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Fermion Masses and Mixing in SUSY Grand Unified Gauge Models with Extended Gut Gauge Groups

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Abstract

We discuss a class of supersymmetric (SUSY) grand unified gauge (GUT) models based on the extended GUT symmetry $G \times G$ or $G \times G \times G$, where G denotes the GUT group that has the Standard Model symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ embedded as a subgroup. As motivated from string theory, these models are constructed without introducing any Higgs field of rank two or higher. Thus all the Higgs fields are in the fundamental representations of the extended GUT symmetry or, when $G = SO(10)$, in the spinorial representation. These Higgs fields, when acquiring their vacuum expectation values, would break the extended GUT symmetry down to the Standard Model symmetry.

In this dissertation, we argue that the features required of unified models, such as the Higgs doublet-triplet splitting, proton stability, and the hierarchy of fermion masses and mixing angles, could have natural explanations in the framework of the extended SUSY GUTs. Furthermore, we argue that the frameworks used previously to construct $SO(10)$ GUT models using adjoint Higgs fields can naturally arise from the $SO(10) \times SO(10)$ and $SO(10) \times SO(10) \times SO(10)$ models by integrating out heavy fermions. This observation thus suggests that the traditional SUSY GUT $SO(10)$ theories can be viewed as the low energy effective theories generated by breaking the extended GUT symmetry down to the $SO(10)$ symmetry.

Preface

This thesis consists of five chapters. In the first three chapters, I describe the fermion mass and mixing problem in Nature and then briefly review the problem under the framework of supersymmetric (SUSY) grand unified (GUT) gauge models. The last two chapters are based on two of my papers that discuss the problem under the SUSY framework with extended GUT symmetry.

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Chapter 1

Introduction

1.1 Fermion masses and mixing problem in the Standard Model

The Standard Model (SM) [1] has long been thought as a successful theory in describing the physical world up to the weak interaction mass scale. Over the past two decades, a myraid of experiments have been carried out to test the SM and these have found no inconsistency. However, despite its success, the SM can only be viewed as an effective theory at low energy, because some 18 parameters are required to fit the experiment data [2]. Among these 18 parameters, three are the gauge couplings for the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, 13 are fermion masses and mixing angles, and the last two are a Higgs vacuum expectation value (VEV) and mass. These parameters are not equally well understood. The weak mixing angle $\sin^2\theta_W$, the QED fine structure constant α, the charged lepton masses m_e, m_μ and m_τ, and the Z^0 boson mass are precisely measured to better than 0.1 percent accuracy. The quark mixing angle V_{us} for the first two families, the charm quark mass m_c, and the bottom quark mass m_b are less accurately determined to be within 5 percent accuracy. The remaining parameters are either roughly known at the 10 percent accuracy level [2, 3, 4] or, like the quark mixing between the first and third families, very poorly known.

However, the SM parameters are not random numbers. Instead, they are in some way hierarchical and show interesting patterns. For example, the fermion mass spectrum that ranges from MeV to 100 GeV is listed in the following table [2]:

$$\begin{aligned} m_t &= 173.8 \pm 5.2 \text{ GeV}, \quad m_c = 1.1 \sim 1.4 \text{ GeV}, \quad m_u = 1.5 \sim 5 \text{ MeV} \\ m_b &= 4.1 \sim 4.4 \text{ GeV}, \quad m_s = 60 \sim 170 \text{ MeV}, \quad m_d = 3 \sim 9 \text{ MeV} \\ m_\tau &= 1.78 \text{ GeV}, \quad m_\mu = 105.6 \text{ MeV}, \quad m_e = 0.511 \text{ MeV} \end{aligned} \tag{1.1}$$

The spectrum is easily classified into following groups

$$\begin{aligned} m_t \sim O(10^2) \text{ GeV} &\gg m_b, m_c, m_\tau \sim O(1) \text{ GeV} \\ &\gg m_s, m_\mu \sim O(10^2) \text{ MeV} \\ &\gg m_u, m_d, m_e \sim O(1) \text{ MeV}, \end{aligned} \tag{1.2}$$

or classified horizontally by fermion flavor

$$\begin{aligned} m_t : m_c : m_u &\sim 1 : \varepsilon_u : \varepsilon_u^2 \\ m_b : m_s : m_d &\sim 1 : \varepsilon_d : \varepsilon_d^2 \\ m_\tau : m_\mu : m_e &\sim 1 : \varepsilon_d : \varepsilon_d \varepsilon_u. \end{aligned} \tag{1.3}$$

where $\varepsilon_d \sim O(10^{-1})$ and $\varepsilon_u \sim O(10^{-2})$ are small numbers.

On the other hand, the quark mixing effects are also hierarchical in the SM. This is shown in the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix that involves in the weak interaction of the quark sector as:

$$W_u^- J_W^{u+} = \frac{1}{\sqrt{2}} W_u^- \bar{u}_m [V_{CKM}] d_m, \tag{1.4}$$

where

$$V_{CKM} \equiv V_{Lu} V_{Ld}^+ = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{1.5}$$

In Eq. (1.4), u_m and d_m denote the up and down quark mass eigenstates which come from the mixing of the corresponding weak eigenstates u_L and d_L as follows:

$$u_m = V_{Lu} \cdot u_L, \qquad d_m = V_{Ld} \cdot d_L. \tag{1.6}$$

Here V_{Lu} and V_{Ld} denote the transformation matrices for the left handed up and down quarks respectively. Experimental data [2] shows that there is a hierarchical pattern for quark mixing effects among different families

$$|V_{us}| : |V_{cb}| : V_{ub}| \sim \lambda : \lambda^2 : \lambda^3. \tag{1.7}$$

where $\lambda \approx 0.22$ is the Cabibbo angle (V_{us}) for the quark mixing between the first two flavors of fermions.

From Eqs. (1.1) and (1.7), the highly organized numbers including nine fermion masses and three CKM angles make up twelve out of the 18 input parameters for Standard Model. That is, most of the unsolved puzzles in the SM are actually due to our lack of knowledge on fermion flavors. Therefore, it is a key problem for modern physics to understand the physics of flavor.

In the Standard Model, fermion masses are generated after spontaneous breaking of the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ down to electromagnetic gauge symmetry $U(1)_{em}$. Generically, we need at least one complex scalar Higgs field ϕ which transforms as an $SU(2)_L$ doublet with hypercharge $Y = 1/2$ for breaking the electroweak symmetry.

$$V(\phi) = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi^2|^2. \tag{1.8}$$

Eq. (1.8) shows a typical potential function for the Higgs field. By minimizing the potential $V(\phi)$, ϕ develops a non-zero vacuum expectation value (VEV) $< \phi >$ that preserves the $U(1)_{em}$ subgroup of $SU(2)_L \times U(1)$. From Eq. (1.8), without changing the minimum value of $V(\phi)$, $< \phi >$ can be rotated to:

$$< \phi > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad v = \sqrt{\frac{\mu^2}{\lambda}}. \tag{1.9}$$

To give the neutral weak boson Z^0 a weak scale mass in the SM, v must be about 246 GeV.

The SM can accommodate two or more Higgs doublet fields with non-zero VEVs [5]. For instance, the mass hierarchy between up quark masses and down quark masses can be the result of coupling different Higgs doublet fields to up quarks and down quarks, respectively, in a two-Higgs models, if one Higgs doublet acquires a large VEV v_2 while the other acquiring a smaller VEV v_1. Therefore, in typical two-Higgs models, the fermion mass hierarchy problem is turned into the hierarchy problem of Higgs VEVs, and additional parameter such as $\tan\beta \equiv v_2/v_1$ is introduced.

For the SM with minimal field content, there are only three gauge bosons, one Higgs doublet ϕ of $SU(3)_C \times SU(2)_L \times U(1)_Y$ with quantum numbers $(1, 2, 1/2)$, three families of quarks with quantum numbers $Q_i(3, 2, 1/6)$, $\bar{u}_i^c(\bar{3}, 1, -2/3)$, $\bar{d}_i^c(\bar{3}, 1, 1/3)$, and three families of leptons with quantum numbers $L_i(1, 2, -1/2)$ and $\bar{e}_i^c(1, 1, 1)$. Here the fermions are listed as left handed 2-component fields, i stands for the flavor/family index and f^c denote the charged conjugate states of the right handed fields f^R. Right handed neutrinos are assumed to be absent or have superheavy Majorana masses. Based on the minimal set of fields, the most general Yukawa couplings at the renormalizable level are given as:

$$L_{Yukawa} = \lambda_u^{ij} Q_i \bar{u}_j^c \phi + \lambda_d^{ij} Q_i \bar{d}_j^c \phi^* + \lambda_e^{ij} L_i e_j^c \phi^*, \tag{1.10}$$

where ϕ^* denotes the complex conjugate of the Higgs doublet field ϕ. No neutrino masses can be generated at the renormalizable level. They can only be generated either by introducing higher dimension operators into the model or by addition of heavy right handed neutrinos. Either case implies very small neutrino masses $m_\nu \approx v^2/M_R$ which evalutes to 10^{-6} GeV when M_R is taken to be the other natural scale in the SM, the Planck scale M_{Pl}. On the other hand, the SM sets no restrictions on the Yukawa coupling constants λ^{ij}. That is, all Yukawa coupling constants could be of $\mathcal{O}(1)$ and spoil the mass pattern/hierarchy shown in Eq.s (1.2) and (1.3). Furthermore, the Yukawa matrices $\lambda_{u,d,e}^{ij}$ are not necessarily proportional to the unity matrix. Thus in principle the fermion mixing effects could be maximal, and large CKM angles could occur. Generally, fermion mass eigenstates need not be the same as weak eigenstates in the SM.

The quark and charged lepton mass matrices $M_{u,d,e}$, which are obtained from the Yukawa couplings in Eq. (1.10) after the breaking of electroweak symmetry, can always be diagonalized by a bi-unitary transformation:

$$\begin{aligned} M_u^D &= V_{Ru} M_u V_{Lu}^+ \\ M_d^D &= V_{Rd} M_d V_{Ld}^+ \\ M_e^D &= V_{Re} M_e V_{Le}^+. \end{aligned} \tag{1.11}$$

Here M^D denote the diagonalized fermion mass matrices, $V_{Ru,d,e}$ denote the transformation matrices for the right handed fermions and $V_{Lu,d,e}$ denote the transformation matrices for the left handed fermions. Since there are no right handed neutrinos in the minimal SM, the charged lepton mass matrix M_e can always be diagonalized without giving any physical lepton mixing effects in weak current interactions.

As seen from Eq. (1.5), V_{CKM} is an unitary matrix due to the unitarity of V_{Lu} and V_{Ld}. Since there are only six quarks in the SM, five of their phases in V_{CKM} thus can be rotated away leaving only four independent parameters in the CKM matrix, i.e., three CKM angles and one CP violating phase. In a convenient parameterization it has the form:

$$V_{CKM} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{1.12}$$

where δ is the CP-violating phase, s_{ij} and c_{ij} denote $\sin\theta_{ij}$ and $\cos\theta_{ij}$ respectively, and θ_{ij} stands for the three mixing angles. Experimental data [2] show that there is a hierarchy among the entries of CKM matrix. This is easily seen from the Wolfenstein

[6] parameterization of the CKM matrix:

$$V_{CKM} \approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho+i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho+i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{1.13}$$

where $\lambda \approx 0.22$ and unitary to $\mathcal{O}(\lambda^4)$, which fits the data with A, ρ and the CP violation parameter η of order $\mathcal{O}(1)$. Thus a hierarchical pattern is also present in the quark mixing efffects. This is not explained by the SM.

Conclusively, the Standard Model can only be considered as an effective theory at low energy with minimal particle content. To have a better understanding of its input parameters, one needs to go beyond the Standard Model. Technically, either more symmetries such as supersymmetry (SUSY), larger gauge symmetries [7, 8] and/or discrete symmetries, or non-minimal field content would be needed in these models. In the next section, we will briefly review some ansatze for the Yukawa matrices that can explain the fermion mass pattern and hierarchy in Nature. These ansatze may be used for constructing realistic particle models in both supersymmetric and non-supersymmetric gauge theories.

1.2 Ansatze for Yukawa matrices

From the previous section, we conclude that the flavor sector of physics can only be understood by theories beyond the Standard Model. Before discussing these theories, we discuss a class of phenomenologically viable *ansatze* that assume *zero textures* (i.e., zero entries) in the Yukawa matrices without providing their origins. Typically these ansatze predict relations among the quark and charged lepton masses as well as CKM angles. These ansatze, if consistent with experiment data, could provide useful clues to the discovery of a more fundamental model. In the following, we give a brief introduction to several well-known fermion mass ansatz.

1.2.1 Fritzsch ansatz

In order to understand the patterns as well as the hierarchies in the flavor sector of the SM, some "zero textures" for Yukawa matrices have been suggested to reduce free parameters in the matrices [9, 10, 11]. For example, Fritzsch [9] suggested the following forms for Yukawa matrices at the weak scale

$$\lambda_f^{Yukawa} = \begin{pmatrix} 0 & C_f & 0 \\ C_f & 0 & B_f \\ 0 & B_f & A_f \end{pmatrix}, \qquad f = u, d, e \tag{1.14}$$

In this thesis, we define the coupling matrices with the doublet fields to the right of the matrices. As suggested by the hierarchical CKM angles, we take the coefficients A, B, and C to have the ordering $A \gg B \gg C$. The two lighter families of fermions get their masses through the quark mixing effects. As a result, the fermion masses have the following hierarchical pattern

$$m_{f3} : m_{f2} : m_{f1} = |A_f| : |\frac{B_f^2}{A_f}| : |\frac{A_f C_f^2}{B_f^2}|. \tag{1.15}$$

The Fritzsch ansatz has in all 6 complex parameters $A_{u,d}$, $B_{u,d}$ and $C_{u,d}$, in the quark sector. All but two phases in these 6 complex parameters can be rotated away by redefining the quark fields. Thus there are only 6 real numbers plus two phases to describe the 6 quark masses and 4 mixing angles in the CKM matrix. Therefore, two predictions can be made from the Fritzsch ansatz. By fitting 5 quark masses and three mixing angles, Gilman and Nir [12] found that the top quark mass should range from 77 GeV to 96 GeV. Recent CDF [4] experiment certainly rules out the Fritzsch ansatz as an acceptable model because the discovery of the top quarks at a mass $m_t \sim 174$ GeV, which is much heavier than the predicted mass. In addition to the unsuccessful prediction for the top quark mass, the Fritzsch ansatz also predicts the following relations for the quark mixing angles and the quark masses (Yukawa couplings)

$$|V_{us}| = |\sqrt{\frac{\lambda_d}{\lambda_s}} - e^{i\delta}\sqrt{\frac{\lambda_u}{\lambda_c}}|, \quad |V_{cb}| = |\sqrt{\frac{\lambda_s}{\lambda_b}} - e^{i\kappa}\sqrt{\frac{\lambda_c}{\lambda_t}}|, \quad |V_{ub}| = |V_{cb}|\sqrt{\frac{\lambda_u}{\lambda_c}}, \quad (1.16)$$

where δ denotes the CP-violating angle and κ is some unknown phase. Again, the quark mixing angle $V_{cb} \approx 0.04$ cannot be consistent with a large top quark mass in the Fritsch model.

However, there are some modified versions [13] of the Fritzsch ansatz that could be consistent with experiment data. One modification is to relax the condition of requiring symmetric mass matrices. For example, if the Yukawa matrices have the following textures:

$$\lambda_u = \begin{pmatrix} 0 & C_u & 0 \\ -C_u & 0 & B_u \\ 0 & -B_u & A_u \end{pmatrix}, \lambda_d = \begin{pmatrix} 0 & C_d & 0 \\ -C_d & 0 & 2B_d \\ 0 & B_d & A_d \end{pmatrix}. \quad (1.17)$$

then we can have V_{cb} value to be consistent with a heavy top quark by giving the new relation

$$|V_{cb}| = |\sqrt{\frac{\lambda_s}{2\lambda_b}} - e^{i\kappa}\sqrt{\frac{\lambda_c}{\lambda_t}}|. \quad (1.18)$$

In general, the modified Fritzsch models differ from the original model by making the Yukawa matrix asymmetric. Detail analysis on the modified Fritzsch ansatz can be found in the literature [13], and will be omitted in this report.

1.2.2 Georgi-Jarlskog ansatz

In addition to Fritzsch ansatz, Georgi and Jarlskog (GJ) [11] proposed an interesting ansatz for the Yukawa matrices at the GUT scale. In contrast to the Fritzsch ansatz, the GJ ansatz has different "zero-textures" for the up and down quark mass matrices. In addition, the charged lepton mass matrix is assumed to be equal to the down quark mass matrix at the GUT scale except for the 22 entry in the mass matrix. Originally, the fermion mass matrices have the following symmetric forms:

$$M_u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, M_d = \begin{pmatrix} 0 & Fe^{i\phi} & 0 \\ Fe^{-i\phi} & E & 0 \\ 0 & 0 & D \end{pmatrix}, M_e = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}. \quad (1.19)$$

Notice that typical $SO(10)$ GUT models predict symmetric Yukawa matrices by the Yukawa coupling terms $16_i 10_H 16_j$, where 16_i denote matter multiplets of the th family in the spinor representation, and 10_H denotes the Higgs field in the fundamental representation of $SO(10)$. The factor 3 in the charged lepton mass matrix can also be obtained as the Clebsch-Gordan coefficient in the $SO(10)$ models.

After diagonalizing the down quark and charged lepton mass matrices, mass relations at the GUT scale are obtained:

$$m_b = m_\tau, \quad m_\mu = 3m_s, \quad , m_e = \frac{1}{3} m_d \quad (1.20)$$

The factor 3 appearing in the 22 entry of the charged lepton mass matrix give a mass hierarchy very close to the correct one since $m_\mu / m_e \approx 10 m_s / m_d$.

In a typical supersymmetric model, the flavor sector usually contains 14 parameters including 9 fermion masses, 4 CKM angles and one Higgs VEV ratio $\tan\beta$. Since 8 free parameters including $\tan\beta$ are used in the supersymmetric version of the GJ ansatz, it leads to 6 predictions by fitting the measured fermion masses (excluding top quark mass) and CKM angles. Anderson *et al.* [14] gave a detail one-loop renormalization group (RG) analysis on the GJ ansatz and found the top quark mass to lie in the range $m_t = 179 \pm 4$ GeV. This is in remarkable agreement with recent top quark experiments by CDF [4]. CKM mixing angles at the GUT scale are predicted in the GJ model as

$$|V_{us}| = |\sqrt{\frac{\lambda_d}{\lambda_s}} - e^{i\delta}\sqrt{\frac{\lambda_u}{\lambda_c}}|, \quad |V_{cb}| = \sqrt{\frac{\lambda_c}{\lambda_t}}, \quad |V_{ub}| = \sqrt{\frac{\lambda_u}{\lambda_c}} |V_{cb}|. \quad (1.21)$$

Similarly, predictions for the CKM mixing angles at low energy regime can be made after accounting the RG running effects.

Although the GJ ansatz seems promising to be a realistic ansatz for the Yukawa matrices, however, by diagonalizing the mass matrices M_d and M_e, we find the following relation

$$\frac{m_s}{m_d}(1-\frac{m_d}{m_s})^2 = \frac{1}{9}\frac{m_\mu}{m_e}(1-\frac{m_e}{m_\mu})^2, \tag{1.22}$$

which predicts $m_s/m_d = 25.15$ which is disfavored by experiments [2, 15]. Therefore, the original Georgi-Jarlskog ansatz for Yukawa matrices may not be correct or, at least, needs to be modified.

Modification for the Georgi-Jarlskog ansatz under the GUT $SO(10)$ framework has been fully discussed and analyzed in the literatures [16, 17, 18]. Generally, the modification changes the zero 23 and 32 entries of the down quark and charged lepton mass matrices into non-zero entries. The changes can either be induced by RG evolution or simply by non-zero Yukawa coupling terms for the 23 and 32 entries. These changes, however, would mostly affect the predictions for the m_s/m_d ratio as well as the CKM angle V_{cb}, but not the fermion masses. For instance, the modified Georgi-Jarlskog models have the Yukawa matrices under the GUT $SO(10)$ framework as

$$\lambda_f = \begin{pmatrix} 0 & z'_f C & 0 \\ z_f C & y_f E^{i\phi} & x'_f B \\ 0 & x_f B & A \end{pmatrix}, \qquad f = u, d, e. \tag{1.23}$$

Notice that A, B, E, and C are all dimensionless. In order to fit experiment data, they must be hierarchical so that $A \gg B \gg E \gg C$. The phase ϕ is the only phase angle that survives after redefining the fermion fields. The flavor-dependent coefficients z_f, z'_f, y_f, x_f, and x'_f actually come from the Clebsch-Gordan coefficients for the corresponding fermion states in the GUT $SO(10)$ theory. It is possible that y_u can still be exactly zero at the GUT scale due to a zero Clebsch-Gordan coefficient.

From the Yukawa matrices in Eq. (1.23), with $y_u = 0$, the following GUT relations can be obtained:

$$|V_{us}| \approx (|\frac{z_d}{z'_d}|)^{1/2}(|\frac{\lambda_d}{\lambda_s}|)^{1/2} \tag{1.24}$$

$$|V_{cb}| \approx \chi(|\frac{\lambda_c}{\lambda_t}|)^{1/2} \qquad \text{with } \chi = \frac{|x_u - x_d|}{\sqrt{|x_u x'_u|}} \tag{1.25}$$

$$|V_{ub}| \approx s_2|V_{cb}| \qquad \text{with } s_2 = (|\frac{z_u}{z'_u}|)^{1/2}(|\frac{\lambda_u}{\lambda_c}|)^{1/2}. \tag{1.26}$$

Notice that $|V_{us}|$ and s_2 are RG invariants, while $|V_{cb}|$ is not. The favored smaller value of V_{cb} could be due to a smaller χ value, $\chi < 1$. As analyzed in [16], the parameter χ should range from $0.55 < \chi < 0.92$. In addition, the mass ratio m_s/m_d is also lowered due to the non-zero 23 and 32 entries in the down quark and charged lepton Yukawa matrices.

1.3 Tools for fermion masses and mixing: A brief introduction

In this section, we briefly review the mechanisms for implementing hierarchical Yukawa matrices and broadly classify the mechanisms into 3 classes: Radiative, High order operators, and Extra dimensions.

1.3.1 Radiative mechanism

It was argued in [19] that the masses of the light fermions, although absent in the tree-level couplings, could arise from the radiative effects from the tree-level masses of the heavy families. However, rather than provide quantitative predictions on fermion mass spectrum and mixing, these constructions gave generically qualitative pictures for the flavor physics and have the naturalness problem in generating fermion mass hierarchy. Moreover, it is usually not easy to avoid dangerous flavor changing neutral currents (FCNC) [20] in these models.

1.3.2 High order operators

One feasible way for generating hierarchies in the fermion mass matrices is to forbid the Yukawa couplings of lighter fermions at the renormalizable level. Although in supersymmetry in particular, nonrenormalization of the superpotential, allows one to drop the unwanted superpotential terms by hand at some heavy scale and protect their absence down to the SUSY breaking scale, usually, the absence of the unwanted couplings is either achieved by imposing some symmetries, global [21, 22, 23, 24] or gauged [25, 26], or by the representation content of the fields in the model [27].

For example, we could construct a supersymmetric gauge model with a non-Abelian flavor group $G_f = U(2)$ [22, 28]. The three generations of matter fields $\Psi_i = \Psi_a, \Psi$, with $a = 1, 2$, transform as **2+1** under G_f. The Higgs fields H are G_f singlets. In order to completely break the flavor group G_f, we introduce a G_f doublet field ϕ^a and an antisymmetric tensor A^{ab} with the following VEVs

$$< \phi^a >= \begin{pmatrix} 0 \\ V \end{pmatrix}, \qquad < A^{ab} >= v\epsilon^{ab}, \tag{1.27}$$

where $\epsilon^{ab} = i\sigma^2$ is the antisymmetric rank two Pauli matrix.

The most general superpotential that gives masses to fermions, and is linear in H and bilinear in the matter fields Ψ_i, is:

$$W = \lambda_1 H\Psi\Psi + \frac{\lambda_2}{M} H\Psi\phi^a\Psi_a + \frac{\lambda_3}{M} H\Psi_a A^{ab}\Psi_b + \frac{\lambda_4}{M^2} H\Psi_a\phi^a\phi^b\Psi_b. \tag{1.28}$$

where higher dimension operators are suppressed by the superheavy scale M according to dimensional analysis. Given that the superheavy scale M is much larger than the G_f breaking scales $M \gg V \gg v$, by inserting the VEVs in Eq. (1.27), the following Yukawa matrices can be obtained:

$$\lambda^f = \begin{pmatrix} 0 & D^f & 0 \\ -D^f & 0 & B^f \\ 0 & C^f & A^f \end{pmatrix}, \qquad f = u, d, e, \tag{1.29}$$

where the Yukawa couplings are hierarchical with $A^f \sim O(1)$, $B^f \sim C^f \sim O(V/M)$, and $D^f \sim O(v/M)$.

In general, nonrenormalizable operators could be induced by the heavy fermion exchanges (HFE) mechanism in the models. For example, the nonrenormalizable interaction $\frac{\lambda_2}{M}H\Psi\phi^a\Psi_a$ can arise from integrating out a aingle pair of the superheavy vector-like fields χ^a and $\bar{\chi}_a$ in a renormalizable superpotential

$$M\chi^a\bar{\chi}_a + H\Psi_a\chi^a + \phi^a\Psi\bar{\chi}_a \longrightarrow \frac{1}{M}H\Phi^a\Psi\Psi_a. \tag{1.30}$$

The induced nonrenormalizable interaction is naturally suppressed by the heavy mass M. Notice that M may be larger than the breaking scale of some symmetries, e.g., the $U(2)$ breaking scale in Eq. (1.27), but need not to be the Planck scale M_{PL}. The HFE mechanism provides a more predictive framework for the fermion mass model building than the nonrenormalizable operators scheme designed by hand. Generically, it induces the higher order operators in a rather selective way such that not all the allowed nonrenormalizable interactions could appear in the tree-level superpotential. Therefore, it provides an understanding for the absence of the unwanted higher order operators.

1.3.3 Extra dimensions mechanism

Traditional mechanisms for generating hierarchical fermion mass spectrum in a low-energy effective theory usually associate with some symmetries in a more fundamental high-energy theory. The symmetries allow the existences of some coupling terms and disallow some others. The key points in constructing the traditional models thus are to discover the symmetries and understand how they are broken [3, 7, 8, 23, 21, 25, 24].

Recently Arkani-Hamed and Schmaltz [29] suggested that fermion masses may come from the localization of the Standard Model fields in the "thick" wall in extra spacetime dimensions. The fermions can freely propagate in $3+1$ dimensions but are stuck at different locations in the extra dimensions. The Yukawa couplings could be exponentially small due to the overlap of the Gaussian wave functions of fermions. For instance, an overlap of two Gaussian functions in a 5-dimensional theory is

$$\int dx_5\phi_1(x_5)\phi_2(x_5) = \frac{\sqrt{2}\mu}{\sqrt{\pi}}\int dx_5 e^{-\mu^2x_5^2}e^{-\mu^2(x_5-r)^2} = e^{-\mu^2r^2/2}. \tag{1.31}$$

Here we use $\phi_1(x_5)$ and $\phi_2(x_5)$ to denote two wavefunctions for fermions and r stands for the distance between the centers of the two fields in the extra fifth dimension. Therefore, the hierarchy problem in Yukawa couplings turns into the "distance problem" in the extra dimension physics. The proton stability could also be understood as the relatively large separation between the associated baryon and lepton fields in the extra dimensions.

Besides the mechanism of Arkani-Hamed and Schmaltz, Cheng [30] implemented the $SU(6)$ pseudo-Goldstone mechanism in the scenario with extra dimensions and branes. By localizing two kinds of Higgs fields $\Sigma(35)$ and $H(6), \bar{H}(\bar{6})$ on the two separate branes, the Higgs doublet-triplet splitting problem is solved by the pseudo-Goldstone boson mechanism. That is, the light Higgs doublet fields are identified with the two linear combinations of the doublets in Σ, H and $\bar{H}$ after the breakdown of the gauge symmetry. The top quark, which lives in the **20** representation, is assumed to reside on the same brane in which Σ lives, so that the $\mathcal{O}(1)$ top Yukawa coupling can be obtained from a tree level superpotential term

$$\lambda_t \mathbf{20}\Sigma\mathbf{20}. \tag{1.32}$$

All other matter multiplets are assumed to live in the bulk with the heavy vector-like fields.

In such a scenario, the couplings of a bulk (external space-time dimensions) field to the brane fields are suppressed by the volume factor of the extra dimensions

$$\varepsilon \equiv (M_* R)^{-n/2}, \tag{1.33}$$

where R denotes the radius of the compactified spacetime which is assumed to be larger than the GUT scale M_G, M_* is the fundamental Planck scale scale in the $4+n$-dimensional theory and is smaller than the effective four-dimensional Planck scale M_{PL}, n is the dimensionality of the extra dimensions, and the small dimensionless factor ε stands for the suppression of the couplings. The coupling of an operator is thus suppressed by powers of ε, and by the heavy masses for the vector-like fields.

The realistic fermion mass spectrum in this scenario was generated by integrating out vector-like bulk fields, therefore is similar to the Heavy Fermion Exchanges mechanism mentioned in the previous section.

Chapter 2

General aspects of SUSY GUTs

2.1 Introduction

In the previous chapter, we have argued that the Standard Model can only be viewed as a low-energy effective theory. This is for not because of the SM has some 18 free input parameters, but also because of the incompleteness of the model. From the theoretical point of view, the SM is not a complete theory since it does not provide an explanation of the following observation:

1. Hierarchical patterns shown in Yukawa matrices.
2. The SM is a chiral theory.
3. There are three families of matter multiplets.
4. There are three gauge forces of Yang-Mills gauge groups.
5. The charge quantization of matter fields.
6. The existence of the scalar Higgs fields.

In addition, the SM contains the so-called "hierarchy problem" [31]: the the light scalar fields may receive a quadratically divergent contribution from the one-loop corrections shown in Fig.(2.1) [32, 33, 7]. That is, in the SM, the mass of the light Higgs doublet scalar field is

$$m_H^2 = (m_H^2)_0 + \frac{\alpha_2}{4\pi}\Lambda^2, \tag{2.1}$$

Figure 2.1: The mass of the scalar Higgs field receives one-loop corrections from the $SU(2)_L$ gauge sector.

where Λ denotes some large cutoff scale, and α_2 is the gauge coupling for the $SU(2)_L$ gauge group. Thus if the world we live in has at least two fundamental scales. *i.e.*. the weak scale and the Planck scale M_{PL}, then a very delicate balance (to the order of $O(10^{-16})$) between the two dimensionalful quantities of the order of the Planck scale Λ and $(m_H)_0$ must occur to get a weak scale Higgs mass m_H. This is an extreme fine tuning of the model parameters.

Based on the above observation. models beyond the SM which address this problem are seriously discussed. Typically, more symmetries such as a larger gauge symmetry and supersymmetry are imposed in these models. Supersymmetry (SUSY) has long been thought as a possible framework for constructing realistic models beyond the Standard Model. Based on the Coleman-Mandula theorem [34]. among all the graded Lie algebras only the supersymmetry algebra generates symmetries of the S-matrix consistent with relativistic quantum field theory [35].

Although some of the unexplained mysteries, such as the number of matter families, still remain unexplained in SUSY gauge models. SUSY does, eliminate the quadratic divergence in scalar masses and naturally solve the gauge hierarchy problem. It acommodates scalar fields naturally as the scalar components of the chiral superfields. Furthermore, in the minimal supersymmetric standard model (MSSM). the observed gauge couplings can be extrapolated by the RGEs to a common unified value. The successful unification of gauge couplings in the MSSM thus motivates further the exploration of the grand unified theories under SUSY framework.

In the following sections in this chapter, we briefly describe how to construct a SUSY GUT model and discuss some potential problems in the model building.

Superfield	Gauge quantum number
Quarks Q	$(3, 2, 1/6)$
Antiquarks $\bar{u}$	$(\bar{3}, 1, -2/3)$
Antiquarks $\bar{d}$	$(\bar{3}, 1, 1/3)$
Leptons L	$(1, 2, -1/2)$
Antileptons $\bar{e}$	$(1, 1, 1)$
Higgs H_u	$(1, 2, 1/2)$
Higgs H_d	$(1, 2, -1/2)$

Table 2.1: The particle content of the MSSM. Gauge superfields are not included in the Table.

2.2 Unified matter multiplets

The MSSM is the minimal SUSY extension of the non-supersymmetric Standard Model. It has the field content that includes all the SM particles and their supersymmetric partners [36], and an extra Higgs superfield required to make the MSSM an anomaly free model. The particle content added to the the gauge superfields of the MSSM is summarized in Table 2.1.

The successful gauge coupling unification in the MSSM strongly implies the existence of a grand unified gauge group at the unification scale $M_G \sim 2 \times 10^{16}$ GeV. The SM gauge group is an embedded subgroup of the GUT gauge group. Among all possible GUT groups, the $SU(5)$ group is the smallest possible simple Lie group that contains the SM group as its subgroup.

Upon embedding the SM group into the SU(5) group, it is easy to check that the fields Q, $\bar{u}$ and $\bar{e}$ can fit into the $\mathbf{10}$ of SU(5), and the fields L and $\bar{d}$ fit into the $\bar{5}$ of SU(5) as follows [37]:

$$\bar{\mathbf{5}} = \begin{pmatrix} \bar{d} \\ L \end{pmatrix}, \qquad \mathbf{10} = \begin{pmatrix} 0 & -\bar{u}_3 & \bar{u}_2 & u_1 & d_1 \\ & 0 & -\bar{u}_1 & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & \bar{e} \\ & & & & 0 \end{pmatrix} \tag{2.2}$$

However, the Higgs doublets H_u and H_d cannot be fit into any representation of the SU(5) without adding more superfields into the particle content. If we identify H_u and H_d as the $SU(2)_L$ parts of the **5** and $\bar{\mathbf{5}}$ of the SU(5) respectively, then a pair of the Higgs triplet superfields H_3 and $\bar{H}_3$ must be introduced into the particle content:

$$H = \begin{pmatrix} H_3 \\ H_u \end{pmatrix}, \qquad \bar{H} = \begin{pmatrix} \bar{H}_3 \\ H_d \end{pmatrix}. \tag{2.3}$$

The introduction of the Higgs triplet fields in the SUSY models can lead to serious problems such as the doublet-triplet splitting problem and the proton stability problem. We will come to these in later sections.

The other well-known GUT group is $SO(10)$. $SO(10)$ is the smallest group in which all the matter fields in one generation can fit into one irreducible representation. the spinorial representation **16** in $SO(10)$. This representation also includes a right-handed neutrino (**1** of $SU(5)$). A **16** includes the SU(5) multiplets as:

$$(\mathbf{1} + \bar{\mathbf{5}} + \mathbf{10})_{SU(5)} \longrightarrow \mathbf{16} \tag{2.4}$$

It is thus tempting to unify the matter fields into spinorial representations in the GUT $SO(10)$ models. The Higgs superfields, can be integrated into the fundamental representation of the $SO(10)$ group as:

$$(H + \bar{H})_{SU(5)} \longrightarrow 10_H. \tag{2.5}$$

It seems that the matter fields as well as the Higgs fields can be unified under the GUT framework as described in the above discussion. Different unifications, however, are still possible. For instance, some of the left handed down quarks may live in the **15** representation with other superheavy particles while the left handed up quarks still living in the **10** representation under the SUSY GUT $SU(5)$ framework [39]. The light Higgs doublets H_u and H_d could live within different fields in the **10** representation under the SUSY GUTs $SO(10)$. Furthermore, the light Higgs doublets need not even to be identified as parts of the fundamental representations in the SUSY GUT models.

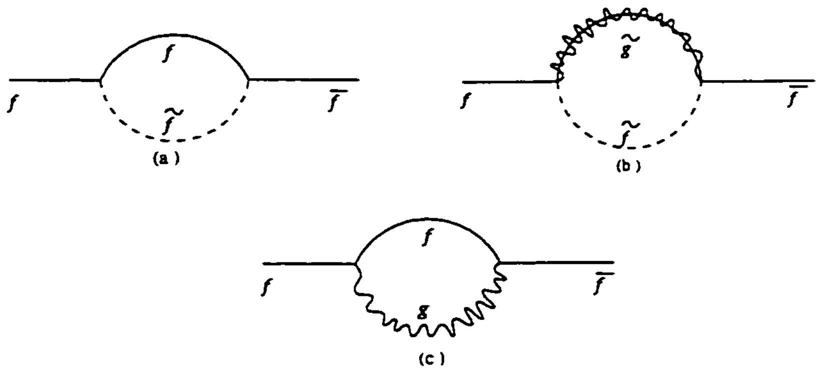

Figure 2.2: The one-loop renormalization of the fermion field.

For example, we can identify the light Higgs doublets as the pseudo-Goldstone bosons in the SUSY $SU(6)$ GUT model [27]. We will briefly describ the pseudo-Goldstone mechanism in a later section of this chapter.

2.3 One-loop RGEs for Gauge and Yukawa couplings

. In this section, we briefly describe the renormalization group equations (RGE) for the gauge and Yukawa couplings [40, 7, 41]. Typically in a quantum field theory the couplings, dimensionless or not, at high energy scales are not necessary the same as in the low energy regime. They receive loop corrections and evolve according to the RGEs. For instance, the one-loop RGEs for the gauge couplings are:

$$\frac{d\ln g_i}{dt} = \frac{b_i}{16\pi^2} g_i^2 \tag{2.6}$$

where g_i denote the gauge couplings and $t = \ln(u/M)$ is the logarithmic function of the energy scale u with cutoff scale M. The β coefficients b_i receive contributions from both the gauge sector and the other parts of the particle content in the model. In the SUSY gauge theory with gauge group G_c and chiral superfields in the representations

r_k, the one-loop β coefficients b_i are given explicitly as:

$$b_i = -3C_2(G_c) + \sum_k C(r_k), \qquad G_c : \text{adjoint representation.} \tag{2.7}$$

where $C_2(r)\mathbf{1} = (T^aT^a)_r$ is the quadratic Casimir operator and $C(r)\delta^{ab} = tr_r[T^aT^b]$. It is thus easy to see that the β coefficients b_i in the MSSM are $b_i = (33/5, 1, -3)$ for gauge groups $U(1)_Y$, $SU(2)$, and $SU(3)_C$ respectively.

Again, typical Yukawa coupling terms in the superpotential in the MSSM are:

$$W_{Yukawa} = \lambda_u^{ij} Q_i \bar{u}_i H + \lambda_d^{ij} Q_i \bar{d}_j \bar{H} + \lambda_e^{ij} L_i \bar{e}_j \bar{H}. \tag{2.8}$$

The SUSY nonrenormalization theorem guarantees that no new superpotential terms will be induced in the superpotential as long as SUSY is not broken. Therefore, the Yukawa couplings receive one-loop corrections only from the contribution of the renormalization of superfields. The one-loop RGEs for the Yukawa couplings are given in the following:

$$\begin{aligned} 16\pi^2 \frac{d\ln\lambda_u^{ab}}{dt} &= \sum_{i,j} 3|\lambda_u^{ij}|^2 + \sum_{i=1}^{3}\{|\lambda_u^{ai}|^2 + 2|\lambda_u^{ib}|^2 + |\lambda_d^{ai}|^2\} \\ &- (\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \end{aligned} \tag{2.9}$$

$$\begin{aligned} 16\pi^2 \frac{d\ln\lambda_d^{ab}}{dt} &= \sum_{i,j}\{3|\lambda_d^{ij}|^2 + |\lambda_e^{ij}|^2\} + \sum_{i=1}^{3}\{|\lambda_u^{ai}|^2 + 2|\lambda_d^{ib}|^2 + |\lambda_d^{ai}|^2\} \\ &- (\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2) \end{aligned} \tag{2.10}$$

$$16\pi^2 \frac{d\ln\lambda_e^{ab}}{dt} = \sum_{i,j}\{3|\lambda_d^{ij}|^2 + |\lambda_e^{ij}|^2\} + \sum_{i=1}^{3}\{2|\lambda_d^{ai}|^2 + |\lambda_e^{ib}|^2\} - (\frac{9}{5}g_1^2 + 3g_2^2) \tag{2.11}$$

The evolution of the Yukawa couplings depend on both the Yukawa and gauge couplings. Suppose the Yukawa coupling λ receives its dominant contribution from the gauge sector and negligible contribution from the Yukawa sector, *i.e.*,

$$16\pi^2 \frac{d\ln\lambda}{dt} \approx \sum_i k_i g_i^2, \tag{2.12}$$

where k_i denote the associated coefficients in Eq.s (2.9 $\sim$ 2.11), then we derive the following relation that relates the parameter λ at high scale (M) and the parameter λ at the low-energy scale (u)

$$\frac{\lambda(M)}{\lambda(u)} \approx \prod_i \left[\frac{g_i(M)}{g_i(u)}\right]^{\frac{k_i}{b_i}} = \prod_i \left[\frac{\alpha_i(M)}{\alpha_i(u)}\right]^{\frac{k_i}{2b_i}} . \tag{2.13}$$

At a high level of accuracy, RGEs at the two-loop level and the threshold corrections may also be important in the GUT model building [42, 43]. The discussion of the topics is beyond the scope of this thesis, and is thus omitted.

2.4 Higgs doublet-triplet splitting

The concept of SUSY grand unification provides a theoretical framework that unifies not only the gauge forces, but also the matter content and gives a derivation of the bizzare charge quantization for the matter fermions. It also provides an origin for the existence of scalar fields in the quantum field theories and, most importantly, solves the so-called "gauge hierarchy" problem for the scalar masses. On the other hand, SUSY grand unification creates new problems that must be solved. One of the new problems is the mechanism for Higgs doublet-triplet splitting [44, 45, 46, 47].

Suppose the SUSY GUT SU(5) (the smallest GUT group) model exists at some GUT scale M_G. If we identify the light Higgs doublets, which give masses to matter fermions after electroweak breaking, to reside in the $SU(2)_L$ blocks of the fundamental Higgs $\mathbf{H}$ and $\bar{\mathbf{H}}$ of SU(5), then we must explain why the Higgs triplets are much heavier than the doublets. Since these Higgs triplets can mediate proton decay, their masses must be at least 10^{16} GeV. We will come to this point again in the next section.

Several mechanisms have been invented for SUSY GUTs to split the heavy Higgs triplets from the light doublets. They are the "sliding-singlet mechanism" [46, 48], the "Dimopoulos-Wilczek mechanism" [47], the "missing partner mechanism" [49] and the "pseudo-Goldstone mechanism" [50, 27]. Among these mechanisms, only the "sliding-singlet mechanism" and the "missing partner mechanism" work for SUSY GUT $SU(5)$

models. In the remainder of this section, we will review these mechanisms.

The missing partner mechanism requires the existence of higher rank tensor Higgs fields in the representations such as $\Psi(\mathbf{50})$, $\bar{\Psi}(\overline{\mathbf{50}})$ and $\Phi(\mathbf{75})$ [49]. For example, the 75 − plet Φ breaks the $SU(5)$ symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by acquiring its VEV of the following forms:

$$\begin{aligned} <\Phi>^{ij}_{kp} &= \frac{1}{2}(\delta^i_k\delta^j_p - \delta^i_p\delta^j_k)V_\Phi \\ <\Phi>^{ab}_{cd} &= \frac{3}{2}(\delta^a_c\delta^b_d - \delta^a_d\delta^b_c)V_\Phi \\ <\Phi>^{ai}_{bj} &= \frac{-1}{2}(\delta^a_b\delta^i_j)V_\Phi, \end{aligned} \tag{2.14}$$

where $a, b, c,$ and d are the $SU(2)_L$ indices, and $i, j, k,$ and p denote the $SU(3)_C$ indices. The triplets in H and $\bar{H}$ thus get a superheavy mass via mixing to the triplets in Ψ and $\bar{\Psi}$ by the following superpotential:

$$M_\Psi \Psi\bar{\Psi} + \bar{H}\Phi\Psi + H\Phi\bar{\Psi}. \tag{2.15}$$

On the other hand, since the 50-plets do not contain doublet states, no heavy masses for the doublet states in H and $\bar{H}$ would be obtained from Eq. (2.15).

Although the missing partner mechanism does provide an elegant solution to the doublet-triplet splitting problem in the SUSY $SU(5)$ model, the requirement of the high rank tensors makes it difficult to produce the $SU(5)$ model from a simple string construction [63].

The other doublet-triplet mechanism for the SUSY GUT $SU(5)$ framework is the "sliding-singlet mechanism". In addition to a pair of the fundamental Higgs superfields H and $\bar{H}$, only one extra adjoint superfield Σ is needed to implement the mechanism. The adjoint Σ breaks the $SU(5)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ after acquiring a GUT scale VEV

$$<\Sigma>= \Sigma_0 \cdot \mathrm{diag}(2/3, 2/3, 2/3, -1, -1). \tag{2.16}$$

The superpotential that is responsible for the doublet-triplet splitting is given in the following:

$$W(H,\bar{H}) = SH\bar{H} + H\Sigma\bar{H}. \tag{2.17}$$

Then the F-term condition $F_{\bar{H}_2} = 0$ leads to

$$(< S > -\Sigma_0)\cdot < H_2 >= 0. \tag{2.18}$$

Since $< H_2 >$ gets a weak scale VEV v_2, it thus leads to $< S >= \Sigma_0$ by Eq. (2.18). From Eq. (2.17), the triplet Higgs bosons obtain large masses $5/3\Sigma_0$ and we have a pair of the light Higgs doublet fields.

The sliding mechanism for the SUSY GUT $SU(5)$ models is not flawless. If the potential of S is flat excpet for the term associated with Eq. (2.18), this mechanism breaks down when the SUSY breaking effects turn on [51, 46, 52]. We will grapple with this problem when discussing the $SU(5) \times SU(5) \times SU(5)$ model in the chapter 4.

Regarding the SUSY GUT $SO(10)$ models, there is one additional mechanism. the Dimopoulos-Wilczek (DW) mechanism [47], that implements the doublet-triplet splitting in the $SO(10)$ models. Basically, the DW mechanism needs one adjoint **45** whose VEV is in the $B - L$ direction and two Higgs fields 10_H and $10_{H'}$ in the fundamental representation. Consider the following superpotential $W(10_H, 10_{H'}, \Sigma_{45})$

$$W(10_H, 10_{H'}, \Sigma_{45}) = \lambda 10_H \Sigma_{45} 10_{H'} + M_{H'} 10_{H'} 10_{H'}. \tag{2.19}$$

with

$$< \Sigma >= v \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(1,1,1,0,0) \tag{2.20}$$

All the Higgs triplet superfields T_1, $\bar{T}_1$ and T_2, $\bar{T}_2$ residing in $\mathbf{10}_H$ and $10_{H'}$ receive heavy masses through the mixing of triplet states. To be explicit, the mass matrix for the Higgs triplets is

$$(\bar{T}_1, \bar{T}_2) \begin{pmatrix} 0 & \lambda v \\ -\lambda v & M_{H'} \end{pmatrix} (T_1, T_2). \tag{2.21}$$

On the other hand, the $B-L$ form for the VEV of Σ_{45} forces the pair of doublet states in 10_H to be light.

Notice that only one adjoint Σ_{45} acquiring a VEV along the $B-L$ direction is not enough to completely breaking the $SO(10)$ down to the Standard Model. One needs at least one pair of spinorial (**16**) Higgs fields whose VEVs preserve the SU(5)-singlet direction [53]. For instance, if a pair of spinorial Higgs fields Ψ and $\bar{\Psi}$ are introduced to the model with the associated superpotential

$$W_{16} = X(\Psi\bar{\Psi})^2/M^2 + f(X), \tag{2.22}$$

where X is the singlet superfield and $f(X)$ contains a linear term in X, then Ψ and $\bar{\Psi}$ could get VEVs along the $SU(5)$-preserving direction.

In general, complicated superpotential terms must be introduced to implement the necessary GUT group breaking and prevent light pseudo-Goldstone fields from being generated by the breaking. The pseudo-Goldstone fields, if they exist, would contribute to the RGEs and affect the unification for gauge couplings. Detailed information of how to remove pseudo-Goldstone modes in the GUT models can be found in the literature [53, 54]. We will provide a very brief discussion on the Goldstone-mode problem when describing the "One Adjoint Higgs Model" under the SUSY GUT $SO(10)$ framework in the next chapter.

The last doublet-triplet splitting mechanism we want to discuss is the GIFT (Goldstones Instead of Fine Tuning) mechanism [50, 27]. Barbieri *et al.* [27] suggested in an elegant paper that the light Higgs doublet fields may be identified as pseudo-Goldstone bosons after spontaneously breaking the global $SU(6)\times SU(6)$ symmetry in their SUSY GUT $SU(6)$ model. In the model, the Higgs sector consists of one adjoint Higgs field (Σ) in the **35** representation, and a pair of Higgs fields H and $\bar{H}$ in the fundamental representations. The superpotential of the Higgs sector respects the global $SU(6)\times SU(6)$ symmetry and has the following form:

$$W_{Higgs} = W(\Sigma) + W(H, \bar{H}). \tag{2.23}$$

By minimizing the superpotential in Eq. (2.23), the superfields Σ, H and $\bar{H}$ acquire their VEVs as below:

$$< \Sigma > \; = \; V_\Sigma \cdot \mathrm{diag}(1,1,1,1,-2,-2) \tag{2.24}$$

$$< H > \; = \; < \bar{H} > = V_H \cdot (1,0,0,0,0,0). \tag{2.25}$$

The two VEVs then break the $SU(6)$ gauge group down to $SU(4) \times SU(2) \times U(1)$ and $SU(5)$ respectively. The relative largeness of the two breaking scales V_Σ and V_H will determine the consequent $SU(6)$ breaking pattern as well as the prediction of the $\sin^2\theta_W$ value. For a successful $\sin^2\theta_W$ prediction, it must be true that $V_H > V_\Sigma$.

By counting the numbers of Goldstone modes and the broken gauge generators, it is found that two linear combinations of the doublets in Σ, H, and $\bar{H}$ do not combine with gauge bosons and become massive but rather remain light after the breaking. The light Higgs doublet fields are:

$$h_1 = \frac{V_H h_\Sigma - 3V_\Sigma h_H}{\sqrt{V_H^2 + 9V_\Sigma^2}}, \qquad h_2 = \frac{V_H \bar{h}_\Sigma - 3V_\Sigma \bar{h}_H}{\sqrt{V_H^2 + 9V_\Sigma^2}}, \tag{2.26}$$

where $h_\Sigma, \bar{h}_\Sigma$ and $h_H, \bar{h}_H$ denote the doublet fields residing in Σ and $H, \bar{H}$ respectively.

The "pseudo-Goldstone boson" mechanism solves the doublet-triplet problems in an elegant way without any fine tuning. However, in order to construct experimentally acceptable fermion mass matrices, the totally antisymmetric rank 3 tensor field, *i.e.*, the **20** representation (rank 3) must be introduced into the model [27]. Again, there is nothing wrong with introducing high rank tensors to the model, but it makes the model more difficult to arise from a simple string theory construction.

2.5 Proton stability

The GUT construction, which puts both quarks and leptons in the same multiplets [37], leads to the prediction of proton decay. GUT models, either supersymmetric or non-supersymmetric, all predict or suffer from, the decay of the proton in some way. Since the experimental data [2] show no sign of proton decay for lifetimes up to 10^{32} years, any successful GUT model must not predict proton decay faster than this experimental bound. In the following discussion we simply assume that the matter parity ($f \longrightarrow -f$), where f denote the matter multiplets, is a symmetry in the GUT models, otherwise the nucleons would decay via dimension 4 operators, giving a very short lifetime.

For a typical SUSY GUT model, there are three classes of mechanisms that lead to proton decay: proton decay through the exchanges of heavy gauge bosons, proton decay through the exchanges of the heavy Higgs triplet bosons, and proton decay through the exchanges of Higgsino triplet states. Typically, the third mechanism is dominant.

Qualitatively, it is easy to understand why the Higgsino-exchanging mechanism would dominate the proton decay processes. The proton decay processes that come from the exchanges of bosons are strongly suppressed by the GUT mass square M_G^2. On the other hand, the Higgsino-exchanging processes are less suppressed by the mass factor M_G.

Fig.(2.3a) shows a typical supergraph generating the effective dimension five operators that mediate proton decay. Notice that the effective dimension five operators must involve quark superfields of different generations. This is the consequence of gauge invariance and the Bose symmetry among the quark superfields. The proton then decays dominantly through the effect of this operator combined with an additional gaugino exchange, as shown in Fig.(2.3), with decay amplitude [38] $\mathcal{M}$:

$$\mathcal{M} \simeq \frac{\lambda}{M_T} \frac{m_{\tilde{g}} g^2}{16\pi^2 m_{\tilde{q}}^2}, \tag{2.27}$$

where λ comes from the product of the Yukawa couplings of up and down quarks. M_T is the mass of the Higgsino triplets in Fig. 2.3(a), $m_{\tilde{g}}$ is the gaugino mass, g denotes the color coupling constant, and $m_{\tilde{q}}$ stands for the squark mass. The current experimental limit requires the coupling strength λ/M_T to be smaller than 10^{-24} GeV. For instance if $\lambda = h_u h_d \sim 10^{-8}$, then the Higgsino triplets must have masses larger than the typical GUT scale 10^{16} GeV. This is the origin of the constraint introduced in section 2.4.

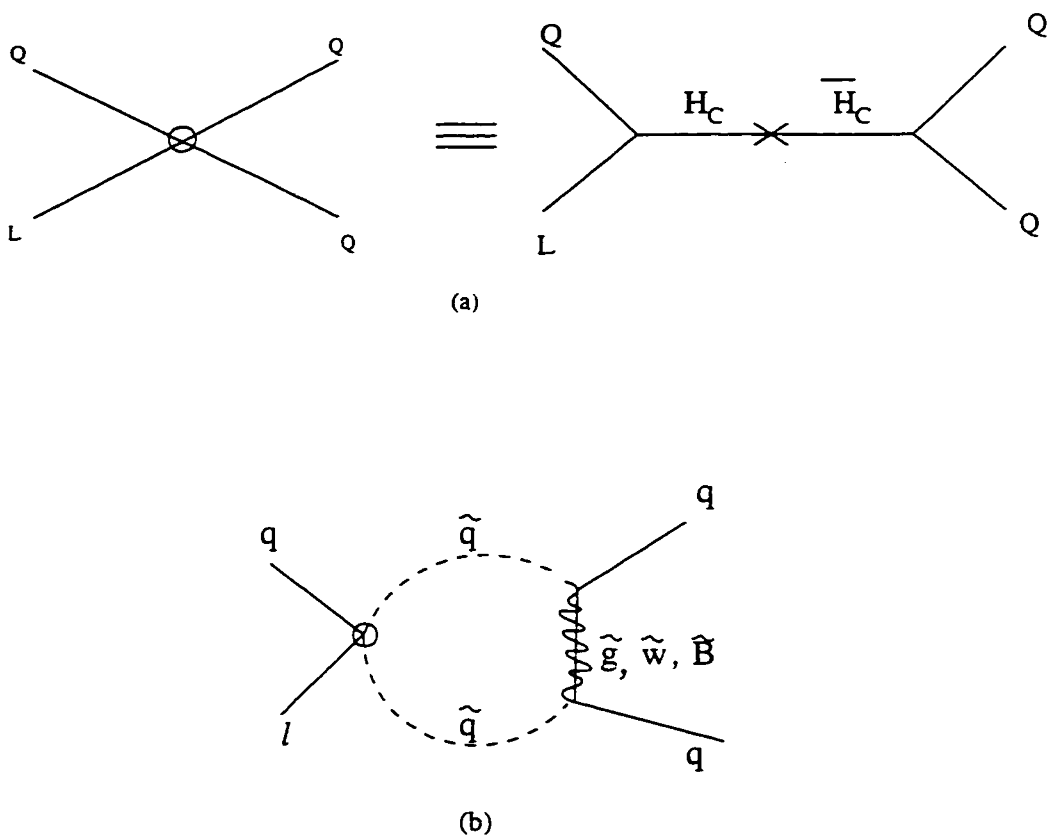

Figure 2.3: The dangerous dimension five operators that mediate Proton decay in the SUSY gauge theory. Proton then decays by exchanging the superpartners of gauge bosons as shown in the figure (*b*).

Chapter 3

Fermion masses and mixing in SUSY GUT models

Until now, we have not discussed the fermion mass and mixing problem under the SUSY grand unification framework. Generically, the concept of the matter unification at the GUT scale implies some GUT scale relations for the Yukawa couplings and the fermion masses [37]. At first glance, the idea of supersymmetry seems to have little to do with the fermion masses problem. However, as mentioned in the previous chapter, the introduction of supersymmetry to the GUT models alters the RG evolution of the Yukawa coupling constants from the GUT scale to the low-energy SUSY breaking scale. That is, starting with the same set of GUT fermion mass relations at the GUT scale, the SUSY GUT model and its non-SUSY version may draw drastically different conclusions on the size of the fermion masses in the low energy regime.

In the following sections, we briefly describe how the fermion masses and mixing problem is handled within the SUSY GUT models.

3.1 SUSY SU(5) models

In the typical GUT $SU(5)$ models, the right handed down quarks and the left handed leptons of the same family are integrated into one multiplet as shown in Eq. (2.2)[1]. In this type of unification, there is only one possible superpotential term leading a dimension-4 Yukawa coupling:

$$\Delta W = \lambda_{\bar{5}}^{ij} 10^{ij} \bar{5}_i \bar{H}_j \tag{3.1}$$

with $i, j = 1, ..., 5$. This predicts the equality of Yukawa matrices at the GUT scale:

$$\lambda_d = \lambda_e. \tag{3.2}$$

RG evolution to the weak scale changes the overall scale of quark vs. lepton masses but preserves the prediction

$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu}, \tag{3.3}$$

which is incorrect by about a factor of 10.

How can we escape this prediction? The potential relation Eq. (3.2) can be avoided by replacing Eq. (3.1) with a superpotential that includes non-renormalizable operators for the first two families of down quarks and leptons. In other words, the coupling strengths $\lambda_{\bar{5}}^{ij}$ may not be fundamental and could be come from the VEVs of some superfields.

In the following, we describe two SUSY GUT $SU(5)$ models which lead to viable fermion mass matrices. These examples, illustrate that we need to introduce higher rank tensor Higgs fields or impose some flavor symmetry that, when broken, would generate the desired patterns as well as the hierarchy in the fermion mass matrices.

[1]Some unconventional GUT SU(5) models have the left handed up and down quarks of the same family belong to different multiplets [39].

3.1.1 SUSY GUT $SU(5)$ with an extra $\overline{\mathbf{45}}$ Higgs field

The minimal GUT $SU(5)$ model has the matter fermions transforming as $(\bar{\mathbf{5}} + \mathbf{10})_i$ and the Higgs superfields transforming as $\mathbf{5} + \bar{\mathbf{5}}$ under the $SU(5)$. In addition, in order to break the $SU(5)$ group down to the SM group, it is common to introduce the adjoint superfield Σ that has the VEV as described in Eq. (2.16).

On the other hand, it is not easy to construct a realistic fermion mass matrices from the minimal particle content of the $SU(5)$ model without giving the problematic GUT mass relation $m_s/m_d = m_\mu/m_e$. One way to solve the puzzle is to invoke more Higgs superfields in representations of higher dimensionality such as the **10**, **45**, and **50** [37]. However since the **10** and **50** have no neutral color singlet component, they cannot be applied directly to the fermion sector and give masses to fermions.

The other possibility is to introduce a **45** Higgs field S. From Young tableaux, it is easy to form the **45** from the direct product of the $\overline{\mathbf{10}}$ and **5**

$$\mathbf{5} \times \overline{\mathbf{10}} = \bar{\mathbf{5}} + \mathbf{45}. \tag{3.4}$$

If S develops a VEV along the $SU(3) \times U(1)_{em}$ direction as

$$< S^b_{a5} >= V \cdot (\delta^b_a - 4\delta^b_4\delta^4_a), \qquad a, b = 1 \cdots 4. \tag{3.5}$$

then the useful GUT mass relation $m_\mu = 3m_s$ can be obtained from the following Yukawa coupling term

$$W_{Yukawa} \supset \lambda^{22}_d S 10_2 \bar{5}_2. \tag{3.6}$$

Actually, fermion mass matrices that are similar to the Georgi-Jarlskog mass matrices in Eq. (1.19) may arise from the following renormalizable superpotential

$$\begin{aligned} W_{Yukawa} &= A10_3 10_3 H + B10_2 10_3 H + C10_1 10_2 H + D10_3 \bar{5}_3 \bar{H} \\ &+ ES10_2 \bar{5}_2 + F10_2 \bar{5}_1 \bar{H} + F' 10_1 \bar{5}_2 \bar{H}, \end{aligned} \tag{3.7}$$

where A, B, C, D, E, F and F' are the coupling strengths. The symmetry decreases from $SO(10)$ to $SU(5)$ allows $F \neq F'$ but preserves the other relations in Eq. (1.19).

In general, the form of the superpotential in Eq. (3.7) could be enforced by imposing some unspecified symmetries at the GUT scale. The famous factor 3 in the Georgi-Jarlskog mass ansatz also gets a simple explanation from the VEV of the **45** Higgs field: it merely reflects the fact that there are three down quarks for every charged lepton in the model.

Although the useful GUT relation $m_\mu = 3m_s$ is obtained in the $SU(5)$ model with the **45** Higgs field, the fermion mass hierarchy is still left unexplained. One way to generate the fermion mass hierarchy under the $SU(5)$ framework is to interpret the smallness of the coupling strengths B, C, E, F and F' as coming from the non-renormalizable operators that exist above the GUT scale.

Suppose we have the $75-$plet Higgs field Φ instead of the $24-$plet Higgs field Σ in the SUSY GUT $SU(5)$. The following superpotential terms [49, 8] W_{Yukawa} for the fermion masses can arise at the GUT scale with its form enforced by some inter-family symmetry.

$$\begin{aligned} W_{Yukawa} &= 10_3 10_3 H + \frac{1}{M} 10_2 (\Phi H) 10_3 + \frac{X}{M^2} 10_1 (\Phi H) 10_2 H \\ &+ 10_3 \bar{5}_3 \bar{H} + \frac{1}{M} 10_2 (\Phi \bar{H}) \bar{5}_2 + \frac{X}{M^2} (10_1 \bar{5}_2 \bar{H} + 10_2 \bar{5}_1 \bar{H}). \end{aligned} \tag{3.8}$$

Here the singlet X is introduced to generate the hierarchy between the masses of the first and the second family fermions. Since the tensor products (ΦH) and $(\Phi \bar{H})$ will induce the fermion masses only via the $\overline{\mathbf{45}}$ and **45** channels, the Clebsch-Gordan factor -3 appears in the 22 entry of the charged lepton mass matrix. From Eq. (3.8), it is easy to check that the Georgi-Jarlskog type of fermion mass matrices are generated with the mass hierarchy parameterized by the scale ratios $< X > /M$ and $< \Phi > /M$.

In the following section, we describe a class of SUSY grand unified $SU(5)$ models [25, 26, 21, 22, 23, 24, 55] with flavor symmetries. The fermion mass hierarchy is generated simply by the hierarchical breaking of the flavor symmetry in the models.

3.1.2 SUSY GUT $SU(5)$ with flavor symmetry: an $SU(3)_H$ case

It has long been suggested that the Standard Model can be viewed to have a $SU(3)$ flavor symmetry when all the fermion masses are set to be zero. This idea provides a qualitative picture that explains the smallness of the fermion masses in the Standard model, and also motivates the suggestion for the existence of a flavor symmetry in the high energy regime. In general, the assumed flavor symmetry would prevent the presence of some unwanted couplings in the model. Some of the renormalizable tree level Yukawa couplings that give masses to fermions may also be forbidden by the flavor symmetry. Therefore, the fermion mass hierarchy may be due to the suppression of the mass operators for the first and second family of fermions. The Yukawa couplings that give small masses to light fermions thus arise from the higher order operators (non-renormalizable operators) allowed by the flavor symmetry.

There are plenty of SUSY GUTs with flavor symmetries in their models [25. 26. 21. 22. 23. 24, 55]. For instance. the $U(2)$ flavor symmetry model presented in the chapter one generates the fermion mass hierarchy through the two-step breakdown of the $U(2)$ flavor group. In this section, we describe a SUSY GUT $SU(5)$ model with the non-Abelian horizontal $SU(3)_H$ symmetry constructed by Berezhiani [56, 57].

Consider the SUSY GUT $SU(5)$ model with the flavor symmetry $SU(3)_H$. The chiral $SU(3)_H$ symmetry unifies all the fermions in the horizontal triplets $F(10,3)$ and $\bar{f}(\bar{5},3)$, where F_i and $\bar{f}_i$ denote the **10** and the $\bar{\mathbf{5}}$ representations for the *ith* family of fermions respectively. Since the terms such as $F\bar{f}\bar{H}$ and FFH transform non-trivially under the horizontal $SU(3)_H$, all renormalizable Yukawa couplings are forbidden and we need some higher order operators for the Yukawa matrices.

It was suggested in Ref. [56] that a set of the horizontal Higgs fields (gauge singlets), the sextets $\chi_n^{\{ij\}}$ and the triplet $\chi_n^{[ij]} \sim \epsilon^{ijk}\chi_k$ with $SU(3)_H$ indices i and j. are sufficient to form the effective Yukawa couplings via higher order operators:

$$\frac{(\lambda_n\chi_n^{\{ij\}} + \lambda'_n\chi_n^{[ij]})}{M}10_i10_jH, \qquad \frac{(\gamma_n\chi_n^{\{ij\}} + \gamma'_n\chi_n^{[ij]})}{M}10_i\bar{5}_j\bar{H} \tag{3.9}$$

Eq. (3.9) suggests that the fermion mass hierarchy may arise from the hierarchical breakdown of the flavor group $SU(3)_H$. If we assume only one of the sextet χ and two of the triplets η and ξ would get non-vanishing VEVs of the form:

$$\begin{aligned} <\chi^{ij}> &= A\delta_3^i\delta_3^j \\ <\eta_i> &= B\delta_i^1, \qquad <\xi_i>= B\delta_i^3. \end{aligned} \tag{3.10}$$

From Eqs (3.9) and (3.10), it is clear that the model would have the Fritzsch-type of fermion mass matrices after the flavor group breaking

$$U(3)_H \xrightarrow{\chi} U(2) \xrightarrow{\eta} U(1) \xrightarrow{\xi} I, \quad \text{with } A \gg B. \tag{3.11}$$

In this section, we have given an example of how to obtain the realistic fermion mass pattern from breaking the non-Abelian horizontal symmetry $SU(3)_H$. Notice that other flavor symmetries, either Abelian [25, 26] or non-Abelian [22, 23, 24], can also be used in building realistic fermion mass matrices under the SUSY GUT framework. Instead of describing those models in the paper, we refer the interested readers to the literatures [25, 26, 22, 23, 24].

3.2 SUSY GUT SO(10) models

In this chapter, we will describe how the fermion mass and mixing problem is treated under the SUSY GUT $SO(10)$ framework.

3.2.1 Top-bottom-tau unification

As mentioned in chapter 2, $SO(10)$ is the smallest group in which all the fermions (including right handed neutrinos) in the same family can be integrated into one representation, the **16** of the $SO(10)$ group. Based on this observation, if we also assume that only the third family of fermions could get order $\mathcal{O}(1)$ masses through the renormalizable operator $16_3 16_3 10_H$, then the following unified Yukawa couplings

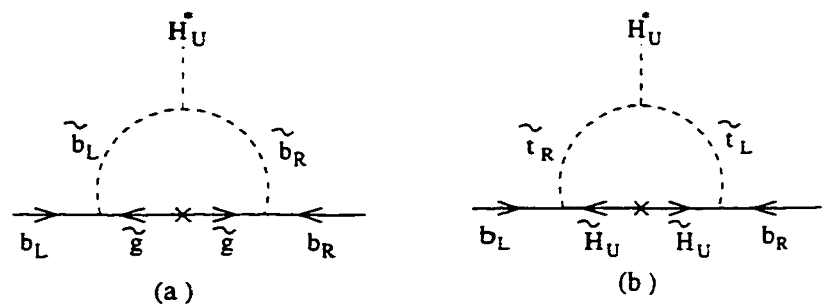

Figure 3.1: The leading one-loop MSSM corrections to the bottom quark mass.

are predicted at the GUT scale:

$$\lambda_t^G = \lambda_b^G = \lambda_\tau^G. \tag{3.12}$$

This relation, the unification of the third family Yukawa coupling constants, sets stringent constraints on the ratio of the VEVs of the two lightest Higgs doublets, the parameter $\tan\beta$. Several studies [58, 16, 14] of the $\tan\beta$ value in the SUSY $SO(10)$ model found that the acceptable $\tan\beta$ should range from $50 \sim 60$. This large $\tan\beta$ value is needed to explain the observed small ratio of the bottom quark mass to the top quark mass $m_b/m_t \sim 1/40$, through the mass relations:

$$\begin{aligned} m_t &= \frac{1}{\sqrt{2}}\lambda_t v_2, \qquad m_b = \frac{1}{\sqrt{2}}\lambda_b v_1 \\ \tan\beta &\equiv \frac{v_2}{v_1}, \end{aligned} \tag{3.13}$$

with $\lambda_t \sim \lambda_b \sim 1$ the top and bottom quark Yukawa couplings at the low scale.

On the other hand, also due to the large $\tan\beta$ value, the bottom quark mass m_b could receive large corrections when we include SUSY breaking effects. Let μ denote the SUSY coupling of the two Higgs doublets, A_t denote the trilinear soft SUSY breaking coupling of the scalar top quark (stop) to the up-type Higgs, $m_{\tilde{g}}$ be the gaugino mass, and m_0 stand for the universal sfermion mass. From Fig.(3.1), Banks [59] derived the following formulae:

$$m_b = \frac{\lambda_b}{\sqrt{2}} v_D [1 + \frac{\tan\beta}{16\pi^2} (\frac{8}{3} g_3^2 \frac{m_{\tilde{g}}\mu}{2m_0^2} + \lambda_t^2 \frac{\mu A_t}{2m_0^2})] \tag{3.14}$$

This is a finite one-loop threshold correction to m_b. It is easily seen that the radiative correction in Eq. (3.14) may be comparable to the tree level mass. When the radiative correction is large, by fitting Eq. (3.14) to the observed bottom quark mass m_b, we extract the λ_b value to be smaller than expected and predict a smaller top quark mass. On the other hand, the sfermion mass m_0 may be heavy enough to suppress the radiative corrections so that a heavier top quark is consistent with the top-bottom-tau unification.

Notice that the threshold corrections may also be induced at the GUT scale by the presence of some superheavy particles at the scale. For example, the top Yukawa coupling λ_{tG} at M_G receives one-loop threshold corrections as [58]

$$\lambda_{tG} = \lambda_G [1 + \frac{1}{4\pi} \sum_{\alpha} (\frac{\lambda_G^2}{4\pi} K_t^\alpha + \frac{g_G^2}{4\pi} L_t^\alpha + \sum_A \frac{\lambda_A^2}{4\pi} K'^{\alpha}_{tA}) \ln(\frac{M_\alpha}{M_G})]. \tag{3.15}$$

where the coefficients K_t^α come from the one-loop corrections mediated by the matter superfields in the associated superfields (*i.e.*, 10_H and 16_3) renormalizations, L_t^α come from the one-loop corrections mediated by gauge superfields, and K'^{α}_{tA} come from the non-MSSM couplings λ_A associated with some superheavy superfields.

The gauge coupling α_{iG} at the GUT scale also receives threshold corrections from the SUSY scale physics in the MSSM. In general, the threshold corrections are determined by the spectrum of the SM superpartners in the MSSM. For example, the gauge coupling α_{Gi} is corrected by the SUSY threshold effects as:

$$\alpha_{iG} = \alpha_G [1 - \frac{\alpha_G}{4\pi} (\sum_a C_{ia} \ln(\frac{M_a}{M_Z}))], \tag{3.16}$$

where the coefficients C_{ia} come from the contributions by the scalar quarks $a = \tilde{q}$, the scalar leptons $a = \tilde{\ell}$, the gauginos $a = \tilde{g}$, and the Higgsinos $a = \tilde{h}$.

In conclusion, a successful prediction of the top quark mass can be obtained under the SUSY GUT $SO(10)$ framework [58, 14, 16]. The necessary mass splitting between the observed bottom quark and the tau lepton masses is also predicted under the SUSY $SO(10)$ framework after taking into account the RG effects. On the other hand, the mass hierarchy among different fermion families also needs to be explained in the SUSY GUTs $SO(10)$, and we will come to this in the following section.

3.2.2 Mass hierarchy among different fermion families

As a first step, one might expect the Yukawa matrices in SUSY $SO(10)$ to come from the following renormalizable operators:

$$\lambda^{ij} 16_i 16_j 10_H. \tag{3.17}$$

These operators, if all present in superpotential, would give three totally symmetric and identical Yukawa matrices $\lambda_u = \lambda_d = \lambda_e$. However, as discussed in the chapter 1. we do need to see some asymmetries between the three Yukawa matrices to predict acceptable fermion masses. Furthermore, the fermion mass hierarchy is not explained by Eq. (3.17). Therefore, a better mechanism for generating the fermion mass pattern and hierarchy is required to solve the puzzles naturally.

Several mechanisms have been suggested [16, 53, 18, 60] to implement realistic fermion mass matrices under the SUSY $SO(10)$ framework. Some of them [53, 60] use only one adjoint **45** in their model building to fit in the requirement by the simple string construction [61, 62]. However, in addition to the adjoint and the fundamental Higgs fields, these usually require a complex Higgs sector including pairs of the $\mathbf{16} + \overline{\mathbf{16}}$, additional Higgs fields in the **10** representation and several singlet Higgs superfields, to generate viable fermion mass pattern.

Anderson *et.al.* [16] suggested that the Georgi-Jarlskog ansatz may be realized in the SUSY $SO(10)$ framework by the Heavy Fermion Exchanges (HFE) mechanism. In the remainder of the section, we will describe how the GJ ansatz was constructed as an outcome of their model.

Their model [16] requires the Higgs superfields to be in the **45** representation of $SO(10)$. Typically several superheavy vector-like Higgs fields in the $\mathbf{16} + \overline{\mathbf{16}}$ are also introduced to couple to the VEV-acquiring adjoint Higgs fields. The adjoint Higgs fields, may develop VEVs along the following directions:

$$\begin{aligned}
< \Sigma_X > &= v_{10} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(2,2,2,2,2) = v_{10} \cdot \mathbf{45_X} \\
< \Sigma_Y > &= v_5 \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(2/3,2/3,2/3,-1,-1) = v_5 \cdot \mathbf{45_Y} \\
< \Sigma_{B-L} > &= a v_5 \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(2/3,2/3,2/3,0,0) = a v_5 \cdot \mathbf{45_{B-L}} \\
< \Sigma_{T_{3R}} > &= b v_5 \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(0,0,0,1/2,1/2) = b v_5 \cdot \mathbf{45_{T_{3R}}}
\end{aligned} \tag{3.18}$$

Notice that the VEV $< \Sigma_X >$ breaks $SO(10)$ down to its subgroup

$$SO(10) \xrightarrow{\mathbf{45_X}} SU(5) \times U(1)_X. \tag{3.19}$$

To break the $SU(5) \times U(1)_X$ down to the SM group, we need the VEV of a $\mathbf{16} + \overline{\mathbf{16}}$ in the $\tilde{\nu}$ direction to break the $U(1)_X$ group and any one of the $< \Sigma_Y >$, $< \Sigma_{B-L} >$ and $< \Sigma_{T_{3R}} >$ to break the $SU(5)$ down to the SM group. In this section, we assume that $v_{10} \gg v_5$. We will use all of the nonzero VEVs in Eq. (3.18) with $a, b \sim \mathcal{O}(1)$.

To obtain nonzero entries in the Gerogi-Jarlskog pattern, we need at least four operators O_{33}, O_{23}, O_{22} and O_{12} to generate non-zero entries in the matrices

$$\lambda_i = \begin{bmatrix} 0 & z'_i C & 0 \\ z_i C & y_i E^{i\phi} & x'_i B \\ 0 & x_i B & A \end{bmatrix}, \tag{3.20}$$

where $i = u, d, e$. Notice that the 22 entry of the matrix λ_u must have $y_u = 0$ as required by the GJ ansatz. Detailed analysis [14] of RG evolution to the weak scale suggested that we must have the following relations for building viable Yukawa matrices consistent with experimental data.

$$\left|\frac{z_u z'_u}{z_d z'_d}\right| \approx 10^{-3} \tag{3.21}$$

$$\left[\frac{\lambda_c}{\lambda_t}\right]_G = |x_u x'_u| (\frac{B}{A})^2 \tag{3.22}$$

$$V_{cbG} = \chi\left[|\frac{\lambda_c}{\lambda_t}|\right]_G^{1/2}, \qquad \chi \equiv \frac{|x_u - x'_u|}{\sqrt{|x_u x'_u|}}. \tag{3.23}$$

Here we briefly explain how the relations (3.21 ~ 3.23) are obtained. First, we observe that the mass ratio m_u/m_d would be naively estimated from Eq. (3.20) as

$$\frac{m_u}{m_d} \sim \frac{EA\tan\beta}{B^2} \sim \frac{m_t^2 m_s}{m_b^2 m_c} \sim 200. \tag{3.24}$$

Since the true valus is about 1/2, the suppression (3.21) must be supplied. Eqs (3.22) and (3.23) are easily obtained after diagonalizing the up quark and the down quark mass matrices. To fit the experimental data, the GUT scale values $(\lambda_c/\lambda_t)_G = 0.003 \sim 0.0012$ and $V_{cb} = 0.040 \sim 0.024$ are needed. The observed values for m_t, m_c and V_{cb} thus suggest the parameter χ ranges from $0.55 < \chi < 0.92$.

As suggested by Georgi and Jarlskog [11], the 22 entries of the Yukawa matrices should have the following relation $|y_u| : |y_d| : |y_e| = 0 : 1 : 3$. Along this general analysis of experimental constraints presented in Eq.s(3.21 ~ 3.23), Anderson *et al.* constructed nine explicit models of the Yukawa matrices. Each of the nine models has different set of the Yukawa coupling operators O_{ij} to generate consistent GJ mass matrices for fermions. To illustrate the principles, we choose one model and describe how the model is constructed by exchanging heavy fermions in the model.

A set of the operators O_{ij} that give the GJ fermion matrices at the GUT scale M_G are listed below:

$$O_{33} = 16_3 16_3 10_H \tag{3.25}$$

$$O_{23} = 16_2 10_H (\frac{\Sigma_{B-L}^2}{\Sigma_X^2}) 16_3 \tag{3.26}$$

$$O_{22} = 16_2 10_H (\frac{M^* \Sigma_{B-L}}{\Sigma_X^2}) 16_2 \tag{3.27}$$

$$O_{12} = 16_1 (\frac{\Sigma_X}{M})^3 10_H (\frac{\Sigma_X}{M})^3 16_2, \tag{3.28}$$

where M^* and M denote some superheavy scales which need not be the GUT scale M_G. According to Table (3.1), different fermion states in the **16** representation may

	X	$B-L$	T_{3R}	Y
u	1	1	0	1/3
$\bar{u}$	1	-1	-1/2	-4/3
d	1	1	0	1/3
$\bar{d}$	-3	-1	-1/2	2/3
e	-3	-3	0	-1
$\bar{e}$	1	3	1/2	2
ν	-3	-3	0	-1
$\bar{\nu}$	5	3	-1/2	0

Table 3.1: Quantum numbers of the adjoint **45** VEVs on fermion states.

acquire different quantum numbers when coupling to the **45** Higgs fields. It is thus easy to check that the operators O_{ij} in Eqs (3.25~ 3.28) give the Clebsch-Gordan factors to the Yukawa matrices. For example, the operator Σ_X^3/M^3 give Clebsch-Gordan coefficients 1^3 and $(-3)^3$ to the fermion states $\bar{u}_2$ and $\bar{d}_2$ respectively. This thus leads to the relation $z_u = z'_u = -z_d/27$. Similarly, the following relations $z_d = z'_d = z_e = z'_e$, $y_u : y_d : y_e = 0 : 1 : 3$ and $\chi = 8/9$ are also obtained by the given set of the operators O_{ij}.

However, it is also important to understand the origins of these higher order mass operators O_{ij}. As discussed in Chapter 1, higher order operators can be generated by the Heavy Fermion Exchange (HFE) mechanism. The terms are naturally suppressed by the heavy masses of the heavy states. For simplicity, we only describe how one illustrative operator is formed by the HFE mechanism. Let O'_{23} be given by

$$O'_{23} \equiv 16_2(\frac{\Sigma_1}{\Sigma_2})10_H(\frac{\Sigma_4}{\Sigma_3})16_3, \tag{3.29}$$

We consider the SUSY GUT $SO(10)$ model which has two matter multiplets 16_3 and 16_2 and one fundamental Higgs field 10_H. There are also 2 pairs of the vector-like $\mathbf{16}+\overline{\mathbf{16}}$ suprefields Ψ_i and $\bar{\Psi}_i$ which acquire their superheavy masses through coupling to the VEV-acquiring Σ Higgs superfields. In the high energy regime, we assume the superpotential to have the following form:

$$W_{op} = 16_3 16_3 10_H + 16_3 \Sigma_4 \bar{\Psi}_2 + 16_2 \Sigma_1 \bar{\Psi}_1 + \bar{\Psi}_1 \Sigma_2 \Psi_1 + \Psi_1 10_H \Psi_2 + \Psi_2 \Sigma_3 \bar{\Psi}_2. \quad (3.30)$$

When the Σ superfields develop VEVs, the spinorial fields Ψ_i and $\bar{\Psi}_i$ acquire masses and the Ψ_i fields mix with the fields 16_2 and 16_3. More explicitly, the 2×4 mass matrix for the spinorial states is given by:

$$(\bar{\Psi}_1, \bar{\Psi}_2) \begin{pmatrix} <\Sigma_2> & 0 & <\Sigma_1> & 0 \\ 0 & <\Sigma_3> & 0 & <\Sigma_4> \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ 16_2 \\ 16_3 \end{pmatrix} \quad (3.31)$$

From Eq. (3.31), we easily find the mass eigenstates with nonzero heavy masses as

$$\begin{aligned} \Psi_{m1} &= \frac{<\Sigma_2> \Psi_1 + <\Sigma_1> 16_2}{\sqrt{<\Sigma_2>^2 + <\Sigma_1>^2}} \\ \Psi_{m2} &= \frac{<\Sigma_3> \Psi_2 + <\Sigma_4> 16_3}{\sqrt{<\Sigma_3>^2 + <\Sigma_4>^2}}. \end{aligned} \quad (3.32)$$

The massless states are also found to be

$$\begin{aligned} \Psi_{m3} &= \frac{- <\Sigma_1> \Psi_1 + <\Sigma_2> 16_2}{\sqrt{<\Sigma_2>^2 + <\Sigma_1>^2}} \\ \Psi_{m4} &= \frac{- <\Sigma_4> \Psi_2 + <\Sigma_3> 16_3}{\sqrt{<\Sigma_3>^2 + <\Sigma_4>^2}}. \end{aligned} \quad (3.33)$$

Therefore, we can write down the spinorial states Ψ_1, Ψ_2, 16_2 and 16_3 in terms of the mass eigenstates as

$$\begin{aligned} \Psi_1 &= \frac{<\Sigma_2> \Psi_{m1} - <\Sigma_1> \Psi_{m3}}{\sqrt{<\Sigma_2>^2 + <\Sigma_1>^2}} \\ \Psi_2 &= \frac{<\Sigma_3> \Psi_{m2} - <\Sigma_4> \Psi_{m4}}{\sqrt{<\Sigma_3>^2 + <\Sigma_4>^2}} \\ 16_2 &= \frac{<\Sigma_1> \Psi_{m1} + <\Sigma_2> \Psi_{m3}}{\sqrt{<\Sigma_2>^2 + <\Sigma_1>^2}} \end{aligned}$$

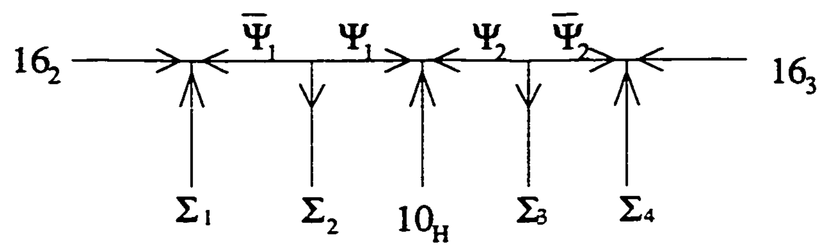

Figure 3.2: Feynman diagram for the operator O'_{23}. The operator is formed by exchanging heavy states Ψ_i and $\bar{\Psi}_i$.

$$16_3 = \frac{<\Sigma_4> \Psi_{m2} + <\Sigma_3> \Psi_{m4}}{\sqrt{<\Sigma_3>^2 + <\Sigma_4>^2}}. \tag{3.34}$$

By identifying the massless states $\Psi_{m3} \equiv 16'_2$ and $\Psi_{m4} \equiv 16'_3$ as the matter multiplets for the second and the third family in the effective theory, from the tree level term $\Psi_1 10_H \Psi_2$, we have the effective operator O'_{23}:

$$16'_2 (\frac{<\Sigma_1>}{<\Sigma_2>})(\frac{1}{\sqrt{1+\frac{<\Sigma_1>^2}{<\Sigma_2>^2}}}) 10_H (\frac{<\Sigma_4>}{<\Sigma_3>})(\frac{1}{\sqrt{1+\frac{<\Sigma_4>^2}{<\Sigma_3>^2}}}) 16'_3. \tag{3.35}$$

The above diagonaliation process can be better described by the Feynman diagram in Fig.3.2. The other higher order operators listed in Eqs (3.25 $\sim$ 3.28) can also be generated by exchanging heavy states as in Fig.3.2. However, the diagonalization processes may be laborious in the models with more matter multiplets and superheavy states.

We now turn to the discussion of the CKM angles following Ref. [16]. As seen from the Yukawa matrices as in Eq. (3.20), there is only one phase angle , the angle ϕ, after redefining the phases of the matter multiplets. The dimensionless coefficients A, B, E and C must be hierarchical to generate acceptable fermion mass pattern. We require the relations $A \gg B$ and $E \gg C$.

The diagonal forms for the Yukawa matrices can be obtained by rotating both the left-handed and the right-handed fermions. On the other hand, the CKM matrix is only concerned with the transforming matrices of those left hand fermions. Thus.

without giving the transforming matrices for the right-handed fermions, we give the approximate transforming matrices for the left-handed quarks as:

$$V_u = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\Phi_u} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{bmatrix}, \tag{3.36}$$

$$V_d = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\Phi_d} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_4 & s_4 \\ 0 & -s_4 & c_4 \end{bmatrix}, \tag{3.37}$$

where $s_3 = -x_u B/A$ and $s_4 = -x_d B/A$ denote the rotation angles in the heavy sectors, and $s_2 = z_u CA/(x_u x'_u B^2)$, $s_1 = z_d C/(y_d E)$ denote the rotation angles in the light sectors, of the up-quark and down-quark Yukawa matrices. The parameters c_i are the corresponding cosine functions for the rotation angles s_i. From Eqs (3.36) and (3.37), the CKM matrix is:

$$V_{CKM} = V_u V_d^{+} = \begin{pmatrix} c_1 c_2 - s_1 s_2 e^{-i\phi} & s_1 + c_1 s_2 e^{-i\phi} & s_2 s \\ -c_1 s_2 - s_1 e^{-i\phi} & c_1 c_2 c e^{-i\phi} - s_1 s_2 & c_2 s \\ s_1 s & -c_1 s & c e^{i\phi} \end{pmatrix}. \tag{3.38}$$

where the angle $\phi = \Phi_u - \Phi_d$; $s = \sin(\theta_3 - \theta_4)$ and $c = \cos(\theta_3 - \theta_4)$.

We now discuss the m_s/m_d value predicted by the model with the Yukawa matrices in Eq. (3.20). By diagonalizing the Yukawa matrices, several GUT scale relations can be obtained by:

$$\begin{aligned} \left|\frac{\lambda_s}{\lambda_b}\right| &= \left|\frac{E e^{i\bar{\phi}} - x_d x'_d B^2/A}{A}\right| = \frac{E}{A}\sqrt{1 - 2\delta_d \cos\bar{\phi} + \delta_d^2} \\ \left|\frac{\lambda_\mu}{\lambda_\tau}\right| &= \left|\frac{3E e^{i\bar{\phi}} - x_e x'_e B^2/A}{A}\right| = \frac{3E}{A}\sqrt{1 - 2\delta_e \cos\bar{\phi} + \delta_e^2} \\ \cos\phi &= \frac{\sin^2\theta_C - s_1^2 - s_2^2 c_1^2}{2 s_1 s_2 c_1} \end{aligned} \tag{3.39}$$

where $\sin\theta_C = |s_1 + c_1 s_2 e^{-i\phi}|$ denotes the Cabibbo angle, and δ_e and δ_d are defined as $\delta_d = x_d {x'}_d B^2/(AE)$ and $\delta_3 = x_e {x'}_e B^2/(3AE)$.

On the other hand, the model also implies the following relations

$$\phi_u \ll \phi_d, \qquad \bar{\phi} \approx \phi_d, \tag{3.40}$$

thus we obtain the m_s/m_d ratio from diagonalizing the Yukawa matrices

$$\frac{m_s}{m_d}(1-\frac{m_d}{m_s})^2 = \frac{1}{9}\frac{m_\mu}{m_e}(1-\frac{m_e}{m_\mu})^2\frac{1-2\delta_d\cos\phi+\delta_d^2}{1-2\delta_e\cos\phi+\delta_e^2}. \tag{3.41}$$

Eq. (3.41) shows that the predicted m_s/m_d value depends on the signs as well as the relative magnitudes of the difference of the parameters $\delta_d - \delta_e$ in the small $\delta_{d,e}$ limit. For instance, if $\delta_d - \delta_e = 0.3$ and $\cos\phi = 0.2$, the experimentally favored ratio $m_s/m_d = 22.45$ is obtained.

3.3 $SO(10)$ with a single adjoint Higgs field

As discussed in the previous sections in this chapter, SUSY GUT $SO(10)$ models possess several merits such as the complete quark-lepton unification for each family, promising mechanisms for explaining the fermion mass pattern and the existence of right-handed neutrinos. In order to explain the GUT group breaking as well as the pattern for quark and lepton masses, typically some higher rank tensor fields are thus introduced to the models for the purposes. However, it has been argued that string theory has difficulty in producing a single scalar field in the adjoint representation [63, 62, 64]. This makes the successful GUT $SO(10)$ model that we described in the previous section a complex framework to be constructed from a simple string construction.

Based on the string consideration, the GUT $SO(10)$ models with a single adjoint Higgs field have been suggested and studied [53]. In such a model, there are two Higgs fields in the fundamental representation. The Higgs doublet-triplet splitting mechanism is realized and stabilized by introducing two pairs of the Higgs fields in the spinor $(16+\overline{16})$ representation. We call these spinors C, $\bar{C}$ and C', $\bar{C}'$ in this paper.

The adjoint Higgs field Σ could develop a VEV along the B-L direction from the superpotential W_Σ

$$W_\Sigma = M_1 tr\Sigma^2 + \frac{1}{M_2} tr\Sigma^4. \tag{3.42}$$

Quite generally, we could split the Higgs doublets from their triplet partners by the following superpotential

$$W_{DT} = 10_H \Sigma 10'_H + M 10'_H 10'_H, \tag{3.43}$$

where M stands for the mass.

Since $< \Sigma >$ breaks the $SO(10)$ group down to $SU(3) \times U(1) \times SO(4)$ but not the SM group, to completely break the GUT group, we thus need the C and $\bar{C}$ to acquire their VEVs along the $SU(5)$-singlet direction. The superpotential that could give VEVs to $C + \bar{C}$ is:

$$\frac{Y(C\bar{C})^2}{M_c^2} + W(Y), \tag{3.44}$$

where $W(Y)$ denotes the superpotential which include at least a linear term in the singlet superfield Y.

Again, more superpotential terms are needed to remove the would-be Goldstone modes in the fields Σ, C and $\bar{C}$ after breaking the GUT group. Barr and Raby [54] suggested that the following superpotential terms $W_{C\Sigma}$ can be added to the model

$$W_{C\Sigma} = \bar{C}'(P\Sigma/M_3 + Z_1)C + \bar{C}(P\Sigma/M_4 + Z_2)C'. \tag{3.45}$$

Here P, Z_1 and Z_2 are singlet superfields. C' and $\bar{C}'$ have vanishing VEVs, thus ensure the stability of the DW form which presents in the VEV of the adjoint Higgs Σ.

Finally, the superpotential W_{HC} is added to give C' a weak scale VEV.

$$W_{HC} = \lambda \bar{C}\bar{C} 10_H \tag{3.46}$$

The value of $< C' >$ is obtained from the F-flatness condition $F_{\bar{C}} = 0$, which gives

$$2\lambda 10_H \bar{C} + (P\Sigma/M_3 + Z_2)C' = 0 \tag{3.47}$$

Since 10_H is assumed to acquire a weak scale VEV in the $SU(2)_L$ direction, therefore C' must also develop a weak scale VEV in the $SU(2)_L$ direction.

The fermion mass matrices were constructed by Albright and Barr [53] to have the following forms:

$$M_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & F \\ 0 & -F & E \end{pmatrix} \qquad M_d \approx \begin{pmatrix} 0 & 0 & G' \\ 0 & 0 & F+G \\ 0 & -F & E \end{pmatrix}$$
$$M_e \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -3F \\ G' & 3F+G & E \end{pmatrix} \tag{3.48}$$

Again, we see the Clebsch factor 3 appearing in the charged lepton mass matrix in the 23 and 32 entries. This factor actually reflects the fact that for each charged lepton there are three down quarks in the same family.

The matrices in Eq. (3.48) can be generated by the HFE mechanism. Suppose we have a tree level superpotential that is responsible for generating the Yukawa couplings after integrating out some heavy states as

$$\begin{aligned} W_{Yukawa} &= 16_3 16_3 10_H + 16\overline{16}P + 16_3\overline{16}\Sigma + \gamma_i 16_i 16\Sigma \\ &+ 10_H 10'_H \bar{C} C/M_P + c_i 16_i 10_H C + 16_3 10'_H C'. \end{aligned} \tag{3.49}$$

When P, Σ C and $\bar{C}$ all get their superheavy VEVs, some of the spinor states in Eq. (3.49) become superheavy and mix with each others. After diagonalizing the mass matrix for spinors, we thus find the massless eigenstates and identify them as the matter spinors in the low energy effective theory. Since the discussion here is quite similar to the discussion in the previous section, we skip the details of the fermion mass operators and focus on the following generated effective mass operators O_{i3},

$i = 1, 2$.

$$O_{i3} = \frac{\gamma_i}{M_G} 16_i < 10_H >< \Sigma > 16_3 \tag{3.50}$$

As seen from Eq. (3.50) and Table (3.1), $< \Sigma >$ (in the B-L direction) would give different quantum numbers on the fermion states within the matter multiplet 16_3. For instance, the quantum numbers of the fermion states $u_3, \bar{u}_3, d_3, \bar{d}_3, e_3$ and $\bar{e}_3$ have the following ratios:

$$N_u : N_{\bar{u}} : N_d : N_{\bar{d}} : N_e : N_{\bar{e}} = 1 : -1 : 1 : -1 : -3 : 3 \tag{3.51}$$

Therefore, we have the Clebsch factor 3 in the lepton Yukawa matrix.

Clearly, when imposing the requirement of a single adjoint Higgs on the particle content of the SUSY GUTs, more spinors and the fundamental Higgs fields must be introduced. and the predictive power of the conventional GUT $SO(10)$ is lost. There are two ways to avoid this kind of difficulty: First is to put the string theory considerations aside and not to take them seriously. But there is another way out: One can extend the GUT symmetry and introduce Higgs fields in product representation of the extended gauge group. These can fill the role of the adjoint Higgs fields. In the next chapter, we will adopt the second approach and construct our models.

Chapter 4

SUSY GUTs based on the gauge group $SU(5)^3$

4.1 Introduction

As discussed in the previous chapters, it is generally true that one needs to introduce some higher rank tensor Higgs fields into SUSY GUT models so that the GUT gauge group can be broken down to the Standard Model group. In addition, these higher rank tensor Higgs fields also help to realize viable fermion mass matrices in the models. For example, some $SO(10)$ GUTs have the Higgs fields in the **45** representation to generate the Georgi-Jarlskog fermion mass matrices. Similarly, some $SU(5)$ GUTs also need the Higgs fields in the **45** and **75** representations.

On the other hand, it has been known for some time that the string construction at the affine Lie algebra level 1 does not allow for scalars fields in the adjoint representation [65, 64], necessary for the breaking of GUT symmetry. Technically, higher k level construction in the string theory may help to produce higher rank tensor fields, however, the construction is complicated.

Barbieri *et al.* [66] suggested that a class of supersymmetric models based on the gauge group $G \times G$, with $G \supseteq SU(5)$, may be obtainable from simple string theory constructions with only Higgs fields in the fundamental representation. The fundamental Higgs fields then break the GUT symmetry after acquiring their VEVs in the diagonal subgroup directions of the GUT symmetry. For an illustration, we describe their $SU(5)_1 \times SU(5)_2$ model in the following.

Suppose the GUT symmetry is the product group $SU(5)_1 \times SU(5)_2$ above the some high scale M_G. The full breaking of the GUT symmetry can be implemented by a set of the Higgs fields Z_i and $\bar{Z}_i$ in the representations $(\bar{5}, 5)$ and $(5, \bar{5})$ respectively which acquire their VEVs along the following directions:

$$< Z_1 >= V_1 \cdot \mathrm{diag}(1,1,1,1,1), \qquad < \bar{Z}_1 >=< Z_1 >, \tag{4.1}$$

$$< Z_2 >= V_2 \cdot \mathrm{diag}(0,0,0,1,1), \qquad < \bar{Z}_2 >=< Z_2 >, \tag{4.2}$$

$$< Z_3 >= V_3 \cdot \mathrm{diag}(1,1,1,0,0), \qquad < \bar{Z}_3 >=< Z_3 >, \tag{4.3}$$

$$< Z_4 >= V_4 \cdot \mathrm{diag}(1,1,1,a,a), \qquad < \bar{Z}_4 >=< Z_4 >, \tag{4.4}$$

The conditions $< Z_i >=< \bar{Z}_i >$ are required to preserve a supersymmetric vacuum in the model and are thus energetically favored. In principle, these VEVs can come from stabilizing a tree level superpotential $W(Z)$ of the following form

$$\begin{aligned} W(Z) &= M_1 \sum_i (Z_i \bar{Z}_i) + \frac{1}{M_2} \sum_{i \neq j} (Z_i \bar{Z}_i)(Z_j \bar{Z}_j) + \frac{1}{M_3} \sum_j (Z_i \bar{Z}_i)^2 \\ &+ \frac{1}{M_4} \sum_{i \neq j} (Z_i \bar{Z}_i Z_j \bar{Z}_j) + \frac{1}{M_5} \sum_i (Z_i \bar{Z}_i Z_i \bar{Z}_i). \end{aligned} \tag{4.5}$$

As usual, this form of the superpotential could be enforced by some symmetry. For example, a set of the discrete symmeties that have the Z fields transforming as $Z_i \longrightarrow -Z_i$ and $\bar{Z}_i \longrightarrow -\bar{Z}_i$ would do the job. Typically, by solving the F-flatness conditions:

$$\frac{\partial W}{\partial Z_i} = \frac{\partial W}{\partial \bar{Z}_i} = 0, \tag{4.6}$$

many SUSY vacua may be obtained. The SUSY vacuum given in Eq. (4.1 $\sim$ 4.4) is just one of them.

Therefore, each of the $< Z >$ VEVs leads to the breaking of the GUT symmetry with one of the following patterns:

$$SU(5) \tag{4.7}$$

$$SU(3)_1 \times SU(3)_2 \times SU(2) \times U(1) \tag{4.8}$$

$$SU(3) \times U(1) \times SU(2)_1 \times SU(2)_2 \tag{4.9}$$

$$SU(3) \times SU(2) \times U(1) \quad . \tag{4.10}$$

Here the diagonal subgroups of the $SU(5)_1 \times SU(5)_2$ are represented by the factors without subscripts. Notice that the $SU(5)$ GUT group can be fully broken either by a single $< Z_4 >$ or by the $< Z_2 >$ and $< Z_3 >$ together.

The Higgs doublet-triplet splitting problem could receive a natural solution in the $SU(5)_1 \times SU(5)_2$ models. Consider that two pairs of the Higgs fields $H(5,1)+\bar{H}(\bar{5},1)$ and $H'(1,5)+\bar{H}'(1,\bar{5})$ exist in the model. The Z VEVs in Eqs. (4.1 $\sim$ 4.4) suggest that all the Higgs states in the H, $\bar{H}$, H' and $\bar{H}'$, except for one pair of the doublets , could be made heavy by the following superpotential W_{DT}

$$W_{DT} = HZ_3\bar{H}' + \bar{H}\bar{Z}_3H' + \bar{H}Z_1H'. \tag{4.11}$$

From Eq. (4.11), the light Higgs doublets reside in H and $\bar{H}'$. Needless to say, some discrete symmetries must typically be introduced to restrict the couplings between the Z fields and the H fields to enforce the form of the superpotential W_{DT}. For example, when we include possible nonrenormalizable terms in the superpotential, the identified light Higgs doublets would no longer be light enough if the following high order operators

$$(HZ^n\bar{H}')Tr(Z^{m+1}), \qquad n, m \text{ are odd integer.} \tag{4.12}$$

are not sufficiently suppressed.

Generally, the fermion mass spectrum is quite model dependent under the GUT $SU(5) \times SU(5)$ framework since we are free to assign the matter multiplets to transform under either the gauge group $SU(5)_2$ or the $SU(5)_1$. This feature can be used to our advantage. For instance, if all the matter multiplets transform as singlets under the gauge group $SU(5)_2$, then the down quark masses are suppressed by the heavy mass M and a hierarchy between the up and down quark masses is generated

$$\lambda^{ij} 10_i 10_j H + \frac{\gamma^{ij}}{M} \bar{5}_i 10_j \bar{Z}_2 \bar{H}'. \tag{4.13}$$

Notice that Eq. (4.13) is not sufficient to explain the mass hierarchy among different matter families and the asymmetries among the Yukawa matrices. These may be explained by introducing higher rank Higgs fields into the model, but that violates the motivation for this construction. It is possible to generate the required hierarchies in an $SU(5) \times SU(5)$ model by using a more complicated set of fundamental Higgs fields. We will come to this discussion in the later of this section.

For the $G = SO(10)$ case, in addition to the GUT-breaking Higgs fields in the fundamental $(10, 10)$ representation, we also need at least one pair of vector-like Higgs fields in the spinor representation to fully break the GUT symmetry down to the SM group. Barbieri *et al.* constructed an $SO(10) \times SO(10)$ model in a manner similar to their $SU(5) \times SU(5)$ model. We thus omit the discussion of their $SO(10) \times SO(10)$ model in this section.[1]

Following their previous work on the $G \times G$ GUT model, Barbieri and his collaborators then generalized the GUT symmetry to the product gauge group $G \times G \times G$. Similar to their $SU(5) \times SU(5)$ model, the $SU(5) \times SU(5) \times SU(5)$ model also possesses a set of the VEV-acquiring Higgs fields *i.e.*, the Z fields, to break down the GUT symmetry. In their paper, the Z fields carry the gauge quantum numbers in a permutative way. That is, there are three kinds of the Z fields, the Z_{12} transforms as $(\bar{5}, 5, 1)$, the Z_{23} transform as $(1, \bar{5}, 5)$ and the Z_{13} transforms as $(\bar{5}, 1, 5)$ under the GUT symmetry. The fields $\bar{Z}_{ij}$ are assumed to be the conjugate fields of the corresponding Z_{ij} fields. These Z_{ij} and $\bar{Z}_{ij}$ Higgs fields then develop their VEVs and break the GUT symmetry.

On the other hand, each of the matter multiplets is assigned to transform under

[1]We argue in the last chapter that the $SO(10) \times SO(10)$ case can be quite different from the $SU(5)^2$ model. This is due to the fact that the $(10, 10)$ Higgs fields can acquire their VEVs with the symmetric or the antisymmetric gauge indices.

different gauge group $SU(5)_i$. For example, the first family of matter $f_1(10 \oplus \bar{5})_1$ transforms under the gauge $SU(5)_1$, f_2 transforms under the $SU(5)_2$, and so on. To explore possible fermion mass patterns in the model, the knowledge of how the Higgs fields couple to the matter multiplets is very important. In general, we can have any numbers of the Higgs fields that couple to matter multiplets as long as the gauge anomaly is canceled. However, in their $SU(5)^3$ model, 3 pairs of the Higgs fields $H_i + \bar{H}_i$ are given and each pair also transform under different gauge group $SU(5)_i$. Among these H and $\bar{H}$ fields, only the doublets in H_3 and $\bar{H}_3$ would remain light by giving the following coupling terms in the superpotential

$$W(H,Z) = (\bar{H}_1, \bar{H}_2, \bar{H}_3) \begin{pmatrix} 0 & \bar{Z}_{12} & \bar{Z}_{13} \\ Z_{12} & 0 & \bar{Z}_{23} \\ Z_{13} & Z_{23} & 0 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}. \qquad (4.14)$$

As easily seen from Eq. (4.14), the pair of the doublets in H_3 and $\bar{H}_3$ would remain light only if the Z_{12} and $\bar{Z}_{12}$ fields have non-vanishing VEVs in the two lower entries.

The above field content of their model immediately leads to several problems. First is the lack of sufficient asymmetry between the up and down quark mass matrices. The off-diagonal entries of the Yukawa coupling matrices are always more suppressed than the corresponding diagonal entries. For instance, the 22 and the 23 entries for the up and down quark Yukawa matrices are of the following forms:

$$\lambda_{22}^{u,d} = 10_2 10_2 \bar{Z}_{23} H_3, \quad 10_2 \bar{5}_2 Z_{23} \bar{H}_3, \qquad (4.15)$$

$$\lambda_{23}^{u,d} = 10_2 10_3 Z_{23}^2 H_3, \quad 10_2 \bar{5}_3 Z_{23}^2 \bar{H}_3. \qquad (4.16)$$

This is embarrassing since it predicts that the up and down quark masses would have the same mass hierarchy among different fermion families. That is, it predicts the following unacceptable mass relations $m_t/m_b \approx m_c/m_s \approx m_u/m_d$.

Second, the proton would decay too quickly since the heavy Higgs triplet has a large coupling to the first generation of matter.

In order to rescue their $SU(5) \times SU(5) \times SU(5)$ GUT model, Barbieri *et al.* suggested the following solution: They observed that realistic fermion mass patterns and proton stability can be obtained if one introduces two replicas of the Z fields and four replicas of the H fields into the model. They call them Z, Z', Y, Y' and H, H' in the model. Among those Z, Z', Y and Y' fields, only Y and Y' will couple to the up and down quarks respectively. As usual, some discrete symmetries are required to prevent the Y' fields from coupling to the matter fields 10_i, and to prevent the Y from coupling to the $\bar{5}_i$. The Z and Z' fields only couple to the H and H', and are responsible for splitting the Higgs doublets from the triplets. As a result, the necessary asymmetry between the up and down quark mass matrices can be obtained.

Clearly, this viable $SU(5) \times SU(5) \times SU(5)$ model has a very complicated Higgs sector and provides no better understanding on the fermion mass problem than the conventional $SU(5)$ models. When $G = SO(10)$, their $SO(10) \times SO(10) \times SO(10)$ model suffers the same weakness. Even worse, the special $SO(10)$ relations such as top-bottom-tau Yukawa unification are lost and the $SO(10) \times SO(10) \times SO(10)$ model looks more like an effective $SU(5) \times SU(5) \times SU(5)$ model.

In the remainder of this thesis, we present our constructions of the SUSY GUTs based on the $G \times G$ and $G \times G \times G$ GUT symmetries. We show that viable fermion mass patterns can be obtained with a simple Higgs sector. In the $G = SU(5)$ case [67], the needed asymmetry between the up and down quark Yukawa couplings is due to the asymmetric structure of the Higgs sector in the field content. In the $G = SO(10)$ cases [68], we show that the effective adjoint fields can arise from combining two of the GUT symmetry-breaking Higgs superfields. This is interesting since traditional GUT $SO(10)$ theories of high predictive power in the Yukawa matrices can now be viewed as the effective theories of our $SO(10) \times SO(10)$ and $SO(10) \times SO(10) \times SO(10)$ models. We also give examples showing how to obtain Georgi-Jarlskog mass matrices through the Heavy Fermion Exchange mechanism in the model construction.

4.2 An explicit $SU(5) \times SU(5) \times SU(5)$ GUT model

The Standard Model (SM) is now considered to be completely successful in describing the physical world up to the weak scale. However, it requires some 18 parameters which are input by hand to fit the experiment data. Most of these undetermined parameters reflect our lack of understanding of flavor physics. The SM provides no explanation of why there is a mass hierarchy among the fermion masses and no explanation of the CKM angles. It seems that Nature includes some classification which goes beyond the structure of the SM [69]. Thus, in order to solve these puzzles. we have to go beyond the SM. The Minimal Supersymmetric Standard Model (MSSM) [70] has considered as one of the possible extended theories beyond the SM. Despite of its success in providing true gauge coupling unification [71], it also has flavor problems [72] at least as severe as those in the SM. The fermion mass hierarchy is still left unexplained in the MSSM framework. Even worse, new problems such as Flavor Changing Neutral Currents [20] occur.

Many solutions have been proposed for the flavor problem, either within a supersymmetric framework [73, 74] or in non-supersymmetric theories. Most of these attempts assume that some flavor symmetries, gauged or global, exist above the grand unification scale M_x. The flavor symmetries typically restrict the possible Yukawa coupling terms in the superpotential and provide textures and hierarchy patterns of the fermion mass matrices [75, 76, 74]. This idea is often combined with that of grand unification. For example, one can introduce higher rank tensors such as the 126 in the $SO(10)$ grand unified theories (GUT's) [77] and the 45 in the $SU(5)$ GUT's [78] in order to create specific textures in the quark and lepton mass matrices. Theoretically, there is nothing wrong with introducing high rank tensors. However, the affine level 1 constructions in string theory do not allow string-derived GUT's having tensor fields with rank higher than 2 [63]. This result makes the ordinary SUSY $SU(5)$ [79] and $SO(10)$ [80] GUT theories difficult to obtain from the affine level 1 constructions in string theory. In response to this situation, Barbieri et. al. [66] pointed out that extending the GUT gauge group to be $G \times G$, where G could be some GUT groups such as the $SU(5)$ or the $SO(10)$ group, makes it possible to construct GUT models which break the product gauge groups down to the SM gauge group without introducing high rank tensor fields. The GUT gauge group in these theories could be broken by fields which carry fundamental and antifundamental representations under two different gauge groups. For examples, the $(5,\bar{5})$ and the $(\bar{5},5)$ can break the $SU(5) \times SU(5)$ GUT theories down to the SM gauge group. The same logic applies to theories with gauge group $G \times G \times G$. Furthermore, as pointed out by Barbieri et. al [81], if we choose to have each family of matter fields transforming under different gauge group G, then a flavour theory could be constructed without the need for an explicit flavour symmetry group.

In this paper, we follow the idea of using $SU(5) \times SU(5) \times SU(5)$ as the SUSY GUT gauge group. However, instead of symmetrically assigning each family of matter fields (10+$\bar{5}$) to its own gauge group $SU(5)$, we assign the matter fields transforming under these gauge $SU(5)$'s in an asymmetrical way. In Section 4.2.1, we describe our model and suggest a suitable vacuum for those fields which break the GUT gauge group. A $Z_2 \times Z_3$ discrete symmetry at the superheavy scale is introduced to suppress the dangerous operators as well as to obtain a weak-scale μ value in the model. This gives the full set of assumptions of our construction. In the remainder of the paper, we show that these assumptions lead to many interesting consequences. In Section 4.2.2, we derive the fermion mass matrices and demonstrate a mass hierarchy which follows from the gauge structure of our model. We show that our model can account for the observed fermion mass matrices and CKM angles. In Section 4.2.3, we discuss proton decay in this model. The proton lifetime predicted in this model is consistent with the limit set by the SuperKamiokande experiment. In Section 4.2.4, we present some conclusions.

4.2.1 The model

Our model is based on the SUSY GUT gauge group $SU(5)_1 \times SU(5)_2 \times SU(5)_3$. We identify the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as lying in a diagonal $SU(5)$ subgroup of above product group. To break the GUT gauge group down to the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$, we require the exotic Higgs fields T_1, T_2 and T_3 in the representations $(1, 5, \bar{5})$, $(\bar{5}, 1, 5)$ and $(5, \bar{5}, 1)$. We will find it useful to add two more multiplets, Σ in the $(1, 5, \bar{5})$ and $\bar{\Sigma}$ in the $(1, \bar{5}, 5)$. We assign the three 10's of $SU(5)$ to the three different $SU(5)$ groups and we associate the 5 and $\bar{5}$ Higgs fields with different groups. However, we assign two $\bar{5}$ matter multiplets to the same $SU(5)$. The complete set of assignment is shown in Table 4.1. According to the assignment

	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Σ		5	$\bar{5}$
$\bar{\Sigma}$		$\bar{5}$	5
T_1		5	$\bar{5}$
T_2	$\bar{5}$		5
T_3	5	$\bar{5}$	
10_3			10
10_2		10	
10_1	10		
$\bar{5}_3$			$\bar{5}$
$\bar{5}_2$	$\bar{5}$		
$\bar{5}_1$			$\bar{5}$
H			5
$\bar{H}$		$\bar{5}$	

Table 4.1: The field content of the $SU(5)_1 \times SU(5)_2 \times SU(5)_3$ model.

in Table 4.1, there is already some interesting physics at the level of lower dimension operators. The ordinary μ term

$$\mu H \bar{H} \tag{4.17}$$

is forbidden from appearing in the fundamental Lagrangian by gauge invariance. The leading contribution to the μ term potentially comes from high dimension operators in the superpotential and will be analyzed further in the later of this section.

As one can see from the table, this model contains no fields in the adjoint representation, and no fields with rank higher than 2. All of these fields can appear in a string construction with the gauge group realized at the affine level $k = 1$ [63]. The breaking of the GUT gauge group can be accomplished by the vacuum expectation values (VEV's) of the fields T_1, T_2, T_3, Σ and $\bar{\Sigma}$. The symmetry-breaking ground state could be either by a stabilized tree-level superpotential or by effects of a strongly coupled SUSY gauge theory. Here, before discussing an explicit potential, we would like to propose a possible vacuum which can break the gauge group $SU(5)_1 \times SU(5)_2 \times SU(5)_3$ down to the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

We assume that the symmetry is broken in two steps. First, $SU(5)_1 \times SU(5)_3$ is broken to the diagonal subgroups by an expectation value of T_2.

$$\langle T_2 \rangle = \Lambda_2 \cdot \text{diag}(1,1,1,1,1) \tag{4.18}$$

Then the remaining symmetry $SU(5)_{D31} \times SU(5)_2$ is broken to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ by the expectation values of T_1 and Σ.

$$\langle T_1 \rangle = \Lambda_1 \cdot \text{diag}(0,0,0,1,1) \tag{4.19}$$

$$\langle \Sigma \rangle = \Omega \cdot \text{diag}(1,1,1,0,0) \tag{4.20}$$

Finally, the remaining fields get their expectation values along the $SU(3)_C \times SU(2)_L \times U(1)_Y$ direction. Only relatively small hierarchies between these scales are needed to produce large hierarchies in the quark mass matrices. We will show this in Section 4.2.2. The complete pattern of VEV's consistent with the symmetry breaking pattern just described is:

$$\begin{aligned}
\langle \Sigma \rangle &= \Omega \cdot \text{diag}(1,1,1,0,0) \qquad \langle \bar{\Sigma} \rangle = \bar{\Omega} \cdot \text{diag}(1,1,1,a,a) \\
\langle T_1 \rangle &= \Lambda_1 \cdot \text{diag}(0,0,0,1,1) \qquad \langle T_3 \rangle = \Lambda_3 \cdot \text{diag}(1,1,1,s,s) \\
\langle T_2 \rangle &= \Lambda_2 \cdot \text{diag}(1,1,1,1,1) + \Lambda_3 \cdot \text{diag}(0,0,0,b,b)
\end{aligned} \tag{4.21}$$

Due to the $SU(5)_1$ D-term condition, the VEV $\langle T_2 \rangle$ will receive a correction of order $O(\Lambda_3)$. The constants a, b and s are assumed to be nonzero and would be determined by minimizing the potential. We will show below that the zeros in Σ and T_1 can be exact, up to the point where SUSY is spontaneously broken. As in conventional GUT models, we also require a discrete symmetry to forbid dangerous operators such as $H\bar{5}_1$, $H\bar{5}_3$, $10_3\bar{5}_3\bar{5}_1$ and $T_1T_3H\bar{5}_2$ in the tree-level superpotential. Specifically, we assume a $Z_2^{matter} \times Z_3$ symmetry

$$
\begin{aligned}
Z_2^{matter} : (10_1, 10_2, 10_3, \bar{5}_1, \bar{5}_2, \bar{5}_3) &\longrightarrow -1(10_1, 10_2, 10_3, \bar{5}_1, \bar{5}_2, \bar{5}_3) \\
Z_3 : (H, \bar{H}, \Sigma, \bar{\Sigma}, 10_3) &\longrightarrow (H, \bar{H}, \Sigma, \bar{\Sigma}, 10_3) \\
(T_1, T_3, 10_2, \bar{5}_2) &\longrightarrow e^{i2\pi/3}(T_1, T_3, 10_2, \bar{5}_2) \\
(T_2, 10_1, \bar{5}_1, \bar{5}_3) &\longrightarrow e^{i4\pi/3}(T_2, 10_1, \bar{5}_1, \bar{5}_3)
\end{aligned} \tag{4.22}
$$

The dangerous dimension five operators that could make the proton decay too rapidly are also suppressed by the Z_3 symmetry. We will discuss this in Section 4.2.3. We now discuss the spectra of Higgs masses and the μ parameter. Applying the $Z_2^{matter} \times Z_3$ symmetry, we can easily write all possible leading terms up to dimension 10 level that are bilinear in H and $\bar{H}$.

$$
\begin{aligned}
W_{H\bar{H}} &= \Sigma H\bar{H}\{1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{\Sigma T_2 T_3}{M^3} + \frac{(\Sigma\bar{\Sigma})^2}{M^4} + \frac{\Sigma^5 + \Sigma^2 T_1^3}{M^5} + \frac{(\Sigma\bar{\Sigma})(\Sigma T_2 T_3)}{M^5} \\
&+ \frac{(\Sigma\bar{\Sigma})^3 + (T_1\bar{\Sigma})^3 + (\Sigma T_2 T_3)^2}{M^6} + \sum_{k=0}^{5} \frac{1}{M^{5-k}} \bar{\Sigma}^k (T_2 T_3)^{5-k}\} \\
&+ T_1 H\bar{H}\{\frac{(T_1\bar{\Sigma})^2}{M^4} + \frac{\Sigma^3 T_1^2 + T_1^5 + T_3^5 + (T_1\bar{\Sigma})(T_1 T_2 T_3)}{M^5} \\
&+ \frac{(T_1 T_2 T_3)^2}{M^6}\}
\end{aligned} \tag{4.23}
$$

From the vacuum state described in Eq. (4.21), not all terms in Eq. (4.23) would have non-zero contributions to the Higgs triplet mass and the μ value. The Higgs triplets get a superheavy mass Ω which is shown to be of $O(10^{16})$ GeV in the next section. The leading terms that give μ a nonzero value are

$$
\mu \approx \langle T_1[\frac{(T_1\bar{\Sigma})^2}{M^4} + \frac{\Sigma^3 T_1^2 + T_3^5 + (T_1\bar{\Sigma})(T_1 T_2 T_3)}{M^5} + \frac{(T_1 T_2 T_3)^2}{M^6}]\rangle. \tag{4.24}
$$

Eq. (4.24) is highly suppressed by $1/M^4$. When we estimate the various parameters in the next section, we will see that μ obtains a weak-scale μ value.

It is important to ask whether the pattern of VEV's that we have considered in the Eq. (4.21) can follow from a tree-level superpotential. There is an example of a superpotential that can lead to this structure which incorporates the constraints of $Z_2^{matter} \times Z_3$ symmetry.

$$
\begin{aligned}
W(\Sigma,\bar{\Sigma},T_1,T_2,T_3) &= \frac{1}{M^3}Y_1(\Sigma^3 T_1^2 - \phi_1^5) + \frac{1}{M^3}Y_2(T_2^5 - \phi_2^5) + \frac{1}{M^3}Y_3\Sigma^4 T_1 \\
&+ \frac{1}{M^3}Y_4(\Sigma^2 T_1^3) + \frac{1}{M}Y_5(\phi_1^3 - \Lambda^3) + \frac{1}{M}Y_6(\phi_2^3 - \Lambda_2^3) \\
&+ \sum_{i=1}^{6} X_i Y_i^2 + Y_7\Sigma\bar{\Sigma} + \frac{1}{M}Y_8\Sigma T_2 T_3 + MX_7Y_7 + MX_8Y_8 \\
&+ \frac{A_1}{M}(\Sigma\bar{\Sigma})^2 + \frac{B_1}{M^2}(\Sigma\bar{\Sigma})(\Sigma T_2 T_3) + \frac{B_2}{M^3}(\Sigma T_2 T_3)^2 \\
&+ \sum_{i,j\geq 0}^{3} C_{ij}\frac{(T_1\bar{\Sigma})^i (T_1 T_2 T_3)^j (T_2^5)^{3-i-j}}{M^{12-3i-2j}}
\end{aligned}
\tag{4.25}
$$

Here A_i, B_i and C_{ij} are understood as the unspecified coefficients and M is the super-high scale. The gauge singlets ϕ_i, Y_i and X_i are needed to produce the following constraints

$$
\langle \Sigma^3 T_1^2 \rangle = \Lambda^5, \qquad \langle T_2^5 \rangle = \Lambda_2^5 \tag{4.26}
$$

$$
\langle \Sigma^4 T_1 \rangle = 0, \qquad \langle \Sigma^2 T_1^3 \rangle = 0. \tag{4.27}
$$

These lead to the zero texture patterns in the VEV's of Σ and T_1. The F-term conditions from the superpotential (4.25) as well as the D-term conditions of the GUT gauge groups would determine the possible vacua of this model. The $SU(5)_3$ D-term as well as the $SU(5)_2$ D-term conditions could force the scales Λ_1 and Ω to have approximately equal value $\Lambda_1 \approx \Omega$ if Λ_1 is much larger than Λ_3 and $\bar{\Omega}$. Typically, solving for the minima of a potential would give rise to many discretely degenerate vacua. This is generic to most SUSY GUT theories [81, 66, 82] if a tree-level superpotential is responsible for breaking the GUT gauge group.

The above Higgs triplet-doublet splitting mechanism is similar to the sliding-singlet mechanism [46]. The Higgs triplets and doublets split when the field Σ get superheavy VEV's in its $SU(3)$ block, while keeping vanishing VEV's in its $SU(2)$ block. This description applies to the theory before supersymmetry breaking. It is a well-known difficulty of the sliding-singlet mechanism that SUSY breaking effects could bring corrections to the VEV of Σ and may destroy the gauge hierarchy [51]. We will now argue that this is not a problem in our model.

To be explicit, the problem resides in [51] is that the low energy effective singlet field Σ_s that comes from the field Σ couples to the superheavy heavy triplets in H and $\bar{H}$. If we turn on SUSY breaking effects, this would give rise to one-loop tadpole graphs which induce the following two terms in the low energy effective theory.

$$c_1 m_g^2 M_G \Sigma_s + h.c. \tag{4.28}$$

$$c_2 m_g M_G F_{\Sigma_s} + h.c. \tag{4.29}$$

Here m_g represents the gaugino mass and M_G represents the GUT mass scale. These terms shift the VEV's of Σ and T_1. Adding Eq. (4.28) to the effective theory, the piece of the potential that could shift the VEV's of Σ in its $SU(2)$ block is given by

$$\begin{aligned} V &= (|\langle H\rangle|^2 + |\langle \bar{H}\rangle|^2)|\langle\Sigma\rangle|^2 + |\langle \frac{1}{M^3}\Sigma^4 T_1\rangle|^2 + |\langle \frac{1}{M^3}(\Sigma^3 T_1^2 - X^5)\rangle|^2 \\ &+ |\langle \frac{\Sigma^2 T_1^3}{M^3}\rangle|^2 + |\langle \Sigma\bar{\Sigma} + MX_7\rangle|^2 + |\langle \frac{1}{M}\Sigma T_2 T_3 + MX_8\rangle|^2 + c_1 m_g^2 M_G \Sigma_s \\ &+ \cdots . \end{aligned} \tag{4.30}$$

Inserting $\Sigma \longrightarrow \Sigma + \Delta\Sigma$ into Eq. (4.30), we find the possible shift of the $SU(2)$ VEV's put

$$\Delta\Sigma_2 \lesssim \frac{c_1 m_g^2 M_G}{2|\frac{4}{M^3}\Sigma^3 T_1|^2} \sim O(10^2)\ \text{GeV} \tag{4.31}$$

For the same reason, the VEV's in the $SU(3)$ block of the field T_1 could also receive an order 10^2 GeV correction. As one can see from Eq. (4.31), the shift of the VEV $\langle\Sigma\rangle$ in its $SU(2)$ block is bounded and would not destroy the gauge hierarchy. The same strategy can be applied to the term in Eq. (4.29). After eliminating the auxiliary field F_{Σ_s}, this term gives a potential of the form

$$|Y_7\bar{\Sigma} + \frac{Y_8 T_2 T_3}{M} + \frac{4Y_3\Sigma^3 T_1}{M^3} + \frac{3Y_1\Sigma^2 T_1^2}{M^3} + \frac{2Y_4\Sigma T_1^3}{M^3} + H\bar{H} + c_2 m_g M_G + \cdots|^2. \quad (4.32)$$

This modification can shift the VEV's of the singlets Y_1, Y_3 by an amount of order 10^9 Gev or shift the VEV's of the singlets Y_7 and Y_8 by an amount of order of 10^4 and 10^5 GeV; this gives a small correction to the potential which is consistent with the hierarchy.

In this section, by extending the GUT group from the commonly used $SU(5)$ group to $SU(5)_1 \times SU(5)_2 \times SU(5)_3$, we are allowed to solve the Higgs triplet-doublet splitting problem and give μ a weak-scale value. It seems that having the H and $\bar{H}$ transform under different $SU(5)$ gauge groups gives a natural mechanism for solving these problems. Thus it is well-motivated to introduce product groups like $SU(5) \times SU(5)$ or the $SU(5) \times SU(5) \times SU(5)$ group as potential SUSY GUT gauge groups.

4.2.2 Fermion mass matrices

Now we examine the structure of the Yukuwa couplings in our model. Just as we constructed the terms bilinear in Higgs fields, it is straightforward to write the terms bilinear in quark and lepton fields. For the up quark masses, there are terms that apply $H 10_i 10_j$ to various combinations of the GUT-level Higgs fields.

$$\begin{aligned} W_{up} &= H\{10_3 10_3 + \frac{T_1}{M} 10_2 10_2 + \frac{T_2^2}{M^2} 10_3 10_1 + \frac{T_1 T_3}{M^2} 10_1 10_1 + \\ &+ 10_3 10_2 (\frac{T_1^2 \Sigma}{M^3}) + 10_2 10_1 (\frac{T_1^2 T_2^2 \Sigma}{M^5}) + \cdots\} \end{aligned} \quad (4.33)$$

In the above superpotential W_{up}, we list only the leading terms to various combinations bilinear in fields 10_i. The omitted terms in Eq. (4.33) represent possible next to leading order combinations. For the down quark and lepton masses, we find terms that include $\bar{H} 10_i \bar{5}_j$ contracted with various combinations of the GUT-level Higgs fields.

$$\begin{aligned} W_{down-lepton} &= \bar{H}\{\frac{T_1}{M}10_3\bar{5}_3 + (\frac{T_1(\Sigma\bar{\Sigma})}{M^3} + \frac{T_1(\Sigma T_2 T_3)}{M^4})10_3\bar{5}_1 \\ &+ 10_3\bar{5}_2[\frac{(T_1\bar{\Sigma})(\Sigma^2 T_3)}{M^5})] + \frac{T_3T_2+\bar{\Sigma}}{M^2}10_2\bar{5}_3 + \frac{T_3}{M}10_2\bar{5}_2 \\ &+ [\frac{T_3T_2}{M^2} + \frac{\bar{\Sigma}}{M}]10_2\bar{5}_1 + \frac{T_1T_2^2}{M^3}10_1\bar{5}_3 + \frac{T_2T_1}{M^2}10_1\bar{5}_2 + \frac{T_1T_2^2}{M^3}10_1\bar{5}_1 \\ &+ \cdots\} \end{aligned} \quad (4.34)$$

We have defined the two matter fields 5_1 and 5_3, which have the same gauge and $Z_2^{matter} \times Z_3$ quantum numbers, so that the first term of Eq. (4.34) contains only $\bar{5}_3$. and $\bar{5}_1$ is the orthogonal linear combination. We have ignored all the coefficients that could appear in front of each coupling term in Eq.s (4.33) and (4.34). Terms such as $T_2^4 10_1 10_1 H/M^4$ and $\bar{\Sigma}^4 10_3\bar{5}_3\bar{H}/M^4$ in the superpotentials W_{up} and $W_{down-lepton}$ are not listed because they are the higher-order contributions to the entries of the fermion mass matrices. We will see this point much clearly in the later discussion of this section. However, as we will see in Section 4.2.3, the term $(T_2^4/M^4)10_1 10_1 H$ cannot be ignored in the discussion of the proton decay in the model. As is typical in GUT theories based on $SU(5)$ unification [79], the up-type fermion masses are seen to be unrelated to the down- and lepton-type fermion masses.

According to Eq.s (4.33) and (4.34), only the top quark will receive a weak-scale mass. All other fermion masses arise from nonrenormalizable couplings and thus are suppressed by powers of $1/M$. These powers, together with the various VEV's in Eq. (4.21), lead to a hierarchy of Yukawa couplings. To exhibit this hierarchy, define the small parameters $\rho = \Omega/M$, $\bar{\rho} = \bar{\Omega}/M$, $\xi_1 = \Lambda_1/M$, $\xi_2 = \Lambda_2/M$ and $\xi_3 = \Lambda_3/M$. Then the leading contributions to each element of the Yukawa matrix is

$$(Up)_{\bar{u}_i u_j} = \begin{pmatrix} s\xi_1\xi_3 & 0 & \xi_2^2 \\ \xi_1^2\xi_2^2\rho & \xi_1 & \xi_1^2\rho \\ \xi_2^2 & 0 & 1 \end{pmatrix} \quad (4.35)$$

$$(Down)_{\bar{d}_i d_j} = \begin{pmatrix} \xi_1\xi_2^2 & \xi_2\xi_3 + \bar{\rho} & \xi_1\rho\bar{\rho} \\ \xi_1\xi_2 & \xi_3 & 0 \\ \xi_1\xi_2^2 & \xi_2\xi_3 + \bar{\rho} & \xi_1 \end{pmatrix} \quad (4.36)$$

$$(Lepton)_{\bar{e}_i L_j} = \begin{pmatrix} \xi_1\xi_2^2 & (s)\xi_2\xi_3 + (a)\bar{\rho} & \xi_1\rho\bar{\rho} \\ \xi_1\xi_2 & (s)\xi_3 & 0 \\ \xi_1\xi_2^2 & (s)\xi_2\xi_3 + (a)\bar{\rho} & \xi_1 \end{pmatrix} \quad (4.37)$$

From the above mass matrices, a approximate texture zero structure [10] would be seen in the up quark mass matrix after determining the scale ratios. The down quark mass matrix and the lepton mass matrix are identical, except that the (1, 2), (2, 2) and (3, 2) entries of the lepton mass matrix have different coefficients. These differences are due to the VEV patterns of $\langle\bar{\Sigma}\rangle$ and $\langle T_3\rangle$.

Before making further comments on the mass matrices, we would like to point out that if the introduced Z_3 symmetry is disabled in the model, then the forbidden terms such as $(\bar{\Sigma}^2/M^2 + T_2^2T_3^2/M^4)10_3 10_2 H$, $(T_1T_3^3/M^4 + T_1^3T_2^2/M^5)10_2 10_1 H$ and $(T_1^2T_3/M^3)10_3\bar{5}_2\bar{H}$ will give additional contributions to W_{up} and $W_{down-lepton}$. We list these terms in the Appendix, Eq. (4.63) and (4.64). These new terms show the same hierarchy in powers of the small parameters ρ, $\bar{\rho}$ and ξ_i. In other words, the mass hierarchy is merely determined by the gauge structure but not by the global discrete symmetry in the model.

Since this model cannot predict the coefficients for the coupling terms in superpotential, we assume these to be of order O(1) and ignore all coefficients in the above mass matrices. The zero entries in the up quark mass matrices are only approximate and could be replaced by those ignored subleading terms in Eq. (4.33). In fact, by the estimation made in later in this section, these "zeros" are such small numbers that they should be smaller than 10^{-11}. Therefore, we can just ignore them in the later discussion.

Although we do not know the coupling term coefficients, however, we can still extract some interesting points from Eq. (4.35 - 4.37). First, this model requires a low value of $\tan\beta$ because the top Yukawa coupling is much larger than the bottom Yukawa coupling.

We also observe that because of the VEV structures of $\langle\bar{\Sigma}\rangle$ and $\langle T_3\rangle$, the terms $(T_2T_3/M^2 + \bar{\Sigma}/M)10_2\bar{5}_1\bar{H}$ and $(T_3/M)10_2\bar{5}_2\bar{H}$ have different contributions to the down-quark mass matrix and the lepton mass matrix. It has long been a problem for $SU(5)$ grand unification that the mass relation $m_l = m_d$ at the GUT scale cannot be obeyed for all three generations. Georgi and Jarlskog [11, 83], proposed a solution which has been used in models of SUSY $SO(10)$ grand unification [84]. Despite the successful experimental data fitting in their model, the low energy mass relation $m_s/m_d = 25.15$ predicted in their model is two standard deviations away from the average value obtained by sum rule and chiral perturbation methods [15. 16]. Our scheme does not give a definite prediction for the mass relations, but it does give some required extra freedom. For example, if the coefficient s is taken to be 3, then we obtain the GUT scale mass relations

$$m_\tau = m_b \tag{4.38}$$

$$m_\mu \approx 3m_s \tag{4.39}$$

These GUT mass relations could lead to acceptable m_b/m_τ and m_μ/m_s mass relations [16, 10] at the weak scale.

A specific choice of the parameters that gives an acceptable representation of all of the experimental data on fermion masses is the following:

$$\frac{m_c}{m_t} \sim \xi_1 \sim O(10^{-2}), \tag{4.40}$$

$$\frac{m_u}{m_c} \sim s\cdot\xi_3 \sim O(10^{-2}) \tag{4.41}$$

$$\frac{m_s}{m_b} \sim \frac{\xi_3}{\xi_1} \sim O(10^{-1}) \tag{4.42}$$

$$\frac{m_e}{m_\mu} \sim \frac{\xi_1\xi_2^2}{s\xi_3} + \frac{a\xi_1\xi_2(\bar{\rho}+\xi_2\xi_3)}{(s\xi_3)^2} \sim O(10^{-2}) \tag{4.43}$$

$$\frac{m_d}{m_s} \sim \frac{\xi_1\xi_2^2}{\xi_3} + \frac{\xi_1\xi_2(\bar{\rho}+\xi_2\xi_3)}{\xi_3^2} \sim O(10^{-1}) \tag{4.44}$$

The above relations allow us to choose the scale ratios as

$$\xi_1 \sim \rho \sim \frac{1}{3} \times 10^{-2} \tag{4.45}$$
$$\xi_2 \sim 3 \times 10^{-2} \tag{4.46}$$
$$\xi_3 \sim \frac{1}{3} \times 10^{-3} \tag{4.47}$$
$$\bar{\rho} \sim \frac{1}{2} \times 10^{-4}. \tag{4.48}$$

From the μ value equation in (4.24), it can be easily checked that these values would give rise to a weak-scale μ value. Based on the given scale ratios, we can also estimate the CKM mixing angles s_{12}, s_{23} and s_{13} by

$$s_{12} : s_{23} : s_{13} \sim \frac{\xi_1 \xi_2}{\xi_3} : \frac{\bar{\rho} + \xi_2 \xi_3}{\xi_1} : \xi_2^2 \sim O(10^{-1}) : O(10^{-2}) : O(10^{-3}). \tag{4.49}$$

which is consistent in order of magnitude with the experimental data. The GUT-group breaking scales are now determined to have the relation $\Lambda_2 > \Lambda_1 > \Lambda_3$. This confirms the breaking pattern described in Section 4.2.1.

According to the scale ratio estimations, there are approximate texture zero structures in the fermion mass matrices.

$$(Up)_{\bar{u}_i u_j} = \begin{pmatrix} s\xi_1\xi_3 & 0 & \xi_2^2 \\ 0 & \xi_1 & \xi_1^2 \rho \\ \xi_2^2 & 0 & 1 \end{pmatrix} \tag{4.50}$$

$$(Down)_{\bar{d}_i d_j} = \begin{pmatrix} \xi_1\xi_2^2 & \xi_2\xi_3 + \bar{\rho} & \xi_1 \rho \bar{\rho} \\ \xi_1 \xi_2 & \xi_3 & 0 \\ \xi_1 \xi_2^2 & \xi_2 \xi_3 + \bar{\rho} & \xi_1 \end{pmatrix} \tag{4.51}$$

$$(Lepton)_{\bar{e}_i L_j} = \begin{pmatrix} \xi_1\xi_2^2 & (s)\xi_2\xi_3 + (a)\bar{\rho} & \xi_1 \rho \bar{\rho} \\ \xi_1 \xi_2 & (s)\xi_3 & 0 \\ \xi_1 \xi_2^2 & (s)\xi_2 \xi_3 + (a)\bar{\rho} & \xi_1 \end{pmatrix} \tag{4.52}$$

The zero entries are only approximate and represent values smaller than 10^{-10}. Unlike the case in conventional SUSY flavor models [72, 10, 16, 79], these texture zeros are the natural outcome of the gauge structure as well as the scale ratios given in the model. In other words, they could arise without flavour symmetry.

In this section, we have estimated the possible scale ratio values needed to obtain acceptable fermion mass structures. The GUT gauge group $SU(5)_1 \times SU(5)_2 \times SU(5)_3$ would undergo a two-step breaking down to the SM group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The SM gauge couplings unify at the scale of 10^{16} Gev if we take the superheavy scale M to be the reduced Planck scale. The Higgs triplets H_c and $\bar{H}_c$ would obtain GUT scale masses of order of 10^{16} Gev due to the superpotential term $\Sigma H\bar{H}$. Although we did not discuss the possible threshold effects [43] caused by those exotic Higgs fields as well as the heavy Higgs triplets, it is quite interesting that we find naturally a hierarchical pattern for the fermion mass matrices.

4.2.3 Proton decay

We have already introduced a Z_2^{matter} symmetry to disable all dangerous dimension three and four operators in Section 4.2.1. However, since we find Higgs triplet masses of order 10^{16} GeV, there is a danger that dimension five operators which violate baryon and lepton number could cause fast proton decay [85]. A dimension five operator in the superpotential could lead to proton decay if it has the form

$$\frac{\lambda}{M^*} Q_1 Q_1 Q_2 L_i. \tag{4.53}$$

Here Q_i and L_i represent the i^{th} generation of the quark and lepton multiplets respectively, M^* represents some high scale, and λ is the coupling constant. This operator leads to proton decay through the mode $p \longrightarrow K^+\bar{\nu}$. The current experiment data have already set the limit $\lambda/M^* \lesssim 10^{-24}$ GeV^{-1} with the naturalness assumption that all squark/slepton masses are no larger than 1 TeV [85, 86]. In principle, operators of the form of Eq. (4.53) could arise from integrating out particles with GUT-scale masses or directly from the higher-dimension operators in the original Lagrangian.

In the Appendix, we analyze these higher-dimension operators and show that they are highly suppressed by powers of $1/M$ due to the gauge structure as well as the Z_3 symmetry of the model. Therefore, the main contributions to proton decay in the model will come from heavy Higgsino exchange processes.

Since the VEV $\langle T_1T_3\rangle$ has vanishing contribution to color triplets, the potentially leading term $(T_1T_3/M^2)10_1 10_1 H$ cannot participate in the heavy Higgsino exchange processes. The same logic also applies to the terms such as $(T_1/M)10_2 10_2 H$, $(T_1T_2/M^2)10_1\bar{5}_2\bar{H}$ and $(T_1T_2^2/M^3)10_1\bar{5}_1\bar{H}$. Therefore, by taking the quark mixing into account, the leading terms in the superpotential that contribute to the dimension five operators in Eq. (4.53) are the following:

$$10_3 10_3 H \tag{4.54}$$

$$\{\frac{T_2^2}{M^2}\}10_1 10_3 H \tag{4.55}$$

$$\{\frac{T_2^4}{M^4}\}10_1 10_1 H \tag{4.56}$$

$$\{\frac{\bar{\Sigma}}{M} + \frac{T_2T_3}{M^2}\}10_2\bar{5}_1\bar{H} \tag{4.57}$$

$$\{\frac{T_3}{M}\}10_2\bar{5}_2\bar{H} \tag{4.58}$$

From Eq.s (4.54 - 4.58), the leading dimension five operators that come from integrating out heavy Higgs triplets are shown in Fig. (4.1). We find that the figure (a) in Fig. (4.1) should dominate the proton decay in the model with the decay mode $p \to K^+\bar{\nu}_\mu$. There are two contributions to Fig. 4.1(a), with coupling strengths

$$\frac{\lambda}{M^*} \sim \frac{1}{M_{H_c}} \times \frac{\langle T_2^4\rangle}{M^4} \times \frac{\langle T_3\rangle}{M} \lesssim 10^{-25}\ \mathrm{Gev}^{-1} \tag{4.59}$$

$$\frac{\lambda}{M^*} \sim \frac{\sin\theta_{13}}{M_{H_c}} \times \frac{\langle T_2^2\rangle}{M^2} \times \frac{\langle T_3\rangle}{M} \lesssim 10^{-25}\ \mathrm{Gev}^{-1}. \tag{4.60}$$

In Eq. (4.59), the factor $\langle T_2^4/M^4\rangle$ comes from the next leading term $T_2^4 10_1 10_1 H/M^4$ and the factor $\langle T_3\rangle/M$ comes from $(T_3/M)10_2\bar{5}_2\bar{H}$ in superpotential. In Eq. (4.60),

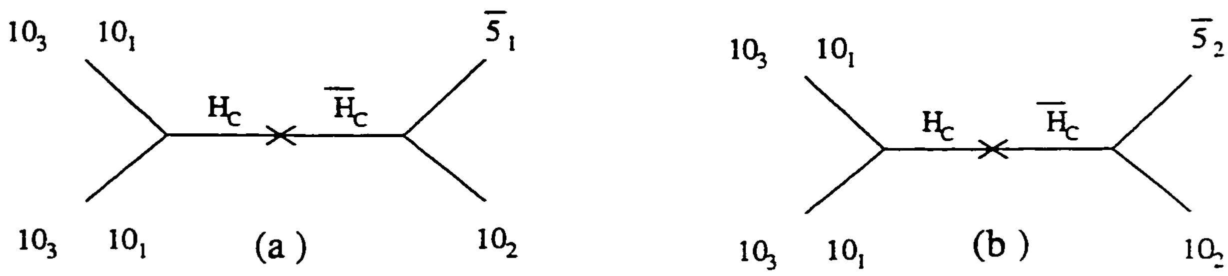

Figure 4.1: Dimension five operators produced by integrating out heavy Higgs triplets. These two operators would dominate the proton decay due to their relatively large coupling strengths.

the factor $\langle T_2^2/M^2\rangle$ comes from the term $(T_2^2/M^2)10_1 10_3 H$ and $\sin\theta_{13}$ represents the mixing angle between the first and the third generation up-type quarks. The above coupling strength estimations show that the proton lifetime in the model should be no less than 10^{34} years. This result is about 100 times longer than the current experiment limit [2]. It is observed to the future experiment limit that could be set by SuperKamiokande.

Although there are uncertainties in determining the coefficients of the Yukawa coupling terms in the superpotential, however, the branching ratio between the $p \to K^+\bar{\nu}_\mu$ channel and the $p \to K^+\bar{\nu}_e$ could be definitely given by

$$\frac{BR(p \to K^+\bar{\nu}_e)}{BR(p \to K^+\bar{\nu}_\mu)} = (\frac{\bar{\rho} + \xi_2\xi_3}{\xi_3})^2 \sim 10^{-2}. \quad (4.61)$$

This branching ratio prediction is generic to some SUSY models [87] that have the down quark mass generated by the seesaw mechanism. It is not clear to us how this prediction could be tested.

4.2.4 Conclusion

In this paper we have presented a supersymmetric GUT model based on the gauge group $SU(5)_1 \times SU(5)_2 \times SU(5)_3$. The Higgs fields and the matter fields are assigned to transform under the different $SU(5)$ groups in asymmetrical pattern. Exotic Higgs fields Σ, $\bar{\Sigma}$, T_1, T_2 and T_3 are needed to break the GUT gauge group down to the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, and also to relate matter fields which transform under different gauge $SU(5)$'s. The discrete global symmetry $Z_2^{matter} \times Z_3$ is imposed at the reduced Planck scale in such a way that some dangerous terms in superpotential are disabled and a weak-scale μ value for light Higgs doublets can be obtained. However, this discrete symmetry is the only flavour symmetry required in our scheme. The fermion mass hierarchy is a natural outcome of the gauge structure presented in this model. That is, it is the breaking of GUT gauge group but not the breaking of flavour symmetry that generates the fermion mass hierarchy in our model. In Section 4.2.1, we have shown that realistic fermion mass matrices can be the result of this mechanism. The fermion mass relations and the CKM angles are estimated to be consistent with measured experiment data at low energy. The exotic Higgs fields also play important roles in predicting realistic down-quark and lepton mass relations. The fields $\bar{\Sigma}$ and T_3 allow us to obtain the Georgi-Jarlskog relation between the leptons and down quark masses, and also more general relations that may be required by experiment.

This model does not forbid the dimension five operators that could result in nucleon decays. In fact, there are allowed tree-level dimension five operators in the superpotential. However, these tree-level terms are suppressed by powers of the superheavy scale M and thus are not important in discussing the proton decay. The proton decay in the model is mainly due to Higgsino-exchange processes. The dominant mode of proton decay in the model is the process $p \to K^+\bar{\nu}_\mu$, the same dominant mode as in minimal SUSY SU(5) model. Due to the VEV pattern of the field T_1, the leading term $(T_1T_3/M^2)10_1 10_1 H$ terms in the superpotential does not participate in the heavy Higgs triplet exchange process and thus gives zero contribution to the proton decay. The next leading order contributions of proton decay come from the term $(T_2^4/M^4)10_1 10_1 H$ and quark mixing effects, which are more suppressed than the leading order term $(T_1T_3/M^2)10_1 10_1 H$. Therefore, proton decay in this model is highly sensitive to the changes of the scale ratio $\langle T_2 \rangle / M$. The proton lifetime is estimated to be larger than 10^{34} years, depending on the exact $\langle T_2 \rangle / M$ value and the unknown coefficients of coupling terms in superpotential.

Models with product SU(5) groups were originally introduced with motivations from string theory. Our model shows that this structure may be interesting in its own right as a possible explanation of the fermion mass spectra.

4.2.5 Appendix

If the Z_3 symmetry is not introduced to the model, then all possible operators bilinear in H and $\bar{H}$ that are up to the dimension 10 level are given as follows:

$$\begin{aligned}
W_{H\bar{H}} &= \Sigma H\bar{H}\{1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \frac{T_1T_2T_3}{M^3} + \frac{\Sigma T_2T_3}{M^3} + \frac{(\Sigma\bar{\Sigma})^2}{M^4} + \frac{(T_1\bar{\Sigma})^2}{M^4} \\
&+ \frac{(\Sigma\bar{\Sigma})(T_1\bar{\Sigma})}{M^4} + \frac{1}{M^5}[(\Sigma\bar{\Sigma})(\Sigma T_2T_3 + T_1T_2T_3) + (T_1\bar{\Sigma})(\Sigma T_2T_3 + T_1T_2T_3) \\
&+ \sum_{k=0}^{5} \frac{1}{M^{5-k}}\bar{\Sigma}^k(T_2T_3)^{5-k} + \sum_{k=0}^{5} \Sigma^k T_1^{5-k} + T_2^5 + T_3^5] \\
&+ \frac{1}{M^6}[(T_1T_2T_3)^2 + (T_1T_2T_3)(\Sigma T_2T_3) + (\Sigma T_2T_3)^2 + \sum_{k=0}^{3}(T_1\bar{\Sigma})^k(\Sigma\bar{\Sigma})^{3-k}]\} \\
&+ T_1H\bar{H}\{1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \frac{T_1T_2T_3}{M^3} + \frac{\Sigma T_2T_3}{M^3} + \frac{(\Sigma\bar{\Sigma})^2}{M^4} + \frac{(T_1\bar{\Sigma})^2}{M^4} \\
&+ \frac{(\Sigma\bar{\Sigma})(T_1\bar{\Sigma})}{M^4} + \frac{1}{M^5}[(\Sigma\bar{\Sigma})(\Sigma T_2T_3 + T_1T_2T_3) + (T_1\bar{\Sigma})(\Sigma T_2T_3 + T_1T_2T_3) \\
&+ \sum_{k=0}^{5} \frac{1}{M^{5-k}}\bar{\Sigma}^k(T_2T_3)^{5-k} + \sum_{k=0}^{5} \Sigma^k T_1^{5-k} + T_2^5 + T_3^5] + \frac{1}{M^6}[(T_1T_2T_3)^2 \\
&+ (T_1T_2T_3)(\Sigma T_2T_3) + (\Sigma T_2T_3)^2 + \sum_{k=0}^{3}(T_1\bar{\Sigma})^k(\Sigma\bar{\Sigma})^{3-k}]\} \qquad (4.62)
\end{aligned}$$

The leading Yukawa coupling terms that give masses to fermions are also listed below:

$$\begin{aligned}
W_{up} &= H\{10_3 10_3 + \frac{T_1}{M}10_2 10_2 + \frac{T_2^2}{M^2}10_3 10_1 + \frac{T_1T_3}{M^2}10_1 10_1 + \\
&+ 10_3 10_2[\frac{\bar{\Sigma}^2}{M^2} + \sum_{k=0}^{3} \frac{\Sigma^{3-k}T_1^k}{M^3} + \frac{(T_2T_3)^2}{M^4}]
\end{aligned}$$

$$
\begin{aligned}
&+ \quad 10_2 10_1 [\frac{T_1 T_3^3}{M^4} + \frac{T_2^2 \bar{\Sigma}^2}{M^4} + \sum_{k=0}^{3} \frac{T_1^k T_2^2 \Sigma^{3-k}}{M^5}] \\
&+ \quad \cdots \}
\end{aligned} \tag{4.63}
$$

$$
\begin{aligned}
W_{down-lepton} &= \bar{H}\{\frac{T_1}{M} 10_3 \bar{5}_3 + \frac{T_1^2 T_3 + \Sigma^2 T_3}{M^3} 10_3 \bar{5}_2 + \frac{T_3 T_2 + \bar{\Sigma}}{M^2} 10_2 \bar{5}_3 + \frac{T_3}{M} 10_2 \bar{5}_2 \\
&+ \quad 10_3 \bar{5}_1 (\frac{T_1}{M}[\frac{T_1 \bar{\Sigma} + \Sigma \bar{\Sigma}}{M^2} + \frac{T_1 T_2 T_3 + \Sigma T_2 T_3}{M^3} \\
&+ \quad \frac{(T_1 \bar{\Sigma})^2 + (T_1 \bar{\Sigma})(\Sigma \bar{\Sigma}) + (\Sigma \bar{\Sigma})^2}{M^4}] \\
&+ \quad \frac{\bar{\Sigma}^4}{M^4}) + (\frac{T_3 T_2}{M^2} + \frac{\bar{\Sigma}}{M}) 10_2 \bar{5}_1 + \frac{T_1 T_2^2}{M^3} 10_1 \bar{5}_3 \\
&+ \quad \frac{T_2 T_1}{M^2} 10_1 \bar{5}_2 + \frac{T_1 T_2^2}{M^3} 10_1 \bar{5}_1 + \cdots \}.
\end{aligned} \tag{4.64}
$$

From (4.63) and (4.64). a hierarchical and texture of fermion masses is still present in the model even without introducing the Z_3 symmetry. This can be seen by the following fermion mass matrices.

$$
(\mathrm{Up})_{\bar{u}_i u_j} = \begin{pmatrix} (s)\xi_1\xi_3 & (s)\xi_1\xi_3^3 + \xi_2^2\bar{\rho}^2 + \xi_1\xi_2^2\rho^2 & \xi_2^2 \\ (s^2)\xi_1\xi_3^3 + \xi_1^2\xi_2^2\rho & \xi_1 & (a)\bar{\rho}^2 + \xi_1\rho^2 + \xi_1^2\rho \\ \xi_2^2 & \bar{\rho}^2 + \xi_1\rho^2 + \xi_1^2\rho & 1 \end{pmatrix} \tag{4.65}
$$

$$
(\mathrm{Down})_{\bar{d}_i u_j} = \begin{pmatrix} \xi_1\xi_2^2 & \xi_2\xi_3 + \bar{\rho} & \xi_1^2\bar{\rho} + \xi_1\rho\bar{\rho} \\ \xi_1\xi_2 & \xi_3 & \rho^2\xi_3 \\ \xi_1\xi_2^2 & \xi_2\xi_3 + \bar{\rho} & \xi_1 \end{pmatrix} \tag{4.66}
$$

$$
(\mathrm{Lepton})_{\bar{e}_i L_j} = \begin{pmatrix} \xi_1\xi_2^2 & (s)\xi_2\xi_3 + (a)\bar{\rho} & \xi_1^2\bar{\rho} + \xi_1\rho\bar{\rho} \\ \xi_1\xi_2 & (s)\xi_3 & \xi_1^2\xi_3 \\ \xi_1\xi_2^2 & (s)\xi_2\xi_3 + (a)\bar{\rho} & \xi_1 \end{pmatrix} \tag{4.67}
$$

From the above matrices, the approximate texture zero structures will be present as a result of the gauge structure of the model. The up quark mass matrix (4.65) becomes slightly asymmetrical due to the VEV structures given in Eq. (4.21) and the gauge structure of this model. The coefficients (s), (s^2) and (a) in the above matrices indicate the additional factors that come from the constants s and a in the VEV $\langle T_3 \rangle$ and $\langle \bar{\Sigma} \rangle$. All together, these make the down quark and the lepton mass matrices different from each other even though they arise from the same superpotential $W_{down-lepton}$.

Without imposing Z_3 symmetry onto this model, if we forbid possible dangerous dimension three and four operators by introducing Z_2^{matter} symmetry, there could still exist some leading tree level operators that would mediate proton decay.

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{T_3^2}{M^3}10_1 10_1 10_2 \bar{5}_2, \tag{4.68}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{(\bar{\Sigma}T_2)^3}{M^7}10_1 10_1 10_2 \bar{5}_2, \tag{4.69}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{T_3\bar{\Sigma}}{M^3}10_1 10_1 10_2 \bar{5}_1, \tag{4.70}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{T_2^4\bar{\Sigma}^2}{M^7}10_1 10_1 10_2 \bar{5}_1, \tag{4.71}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{\Sigma T_2}{M^3}10_1 10_2 10_2 \bar{5}_2, \tag{4.72}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{T_2^3 T_1^2 \Sigma}{M^7}10_1 10_1 10_2 \bar{5}_2, \tag{4.73}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{\Sigma T_3}{M^3}10_1 10_1 10_3 \bar{5}_1, \tag{4.74}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{T_2^3}{M^4}10_1 10_1 10_3 \bar{5}_2, \tag{4.75}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{\bar{\Sigma}^2}{M^3}10_2 10_3 10_3 \bar{5}_1, \tag{4.76}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{T_3\bar{\Sigma}}{M^3}10_2 10_3 10_3 \bar{5}_2, \tag{4.77}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{\Sigma}{M^2}10_2 10_2 10_3 \bar{5}_1, \tag{4.78}$$

$$(1 + \frac{\Sigma\bar{\Sigma}}{M^2} + \frac{T_1\bar{\Sigma}}{M^2} + \cdots)\frac{\Sigma T_3 T_1}{M^4}10_2 10_2 10_3 \bar{5}_2 \tag{4.79}$$

The above non-renormalizable operators, if they exist in our model, would give effective dimension five operators that violate baryon and lepton numbers. By the scale ratios given in Section 4.2.2, we find the largest two coupling strengths in the list to come from Eq.s (4.72) and (4.78)

$$\begin{aligned} \sin\theta_{23}\sin\theta_c\frac{\langle\Sigma\rangle}{M^2} \sim \sin\theta_c\frac{\langle\Sigma T_2\rangle}{M^3} \sim \frac{10^{-5}}{M} &\sim O(10^{-23})\ \mathrm{Gev}^{-1} \\ &> O(10^{-24})\ \mathrm{GeV}^{-1}. \end{aligned} \tag{4.80}$$

where the superheavy scale M is taken to be the reduced Planck scale. This result would predict a proton lifetime which is about 10^2 times shorter than the current experiment limit. Fortunately, if the Z_3 symmetry is introduced, some of the tree-level terms in Eq.s (4.68 $\sim$ 4.79) are forbidden. We are thus left with the leading tree-level terms of Eq.s (4.68), (4.70) and (4.77).

$$\frac{(T_1\bar{\Sigma})}{M^2}\frac{T_3^2}{M^3}10_1 10_1 10_2 \bar{5}_2, \qquad \frac{\lambda}{M^{\cdot}} \sim \frac{\langle(T_1\bar{\Sigma})T_3^2\rangle}{M^5} \sim \frac{10^{-13}}{M} \tag{4.81}$$

$$\frac{(T_1\bar{\Sigma})}{M^2}\frac{T_3\bar{\Sigma}}{M^3}10_1 10_1 10_2 \bar{5}_1, \qquad \frac{\lambda}{M^{\cdot}} \sim \frac{\langle T_3\bar{\Sigma}(T_1\bar{\Sigma})\rangle}{M^4} \sim \frac{10^{-14}}{M} \tag{4.82}$$

$$\frac{T_3\bar{\Sigma}}{M^3}10_2 10_3 10_3 \bar{5}_2, \qquad \frac{\lambda}{M^{\cdot}} \sim \sin\theta_{13}^2\frac{\langle T_3\bar{\Sigma}\rangle}{M^3} \sim \frac{10^{-13}}{M}. \tag{4.83}$$

These terms are much less important than the Higgsino-exchange processes in Eq. (4.59) and (4.60). Therefore, they could just be ignored in discussing the proton decay in this model.

Chapter 5

SUSY GUTs based on $SO(10)^2$ and $SO(10)^3$

5.1 Introduction

The Standard Model (SM) provides a successful description of physics up to the weak scale. However, it provides some 18 parameters which are input by hand to fit experiment data. Most of these input parameters are associated with flavor physics and are included to parameterize the fermion mass hierarchy, Cabibbo-Kobayashi-Maskawa (CKM) angles, and neutrino oscillations. Many theories, either supersymmetric (SUSY) or non-supersymmetric, are constructed to address the flavor problem and, hopefully, make predictions on new physics. Among these theories beyond the Standard Model (SM), supersymmetric grand unification provides an elegant framework that explains not only the gauge quantum numbers of fermions transforming under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, but also the prediction of $\alpha_s(M_Z)$. This remarkable success of the prediction of $\alpha_s(M_Z)$ motivates further exploration of SUSY grand unification [36].

Among the ideas of grand unification, gauge groups such as $SU(5)$, E_6, and $SO(10)$ are frequently used in GUT model construction [7, 37]. However, there are reasons that make $SO(10)$ theories more attractive than others. First, $SO(10)$ is the smallest group in which all matter fields in one family can fit into one irreducible representation. Second, the two light Higgs doublets needed in any SUSY theory fit into one **10** of SO(10). This allows the Yukawa couplings of up-type and down-type quarks to be determined by Clebsch-Gordan coefficients, thus making $SO(10)$ theories more predictive.

There is a problem with this approach, however. Typical SUSY $SO(10)$ models need to use Higgs fields in higher representations, the **126** or **45**, to achieve successful GUT relations for Yukawa matrices. These representations are complex in their own right, and theories which contain tensor fields of rank higher than two cannot be constructed from the simplest string-derived GUT theories [63, 64]. This motivates the use of extended GUT gauge groups such as $G \times G$ or $G \times G \times G$, where G denotes the usual GUT group, in SUSY GUT model construction [66, 81].

Supersymmetric GUT models based on the gauge groups $SO(10) \times SO(10)$ and $SO(10) \times SO(10) \times SO(10)$ have been discussed in the literature [66, 81]. In these models, the breaking of the GUT gauge group was done when fundamental Higgs fields in the (10, 10) representation, acquire their vacuum expectation values (VEVs) along the embedded diagonal subgroup directions of $SO(10)^2$ and $SO(10)^3$, while the spinorial Higgs fields $\Psi_i, \bar{\Psi}_i$ acquire VEVs along $SU(5)$-preserving directions. Four sets of the (10, 10) fields carrying charges of different discrete symmetries were introduced; the large number of fields is needed not only to achieve the desirable Higgs doublet-triplet splitting, but also give the desirable asymmetry between the up and down quark mass matrices. As a result, typical predictions of SUSY GUT $SO(10)$ models, such as the top-bottom Yukawa unification $\lambda_t = \lambda_b$, and Clebsch-Gordan relations in Yukawa matrices are not valid in their models.

In this paper, we follow the idea of using $SO(10) \times SO(10)$ and $SO(10) \times SO(10) \times SO(10)$ as the SUSY GUT gauge groups. However, we show that the traditional merits of the SUSY GUT $SO(10)$ models can be preserved in our $SO(10)^2$ and $SO(10)^3$ model construction. Although it is motivated from the string constructions, our model construction is self-contained and does not make explicit reference to string theories. In our models, all Higgs fields are in the fundamental representations of the gauge groups and no rank two tensors of any SO(10) gauge group are required.

In section 5.2, we show that the extended GUT gauge group breaking can be implemented when Higgs fields acquire VEVs along diagonal $SO(10)_D$ directions, diagonal $SU(5)_D \times U(1)_D$ directions, or other diagonal directions. Most importantly, we argue that the effective adjoint fields for each $SO(10)_i$ group can be formed by combining two VEV-acquiring Higgs fields. In section 5.3, we construct an explicit model based on $SO(10)^2$. We show that the Higgs doublet-triplet problem is naturally solved through the Dimopoulos-Wilczek mechanism [47] without destabilizing the gauge hierarchy. The doublet-triplet splitting mechanism also guarantees strong suppression of proton decay, since the contributions from heavy Higgsino triplet exchange diagrams are absent or highly suppressed. We also show that this model gives Yukawa matrices of the type similar to Georgi-Jarlskog ansatz. An explicit model which was analyzed by Anderson *et al.* [16] is constructed by using effective adjoint operators. In section 5.4, we present an $SO(10) \times SO(10) \times SO(10)$ model with each family of matter multiplets transforming under different $SO(10)$ groups. In section 5.5, we make our conclusion.

5.2 Effective adjoint operators for $SO(10)$

As pointed out in the literature [66, 81], the breaking of extended GUT gauge groups $G \times G$ and $G \times G \times G$ can be achieved by a set of Higgs fields in the fundamental representation. For example, an $SO(10)_1 \times SO(10)_2$ model breaks down to its diagonal subgroups when fields in the fundamental representation $(10, 10)$ develop VEVs. We will denote $(10, 10)$ fields in this paper as S or Z depending on the VEV patterns. We denote fields with the following three canonical patterns of VEVs $< S_X >$, $< S_{B-L} >$, $< S_{T_{3R}} >$, corresponding to

$$< S_X >= \frac{v_D}{\sqrt{10}} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \mathrm{diag}(1, 1, 1, 1, 1) \tag{5.1}$$

$$< S_{B-L} >= \frac{v_G}{\sqrt{10}} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \mathrm{diag}(a, a, a, 0, 0) \tag{5.2}$$

$$< S_{T_{3R}} >= \frac{v_G}{\sqrt{10}} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \mathrm{diag}(0,0,0,b,b). \tag{5.3}$$

The VEVs of S_X, S_{B-L}, and $S_{T_{3R}}$ break $SO(10)_1 \times SO(10)_2$ down to its embedded diagonal subgroups $SO(10)_D$, $SO(6)_D \times SO(4)_1 \times SO(4)_2$, and $SO(6)_1 \times SO(6)_2 \times SO(4)_D$ respectively. Usually, a tree level superpotential has many SUSY vacua which include the VEVs in Eq. (5.3); a typical form includes

$$W \supset \frac{\lambda M}{2}\mathrm{Tr}(SS^T) + \frac{A}{4M}(\mathrm{Tr}(SS^T))^2 + \frac{B}{4M}\mathrm{Tr}(SS^TSS^T). \tag{5.4}$$

However, there are other SUSY vacua which lie along the direction of the embedded diagonal $SU(5)_D \times U(1)_D$, or other directions such as $SU(3)_D \times U(1)_D \times SO(4)_1 \times SO(4)_2$ and $SO(6)_1 \times SO(6)_2 \times SU(2)_D \times U(1)_D$. We denote the associated (10, 10) fields as Z and again refer to the VEV patterns using subscripts:

$$\begin{aligned}
< Z_X > &= \frac{v_{10}}{\sqrt{10}} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \mathrm{diag}(2,2,2,2,2) \\
< Z_{B-L} > &= \frac{v_5}{\sqrt{10}} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \mathrm{diag}(2a/3, 2a/3, 2a/3, 0, 0) \\
< Z_{T_{3R}} > &= \frac{v_5}{\sqrt{10}} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \mathrm{diag}(0,0,0,b/2,b/2).
\end{aligned} \tag{5.5}$$

As an alternative to generating scales or VEVs by minimizing a tree level superpotential, it has been shown that the scale $< S_X >$ could also be dynamically generated through a strongly coupled supersymmetric dynamics [88]. Following the same line of thinking, we can introduce two supersymmetric gauge groups $SU(N_c)$ and $Sp(n_c)$ with fields in their fundamental representations $q(N_c, 1, 10, 1)$, $\bar{q}(\bar{N}_c, 1, 1, 10)$, and $Q(1, 2n_c, 1, 10)$, where the numbers in brackets denote the dimensionality of each field under the two strong groups and the GUT gauge group $SO(10)_1 \times SO(10)_2$. With the imposition of some discrete symmetry, say $Z_N \times Z_K$, that keeps the field S_X from coupling directly to Z_X, the lowest order of tree level superpotential is given by

$$W_{tree} = \frac{A}{2NM^{2N-3}} S_X^{2N} + \frac{B}{2KM^{2K-3}} Z_X^{2K} + \lambda_1 S_X q\bar{q} + \lambda_2 (S_X Z_X)^{ab} Q^a Q^b \tag{5.6}$$

where A and B are coefficients, M is the superheavy scale or Planck scale. To this should be added the dynamical superpotential resulting from the strong dynamics.

$$W_{dyn} = \frac{C}{N_c - 10} \left[\frac{\Lambda_1^{3N_c - 10}}{\det q\bar{q}} \right]^{\frac{1}{N_c - 10}} + \frac{D}{n_c + 1 - 5} \left[\frac{\Lambda_2^{b_0/2}}{\mathrm{Pf}(QQ)} \right]^{\frac{1}{n_c + 1 - 5}} . \tag{5.7}$$

By stabilizing the superpotential in Eqs (5.6) and (5.7) along the $< S_X >$ and $< Z_X >$ directions, we obtain the following equations for the VEVs:

$$\frac{A}{M^{2N-3}} s^{2N-1} + \frac{5C}{(n_c + 1)s} \left[\frac{\lambda_2 s z}{M \Lambda_1} \right]^{\frac{5}{n_c + 1}} \Lambda_1^3 + \frac{10 D}{N_c s} \left[\frac{\lambda_1 s}{\Lambda_2} \right]^{\frac{10}{N_c}} \Lambda_2^3 = 0 \tag{5.8}$$

$$\frac{B}{M^{2K-3}} z^{2K-1} + \frac{5C}{(n_c + 1) z} \left[\frac{\lambda_2 s z}{M \Lambda_1} \right]^{\frac{5}{n_c + 1}} \Lambda_1^3 = 0. \tag{5.9}$$

where $s = v_D/\sqrt{10}$ and $z = 2v_{10}/\sqrt{10}$. It is easily seen that solving Eq.s (5.8) and (5.9) would lead to nonzero v_D and v_{10}, and thus the desirable VEVs $< S_X >$ and $< Z_X >$.

Given VEVs of the S and Z fields, we can form effective rank two tensors which carry quantum numbers of the gauge group $SO(10)_2$ by combining any two of the S and Z fields. In this way, we can form effective adjoint operators of $SO(10)_2$, which we call Σ and Σ', given by

$$\begin{aligned}
\Sigma_X^{bc} &\equiv \mathrm{Tr}_1(Z_X^T S_X) = Z_X^{ab} S_X^{ac}, \\
\Sigma_{B-L}^{bc} &\equiv \mathrm{Tr}_1(Z_{B-L}^T S_X) = Z_{B-L}^{ab} S_X^{ac}, \qquad \Sigma'^{bc}_{B-L} \equiv \mathrm{Tr}_1(S_{B-L}^T Z_X) = S_{B-L}^{ab} Z_X^{ac} \\
\Sigma_{T_{3R}}^{bc} &\equiv \mathrm{Tr}_1(Z_{T_{3R}}^T S_X) = Z_{T_{3R}}^{ab} S_X^{ac}, \qquad \Sigma'^{bc}_{T_{3R}} \equiv \mathrm{Tr}_1(S_{T_{3R}}^T Z_X) = S_{T_{3R}}^{ab} Z_X^{ac}
\end{aligned} \tag{5.10}$$

We can also form effective identity operators of $SO(10)_2$, such as $I = \mathrm{Tr}_1(S_X^T S_X)$ or

$I' = \mathrm{Tr}_1(Z_X^T Z_X)$. Reciprocally, we can form effective adjoint and identity operators of $SO(10)_1$. All of these effective tensors can arise physically from integrating out heavy states which transform under one of the $SO(10)$'s. For example, we can generate the structure $\mathrm{Tr}_1(Z^T Z')$ by integrating out the heavy states 10_1 and $10'_1$ from the following superpotential:

$$M_1 10_1 10'_1 + 10_1 Z 10_2 + 10'_1 Z' 10'_2 \longrightarrow \frac{1}{M} 10_2 \mathrm{Tr}_1(Z^T Z') 10'_2 \tag{5.11}$$

Once we are equipped with these effective rank two tensors, it is possible to construct supersymmetric GUT models with realistic fermion masses and CKM angles. A systematic analysis of the construction of $SO(10)$ GUT models has been done by Anderson *et al.* [16]. Our treatment, with the Σ, Σ', I and I' effective fields, now maps directly onto that analysis.

5.3 A SUSY $SO(10) \times SO(10)$ GUT model

In this section, we present an example based on the S_X and Z VEVs which demonstrates that typical SUSY $SO(10)$ GUT predictions can actually be preserved in $SO(10)_1 \times SO(10)_2$ gauge theories with experimentally acceptable Yukawa matrices. We assume four fundamental Higgs fields S_X, Z_X, Z_{B-L}, and $Z_{T_{3R}}$ of representation dimensionality $(10, 10)$ in our $SO(10)^2$ GUT model. We construct the superpotential so that each of the $(10, 10)$ Higgs fields acquires a VEV along the indicated direction as described in Section 5.2.

5.3.1 Higgs doublet-triplet splitting

The Higgs structure is constructed by the requirement of Higgs doublet-triplet splitting. Higgs triplets, if they are not heavy enough, could contribute to the evolution of the gauge couplings, and thus spoil the unification of the gauge couplings. In addition, Higgsino triplets may also mediate fast proton decay. So we might begin by analyzing the constraints imposed by the splitting mechanism.

In conventional $SO(10)$ models, Higgs triplet fields may acquire heavy masses by coupling to the adjoint fields which have their VEVs along the $B-L$ direction

$$\begin{aligned} W(H_1, H_2) &= H_1 A H_2, \qquad \text{with} \\ < A > &= V \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \mathrm{diag}(1,1,1,0,0), \end{aligned} \tag{5.12}$$

where H_1 and H_2 are the fundamental Higgs fields, and A denotes the adjoint Higgs field which acquires its VEV of the Dimopolous-Wilczek forms. As seen from Eq. (5.12), the triplet fields in H_1 and H_2 get heavy masses V and splitted from their doublet partners.

In our $SO(10) \times SO(10)$ model, among the four fundamental Higgs, Z_{B-L} and $Z_{T_{3R}}$ acquire their VEVs of the Dimopoulos-Wilczek (DW) forms through the stabilization of a tree level superpotential as in Eq. (5.4). However, the DW forms of VEVs may be seriously destabilized when some cross coupling terms, such as $\mathrm{Tr}(Z^T_{B-L} Z_{T_{3R}}) \equiv Z^{ab}_{B-L} Z^{ab}_{T_{3R}}$, $\mathrm{Tr}(Z^T_X Z_{B-L})$, $\mathrm{Tr}(Z^T_X Z_{T_{3R}})$, and $\mathrm{Tr}(Z^T_X Z_{B-L})$ are present in the superpotential. For instance, the presence of the term $\mathrm{Tr}(Z^T_X Z_{T_{3R}})$ would destabilizes the gauge hierarchy in $Z_{T_{3R}}$ since the F-flatness condition $F_{Z_{T_{3R}}} = 0$ would give a term proportional to Z_X. As a result, these cross coupling terms must be excluded to implement the DW mechanism for the Higgs doublet-triplet splitting problem. Although SUSY allows unwanted superpotential terms to be dropped by hand, it is less arbitrary to forbid them by a discrete symmetry.

Barr [82] has suggested that a discrete symmetry may do the job of forbidding the above cross coupling terms. In our model, there is a possible choice $K = Z_2^{T_{3R}} \times Z_2^{T'_{3R}} \times Z_2^{B-L} \times Z_5^1 \times Z_5^2$, and under which the various Z fields transform as

$$\begin{aligned} Z_2^{T_{3R}} &: \quad Z_{T_{3R}} \to -Z_{T_{3R}} \\ Z_2^{T'_{3R}} &: \quad Z_{T_{3R}} \to -Z_{T_{3R}} \\ Z_2^{B-L} &: \quad Z_{B-L} \to -Z_{B-L} \\ Z_5^1 &: \quad S_X \to e^{6\pi i/5} S_X \\ Z_5^2 &: \quad (S_X, Z_X) \to e^{2\pi i/5}(S_X, Z_X). \end{aligned} \tag{5.13}$$

The Z_2^{B-L} and $Z_2^{T_{3R}}$ symmetries in Eq. (5.13) are designed to forbid the dangerous cross coupling superpotential terms noted above but still allow the coupling terms at the quartic level

$$\begin{aligned} \mathrm{Tr}(Z_{B-L}Z_{B-L}^T)\mathrm{Tr}(Z_{T_{3R}}Z_{T_{3R}}^T) \quad &, \quad [\mathrm{Tr}(Z_{B-L}Z_{T_{3R}}^T)]^2 \\ \mathrm{Tr}(Z_{B-L}Z_{B-L}^T Z_{T_{3R}}Z_{T_{3R}}^T) \quad &, \quad \mathrm{Tr}(Z_{B-L}Z_{T_{3R}}^T Z_{B-L}Z_{T_{3R}}^T). \end{aligned} \tag{5.14}$$

The terms in Eq. (5.14) might change the values of the scales appearing in $< Z_{B-L} >$ and $< Z_{T_{3R}} >$, but they do not destabilize the DW forms of VEVs. The last two terms of Eq. (5.14) would have zero contribution to the F-flatness conditions. However, they remove the would-be Goldstone modes that are not eaten by the gauge bosons in the fields Z_{B-L} and $Z_{T_{3R}}$. The $Z_2^{T_{3R}'}$ discrete symmetry prevents the effective identity operator $\mathrm{Tr}_1(Z_X^T Z_{T_{3R}})$ from coupling to the spinorial superheavy states Ψ_1 and $\bar{\Psi}_7$ in our model. However, this $Z_2^{T_{3R}'}$ symmetry is basically construction-dependent and may not be necessarily introduced into our $SO(10)^2$ model. We will come to this point again when discussing fermion spectrum in the next subsection. In general, the symmetry K would keep fields S_X and Z_X from coupling to Z_{B-L} and $Z_{T_{3R}}$ up to a very high order, *e.g.* $\mathrm{Tr}(Z_X Z_{B-L})\mathrm{Tr}(Z_X Z_{B-L} S_X^8)$ as implied by Table 5.1. Thus the DW forms of VEVs are protected up to corrections of the order of the weak scale when the GUT gauge group breaking parameters v_D/M, v_{10}/M and v_5/M are sufficiently small.

An explicit superpotential giving Higgs doublet-triplet splitting by the above mechanism is:

$$W_{DT} = 10_H Z_{B-L} 10_{H'} + 10_{H'} S_X Z_{T_{3R}} 10_{H''} + X 10_{H''} 10_{H''} \tag{5.15}$$

where 10_H, $10_{H'}$ and $10_{H''}$ denote Higgs fields in (1,10), (10,1), (10,1) representations respectively. X is a gauge singlet[1] that acquires a GUT scale VEV and this makes $10_{H''}$ superheavy. The introduction of the singlet X is required by the fact that if

[1]The singlet X may or may not be the effective rank two tensor fields I or I' depending on how the K symmetry is chosen in our model.

$10_{H''}10_{H''}$ is a singlet and present in superpotential, so is the non-renormalizable term $S_X S_X 10_{H'}10_{H'}$. This term $S_X S_X 10_{H'}10_{H'}$, if exists, will give superheavy mass to the triplet states living in $10_{H'}$ and generates heavy Higgsino triplets exchange diagrams that mediate proton decay and spoil the strong suppression of proton decay. As in generating the effective rank two tensors, the non-normalizable term in Eq. (5.15) may rise from integrating out heavy states in the (1, 10) representation. The insertion of the field S_X in this term is designed to protect 10_H from coupling $10_{H''}$ to a high order level. In order to achieve DW mechanism, these Higgs fields must transform non-trivially under the discrete symmetry K. In general, there are many possible ways of assigning K charges to all fields in our model. One assignment for the K charges is given and can be found in Table 5.1

Here it is clear that the discrete symmetry $Z_5^1 \times Z_5^2$ would play the role of forbiding unwanted terms in superpotential. According to Table 5.1, the Higgs mass terms $M_{HH}10_H 10_H$ and $M_{H'H'}10_{H'}10_{H'}$ are forbidden by this discrete symmetry up to $(< XS_X^8 > /M^8 + < X^3 S_X^2 Z_X^2/M^6 >)10_H 10_H$ and $< S_X^2 X^4 > /M^5 10_{H'}10_{H'}$ respectively, and $M_{HH''}10_H 10_{H''}$ are very highly suppressed by the discrete symmetry K. Therefore, up to the order of weak scale, the mass matrix M_{H_T} and M_{H_D} for Higgs triplets and doublets are given as

$$M_{H_T} = \begin{pmatrix} 0 & < Z_{B-L} > & 0 \\ < Z_{B-L} > & \frac{<S_X^2 X^4>}{M^5} & 0 \\ 0 & 0 & < X > \end{pmatrix}, M_{H_D} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{<S_X^2 X^4>}{M^5} & < \frac{S_X Z_{T_{3R}}}{M} > \\ 0 & < \frac{S_X Z_{T_{3R}}}{M} > & < X > \end{pmatrix}. \tag{5.16}$$

Therefore, as from Eq. (5.16), only one pair of the doublets in 10_H would remain light after the breaking of the GUT gauge group. However, as seen from Eq. (5.16), one pair of the heavy Higgs triplets may receive a GUT scale $M_G \sim v_5$ mass, while the corresponding Higgs doublets fields receive a mass $v_D^2 v_5^2 / < X >$ which is less than the scale v_5. This may affect the gauge unification in our model depending upon the scale hierarchy between the two masses. Actually, this mass discrepancy results from forbidding dangerous high order nonrenormalizable operators which also contribute to the mass matrices M_{H_T} and M_{H_D}. If we assume that only renormalizable terms in superpotential are allowed at the superheavy scale M and all high order terms are generated from the Heavy Fermion Exchange mechanism (HFE) [89], than the scale ratio $< S_X > /M$ can be of order $O(1)$ thus all heavy Higgs states, doublets and triplets, would have GUT scale masses $m \sim v_5$ and we have the gauge unification as that of the minimal supersymmetric model (MSSM). In this paper, we simply assume negligible effects on gauge unification caused by the mass discrepancy among the heavy Higgs multiplets.

Finally, we discuss the implications for proton decay. Eq. (5.16) also implies a strong suppression of proton decay. Since the high order operator $S_X^2 X^4 10_{H'} 10_{H'}/M^5$ is present, then the dimension 5 operators (dimension 4 in superpotential) that mediate proton decay are formed by exchanging heavy Higgsino triplets

$$\frac{\lambda}{M^*} QQQL.., \tag{5.17}$$

with the effective mass M^*

$$M^* \approx \frac{M^5 < Z_{B-L} >^2}{< S_X^2 X^4 >} \sim 10^{31} \text{ GeV} >> M_{pl}. \tag{5.18}$$

Here we use Q and L to represent the associated quarks and leptons in proton decay processes. The estimated value for M^* in Eq. (5.17) is obtained by assuming $M \sim M_{pl}$, $< Z_{B-L} > / < S_X > \sim 10^{-2}$, and $< X > /M \sim 10^{-4}$. The coupling strength parameter $\lambda \sim 10^{-7}$ comes from multiplying the associating Yukawa coupling constants in the color-Higgs exchange Feynman diagrams. To saturate current experiment limits on proton decay [2], , the coupling strength λ/M^* for the dimension 5 operators should be no large than about 10^{-24} GeV^{-1}. Obviously, the estimated strength in Eq. (5.17) is far more less than the limit, therefore proton decay is highly suppressed in our model.

Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6
(+,+,2,-2)	(-,-,2,-1)	(-,+,2,-2)	(+,+,-1,-2)	(-,+,1,-2)	(-,+,-2.2)
$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	$\bar{\Psi}_4$	$\bar{\Psi}_5$	$\bar{\Psi}_6$
(-,+,0,0)	(+,-,0,-1)	(-,+,2,0)	(+,+,0,0)	(-,+,-2,0)	(-,+,1,1)
Ψ_7	Ψ_8	S_X	Z_X	Z_{B-L}	$Z_{T_{3R}}$
(+,+,1,2)	(-,+,-1,2)	(+,+,3,1)	(-,+,0,1)	(+,-,0,0)	(-,+,0,0)
$\bar{\Psi}_7$	$\bar{\Psi}_8$	X_S	16_1	16_2	16_3
(+,+,-2,1)	(-,+,0,1)	(+,-,-2,-1)	(+,+,-2,0)	(+,+,2,2)	(+,+,2,0)
X	10_H	$10_{H'}$	$10_{H''}$		
(+,+,-1,2)	(+,+,1,0)	(+,-,-1,0)	(-,-,-2,-1)		

Table 5.1: Fields transforming under the discrete symmetry $Z_2^{T_{3R}} \times Z_2^{B-L} \times Z_5^1 \times Z_5^2$. All fields are $Z_2^{T'_{3R}}$ singlets except for the fields $Z_{T_{3R}}$ and $10_{H''}$.

5.3.2 Fermion masses

Anderson *et al.* [16] showed that, with adjoint operators Σ in a SUSY GUT $SO(10)$ gauge theory, experimentally acceptable fermion mass spectrum as well as CKM angles can be obtained when these fields acquire their VEVs and break the GUT $SO(10)$ gauge group down to Standard Model gauge group [16]. We can generate the same Yukawa matrices by using effective higher dimension operators. These can be obtained by integrating out heavy fields. Then, following the choices made by Anderson *et al.*, we show that viable fermion mass matrices, such as those incorporating Georgi-Jarlskog ansatz [16, 11], can be constructed in the $SO(10)_1 \times SO(10)_2$ model.

We need to introduce additional heavy fields in the **16** and $\overline{\mathbf{16}}$ of $SO(10)_2$. We assume that all other matter multiplets also transform under the gauge group $SO(10)_2$. From Table 5.1 , it is easy to see that non-renormalizable terms at the quartic level, for instance the $\Psi_1 \mathrm{Tr}_1(Z_X^T S_X)\bar{\Psi}_1$, are allowed to occur in our $SO(10)_1 \times SO(10)_2$ model. This term may come from integrating out a pair of superheavy spinorial fields $\Psi'_1(16, 1)$ and $\bar{\Psi}'_1(\overline{16}, 1)$ from the renormalizable superpotential

$$W \supset M'_1 \Psi'_1 \bar{\Psi}'_1 + \Psi_1 S_X \bar{\Psi}'_1 + \Psi'_1 Z_X \bar{\Psi}_1. \tag{5.19}$$

where M_1' denotes the super-heavy mass of Ψ_1' and $\bar{\Psi}_1'$. At the renormalizable level with the generated quartic terms, the most general tree level superpotential consistent with the discrete symmetry K in Table 5.1 and responsible for giving masses to quarks and leptons has the form

$$\begin{aligned} W_{mass} \supset \; & 16_3 16_3 10_H + 16_3 \Sigma_{B-L} \bar{\Psi}_2 + 16_2 \Psi_1 10_H + 16_2 \Sigma_X \bar{\Psi}_8 + 16_1 \Sigma_X \bar{\Psi}_3 \\ + \; & \Psi_1 \Sigma_X \bar{\Psi}_1 + \Psi_2 \Sigma_X \bar{\Psi}_2 + \Psi_2 \Sigma_{B-L} \bar{\Psi}_1 + \Psi_3 \Sigma_X \bar{\Psi}_4 + \Psi_4 \Sigma_X \bar{\Psi}_5 + \Psi_5 \Psi_6 10_H \\ + \; & \bar{\Psi}_6 \Sigma_X \Psi_7 + \bar{\Psi}_7 \Sigma_X \Psi_8 + X_S 16_2 \bar{\Psi}_2 + \sum_{i=3}^{8} \Psi_i \cdot I \cdot \bar{\Psi}_i, \end{aligned} \tag{5.20}$$

where the gauge singlet field X_S is introduced to give mass to 16_2 and $\bar{\Psi}_2$ when acquiring a superheavy VEV.

From Eq. (5.20), only the third family matter multiplet 16_3 could get a mass of weak scale due to the discrete symmetry K. When the effective adjoint operators Σ_X and Σ_{B-L} acquire their VEVs, the spinorial fields $\Psi_i, \bar{\Psi}_i$ become heavy and can be integrated out in the low energy effective theory. The higher dimension operators O_{ij} that give masses to matter quarks and leptons are thus generated after diagonalizing the mass matrices of these superheavy spinorial fields [16].

$$\begin{aligned} O_{23} &= 16_2 10_H \frac{\Sigma_{B-L}^2}{\Sigma_X^2} 16_3 \\ O_{22} &= 16_2 10_H \frac{X_S \Sigma_{B-L}}{\Sigma_X^2} 16_2 \\ O_{12} &= 16_1 (\frac{\Sigma_X}{I})^3 10_H (\frac{\Sigma_X}{I})^3 16_2. \end{aligned} \tag{5.21}$$

The generation for the O_{ij} operators is much easier to be seen from the diagrams in Fig.(5.1). As seen from Eq. (5.21), fermion mass hierarchy is explained due to the hierarchy of the GUT breaking scales $M > v_D > v_{10} > v_5$. The effective adjoint operators Σ_X and Σ_{B-L} act on fermion states and give different quantum numbers to the states as described in Table 3.1. As a result, Eq. (5.21) leads to the following

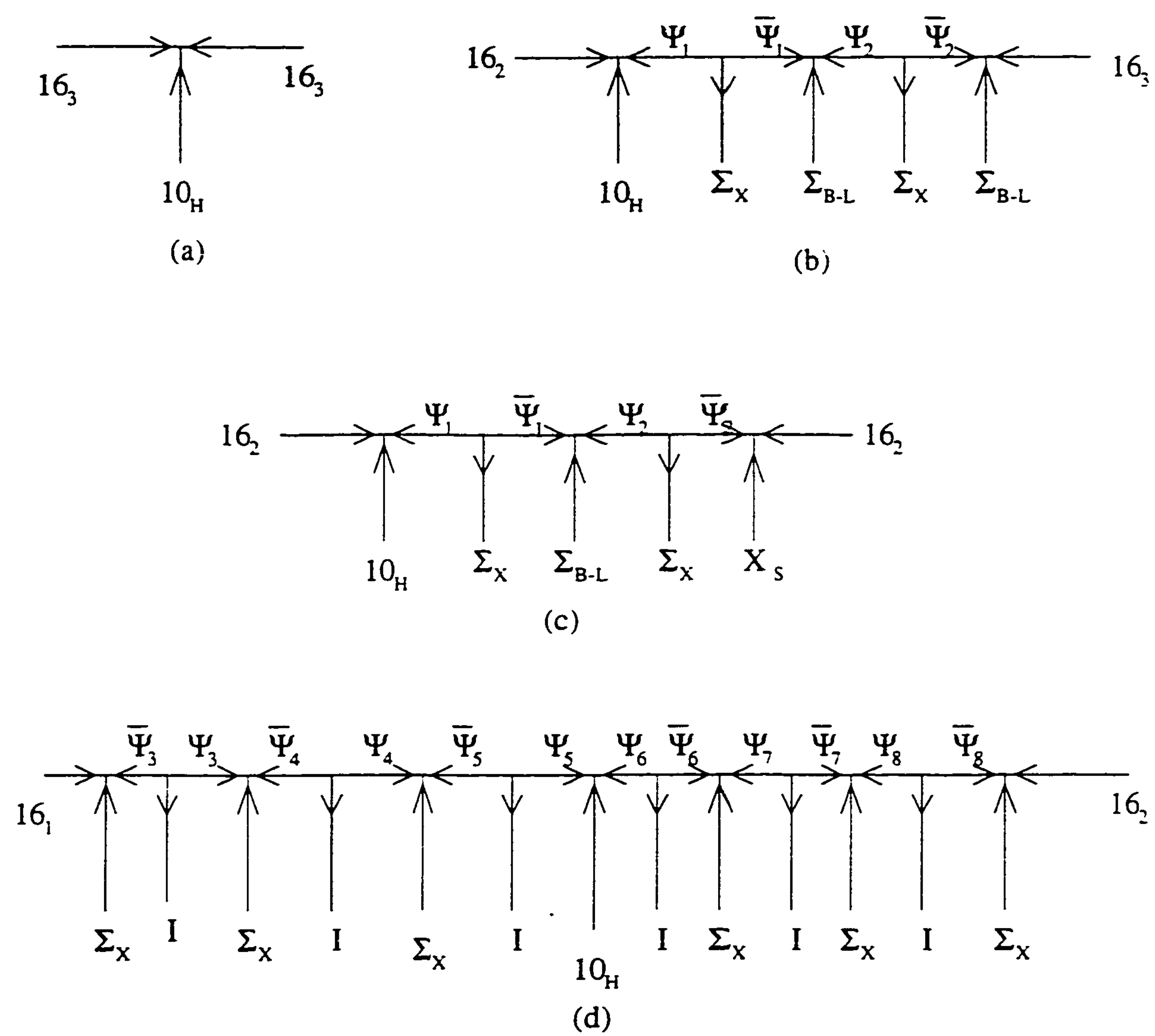

Figure 5.1: Operators O_{ij} that give Yukawa matrices are formed by exchanging heavy fermion states.

typical Georgi-Jarlskog ansatz for the Yukawa matrices at GUT scale

$$M_u = <H> \begin{bmatrix} 0 & \frac{1}{27}C & 0 \\ \frac{1}{27}C & 0 & B \\ 0 & B & A \end{bmatrix}, \qquad M_d =<\bar{H}> \begin{bmatrix} 0 & -C & 0 \\ -C & E & B \\ 0 & \frac{1}{9}B & A \end{bmatrix}.$$

$$M_e = <\bar{H}> \begin{bmatrix} 0 & -C & 0 \\ -C & 3E & B \\ 0 & 9B & A \end{bmatrix}, \tag{5.22}$$

with $A \approx O(1)$, $B \approx v_5^2/v_{10}^2$, $C \approx 27(v_{10}^6/v_D^6)$, and $E \approx v_5 M < X_S > /(v_D v_{10}^2)$. As seen from Eq. (5.22), we have the following successful GUT relations

$$\lambda_t = \lambda_b = \lambda_\tau \tag{5.23}$$

$$\lambda_{22}^u : \lambda_{22}^d : \lambda_{22}^e = 0 : 1 : 3 \tag{5.24}$$

$$m_\tau = m_b, \quad , m_\mu \approx 3m_s, \quad m_d \approx 3m_e. \tag{5.25}$$

where λ's denote the effective Yukawa coupling constants for corresponding mass operators.

Conclusively, it is suggested that the breaking of our GUT model is arranged as

$$SO(10)_1 \times SO(10)_2 \xrightarrow{v_D} SO(10)_D \xrightarrow{v_{10}} SU(5)_D \times U(1)_D$$
$$\xrightarrow{v_5} SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{5.26}$$

with approximate ratios $v_D/M \sim 1/30$, $v_{10}/v_D \sim O(10^{-1})$, $v_5/v_{10} \sim O(10^{-1})$, and $< X_S >=< X >\approx v_D v_5/M$. Detailed analysis for the mass operators O_{ij} can be found in [16], and will not be discussed in this paper.

As in more familiar GUT $SO(10)$ models, we can also analyze the neutrino masses in our $SO(10)_1 \times SO(10)_2$ model. First we observe that the matrix $M_{\nu^c\nu}$ for Dirac masses of neutrinos has a nonzero 22 entity also coming from O_{22}, and is far from identical to the up-quark mass matrix.

$$M_{\nu^c\nu} = <H> \cdot \begin{bmatrix} 0 & -125C & 0 \\ -125C & -\frac{6}{25}E & B \\ 0 & \frac{9}{25}B & A \end{bmatrix} \tag{5.27}$$

Since $125C$ is almost the same order of magnitude as A, the Dirac mass matrix for neutrino is no longer as hierarchical as quark and charged lepton mass matrices. To form Majorana mass for the right handed neutrinos, we introduce a set of spinorial Higgs fields $\Psi_{V_i}(1,16)$, $\bar{\Psi}_{V_i}(1,\overline{16})$ which VEVs preserve the $SU(5)_2$ subgroup of $SO(10)_2$. In general, the following neutrino mass operators can also be formed from heavy fermion exchanges

$$\frac{1}{M}\sum_{i,j}(\bar{\Psi}_{V_i}\Gamma_a^{\overline{(126)}}\bar{\Psi}_{V_j})(16_i\Gamma_a^{(126)}16_j). \tag{5.28}$$

where i, j are flavor indices. For simplicity, we would assume the Majorana mass matrix M_R for right handed neutrinos to be a diagonal matrix. Thus, from Eqs (5.27) and (5.28), the effective left handed Majorana mass matrix is

$$M_{\nu\nu} \approx M_{\nu^c\nu}^{+} M_R^{-1} M_{\nu^c\nu}. \tag{5.29}$$

Taking $C/E \approx 0.22 \approx 6/25$ as implied by the Cabibbo angle, it thus lead to the following three Majorana eigenmasses for left handed neutrinos

$$m_{\nu_\tau} \approx \frac{m_t^2}{m_{R_3}}, \quad m_{\nu_\mu} \approx \frac{(125 m_d tan\beta)^2}{m_{R_2}}, \quad m_{\nu_e} \approx \frac{(125 m_d tan\beta)^2}{m_{R_1}}. \tag{5.30}$$

Although all the VEVs of Ψ_{V_i} need not to be the same, we might take all $< \Psi_{V_i} >$ to be equal to v_{10} for illustration. This gives $m_{R_i} = m_R \approx 2 \times 10^{14}$ GeV and leads to $m_{\nu_\tau} \sim 1/20$ eV, $m_{\nu_\mu} \approx m_{\nu_e} \sim O(10^{-3}$ eV) for $\tan\beta = 45$. Visible $\nu_\tau - \nu_\mu$ and $\nu_\tau - \nu_e$ oscillations with very small neutrino oscillation angles $\sin^2 2\theta_{\tau\mu}^{osc}$ and $\sin^2 2\theta_{\tau e}^{osc}$ are favored when taking such assumption. However, other mass spectra for left handed neutrinos as well as large neutrino mixing angles may be obtained [90, 25]. This is because the right handed neutrino mass matrix M_R may itself be nontrivial and have

	$SO(10)_1$	$SO(10)_2$	$SO(10)_3$
S_X		10	10
Z_X		10	10
Z_{B-L}		10	10
$Z_{T_{3R}}$		10	10
Z'_{B-L}	10	10	
$Z'_{T_{3R}}$	10	10	
10_H			10
$10'_H$		10	
$10''_H$		10	
16_1	16		
16_2		16	
16_3			16

Table 5.2: The field content of the $SO(10)_1 \times SO(10)_2 \times SO(10)_3$ model.

a hierarchical structure in our $SO(10)^3$ model.

5.4 An SO(10) × SO(10) × SO(10) model

It is straightforward to extend the GUT gauge group to $SO(10)_1 \times SO(10)_2 \times SO(10)_3$ and have all matter multiplets transform under one of the $SO(10)$ gauge groups. However, this extension is basically a replication of the SUSY GUT $SO(10)_1 \times SO(10)_2$ model described in the previous sections. Different from the above direct generalization, in this section, we assign each matter multiplet 16_i to transform under different gauge group $SO(10)_i$. We also assume the existence of the three Higgs fields $10_H(1,1,10)$, $10_{H'}(1,10,1)$, and $10_{H''}(1,10,1)$, and a set of fundamental Higgs fields $S_X(1,10,10)$, $Z_X(1,10,10)$, $Z_{B-L}(1,10,10)$, $Z_{T_{3R}}(1,10,10)$, $Z'_{B-L}(10,10,1)$, and $Z'_{T_{3R}}(10,10,1)$ for implementing the DW mechanism. The complete set of assignment is shown in Table 5.2.

The fundamental Higgs fields acquire their VEVs along some GUT breaking directions as described in the previous sections. To protect the DW forms of the VEVS, some discrete symmetries above the GUT scales must typically be expected to restrict possible tree level superpotential terms. Without giving the discrete symmetries explicitly, we note that the superpotential responsible for giving heavy masses to Higgs triplet fields must be restricted to the following form:

$$10_H Z_{B-L} 10_{H'} + \frac{1}{M^2} 10_{H'} S_X Z_{T_{3R}} 10_{H''} + X 10_{H''} 10_{H''}, \tag{5.31}$$

where M denotes the superheavy scale and X is again a gauge singlet with a GUT scale VEV. By the HFE mechanism mentioned in Section 5.3. the second term in Eq. (5.31) may also come from integrating out some superheavy states. In the worst case. if all allowed nonrenormalizable operators are present in superpotential. the gauge hierarchy as well as the DW forms of VEVs could still be protected up to a very high order by some discrete symmetries. As a result, only the pair of the Higgs doublets states in 10_H remain light down to the weak scale and proton decay could be suppressed strongly.

In the following, we will briefly discuss the construction of realistic fermion mass matrices without going into details of how the fields transform under the needed discrete symmetries.

As usual, only the third family of matter multiplet 16_3 gets a weak scale mass through the tree level dimension four operator $O_{33} = 16_3 16_3 10_H$. Other O_{ij} operators are generically nonrenormalizable because 16_1 and 16_2 both carry no $SO(10)_3$ gauge quantum numbers. However, it is impossible to form O_{ij} operators for the off-diagonal entries of fermion mass matrices by simply using matter multiplets and the Higgs fields in fundamental representations. A set of additional heavy fields in the $\mathbf{16} + \overline{\mathbf{16}}$, Ψ_{V_i} and $\bar{\Psi}_{V_i}$, which transform under the GUT gauge group as $(16, 1, 1)$, $(\overline{16}, 1, 1)$, $(1, 16, 1)$, $(1, \overline{16}, 1)$, $(1, 1, 16)$ and $(1, 1, \overline{16})$, must be introduced into the model and acquire VEVs along the $SU(5)_i$ singlet directions. The VEV's can be stabilized by the superpotential of the following form

$$Y(\Psi_V \bar{\Psi}_V)^2 / M_V^2 + f(Y). \tag{5.32}$$

where Y is a singlet field and $f(Y)$ is a polynomial function that contains a linear term. Notice that the would-be Goldstone modes in the spinors Ψ_{V_i} and $\bar{\Psi}_{V_i}$ can be removed by adding superpotential terms similar to Eq. (3.45).

Although there are many possible nonrenormalizable operators which may or may not survive from imposing the discrete symmetries, the following high dimensional operators could also arise from the HFE mechanism, and are interesting because they may help to realize the Gerogi-Jarlskog type of Yukawa matrices in the model.

$$O_{23} = (\Psi_{V_2} \cdot S_X \cdot S_X \cdot 16_2) \cdot (\Psi_3 \cdot 10_H \cdot \frac{1}{\Sigma_X^2} 16_3) \tag{5.33}$$

$$O_{22} = (16_2 \cdot S_X \cdot \frac{\Sigma_{B-L}}{\Sigma_X^2} 16_2) \cdot 10_H \tag{5.34}$$

$$O_{12}^{(1)} = (\Psi_{V_1} \cdot Z'_{B-L} \cdot 16_1) \cdot (\Psi_{V_2} \cdot S_X \cdot 16_2) \cdot 10_H \tag{5.35}$$

$$O_{12}^{(2)} = (\Psi'_{V_1} \cdot Z'_{T_{3R}} \cdot 16_1) \cdot (\Psi_{V_2} \cdot S_X \cdot 16_2) \cdot 10_H \tag{5.36}$$

$$O_{12}^{(3)} = (\Psi_{V_1} \cdot Z'_{B-L} \cdot Z'_{B-L} \cdot 16_1) \cdot (\Psi'_{V_2} \cdot S_X \cdot \frac{\Sigma_X^3}{I^3} 16_2) \cdot 10_H \tag{5.37}$$

$$O_{12}^{(4)} = (\Psi_{V_1} \cdot Z'_{T_{3R}} \cdot Z'_{T_{3R}} \cdot 16_1) \cdot (\Psi'_{V_2} \cdot S_X \cdot \frac{\Sigma_X^3}{I^3} 16_2) \cdot 10_H \tag{5.38}$$

$$O_{12}^{(5)} = (\Psi'_{V_1} \cdot Z'_{B-L} \cdot Z'_{T_{3R}} \cdot 16_1) \cdot (\Psi'_{V_2} \cdot S_X \cdot \frac{\Sigma_X^3}{I^3} 16_2) \cdot 10_H \tag{5.39}$$

Again, the effective adjoint operator Σ_X of the gauge group $SO(10)_2$ gives different quantum numbers to the fermion states in the matter multiplets 16_i. Since the Higgs fields Z'_{B-L} and $Z'_{T_{3R}}$ must at least carry different charges of some global Z_2 symmetry to avoid the breaking of gauge hierarchy, we thus need two additional VEV-acquiring spinors Ψ'_{V_i} and $\overline{\Psi'}_{V_i}$, where $i = 1, 2$, to make the operators O_{12} respect the Z_2 symmetry.

Let us parametrize the contributions of the operators $16_3 16_3 10_H$ and the O_{ij} as $A, B, E, \ldots\ldots$ In the case that only $16_3 16_3 10_H$ and the O_{ij} operators give dominant contributions to fermion masses, the fermion mass matrices become

$$
\begin{aligned}
M_u &= <H> \begin{bmatrix} 0 & C^{(3)} & 0 \\ C^{(5)} & 0 & B \\ 0 & B & A \end{bmatrix}, \qquad M_d =<\bar{H}> \begin{bmatrix} 0 & C^{(1)} & 0 \\ 27C^{(5)} & E & 0 \\ 0 & \frac{1}{9}B & A \end{bmatrix}, \\
M_e &= <\bar{H}> \begin{bmatrix} 0 & 27C^{(4)} & 0 \\ C^{(2)} & 3E & \frac{1}{9}B \\ 0 & 0 & A \end{bmatrix}, \\
M_{\nu^c\nu} &= <H> \cdot \begin{bmatrix} 0 & 27(C^{(3)}+C^{(4)}+C^{(5)}) & 0 \\ C^{(2)} & -\frac{6}{25}E & \frac{1}{9}B \\ 0 & 0 & A \end{bmatrix}.
\end{aligned} \tag{5.40}
$$

where A, B, E, and $C^{(i)}$ come from the contribution of the operators O_{33}, O_{23}, O_{22}, and $O_{12}^{(i)}$ respectively. Again, the numbers shown in Eq. (5.40) are Clebsch-Gordan coefficients. From the mass ratio m_u/m_d, we may estimate that the ratio $C^{(3)}/C^{(1)} \approx 1/27$. Therefore, to realize an experimentally acceptable fermion mass matrix, as implied from Eq. (5.40), the breakdown of the GUT gauge group $SO(10)_1 \times SO(10)_2 \times SO(10)_3$ may take the following steps

$$
\begin{aligned}
SO(10)^3 &\longrightarrow SU(5)_1 \times SU(5)_2 \times SU(5)_3 \quad \text{at } <\Psi_{V_i}> \sim M \\
&\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \quad \text{at } v_5 \approx \frac{1}{30}M,
\end{aligned} \tag{5.41}
$$

where $M \approx 6 \times 10^{17}$ GeV, $v_D \approx v_{10} \approx v_5$, and $<X_S>/v_5 \sim <X_S'>/v_5 \sim 10^{-1}$ are assumed. In this GUT group breaking scenario, the $SO(10)^3$ GUT gauge group would first be broken down to $SU(5)^3$ by the spinorial Higgs fields Ψ_{V_i} and $\bar{\Psi}_{V_i}$, and then breaks into the embedded diagonal subgroup $SU(3)_C \times SU(2)_L \times U(1)_Y$ at the GUT scale $M_G \approx v_5$.

Neutrinos may acquire masses by the same mechanism described in the previous section. A set of spinorial Higgs fields with non-vanishing VEVs along the $SU(5)_i$-preserving directions are necessary for giving Majorana masses to right handed neutrinos. However, none of the spinorial Higgs fields used in constructing the operators O_{ij} can be used in giving a Majorana mass to right handed τ neutrino ν_τ since, otherwise, we would get the Majorana mass for left handed τ neutrino $m_{\nu_\tau} \approx m_t^2/M \approx 1/6 \times 10^{-4}\mathrm{eV} \gg m_{\nu_\mu}, m_{\tau_e}$, which is disfavored by recent SuperKamiokande data [91]. Thus a new pair of spinorial Higgs fields Ψ'_{V_3} and Ψ'_{V_3} would be needed to give an acceptable mass to $\nu_\tau^c \nu_\tau^c$

$$\frac{1}{M}(\bar{\Psi}'_{V_3}\Gamma_a^{\overline{(126)}}\bar{\Psi}'_{V_3})(16_3\Gamma_a^{(126)}16_3). \tag{5.42}$$

with $< \Psi'_{V_3} >=< \bar{\Psi}'_{V_3} >\approx v_5$.

As before, a non-trivial Majorana mass matrix M_ν^R for right handed neutrinos may be present in the model, and heavily influence the Majorana mass spectrum as well as neutrino mixing angles of left handed neutrinos. We will not discuss this problem in detail in this paper.

5.5 Conclusion

Typical SUSY $SO(10)$ GUT models require a variety of rank two tensor fields, such as the fields in **45** and **54** representations, to be phenomenologically successful. These rank two tensors, when they acquire their VEVs and break the gauge $SO(10)$ group, also play important roles in implementing the Dimopoulos-Wilczek mechanism and in deriving experimentally acceptable Yukawa matrices. However, these representations are complicated, and it is usually difficult for all the needed rank two tensor fields to be generated by a simple string construction.

In this paper, we have shown that it is possible not only to implement the DW mechanism but also to provide experimentally acceptable Yukawa matrices. In our $SO(10)^2$ and $SO(10)^3$ GUT models, without introducing any rank two tensor fields, the Higgs doublet-triplet splitting problem is naturally solved with strong suppression of proton decay when some Higgs fields of fundamental representations acquiring their VEVs in Dimopoulos-Wilczek forms. Also, unlike other $SO(10)^2$ and $SO(10)^3$ models in the literatures [66, 81], effective adjoint operators of at least one of the $SO(10)$ gauge group can be formed when combining the S and one of the Z fields in our model. This allows us to construct realistic fermion mass matrices with successful GUT relations such as top-bottom-tau unification $\lambda_t = \lambda_b = \lambda_\tau$, $m_\mu = 3m_s$, and $m_d = 3m_e$.

On the neutrino mass problem, as in conventional $SO(10)$ theories, some spinorial Higgs fields in the **16** representation of the corresponding $SO(10)_i$ gauge group are necessary for making effective $\nu^c\nu^c$ mass operators. When acquiring VEVs that preserve subgroups $SU(5)_i$ for each corresponding $SO(10)_i$, these spinorial Higgs fields give superheavy Majorana masses to right handed neutrinos. Small Majorana masses for left handed neutrinos are thus generated from see-saw mechanism. However, further understandings on the neutrino sector will be needed in our models for constructing the mass matrix for right handed neutrinos, and also for understanding the mass hierarchy/splitting as well as the mixing angles among left handed neutrinos.

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