressed by the expansion parameter $M_{\rm susy}^2/{M_p}^2$, with $M_p \sim 10^{18}$ GeV. If for some reason the leading order and one-loop contributions vanish (which is not so hard to arrange), then $\rho_{\rm vac} \sim M_{\rm susy}^8/{M_p}^4$ is close to the observed value of ρ_Λ [5].

On the other hand, there is an independent approach that we can pursue. Since the root of the problem is the dual interpretation of the cosmological constant as both a vacuum energy in particle physics and a source term in Einstein's equations, the problem would be solved if we could somehow sever this link. This is possible via a mechanism called self-tuning.

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[2] $M_{\rm cutoff}$ can be much smaller than M_p if new physics sets in well below the Planck scale. For example, it is often assumed that $M_{\rm cutoff} \sim$ TeV.

The starting point for the self-tuning mechanism is the brane world scenario [11,12,13,14,15,16,17]. In this scenario, the standard model lives on a (3 + 1)-dimensional hyperplane embedded in a higher dimensional bulk in which gravity is free to propagate. Four dimensional gravity arises from the zero mode of the higher dimensional graviton [18]. The hyperplane may or may not be composed of the branes of string theory. For concreteness, consider a four-dimensional domain wall embedded in five-dimensional space, and coupled to a bulk scalar. Given generic couplings of the domain wall to the scalar, it is possible to show that a solution exists in which the scalar responds in just the right way to keep the domain wall Minkowskian even when the tension of the domain wall (4D vacuum energy) is nonzero. This is what we will refer to as the self-tuning mechanism.

A complete discussion of the self-tuning mechanism is given in Chaps. 2 and 3. For now, we simply note that while the mechanism does successfully eliminate the four-dimensional gravitational effects of the vacuum energy, it also comes with at least two undesirable properties. (See [19] for further discussion of the cosmological viability of self-tuning models). First, although the mechanism renders Minkowski space stable to quantum mechanical corrections to the vacuum energy, there also exist de Sitter and Anti-de Sitter solutions. There is no reason why a flat solution should be preferred. Second, all solutions that satisfy the dominant energy condition at the domain wall (in particular $\rho + p > 0$) have naked curvature singularities at a finite distance from the domain wall.

We will have the opportunity to discuss (Anti-)de Sitter self-tuning solutions at length in Chapter 2. Since similar attention will not be devoted to to naked singularities, we will now devote the remainder of this section to saying a few words about singularities.

The naked singularitites of the self-tuning solutions are excluded by the cosmic censorship conjecture.[3] Similar singularities do exist in Hořava-Witten theory and in the supergravity description of orientifold planes. However, if the singularities of our self-tuning solutions have a stringy resolution in terms of quantized objects

[3] There are two forms of this conjecture. The weak cosmic censorship conjecture states that, given physically reasonable initial conditions, the singularities produced by gravitational collapse must be hidden behind black hole horizons. The strong cosmic censorship conjecture forbids all timelike singularities, including those hidden behind horizons. For a review of the status of cosmic censorship, see [20].

of string theory, then we might expect a specific boundary condition to be required at the singularity, which we neglected to impose. Still, the situation is far from understood. One possibility is that there is a "discretuum" of allowed boundary conditions in string theory that approaches a continuum in the supergravity limit [21]. It is also possible that the self-tuning singularities are simply unphysical and are not resolved in string theory. Soon after [5] and [3] appeared, several proposals were given for deciding whether a singularity is unphysical. According to the first proposal, gravity backgrounds with "good" singularities can always be obtained as singular limits of thermal solutions in which the singularities are hidden behind event horizons [22]. It is unclear what this criterion implies for the self-tuning solutions. The gravity solutions on either side of a self-tuning domain wall *can* be deformed to hide the singularities behind horizons. But, the two deformed solutions cannot be patched together without violating the weak energy condition at the domain wall [23]. The proposal was soon retracted by its author and replaced by a weaker one involving the behavior of the bulk potential for the scalar [24]. While interesting, this new criterion does not apply to our model in the main case of interest, with bulk supersymmetry and vanishing 5D cosmological constant.

Whatever the status of the self-tuning singularities, a widely held belief is that every singularity of classical general relativity is either excluded or resolved in string theory. This includes not only the timelike naked singularities just discussed, but also cosmological singularities such as those at a big bang or big crunch. Formulating a string theoretical classification of classical singularities is an interesting but so far elusive direction of research.

1.3 Moduli Stabilization

Massless scalar fields, or moduli, are ubiquitous in string theory compactifications. Obviously, any light scalars that are charged under standard model gauge fields would have been observed in collider experiments and are therefore excluded. However, the moduli that we encounter in string theory couple to standard model fields only through Planck suppressed interactions, even when they couple directly to these fields through the moduli-dependence of Yukawa or gauge couplings. Therefore, these string moduli are not excluded by collider experiments.

On the other hand, exact moduli *are* excluded, and approximate moduli constrained, for at least two reasons. First of all, the moduli would mediate long-range interactions that have been excluded by precision tests of Newtonian gravity (alternatively referred to as fifth-force experiments or tests of the equivalence principle) [25,26]. These experiments also exclude light enough massive approximate moduli [27]. In addition, approximate moduli are constrained by the *Polonyi* or *cosmological moduli problem* [28]. If, in the early universe, such fields were initially far from the minima of their potentials, their oscillations would give rise to an energy density like that of nonrelativistic matter rather than radiation. This would ultimately cause deviations from the successful predictions of big bang nucleosynthesis.

Therefore, the moduli must be lifted. There are traditionally two ways in which this lifting can take place. Either nonperturbative string theory corrections can generate a potential that lifts the moduli, giving them Planck scale masses, or else the moduli can remain massless down to the supersymmetry breaking scale. In the context of perturbative $\mathcal{N} = 1$ heterotic compactifications, we starts out with perturbatively massless moduli, and then generate a potential by one of the possible sources of nonperturbative corrections. The two most well understood are worldsheet instantons and Euclidean NS5-brane instantons. But even for these nonperturbative effects, it has proven prohibitively difficult to perform a calculation. There are no examples in which anyone has computed a worldsheet instanton sum and found supersymmetric (or nonsupersymmetric) vacua in the resulting potential [29].

In contrast, a more computationally amenable class of four-dimensional $\mathcal{N} = 1$ string vacua has recently come to be appreciated. This class of vacua involves orientifolds of $\mathcal{N} > 1$ compactifications of Type IIB string theory, in which NS and RR three-form flux is turned on through nontrivial three-cycles in the compactification manifold. The fluxes generate a *perturbative* superpotential that generically lifts all of the complex structure moduli, plus the dilaton-axion and some Kähler moduli as well.

These compactifications frequently also have a useful holographic interpretation. For example, there is a holographic duality between Type IIB string theory on a warped deformed conifold with flux, and four-dimensional $\mathcal{N} = 1$ gauge theory with chiral symmetry breaking [30]. In this duality, the scale of chiral symmetry breaking corresponds to the size of a minimal three-sphere in the warped deformed conifold. More recently, it was observed that this construction can be embedded within a Type IIB orientifold, where the internal manifold is a true compact Calabi-Yau, near a conifold point in its complex structure moduli space [31]. In this case, the superpotential is computable and the equations of motion soluble, at least near the conifold point. By solving the equations of motion, one finds that the size of the minimal three-sphere mentioned above is naturally exponentially small in a ratio of fluxes. A similar statement can be made for the minimum of the warp factor that appears in the metric. Therefore, this model provides a stringy realization of the Randall-Sundrum approach to solving the hierarchy problem [32].[4]

A shortcoming of the model just described is that it relies on an an approximate analysis near special points in the moduli space of complex structure. To address this shortcoming, we will study a different model in Chapter 4. Instead of considering a Calabi-Yau orientifold, we examine the computationally simpler compactification of Type IIB string theory on the T^6/Z_2 orientifold. This geometry arises in the T-dual description of Type I theory on T^6, and one normally introduces 16 space-filling D3-branes to cancel the RR tadpoles. Here, we instead cancel the RR tadpoles either partially or fully by turning on three-form flux in the compact geometry. The resulting (super)potential for moduli is globally calculable and the equations of motion soluble. We will see that we can find many explicit examples of $\mathcal{N}=1$ supersymmetric vacua with greatly reduced numbers of moduli. In addition, a few examples with $\mathcal{N}>1$ supersymmetry or complete supersymmetry breaking will also be discussed.

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[4] Briefly, the gauge hierarchy problem is a fine-tuning problem somewhat similar to the cosmological constant problem. We believe that $m_{\text{Higgs}} \sim 10^2$ GeV. On the other hand, from one-loop corrections to the Higgs self-energy, we expect $m_{\text{Higgs}} = m^2_{\text{bare}} + \delta m^2$, where $\delta m^2 \sim M^2_{\text{cutoff}}$ and M_{cutoff} is the ultraviolet cutoff. If M_{cutoff} is either Planck scale, or at least much greater than m_{Higgs}, an unexpected miraculous cancellation between m^2_{bare} and δm^2 is required. However, unlike the cosmological constant problem, the hierarchy problem is solved by the MSSM with TeV scale cutoff. In the Randall-Sundrum approach to the hierarchy problem [32], the Planck scale is at a TeV, but an exponential hierarchy of mass scales arises from the warp factor $e^{2A(y)}$ that appears in the metric $ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + g(y)_{mn}dy^m dy^n$.

1.4 Vacuum Connectedness

In the previous section we discussed the lifting of moduli by perturbative and nonperturbative superpotentials. However. even after these superpotentials are taken into account. string theory does not have a unique vacuum state. It is therefore natural to ask how the particular vacuum in which we live is chosen. A logical possibility is that nature has ended up in a particular vacuum state through no more than a historical accident. But. this would be disappointing. We would like to think that string theory provides a physical mechanism for vacuum selection. Ideally. this mechanism would not only single a particular four-dimensional vacuum. but would also predict that exactly four of the dimensions of spacetime are large. At present. very little is known about the dynamics of vacuum selection. A number of ideas have been proposed by T. Banks and collaborators. (See for example [33.34]). We will not discuss any of these ideas here. Instead. we will focus on the related issue of vacuum connectedness.

It is likely that any mechanism in string theory for vacuum selection acts separately on a number of disjoint subsets of vacua. or superselection sectors. Understanding the possible notions of vacuum connectedness is therefore a logical first step toward understanding vacuum selection. The strongest notion of vacuum connectedness is connectedness in moduli space. While we do know a fair amount about this type of connectedness. this notion is almost certainly too strong. Cosmology samples more than the minima of potentials. Though we do not understand string field theory well enough to make precise statements about the potential barriers that connect different string vacua, we expect by analogy to point-particle field theory that such barriers exist, and that two vacua are dynamically connected if the barrier that separates them is much smaller than the four-dimensional Planck scale. In such a case, there should exist solutions with nonstatic domain walls,[5] or bubble walls, that interpolate between the two vacua. Therefore, we will adopt as a tentative notion of vacuum connectedness the existence of bubble solutions interpolating between two vacua, with bubble tension small in Planck units.[6]

[5] The results of [35] show that there do not exist static BPS domain walls that interpolate between two nonsingular supersymmetric vacua.

[6] The last part of this definition is explained in more detail in Chap. 5. In short, it is necessary to have sub-Planckian bubble tension for the bubbles to be macroscopic but at the same time not be black holes.

As a specific application of this definition, we ask whether four-dimensional vacua that preserve different amounts of supersymmetry can be connected. This question is interesting since, on the one hand, we suspect that the vacuum in which we live is an $\mathcal{N} = 1$ string background with spontaneously broken supersymmetry, and on the other hand, we know that four-dimensional vacua with different amounts of supersymmetry are *not* connected in moduli space.

In Chap. 5, we provide a positive answer to this question by showing that all of the Type IIB flux vacua of Chap. 4, including those that preserve different amounts of supersymmetry, can be connected to one another by nonstatic spherical domain walls. The tension of these bubble walls is tunably lower than the four-dimensional Planck scale, and the stringy description of these walls is known in terms of wrapped D5 and NS5 branes. This construction allows us to connect vacua with anywhere from $\mathcal{N} = 1$ to $\mathcal{N} = 4$ supersymmetry in four dimensions.

2. Self-Tuning Flat Domain Walls

Some time ago, it was suggested that the cosmological constant problem could become soluble in models where our world is a topological defect in a higher dimensional spacetime [11]. Recently such models have come under renewed investigation. This has been motivated both by brane world scenarios [12,13,14,15,16,17], and by the suggestion of Randall and Sundrum [18] that the four-dimensional graviton might be a bound state of a 5D graviton to a 4D domain wall. At the same time, new ideas relating 4D renormalization group flows to 5D AdS gravity via the AdS/CFT correspondence [36,37,38] have inspired related approaches to explaining the near-vanishing of the 4D cosmological term [39,40]. The latter authors have suggested (following [11]) that quantum corrections to the 4D cosmological constant could be cancelled by variations of fields in a five-dimensional bulk gravity solution. The results of this chapter may be regarded as a concrete partial realization of this scenario, in the context of 5D dilaton gravity and string theory. A different AdS/CFT motivated approach to this problem was discussed in [41].

In the thin wall approximation, we can represent a domain wall in 5D gravity by a delta function source with some coefficient $f(\phi)$ (where ϕ is a bulk scalar field, the dilaton), parametrizing the tension of the wall. Quantum fluctuations of the fields with support on the brane should correct $f(\phi)$. In this chapter, we present a concrete example of a 5D dilaton gravity theory where one can find Poincare invariant domain wall solutions for *generic* $f(\phi)$. The constraint of finding a finite 4D Planck scale then restricts the sign of f and the value of f'/f at the wall to lie in a range of order one. Thus fine-tuning is not required in order to avoid

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having the quantum fluctuations which correct $f(\phi)$ generate a 4D cosmological constant. One of the requirements we must impose is that the 5D cosmological constant Λ should vanish.[7] This would be natural in scenarios where the bulk is supersymmetric (though the brane need not be), or where quantum corrections to the bulk are small enough to neglect in a controlled expansion.

For suitable choices of $f(\phi)$, this example exhibits the precise dilaton couplings which naturally arise in string theory. There are two interesting and distinct contexts in which this happens. One is to consider $f(\phi)$ corresponding to tree-level dilaton coupling ($Ve^{-2\phi}$ in string frame, for some constant V). This form of the dilaton coupling is not restricted to tree-level *perturbative* string theory—it occurs for example on the worldvolumes of Neveu-Schwarz (NS) branes in string theory. There, the dynamics of the worldvolume degrees of freedom does not depend on the dilaton—the relevant coupling constant is dilaton independent. Therefore, quantum corrections to the brane tension due to dynamics of worldvolume fields would be expected to maintain the "tree-level" form of $f(\phi)$, while simply shifting the coefficient V of the (string frame) $e^{-2\phi}$. The other form of $f(\phi)$ natural in string theory involves a power series in e^{ϕ}. This type of coupling occurs when quantum corrections are controlled by the dilaton in string theory.

In either case, as long as we only consider quantum corrections which modify $f(\phi)$ but maintain the required form of the bulk 5D gravity action, this means that quantum corrections to the brane tension do not destabilize flat space: they do not generate a four-dimensional cosmological constant. We will argue that some of our examples should have a microscopic realization in string theory with this feature, at leading order in a controllable approximation scheme. It is perhaps appropriate to call this "self-tuning" of the cosmological constant because the 5D gravity theory and its matter fields respond in just the right way to shifts in the tension of the brane to maintain 4D Poincare invariance. Note that here, as in [18], there is a distinction between the brane tension and the 4D cosmological constant.

There are two aspects of the solutions we find which are not under satisfactory control. Firstly, the curvature in the brane solutions of interest has singularities at finite distance from the wall; the proper interpretation of these singularities will likely

[7] It is possible that an Einstein frame bulk cosmological term which is independent of ϕ will also allow for similar physics [4].

be crucial to understanding the mechanism of self-tuning from a four-dimensional perspective. We cut off the space at these singularities. The wavefunctions for the four-dimensional gravitons in our solutions vanish there. Secondly, the value of the dilaton ϕ diverges at some of the singularities: this implies that the theory is becoming strongly coupled there. However, the curvature and coupling can be kept arbitrarily weak at the core of the wall. Therefore, some aspects of the solutions are under control and we think the self-tuning mechanism can be concretely studied. We present some preliminary ideas about the microscopic nature of the singularities in Sec. 2.4.

A problem common to the system studied here and that of [18] is the possibility of instabilities, hidden in the thin wall sources, that are missed by the effective field theory analysis. Studying thick wall analogues of our solutions would probably shed light on this issue. We do not resolve this question here. But taking advantage of the stringy dilaton couplings possible in our set of self-tuned models, we present a plausibility argument for the existence of stringy realizations, a subject whose details we leave for future work [4].

Another issue involves solutions where the wall is not Poincare invariant. This could mean it is curved (for example, de Sitter or anti-de Sitter spacetime). However it could also mean that there is a nontrivial dilaton profile along the wall (one example being the linear dilaton solution in string theory, which arises when the tree-level cosmological constant is nonvanishing). This latter possibility is *a priori* as likely as others, given the presence of the massless dilaton in our solutions.

Our purpose in this chapter is to argue that starting with a Poincare invariant wall, one can find systems where quantum corrections leave a Poincaré-invariant wall as a solution. However one could also imagine starting with non Poincaré-invariant wall solutions of the same 5D equations (and preliminary analysis suggests that such solutions do exist in the generic case, with finite 4D Planck scale). We are in the process of systematically analyzing the fine tuning of initial conditions that considering a classically Poincaré-invariant wall might entail [4].

The chapter is organized as follows. In Sec. 2.1, we write down the 5D gravity + dilaton theories that we will be investigating. We solve the equations of motion to find Poincaré-invariant domain walls, both in the cases where the 5D Lagrangian has couplings which provide the self-tuning discussed above, and in more general cases. In Sec. 2.2, we describe several possible embeddings of our results into a more microscopic string theory context. We close with a discussion of promising directions for future thought in Sec. 2.3.

There have been many interesting recent papers which study domain walls in 5D dilaton gravity theories. We particularly found [42] and [43] useful, and further references may be found there.

This research was inspired by very interesting discussions with O. Aharony and T. Banks. While our work on Poincaré-invariant domain walls and self-tuning was in progress, we learned that very similar work was in progress by Arkani-Hamed, Dimopoulos, Kaloper and Sundrum [5]. In particular, before we had obtained the solutions in Sec. 2.1.3 and Sec. 2.1.4, R. Sundrum told us that they were finding singular solutions to the equations and were hoping the singularities would "explain" a breakdown of 4D effective field theory on the domain wall.

2.1 Poincaré-Invariant 4D Domain Wall Solutions

2.1.1 Basic Setup and Summary of Results

Let us consider the action

$$S = \int d^5x\sqrt{-G}\left[R - \frac{4}{3}(\nabla\phi)^2 - \Lambda e^{a\phi}\right] + \int d^4x\sqrt{-g}\left[-f(\phi)\right] \qquad (2.1.1)$$

describing a scalar field ϕ and gravity living in five dimensions coupled to a thin four-dimensional domain wall. Let us set the position of the domain wall at $x_5 = 0$. Here we follow the notation of [18] so that the metric $g_{\mu\nu}$ along the four-dimensional slice at $x_5 = 0$ is given in terms of the five-dimensional metric G_{MN} by

$$\begin{aligned} g_{\mu\nu} &= \delta_\mu^M \delta_\nu^N G_{MN}(x_5 = 0) \\ \mu, \nu &= 1, \ldots, 4 \\ M, N &= 1, \ldots, 5 \end{aligned} \qquad (2.1.2)$$

For concreteness, in much of our discussion we will make the choice

$$f(\phi) = Ve^{b\phi} \qquad (2.1.3)$$

However, most of our considerations will *not* depend on this detailed choice of $f(\phi)$ (for reasons that will become clear). With this choice, (2.1.1) describes a family of theories parameterized by V, Λ, a, and b. If $a = 2b = 4/3$, the action (2.1.1) agrees with tree-level string theory where ϕ is identified with the dilaton. (That is, the 5D cosmological constant term and the 4D domain wall tension term both scale like $e^{-2\phi}$ in string frame.) In Sec. 2.2 we will discuss a very natural context in which this type of action arises in string theory, either with the specific form (2.1.3) or with more general $f(\phi)$.

In the rest of this section we will derive the field equations arising from this action and construct some interesting solutions of these equations. In particular, we will be interested in whether there are Poincaré-invariant solutions for the metric of the four-dimensional slice at $x_5 = 0$ for generic values of these parameters (or more generally, for what subspaces of this parameter space there are Poincaré-invariant solutions in four dimensions). We will also require that the geometry is such that the four-dimensional Planck scale is finite. Our main results can be summarized in three different cases as follows:

(I) For $\Lambda = 0$, $b \neq \pm\frac{4}{3}$ but otherwise arbitrary, and arbitrary magnitude of V we find a Poincaré-invariant domain wall solution of the equations of motion. For $b = 2/3$, which is the value corresponding to a brane tension of order $e^{-2\phi}$ in string frame, the sign of V must be positive in order to correspond to a solution with a finite four-dimensional Planck scale, but it is otherwise unconstrained. This suggests that for fixed scalar field coupling to the domain wall, quantum corrections to its tension V do not spoil Poincaré-invariance of the slice. In §2.2 we will review examples in string theory of situations where worldvolume degrees of freedom contribute quantum corrections to the $e^{-2\phi}$ term in a brane's tension. Our result implies that these quantum corrections do not need to be fine-tuned to zero to obtain a flat four-dimensional spacetime.

For a generic choice of $f(\phi)$ in (2.1.1) (including the type of power series expansion in e^{ϕ} that would arise in perturbative string theory), the same basic results hold true: We are able to find Poincaré-invariant solutions without fine-tuning f. Insisting on a finite 4D Planck scale gives a furthur constraint on f'/f at the wall, forcing it to lie in a range of order one.

Given a solution with one value of V and $\Lambda = 0$, a self-tuning mechanism is in fact clear from the Lagrangian (for $b \neq 0$). In (2.1.1) we see that if $\Lambda = 0$ (or $a = 0$), the only non-derivative coupling of the dilaton is to the brane tension term, where it appears in the combination $(-V)e^{b\phi}$. Clearly given a solution for one value of V, there will be a solution for any value of V obtained by absorbing shifts in V with shifts in ϕ. With more general $f(\phi)$, similar remarks hold: the dilaton zero mode appears only in f, and one can absorb shifts in V by shifting ϕ.

However, in the special case $b = 0$ (where $f(\phi)$ is just a constant), we will also find flat solutions for generic V. This implies that the freedom to vary the dilaton zero mode is not the only mechanism that ensures the existence of a flat solution for arbitrary V.

(II) For $\Lambda = 0$, $b = \pm 4/3$, we find a different Poincaré-invariant solution (obtained by matching together two 5D bulk solutions in a different combination than that used in obtaining the solutions described in the preceding paragraph (I)). A solution is present for any value of V. This suggests that for fixed scalar field coupling to the domain wall, quantum corrections to its tension V do not spoil Poincaré-invariance of the slice. Again the sign of V must be positive in order to have a finite four-dimensional Planck scale.

(III) We do not find a solution (nor do we show that none exists) for general Λ, V, a, and b (in concordance with the counting of parameters in [42]). However, for each Λ and V there is a choice of a and b for which we do find a Poincaré-invariant solution using a simple ansatz.

For $a = 0$, and general b, Λ, and V we are currently investigating the existence of self-tuning solutions. Their existence would be in accord with the fact that in this case, as in the cases with $\Lambda = 0$, the dilaton zero mode only appears in the tension of the wall. This means again that shifts in V can be absorbed by shifting ϕ, so if one finds a Poincaré-invariant solution for any V, one does not need to fine-tune V to solve the equations.

2.1.2 Equations of Motion

The equations of motion arising for the theory (2.1.1), with our simple choice for $f(\phi)$ given in Eq. (2.1.3), are as follows. Varying with respect to the dilaton gives:

$$\sqrt{-G}\left(\frac{8}{3}\nabla^2\phi - a\Lambda e^{a\phi}\right) - bV\delta(x_5)e^{b\phi}\sqrt{-g} = 0 \qquad (2.1.4)$$

The Einstein equations for this theory are:

$$\begin{aligned}&\sqrt{-G}\Big(R_{MN}-\frac{1}{2}G_{MN}R\Big)\\&-\frac{4}{3}\sqrt{-G}\Big[\nabla_M\phi\nabla_N\phi-\frac{1}{2}G_{MN}(\nabla\phi)^2\Big]\\&+\frac{1}{2}\Big[\Lambda e^{a\phi}\sqrt{-G}G_{MN}+\sqrt{-g}Vg_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(x_5)\Big]=0\end{aligned}\tag{2.1.5}$$

We are interested in whether there are solutions with Poincaré-invariant four-dimensional physics. Therefore we look for solutions of Eqs. (2.1.4) and (2.1.5) where the metric takes the form

$$ds^2=e^{2A(x_5)}\big(-dx_1^2+dx_2^2+dx_3^2+dx_4^2\big)+dx_5^2\tag{2.1.6}$$

With this ansatz for the metric, the equations become

$$\frac{8}{3}\phi''+\frac{32}{3}A'\phi'-a\Lambda e^{a\phi}-bV\delta(x_5)e^{b\phi}=0\tag{2.1.7}$$

$$6(A')^2-\frac{2}{3}(\phi')^2+\frac{1}{2}\Lambda e^{a\phi}=0\tag{2.1.8}$$

$$3A''+\frac{4}{3}(\phi')^2+\frac{1}{2}e^{b\phi}V\delta(x_5)=0\tag{2.1.9}$$

where $'$ denotes differentiation with respect to x_5. The first one (2.1.7) is the dilaton equation of motion, the second (2.1.8) is the 55 component of Einstein's equations, and the last (2.1.9) comes from a linear combination (the difference) of the $\mu\nu$ component of Einstein's equation and the 55 component.

We will mostly consider the simple ansatz

$$A'=\alpha\phi'.\tag{2.1.10}$$

However for the case $a=0$, $\Lambda\neq 0$ we will integrate the equations directly.

2.1.3 $\Lambda=0$ *Case*

Let us first consider the system with $\Lambda=0$. We will first study the bulk equations of motion (i.e. the equations of motion away from $x_5=0$) where the δ-function terms in Eqs. (2.1.7) and (2.1.9) do not come in. Note that because the delta function terms do not enter, the bulk equations are independent of our choice of $f(\phi)$ in Eq. (2.1.1). We will then consider the conditions required to match two bulk solutions on either side of the domain wall of tension $Ve^{b\phi}$ at $x_5 = 0$. We will find two qualitatively different ways to do this, corresponding to results (I) and (II) quoted above. We will also find that for fairly generic $f(\phi)$, the same conclusions hold.

Bulk Equations: $\Lambda = 0$

Plugging the ansatz (2.1.10) into Eq. (2.1.8) (with $\Lambda = 0$) we find that

$$6\alpha^2(\phi')^2 = \frac{2}{3}(\phi')^2 \tag{2.1.11}$$

which is solved if we take

$$\alpha = \pm\frac{1}{3} \tag{2.1.12}$$

Plugging this ansatz into Eq. (2.1.7) we obtain

$$\frac{8}{3}\left[\phi'' + 4(\pm\frac{1}{3})(\phi')^2\right] = 0 \tag{2.1.13}$$

Plugging it into Eq. (2.1.9) we obtain

$$3(\pm\frac{1}{3})\phi'' + \frac{4}{3}(\phi')^2 = 0 \tag{2.1.14}$$

With either choice of sign for α, these two equations become identical in bulk. For $\alpha = \pm\frac{1}{3}$, we must solve

$$\phi'' \pm \frac{4}{3}(\phi')^2 = 0 \tag{2.1.15}$$

in bulk. This is solved by

$$\phi = \pm\frac{3}{4}\log\left|\frac{4}{3}x_5 + c\right| + d \tag{2.1.16}$$

, where c and d are arbitrary integration constants.

Note that there is a singularity in this solution at

$$x_5 = -\frac{3}{4}c \tag{2.1.17}$$

Our solutions will involve regions of spacetime to one side of this singularity; we will assume that it can be taken to effectively cut off the space. At present we do not have much quantitative to say about the physical implications of this singularity. The results we derive here (summarized above) strongly motivate further exploring the effects of these singularities on the four-dimensional physics of our domain wall solutions.

At $x_5 = 0$ there is localized energy density leading to the δ-function terms in Eq. (2.1.7) and (2.1.9). We can solve these equations by introducing appropriate discontinuities in ϕ' at the wall (while insisting that ϕ itself is continuous). We will now do this for two illustrative cases (the first being the most physically interesting).

Solution (I):

Let us take the bulk solution with $\alpha = +\frac{1}{3}$ for $x_5 < 0$. and the one with $\alpha = -\frac{1}{3}$ for $x_5 > 0$. So we have

$$\phi(x_5) = \phi_1(x_5) = \frac{3}{4}\log\left|\frac{4}{3}x_5 + c_1\right| + d_1. \quad x_5 < 0 \tag{2.1.18}$$

$$\phi(x_5) = \phi_2(x_5) = -\frac{3}{4}\log\left|\frac{4}{3}x_5 + c_2\right| + d_2. \quad x_5 > 0 \tag{2.1.19}$$

where we have allowed for the possibility that the (so far) arbitrary integration constants can be different on the two sides of the domain wall.

Imposing continuity of ϕ at $x_5 = 0$ leads to the condition

$$\frac{3}{4}\log|c_1| + d_1 = -\frac{3}{4}\log|c_2| + d_2 \tag{2.1.20}$$

This equation determines the integration constant d_2 in terms of the others.

To solve (2.1.7) we then require

$$\frac{8}{3}(\phi_2'(0) - \phi_1'(0)) = bVe^{b\phi(0)} \tag{2.1.21}$$

while to solve (2.1.9) we need

$$3\Big(\alpha_2\phi_2'(0) - \alpha_1\phi_1'(0)\Big) = -\frac{1}{2}Ve^{b\phi(0)} \tag{2.1.22}$$

(where $\alpha_1 = +\frac{1}{3}$ and $\alpha_2 = -\frac{1}{3}$). These two matching conditions become

$$-\frac{8}{3}\left(\frac{1}{c_1} + \frac{1}{c_2}\right) = bVe^{bd_1}|c_1|^{\frac{3}{4}b} \tag{2.1.23}$$

and

$$\frac{1}{c_2} - \frac{1}{c_1} = -\frac{1}{2} V e^{bd_1} |c_1|^{\frac{3}{4}b} \tag{2.1.24}$$

Solving for the integration constants c_1 and c_2 we find

$$\frac{2}{c_2} = \left[-\frac{3b}{8} - \frac{1}{2} \right] V e^{bd_1} |c_1|^{\frac{3}{4}b} \tag{2.1.25}$$

$$\frac{2}{c_1} = \left[-\frac{3b}{8} + \frac{1}{2} \right] V e^{bd_1} |c_1|^{\frac{3}{4}b} \tag{2.1.26}$$

Note that as long as $b \neq \pm\frac{4}{3}$, we here find a solution for the integration constants c_1 and c_2 in terms of the parameters b and V which appear in the Lagrangian and the integration constant d_1. (As discussed above, the integration constant d_2 is then also determined).[8] In particular, for scalar coupling given by b, there is a Poincaré-invariant four-dimensional domain wall for any value of the brane tension V; V does not need to be fine-tuned to find a solution. As is clear from the form of the 4D interaction in (2.1.1), one way to understand this is that the scalar field ϕ can absorb a shift in V since the only place that the ϕ zero mode appears in the Lagrangian is multiplying V. However since we can use these equations to solve for $c_{1,2}$ without fixing d_1, a more general story is at work; in particular, even for $b = 0$ we find solutions for arbitrary V.

A constraint on the sign of V arises, as we will now discuss, from the requirement that there be singularities (2.1.17) in the bulk solutions, effectively cutting off the x_5 direction at finite volume.

More General $f(\phi)$

If instead of Eq. (2.1.3) we include a more general choice of f in the action (2.1.1), the considerations above go through unaltered. The choice of f only enters in the matching conditions (2.1.21) and (2.1.22) at the domain wall. The modified equations become

$$\frac{8}{3}\left(\phi_2'(0) - \phi_1'(0)\right) = \frac{\partial f}{\partial \phi}\left(\phi(0)\right) \tag{2.1.27}$$

$$3\left(\alpha_2 \phi_2'(0) - \alpha_1 \phi_1'(0)\right) = -\frac{1}{2} f\left(\phi(0)\right) \tag{2.1.28}$$

[8] We will momentarily find a disjoint set of $\Lambda = 0$ domain wall solutions for which b will be forced to be $\pm 4/3$, so altogether there are solutions for any b.

In terms of the integration constants, these become:

$$-\frac{8}{3}\left(\frac{1}{c_1}+\frac{1}{c_2}\right)=\frac{\partial f}{\partial\phi}\left(\frac{3}{4}\log|c_1|+d_1\right) \tag{2.1.29}$$

$$\frac{1}{c_2}-\frac{1}{c_1}=-\frac{1}{2}f\left(\frac{3}{4}\log|c_1|+d_1\right) \tag{2.1.30}$$

Clearly for generic $f(\phi)$, one can solve these equations.

Obtaining a Finite 4D Planck Scale

Consider the solution (2.1.18) on the $x_5<0$ side. If $c_1<0$, then there is never a singularity. Let us consider the four-dimensional Planck scale. It is proportional to the integral [18]

$$\int dx_5\; e^{2A(x_5)} \tag{2.1.31}$$

In the $x_5<0$ region, this goes like

$$\int dx_5\sqrt{\left|\frac{4}{3}x_5+c_1\right|} \tag{2.1.32}$$

If $c_1<0$, then there is no singularity, and this integral is evaluated from $x_5=-\infty$ to $x_5=0$. It diverges. If $c_1>0$, then there is a singularity at (2.1.17). Cutting off the volume integral (2.1.32) there gives a finite result. Note that the ansatz (2.1.10) leaves an undetermined integration constant in A, so one can tune the actual value of the 4D Planck scale by shifting this constant.

In order to have a finite 4D Planck scale, we therefore impose that $c_1>0$. This requires $V(\frac{1}{2}-\frac{3b}{8})>0$. For the value $b=2/3$, natural in string theory (as we will discuss in §2.2), this requires $V>0$. With this constraint, there is similarly a singularity on the $x_5>0$ side which cuts off the volume on that side.

These conditions extend easily to conditions on $f(\phi)$ in the more general case. We find

$$\begin{aligned}-\frac{3}{8}\frac{\partial f}{\partial\phi}(\phi(0))-\frac{1}{2}f(\phi(0))<0\\ -\frac{3}{8}\frac{\partial f}{\partial\phi}(\phi(0))+\frac{1}{2}f(\phi(0))>0\end{aligned} \tag{2.1.33}$$

This means that $f(\phi)$ must be positive at the wall (corresponding to a positive tension brane), and that

$$-\frac{4}{3}<\frac{f'}{f}<\frac{4}{3} \tag{2.1.34}$$

So although f does not need to be fine-tuned to achieve a solution of the sort we require, it needs to be such that f'/f is in the range (2.1.34).

Let us discuss some of the physics at the singularity. Following [18,42], we can compute the x_5-dependence of the four-dimensional graviton wavefunction. Expanding the metric about our solution (taking $g_{\mu\nu} = e^{2A}\eta_{\mu\nu} + h_{\mu\nu}$), we find

$$h_{\mu\nu} \propto \sqrt{\left|\frac{4}{3}x_5 + c\right|} \tag{2.1.35}$$

At a singularity, where $|\frac{4}{3}x_5 + c|$ vanishes, this wavefunction also vanishes. Without understanding the physics of the singularity, we cannot determine yet whether it significantly affects the interactions of the four-dimensional modes.

It is also of interest to consider the behavior of the scalar ϕ at the singularities. In string theory this determines the string coupling. In our solution (I), we see that

$$\begin{aligned} x_5 &\to -\frac{3}{4}c_1 \Rightarrow \phi \to -\infty \\ x_5 &\to -\frac{3}{4}c_2 \Rightarrow \phi \to \infty \end{aligned} \tag{2.1.36}$$

So in string theory, the curvature singularity on the $x_5 < 0$ side is weakly coupled, while that on the $x_5 > 0$ side is strongly coupled. It may be possible to realize these geometries in a context where supersymmetry is broken by the brane, so that the bulk is supersymmetric. In such a case the stability of the high curvature and/or strong-coupling regions may be easier to ensure. In any case we believe that the results of this section motivate further analysis of these singular regions, which we leave for future work.

Putting everything together, we have found the solution described in case (I) above. It should be clear that since $f(\phi)$ only appears in Eq. (2.1.1) multiplying the delta function "thin wall" source term, we can always use the choice (2.1.3) in writing matching conditions at the wall for concreteness. To understand what would happen with a more general f, one simply replaces $Ve^{b\phi(0)}$ with $f(\phi(0))$ and $bVe^{b\phi(0)}$ with $\frac{\partial f}{\partial \phi}(\phi(0))$ in the matching equations. We will not explicitly say this in each case, but it makes the generalization to arbitrary f immediate.

Solution (II):

A second type of solution with $\Lambda = 0$ is obtained by taking α to have the same sign on both sides of the domain wall. So we have

$$\phi(x_5) = \phi_1(x_5) = \pm\frac{3}{4}\log\left|\frac{4}{3}x_5 + c_1\right| + d_1, \quad x_5 < 0 \tag{2.1.37}$$

$$\phi(x_5) = \phi_2(x_5) = \pm\frac{3}{4}\log\left|\frac{4}{3}x_5 + c_2\right| + d_2, \quad x_5 > 0 \tag{2.1.38}$$

The matching conditions then require $b = \mp\frac{4}{3}$ for consistency of Eqs. (2.1.7) and (2.1.9) (in the case with more generic $f(\phi)$, this generalizes to the condition $\frac{\partial f}{\partial \phi}(\phi(0)) = \mp\frac{4}{3}f(\phi(0))$). This is not a value of b that appears from a dilaton coupling in perturbative string theory. It is still interesting, however, as a gravitational low-energy effective field theory where V does not have to be fine-tuned in order to preserve four-dimensional Poincaré-invariance. We find a solution to the matching conditions with

$$\begin{aligned} c_1 &= c, \quad x_5 > 0 \\ c_2 &= -c, \quad x_5 < 0 \\ d_1 &= d_2 = d \\ e^{\mp\frac{4}{3}d} &= \frac{4}{V}\frac{c}{|c|} \end{aligned} \tag{2.1.39}$$

for some arbitrary constant c, and any V. This gives the results summarized in case (II) above. The value $b = \mp 4/3$, which is required here, was excluded from the solutions (I) derived in the last section.

As long as we choose c such that there are singularities on both sides of the domain wall, we again get finite 4D Planck scale. As we can see from Eqs. (2.1.37) and (2.1.38), having singularities on either side of the origin requires c to be positive. Then we see from (2.1.39) that we can find a solution for arbitrary positive brane tension V.

Let us discuss the physics of the singularities in this case. As in solutions (I), the graviton wavefunction decays to zero at the singularity like $(x - x_{\text{sing}})^{\frac{1}{2}}$. For $b = -4/3$, $\phi \to -\infty$ at the singularities on both sides, while for $b = \frac{4}{3}$, $\phi \to \infty$ at the singularities on both sides.

Putting solutions (I) and (II) together, we see that in the $\Lambda = 0$ case one can find a Poincaré-invariant solution with finite 4D Planck scale for any positive tension V and any choice of b in Eq. (2.1.1). As we have seen, this in fact remains true with Eq. (2.1.3) replaced by a more general dilaton dependent brane tension $f(\phi)$.

Two-Brane Solutions

One can also obtain solutions describing a pair of domain walls localized in a compact fifth dimension. In case (I), one can show that such solutions always involve singularities. In case (II), there are solutions which avoid singularities while maintaining the finiteness of the four-dimensional Planck scale. They however involve extra moduli (the size of the compactified fifth dimension) which may be stabilized by for example the mechanism of [44]. The singularity is avoided in these cases by placing a second domain wall between $x_5 = 0$ and the would-be singularity at $\frac{4}{3}x_5 + c = 0$. This allows us in particular to find solutions for which ϕ is bounded everywhere, so that the coupling does not get too strong. This is a straightforward generalization of what we have already done and we will not elaborate on it here.

2.1.4 $\Lambda \neq 0$ *(Solution III)*

More generally we can consider the entire Lagrangian (2.1.1) with parameters Λ, V, a, b. In this case, plugging in the ansatz (2.1.10) to Eqs. (2.1.7)-(2.1.9), we find a bulk solution

$$\begin{aligned} \phi &= -\frac{2}{a}\log(\frac{a(\mp\sqrt{B})}{2}x_5 + d) \\ B &= \frac{\Lambda}{\frac{4}{3} - 12\alpha^2} \\ \alpha &= -\frac{8}{9a} \end{aligned} \tag{2.1.40}$$

We find a domain wall solution by taking one sign in the argument of the logarithm in Eq. (2.1.40) for $x_5 < 0$ and the opposite sign in the argument of the logarithm for $x_5 > 0$. Say for instance that $a > 0$. Then we could take the $-$ sign for $x > 0$ and the $+$ sign for $x < 0$, and find a solution which terminates at singularities on both sides if we choose $d > 0$.

The matching conditions then require

$$V = -12\alpha\sqrt{B} \tag{2.1.41}$$

and

$$b = -\frac{4}{9\alpha} \tag{2.1.42}$$

So we see that here V must be fine-tuned to the Λ-dependent value given in Eq. (2.1.41). This is similar to the situation in [18], where one fine-tune is required to set the four-dimensional cosmological constant to zero. Like in our solutions in Sec. 2.1.1, there is one undetermined parameter in the Lagrangian. But here it is a complicated combination of Λ and V (namely, $V/\sqrt{\Lambda}$), and we do not have an immediate interpretation of variations of this parameter as arising from nontrivial quantum corrections from a sector of the theory.

The fact, apparent from Eqs. (2.1.40) and (2.1.42), that $b = a/2$ in this solution makes its embedding in string theory natural, as we will explain in the next section.

$\Lambda \neq 0,\ a = 0$

In this case, the bulk equations of motion become (in terms of $h \equiv \phi'$ and $g \equiv A'$)

$$\begin{aligned} &h' + 4hg = 0 \\ &6g^2 - \frac{2}{3}h^2 + \frac{1}{2}\Lambda = 0 \\ &3g' + \frac{4}{3}h^2 = 0 \end{aligned} \tag{2.1.43}$$

We can solve the second equation for g in terms of h, and then integrate the first equation to obtain $h(x_5)$. For $g \neq 0$ the third equation is then automatically satisfied. We will not need detailed properties of the solution, so we will not include it here. The solutions are more complicated than those of Sec. 2.1.3. We are currently exploring under what conditions one can solve the matching equations to obtain a wall with singularities cutting off the x_5 direction on both sides [4]. If such walls exist, they will also exhibit the self-tuning phenomenon of Sec. 2.1.3, since the dilaton zero mode can absorb shifts in V and doesn't appear elsewhere in the action.

2.2 Toward a String Theory Realization

2.2.1 $\Lambda = 0$ *Cases*

Taking $\Lambda = 0$ is natural in string theory, since the tree-level vacuum energy in generic critical closed string compactifications (supersymmetric or not) vanishes. One would expect bulk quantum corrections to correct Λ in a power series in $g_s = e^{\phi}$. However, the analysis of Sec. 2.1.3 may still be of interest if the bulk corrections to Λ are small enough. This can happen for instance if the supersymmetry breaking is localized in a small neighborhood of the wall and the x_5 interval is much larger, or more generally if the supersymmetry breaking scale in bulk is small enough.

General $f(\phi)$

The examples we have found in Sec. 2.1 which "self-tune" the 4D cosmological constant to zero have $\Lambda = 0$ with a broad range of choices for $f(\phi)$. We interpret this as meaning that quantum corrections to the brane tension, which would change the form of f, do not destabilize the flat brane solution. The generality of the dilaton coupling $f(\phi)$ suggests that our results should apply to a wide variety of string theory backgrounds involving domain walls. We now turn to a discussion of some of the features of particular cases.

D-branes

In string theory, one would naively expect codimension-1 D-branes (perhaps wrapping a piece of some compact manifold) to have $f(\phi)$ given by a power series of the form

$$f(\phi) = e^{\frac{5}{3}\phi} \sum_{n=0}^{\infty} c_n e^{n\phi} \tag{2.2.1}$$

The c_0 term represents the tree-level D-brane tension (which goes like $1/g_s$ in string frame). The higher order terms in Eq. (2.2.1) represent quantum corrections from the Yang-Mills theory on the brane, which has coupling $g_{\rm YM}^2 = e^{\phi}$.

If one looks for solutions of the equations which arise with the choice (2.1.3) for $f(\phi)$ with positive V and $b = 5/3$ (the tree level D-brane theory), then there are no solutions with finite 4D Planck scale. The constraints of Sec. 2.1.3 cannot be solved to give a single wall with singularities on both sides cutting off the length in the x_5 direction. However, including quantum corrections to the D-brane theory to get a more generic f as in (2.2.1), there is a constraint on the magnitude of $\frac{\partial f}{\partial \phi}(\phi(0))$ divided by $f(\phi(0))$ which can be obeyed. Therefore, one concludes that for our mechanism to be at work with D-brane domain walls, the dilaton ϕ must be stabilized away from weak coupling—the loop corrections to Eq. (2.2.1) must be important.

The Case $f(\phi) = Ve^{\frac{2}{3}\phi}$ and NS Branes

Another simple way to get models which could come out of string theory is to set $b = 2/3$ in (2.1.3), so

$$f(\phi) = Ve^{\frac{2}{3}\phi} \tag{2.2.2}$$

Then (2.1.1) becomes precisely the Einstein frame action that one would get from a "3-brane" in string theory with a string frame source term proportional to $e^{-2\phi}$. In this case, ϕ can also naturally be identified with the string theory dilaton. This choice of b is possible in solutions of the sort summarized in result (I) in Sec. 2.1.1.

However, after identifying ϕ with the string theory dilaton, if we really want to make this specific choice for $f(\phi)$ we would also like to find branes where it is natural to expect that quantum corrections to the brane tension (e.g. from gauge and matter fields living on the brane) would shift V, but not change the overall ϕ dependence of the source term. This can only happen if the string coupling $g_s = e^{\phi}$ is *not* the field-theoretic coupling parameter for the dynamical degrees of freedom on the brane.

Many examples where this happens are known in string theory. For example, the NS five-branes of Type IIB and heterotic string theory have gauge fields on their worldvolume whose Yang-Mills coupling does not depend on g_s [45,46,47]. This can roughly be understood from the fact that the dilaton grows to infinity down the throat of the solution, and its value in the asymptotic flat region away from this throat is irrelevant to the coupling of the modes on the brane. Upon compactification, this leads to gauge couplings depending on sizes of cycles in the compactification manifold (in units of α') [46,48,49]. For instance, in [48,49] gauge groups which arise "non-perturbatively" in singular heterotic compactifications (at less supersymmetric generalizations of the small instanton singularity [45]) are discussed. There, the 4D gauge couplings on a heterotic NS five-brane wrapped on a two-cycle go like

$$g_{\rm YM}^2 \sim \frac{\alpha'}{R^2} \tag{2.2.3}$$

Here R is the scale of this two-cycle in the compactification manifold. In [48,49], this was used to interpret string sigma model worldsheet instanton effects, which go like $e^{-R^2/\alpha'}$, in terms of nonperturbative effects in the brane gauge group, which go like $e^{-8\pi^2/g_{YM}^2}$. So this is a concrete example in which nontrivial dilaton-independent quantum corrections to the effective action on the brane arise. One can imagine analogous examples involving supersymmetry breaking. In such cases, perturbative shifts in the brane tension due to brane worldvolume gauge dynamics would be a series in α'/R^2 and not $g_s = e^{\phi}$.

In particular, one can generalize such examples to cases where the branes are domain walls in 5D spacetime (instead of space-filling in 4D spacetime as in the examples just discussed), but where again the brane gauge coupling is not the string coupling. Quantum corrections to the brane tension in the brane gauge theory then naturally contribute shifts

$$e^{\frac{2}{3}\phi}V \rightarrow e^{\frac{2}{3}\phi}(V + \delta V) \tag{2.2.4}$$

to the (Einstein frame) $b = 2/3$ source term in Eq. (2.1.1), without changing its dilaton dependence.

Most of our discussion here has focused on the case where ϕ is identified with the string theory dilaton. However, in general it is possible that some other string theory modulus could play the role of ϕ in our solutions, perhaps for more general values of b.

Resemblance to Orientifolds

In our analysis of the equations, we find solutions describing a 4D gravity theory with zero cosmological constant if we consider singular solutions and cut off the fifth dimension at these singularities. The simplest versions of compactifications involving branes in string theory also include defects in the compactification which absorb the charge of the branes and cancel their contribution to the cosmological constant in four dimensions, at least at tree level. Examples of these defects include orientifolds (in the context of D-brane worlds), S-duals of orientifolds (in the context of NS brane worlds), and Horava-Witten "ends of the world" (in the context of the strongly coupled heterotic string).

Our most interesting solutions involve two different behaviors on the two sides of the domain wall. On one side the dilaton goes to strong coupling while on the other side it goes to weak coupling at the singularity. This effect has also been seen in brane-orientifold systems [50].

It would be very interesting to understand whether the singularities we find can be identified with orientifold-like defects, as these similarities might suggest. Then their role (if any) in absorbing quantum corrections to the 4D cosmological constant could be related to the effective negative tension of these defects. However, various aspects of our dilaton gravity solutions are not familiar from brane-orientifold systems. In particular, the existence of solutions with curved 4D geometry on the same footing as our flat solutions does not occur in typical perturbative string compactifications. In any case, note that (as explained in Sec. 2.2.1) our mechanism does not occur in the case of weakly coupled D-branes and orientifolds.

2.2.2 $\Lambda \neq 0$ *Cases*

Some of the $\Lambda \neq 0$ cases discussed in Sec. 2.1.4 could also arise in string theory. As discussed in [51,49] one can find closed string backgrounds with nonzero tree level cosmological constant $\Lambda < 0$ by considering subcritical strings. In this case, the cosmological term would have dilaton dependence consistent with $a = 4/3$ in bulk. Using Eqs. (2.1.40) and (2.1.42), this implies $b = 2/3$, which is the expected scaling for a tree-level brane tension in the thin-wall approximation as well.

One would naively expect to obtain vacua with such negative bulk cosmological constants out of tachyon condensation in closed string theory [51,49]. It is then natural to consider these domain walls (in the $a = 4/3, b = 2/3$ case) as the thin wall approximation to "fat" domain walls which could be formed by tachyon field configurations which interpolate between different minima of a closed string tachyon potential. In the context of the Randall-Sundrum scenario, such "fat" walls were studied for example in [42,52,53].

It would be interesting to find cases where the $\Lambda \neq 0$, $a = 0$ solutions arise from a more microscopic theory. However, it is clear that the dilaton dependence of Eq. (2.1.1) is then not consistent with interpreting ϕ as the string theory dilaton. Perhaps one could find a situation where ϕ can be identified with some other string theoretic modulus, and Λ can be interpreted as the bulk cosmological constant after other moduli are fixed.

2.3 Discussion

The concrete results of Sec. 2.1 motivate many interesting questions. which we have only begun to explore. Answering these questions will be important for understanding the four-dimensional physics of our solutions.

The most serious question has to do with the nature of the singularities. There are many singularities in string theory which have sensible physical resolutions. either due to the finite string tension or due to quantum effects. Most that have been studied (like flops [54.55] and conifolds [56]) involve systems with some supersymmetry. though some (like orbifolds [57]) can be understood even without supersymmetry. We do not yet know the proper interpretation of our singularities. though as discussed in Sec. 2.2 there are intriguing similarities to orientifold physics in our system. After finding the solutions. we cut off the volume integral determining the four-dimensional Planck scale at the singularities. It is important to determine whether this is a legitimate operation.

It is desirable (and probably necessary in order to address the question in the preceding paragraph) to embed our solutions microscopically into M theory. As discussed in Sec. 2.2. some of our solutions appear very natural from the point of view of string theory, where the scalar ϕ can be identified with the dilaton. It would be interesting to consider the analogous couplings of string-theoretic moduli scalars other than the dilaton. Perhaps there are other geometrical moduli which couple with different values of a and b in Eq. (2.1.3) than the dilaton does.

It is also important to understand the effects of quantum corrections to quantities other than $f(\phi)$ in our Lagrangian. In particular, corrections to Λ and corrections involving different powers of e^{ϕ} in the bulk (coming from loops of bulk gravity modes) will change the nature of the equations. It will be interesting to understand the details of curved 4D domain wall solutions to the corrected equations [58,42,4]. More specifically, it will be of interest to determine the curvature scale of the 4D slice, in terms of the various choices of phenomenologically natural values for the Planck scale. Since the observed value of the cosmological constant is nonzero according to studies of the mass density, cosmic microwave background spectral distribution, and supernova events [59], such corrected solutions might be of physical interest.

Perhaps the most intriguing physical question is what happens from the point of view of four-dimensional effective field theory (if such a description in fact exists).

Understanding the singularity in the 5D background is probably required to answer this question. One possibility (suggested by the presence of the singularity and by the self-tuning of the 4D cosmological constant discovered here) is that four-dimensional effective field theory breaks down in this background, at least as far as contributions to the 4D cosmological constant are concerned. In [18] and analogous examples, there is a continuum of bulk modes which could plausibly lead to a breakdown of 4D effective field theory in certain computations. In our theories, cutting off the 5D theory at the singularities leaves finite proper distance in the x_5 direction. This makes it unclear how such a continuum could arise (in the absence of novel physics at the singularities, which could include "throats" of the sort that commonly arise in brane solutions). So in this system, any breakdown of 4D effective field theory is more mysterious.

3. Bounds on Curved Domain Walls

In this chapter. we extend the results of the previous chapter to curved domain walls. In Chap. 2. we concentrated for the most part on 5D gravity theories with a bulk scalar dilaton ϕ. and action

$$S = M_*^3 \int d^5x\sqrt{-G}\left[R - \frac{4}{3}(\nabla\phi)^2\right] + \int d^4x\sqrt{-g}(-f(\phi)) \tag{3.0.1}$$

(with vanishing bulk cosmological term). In Eq. (3.0.1), G is the 5D bulk metric, while g is the induced metric on the domain wall. which is located at $x_5 = 0$. We demonstrated that one can find flat domain wall solutions for fairly generic thin wall δ-function sources $f(\phi)$, i.e.. without "fine-tuning" the brane tension $f(\phi)$. This is important because quantum loops of brane matter fields will in the most general circumstances correct the form of $f(\phi)$; it demonstrates some insensitivity of the existence of a flat 4D world to brane quantum loops. However. we did not address the issue of *curved* (de Sitter or anti-de Sitter) solutions to the same 5D equations of motion.

Here we find curved solutions with maximal symmetry in four dimensions. More specifically, for both negatively curved and positively curved deformations we find that the largest scale of curvature possible is given by the scale set by the inverse proper length of the fifth dimension. In particular, the curvature can at most reach the mass scale of Kaluza-Klein modes in the fifth dimension. Unfortunately this upper bound is essentially equivalent to a 4D vacuum energy of the order of the scale of brane physics.

The organization of this chapter is as follows. In Sec. 3.1 we describe the bulk gravity solutions with maximally symmetric curved domain walls and the matching

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boundary conditions at the domain wall. In Sec. 3.2 we explain the bounds on the curvature of curved solutions that result from these solutions. In Sec. 3.3. we discuss some additional issues of interest in analyzing the physics of the solutions discussed here and in [3,5], including the singularities.

3.1 Curved Solutions and Matching Conditions

We will make the following ansatz for the metric (following [42,58]):

$$ds^2 = e^{2A(x_5)}\bar{g}_{\mu\nu}dx^\mu dx^\nu + dx_5^2, \tag{3.1.1}$$

where

$$\bar{g}_{\mu\nu} = \text{diag}(-1, e^{2\sqrt{\bar{\Lambda}}x_1}, e^{2\sqrt{\bar{\Lambda}}x_1}, e^{2\sqrt{\bar{\Lambda}}x_1})$$

for de Sitter space and

$$\bar{g}_{\mu\nu} = \text{diag}(-e^{2\sqrt{-\bar{\Lambda}}x_4}, e^{2\sqrt{-\bar{\Lambda}}x_4}, e^{2\sqrt{-\bar{\Lambda}}x_4}, 1)$$

for anti-de Sitter space.

Plugging this ansatz into the dilaton equations and Einstein's equations gives

$$\frac{8}{3}\phi'' + \frac{32}{3}A'\phi' - \frac{\partial f}{\partial\phi}\delta(x_5) = 0 \tag{3.1.2}$$

$$6(A')^2 - \frac{2}{3}(\phi')^2 - 6\bar{\Lambda}e^{-2A} = 0 \tag{3.1.3}$$

$$3A'' + \frac{4}{3}(\phi')^2 + 3\bar{\Lambda}e^{-2A} + \frac{1}{2}f(\phi)\delta(x_5) = 0 \tag{3.1.4}$$

Note here that the zero mode $A(0)$ always appears together with $\bar{\Lambda}$ here; we will take $A(0) = 0$ in what follows.

Integrating the first equation in the bulk gives

$$\phi' = \gamma e^{-4A} \tag{3.1.5}$$

for some integration constant γ. Substituting this into the second equation gives

$$A' = \epsilon\sqrt{\tfrac{1}{9}\gamma^2 e^{-8A} + \bar{\Lambda}e^{-2A}} \tag{3.1.6}$$

where $\epsilon = \pm 1$ determines the branch of the square root that we choose in the solution. Note here that this solution only makes sense when the argument of the square root in Eq. (3.1.6) is positive: for anti-de Sitter slices (negative $\bar{\Lambda}$) this gives a constraint on $\bar{\Lambda}$ which we will discuss in §3.

This equation can be integrated to yield

$$\int^{A} \frac{\epsilon\, dA}{\sqrt{\frac{1}{9}\gamma^2 e^{-8A} + \bar{\Lambda} e^{-2A}}} = x_5 + \frac{3}{4}c. \tag{3.1.7}$$

The left-hand side of Eq. (3.1.7) is

$$\frac{3}{4}\epsilon \frac{1}{|\gamma|} e^{4A}\, {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, -\frac{9\bar{\Lambda}}{\gamma^2} e^{6A}\right) = x_5 + \frac{3}{4}c. \tag{3.1.8}$$

where ${}_2F_1(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, z) \equiv F(z)$ is a hypergeometric function. It is analytic on $\mathbf{C} - \{[1, \infty) \subset \mathbf{R}\}$ and increases monotonically from $F(-\infty) = 0$ through $F(0) = 1$ until it attains its maximum at

$$F(1) = F_{\max} = \frac{\Gamma(\frac{5}{3})\Gamma(\frac{1}{2})}{\Gamma(\frac{7}{6})} = 1.725.$$

Because $F \geq 0$, the solution (3.1.8) is valid only on one side of $x_5 = -\frac{3}{4}c$ (determined by the sign ϵ). At $x_5 = -c$ there is a curvature singularity. As in [3.5], we make the assumption that the space can be truncated at this singularity, at least as far as low-energy physics is concerned.

Let us now introduce a domain wall at $x_5 = 0$. We must match the bulk solutions (given implicitly in Eqs. (3.1.5) and (3.1.8)) on the two sides of the wall, consistent with the δ-function terms in Eqs. (3.1.2) and (3.1.4). Let us denote the integration constants on the left ($x_5 < 0$) side of the wall by c_1, γ_1, d_1 and those on the right ($x_5 > 0$) side by c_2, γ_2, d_2. Here d_i refers to the zero mode $\phi(0)$ of the dilaton field on the ith side of the wall. Imposing continuity of ϕ at the wall fixes d_2.

Defining $\tilde{c}_i = c_i/F|_{x_5=0}$, we find the matching conditions

$$-\frac{8}{3}M_*^3\left(\frac{1}{\tilde{c}_1} + \frac{1}{\tilde{c}_2}\right) = \frac{\partial f}{\partial \phi}(\phi(0)), \tag{3.1.9}$$

$$M_*^3\left(\sqrt{\left(\frac{1}{\tilde{c}_1}\right)^2 + 9\bar{\Lambda}} + \sqrt{\left(\frac{1}{\tilde{c}_2}\right)^2 + 9\bar{\Lambda}}\right) = \frac{1}{2}f(\phi(0)). \tag{3.1.10}$$

We here used the fact, which follows from Eq.(3.1.8) evaluated at $x_5 = 0$ with $A(0) = 0$, that $|\gamma_i||\tilde{c}_i| = 1$.

3.2 Bounds on Curved Deformations

3.2.1 Asymmetric Solutions (I) (General $f(\phi)$)

We have now gathered the information we need to determine the extent of curvature of these curved-slice deformations of the flat solutions of [3.5]. We will first discuss a bound on $|\bar{\Lambda}|$, which basically constrains it to be less than the inverse proper length of the fifth dimension, that applies to both signs of $\bar{\Lambda}$. We will then discuss a tighter bound that arises in the case of positive $\bar{\Lambda}$.

Consider Eq. (3.1.8) at $x_5 = 0$:

$$\epsilon|\bar{c}|F\left(-9\Lambda(\bar{c})^2\right) = c. \tag{3.2.1}$$

Defining $y \equiv \sqrt{9|\bar{\Lambda}|}|\bar{c}|$, this equation implies

$$|yF(-y^2)| = |c|\sqrt{9|\bar{\Lambda}|}. \tag{3.2.2}$$

Now the quantity $yF(-y^2)$ is bounded. In fact, its maximum value (attained as $y \to \infty$) is 4. We have from Eq. (3.2.2) that

$$\sqrt{9|\bar{\Lambda}|} < 4\left|\frac{1}{c_i}\right| \tag{3.2.3}$$

where we added the index to c_i since this bound applies on either side of the domain wall.

Note from the metric (3.1.1) that $|c_i|$ is the proper distance to the singularity on the ith side of the wall. So for either sign of $\bar{\Lambda}$, we find that the effective 4D cosmological constant of the curved solutions is bounded to be smaller than the Kaluza-Klein scale in the bulk.

This reflects the same physical point made in [3,5]: there is no contribution from physics localized on the brane to the 4D cosmological constant. A brane-scale cosmological constant would have manifested itself in a contribution to Eq. (3.2.3) which depends on $f(\phi(0))$, and such terms are absent. These bounds arise from the matching conditions, but note that it is not the case that the singularities recede to ∞ (or come in to the origin) as one saturates the bound.

The largest phenomenologically viable value for the proper distance c is roughly a millimeter [15,16]. This would give us a bound on $\bar{\Lambda}$ of about 10^{-6} eV^2. This is much larger than the observed value $\bar{\Lambda} \sim 10^{-64}$ eV^2 of the cosmological constant.

Note that we here are using "general relativity" conventions for the cosmological constant $\bar{\Lambda}$; the standard "particle physics" cosmological constant is $\Lambda_4 = M_4^2\bar{\Lambda} \sim$ mm^{-4}. Unfortunately this is within a couple of orders of magnitude of the standard model scale of TeV^4. In a model with supersymmetry spontaneously broken at the TeV scale, this would be the scale of a brane cosmological constant.

For positive $\bar{\Lambda}$ this $1/c$ scale is itself bounded by a further constraint. Consider the matching condition (3.1.10). It implies that

$$\frac{1}{\tilde{c}_i^2} < \frac{1}{2M_*^3} f(\phi). \tag{3.2.4}$$

Therefore, since $c = \tilde{c}/F|_0$, we can extend Eq. (3.2.3) to

$$\sqrt{9\bar{\Lambda}} < 4F_{\max}\frac{1}{2M_*^3} f(\phi) \tag{3.2.5}$$

In fact we can do better than Eq. (3.2.5). The 4D Planck scale M_4 is given by

$$M_4^2 = M_*^3 \int dx_5\, e^{2A} = \frac{M_*^3}{9\bar{\Lambda}}\left(\sqrt{\frac{1}{|\tilde{c}_1|^2}+9\bar{\Lambda}}+\sqrt{\frac{1}{|\tilde{c}_2|^2}+9\bar{\Lambda}}-\frac{1}{|\tilde{c}_1|}-\frac{1}{|\tilde{c}_2|}\right). \tag{3.2.6}$$

Multiplying Eq. (3.2.6) by $\bar{\Lambda}$, and dividing by M_4^2, we get an equation for $\bar{\Lambda}$. For positive $\bar{\Lambda}$, we can use the matching condition (3.1.10) to replace the first two terms in the parentheses in Eq. (3.2.6) with $\frac{1}{2}f(\phi(0))$. We then obtain the inequality

$$\bar{\Lambda} < \frac{1}{18}\frac{f(\phi(0))}{M_4^2} \tag{3.2.7}$$

(for negative $\bar{\Lambda}$, we would not obtain such an inequality). So for instance if the value of $f(\phi(0))$ is TeV scale, which is natural if we take the standard model (cut off at about a TeV) to live on the brane, then

$$\bar{\Lambda} < 10^{-33}\ \mathrm{TeV}^2. \tag{3.2.8}$$

This is of the same order as the contribution of a brane with supersymmetry spontaneously broken at a TeV.

3.2.2 Symmetric Solutions (II) $\left[f(\phi) = e^{\pm\frac{4}{3}\phi}\right]$

When we pick $f(\phi) = e^{\pm\frac{4}{3}\phi}$, we find the matching condition (3.1.9) becomes

$$M_*^3\left(\frac{1}{|\bar{c}_1|} + \frac{1}{|\bar{c}_2|}\right) = \pm\frac{1}{2}e^{\pm\frac{4}{3}\phi} \tag{3.2.9}$$

which agrees with the second condition (3.1.10) when $\bar{\Lambda} = 0$. When $\bar{\Lambda} \neq 0$, the two conditions (3.2.9) and (3.1.10) contradict each other, and there are no solutions. This means that the symmetric solutions of [3.5] (solutions (II) in the classification of [3]) do not have any deformations with 4D de Sitter or anti-de Sitter symmetry. This slightly extends the result of Arkani-Hamed *et al.* [5], who observed that such deformations would violate the Z_2 symmetry of the solution, and thus could not appear in a Z_2 orbifold of this solution.

3.3 Discussion and Further Issues

A priori there is a question as to whether the space of integration constants is parameterized by vacuum expectation values of fluctuating fields in four dimensions, or whether instead different members of this family arise from different four-dimensional Lagrangians.[9] The existence of anti-de Sitter and de Sitter deformations (bounded though they are) suggests that these deformations constitute parameters in the 4D effective theory. If the effective 4D cosmological constant were parametrized by a field, then in solving its equations of motion one would end up with one consistent possibility for the value of the 4D cosmological constant. The fact that we find a family of solutions suggests that this is not the case here. Indeed, naive calculation of the coefficient of the kinetic term for the mode which moves one from flat to curved 4D metrics does suggest that it is not a dynamical mode (it has infinite kinetic term). However the divergence in the calculation arises at the singularities, so this conclusion depends sensitively on how the singularities are resolved by microphysics.

To a 4D effective field theorist, the choice of which member of the family to start with constitutes a tuning of the 4D cosmological constant. From the point of view of the microscopic 5D theory, this tuning involves a parameter in the solution

[9] We thank S. Dimopoulos and R. Sundrum for discussions on this point.

and not a parameter in the Lagrangian. If this system can be embedded consistently into string theory, where there are no input parameters in the "Lagrangian," the mere existence of Poincaré-invariant solutions after some quantum corrections have been taken into account would be significant, even if such solutions lie in a family of curved solutions that signal the appearance of fine-tuning at low energies. In any case, our results here indicate that the apparent fine-tuning required to choose a flat slice is independent of standard model physics, though it can arise at the same scale.

Having understood better the situation with respect to this issue of fine-tuning, one is led to consider the main challenge identified in [3,5]: the question of possible microphysical constraints on the (codimension one) singularities in the solutions. The type of analysis we did here might help resolve an issue raised in [24], as we will mention presently, after first discussing the issue in a little more generality.

One possibility is that boundary conditions are required at the singularities, as in the case analyzed in [60]. It is then important to check whether the appropriate boundary conditions, along with the equations of motion and matching conditions, can be solved within the space of curved solutions we have identified [3,5].

There are some singularities in string theory (like conifolds, orbifolds, and brane-orientifold systems) which have a well-understood quantum resolution involving new degrees of freedom at the singularity; in these cases the resolution does not imply any extra boundary conditions in the effective long-wavelength theory.

It has recently been suggested that the singularities that appear in our solutions do not permit a finite-temperature deformation accessible with a long-wavelength general relativistic analysis [24]. This is a criterion that does not appear to contradict the microscopic consistency of orbifolds or conifolds, and the case of orientifolds and their duals must be considered carefully. Because of the large curvatures (and in some cases large couplings) in the backgrounds we consider here, such an analysis is necessarily limited. However, the general question of how finite temperature can be obtained in these backgrounds is an important one.

Within the context of the analysis of [24], it is notable that our solutions lie on the boundary between (conjecturally) allowed and (conjecturally) disallowed singularities. It is important to redo this analysis for solutions which include some bulk corrections. In particular, a nontrivial bulk dilaton potential of the right sign (as in our case (III) [3]) may put us in the allowed region according to the conjectured criterion of [24]. Instead of fine-tuning to obtain 4D Poincaré-invariant slices as we did in case (III) of [3], one can consider curved solutions of the sort given here. In the context of the type (III) situation where there is a bulk potential for ϕ, this is in fact natural if we do not wish to fine-tune the parameters in the 5D Lagrangian in order to obtain a 4D Poincaré-invariant solution. It is possible that this bulk correction will induce a sub-TeV correction to the 4D cosmological constant, while satisfying the conjectured constraints coming from the long-wavelength analysis of [24].

4. Moduli Stabilization from Fluxes

The study of Calabi-Yau orientifold compactifications of Type II string theory (or F-theory compactifications on Calabi-Yau fourfolds), with nontrivial background RR and NS fluxes through compact cycles of the Calabi-Yau manifold, is of interest for several reasons.

Conventional compactifications give rise to models which typically have many moduli. Understanding how these flat directions are lifted is important, both from the point of view of phenomenology and of cosmology. One expects the moduli to be lifted once supersymmetry is broken, but studying this in a calculable way in conventional compactifications has proved challenging so far. In contrast, compactifications with background RR and NS fluxes turned on give rise to a nontrivial low energy potential which freezes many of the Calabi-Yau moduli. Moreover, the potential is often calculable and as a result one can hope to study the stabilization of many moduli in a controlled manner in this setting. Flux-induced potentials for moduli have been discussed before in e.g. [61,62,63,64,65,31,66,67]. (Several other methods of constructing models with few moduli, for instance by considering asymmetric orbifold models [68,69,70] or theories with non-commuting Wilson lines which yield reduced-rank gauge groups as well [71,72], have also appeared previously.)

Compactifications with fluxes have also been proposed as a natural setting for warped solutions to the hierarchy problem [73,74], along the lines of the proposal of Randall and Sundrum [32]. The combination of fluxes and space filling D-branes which often need to be introduced for tadpole cancellation in these models leads to a nontrivial warped metric, with the scale of 4D Minkowski space varying over the compact dimensions. Examples of such models, with almost all moduli stabilized and exponentially large warping giving rise to a hierarchy, appeared in [31]. (See also [75,76]).

Finally, compactifications with fluxes also have interesting (and relatively unexplored) dual descriptions, via mirror symmetry and heterotic/Type II duality. Some examples of these dualities have been discussed in [65,77].

In this chapter, we explore in detail the simplest such compactification which admits supersymmetric vacua with nontrivial NS and RR fluxes: the compactification of Type IIB string theory on the T^6/Z_2 orientifold. The most familiar avatar of this model includes 16 D3-branes which cancel the RR charge of the 64 O3-planes at the 2^6 fixed points of the Z_2 action. However, one is free to replace some (or all) of the D3-branes with appropriate integral RR and NS 3-form fluxes $F_{(3)}$ and $H_{(3)}$. Given such a choice of integral fluxes, one can compute the low-energy superpotential governing the light fields. In a generic Calabi-Yau orientifold in IIB string theory, the periods which are required to determine W would only be computable as approximate expansions about various extreme points in moduli space, making any global and tractable expression for W difficult to obtain. A nice feature of the T^6/Z_2 case is that W is easily computable.

With the superpotential in control we can ask if there are $\mathcal{N} = 1$ supersymmetry preserving minima. It turns out that for generic choices of the fluxes supersymmetry is broken. By suitably choosing the fluxes, however, we find several examples which give rise to stable, $\mathcal{N} = 1$ supersymmetric ground states.[10] In these minima, typically, the dilaton-axion, all complex structure moduli, and some of the Kähler moduli are stabilized. In addition, since some or all of the O3-plane charge is cancelled by the flux, fewer D3-branes are present, and the number of moduli coming from the open string sector is also reduced. The conventional IIB compactification on this T^6/Z_2 orientifold has 67 (complex) moduli.[11] Once fluxes are turned on, it is easy to find examples with far fewer moduli (~ 3 in the models we discuss here, and fewer in the class of models described in [31]).

The organization of this chapter is as follows. In Sec. 4.1, we review basic facts about vacua with flux and about the moduli of the T^6/Z_2 orientifold, and parametrize the possible choices of flux. In Sec. 4.2, we discuss the constraints that

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[10] We work under the assumption that the full superpotential is given by the flux-induced contribution. Therefore, we neglect the possibility that e.g. Euclidean wrapped branes make an additional (Kähler moduli-dependent) contribution in our examples.

[11] The model has 16 D3-branes each of which give rise to 3 moduli. In addition there are 19 moduli coming from the closed string sector.

must be imposed to find a supersymmetric vacuum, following [78,79], and write down a formula for the superpotential as a function of the T^6 moduli. In Sec. 4.3, we exhibit many examples which lead to $\mathcal{N}=1$ supersymmetric solutions. We also analyze some cases which turn out to have $\mathcal{N}=3$ supersymmetry and make some comments about finding the most general supersymmetric solution. In Sec. 4.4 we discuss the conditions under which two apparently distinct solutions are nevertheless equivalent (using the reparametrization symmetries of the torus and U-duality). In Sec. 4.5 we describe how, starting from a supersymmetric solution, additional physically distinct ones can be found using rescalings and $GL(2,\mathbf{Z}) \times GL(6,\mathbf{Z})$ transformations. In Sec. 4.6, we derive the conditions which must be imposed on the $G_{(3)}$ flux to find $\mathcal{N}=2$ supersymmetric solutions, and consider one illustrative example. Sec. 4.7 contains some examples of nonsupersymmetric solutions. In Sec. 4.8 we discuss the dynamics on the D3 branes which one should insert into many of our vacua, to saturate the D3 tadpole. We close with a brief description of directions for future research in Sec. 4.9, and some important details are relegated to Appendices 4.A–D.

While this work was in progress, we learned of a related work exploring novel 4D $\mathcal{N}=3$ supersymmetric vacua which can be found from special flux configurations on T^6/Z_2 [80]. We are grateful to the authors of [80] for providing us with an early version of their paper, and for helpful comments.

4.1 Preliminaries

4.1.1 D3-brane Charge from 3-form Flux

The Type IIB supergravity action in Einstein frame is [81]

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}{}^2}\int d^{10}x\sqrt{-g}\left(R - \frac{\partial_M\phi\partial^M\bar{\phi}}{2(\mathrm{Im}\phi)^2} - \frac{G_{(3)}\cdot\bar{G}_{(3)}}{2\cdot 3!\mathrm{Im}\phi} - \frac{\tilde{F}_{(5)}{}^2}{4\cdot 5!}\right) + \frac{1}{2\kappa_{10}{}^2}\int\frac{C_{(4)}\wedge G_{(3)}\wedge\bar{G}_{(3)}}{4i\mathrm{Im}\phi} + S_{\mathrm{local}}. \tag{4.1.1}$$

Here,

$$\phi = C_{(0)} + i/g_s, \qquad G_{(3)} = F_{(3)} - \phi H_{(3)}, \tag{4.1.2}$$

and

$$\tilde{F}_{(5)} = F_{(5)} - \tfrac{1}{2}C_{(2)}\wedge H_{(3)} + \tfrac{1}{2}F_{(3)}\wedge B_{(2)}, \quad \text{with } *\tilde{F}_{(5)} = \tilde{F}_{(5)}. \tag{4.1.3}$$

where $F_{(3)} = dC_{(2)}$ and $H_{(3)} = dB_{(2)}$. If one compactifies on a six dimensional compact manifold, $\mathcal{M}_6$, and includes the possibility of space-filling D3-branes and O3-planes, then the equation of motion/Bianchi identity for the 5-form field strength is

$$d\tilde{F}_{(5)} = d * \tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} + 2{\kappa_{10}}^2 \mu_3 \rho_3^{\text{local}}. \tag{4.1.4}$$

Here μ_3 is the charge density of a D3-brane and ρ_3^{local} is the number density of local sources of D3-brane charge on the compact manifold. We can integrate this equation over $\mathcal{M}_6$ to give the condition

$$\frac{1}{2{\kappa_{10}}^2\mu_3}\int_{\mathcal{M}_6} H_{(3)} \wedge F_{(3)} + Q_3^{\text{local}} = 0. \tag{4.1.5}$$

In condition (4.1.5), Q_3^{local} is the sum of contibutions $+1$ for each D3-brane and $-1/4$ for each normal O3-plane. As discussed in [82] and [71], there are actually three other types of O3-plane, each characterized by the presence of discrete RR and/or NS flux at the orientifold plane. These exotic O3-planes each contribute $+1/4$ to Q_3^{local}.

We will be interested in the case that $\mathcal{M}_6$ is the T^6/Z_2 orientifold. There are 2^6 O3-planes in this compactification, with a total contribution of $-16 + \frac{1}{2}N_{\text{O3}'}$ units of D3-brane charge to Q_3^{local}, where $N_{\text{O3}'}$ is the number of exotic O3-planes. Therefore, (4.1.5) takes the form

$$\frac{1}{2}N_{\text{flux}} + N_{\text{D3}} + \frac{1}{2}N_{\text{O3}'} = 16. \tag{4.1.6a}$$

Here

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int_{T^6} H_{(3)} \wedge F_{(3)}. \tag{4.1.6b}$$

The factor of $\frac{1}{2}$ multiplying N_{flux} compensates for the fact that the integration is over T^6 rather than T^6/Z_2. We have also replaced the prefactor, $1/(2{\kappa_{10}}^2\mu_3)$, with its explicit value in terms of α'. It is clear from (4.1.6) that appropriately chosen three-form fluxes can carry D3-brane charge. The fluxes obey a quantization condition

$$\frac{1}{(2\pi)^2\alpha'}\int_\gamma F_{(3)} = m_\gamma \in \mathbf{Z}, \qquad \frac{1}{(2\pi)^2\alpha'}\int_\gamma H_{(3)} = n_\gamma \in \mathbf{Z}. \tag{4.1.7}$$

where γ is an arbitrary class in $H_3(T^6, \mathbf{Z})$. There is a subtlety in arguing that these are the correct quantization conditions for T^6/Z_2 [80].[12] This is because there are additional three cycles in T^6/Z_2, which are not present in the covering space T^6. If some of the integers m_γ (n_γ) are odd, additional discrete RR (NS) flux needs to be turned on at appropriately chosen orientifold planes to meet the quantization condition on these additional cycles. (See Appendix A for more discussion of this condition). In practice it is quite non-trivial to turn on the required discrete flux in a consistent manner without violating the charge conservation condition (4.1.6). We will avoid these complications in this chapter, by restricting ourselves to cases where m_γ, n_γ are even integers, and by not including any discrete flux at the orientifold planes.

Finally, $G_{(3)}$ obeys an imaginary self-duality (ISD) condition, $*_6 G_{(3)} = iG_{(3)}$, as will be shown in the next section. This condition implies that the 3-form flux contributes positively to the total D3-brane charge. To see this note that the ISD condition implies that

$$*_6 H_{(3)}/g_s = -(F_{(3)} - C_{(0)} H_{(3)}). \tag{4.1.8}$$

Since $H_{(3)} \wedge F_{(3)} = H_{(3)} \wedge (F_{(3)} - C_{(0)} H_{(3)})$, we learn that[13]

$$\begin{aligned} \int_{\mathcal{M}_6} H_{(3)} \wedge F_{(3)} &= -\frac{1}{g_s} \int_{\mathcal{M}_6} H_{(3)} \wedge *_6 H_{(3)} \\ &= \frac{1}{g_s} \frac{1}{3!} \int_{\mathcal{M}_6} \sqrt{g_{\mathcal{M}_6}}\, {H_{(3)}}^2 > 0. \end{aligned} \tag{4.1.9}$$

Therefore, in the presence of nontrivial RR and NS fluxes which carry nonzero N_{flux}, the number of D3 branes required to saturate (4.1.6) will always be fewer than 16.[14]. In fact, in some models, one can entirely cancel the tadpole with fluxes.

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[12] We are indebted to A. Frey and J. Polchinski for pointing out this subtlety.

[13] In the conventions of [31], $H_{(3)} \wedge *_6 H_{(3)} = -\frac{1}{3!} H_{mnp} H^{mnp}$ Vol, where m, n, p are real coordinates on $\mathcal{M}_6$ and Vol is the volume form.

[14] We do not allow the presence of anti D3-branes, since our main interest is SUSY solutions. Some aspects of non-supersymmetric vacua with anti D3-branes and fluxes have recently been described in [83]

4.1.2 The Scalar Potential from 3-form Flux

Turning on three-form fluxes gives rise to a potential for some of the moduli. The four dimensional effective theory has a term of the form [31]

$$\mathcal{L}_G = \frac{1}{4\kappa_{10}{}^2} \int_{\mathcal{M}_6} d^6y \frac{G_{(3)} \wedge *_6 \bar{G}_{(3)}}{\mathrm{Im}\phi}. \tag{4.1.10}$$

which arises from the $G_{(3)} \cdot \bar{G}_{(3)}$ term in the ten dimensional action (4.1.1). To understand why this term gives rise to a potential for some moduli it is useful to write

$$G_{(3)} = G^{\mathrm{ISD}} + G^{\mathrm{IASD}}. \tag{4.1.11}$$

where

$$\begin{aligned} *_6 G^{\mathrm{ISD}} &= +iG^{\mathrm{ISD}}, \\ *_6 G^{\mathrm{IASD}} &= -iG^{\mathrm{IASD}}. \end{aligned} \tag{4.1.12}$$

Then,

$$\begin{aligned} -\mathcal{L}_G = & -\frac{1}{2\kappa_{10}{}^2\mathrm{Im}\phi} \int_{\mathcal{M}_6} G^{\mathrm{IASD}} \wedge *_6 \bar{G}^{\mathrm{IASD}} + \frac{i}{4\kappa_{10}{}^2\mathrm{Im}\phi} \int_{\mathcal{M}_6} G_{(3)} \wedge \bar{G}_{(3)} \\ = & \qquad \mathcal{V}_{\mathrm{scalar}} \qquad + \qquad \text{topological.} \end{aligned} \tag{4.1.13}$$

The second term in (4.1.13) is topological. It is proportional to N_{flux} (4.1.6) and independent of moduli. One expects on general grounds that three-form flux configurations, which give rise to D3-brane charge, should also lead to D3-brane tension. This contribution to D3-brane tension is accounted for by the second term.

The first term in (4.1.13) gives rise to the scalar potential and is central to this chapter. It is positive semidefinite and vanishes when the flux meets the imaginary self-duality condition. The moduli dependence enters in two ways. First, $G_{(3)}$ depends on the axion-dilaton (4.1.2). Second, the decomposition of $G_{(3)}$ into ISD and IASD parts, depends on some metric moduli. Requiring that $G_{(3)}$ is imaginary self dual fixes many of these moduli.

4.1.3 IIB on the Orientifold T^6/Z_2

Let us now focus on IIB string theory compactified on a T^6/Z_2 orientifold. The six transverse directions will be denoted as x^i, y^i, $i = 1, \ldots, 3$. The orientifold action can be denoted as $\Omega R(-1)^{F_L}$, where R stands for a reflection of all of the compactified dimensions $(x^i, y^i) \to -(x^i, y^i), i = 1, \cdots, 3$. In fact the model is related to the Type I theory compactified on T^6 by six T-dualities along all the compactified directions. It preserves $\mathcal{N} = 4$ supersymmetry, i.e., 16 supercharges.

The massless fields after compactification arise from the massless fields in the IIB ten dimensional supergravity theory. The bosonic fields in the ten dimensional theory are the metric g_{MN}, the NS 2-form $B_{(2)}$, the dilaton D, and the RR fields $C_{(0)}, C_{(2)}$, and $C_{(4)}$. Their transformation properties under $\Omega(-1)^{F_L}$ are as follows:

$$\begin{array}{rcc} & \Omega & (-1)^{F_L} \\ g_{MN} & + & + \\ B_{(2)} & - & + \\ C_{(2)} & + & - \\ C_{(4)} & - & - \\ C_{(0)} & - & - \\ D & + & + \end{array} \tag{4.1.14}$$

The resulting massless bosonic fields are then:

$$\begin{array}{rcl} g_{\mu\nu} & 1 & \text{graviton} \\ g_{ab} & 21 & \text{scalars} \\ (B_{(2)})_{a\mu} & 6 & \text{gauge bosons} \\ (C_{(2)})_{a\mu} & 6 & \text{gauge bosons} \\ (C_{(4)})_{abcd} & 15 & \text{scalars} \\ C_{(0)} & 1 & \text{scalar} \\ D & 1 & \text{scalar} \end{array} \tag{4.1.15}$$

We see that the massless fields which survive the orientifold projection are the graviton, 12 gauge bosons and 38 scalars, plus their fermionic partners. These are organized into representations of $\mathcal{N} = 4$ supergravity as follows. The graviton, six gauge bosons and the axion-dilaton along with their fermionic partners, lie in a supergravity multiplet [81]. In addition there are six vector multiplets each containing a gauge boson, six scalars and their fermionic partners. Thus, in the absence of 3-form flux, the moduli space of T^6/Z_2 compactifications is parametrized by 38 scalars. When 3-form flux is turned on, some of the scalars from $C_{(4)}$ become charged, which means that they obtain Stuckelberg type kinetic terms $\sim (\partial_\mu \lambda + mA_\mu)^2$, where m is determined by the flux. For generic $\mathcal{N} = 1$ solutions, one can show that twelve of these scalars are eaten by gauge fields though the Higgs mechanism. (See, for example, [84] or [80] for related discussions in somewhat different contexts). Six of these twelve scalars are partners of metric Kähler moduli which also get heavy. The remaining three scalars from $C_{(4)}$ pair up with three metric Kähler moduli to form three $\mathcal{N} = 1$ chiral multiplets which survive in the low-energy theory.

In the T-dual of Type I theory on T^6, one would also include 16 D3-branes, each with a worldvolume $\mathcal{N} = 4$ vector multiplet. We will ignore any brane worldvolume fields for now, and briefly discuss the physics on the branes we must introduce in Sec. 4.8.

We discussed above that turning on fluxes leads to a potential on moduli space. It is important to note that although some of the moduli will gain a mass from this potential, the effective field theory keeping only the fields (4.1.15) from the closed string sector (plus any massless open string fields, if branes are introduced) is valid. This is because the masses generated by the flux-induced potential will scale like $m \sim \alpha'/R^3$, where we have assumed an isotropic torus of size $\sim R$. The KK modes on the Calabi-Yau geometry have masses that scale like $m_{KK} \sim 1/R$, so if we work at sufficiently large radius (where our supergravity considerations are most valid in any case), $m << m_{KK}$, and we are justified in truncating to the field theory of the modes (4.1.15).

It is helpful to regard the torus as a complex manifold and organize the various moduli accordingly. Nine of the twenty-one scalars that arise from the ten-dimensional metric correspond to Kähler deformations, while the remaining twelve scalars correspond to complex structure deformations.

An essential difference between the six-torus and a Calabi-Yau three-fold is the following. For a generic CY_3, Yau's theorem implies that any complex structure or Kähler deformation corresponds to a nontrivial deformation of the Ricci-flat metric. This is not true for the six-torus or the T^6/Z_2 case at hand. In this case, as we will see below, the complex structure is specified by nine complex parameters. Three of these parameters correspond to deformations of the complex structure at fixed metric.

4.1.4 The Complex Structure of a Torus

Nine complex coordinates are needed to describe the complex structure of T^6. We will use the explicit parametrization discussed in [85], which is summarized below. Let x^i, y^i, $i = 1, \ldots, 3$ be six real coordinates on T^6 which are periodic, $x^i \equiv x^i + 1$, $y^i \equiv y^i + 1$, and take the holomorphic 1-forms to be $dz^i = dx^i + \tau^{ij} dy^j$.

The complex structure is completely specified by the period matrix τ^{ij}. We choose the orientation[15]

$$\int dx^1 \wedge dx^2 \wedge dx^3 \wedge dy^1 \wedge dy^2 \wedge dy^3 = 1. \tag{4.1.16}$$

and use the following basis of $H^3(T^6, \mathbf{Z})$:

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3, \\ \alpha_{ij} &= \frac{1}{2}\epsilon_{ilm} dx^l \wedge dx^m \wedge dy^j, \quad 1 \le i, j \le 3, \\ \beta^{ij} &= -\frac{1}{2}\epsilon_{jlm} dy^l \wedge dy^m \wedge dx^i, \quad 1 \le i, j \le 3, \\ \beta_0 &= dy^1 \wedge dy^2 \wedge dy^3. \end{aligned} \tag{4.1.17}$$

This basis satisfies the properties

$$\int_{\mathcal{M}_6} \alpha_I \wedge \beta^J = \delta_I^J, \qquad \int_{\mathcal{M}_6} \alpha_I \wedge \alpha_J = 0, \qquad \int_{\mathcal{M}_6} \beta^I \wedge \beta^J = 0. \tag{4.1.18}$$

Finally, we choose a normalization so that the holomorphic three-form Ω is

$$\Omega = dz^1 \wedge dz^2 \wedge dz^3. \tag{4.1.19}$$

One can show that

$$\Omega = \alpha_0 + \alpha_{ij}\tau^{ij} - \beta^{ij}(\mathrm{cof}\tau)_{ij} + \beta^0(\det \tau), \tag{4.1.20}$$

where

$$(\mathrm{cof}\tau)_{ij} \equiv (\det \tau)\tau^{-1,T} = \frac{1}{2}\epsilon_{ikm}\epsilon_{jpq}\tau^{kp}\tau^{mq}. \tag{4.1.21}$$

4.1.5 The RR and NS flux

The flux that we turn on must be even under the Z_2 orientifold symmetry. The intrinsic parity, under $\Omega(-1)^{F_L}$, of the various fields is given in (4.1.14). One sees that the 3-form field strengths $F_{(3)}$, $H_{(3)}$ that are excited must be proportional to 3-forms of odd intrinsic parity. However, the quantity that must be even is the total parity, which for a p-form on the internal space is the product of this intrinsic parity

[15] This choice of orientation is different that in [85] and is chosen to be consistent with the conventions of [31].

and an explicit $(-1)^p$ from the reflection action on the indices [31]. Therefore, the 3-form field strengths must transform as $(F_{(3)})_{abc} \to (F_{(3)})_{abc}$, $(H_{(3)})_{abc} \to (H_{(3)})_{abc}$ under the Z_2 action. Similarly, the field strength $F_{(5)}$ must be proportional to a 5-form of even intrinsic parity. We will ensure below that the three-forms which are excited have the correct symmetry properties. The resulting 5-form field strength is then determined by the equations of motion (4.1.4), and automatically satisfies the correct symmetry properties.

Note that the Bianchi identities for $F_{(3)}$ and $H_{(3)}$ require that they be closed. They should thus be expressible as a linear combination of the basis vectors of $H^3(T^6, \mathbf{Z})$. All the basis elements, (4.1.17), are three forms of odd parity under the Z_2 action which takes $x^i, y^i \to -x^i, -y^i$. So the symmetry constraint mentioned above is automatically taken care of by expressing the three-forms in this manner. Finally, taking into account the quantization conditions (4.1.7), $F_{(3)}$ and $H_{(3)}$ can be expressed as

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'}F_{(3)} &= a^0\alpha_0 + a^{ij}\alpha_{ij} + b_{ij}\beta^{ij} + b_0\beta^0, \\ \frac{1}{(2\pi)^2\alpha'}H_{(3)} &= c^0\alpha_0 + c^{ij}\alpha_{ij} + d_{ij}\beta^{ij} + d_0\beta^0. \end{aligned} \tag{4.1.22}$$

Here $a^0, \alpha_{ij}, \beta^{ij}, \beta^0$ and c^0, c^{ij}, d_{ij}, d_0 are all integers. We will search for vacua maintaining the ansatz of constant fluxes (4.1.22) on the T^6 throughout the chapter.

4.2 Supersymmetry

4.2.1 Spinor Conditions

In the discussion below our conventions are as follows: The $\gamma_i, i = 0, \cdots, 9$ matrices are all real and satisfy the algebra $\{\gamma^i, \gamma^j\} = \eta^{ij}$. The matrix, γ^0, is anti-hermitian and the others are hermitian. Also,

$$\Gamma^{(4)} \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{4.2.1}$$

and

$$\Gamma^{(6)} \equiv i\gamma_4\gamma_5\gamma_6\gamma_7\gamma_8\gamma_9. \tag{4.2.2}$$

Both $\Gamma^{(4)}, \Gamma^{(6)}$ are hermitian with eigenvalues ± 1. For the rest we follow the conventions of [79]. Denote the spinor ϵ as

$$\epsilon = \epsilon_L + i\epsilon_R. \tag{4.2.3}$$

Here, ϵ_L is a Majorana spinor in ten dimensions. We can write

$$\epsilon_L = u \otimes \chi + u^* \otimes \chi^*. \tag{4.2.4}$$

where * denotes complex conjugation, and $\Gamma^{(4)} u = u$, $\Gamma^{(6)} \chi = -\chi$. The complex conjugate spinors have opposite 4 and 6 dimensional helicity.

Since we are working on a T^6/Z_2 orientifold, the spinor must be invariant with respect to the Z_2 symmetry. The Z_2 action corresponds to $\Omega R_{456789}(-1)^{F_L}$, where R_{456789} stands for a reflection in the six directions. This means that

$$\epsilon_R = -\gamma_4\gamma_5\gamma_6\gamma_7\gamma_8\gamma_9\epsilon_L. \tag{4.2.5}$$

That is

$$i\epsilon_R = -\Gamma^{(6)}\epsilon_L = u \otimes \chi - u^* \otimes \chi^*. \tag{4.2.6}$$

which gives from (4.2.3)

$$\epsilon = 2u \otimes \chi. \tag{4.2.7}$$

So, the spinor consistent with the Z_2 orientifolding symmetry is of type B(ecker).

Now following [79] we are lead to the conditions

$$G_{(3)}\chi = 0, \; G_{(3)}\chi^* = 0, \text{ and } G_{(3)}\gamma^{\bar{i}}\chi^* = 0, \tag{4.2.8}$$

where we have introduced complex coordinates such that

$$\gamma^{\bar{i}}\chi = 0. \tag{4.2.9}$$

The first condition in (4.2.8) gives

$$(G_{(3)})_{ijk} = 0, (G_{(3)})^{j}_{ij} = 0. \tag{4.2.10}$$

The second that:

$$(G_{(3)})_{\bar{i}\bar{j}\bar{k}} = 0, (G_{(3)})^{\bar{j}}_{\bar{i}\bar{j}} = 0, \tag{4.2.11}$$

note the second condition in (4.2.11) kills off the (1, 2) terms of the kind $J \wedge d\bar{z}^a$. Finally the third condition in (4.2.8) gives:

$$(G_{(3)})_{\bar{i}\bar{j}l} = 0 \tag{4.2.12}$$

Putting all this together only primitive (2, 1) terms in $G_{(3)}$ survive. Primitivity means that

$$J \wedge G_{(3)} = 0, \tag{4.2.13}$$

where $J = ig_{i\bar{j}}dz^i \wedge dz^{\bar{j}}$ is the Kähler form. For $G_{(3)}$ of type (2, 1), this is equivalent to requiring that

$$g^{i\bar{j}}(G_{(3)})_{li\bar{j}} = 0. \tag{4.2.14}$$

We turn next to analyzing the requirement that $G_{(3)}$ is of (2, 1) type and then discuss the requirements imposed by primitivity in Sec. 4.2.4.

4.2.2 $G_{(3)}$ of Type (2,1)

Another way to phrase the condition that $G_{(3)}$ be of type (2,1) is that the (0,3), (3,0), and (1,2) terms in $G_{(3)}$ must vanish. We saw above that the moduli space of complex structures for T^6 can be parametrized by the period matrix τ^{ij}. One can show that

$$\partial_{\tau^{ij}}\Omega = k_{ij}\Omega + \chi_{ij}, \tag{4.2.15}$$

where χ_{ij}, $1 \le i, j \le 3$ are a complete set of (2, 1) forms. The condition that $G_{(3)}$ is of (2, 1) type is then equivalent to requiring that

$$\begin{aligned} \int G_{(3)} \wedge \Omega &= 0 \\ \int \bar{G}_{(3)} \wedge \Omega &= 0 \\ \int G_{(3)} \wedge \chi_{ij} &= 0, \quad 1 \le i, j \le 3. \end{aligned} \tag{4.2.16}$$

A convenient way to impose the requirements (4.2.16), is by constructing the superpotential

$$W = \int G_{(3)} \wedge \Omega. \tag{4.2.17}$$

From (4.2.15) we find that

$$\partial_{\tau^{ij}}W = k_{ij}W + \int G \wedge \chi_{ij} \tag{4.2.18}$$

Similarly,

$$\partial_\phi W = -\int H \wedge \Omega = \frac{1}{(\phi - \bar{\phi})} \int (G_{(3)} - \bar{G}_{(3)}) \wedge \Omega \qquad (4.2.19)$$

Thus (4.2.16) is equivalent to demanding that

$$W = 0 \qquad (4.2.20a)$$

$$\partial_\phi W = 0 \qquad (4.2.20b)$$

$$\partial_{\tau^{ij}} W = 0 \qquad (4.2.20c)$$

4.2.3 The Superpotential and Equations for SUSY Vacua

Using (4.1.22) it follows that the superpotential (4.2.17), is:

$$\frac{1}{(2\pi)^2\alpha'} W = (a^0 - \phi c^0) \det \tau - (a^{ij} - \phi c^{ij})(\operatorname{cof} \tau)_{ij} - (b_{ij} - \phi d_{ij})\tau^{ij} - (b_0 - \phi d_0). \qquad (4.2.21)$$

We see from (4.2.21), that it depends on ten complex variables—ϕ and the nine components of τ^{ij}. But, equations (4.2.20) give rise to eleven equations in these variables. Thus, generically all the equations (4.2.20*a*–*c*) cannot be met and supersymmetry is broken.

The explicit equations of motion that follow from (4.2.20) and (4.2.21) are

$$a^0 \det \tau - a^{ij}(\operatorname{cof}\tau)_{ij} - b_{ij}\tau^{ij} - b_0 = 0, \qquad (4.2.22a)$$

$$c^0 \det \tau - c^{ij}(\operatorname{cof}\tau)_{ij} - d_{ij}\tau^{ij} - d^0 = 0, \qquad (4.2.22b)$$

$$(a^0 - \phi\, c^0)(\operatorname{cof}\tau)_{kl} - (a^{ij} - \phi\, c^{ij})\epsilon_{ikm}\epsilon_{jln}\tau^{mn} - (b_{ij} - \phi\, d_{ij})\delta^i_k\delta^j_l = 0. \qquad (4.2.22c)$$

Here, the first equation comes from (4.2.20*a*) minus (4.2.20*b*), the second, from (4.2.20*b*), and the third from (4.2.20*c*).[16] The equations (4.2.22) are coupled non-linear equations in several variables and are difficult to solve in full generality.

It might seem odd at first glance that all nine scalars parametrizing the complex structure can be fixed, even though, as was argued in Sec. 4.1.3, only six of them correspond to components of the metric and enter in the supergravity equations of motion. This happens because the requirements for $\mathcal{N} = 1$ supersymmetric solutions are stronger than the requirements which would follow from searching for generic solutions to the equations of motion.

[16] In deriving the third equation, it is useful to note the relations $\det \tau = \frac{1}{3}\epsilon_{ikm}\epsilon_{jln}\tau^{ij}\tau^{kl}\tau^{mn}$, and $(\operatorname{cof}\tau)_{ij} = \frac{1}{2}\epsilon_{ikm}\epsilon_{jln}\tau^{kl}\tau^{mn}$

4.2.4 Primitivity

Once the complex structure is chosen such that $G_{(3)}$ is of $(2,1)$ type, (4.2.14), imposes the requirement of primitivity. Note that in (4.2.14) the index l can take values $\{1,2,3\}$, so primitivity gives rise to three complex equations or equivalently six real equations. The space of Kähler forms is 9 dimensional to begin with so generically this will leave a three dimensional moduli space of Kähler deformations[17].

Equation (4.2.14) can be thought of as 6 linear equations in the 9 metric components $g^{i\bar{j}}$. Solving these is relatively straightforward. In contrast we saw above that requiring G to be of type $(2,1)$ results in coupled non-linear equations which are considerably harder to work with. In practice, in the examples below, it will sometimes be easier to ensure primitivity by directly imposing the condition (4.2.13) on the Kähler two-form.

It is worth making one more comment at this stage. We mentioned in Sec. 4.1.1 that the equations of motion can be solved if $G_{(3)}$ is an imaginary self-dual three form. This allows $G_{(3)}$ to be of three types: primitive $(2,1)$, $(0,3)$, or $(1,2)$ of the kind $J \wedge d\bar{z}^a$. We also saw in Sec. 4.1.2 that in all these cases, the scalar potential for the moduli was minimized and equal to zero. Supersymmetry on the other hand is preserved if $G_{(3)}$ is purely a primitive $(2,1)$ form. Thus for choices of complex structure and Kähler class where $G_{(3)}$ has $(0,3)$ or $(1,2)$ terms, some auxilary F or D term must get a vev. However, since the potential continues to vanish in these cases, these F- and D-terms cannot be present in the scalar potential. Part of this discussion is already familiar from the study of a generic Calabi Yau manifold [31]. If $G_{(3)}$ has a $(0,3)$ term the F-component of the volume modulus gets a vacuum expectation value, however this F-component does not enter the potential because of the no-scale structure of the four-dimensional supergravity theory. Similarly when $(1,2)$ terms are present auxiliary D-terms must acquire expectation values in general. The absence of these terms in the potential can probably best be understood in the context of the underlying $\mathcal{N}=4$ supersymmetry

[17] The surviving Kähler moduli have axionic partners which come from the C_4 field, together these give rise to three chiral superfields at low energies. The six Kähler moduli which get heavy also have partners, these obtain a mass due to Chern-Simons couplings (4.1.1), (4.1.3).

present in the T^6/Z_2 case. We leave a more systematic analysis of the low-energy supergravity theory along the lines of [86,7,84] for future work; such analyses for the case of generic Calabi-Yau threefolds with fluxes have appeared in e.g. [62,87,88].

4.3 Some Supersymmetric Solutions

The equations which determine the value of the moduli are dificult to solve in general. The main challenge are the coupled non-linear equations (4.2.22) which determine the complex structure of the torus.

We do not solve these equations in their full generality below. Instead in Sec. 4.3.1 we discuss some examples, where the fluxes take simple values that allow for analytic solutions. Already these simpler cases are quite interesting. As we will see, in many cases, stable minima exist where all the complex structure moduli and some of the Kähler moduli are stabilized. Sec. 4.3.2 deals with the inverse problem: we start with some values for the moduli and ask for fluxes which stabilize the moduli at these values consistent with supersymmetry. The inverse problem is sometimes easier to solve. The solutions in Sec. 4.3.1 have $\mathcal{N}=1$ supersymmetry. With a few possible exceptions this should be true of the vacua in Sec. 4.3.2 as well. Sec. 4.3.3 analyses some additional cases where the fluxes lead to tractable solutions. These examples turn out to have $\mathcal{N}=3$ supersymmetry. Finally, some comments related to obtaining a general supersymmetric solution are in Sec. 4.3.4.

Not all of the solutions studied in this section are physically distinct. Sec. 4.4 discusses how solutions related by $SL(2,\mathbf{Z}) \times SL(6,\mathbf{Z})$ transformations should be identified. Starting with some of solutions found in this section, other physically distinct solutions can be obtained by rescaling the fluxes, or carrying out $GL(2,\mathbf{Z}) \times GL(6,\mathbf{Z})$ transformations. This is illustrated in some examples here and discussed more fully in Sec. 4.5.

One final comment before turning to examples. One would like to know if the analysis of $\mathcal{N}=1$ supersymmetric vacua in this section, receives significant α' and g_s corrections. We have not discussed an explicit $\mathcal{N}=1$ superspace description of the the low-energy effective theory in the presence of fluxes in this chapter. But it is clear that such a description would involve both a superpotential (4.2.17), and D-terms.[18] The superpotential must be exact in the α' expansion since the

[18] These play a role in ensuring primitivity of $G_{(3)}$ for example.

partner of volume modulus is an axion which cannot occur in the α' (or string loop) corrections to the superpotential. Quite plausibly, in this case, this is true of the D terms as well, since they are related by the underlying $\mathcal{N} = 4$ supersymmetry to the F-terms. The dilaton in the examples below is typically stabilized at a value of order one. Therefore, it is also important to understand the effect of g_s corrections. In a general example in the $\mathcal{N} = 1$ supersymmetric case, the superpotential is not corrected pertubatively in g_s, but it could receive nonperturbative corrections from Euclidean D-branes (D-instantons) wrapped on even dimensional cycles in the Calabi-Yau. These corrections will depend on the Kähler moduli and the dilaton, and are complementary to the potential for complex structure moduli which we compute below. In models with an F-theory lift (like the one we consider), these corrections should only exist if the F-theory fourfold admits divisors of arithmetic genus one [71]. We would expect them to break the no-scale structure mentioned in Sec. 4.2.4 . For $\mathcal{N} > 1$, these instanton corrections to the superpotential do not appear (as the relevant Euclidean branes would have too many fermion zero modes to correct a superpotential).

4.3.1 Example 1: Fluxes Proportional to the Identity

We begin by studying the case where,

$$(a^{ij}, b_{ij}, c^{ij}, d_{ij}) = (a, b, c, d)\, \delta_{ij}. \tag{4.3.1}$$

that is all the flux matrices are diagonal and proportional to the identity.

The equations determining the complex structure, (4.2.22) will be considered first, followed by the conditions for primitivity.

With the flux matrices of the form (4.3.1), it is easy to see from (4.2.22), that the period matrix must be diagonal.

$$\tau^{ij} = \tau\delta^{ij}. \tag{4.3.2}$$

(In fact this is more generally true if the flux matrices are all diagonal).

The equations of motion (4.2.22) then take the form

$$P_1(\tau) \equiv a^0\tau^3 - 3a\tau^2 - 3b\tau - b_0 = 0, \tag{4.3.3}$$

$$P_2(\tau) \equiv c^0\tau^3 - 3c\tau^2 - 3d\tau - d_0 = 0, \tag{4.3.4}$$

$$(a^0 - \phi c^0)\tau^2 - 2(a - \phi c)\tau - (b - \phi d) = 0. \tag{4.3.5}$$

We are only interested in solutions in which τ is complex (since solutions with $\mathrm{Im}(\tau) = 0$ lie at boundaries of the moduli space). It is straightforward to show that in this case[19].

$$P_1(\tau) = (f\tau + g)P(\tau). \quad P_2(\tau) = (h\tau + k)P(\tau). \tag{4.3.6a}$$

for some

$$P(\tau) = l\tau^2 + m\tau + n. \quad f.g.h.k.l.m.n \in \mathbf{Z}. \tag{4.3.6b}$$

Thus. τ is a root of $P(\tau)$ and ϕ is determined from equation (4.3.5). Note that not every septuple $(f.g.h.k.l.m.n)$ corresponds to integral flux. From the relations

$$\begin{aligned} fm + gl &= -3a. & hm + kl &= -3c. \\ fn + gm &= -3b. & hn + km &= -3d. \end{aligned} \tag{4.3.7}$$

we have consistency conditions modulo 3.

The D3-brane charge of the flux in this solution is given by

$$\begin{aligned} N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int H_{(3)} \wedge F_{(3)} &= (b_0c^0 - a^0d_0) + 3(bc - ad) \\ &= -\frac{1}{3}(fk - gh)(m^2 - 4ln), \end{aligned} \tag{4.3.8}$$

which has the property that it is always $0\,(\mathrm{mod}\,3)$.[20] One can also show that the result (4.3.8) is explicitly positive in our conventions.[21]

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[19] P_1 and P_2 are cubic polynomials with real coefficients, that share a common complex root, τ. Therefore, $\bar{\tau}$ is also a root, and the two equations share a common quadratic factor. This common factor is proportional to $P = c^0P_1 - a^0P_2$, which has integer coefficients. Since P_1 and P_2 also have integer coefficients, it follows that P_1/P and P_2/P are each binomials with rational coefficients. But, a polynomial with integer coefficients that factorizes over the rationals also factorizes over the integers.

[20] To see this, note that (4.3.7) can be written as $\left(\begin{smallmatrix} f & g \\ h & k \end{smallmatrix}\right)\left(\begin{smallmatrix} m & n \\ l & m \end{smallmatrix}\right) = -3\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$. Since $\left(\begin{smallmatrix} m & n \\ l & m \end{smallmatrix}\right) \equiv \left(\begin{smallmatrix} m & -2n \\ -2l & m \end{smallmatrix}\right)$ (mod 3), this means that $\left(\begin{smallmatrix} f & g \\ h & k \end{smallmatrix}\right)\left(\begin{smallmatrix} m & -2n \\ -2l & m \end{smallmatrix}\right) \equiv 0$ (mod 3). Taking the determinant of both sides then gives $(fk - gh)(m^2 - 4ln) \equiv 0$ (mod 9).

[21] Our conventions are $\mathrm{Im}\tau.\mathrm{Im}\phi > 0$. One can show that the factor $(fk - gh)$ in (4.3.8) satisfies $\mathrm{sign}(fk - gh) = \mathrm{sign}(\mathrm{Im}\phi/\mathrm{Im}\tau)$. Therefore it is positive. The other factor, $(m^2 - 4ln)$, is the discriminant of $P(\tau)$. It is negative since the roots are complex.

In summary, starting with fluxes of the form (4.3.1), the neccessary and sufficient condition for a non-singular solution, is the existence of integers (f, g, h, k, l, m, n) which satisfy the conditions. (4.3.7), and which give rise to nonzero three brane charge, (4.3.8).

In practice, determining polynomials of the form (4.3.6), by direct scrutiny is often easier than finding appropriate septuples (f, g, h, k, l, m, n).

As a concrete example, consider the case

$$P_1(\tau) \equiv \tau^3 - 1 = 0 \tag{4.3.9}$$

$$P_2(\tau) \equiv \tau^3 + 3\tau^2 + 3\tau + 2 = 0 \tag{4.3.10}$$

Both polynomials share a common factor $P(\tau) = \tau^2+\tau+1$ and can be expressed as:

$$P_1 \equiv (\tau - 1)P(\tau) = 0 \tag{4.3.11}$$

$$P_2 \equiv (\tau + 2)P(\tau) = 0. \tag{4.3.12}$$

Solving $P(\tau) = 0$ with the condition $\mathrm{Im}(\tau) > 0$, gives

$$\tau = e^{\frac{2\pi i}{3}}. \tag{4.3.13}$$

ϕ is obtained from (4.3.5), and given by

$$\phi = \tau = e^{\frac{2\pi i}{3}}. \tag{4.3.14}$$

We see that the moduli are fixed at a very symmetric point. Since the period matrix is diagonal, the torus factorizes as $T^6 \equiv T^2 \times T^2 \times T^2$ with respect to complex structure. In fact, when viewed in F-theory, this factorization becomes $T^8 \equiv T^2 \times T^2 \times T^2 \times T^2$. Since the eigenvalues of the period matrix are all equal to one another, and to value of the dilaton-axion, all the four 2-tori have the same modular parameter.

From (4.3.11), (4.3.12), we see that the septuple

$$(f, g, h, k, l, m, n) = (1, -1, 1, 2, 1, 1, 1). \tag{4.3.15}$$

Also from (4.3.9), (4.3.10), and (4.3.3), (4.3.4), we see that the integers

$$(a^0, a, b, b_0) = (1, 0, 0, 1) \quad (c^0, c, d, d_0) = (1, -1, -1, -2) \tag{4.3.16}$$

Either way, we find that the three-brane charge carried by the flux is given by

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int H_{(3)} \wedge F_{(3)} = 3. \tag{4.3.17}$$

Notice that most of the non-zero fluxes in (4.3.16) are odd integer. We discussed in Sec. 4.1.1 why consistency on the T^6/Z_2 orientifold requires additional discrete flux to be turned on when odd integer flux is present.

To avoid this complication we can simply choose the fluxes to be twice the values indicated in (4.3.16). That is

$$(a^0, a, b, b_0) = (2, 0, 0, 2) \quad (c^0, c, d, d_0) = (2, -2, -2, -4). \tag{4.3.18}$$

and,

$$(f, g, h, k, l, m, n) = (2, -2, 2, 4, 1, 1, 1). \tag{4.3.19}$$

No discrete flux in needed now. Since doubling all fluxes simply rescales the superpotential by an overall factor, the equations determining the moduli (4.2.22), remain the same and therefore the solutions for the moduli are still given by (4.3.13), (4.3.14).

From (4.3.8), we see that after doubling the fluxes

$$N_{\text{flux}} = 12. \tag{4.3.20}$$

Eq. (4.1.6), now implies that for a consistent solution we need to add ten wandering branes in addition, i.e., $N_{D3} = 10$.

This completes our discussion of how the complex structure moduli are determined, in this case. To complete our analysis we must next impose the requirement that the three flux $G_{(3)}$ is primitive. Before doing so though, let us pause to make two comments.

First, other closely related examples can be obtained by starting with the fluxes (4.3.16), and doing other rescalings. For example, one can double the $H_{(3)}$ flux while increasing the $F_{(3)}$ flux by a factor of four so that the resulting values for the fluxes are:

$$(a^0, a, b, b_0) = (4, 0, 0, 4) \quad (c^0, c, d, d_0) = (2, -2, -2, -4). \tag{4.3.21}$$

Now, it is straightforward to see from (4.3.3), (4.3.4), that the resulting value for τ, ϕ, are:

$$\tau = e^{\frac{2\pi i}{3}}, \quad \phi = 2e^{\frac{2\pi i}{3}}. \tag{4.3.22}$$

The resulting contribution to three brane charge is given by:

$$N_{\text{flux}} = 24. \qquad (4.3.23)$$

so that the $N_{D3} = 4$. The rescalings discussed in (4.3.21), illustrate a more general feature which will be dealt with in more generality in Sec. 4.5: given a solution, additional ones can be obtained by carrying out $GL(6,\mathbf{Z})\times GL(2,\mathbf{Z})$ transformations on the fluxes and the moduli, provided the resulting contribution to D3-brane charge is within bounds.

Second, one would like to know whether there are other solutions with fluxes of the form, (4.3.1), which are not related to those discussed above by $GL(6,\mathbf{Z}) \times GL(2,\mathbf{Z})$ transformations or rescalings. While we do not give all the details here, it is straightforward to tabulate all choices of fluxes (or equivalently choices of the septuple (f,g,h,k,l,m,n)) which meet the requirements for the existence of $\mathcal{N} = 1$ supersymmetric solutions. In all these cases one finds that the resulting values for the moduli are related to (4.3.13), (4.3.14), by a rescaling or $GL(6,\mathbf{Z}) \times GL(2,\mathbf{Z})$ transformations. We have not studied the corresponding fluxes exhaustively, but in several cases they too are related to (4.3.16), by the same rescaling or $GL(6,\mathbf{Z}) \times GL(2,\mathbf{Z})$ transformation.

Primitivity

We must also verify that (at least on some subspace of the Kähler moduli space), the $G_{(3)}$ flux found from the superpotential above is primitive. We will go through this for the flux in Example 1. A similar analysis (without substantially more complexity) would apply to our other examples.

In the case at hand, the flux takes the form (4.3.1). More explicitly,

$$\begin{aligned} F &= a^0 dx^1 \wedge dx^2 \wedge dx^3 + a\big(dx^1 \wedge dx^2 \wedge dy^3 + \text{cyc. perms of } 123\big), \\ &\quad - b\big(dx^1 \wedge dy^2 \wedge dy^3 + \text{ cyc. perms of } 123\big) + b_0 dy^1 \wedge dy^2 \wedge dy^3, \\ H &= c^0 dx^1 \wedge dx^2 \wedge dx^3 + c\big(dx^1 \wedge dx^2 \wedge dy^3 + \text{ cyc. perms of } 123\big), \\ &\quad - d\big(dx^1 \wedge dy^2 \wedge dy^3 + \text{ cyc. perms of } 123\big) + d_0 dy^1 \wedge dy^2 \wedge dy^3. \end{aligned} \qquad (4.3.24)$$

In the present example, it is convenient to impose the requirement of primitivity in the form of (4.2.13),

$$J \wedge G_{(3)} = 0. \qquad (4.3.25)$$

We are interested in the subspace of Kähler forms for which this requirement is met.

Take J to be of the form

$$J = \sum_{a=1}^{3} r_a^2 \, dz^a \wedge d\bar{z}^a \sim \sum_{a=1}^{3} i r_a^2 \, dx_a \wedge dy_a \tag{4.3.26}$$

where the second expression uses the fact that the complex structure τ of all the three T^2s, as given in (4.3.13), are equal. Now, notice that each term in F and H as given in (4.3.24) contains no repeat superscripts: one either chooses dx^a or dy^a for each of $a = 1, 2, 3$, and then wedges the three one-forms together. But the Kähler form in (4.3.26) contains a sum of two-forms, each of which looks like $dx^a \wedge dy^a$. The wedge product of each such term with $G_{(3)}$ will clearly vanish, because it hits either another dx^a or another dy^a in each term in F and H. Therefore, $J \wedge G_{(3)} = 0$ for the most general J of the form (4.3.26).

Is there a larger subspace of Kähler moduli space that preserves the primitivity? Since G is of type (2,1) and J is a (1,1) form, $J \wedge G$ is a (3,2) form. There are three nontrivial (3,2) forms on the T^6, so we expect that requiring $J \wedge G = 0$ will yield three nontrivial complex equations. The space of Kähler forms has real dimension 9, so generically we expect only a three-dimensional subspace of the Kähler moduli space (suitably complexified by the addition of axions in the relevant chiral multiplets) to parametrize flat directions of this $\mathcal{N} = 1$ theory. However, in the case at hand, the $G_{(3)}$ flux is particularly simple and non-generic, and the number of flat directions parametrized by Kähler moduli is 6 instead of 3. One can see the three "extra" flat directions by inspection. For instance, consider the two-form

$$\omega \sim i(dx^1 \wedge dy^2 + dx^2 \wedge dy^1) \tag{4.3.27}$$

One can easily check from (4.3.24) that $\omega \wedge G = 0$. Further, since the complex structure of all three T^2's is the same, it is easy to check that

$$\omega \sim dz^1 \wedge d\bar{z}^2 + dz^2 \wedge d\bar{z}^1, \tag{4.3.28}$$

so that ω is of type $(1,1)$. Analogous perturbations with $\{1,2\}$ replaced by $\{1,3\}$ and $\{2,3\}$ similarly maintain the primitivity of $G_{(3)}$. So the $\mathcal{N} = 1$ vacua persist along a six-dimensional slice of the Kähler moduli space.

One final comment is in order. Our analysis has ensured that the solutions discussed above have at least $\mathcal{N} = 1$ supersymmetry, but it does not preclude the possibility of enhanced supersymmetry. A simple check is the following: enhanced supersymmetry requires that additional choices of complex structure are possible, in which $G_{(3)}$ is still of the kind (2, 1) (and primitive). $\mathcal{N} = 2$ and $\mathcal{N} = 3$ require one and two additional choices of complex structure respectively. In the solutions above, with $T^6 \equiv T^2 \times T^2 \times T^2$, there is a complete permutation symmetry among the three two-tori. This ensures that, upto an overall constant, $G_{(3)}$ must have the form.

$$G_{(3)} \sim (dz^1 \wedge dz^2 \wedge d\bar{z}^3 + dz^2 \wedge dz^3 \wedge d\bar{z}^1 + dz^3 \wedge dz^1 \wedge d\bar{z}^2). \tag{4.3.29}$$

Other choices of complex structure can be made, by taking $z^i \to \bar{z}^i$ for some or all of the three T^2's, but none of them preserve the (2, 1) nature of $G_{(3)}$. So we see that these examples have only $\mathcal{N} = 1$ supersymmetry. A detailed examination of the conditions for $\mathcal{N} = 2$ supersymmetry is presented in Sec. 4.6, and some more comments on this matter can be found there.

4.3.2 The Inverse Problem: Fluxes from Moduli

In the previous section we started with some fluxes and asked what are the resulting values for moduli in an $\mathcal{N} = 1$ susy vacuum. In this section we address the inverse problem, namely: start with some values for the moduli and ask if there are fluxes which can be turned on such that the resulting potential stabilizes the moduli at the values we begin with, while preserving $\mathcal{N} = 1$ susy. The inverse problem is sometimes easier to solve and helpful in understanding the full set of consistent vacuua.

Our discussion will not be exhaustive. Instead we will consider one illustrative case. In Sec. 4.3.1 we started with flux matrices which were all proportional to the identity (4.3.1), then argued that the period matrix must be a multiple of the identity. Here, we start by fixing the period matrix to be a multiple of the identity as in Equation (4.3.2), then ask what values of the fluxes can yield such a solution while preserving $\mathcal{N} = 1$ supersymmetry. Our notation in this section will be chosen to be consistent with Sec. 4.3.1.

We begin by writing

$$a^{ij} = a\delta^{ij} + \tilde{a}^{ij}, \quad \mathrm{tr}\,\tilde{a} = 0, \tag{4.3.30}$$

with similar relations for b, c, d. Equations (4.2.22a) and (4.2.22b) then become

$$a^0\tau^3 - 3a\tau^2 - 3b\tau - b_0 = 0. \tag{4.3.31}$$

$$c^0\tau^3 - 3c\tau^2 - 3d\tau - d_0 = 0. \tag{4.3.32}$$

and $\partial_{\tau^{ij}} W = 0$ becomes

$$\begin{aligned}(a^0 - \phi c^0)\tau^2 - 2(a - \phi c)\tau - (b - \phi d) &= 0.\\ (\tilde{a}^{ji} - \phi \tilde{c}^{ji})\tau - (\tilde{b}_{ij} - \phi \tilde{d}_{ij}) &= 0.\end{aligned} \tag{4.3.33}$$

Eq. (4.3.33) arises by taking the trace and traceless parts of the third equation in (4.2.22). It can be be summarized as

$$\phi = \frac{a^0\tau^2 - 2a\tau - b}{c^0\tau^2 - 2c\tau - d} = \frac{\tilde{a}^{ji}\tau - \tilde{b}_{ij}}{\tilde{c}^{ji}\tau - \tilde{d}_{ij}}. \tag{4.3.34}$$

In the notation of (4.3.3), (4.3.4), and (4.3.6), the first expression for ϕ in (4.3.34) is

$$\frac{P_1(\tau)}{P_2(\tau)} = \frac{\big((f\tau + g)P(\tau)\big)'}{\big((h\tau + k)P(\tau)\big)'}. \tag{4.3.35}$$

where a prime denotes differentiation with respect to τ. At $P(\tau) = 0$, this reduces to $(f\tau + g)/(h\tau + k)$ and (4.3.34) becomes

$$\phi = \frac{f\tau + g}{h\tau + k} = \frac{\tilde{a}^{ji}\tau - \tilde{b}_{ij}}{\tilde{c}^{ji}\tau - \tilde{d}_{ij}}. \tag{4.3.36}$$

So, given a solution with $a^{ij}, b_{ij}, c^{ij}, d_{ij}$ proportional to the identity, we can generate a new solution with the same τ by, for example, turning on

$$\tilde{a}^{ji} = f n_{ij}, \quad \tilde{b}^{ji} = -g n_{ij}, \quad \tilde{c}^{ij} = h n_{ij}, \quad \tilde{d}^{ij} = -k n_{ij}, \tag{4.3.37}$$

with n_{ij} an arbitrary traceless integer-valued matrix. This is still not the most general solution. For each i, j, equation (4.3.36) is two real equations in the four integers $\tilde{a}^{ji}, \tilde{b}_{ij}, \tilde{c}^{ji}, \tilde{d}_{ij}$, for which we have found a $\mathbf{Z}$'s worth of solutions parametrized by $n_{ij} \in \mathbf{Z}$. More complicated solutions will fill out a $\mathbf{Z}^2$'s worth for each i, j. In addition, the requirement that, for example, a and $\tilde{a}^{ij}$ each be integer valued is too strict. We really only require $a\delta^{ij} + \tilde{a}^{ij} = a^{ij}$ to be integer valued, and similarly for b, c, d.

Finally, the D3-brane charge from flux in this solution can be shown to generalize from (4.3.8) to

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int H_{(3)}\wedge F_{(3)} = -\frac{1}{3}\big(1+\frac{1}{3}\sum_{ij} n_{ij}{}^2\big)(fk-gh)(m^2-4ln). \quad (4.3.38)$$

As a concrete example consider starting with the values:

$$(a^0, a, b, b_0) = (2, 0, 0, 2), \quad (c^0, c, d, d_0) = (2, -2, -2, 4), \quad (4.3.39)$$

which were considered in (4.3.18), of Sec. 4.3.1. In this case,

$$(f, g, h, k, l, m, n) = (2, -2, 2, 4, 1, 1, 1). \quad (4.3.40)$$

Since (4.3.31) and (4.3.32), are the same as (4.3.3)and (4.3.4), τ is given by (4.3.13). Also, since the first equation in (4.3.33), is the same as (4.3.5), ϕ is given by (4.3.14).

The D3-brane charge is given by, (4.3.38),

$$N_{\text{flux}} = 12 + 4\sum_{ij} n_{ij}^2. \quad (4.3.41)$$

Now it is easy to find many non-diagonal matrices where $\sum_{ij} n_{ij}^2 = 1, 2, 3, 4, 5$. Each of them gives a consistent solution, with N_{flux} taking values, $N_{\text{flux}} = 12, 16, 20, 24, 28, 32$ respectively. Also, we should point out that since, (f, g, h, k) are even (4.3.40), the resulting values of $\tilde{a}^{ij}, \tilde{b}_{ij}, \tilde{c}^{ij}, \tilde{d}_{ij}$ are all even as well, (4.3.37), and thus all the fluxes are even.

One last comment. We argued towards the end of the previous Sec. 4.3.1 that the examples discussed in it had $\mathcal{N} = 1$ supersymmetry, and no more. The examples in this section are closely related to those in Sec. 4.3.1, and we expect that they too will generically have only $\mathcal{N} = 1$ supersymmetry.

4.3.3 More General Fluxes

Sec. 4.3.1, discussed the case where the flux matrices $(a^{ij}, b_{ij}, c^{ij}, d_{ij})$ are proportional to the identity matrix. Here we would like to consider flux matrices which are diagonal but with unequal eigenvalues. In these cases one can still argue that the period matrix is diagonal, $\tau^{ij} = \text{diag}(\tau_1, \tau_2, \tau_3)$. As viewed from F-theory then, the resulting compactification is a product of four two-tori. but the modular parameters are in general unequal. Unfortunately, solving the equations for the most general set of diagonal flux matrices is a difficult task.

To proceed we need to place additional restrictions on the flux matrices. Let us begin by considering fluxes of the form:

$$\begin{aligned} a^{ij} &= \mathrm{diag}(a_1, a_2, a_2), \qquad c^{ij} = \mathrm{diag}(a^0, c, c), \\ b_{ij} &= \mathrm{diag}(b_1, b_2, b_2), \qquad d_{ij} = -\mathrm{diag}(d_1, a_2, a_2), \\ & \qquad\qquad d_0 = -b_1 \end{aligned} \tag{4.3.42}$$

Setting $\tau^{ij} = \mathrm{diag}(\tau_1, \tau_2, \tau_3)$. the superpotential (4.2.21). is now given by

$$\begin{aligned} W = &- c^0\phi\tau_1\tau_2\tau_3 + a^0(\tau_1+\phi)\tau_2\tau_3 + c(\tau_2+\tau_3)\phi\tau_1 \\ &- a_1\tau_2\tau_3 - d_1\phi\tau_1 - a_2(\tau_1+\phi)(\tau_2+\tau_3) \\ &- b_1(\phi+\tau_1) - b_2(\tau_2+\tau_3) - b_0 \end{aligned} \tag{4.3.43}$$

One can see that the superpotential is symmetric between $\phi \leftrightarrow \tau_1$ and $\tau_2 \leftrightarrow \tau_3$. Thus, for the restricted choice, (4.3.42), one can consistently seek solutions where the four modular parameters take at most two distinct values.

We now turn to describing two examples where additional restrictions lead to tractable solutions.

Example 2

In the first example, we set all trilinear and linear terms in the superpotential, (4.3.43), to be zero, i.e.,

$$a^0 = c = b_1 = b_2 = 0. \tag{4.3.44}$$

In this case the superpotential takes the form

$$W = -c^0\phi\tau_1\tau_2\tau_3 - a_1\tau_2\tau_3 - d_1\phi\tau_1 - a_2(\tau_1+\phi)(\tau_2+\tau_3) - b_0. \tag{4.3.45}$$

Setting $\partial_\phi W = 0, \partial_{\tau_i} W = 0$ shows that

$$\tau_1 = \phi, \quad \tau_2 = \tau_3, \tag{4.3.46}$$

(as expected) and in addtion leads to two equations:

$$-c^0\tau_1\tau_2^2 - d_1\tau_1 - 2a_2\tau_2 = 0, \tag{4.3.47}$$

$$-c^0\tau_1^2\tau_2 - a_1\tau_2 - 2a_2\tau_1 = 0, \tag{4.3.48}$$

where in both equations we have substituted for ϕ, τ_3, using (4.3.46).

These lead to the relation,

$$\tau_2^2 = \frac{d_1}{a_1}\tau_1^2, \tag{4.3.49}$$

i.e.,

$$\tau_2 = \pm\sqrt{\frac{d_1}{a_1}}\tau_1. \tag{4.3.50}$$

Substituting in (4.3.47) gives

$$\tau_1 = i\sqrt{\frac{a_1}{d_1c^0}}\left(d_1 \pm 2a_2\sqrt{\frac{d_1}{a_1}}\right)^{1/2}. \tag{4.3.51}$$

Setting $W = 0$ then leads to a condition determining b_0 in terms of the other flux integers,

$$b_0 = \frac{a_1}{d_1c^0}\left(d_1 \pm 2a_2\sqrt{\frac{d_1}{a_1}}\right)^2. \tag{4.3.52}$$

Finally, the contribution to the three brane charge is then given by

$$N_{\text{flux}} = 2a_1d_1 + 6a_2^2 \pm 4a_2\sqrt{a_1d_1}. \tag{4.3.53}$$

To find a consistent non-singular solution we need to choose integers c^0, a_1, d_1 and a_2 such that τ_1, τ_2 are complex, b_0 is an integer, and the total flux N_{flux} is within bounds.

One solution to these conditions is obtained by taking

$$(a_1, d_1, a_2, c^0) = (1, 1, -1, -1), \tag{4.3.54}$$

and choosing the positive sign in (4.3.50), so that

$$\tau_2 = +\sqrt{\frac{d_1}{a_1}}\tau_1 = \tau_1. \tag{4.3.55}$$

Then from (4.3.51), we find

$$\tau_1 = i \tag{4.3.56}$$

and from (4.3.53),

$$N_{\text{flux}} = 4. \tag{4.3.57}$$

Also, from (4.3.52), $b_0 = -1$ and is indeed an integer.

Notice that the integers (4.3.54) are odd. As in Example 1, Sec. 4.3.1, to avoid complications related to adding discrete flux we can obtain a consistent solution by doubling all the fluxes so that

$$(a_1, d_1, a_2, c^0, b^0) = (2, 2, -2, -2, -2). \tag{4.3.58}$$

The modular parameters are unchanged and given by (4.3.56), (4.3.55), (4.3.46). The total flux is

$$N_{\text{flux}} = 16, \tag{4.3.59}$$

which means 8 dynamical D3-branes need to be added for a consistent solution.

It turns out that the solution above has $\mathcal{N} = 3$ supersymmetry. Vacua with $\mathcal{N} = 3$ are analysed in generality in the recent paper [80]. The possibility of $\mathcal{N} = 3$ supersymmetry was also mentioned in [65]. The solution above is in fact a special case of the examples found in [80]. To see that it has $\mathcal{N} = 3$ supersymmetry, we note that with the flux (4.3.58) and the moduli, $\phi = \tau^i = i$, $G_{(3)}$ takes the form

$$\frac{1}{(2\pi)^2\alpha'} G_{(3)} = 2i d\bar{z}^1 \wedge dz^2 \wedge dz^3. \tag{4.3.60}$$

It then follows that two additional complex structures in which $G_{(3)}$ is still of type (2,1) can be defined by taking the complex coordinates on the three T^2's to be $(w^1, w^2, w^3) = (\bar{z}^1, \bar{z}^2, z^3)$ or $(w^1, w^2, w^3) = (\bar{z}^1, z^2, \bar{z}^3)$. Thus, as per the general discussion in [80](see also Sec. 4.3.1 above), the solution has $\mathcal{N} = 3$ supersymmetry.

Let us also add that additional solutions can be obtained by starting with the (4.3.54), (4.3.55), (4.3.56), and doing $GL(6, \mathbf{Z}) \times GL(2, \mathbf{Z})$ transformations. In particular one can obtain a solution in which $N_{\text{flux}} = 32$, as will be discussed in more detail in the examples of Sec. 4.5.

Example 3

In the next example we again start with flux matrices and superpotential of the form (4.3.42), (4.3.43), respectively, but now set the following additional restrictions on the fluxes:

$$c^0 = 0, c = -a^0, a_2 = 0, d_1 = -a_1, b_2 = -b_1. \tag{4.3.61}$$

The superpotential (4.3.43) then takes the form

$$\begin{aligned} W =& + a^0(\tau_1 + \phi)\tau_2\tau_3 - a^0(\tau_2 + \tau_3)\phi\tau_1 \\ & - a_1\tau_2\tau_3 + a_1\phi\tau_1 \\ & - b_1(\phi + \tau_1) + b_1(\tau_2 + \tau_3) - b_0. \end{aligned} \tag{4.3.62}$$

Solving the equations $\partial_\phi W = 0, \partial_{\tau_i} W = 0$, it is easy to see that

$$\tau_1 = \phi = \tau_2 = \tau_3 \equiv \tau, \tag{4.3.63}$$

with τ given by

$$\tau = \frac{a_1 \pm \sqrt{a_1^2 - 4a^0 b_1}}{2a^0}. \tag{4.3.64}$$

is a solution. Setting $W = 0$ yields the condition that

$$b_0 = 0. \tag{4.3.65}$$

Finally the D3-brane charge contribution is

$$N_{\text{flux}} = 4b_1 a^0 - a_1^2. \tag{4.3.66}$$

Consistent solutions can be found by taking

$$a^0 = 2, b_1 = 2, a_1 = 2. \tag{4.3.67}$$

This yields

$$\tau = \frac{1 \pm i\sqrt{3}}{2} \tag{4.3.68}$$

and

$$N_{\text{flux}} = 12. \tag{4.3.69}$$

Alternatively, one can take

$$a^0 = 2, b_1 = 4, a_1 = 2. \tag{4.3.70}$$

In this case,

$$\tau = \frac{1 \pm i\sqrt{7}}{2} \tag{4.3.71}$$

and

$$N_{\text{flux}} = 28. \tag{4.3.72}$$

Note that unlike Example 2 above, the two-tori in (4.3.68) and (4.3.71) are not square.

Once again, doing general rescalings and $GL(6,\mathbf{Z}) \times GL(2,\mathbf{Z})$ transformations leads to additional solutions in each of these cases.

As in the previous example, the solutions discussed here have $\mathcal{N} = 3$ supersymmetry as well. This follows by the same argument as in the previous example, after noting that in both the cases (4.3.67) and (4.3.70), $G_{(3)}$ can be expressed as

$$\frac{1}{(2\pi)^2\alpha'} G_{(3)} = a^0(d\bar{z}^1 \wedge dz^2 \wedge dz^3). \tag{4.3.73}$$

4.3.4 Toward a General Supersymmetric Solution

Solving the supersymmetric equations of motion (4.2.22) without any simplifying assumptions is a difficult task. However, a couple of observations can make the task easier. First, note that it is possible to re-write equation (4.2.22) as

$$\left(\mathrm{cof}\left(\tau - \frac{1}{A^0}A\right)\right)_{ij} = \frac{1}{A^{0\,2}}(\mathrm{cof}A)_{ij} + \frac{1}{A^0}B_{ij}\,. \tag{4.3.74}$$

This determines τ^{ij} in terms of the flux matrices and the dilaton ϕ, since if $\mathrm{cof}\, x = y$, then $x = \mathrm{cof} y/\sqrt{\det y}$.

Next, we note that one can actually eliminate the τ^{ij} from the $W = 0$ and $\partial_{\tau^{ij}} W = 0$ equations to obtain a quartic equation for ϕ. The quartic is derived in Appendix B, and takes the form

$$(\det A)B_0 - (\det B)A^0 + (\mathrm{cof}A)_{ij}(\mathrm{cof}B)^{ij} + \tfrac{1}{4}(A^0B_0 + A^{ij}B_{ij})^2 = 0, \tag{4.3.75}$$

where $A^0 = a^0 - \phi c^0$, $A^{ij} = a^{ij} - \phi c^{ij}$, and B_0, B_{ij} are defined similarly. A quartic equation is soluble, so one can solve (4.3.75) for the allowed values of ϕ.

This leaves only the equation $\partial_\phi W = 0$, which upon substitution for ϕ and τ^{ij} gives one nonlinear equation in integers. The integer equation is a consistency condition that determines whether the choice of flux can lead to a supersymmetric solution. The hard part is solving this equation. An additional complication is that for each solution to the integer equation, one must determine all consistent configurations of exotic orientifold planes (as described in [80]), if one is to find all supersymmetric solutions.

4.4 Distinctness of Solutions

Not all solutions with different values of ϕ or τ^{ij} are physically distinct. There is an $SL(2,\mathbf{Z})$ symmetry that relates equivalent values of the dilaton-axion, and an $SL(6,\mathbf{Z})$ symmetry that relates equivalent values of the period matrix τ^{ij}.

4.4.1 $SL(2,\mathbf{Z})$ Equivalence

The Type IIB supergravity action (4.1.1) is invariant under the $SL(2,\mathbf{R})$ symmetry,

$$\begin{pmatrix} F_{(3)} \\ H_{(3)} \end{pmatrix} \to \begin{pmatrix} F'_{(3)} \\ H'_{(3)} \end{pmatrix} = m \begin{pmatrix} F_{(3)} \\ H_{(3)} \end{pmatrix}, \qquad \phi \to \phi' = \frac{a\phi + b}{c\phi + d}, \quad m = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbf{R}). \tag{4.4.1}$$

Under this symmetry, the complex 3-form flux transforms as

$$G_{(3)} \to G'_{(3)} = F'_{(3)} - \phi' H'_{(3)}, \tag{4.4.2}$$

which one can check is equivalent to

$$G_{(3)} \to G'_{(3)} = \frac{G_{(3)}}{c\phi + d}. \tag{4.4.3}$$

At the quantum level, only an $SL(2,\mathbf{Z}) \subset SL(2,\mathbf{R})$ survives. Solutions that differ only by $SL(2,\mathbf{Z})$ transformations are equivalent. It is therefore customary to take ϕ to be in the fundamental domain $\mathcal{F}$, of the upper half plane modulo $PSL(2,\mathbf{Z})$:

$$\mathcal{F} = \left\{ \phi \in \mathbf{C} \;\middle|\; \mathrm{Im}\phi > 0, -\tfrac{1}{2} \le \mathrm{Re}\phi \le \tfrac{1}{2}, |\phi| \ge 1 \right\}. \tag{4.4.4}$$

The examples were not chosen in such a way that the solutions would necessarily give $\phi \in \mathcal{F}$. However it is a simple matter to transform them to the fundamental domain using (4.4.1), where now

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbf{Z}). \tag{4.4.5}$$

4.4.2 $SL(6,\mathbf{Z})$ Equivalence

Following [89], let

$$\mathcal{B}_{\mathbf{C}^3} = \left(\mathbf{e}_{(1)}, \mathbf{e}_{(2)}, \mathbf{e}_{(3)} \right) \tag{4.4.6}$$

denote a basis of $\mathbf{C}^3$, and consider a T^6 in which the lattice basis is

$$\mathcal{B}_{T^6} = \left(\mathbf{e}_{(1)},\ \mathbf{e}_{(2)},\ \mathbf{e}_{(3)},\ \mathbf{e}_{(i)}\tau^i{}_1,\ \mathbf{e}_{(i)}\tau^i{}_2,\ \mathbf{e}_{(i)}\tau^i{}_3\right) \tag{4.4.7a}$$

$$= \mathcal{B}_{\mathbf{C}^3}\,\Lambda, \quad \Lambda = \left(1, \tau\right). \tag{4.4.7b}$$

Under a change of lattice basis,

$$\mathcal{B}_{T^6} \to \mathcal{B}'_{T^6} = \mathcal{B}_{T^6} M, \quad M \in SL(6, \mathbf{Z}). \tag{4.4.8}$$

so

$$\Lambda \to \Lambda'' = \Lambda M, \quad M \in SL(6, \mathbf{Z}). \tag{4.4.9}$$

The change of lattice basis does not produce Λ'' in the standard form $(1, *)$. However, under a change of $\mathbf{C}^3$ basis,

$$\Lambda'' \to \Lambda' = N\Lambda'' = N\Lambda M, \quad N \in GL(3, \mathbf{C}). \tag{4.4.10}$$

We can choose $N = N(M, \tau)$, so that Λ' is in standard form,

$$\Lambda' = N\Lambda M = \left(1, \tau'\right). \tag{4.4.11}$$

Two period matrices τ and τ' related by (4.4.7*b*) and (4.4.11), are equivalent. Also, under an $SL(6, \mathbf{Z})$ coordinate transformation M, the fluxes $F_{(3)}, H_{(3)}$, (when regarded as three-forms) must stay the same [22]. This means that two solutions with period matrices τ and τ' related by (4.4.7*b*) and (4.4.11), and which are otherwise identical, are equivalent.

We should make one more comment before turning to an example. In Sec. 4.6 we discuss solutions which break supersymmetry. The analysis above, identifying solutions related by $SL(2, \mathbf{Z}) \times SL(6, \mathbf{Z})$ transformations, applies to these cases as well.

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[22] Under the $SL(6, \mathbf{Z})$ transformation, (4.4.8), the two coordinate systems are related as:

$$M \begin{pmatrix} x'_i \\ y'_i \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix}. \tag{4.4.12}$$

The transformation of $(F_{(3)})_{ijk}, (H_{(3)})_{ijk}$ then follow by requiring that the three forms, $F_{(3)}, H_{(3)}$ stay invariant.

4.4.3 Example

To illustrate the equivalences, consider Example 1 from Sec 4.3. Suppose instead of choosing the two polynomials (4.3.9), (4.3.10), we made the following choices:

$$P_1(\tau) \equiv -(\tau^3 + 1) = 0. \tag{4.4.13}$$

$$P_2(\tau) \equiv 2\tau^3 - 3\tau^2 + 3\tau - 1 = 0. \tag{4.4.14}$$

These two polynomials have a common factor $P(\tau) = \tau^2 - \tau + 1$, and the corresponding values of integers are

$$((a^0)', a', b', b_0') = (-1, 0, 0, 1) \quad ((c^0)', c', d', d_0') = (2, 1, -1, 1). \tag{4.4.15}$$

where the prime superscripts are being used to distinguish the present case from Example 1, in Sec. 4.3. Solving $P(\tau) = 0$ and choosing the solution with $\mathrm{Im}(\tau') > 0$ gives

$$\tau' = e^{\frac{i\pi}{3}}. \tag{4.4.16}$$

Also, solving (4.3.5) with (4.4.15) gives $\phi' = e^{\frac{i2\pi}{3}}$. Finally the total three brane charge in this case is $N_{\text{flux}} = 3$, as follows from (4.3.8), (4.4.15).

This solution is in fact related to the one corresponding to flux, (4.3.16), by an $SL(6, \mathbf{Z})$ transformation.

The $SL(6, Z)$ transformation has the form, $S \otimes S \otimes S$ where, each $S \in SL(2, \mathbf{Z})$, acts on the one of the three T^2's as:

$$S : \tau \rightarrow -\frac{1}{\tau}. \tag{4.4.17}$$

To see this we note first that under (4.4.17), the modular parameter $\tau' = e^{\frac{i\pi}{3}} \rightarrow e^{\frac{2\pi i}{3}}$, which agrees with (4.3.13). Second, one can show that the corresponding matrix M, in (4.4.12), acting on the coordinates of each T^2 has the form $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. From this it follows that in order to be related by the same $SL(6, \mathbf{Z})$ transformation, the flux integers, (4.3.16), (4.4.15), must satisfy the conditions:

$$((a^0)', a', b', b_0') = (-b_0, -b, a, a^0), \tag{4.4.18}$$

$$((c^0)', c', d', d_0') = (-d_0, -d, c, c^0). \tag{4.4.19}$$

Comparing, (4.3.16), and (4.4.15), we see that these conditions are in fact true. Finally, the two solutions have the same value for the dilaton and agree in the value for N_{flux}. Thus, as per our general discussion above, they are identical.

4.5 New Solutions Using $GL(2, \mathbf{Z}) \times GL(6, \mathbf{Z})$ Transformations

In various examples of Sec. 4.3 we have seen that starting with a given solution, additional ones can be generated by appropriately rescaling the fluxes. Here we discuss this in more generality and show how additional solutions can be obtained by using $GL(2, \mathbf{Z}) \times GL(6, \mathbf{Z})$ transformations. The resulting solutions are physically distinct in general, with a different flux contribution to three brane charge. Solving the tadpole condition (4.1.6) without anti-branes requires that the value of N_{flux} for the new solutions is ≤ 32, and that the required number of wandering D3-branes are added in each case.

The general discussion in this section is applied to some examples at the end. These illustrate that starting with a diagonal period matrix physically distinct solutions can be obtained with a non-diagonal period matrix using the $GL(\mathbf{Z})$ transformations. The examples also yield solutions where all the three brane charge is cancelled by fluxes alone, leaving in one instance, four flat directions in Kähler moduli space. These solutions are of the kind mentioned in the introduction and are good illustrations of the reduced number of moduli that survive once fluxes are turned on.

4.5.1 $GL(2, \mathbf{Z})$ Transformations

Consider a solution to the $\mathcal{N} = 1$ susy equations which has flux, $F_{(3)}, H_{(3)}$, and moduli fixed at values ϕ, τ^{ij}. Now transform the fluxes as follows:

$$\begin{pmatrix} F_{(3)} \\ H_{(3)} \end{pmatrix} \to \begin{pmatrix} F'_{(3)} \\ H'_{(3)} \end{pmatrix} = m \begin{pmatrix} F_{(3)} \\ H_{(3)} \end{pmatrix}, \tag{4.5.1}$$

where the matrix $m \in GL(2, \mathbf{Z})$[23].

One can show that a solution to the the supersymmetry conditions for the new fluxes is obtained by taking the moduli to be at the values

$$\phi' = \frac{a\phi + b}{c\phi + d}, \quad (\tau^{ij})' = \tau^{ij}. \tag{4.5.2}$$

[23] By this we mean that $m = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a, b, c, d \in \mathbf{Z}$. In particular $\det(m)$ need not be 1.

To see this note that under the transformation (4.5.1),

$$G_{(3)} \to G'_{(3)} = \det(m)\frac{G_{(3)}}{c\phi + d}. \tag{4.5.3}$$

The resulting superpotential, (4.2.17), transforms to

$$W \to W'[\phi', \tau] = \int \Omega \wedge G'_{(3)} = \det(m)\frac{W[\phi, \tau]}{c\phi + d}. \tag{4.5.4}$$

where the dependence of the superpotential on the moduli has been explicitly indicated above.

It now follows that if W satisfied the supersymmetry equations, (4.2.20), when the moduli take values ϕ, τ^{ij}, then W' will also meet the susy eqautions for the transformed values, (4.5.2).

Finally, note that under the transformation of the fluxes, (4.5.1), the flux contribution to three brane charge becames

$$N_{\text{flux}} \to N'_{\text{flux}} = \det(m) N_{\text{flux}}. \tag{4.5.5}$$

Starting with a solution, where $N_{\text{flux}} > 0$ we are therefore restricted to $GL(2, \mathbf{Z})$ transformations with $\det(m) > 0$. Also as mentioned above, we need to ensure that $N'_{\text{flux}} \leq 32$, (4.1.6).

4.5.2 $GL(6, \mathbf{Z})$ Transformations

Our starting point is once again a $\mathcal{N} = 1$ susy preserving solution with flux, $F_{(3)}, H_{(3)}$ and moduli fixed at values ϕ, τ^{ij}. But this time we consider transforming the flux by a $GL(6, \mathbf{Z})$ transformation. The transformation can be described explicitly as follows. We fix a basis of one forms (dx^i, dy^i) as in Sec. 4.1.4. The components of $F_{(3)}$ in this basis then transform as

$$(F_{(3)})_{abc} \to (F'_{(3)})_{abc} = (F_{(3)})_{def} M^d_a M^e_b M^f_c, \tag{4.5.6}$$

and similarly for $H_{(3)}$. As a result the components of $G_{(3)}$ in this basis also then transform under $GL(6, \mathbf{Z})$ as :

$$(G_{(3)})_{abc} \to (G'_{(3)})_{abc} = (G_{(3)})_{def} M^d_a M^e_b M^f_c. \tag{4.5.7}$$

In (4.5.6), (4.5.7), M^a_b are the elements of a matrix, $M \in GL(6, \mathbf{Z})$.

We will see that the new fluxes lead to the moduli being stabilized at values ϕ', τ' where $\phi' = \phi$ and

$$(1, \tau') = N(1, \tau)M. \tag{4.5.8}$$

In (4.5.8), M is the same matrix that appears in (4.5.7), and $N \in GL(3, C)$ is a matrix that is chosen so that the left hand side has the form $(1, *)$. In Appendix C, we show that the superpotential for the transformed flux, (4.5.7), is related to the original superpotential by

$$W'[\tau', \phi] = \det(N)\det(M)W[\tau, \phi] \tag{4.5.9}$$

where τ', τ are related as in (4.5.8). It then follows that if τ, ϕ solve the supersymmetry equations (4.2.20) for the original fluxes, τ', ϕ' are the solutions for the transformed fluxes.

Let us also note that under the transformation (4.5.7), the contribution to the three brane charge for the new flux is given by

$$N_{\text{flux}} \to N'_{\text{flux}} = \det(M)N_{\text{flux}}. \tag{4.5.10}$$

Once again we must ensure that the resulting value of three brane charge meets the consistency checks.

Two more comments are worth making at this stage. First, suppose the solution one began with had a diagonal period matrix τ. Then it is possible by an appropriate choice of the matrix M to obtain other solutions where the resulting period matrix τ', (4.5.8), is non-diagonal. A specific example will be given in the next section. Second, in the discussion above we took $M \in GL(6, \mathbf{Z})$. In fact, this is not necessary. All that is required is that the transformed fluxes (4.5.7), have integer components in the cohomology basis (4.1.17).[24] For example choosing $M^a_b = \lambda\delta^a_b$, where $\lambda^3 = 2$ is perfectly acceptable. In this case, we learn from (4.5.8), that $N = \lambda^3 \mathbf{1}_{3\times3}$, and $\tau' = \tau$. We have already encountered this case in Sec. 4.3.1: doubling the flux rescales the superpotential and leaves the moduli fixed.

[24] In fact, the coefficients should be even integers if discrete flux is not being turned on.

4.5.3 An Example

For an example we start with the a solution discussed in Example 2 of Sec. 4.3. The fluxes are given by (4.3.58), and the resulting moduli are stabilized at $\phi = i$ and

$$\tau^{ij} = i\delta^{ij}, \tag{4.5.11}$$

(4.3.56), (4.3.55), and (4.3.46). The solution has $N_{\mathsf{flux}} = 16$.

Now take the matrix M, (4.5.7), to be

$$M = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & D \end{pmatrix}. \tag{4.5.12}$$

Here $M \in GL(6, \mathbf{Z})$, $\mathbf{1}$ is the 3×3 identity matrix and $D \in GL(3, \mathbf{Z})$. The resulting values for the fluxes can be worked out using (4.5.7), but we will not do so explcitly here.

The general discussion of the previous section then tells us that the moduli are stabilized at $\phi' = \phi = i$ and τ', where τ' is given in terms of the original period matrix (4.5.11) as discussed in (4.5.8). Given M in (4.5.12), and τ in (4.5.11), it is easy to show that the matrix N in (4.5.8) is

$$N = \mathbf{1}_{3\times 3}. \tag{4.5.13}$$

Therefore,

$$\tau' = iD. \tag{4.5.14}$$

The flux contribution to the three brane charge in this case is given by (4.5.10).

$$N'_{\mathsf{flux}} = \det(D) N_{\mathsf{flux}} = 16 \det(D). \tag{4.5.15}$$

Since $N'_{\mathsf{flux}} \leq 32$, we learn that $\det(D) = 2$ is the only possibility (cases with $\det(D) = 1$ give rise to solutions related to the original one by $SL(6, \mathbf{Z})$ transformations, which by the discussion in Sec. 4.4.2 are not physically distinct).

As examples for D, two possibilities are

$$D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.5.16}$$

in which case the resulting period matrix is still diagonal (4.5.14), but the eigenvalues are unequal. Or,

$$D = \begin{pmatrix} 1 & -3 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4.5.17}$$

in which case the resulting period matrix is not diagonal. In the latter case we see that starting with a diagonal period matrix we have found an example where τ is fixed at a non-diagonal value.

It is also useful to briefly revisit the primitivity constraint in the example (4.5.16). Since the complex structure is diagonal, it is straightforward to verify that three Kähler deformations of the type (4.3.26), survive as flat directions. In addition to these deformations, the deformation with $w \sim dx^2 \wedge dy^3 + dx^3 \wedge dy^2$ is also now of type (1,1). Thus, altogether there are four Kähler flat directions. This is an example of the kind mentioned in the introduction. The fluxes contribute $N_{\rm flux} = 32$, so no extra D3 branes are needed to soak up the orientifold three plane charge. The dilaton-axion and all complex structure moduli are lifted, leaving four surviving moduli which are Kähler deformations.

4.6 Solutions with $\mathcal{N} = 2$ Supersymmetry.

In this section we discuss the conditions which the $G_{(3)}$ flux must satisfy to preserve $\mathcal{N} = 2$ supersymmetry. We will illustrate the discussion with one example at the end of this section. A more extensive study of $\mathcal{N} = 2$ preserving vacua is left for the future.

An $\mathcal{N} = 2$ theory has an $SU(2)_R$ R-symmetry. $SU(2)_R$ is embedded in $SO(6)$, the group of rotations along the six dimensional compactified directions, as follows [25] :

$$SU(2)_R \subset SU(2)_L \times SU(2)_R \subset SO(4) \times U(1) \subset SO(6). \tag{4.6.1}$$

We choose conventions so that the spinor representation, 4, of $SO(6)$ transforms as a $(2,1)_{+1} + (1,2)_{-1}$ under $SU(2)_R \times SU(2)_L \times U(1)$, and the 6 of $SO(6)$ as $(2,2)_0 + (1,1)_{+2} + (1,1)_{-2}$. In the discussion below we will use indices a, b to denote

[25] This embedding follows by noting that the spinor ϵ, under which the dilatino and gravitino variations vanish, must be a doublet of $SU(2)_R$.

an element of the 6 which transforms as $(2,2)_0$ and indices l,m to denote elements transforming as $(1,1)_2,(1,1)_{-2}$.

Since $SU(2)_R$ is a symmetry of the $\mathcal{N}=2$ theory, it must be left unbroken by the compactification. This means in particular that $G_{(3)}$ must leave an $SU(2)$ subgroup of $SO(6)$ unbroken. $G_{(3)}$ transforms as $[6\times 6\times 6]_A$ under $SO(6)$. With respect to $SU(2)_R\times SU(2)_L\times U(1)$ this decomposes as

$$[6\times 6\times 6]_A=(2,2)_0+(2,2)_0+(3,1)_2+(3,1)_{-2}+(1,3)_2+(1,3)_{-2}. \qquad (4.6.2)$$

For $G_{(3)}$ to preserve $SU(2)_R$ it can only have components along the $(1,3)_{\pm 2}$ representations. A little thought shows that this means $G_{(3)}$ has index structure $(G_{(3)})_{abl}$, in the notation introduced above.

A detailed analysis of the spinor conditions will be presented in Appendix D. The conclusion is the following: in order to preserve $\mathcal{N}=2$ supersymmetry $G_{(3)}$ must only take values in the $(1,3)_2$ representation of $SU(2)_R\times SU(2)_L\times U(1)$. In other words, the $(1,3)_{-2}$ representation, which would have also preserved $SU(2)_R$, must be absent.

Let us check that this condition on $G_{(3)}$ leads to a solution of the equations of motion. The ISD condition (4.1.12), can be written in the present case as

$$\epsilon_{abmcdl}G^{cdl}_{(3)}=i(G_{(3)})_{abm}, \qquad (4.6.3)$$

which can also be expressed as

$$\epsilon_{abcd}\epsilon_{ml}G^{cdl}_{(3)}=i(G_{(3)})_{abm}. \qquad (4.6.4)$$

The two ϵ symbols above refer to the four directions on which the $SO(4)$ acts and the two directions on which the $U(1)$ acts respectively. Since $G_{(3)}$ is a tensor transforming as a $(1,3)$ representation of $SU(2)_R\times SU(2)_L$ it corresponds to the self dual representation of $SO(4)$ and therefore satisfies the condition $\epsilon_{abcd}(G_{(3)})^{cdl}=G^{l}_{ab}$. Further, one can show, in our choice of conventions, that a charge 2 representation of the $U(1)$ satisfies $\epsilon_{ml}(G_{3)})^{cdl}=i(G_{(3)})^{cd}_{m}$. From this, we see that if $G_{(3)}$ is of the $(1,3)_2$ kind, it satisfies the ISD requirement.

It is useful to relate the discussion above to that in Sec. 4.2.1 where we saw that $G_{(3)}$ must be primitive and $(2,1)$ to preserve $\mathcal{N}=1$ susy. Requiring $\mathcal{N}=2$ supersymmetry must impose extra conditions on the $G_{(3)}$ flux. The requirements for $\mathcal{N}=1$ supersymmetry mean that under an $SU(3) \subset SO(6)$, $G_{(3)}$ transforms as a $\bar{6}$. The $SU(2)_L$ discussed above is a subgroup of this $SU(3)$ (since ϵ is a singlet under it), so the $\bar{6}$ representation of $SU(3)$ transforms under $SU(2)_L$ as $3+2+1$. As a result , we learn that $\mathcal{N}=1$ susy alone allows $G_{(3)}$ to take values in the $3+2+1$ representations of $SU(2)_L$. $\mathcal{N}=2$ susy imposes the additional requirement that the doublet and singlet components are missing and $G_{(3)}$ transforms purely as a triplet under $SU(2)_L$. For completeness let us also mention that for unbroken $\mathcal{N}=3$ one must impose yet a further restriction: $G_{(3)}$ must have only one non-zero component proportional to a highest weight state of the triplet representation.

These conditions can be visualised as follows. The weight diagram of the $\bar{6}$ representation is a triangle. (See, e.g., (IX.iii) of [90]). Each state in the $\bar{6}$ representation is denoted by a point in this diagram. $\mathcal{N}=3$ supersymmetry requires $G_{(3)}$ to be proportional to any one of the three vertices of the triangle. $\mathcal{N}=2$ requires that the components of $G_{(3)}$ all lie along an edge of the triangle, and finally, $\mathcal{N}=1$ supersymmetry allows components along all six points in the diagram.

In the example below it will be useful to first impose the conditions for $\mathcal{N}=1$ supersymmetry, then check if the extra restrictions for $\mathcal{N}=2$ supersymmetry are met.

4.6.1 An $\mathcal{N}=2$ Example

As an example choose the fluxes to be:

$$\begin{aligned} H_{135} = H_{245} = F_{136} = F_{246} &= (2\pi)^2\alpha' a^0 \\ F_{135} = F_{245} = -H_{246} = -H_{136} &= (2\pi)^2\alpha' a^0, \end{aligned} \tag{4.6.5}$$

where we are working in the coordinates x^i, y^i introduced in Sec. 4.1.4. Each index above takes six possible values; $i = 1,3,5$, denote components along x^1, x^2, x^3 directions, $i = 2,4,6$, along y^1, y^2, y^3. Also in (4.6.5), a^0 is an integer. In the cohomology basis, (4.1.17), the fluxes can be expressed as

$$\frac{1}{(2\pi)^2\alpha'}F_{(3)} = a^0\alpha_0 + a^0\beta^0 - a^0\beta^{33} + a^0\alpha_{33} \tag{4.6.6}$$

and

$$\frac{1}{(2\pi)^2\alpha'}H_{(3)} = a^0\alpha_0 - a^0\beta^0 - a^0\beta^{33} - a^0\alpha_{33}. \tag{4.6.7}$$

The superpotential is then given by

$$W = a^0(1-\phi)\det\tau - a^0(1+\phi)(\mathrm{cof}\tau)_{33} + a^0(1-\phi)\tau^{33} - a^0(1+\phi). \qquad (4.6.8)$$

One can show that the equations for $\mathcal{N} = 1$ supersymmetry (4.2.22), have the solution

$$\tau^{ij} = i\delta^{ij}, \phi = i. \qquad (4.6.9)$$

The contribution to three brane flux is

$$N_{\text{flux}} = 4(a^0)^2. \qquad (4.6.10)$$

Choosing $a^0 = 2$ we have $N_{\text{flux}} = 16$ which is within the acceptable bound, (4.1.6).

With the choice of complex structure in (4.6.9), $G_{(3)}$ can now be expressed as

$$\frac{1}{(2\pi)^2\alpha'}G_{(3)} = \frac{a^0(1-i)}{2}(dz^1 \wedge d\bar{z}^2 \wedge dz^3 + d\bar{z}^1 \wedge dz^2 \wedge dz^3). \qquad (4.6.11)$$

It is clear that the primitivity condition is satisfied if one chooses the Kähler form to be of the form

$$J = i\sum_A r_A^2 dz^A \wedge d\bar{z}^A. \qquad (4.6.12)$$

In addition the perturbation

$$\delta J = i(dx^1 \wedge dy^2 + dx^2 \wedge dy^1) \sim dz^1 \wedge d\bar{z}^2 + dz^2 \wedge d\bar{z}^1 \qquad (4.6.13)$$

satisfies $\delta J \wedge G = 0$. The remaining 5 Kähler moduli are lifted.

So far we have ensured that there is $\mathcal{N} = 1$ supersymmetry. We will now argue that the solution above in fact preserves $\mathcal{N} = 2$ supersymmetry.

Start by first taking the Kähler metric to be $g_{i\bar{j}} = \delta_{i\bar{j}}$. The coordinates x^i, y^i then define a flat coordinate system. Consider an $SO(4) \times U(1)$ subgroup of $SO(6)$ where the $SO(4)$ acts on the x^1, x^2, y^1, y^2, indices and the $U(1)$ refers to rotations in the x^3, y^3, plane. It is easy to see that for the values (4.6.5), $G_{(3)}$ satisfies the relation, $\epsilon_{abcd}G_{(3)}^{cdl} = (G_{(3)})^l_{ab}$, and therefore transforms as a self dual representation of $SO(4)$ (here we are following the notation of the previous section and the indices a, b take values x^1, x^2, y^1, y^2, while l, m range over x^3, y^3). Since we have already verified that $G_{(3)}$ satsifies the $\mathcal{N} = 1$ conditions, it is ISD, and it follows that it must have charge 2 under the $U(1)$. Putting all this together, in the example above we find that $G_{(3)}$ transforms as a $(1,3)_2$ representation under $SU(2)_R \times SU(2)_L \times U(1)$. As per our discussion above, it therefore meets the requirements for $\mathcal{N}=2$ supersymmetry.

Alternatively, working in the complex coordinates $z^i = x^i + iy^i, \bar{z}^i = x^i - iy^i$, let us define $A_{\bar{k}\bar{l}} = (G_{(3)})_{ij\bar{k}}\epsilon^{ij}_{\bar{l}}$. We see that $A_{\bar{1}\bar{1}}$ and $A_{\bar{2}\bar{2}}$ have nonzero values in the above example. Under the $SU(3)$ symmetry, $(\bar{z}^1, \bar{z}^2, \bar{z}^3)$, transforms as a $\bar{3}$ representation. Consider an $SU(2) \subset SU(3)$ which acts on the $\bar{z}^1, \bar{z}^2$, coordinates and leaves $\bar{z}^3$ invariant. $A_{\bar{k}\bar{l}}$ or equivalently $G_{(3)}$ transforms as a triplet of this $SU(2)$.

An additional check, also mentioned in Sec. 4.3.1, is the following: in an $\mathcal{N}=2$ supersymmetric theory one should be able to define another inequivalent complex structure which keeps $G_{(3)}$ of kind $(2,1)$. In the example above it is easy to see that this corresponds to choosing holomorphic coordinates $(w^1, w^2, w^3) = (\bar{z}^1, \bar{z}^2, z^3)$.

Finally, some thought shows that under Kähler deformations of the form (4.6.12), (4.6.13), the conditions for $\mathcal{N}=2$ supersymmetry continue to hold.

4.7 Non-Supersymmetric Solutions

For generic (non-supersymmetric) solutions, we require only that the scalar potential vanish, or equivalently by (4.1.13), that $G_{(3)}$ be ISD. However, it is computationally simpler to consider the subclass of solutions in which $G_{(3)}$ is also primitive. In this case $G_{(3)}$ can only have pieces of type (2,1) and (0,3). The equations that one needs to solve are then

$$\begin{aligned} D_{\tau^{ij}} W &= \partial_{\tau^{ij}} W + (\partial_{\tau^{ij}} \mathcal{K}) W = 0, \\ D_\phi W &= \partial_\phi W + (\partial_\phi \mathcal{K}) W = 0, \end{aligned} \tag{4.7.1}$$

along with the primitivity condition,

$$J \wedge G_{(3)} = 0. \tag{4.7.2}$$

Here $\mathcal{K}$ is the Kähler potential for closed string fields, inherited from the $\mathcal{N}=4$ supersymmetric T^6/Z_2 compactification, and will be defined momentarily. The first set of equations in (4.7.1) imposes the third set of equations appearing in

(4.2.16), and forbids type (1.2) pieces of $G_{(3)}$. The second equation in (4.7.1) is the second equation in (4.2.16), i.e. forbids a (3.0) piece in $G_{(3)}$. Then, equation (4.7.2) kills the possibility of (2.1) IASD pieces in the three-form flux (T^6, unlike a generic Calabi-Yau, has a three-dimensional space of IASD non-primitive (2.1) forms). More generic non-supersymmetric solutions could be found by relaxing the requirement that the (1.2) ISD forms be absent from $G_{(3)}$, but we will not pursue them here.

Fluxes which obey the equations (4.7.1) and (4.7.2) will break supersymmetry iff $G_{(3)}$ contains a nontrivial component of type (0.3). This is easily interpreted in the low-energy supergravity: Since we are looking for solutions which are not necessarily supersymmetric, we no longer need to impose $D_{\rho^\alpha} W \propto W = 0$ for the Kähler moduli, ρ^α. Precisely when $G_{(3)}$ has a non-vanishing (0.3) piece, $W \neq 0$ and supersymmetry is broken, but still with vanishing potential (at leading order in α' and g_s). Examples of such vacua were discussed in [31,91]. Such vacua will suffer a variety of instabilities in perturbation theory (as the "no-scale" structure of the potential will be violated by α' and g_s corrections), which is why we only discuss them briefly here.

The Kähler potential for the τ^{ij} is

$$\mathcal{K} = \mathcal{K}_{\text{dilaton}} + \mathcal{K}_{\text{cpx}}. \tag{4.7.3}$$

Here,

$$\mathcal{K}_{\text{dilaton}} = -\ln\big(-i(\phi - \bar{\phi})\big), \tag{4.7.4}$$

and

$$\begin{aligned} \mathcal{K}_{\text{cpx}} &= -\ln\Big(-i\int_{T^6} \Omega \wedge \bar{\Omega}\Big) \\ &= -\ln\det\big(-i(\tau - \bar{\tau})\big) \\ &= -\ln\big(i\epsilon_{ijk}(\tau - \bar{\tau})^{i1}(\tau - \bar{\tau})^{j2}(\tau - \bar{\tau})^{k3}\big). \end{aligned} \tag{4.7.5}$$

Since both τ^{ij} and $\bar{\tau}^{ij}$ enter into (4.7.1), it is in general difficult to solve the resulting non-holomorphic equations. However, in an ansatz with enough symmetry, the problem becomes tractable.

4.7.1 A Non-Supersymmetric Example

Let us make a simple flux ansatz which is a subcase of the ansatz made in Example 1 of Sec. 4.3. We take $a^{ij} = a\delta^{ij}$, $d_{ij} = -a\delta_{ij}$, and b_0, c_0 to be nonzero, with all other fluxes vanishing. Then we find that the superpotential takes the form

$$\frac{1}{(2\pi)^2\alpha'}W = -c^0\phi \det\tau - a^{ij}(\mathrm{cof}\tau)_{ij} + d_{ij}\phi\tau^{ij} - b_0. \tag{4.7.6}$$

It is easy to compute the D3-charge carried by the fluxes with this ansatz,

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int H_{(3)} \wedge F_{(3)} = b_0c^0 - a^{ij}d_{ij} = b_0c^0 + 3a^2. \tag{4.7.7}$$

From the symmetry of the problem, one can show that $\tau^{ij} = \tau\delta^{ij}$. Let us further assume that

$$\tau = -\bar{\tau}, \quad \phi = \tau. \tag{4.7.8}$$

Then,

$$\partial_\tau \mathcal{K} = -\frac{3}{2\tau}, \quad \partial_\phi \mathcal{K} = -\frac{1}{2\tau}, \tag{4.7.9}$$

so that

$$\begin{aligned} D_\tau W &= \partial_\tau W + (\partial_\tau \mathcal{K})W = -\frac{3}{2\tau}(c^0\tau^4 - b_0) = 0, \\ D_\phi W &= \partial_\phi W + (\partial_\phi \mathcal{K})W = -\frac{1}{2\tau}(c^0\tau^4 - b_0) = 0. \end{aligned} \tag{4.7.10}$$

The equations are both satisfied if

$$\tau(=\phi) = i\left(\frac{b_0}{c^0}\right)^{1/4}, \tag{4.7.11}$$

therefore our assumption was consistent. Finally, since the flux ansatz is a special case of Sec. 4.3 Example 1, we can solve (4.7.2) by taking J to be in the same space that led to $G_{(3)}$ primitive in Sec. 4.3.1. We can also check that the conditions for supersymmetry here are the same as those found earlier. The solution will be supersymmetric if $W = 0$. In the present example,

$$W = -6a\tau^2 - 2b_0 = 2b_0\left(\sqrt{\frac{9a^2}{b_0c^0}} - 1\right). \tag{4.7.12}$$

So, the solutions are non-supersymmetric as long as $9a^2 \neq b_0c^0$. In fact, it turns out there are no solutions which have even fluxes, $9a^2 = b_0c^0$ and $N_{\text{flux}} \leq 32$ in any case.

4.8 Brane Dynamics

In many of the examples of $\mathcal{N}=1$ vacua with flux, one finds that the number of space-filling D3 branes needed to satisfy the tadpole cancellation requirement (4.1.6) is

$$N_{\rm D3} = 16 - \frac{1}{2}N_{\rm O3'} - \frac{1}{2(2\pi)^4(\alpha')^2}\int H_{(3)}\wedge F_{(3)} \; > \; 0 \tag{4.8.1}$$

($N_{D3}\geq 0$ is needed for supersymmetry). Therefore, in addition to the background 3-form flux, one must introduce space-filling D3 branes.

Following the work of Myers [92], it has been recognized that background p-form fields can have interesting effects on brane dynamics. It follows from [92] that the worldvolume potential (working at vanishing RR axion C_0) is given by

$$\mathcal{V}_{\rm open} \sim \frac{1}{g_s}H_{ijk}Tr(X^iX^jX^k) - (*_6F_{(3)})_{ijk}Tr(X^iX^jX^k) + \cdots \tag{4.8.2}$$

where $\cdots$ includes the usual $\mathcal{N}=4$ field theory potential. When $G_{(3)}$ is ISD,

$$*_6F_{(3)} \;=\; \frac{1}{g_s}H_{(3)} \tag{4.8.3}$$

and the first two terms in (4.8.2) exactly cancel.

This is in keeping with the fact that the ISD fluxes mock up D3 brane charge and tension, and satisfy a "no force" condition with the D3 branes [31]. Therefore, at least at large radius (where supergravity intuition applies), the D3 point sources are free to live at arbitrary positions on the T^6. When $k\leq N_{D3}$ branes meet at a generic point, the low-energy physics is that of $SU(k)$ $\mathcal{N}=4$ SYM theory, while k branes meeting at an O3 plane will give rise to an $SO(2k)$ theory, as usual.[26] It would be interesting to determine the leading nontrivial effects of the fluxes on the D-branes, and to find more elaborate types of models where phenomena reminiscent of those observed in [93] can occur. Inclusion of anti-branes in the flux background might also lead to interesting phenomena, as in [83].

It follows from this discussion that inclusion of N_{D3} branes in one of our models adds $3N_{D3}$ complex moduli to the low energy theory. From this perspective, the models with $N_{\rm flux}\simeq 32$ and $N_{D3}\simeq 0$ are the most satisfying.

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[26] Here we have assumed that the O3-plane is of the usual type, with no discrete NS or RR flux, as is the case for the all of the examples considered here. In the case that the O3-plane contains discrete NS or RR flux, enhancement to $Sp(2k)$ or $SO(2k+1)$ is also possible.

4.9 Discussion

IIB compactifications on Calabi-Yau spaces with both RR and NS 3-form fluxes turned on provide a rich class of vacua which are amenable to detailed study. It should be clear that the techniques used here to compute W and study vacua of the T^6/Z_2 orientifold would generalize to many other examples. The main novelty of these examples is that they provide a setting where the stabilization of Calabi-Yau moduli becomes a concrete and tractable problem. These models are also of interest because they give rise to warped compactifications of string theory, and in some cases the low-energy physics has a holographic interpretation via variants of the AdS/CFT duality [73,74,31].

Several natural questions about the T^6/Z_2 models studied here would be suitable for further study. A complete classification of supersymmetric vacua may be possible (although, especially in cases where the additional complications of discrete RR and NS flux arise [80], it could be very difficult to achieve). It is also interesting to ask whether there are any cases where, with a fixed topological class for the fluxes, one finds multiple vacua. Finally, various dual descriptions of these models should exist, and fleshing out these dualities (and in particular, understanding any analogues of mirror symmetry for vacua with nonzero H-flux) seems worthwhile.

4.A Flux Quantization

We follow the conventions of [31] and [81]. A Dp-brane couples to the $(p+2)$-form RR field strength via the action

$$-\frac{1}{2\kappa_{10}^2}\frac{1}{2(p+2)!}\int_{\mathcal{M}_6} d^{10}x\sqrt{-g}\;{F_{(p+2)}}^2 + \mu_p\int C_{p+1}. \tag{4.A.1}$$

The usual quantization condition that follows from this action is

$$\int_\gamma F_{p+2} = (2{\kappa_{10}}^2\mu_{6-p})n_\gamma, \quad n_\gamma \in \mathbf{Z}, \quad \mu_p = \frac{1}{(2\pi)^p}\alpha'^{-\frac{p+1}{2}}, \tag{4.A.2}$$

for an arbitrary 3-cycle $\gamma \in H_3(\mathcal{M}_6, \mathbf{Z})$. Here μ_p is the electric charge of a Dp-brane and μ_{6-p} is the charge of the dual magnetic D$(6-p)$-brane. The product of these two charges is related to the factor $1/2\kappa_{10}{}^2 = (2\pi)^7\alpha'^4$ that multiplies the action, via the Dirac quantization condition

$$\mu_p\mu_{6-p} = \frac{2\pi}{2\kappa_{10}{}^2}. \qquad (4.A.3)$$

From (4.A.2) and (4.A.3),

$$\mu_p \int F_{p+2} = 2\pi n, \quad n \in \mathbf{Z}, \qquad (4.A.4)$$

which, in the case $p = 1$, becomes

$$\frac{1}{2\pi\alpha'} \int F_3 = 2\pi n, \quad n \in \mathbf{Z}. \qquad (4.A.5)$$

Similarly, we know that the electric NS charge of a fundamental string is $\mu_{\rm F1} = 1/2\pi\alpha'$. So, using $\mu_{\rm F1}\mu_{\rm NS5} = 2\pi/2\kappa_{10}{}^2$ together with the analog of the first equation in (4.A.2),

$$\frac{1}{2\pi\alpha'} \int H_3 = 2\pi n, \quad n \in \mathbf{Z}. \qquad (4.A.6)$$

This equation can also be obtained from (4.A.5) by S-duality.

For compactification on T^6/Z_2, it can be shown that the quantization condition is exactly (4.A.2), with $\mathcal{M}_6 = T^6$[80][27]. The 3-cycles on T^6/Z_2 include both the 3-cycles on T^6 and also new cycles, such as

$$\gamma_0 : 0 \le x^1, x^2 \le 1, \quad 0 \le x^3 \le \frac{1}{2}, \quad y^i = 0, \qquad (4.A.7)$$

which are "half-cycles" on T^6. Naively, this would seem to lead to a problem with the quantization condition (4.A.2). Define γ_1 by

$$\gamma_1 : 0 \le x^1, x^2, x^3 \le 1, \quad y^i = 0. \qquad (4.A.8)$$

Then, one has $n_{\gamma_0} = \frac{1}{2}n_{\gamma_1}$, so that $n_{\gamma_0} \notin \mathbf{Z}$ when n_{γ_1} is odd. However, as discussed in [80], the quantization condition is still satisfied in this case, if a half unit of discrete RR flux is turned on at an odd number of the O3-planes that intersect $\gamma_{0,1}$. Similarly, when m_{γ_1} is odd, a half unit of NS flux must be turned on at an odd number of the O3-plane that intersects $\gamma_{0,1}$. When n_{γ_1} (m_{γ_1}) is even, it is also permissible to turn on RR (NS) flux at some of the O3-planes that intersect $\gamma_{0,1}$, but we require that the total number of such O3-planes be even. Because the construction of vacua with these exotic O3 planes is somewhat involved except in the simplest examples, we have focused in this chapter on cases where all of the fluxes in the covering space are even integers, and the naive problem does not arise.

[27] We are indebted to A. Frey and J. Polchinski for providing us with a preliminary draft of their preprint[80]. The remainder of this section summarizes an analogous section in their preprint.

4.B Derivation of Equation (4.3.75)

Write $\tau^{ij} = T^{ij} + A^{ij}/A^0 = T^{ij} + \widetilde{A}^{ij}$, where a tilde denotes division by A^0. Here, $A^{ij} = a^{ij} - \phi c^{ij}$, and B_{ij}, A^0 and B_0 are defined similarly. Then, equation (4.2.22c) becomes

$$\widetilde{W} = \det\tau - \widetilde{A}^{ij}(\mathrm{cof}\tau)_{ij} - \widetilde{B}_{ij}\tau^{ij} - \widetilde{B}_0, \tag{4.B.1}$$

which, after some algebra can be shown to have the T^{ij} expansion

$$\widetilde{W} = \det T - \big((\mathrm{cof}\widetilde{A})_{ij} + \widetilde{B}_{ij}\big)T^{ij} - \big(\widetilde{A}^{ij}\widetilde{B}_{ij} + \widetilde{B}_0 + 2\det\widetilde{A}\big).$$

The analog of equation (4.2.22c) has already been obtained in equation (4.3.74),

$$(\mathrm{cof}T)_{ij} = (\mathrm{cof}\widetilde{A})_{ij} + \widetilde{B}_{ij}. \tag{4.B.2}$$

By virtue of this equation, the previous result becomes

$$\widetilde{W} = -2\det T + (\widetilde{A}^{ij}\widetilde{B}_{ij} + \widetilde{B}_0 + 2\det\widetilde{A}).$$

When $W = 0$,

$$\det T = -\frac{1}{2}(\widetilde{A}^{ij}\widetilde{B}_{ij} + \widetilde{B}_0 + 2\det\widetilde{A}). \tag{4.B.3}$$

Since we have independent expressions (4.B.2) and (4.B.3) for $\mathrm{cof}T$ and $\det T$, respectively, the equality

$$\det\mathrm{cof}T = (\det T)^2 \tag{4.B.4}$$

gives a quartic equation for ϕ. Explicitly, we have

$$\begin{aligned}\det\mathrm{cof}T &= \det\mathrm{cof}\widetilde{A} + (\mathrm{cofcof}\widetilde{A})^{ij}\widetilde{B}_{ij} + (\mathrm{cof}\widetilde{A})_{ij}(\mathrm{cof}\widetilde{B})^{ij} + \det\widetilde{B}\\ &= (\det\widetilde{A})^2 + (\det\widetilde{A})(\widetilde{A}^{ij}\widetilde{B}_{ij}) + (\mathrm{cof}\widetilde{A})_{ij}(\mathrm{cof}\widetilde{B})^{ij} + \det\widetilde{B},\end{aligned} \tag{4.B.5}$$

and

$$(\det T)^2 = (\det\widetilde{A})^2 + (\det\widetilde{A})(\widetilde{A}^{ij}\widetilde{B}_{ij} + \widetilde{B}_0) + \tfrac{1}{4}(A^0B_0 + A^{ij}B_{ij})^2, \tag{4.B.6}$$

so, equating the two and multiplying by $(A^0)^4$,

$$(\det A)B_0 - (\det B)A^0 + (\mathrm{cof}A)_{ij}(\mathrm{cof}B)^{ij} + \tfrac{1}{4}(A^0B_0 + A^{ij}B_{ij})^2 = 0, \tag{4.B.7}$$

which is equation (4.3.75).

4.C Derivation of Equation (4.5.9)

To establish that (4.5.9), is correct, notice first that the transformed superpotential for the new fluxes is given by

$$W'[\tau] = \int G'_{(3)} \wedge \Omega[\tau], \tag{4.C.1}$$

where we have explicitly indicated that the dependence on the complex structure moduli arises from Ω on the right hand side. Using, (4.5.7), (4.C.1), can also be expressed as

$$W'[\tau] = (G_{(3)})_{rst}\Omega[\tau]_{def} M^r_a M^s_b M^t_c \epsilon^{abcdef}. \tag{4.C.2}$$

Now,

$$\Omega[\tau] = dz^1 \wedge dz^2 \wedge dz^3, \tag{4.C.3}$$

where,

$$\begin{pmatrix} dz^1 \\ dz^2 \\ dz^3 \end{pmatrix} = \begin{pmatrix} 1 & \tau \end{pmatrix} \begin{pmatrix} dx^i \\ dy^i \end{pmatrix}. \tag{4.C.4}$$

Under a change of complex structure, $\tau \to \tau'$ (where τ' is given by (4.5.8))

$$\begin{pmatrix} dz^1 \\ dz^2 \\ dz^3 \end{pmatrix} \to \begin{pmatrix} (dz^1)' \\ (dz^2)' \\ (dz^3)' \end{pmatrix} = N \begin{pmatrix} 1 & \tau \end{pmatrix} M \begin{pmatrix} dx^i \\ dy^i \end{pmatrix}. \tag{4.C.5}$$

As a result one finds that[28]

$$\Omega[\tau']_{(def)} = \det(N)\Omega[\tau]_{uvw} M^u_d M^e_b M^w_f. \tag{4.C.6}$$

Substituting in (4.C.2), then leads to

$$W'[\tau'] = \det(N)\det(M) W[\tau]. \tag{4.C.7}$$

4.D The Spinor Conditions for $\mathcal{N} = 2$ Supersymmetry

Throughout this appendix the components for all tensors will be evaluated in a vielbein frame. We will also use the notation introduced in Sec. 4.6. The $SO(6)$

[28] This follows, for example, by noting from (4.C.5) that, up to an overall normalization of $\det(N)$, $\Omega[\tau']$ in the basis $\begin{pmatrix} (dx^i)' \\ (dy^i)' \end{pmatrix} = M \begin{pmatrix} dx^i \\ dy^i \end{pmatrix}$ has the same components as $\Omega[\tau]$ in the basis $\begin{pmatrix} dx^i \\ dy^i \end{pmatrix}$.

group of rotations in the 6 compactified directions has an $SO(4)\times U(1)$ subgroup. In our notation, indices a, b which take four values refer to directions which transform under the $SO(4)$ and indices l, m which take two values refer to directions which are acted on by the $U(1)$ subgroup. The metric in the vielbein frame has components $g_{ab} = \delta_{ab}, g_{lm} = \delta_{lm}, g_{al} = 0$. Also the γ matrices satisfy the relation

$$\{\gamma^l, \gamma^a\} = 0. \tag{4.D.1}$$

Using the fact that an $SU(2)_R$ symmetry group must be left unbroken we argued in Sec. 4.6 that the flux must have the index structure, $(G_{(3)})_{(abl)}$, and further that $G_{(3)}$ must transform as $(1,3)_{\pm 2}$ under the $SU(2)_R \times SU(2)_L \times U(1) \subset SO(6)$ group. Here we will show that the spinor conditions imply that the $(1,3)_{-2}$ terms must be absent and $G_{(3)}$ must only transform as a $(1,3)_2$ representation under this group.

The spinor conditions are given in [79] and (4.2.8).

$$G_{(3)}\epsilon = G_{(3)}\epsilon^* = G_{(3)}\gamma^l\epsilon^* = G_{(3)}\gamma^a\epsilon^* = 0. \tag{4.D.2}$$

In our choice of conventions, the spinor 4 representation of $SO(6)$ transforms as $(2,1)_1 + (1,2)_{-1}$ under $SU(2)_R \times SU(2)_L \times U(1)$. In the $\mathcal{N} = 2$ supersymmetry case, ϵ is a doublet of $SU(2)_R$ and therefore transforms as a $(2,1)_1$ representation of $SU(2)_R \times SU(2)_L \times U(1)$. We are now ready to ask what conditions (4.D.2) imposes on the flux $G_{(3)}$.

We noted above that the flux has index structure G_{abl}. Using (4.D.1), the first condition in (4.D.2) can be explicitly written as

$$(G_{(3)})_{lab}\gamma^l[\gamma^a, \gamma^b]\epsilon = 0. \tag{4.D.3}$$

If $G_{(3)}$ transforms as $(1,3)_{\pm 2}$ under $SU(2)_R \times SU(2)_L \times U(1)$ it is easy to see that $(G_{(3)})_{lab}[\gamma^a, \gamma^b]$ is a generator of $SU(2)_L$ and therefore must annihilate ϵ, which is a singlet of $SU(2)_L$. So (4.D.3) is met.

Similarly, since ϵ^* is also a singlet under $SU(2)_L$, it is also true that $G_{(3)}\epsilon^* = 0$. The third condition in (4.D.2), can be written as

$$\frac{1}{2}(G_{(3)})_{mab}\gamma^m\gamma^l[\gamma^a, \gamma^b]\epsilon^* = 0. \tag{4.D.4}$$

Once again the same argument leading to the first two conditions being met ensures that (4.D.4) is also satisfied.

Finally we come to the last condition in (4.D.2). This can be expressed as

$$(G_{(3)})_{bcl}\gamma^b\gamma^c\gamma^a\gamma^l\epsilon^* = 0. \tag{4.D.5}$$

One can show that (4.D.5) is not met if $G_{(3)}$ has a $(1,3)_{-2}$ component. If this component is absent though, and $G_{(3)}$ is entirely of the $(1,3)_2$ kind, one can show that

$$(G_{(3)})_{bcl}\gamma^l\epsilon^* = 0. \tag{4.D.6}$$

Condition (4.D.5) then follows.

To show that (4.D.6) is satisfied when $G_{(3)}$ transforms as a $(1,3)_2$ state we first note, as was pointed out above, that ϵ has charge $+1$ with respect to the $U(1)$. So ϵ^* has charge -1. As a result, if $G_{(3)}$ is of $(1,3)_2$ kind, $(G_{(3)})_{abl}\gamma^l\epsilon^*$, has charge -3 under the $U(1)$. Also note that the state $(G_{(3)})_{abl}\gamma^l\epsilon^*$ transforms as a 4 spinor under the $SO(6)$ symmetry. But the 4 representation does not have any state with -3 charge under the $U(1)$ symmetry. Thus the left hand side of (4.D.6) must vanish.

In summary, the spinor conditions show that $G_{(3)}$ must transform under $SU(2)_R \times SU(2)_L \times U(1)$ as a $(1,3)_2$ representation, in order to preserve $\mathcal{N} = 2$ supersymmetry.

5. Supersymmetry Changing Bubbles

String theory is known to have many different vacua. An important direction of research aims at understanding whether these different vacua are connected. For compactifications to 4 dimensional Minkowski space the situation is as follows. With $\mathcal{N} = 4$ supersymmetry (susy), it is known that there are several disconnected components of the space of vacua (see e.g. [72]). With $\mathcal{N} = 2$ susy, naively disconnected components are known to be connected up in a large web [94,95,96], although it is premature to say that all such models are connected. For $\mathcal{N} = 1$, less is known,[29] although some classical obstructions to connecting vacua are circumvented by string theory via chirality changing phase transitions [97,98].

In the discussion above, the notion of connectedness relates to moving along a moduli space of degenerate solutions. It is known that Minkowski vacua with *different* amounts of susy can never be connected, in this sense. A theorem to this effect was proved for the perturbative heterotic theory in [99]. However, it is clear that weaker notions of connectedness exist and could be physically relevant. For instance, two vacua can be connected by a finite potential barrier, $V_{\rm bar}$. For $V_{\rm bar}$ much less than the four dimensional (4D) Planck scale M_4, low-energy field theory would correctly describe the dynamics in rolling betwen these vacua. Such a notion of connectedness might be relevant in cosmology. A related weaker notion of connectedness requires the existence of vacuum bubbles of one vacuum inside the other, with the domain wall separating the two having a tension $\sigma \ll (M_4)^3$.

In this letter we show that one *can* unify some vacua with different amounts of supersymmetry in this weaker sense. Our starting point is IIB string theory compactified on the T^6/Z_2 orientifold. This vacuum has $\mathcal{N} = 4$ susy and is a

[29] In this case, we generically expect the moduli to be lifted by quantum corrections. For this reason our discussion will mostly focus on $\mathcal{N} \geq 2$ vacua, but our idea would also apply to any $\mathcal{N} = 1$ models where the flux-generated no-scale potential is the full potential.

dual description of the heterotic theory on T^6. Appropriately turning on RR and NS fluxes yields vacua with $\mathcal{N} = 3, 2, 1$ susy [100,6,80]. We show that the vacua with reduced susy can be connected to the $\mathcal{N} = 4$ vacuum, and to each other, by spherical domain walls. In the ten dimensional string theory, these domain walls are made up of NS and Dirichlet five branes, each of which wrap an internal three cycle, besides spanning the spherical boundary. It is important to note that the tension of the resulting domain walls can be made parametrically lighter than $(M_4)^3$ (by tuning the compactification volume V, as we will demonstrate). This ensures that the vacuum inside the bubble is not shielded from the one outside by a black hole horizon, and is available for inspection from the outside.

The bubble configuration we construct is not BPS and evolves in time, with a trajectory determined primarily by the tension of the domain wall. By tuning the radius of the internal space, one can make the lifetime of the bubble arbitrarily large.

We should emphasise that the vacua under consideration here are quantum mechanically stable, and all of them have zero ground state energy. As a result, the spherical bubbles referred to above are not produced by quantum tunneling, as in the decay of a false vacuum.

In Sec. 5.1, we briefly review the construction of the various vacua in IIB on T^6/Z_2 with fluxes. Sec. 5.2 describes the domain wall brane configurations which interpolate between the different vacua. We discuss the construction of the domain walls from wrapped five branes, their resulting dynamics, and the stability of the walls. As a concrete example we consider a bubble of the standard $\mathcal{N} = 4$ vacuum inside a theory with $\mathcal{N} = 2$ susy. We close with a discussion in Sec. 5.3.

5.1 Vacua with Various $\mathcal{N}$ in IIB on T^6/Z_2

Our starting point is IIB theory compactified on a T^6/Z_2 orientifold. This model is T-dual to Type I theory and preserves $\mathcal{N} = 4$ supersymmetry. Sixteen D3 branes are needed to cancel the RR tadpoles arising from the O3 planes. The resulting low energy theory is $SO(32)$ $\mathcal{N} = 4$ supersymmetric Yang-Mills theory coupled to $\mathcal{N} = 4$ supergravity.

However, this is not the most general possibility. The IIB compactification also admits other superselection sectors in which we turn on quantized fluxes of the three-form field strengths H and F originating from the NS-NS and RR sectors. That is, H and F satisfy the conditions

$$\frac{1}{(2\pi)^2\alpha'}\int_\gamma F = m_\gamma \in \mathbf{Z}, \quad \frac{1}{(2\pi)^2\alpha'}\int_\gamma H = n_\gamma \in \mathbf{Z}, \tag{5.1.1}$$

where γ labels the classes in $H_3(T^6, \mathbf{Z})$. For the case of a six-torus with coordinates x^i and y^i, each of period 1, we can be very explicit about this choice. Let $d\xi^a = dx^i, dy^j, 1 \le i, j \le 3$, denote six one-forms. Then, a basis for $H^3(T^6, \mathbf{Z})$ is given by the twenty three forms, $d\xi^a \wedge d\xi^b \wedge d\xi^c, 1 \le a, b, c \le 6$. For the most general choice of flux, $\frac{1}{(2\pi)^2\alpha'}F$ and, $\frac{1}{(2\pi)^2\alpha'}H$ can be expanded in this basis with integer coefficients.

In the presence of such fluxes, the full tadpole cancellation condition for the D3 brane charge reads:[30]

$$\frac{1}{2(2\pi)^4(\alpha')^2}\int_{T^6} H \wedge F \ + N_{D3} \ = \ 16. \tag{5.1.2}$$

Here we consider only the susy preserving case with no anti-branes. Susy breaking by adding anti-branes and vacuum bubbles in similar backgrounds was studied in [83].

In sectors with non-vanishing flux, one finds an effective (super)potential for the Calabi-Yau complex structure and Kähler moduli [63] (for a detailed derivation, see Appendix A of [31]). Supersymmetric vacua are located at points in complex structure moduli space where $G = F - \phi H$ is of type (2,1) (here ϕ is the IIB axio-dilaton), while the Kähler structure J should be chosen to make G primitive (i.e. satisfy $J \wedge G = 0$). These conditions were studied in detail for the case of T^6/Z_2 in [6], and it was found that for generic choices of the fluxes there are no supersymmetric critical points. However, for suitable non-generic choices of flux, one can find vacua with $\mathcal{N} = 1, 2, 3$ supersymmetry. In these vacua, typically all the complex structure moduli and some of the Kähler moduli are fixed. The dilaton-axion is also typically fixed with $g_s \sim O(1)$. One Kähler modulus, governing the overall volume of compactification V, is never lifted in these models; this will be important in the discussion below.

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30 Here we ignore the possibility of exotic O3 planes and choose the integer coefficients which characterise the flux to be even, as explained in [80].

In Sec. 5.2.3. a specific $\mathcal{N} = 2$ vacuum will be considered. It corresponds to the choice of flux

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'}F &= 2dx^1 \wedge dx^2 \wedge dy^3 + 2dy^1 \wedge dy^2 \wedge dy^3 \\ \frac{1}{(2\pi)^2\alpha'}H &= 2dx^1 \wedge dx^2 \wedge dx^3 + 2dy^1 \wedge dy^2 \wedge dx^3. \end{aligned} \tag{5.1.3}$$

Following [100,6] one easily finds that there is a moduli space of $\mathcal{N} = 2$ supersymmetric vacua with these fluxes. A particular locus in this moduli space has $\phi = i$ and a T^6 which is of the form $(T^2)^3$, where each two-torus has complex structure $\tau = i$. The Kähler form can be chosen to be $J \sim iR^2 \sum_{i=1}^3 dz_i \wedge d\bar{z}_i$. This is just a product of square two-tori with overall volume R^6.

A quick way to see that the vacuum preserves $\mathcal{N} = 2$ supersymmetry is by noticing that along this locus, G takes the form

$$\frac{1}{(2\pi)^2\alpha'}G = -\frac{i}{2}(dz_1 \wedge d\bar{z}_2 \wedge dz_3 + d\bar{z}_1 \wedge dz_2 \wedge dz_3). \tag{5.1.4}$$

$\mathcal{N} = 2$ susy requires that there be another inequivalent choice of complex structure which keeps G of type $(2,1)$; this corresponds to taking $z_{1,2} \to \bar{z}_{1,2}, z_3 \to z_3$.

5.2 Vacuum bubbles from D5 and NS5 Branes

5.2.1 Overview

The key idea in our construction of bubbles is the following: by wrapping D5/NS5 branes on three-cycles of the compact manifold it is possible to construct domain walls in $R^{3,1}$ across which the quantised fluxes in the compact manifold jump. E.g., wrapping a D5 brane on a three-cycle causes the flux of F through the *dual* three-cycle to jump by one unit. Since the vacua reviewed above differ essentially in the RR and NS fluxes along the compact directions, this allows different vacua to be connected.

In fact this idea was used in [63] to construct BPS domain walls between $\mathcal{N} = 1$ vacua in the setting of non-compact Calabi-Yau constructions. Our interest is in compact internal manifolds, resulting in flat 4D spacetime. In this case, we do not expect BPS domain walls to interpolate between vacua with different amounts of supersymmetry for two reasons. First, the central extensions of the supersymmetry algebra do not admit BPS domain walls of nonzero tension between supersymmetric Minkowski vacua in supergravity (see e.g. [43.35]).[31] Second, planar domain walls have codimension one and are often singular in supergravity, see e.g. [101].

With this in mind, we construct non-BPS spherical domain walls in $R^{3,1}$, separating a bubble of one vacuum inside the wall from another vacuum outside. Two requirements must be met by the domain wall to consistently interpolate between the vacua. First, the flux of F, H must jump appropriately across the wall. Second, the moduli must vary smoothly across it. It is clear that any jump in F, H fluxes can be engineered by choosing D5,NS5 branes wrapping three cycles in the appropriate homology classes. We will choose the minimum area three-cycle in each homology class which is consistent with our boundary conditions. The domain wall is then the composite configuration made out of the resulting D5,NS5 branes.

To meet the condition on the moduli, we restrict ourselves here to considering pairs of vacua such that moduli lifted in both vacua are fixed to the same values. The remaining moduli, unfixed in one or both vacua, can then simply be tuned to take the same values on both sides of the wall (we will show that the backreaction of the walls is small enough to make this a good approximation).

In fact, this condition is not very restrictive, and allows our construction to connect several vacua, including many with different susy's. For example, since none of the moduli in the standard $\mathcal{N} = 4$ vacuum are fixed, it can be connected to all the other vacua in the above manner. This is enough to establish that all the vacua of Sec. 5.1 are connected by the above construction.

The two vacua connected by the wall will in general have different numbers of D3 branes (5.1.2). One can verify that the extra D3 branes in one vacuum terminate on the 5 branes making up the domain wall consistently [102]. Also, we note that being composed of 5 branes, the resulting domain walls have a thickness of order the string scale. As a result, in analysing their dynamics below we can work in the thin wall approximation.

Some features of the resulting domain wall dynamics were discussed in the introduction. Let us verify that the tension of the domain wall, compared to $(M_4)^3$, can be lowered by tuning the volume modulus $V \sim R^6$. In the estimate below, we

[31] This is not true in global supersymmetry. The additional constraint in supergravity arises roughly because one needs the superpotential W to vanish for a Minkowski vacuum.

set $g_s \sim O(1)$. A 5-brane wrapping a three-cycle of size R^3 gives rise to a domain wall tension

$$\sigma \sim R^3/(\alpha')^3. \tag{5.2.1}$$

On the other hand, $M_4 \sim (\alpha')^{-2}R^3$, so that $\sigma/(M_4)^3 \sim (\alpha')^3/R^6$. This ratio can be made small by taking R to be large.[32]

The rest of this section is organised as follows. The time dependent dynamics of the domain wall is analysed in Sec. 5.2.2. under the assumption that the the wall moves as a single cohesive unit, driven primarily by its net tension. A specific example of two vacua and the interpolating domain wall is discussed next, in Sec. 5.2.3. Finally in Sec. 5.2.4, the relative forces between branes which make up the wall are analysed. These forces are found to be small, thereby justifying the analysis of Sec. 5.2.2.

5.2.2 Bubble Dynamics

We begin by neglecting the backreaction of the domain wall on the metric and other closed string modes, and analyse its trajectory in flat space. Next, we estimate the backreaction effects and show that they are small most of the time. All along we work with walls of tension $\sigma \ll (M_4)^3$.

A spherical domain wall in flat space is described by the action,

$$S = -\int_{t_i}^{t_f} dt\ 4\pi\sigma\rho^2\sqrt{1-\dot{\rho}^2}. \tag{5.2.2}$$

or equivalently an energy

$$M = \frac{4\pi\sigma\rho^2}{\sqrt{1-\dot{\rho}^2}}. \tag{5.2.3}$$

The dynamics is easy to work out in detail. For fixed M and initial outward radial velocity, the bubble expands to a maximum size

$$4\pi\sigma\rho_{max}^2 = M, \tag{5.2.4}$$

[32] The tension of the domain wall depends on both the volume and the moduli that control the sizes of the relevant three-cycles. We will show that the backreaction of the wall on all moduli, including these, is small.

then recollapses.[33]

Birkhoff's theorem tells us that the spherically symmetric geometry outside the wall is described by the Schwarzschild metric, while that inside the wall is described by flat space. The Schwarzschild radius $R_s \sim G_N M$ (with $G_N \sim M_4^{-2}$ the 4D Newton's constant). The gravitational backreaction is therefore small as long as

$$\rho \gg G_N M. \tag{5.2.5}$$

When $\sigma/(M_4)^3 \ll 1$, (5.2.5) can be met by suitably choosing the initial radius ρ_i and the total energy of the wall. E.g., for a slowly moving wall, $\dot{\rho} \ll 1$, (5.2.5) is met by taking,

$$\rho_i \ll \frac{(M_4)^2}{\sigma} \sim \frac{R^3}{\alpha'}, \tag{5.2.6}$$

where we have used (5.2.3) and (5.2.1). Ultimately, as the bubble recollapses, (5.2.5) will no longer hold and the gravitational backreaction will get significant, potentially leading to the formation of a black hole.[34]

The important thing to emphasize is that even if a black hole eventually forms, by tuning the volume and other moduli, the time for which the wall lies outside the black hole horizon can be made as large as one wishes. E.g., the time it takes starting from an initial radius $\rho_i \leq R^3/\alpha'$ to recollapse back to $\rho_f = \rho_i$ is of order $\Delta t \sim R^3/\alpha'$.

The domain wall also acts as a source for the various moduli that determine its tension. We now show that the back reaction on these moduli is also small as long as the domain wall is well outside its Schwarzschild radius. We denote the canonically normalised modulus under consideration as ψ, and by an additive shift ensure that asymptotically far away, $r \to \infty$, $\psi = 0$ (e.g. for the radius $\psi \sim \big(\log(R) - \log(R_\infty)\big)$. One can show that ψ satsifies the equation:

$$\nabla^2 \psi = \frac{\beta\sigma}{(M_4)^2} \sqrt{1 - \dot{\rho}(t)^2}\ \delta\big(r - \rho(t)\big). \tag{5.2.7}$$

The right hand term arises because the tension, σ, depends on ψ. β is determined by this dependence, and $\rho(t)$ is the radius of the wall.

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[33] We do not consider trajectories where the wall moves in the internal directions. This is a consistent approximation to make.

[34] See however the discussion of stability in Sec. 5.2.4

Since our main concern is the part of the trajectory where the bubble is well outside its Schwarzschild radius, we consider a simplified model for the domain wall's history below. We assume the wall is constructed at time $t = t_i$ and then evolves till $t = t_f$ with the radius, $\rho(t)$, meeting the condition (5.2.5) all along. At time t_f we assume the bubble is destroyed.

In this example, ψ satisfies the following boundary conditions:[35] it vanishes as $r \to \infty$ for all t, and also as $t \to \pm\infty$ for all r. Also, we choose boundary conditions such that $\psi = 0$ inside the bubble.

The resulting solution for ψ is,

$$\psi = \frac{f_+(t+r) + f_-(t-r)}{r}. \tag{5.2.8}$$

$f_\pm$ meet two junction conditions across the wall: $f_+(t+\rho(t)) + f_-(t-\rho(t)) = 0$, and, $f'_+(t+\rho(t)) + f'_-(t-\rho(t)) = \frac{\beta \sigma_0 \rho(t)}{M_4^2}$, with prime indicating derivative with respect to argument. σ_0 is the tension at $\psi = 0$. Using these, we can solve for f_- in terms of the trajectory $\rho(t)$:

$$f_-(t-\rho(t)) = \begin{cases} 0 & t < t_i \\ \frac{\beta\sigma_0}{2M_4^2}\int_{t_i}^{t} dt \rho(t)\left(1-\dot\rho(t)^2\right) & t_i < t < t_f \\ \frac{\beta\sigma_0}{2M_4^2}\int_{t_i}^{t_f} dt \rho(t)\left(1-\dot\rho(t)^2\right) & t_f < t \end{cases}. \tag{5.2.9}$$

f_+, and finally ψ can then be determined from (5.2.8) and the junction conditions above. A small backreaction means $\psi \ll 1$. It is easy to see from (5.2.9), (5.2.8) that this requirement is met when the bubble radius is much larger than the Schwarzschild radius, (5.2.5).

5.2.3 An Example

As a concrete example, consider a spherical bubble of the standard $\mathcal{N} = 4$ vacuum inside the $\mathcal{N} = 2$ vacuum determined by (5.1.3).

The domain wall in this case consists of two kinds of D5 branes and two kinds of NS5 branes. The D5 branes wrap the three-cycles $x_1 = x_2 = y_3 = 0$ and $y_1 = y_2 = y_3 = 0$, respectively, with appropriate orientations. Each of these branes carries two units of D5 brane charge. The NS5 branes, each carrying two units of NS 5-charge, wrap the three-cycles $x_1 = x_2 = x_3 = 0$ and $y_1 = y_2 = x_3 = 0$ respectively.

The compactification also has 64 O3 planes. Finally, the $\mathcal{N} = 4$ vacuum has 16 D3 branes, while the $\mathcal{N} = 2$ vacuum has 12 D3 branes (5.1.2). The extra D3 branes in the $\mathcal{N} = 4$ vacuum terminate on the 5 branes.

35 r, t are the usual radial and time coordinates in flat 4D space.

5.2.4 Stability

Our discussion of the wall dynamics assumed that the different branes making up the wall do not come apart due to relative forces between them. This assumption is worth examining, since the configuration breaks supersymmetry and $g_s \sim O(1)$ in these vacua.

To begin, it is useful to understand the two sources of susy breaking in this configuration. First, there is the curvature of the two sphere in spacetime, which via the bubble tension gives rise to collective motion of the branes. Second, there is the presence of both the branes, and the three-form flux.

To understand the second source, it is helpful to study the example of Sec. 5.2.3. Here, we take the decompactification limit, $R \to \infty$, and consider planar parallel branes in $R^{3,1}$ in this limit (while keeping the orientation of the branes in the internal directions unchanged). The spinor conditions can be analysed as in [103,79]. One finds that the configuration of branes and O3 planes of Sec. 5.2.3 preserves $\mathcal{N} = 1$ susy, i.e. four supercharges. Also, it turns out that any two components, e.g., two kinds of branes or one brane and the O3 planes, preserve $\mathcal{N} = 2$ susy. Breaking to $\mathcal{N} = 1$ requires three kinds of branes/planes.[36]

As $R \to \infty$, the effect of the flux G vanishes and can be neglected. But for finite R, the G flux (5.1.3) contributes additional terms in the spinor equations [79,78]. Now it turns out that the spinor conditions imposed by the brane configuration are in conflict with those imposed by the fluxes. As a result, supersymmetry is completely broken at finite R.

With this example in mind, let us return to the general discussion about relative forces between branes. These are of two kinds. First, the different branes couple to different ambient fluxes (the effect of the flux sourced by a brane should be neglected in this interaction). The electric potential energy of a brane in the flux background is

$$V_{\rm flux}(\rho) \sim \mu \int C_{(6)} \sim (\alpha')^{-2} \rho^3. \qquad (5.2.10)$$

[36] Essentially the same analysis of susy breaking and stability applies to the domain wall obtained by replacing the $\mathcal{N} = 2$ vacuum (5.1.3) with the example in Sec.4.6.1. This latter $\mathcal{N} = 2$ vacuum lifts all complex structure moduli.

where C_6 is the appropriate gauge potential and we have set $g_s \sim O(1)$. μ is order one in string units. The energy in the tension, (5.2.1), is $V_{\text{tension}} \sim \sigma\rho^2 \sim R^3\rho^2/(\alpha')^3$. Comparing, we see that

$$V_{\text{flux}}(\rho)/V_{\text{tension}}(\rho) \sim \alpha'\rho/R^3. \tag{5.2.11}$$

This ratio is small as long as the bubble radius is bigger than the Schwarzschild radius, (5.2.5).

Second, interbrane forces could arise if in the absence of flux, the brane/plane configuration breaks susy completely, or, as in the example above, the brane/plane configuration preserves only $\mathcal{N} = 1$ susy, which allows for a superpotential to be generated. One expects the resulting (super)potential to scale like the common world volume of the branes required to reduce the susy to $\mathcal{N} \leq 1$. To be comparable with V_{tension} the potential must scale like R^3, so all the required branes must be parallel in the internal direction.[37] $\mathcal{N} \leq 1$ susy then leaves only one possibility: a pair of NS and D5 branes. Such a pair of 5 branes breaks all susy's. However, in this case the pair can be replaced by a 5 brane susy preserving bound state carrying both NS and D5 brane charge, which results in the same jump in F, H flux. Thus, by appropriately choosing the components of the domain wall, such forces can be made small.

5.3 Discussion

The general idea of unifying vacua through vacuum bubbles of some fixed tension, σ, was discussed by Banks in [104]. There, the focus was on how gravitational back-reaction makes it difficult to imagine using such bubbles as a diagnostic in gravity theories (as opposed to field theories). In our construction, however, the moduli spaces we are unifying allow us to make σ very small in 4D Planck units, and hence to manufacture bubbles which are large but are not yet black holes. In such a circumstance, we find the notion of "unification through vacuum bubbles" meaningful, even in the presence of gravity.

[37] In Sec. 5.2.3 three branes/planes are neccessary to break susy to $\mathcal{N} = 1$, and the common world volume lies entirely in the $R^{3,1}$ spacetime. Thus the resulting potential energy δV scales like $\delta V \sim \rho^2/(\alpha')^3$ and is small compared to V_{tension}.

Our construction connects vacua with large enough volume. Starting with a vacuum where the volume modulus is small. one can imagine first creating a large region of spacetime where the volume modulus is large. This can be done by a slowly varying, large amplitude wave of the volume modulus. In this region the bubble construction could then proceed as before. While we have not explored such time dependent solutions with moduli waves and spherical bubbles in detail, it seems quite reasonable that they exist.[38]

Moving beyond. as a next step in making our construction useful in the overall scheme of string duality. it would be important to find transitions connecting e.g. $\mathcal{N}=2$ models on T^6/Z_2 to $\mathcal{N}=2$ compactifications on some more generic Calabi-Yau space. In such a case. up to standard dualities. one would have successfully unified the heterotic string on T^6 with the best-understood web of vacua with less supersymmetry.

It would also be interesting to study connectedness by asking if one can find time dependent solutions which roll between vacua separated by a finite potential barrier. The fact that $\sigma \ll M_4^3$ in our construction is suggestive. However. the existence of such solutions cannot be explored in low energy field theory. since the fields which create the five-branes would also have to be excited. Exploring such solutions in string field theory seems difficult. at the moment.

5.A Brane SUSY Spinor Conditions

We follow the conventions of Hanany-Witten [82]. Type IIB string theory preserves 32 supersymmetries corresponding to the 32 degrees of freedom of a 10d positive chirality Weyl spinor ϵ,

A D5 brane stretched along the $012abc$ directions breaks half of these supersymmtries through the spinor conditions

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_a\Gamma_b\Gamma_c\epsilon_R. \quad (5.A.1)$$

[38] A similar construction should also apply to directly connect two vacua in which some of the moduli are fixed to different values. The fluxes give rise to a potential on moduli space, $V_{pot}/(M_4)^4 \sim (\alpha')^6/R^{12}$, which is small for large R. Hence, the resulting bubble should still be accessible from the outside vacuum.

An NS5 brane stretched along the $012abc$ directions breaks a different half of the supersymmetries through the spinor conditions

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_a\Gamma_b\Gamma_c\epsilon_L, \tag{5.A.2L}$$

$$\epsilon_R = -\Gamma_0\Gamma_1\Gamma_2\Gamma_a\Gamma_b\Gamma_c\epsilon_R. \tag{5.A.2R}$$

Finally, an O3 plane stretched along the 0123 directions breaks the same supersymmetries as a D3 brane. The D3 brane spinor conditions are

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_3\epsilon_R. \tag{5.A.3}$$

For the brane configuration discussed in Sec. 5.2.2, the D5 brane conditions become

$$\epsilon_L = -\Gamma_0\Gamma_1\Gamma_2\Gamma_7\Gamma_8\Gamma_6\epsilon_R, \tag{5.A.4}$$

$$\epsilon_L = -\Gamma_0\Gamma_1\Gamma_2\Gamma_4\Gamma_5\Gamma_6\epsilon_R, \tag{5.A.5}$$

where the apparent sign change from (5.A.1) is due to the orientation of the three-cycles. For example, $F \supset dx^1 \wedge dx^2 \wedge dy^3$, so one D5 brane wraps the cycle whose volume form is $\omega = -dy^1 \wedge dy^2 \wedge dx^3$. The minus sign ensures that $F \wedge \omega$ is proportional to the volume form of the torus, with positive proportionality constant. (We take the volume form to be $dx^1 \wedge dx^2 \wedge dx^3 \wedge dy^1 \wedge dy^2 \wedge dy^3$). If one keeps track of the orientations of the other relevant three-cycles as well, then the result is that the second D5 brane wraps $-dx^1 \wedge dx^2 \wedge dx^3$, but the two NS5 branes wrap $dy^1 \wedge dy^2 \wedge dy^3$ and $dx^1 \wedge dx^2 \wedge dy^3$, with no additional sign change from orientation. Therefore, the NS5 brane conditions are

$$\epsilon_{\substack{L\\R}} = \pm\Gamma_0\Gamma_1\Gamma_2\Gamma_7\Gamma_8\Gamma_9\epsilon_{\substack{L\\R}}, \tag{5.A.6}$$

$$\epsilon_{\substack{L\\R}} = \pm\Gamma_0\Gamma_1\Gamma_2\Gamma_4\Gamma_5\Gamma_9\epsilon_{\substack{L\\R}}. \tag{5.A.7}$$

By combining (5.A.4) through (5.A.6), and also using the chirality condition

$$\bar{\Gamma} = \Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\Gamma_7\Gamma_8\Gamma_9 = 1, \tag{5.A.8}$$

it is possible to show that (5.A.3) and (5.A.7) follow automatically. Therefore, there are three independent conditions and supersymmetry is reduced by a factor of 2^3 to four supercharges by the brane configuration. The fluxes will complete break this residual supersymmetry.

5.B Flux SUSY Spinor Conditions

As in Appendix B, we use a notation in which the indices 0123 denote spacetime directions, and the indices 345679 denote the T^6 directions $x^1x^2x^3y^1y^2y^3$. It is useful to discuss the additional conditions imposed by the fluxes in the notation of Appendix D of [6]. (Note the indices a, b take values in 4578 and the indices l, m in 69).

The conditions imposed by the flux are summarized in Equation (D.2) of [6]. The equations

$$G\epsilon = G^*\epsilon = 0 \tag{5.B.1}$$

are equivalent to

$$(F_{(3)})_{abl}\Gamma^a\Gamma^b\Gamma^l\epsilon = (H_{(3)})_{abl}\Gamma^a\Gamma^b\Gamma^l\epsilon = 0. \tag{5.B.2}$$

(Here, $\Gamma^0 = -\Gamma_0$, and $\Gamma^i = \Gamma_i$ for $i \neq 0$). These conditions yield

$$\Gamma^4\Gamma^5\Gamma^7\Gamma^8\epsilon_{L,R} = \epsilon_{L,R}. \tag{5.B.3}$$

The third condition, (D.4), is then automatically met.

The last condition imposed by the flux is (D.6). For the fluxes under consideration here,

$$F_{(3)} = dx^2 \wedge dx^2 \wedge dy^3 + dy^1 \wedge dy^2 \wedge dy^3, \tag{5.B.4}$$

$$H_{(3)} = dx^2 \wedge dx^2 \wedge dx^3 + dy^1 \wedge dy^2 \wedge dx^3, \tag{5.B.5}$$

this gives

$$\Big((F_{(3)})_{ab9}\Gamma^9 - i(H_{(3)})_{ab6}\Gamma^6\Big)\epsilon^* = 0, \tag{5.B.6}$$

which reduces to

$$\Gamma^6\Gamma^9\epsilon_L = \epsilon_R. \tag{5.B.7}$$

It is possible to show that the flux condition (5.B.3) is in direct contradiction with the brane conditions (5.A.4) and (5.A.5). Similarly, (5.B.7) is in direct contradiction with Equations (5.A.4) and (5.A.6). Therefore, the fluxes and branes do not preserve compatible supersymmetries.

References

[1] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].

[2] See, for example: `http://www.slac.stanford.edu/spires/topcites`

[3] S. Kachru, M. B. Schulz and E. Silverstein, "Self-tuning flat domain walls in 5d gravity and string theory," Phys. Rev. D **62**, 045021 (2000) [arXiv:hep-th/0001206].

[4] S. Kachru, M. B. Schulz and E. Silverstein, "Bounds on curved domain walls in 5d gravity," Phys. Rev. D **62**, 085003 (2000) [arXiv:hep-th/0002121].

[5] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, "A small cosmological constant from a large extra dimension," Phys. Lett. B **480**, 193 (2000) [arXiv:hep-th/0001197].

[6] S. Kachru, M. B. Schulz and S. Trivedi, "Moduli stabilization from fluxes in a simple IIB orientifold," arXiv:hep-th/0201028.

[7] N. Kaloper and R. C. Myers, "The $O(d,d)$ story of massive supergravity," JHEP **9905**, 010 (1999) [arXiv:hep-th/9901045].

[8] N. Hitchin, "The geometry of three-forms in six and seven dimensions," arXiv:math.dg/0010054.

[9] S. Kachru, X. Liu, M. B. Schulz and S. P. Trivedi, "Supersymmetry changing bubbles in string theory," arXiv:hep-th/0205108.

[10] S. M. Carroll, "The cosmological constant," Living Rev. Rel. **4**, 1 (2001) [arXiv:astro-ph/0004075].

[11] V. A. Rubakov and M. E. Shaposhnikov, "Extra Space-Time Dimensions: Towards A Solution To The Cosmological Constant Problem," Phys. Lett. B **125**, 139 (1983).

[12] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven dimensions," Nucl. Phys. B **460**, 506 (1996) [arXiv:hep-th/9510209].

[13] P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. B **475**, 94 (1996) [arXiv:hep-th/9603142].

[14] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, "The universe as a domain wall," Phys. Rev. D **59**, 086001 (1999) [arXiv:hep-th/9803235].

[15] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The hierarchy problem and new dimensions at a millimeter," Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].

[16] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[17] Z. Kakushadze and S. H. Tye, "Brane world," Nucl. Phys. B **548**, 180 (1999) [arXiv:hep-th/9809147].

[18] L. Randall and R. Sundrum, "An alternative to compactification," Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[19] S. M. Carroll and L. Mersini, "Can we live in a self-tuning universe?," Phys. Rev. D **64**, 124008 (2001) [arXiv:hep-th/0105007].

[20] R. M. Wald, "Gravitational collapse and cosmic censorship," arXiv:gr-qc/9710068.

[21] R. Bousso and J. Polchinski, "Quantization of four-form fluxes and dynamical neutralization of the cosmological constant," JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].

[22] S. S. Gubser, Stanford ITP Seminar, March 2000

[23] S. Kachru, M. B. Schulz, and E. Silverstein, unpublished.

[24] S. S. Gubser, "Curvature singularities: The good, the bad, and the naked," Adv. Theor. Math. Phys. **4**, 679 (2002) [arXiv:hep-th/0002160].

[25] G. L. Smith, C. D. Hoyle, J. H. Gundlach, E. G. Adelberger, B. R. Heckel and H. E. Swanson, "Short Range Tests Of The Equivalence Principle," Phys. Rev. D **61**, 022001 (2000).

[26] C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner and H. E. Swanson, "Sub-millimeter tests of the gravitational inverse-square law: A search for 'large' extra dimensions," Phys. Rev. Lett. **86**, 1418 (2001) [arXiv:hep-ph/0011014].

[27] S. Dimopoulos and G. F. Giudice, "Macroscopic Forces from Supersymmetry," Phys. Lett. B **379**, 105 (1996) [arXiv:hep-ph/9602350].

[28] A. D. Linde, "Relaxing the Cosmological Moduli Problem," Phys. Rev. D **53**, 4129 (1996) [arXiv:hep-th/9601083].

[29] S. Kachru, private communication.

[30] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and χSB-resolution of naked singularities," JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[31] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," arXiv:hep-th/0105097.

[32] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[33] T. Banks, M. Berkooz and P. J. Steinhardt, "The Cosmological moduli problem, supersymmetry breaking, and stability in postinflationary cosmology," Phys. Rev. D **52**, 705 (1995) [arXiv:hep-th/9501053].

[34] T. Banks, "SUSY Breaking, Cosmology, Vacuum Selection and the Cosmological Constant in String Theory," arXiv:hep-th/9601151.

[35] S. Ferrara and M. Porrati, "Central extensions of supersymmetry in four and three dimensions," Phys. Lett. B **423**, 255 (1998) [arXiv:hep-th/9711116].

[36] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[37] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].

[38] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[39] E. Verlinde, "On RG-flow and the cosmological constant," Class. Quant. Grav. **17**, 1277 (2000) [arXiv:hep-th/9912058].

[40] C. Schmidhuber, "AdS(5) and the 4d cosmological constant," Nucl. Phys. B **580**, 140 (2000) [arXiv:hep-th/9912156].

[41] S. Kachru and E. Silverstein, "4d conformal theories and strings on orbifolds," Phys. Rev. Lett. **80**, 4855 (1998) [arXiv:hep-th/9802183].

[42] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, "Modeling the fifth dimension with scalars and gravity," Phys. Rev. D **62**, 046008 (2000) [arXiv:hep-th/9909134].

[43] M. Cvetic and H. H. Soleng, "Supergravity domain walls," Phys. Rept. **282**, 159 (1997) [arXiv:hep-th/9604090].

[44] W. D. Goldberger and M. B. Wise, "Modulus stabilization with bulk fields," Phys. Rev. Lett. **83**, 4922 (1999) [arXiv:hep-ph/9907447].

[45] E. Witten, "Small Instantons in String Theory," Nucl. Phys. B **460**, 541 (1996) [arXiv:hep-th/9511030].

[46] A. Klemm, W. Lerche, P. Mayr, C. Vafa and N. P. Warner, "Self-Dual Strings and $N = 2$ Supersymmetric Field Theory," Nucl. Phys. B **477**, 746 (1996) [arXiv:hep-th/9604034].

[47] M. Berkooz, M. Rozali and N. Seiberg, "Matrix Description of M-theory on T^4 and T^5," Phys. Lett. B **408**, 105 (1997) [arXiv:hep-th/9704089].

[48] S. Kachru, N. Seiberg and E. Silverstein, "SUSY Gauge Dynamics and Singularities of 4d N=1 String Vacua," Nucl. Phys. B **480**, 170 (1996) [arXiv:hep-th/9605036].

[49] S. Kachru, J. Kumar and E. Silverstein, "Orientifolds, RG flows, and closed string tachyons," Class. Quant. Grav. **17**, 1139 (2000) [arXiv:hep-th/9907038].

[50] J. Polchinski and E. Witten, "Evidence for Heterotic-Type I String Duality," Nucl. Phys. B **460**, 525 (1996) [arXiv:hep-th/9510169].

[51] S. P. de Alwis, J. Polchinski and R. Schimmrigk, "Heterotic Strings With Tree Level Cosmological Constant," Phys. Lett. B **218**, 449 (1989).

[52] M. Gremm, "Four-dimensional gravity on a thick domain wall," Phys. Lett. B **478**, 434 (2000) [arXiv:hep-th/9912060].

[53] C. Csaki, J. Erlich, T. J. Hollowood and Y. Shirman, "Universal aspects of gravity localized on thick branes," Nucl. Phys. B **581**, 309 (2000) [arXiv:hep-th/0001033].

[54] E. Witten, "Phases of $N = 2$ theories in two dimensions," Nucl. Phys. B **403**, 159 (1993) [arXiv:hep-th/9301042].

[55] P. S. Aspinwall, B. R. Greene and D. R. Morrison, "Calabi-Yau moduli space, mirror manifolds and spacetime topology change in string theory," Nucl. Phys. B **416**, 414 (1994) [arXiv:hep-th/9309097].

[56] A. Strominger, "Massless black holes and conifolds in string theory," Nucl. Phys. B **451**, 96 (1995) [arXiv:hep-th/9504090].

[57] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, "Strings On Orbifolds," Nucl. Phys. B **261**, 678 (1985).

[58] N. Kaloper, "Bent domain walls as braneworlds," Phys. Rev. D **60**, 123506 (1999) [arXiv:hep-th/9905210].

[59] N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, "The Cosmic Triangle: Revealing the State of the Universe," Science **284**, 1481 (1999) [arXiv:astro-ph/9906463].

[60] A. G. Cohen and D. B. Kaplan, "Solving the hierarchy problem with noncompact extra dimensions," Phys. Lett. B **470**, 52 (1999) [arXiv:hep-th/9910132].

[61] J. Polchinski and A. Strominger, "New Vacua for Type II String Theory," Phys. Lett. B **388**, 736 (1996) [arXiv:hep-th/9510227].

[62] J. Michelson, "Compactifications of type IIB strings to four dimensions with non-trivial classical potential," Nucl. Phys. B **495**, 127 (1997) [arXiv:hep-th/9610151].

[63] S. Gukov, C. Vafa and E. Witten, "CFT's from Calabi-Yau four-folds," Nucl. Phys. B **584**, 69 (2000) [Erratum-ibid. B **608**, 477 (2001)] [arXiv:hep-th/9906070].

[64] B. R. Greene, K. Schalm and G. Shiu, "Warped compactifications in M and F theory," Nucl. Phys. B **584**, 480 (2000) [arXiv:hep-th/0004103].

[65] K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088].

[66] E. Silverstein, "(A)dS backgrounds from asymmetric orientifolds," arXiv:hep-th/0106209.

[67] G. Curio, A. Klemm, D. Lust and S. Theisen, "On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes," Nucl. Phys. B **609**, 3 (2001) [arXiv:hep-th/0012213].

[68] J. A. Harvey, G. W. Moore and C. Vafa, "Quasicrystalline Compactification," Nucl. Phys. B **304**, 269 (1988).

[69] M. Dine and E. Silverstein, "New M-theory backgrounds with frozen moduli," arXiv:hep-th/9712166.

[70] A. Dabholkar and J. A. Harvey, "String islands," JHEP **9902**, 006 (1999) [arXiv:hep-th/9809122].

[71] E. Witten, "Toroidal compactification without vector structure," JHEP **9802**, 006 (1998) [arXiv:hep-th/9712028].

[72] J. de Boer, R. Dijkgraaf, K. Hori, A. Keurentjes, J. Morgan, D. R. Morrison and S. Sethi, "Triples, fluxes, and strings," Adv. Theor. Math. Phys. **4**, 995 (2002) [arXiv:hep-th/0103170].

[73] H. Verlinde, "Holography and compactification," Nucl. Phys. B **580**, 264 (2000) [arXiv:hep-th/9906182].

[74] C. S. Chan, P. L. Paul and H. Verlinde, "A note on warped string compactification," Nucl. Phys. B **581**, 156 (2000) [arXiv:hep-th/0003236].

[75] P. Mayr, "On supersymmetry breaking in string theory and its realization in brane worlds," Nucl. Phys. B **593**, 99 (2001) [arXiv:hep-th/0003198].

[76] P. Mayr, "Stringy world branes and exponential hierarchies," JHEP **0011**, 013 (2000) [arXiv:hep-th/0006204].

[77] G. Curio, A. Klemm, B. Kors and D. Lust, "Fluxes in heterotic and type II string compactifications," Nucl. Phys. B **620**, 237 (2002) [arXiv:hep-th/0106155].

[78] K. Becker and M. Becker, "M-Theory on Eight-Manifolds," Nucl. Phys. B **477**, 155 (1996) [arXiv:hep-th/9605053].

[79] M. Grana and J. Polchinski, "Gauge/gravity duals with holomorphic dilaton," arXiv:hep-th/0106014.

[80] A. R. Frey and J. Polchinski, "$N = 3$ warped compactifications," arXiv:hep-th/0201029.

[81] J. Polchinski, "String Theory. Vol. 2: Superstring Theory And Beyond," *Cambridge, UK: Univ. Pr. (1998) 531 p.*

[82] A. Hanany and B. Kol, "On orientifolds, discrete torsion, branes and M theory," JHEP **0006**, 013 (2000) [arXiv:hep-th/0003025].

[83] S. Kachru, J. Pearson and H. Verlinde, "Brane/flux annihilation and the string dual of a non-supersymmetric field theory," arXiv:hep-th/0112197.

[84] E. Bergshoeff, M. de Roo and E. Eyras, "Gauged supergravity from dimensional reduction," Phys. Lett. B **413**, 70 (1997) [arXiv:hep-th/9707130].

[85] G. W. Moore, "Arithmetic and attractors," arXiv:hep-th/9807087.

[86] J. Louis and A. Micu, "Heterotic string theory with background fluxes," Nucl. Phys. B **626**, 26 (2002) [arXiv:hep-th/0110187].

[87] G. Dall'Agata, "Type IIB supergravity compactified on a Calabi-Yau manifold with H-fluxes," JHEP **0111**, 005 (2001) [arXiv:hep-th/0107264].

[88] L. Andrianopoli, R. D'Auria and S. Ferrara, "Consistent reduction of $N = 2 \rightarrow N = 1$ four dimensional supergravity coupled to matter," Nucl. Phys. B **628**, 387 (2002) [arXiv:hep-th/0112192].

[89] M. Schlichenmaier "An Introduction to Riemann Surfaces, Algebraic Curves and Moduli Spaces," *Berlin: Springer-Verlag (1989) 148 p.* (Lecture Notes in Physics, 322).

[90] H. Georgi, "Lie Algebras In Particle Physics. From Isospin To Unified Theories," Front. Phys. **54**, 1 (1982).

[91] K. Becker and M. Becker, "Supersymmetry breaking, M-theory and fluxes," JHEP **0107**, 038 (2001) [arXiv:hep-th/0107044].

[92] R. C. Myers, "Dielectric-branes," JHEP **9912**, 022 (1999) [arXiv:hep-th/9910053].

[93] J. Polchinski and M. J. Strassler, "The string dual of a confining four-dimensional gauge theory," arXiv:hep-th/0003136.

[94] B. R. Greene, D. R. Morrison and A. Strominger, "Black hole condensation and the unification of string vacua," Nucl. Phys. B **451**, 109 (1995) [arXiv:hep-th/9504145].

[95] A. C. Avram, P. Candelas, D. Jancic and M. Mandelberg, "On the Connectedness of Moduli Spaces of Calabi-Yau Manifolds," Nucl. Phys. B **465**, 458 (1996) [arXiv:hep-th/9511230].

[96] T. M. Chiang, B. R. Greene, M. Gross and Y. Kanter, "Black hole condensation and the web of Calabi-Yau manifolds," Nucl. Phys. Proc. Suppl. **46**, 82 (1996) [arXiv:hep-th/9511204].

[97] S. Kachru and E. Silverstein, "Chirality-changing phase transitions in 4d string vacua," Nucl. Phys. B **504**, 272 (1997) [arXiv:hep-th/9704185].

[98] B. A. Ovrut, T. Pantev and J. Park, "Small instanton transitions in heterotic M-theory," JHEP **0005**, 045 (2000) [arXiv:hep-th/0001133].

[99] T. Banks and L. J. Dixon, "Constraints On String Vacua With Space-Time Supersymmetry," Nucl. Phys. B **307**, 93 (1988).

[100] K. Dasgupta, K. H. Oh, J. Park and R. Tatar, "Geometric transition versus cascading solution," JHEP **0201**, 031 (2002) [arXiv:hep-th/0110050].

[101] J. Gutowski, "Stringy domain walls of $N = 1$, $D = 4$ supergravity," Nucl. Phys. B **627**, 381 (2002) [arXiv:hep-th/0109126].

[102] A. Strominger, "Macroscopic Entropy of $N = 2$ Extremal Black Holes," Phys. Lett. B **383**, 39 (1996) [arXiv:hep-th/9602111].

[103] A. Hanany and E. Witten, "Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics," Nucl. Phys. B **492**, 152 (1997) [arXiv:hep-th/9611230].

[104] T. Banks, "On isolated vacua and background independence," arXiv:hep-th/0011255.