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ANOMALOUS BARYOGENESIS AT THE WEAK SCALE*

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* Ph.D. thesis

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Stanford University, 1991

Abstract

One of the fundamental constants of nature is the baryon asymmetry of the universe – the ratio of the number of baryons to the entropy. This constant is about 10^{-11}. In baryon-number conserving theories, this was just an initial condition. With the advent of grand unified theories (GUTs), baryon number is no longer conserved, and this asymmetry can be generated dynamically. Unfortunately, however, there are reasons for preferring another mechanism. For example, GUTs predict proton decay which, after extensive searches, has not been found. An alternative place to look for baryogenesis is the electroweak phase transition, described by the standard model, which posses all the necessary ingredients for baryogenesis.

Anomalous baryon-number violation in weak interactions becomes large at high temperatures, which offers the prospect of creating the asymmetry with the standard model or minimal extensions. This can just barely be done if certain conditions are fulfilled. CP violation must be large, which rules out the minimal standard model as the source of the asymmetry, but which is easily arranged with an extended Higgs sector. The baryon-number violating rates themselves are not exactly known, and they must be pushed to their theoretical limits. A more ex-

act determination of these rates is needed before a definitive answer can be given. Finally, the phase transition must be at least weakly first order. Such phase transitions are accompanied by the formation and expansion of bubbles of true vacuum within the false vacuum, much like the boiling of water. As the bubbles expand, they provide a departure from thermal equilibrium, otherwise the dynamics will adjust the net baryon number to zero. The bubble expansion also provides a biasing that creates an asymmetry on the bubble surface. Under optimal conditions, the observed asymmetry can just be produced.

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1. Introduction

With the advent of grand unified theories, there arose the hope of dynamically generating the baryon number of the universe, a quantity that had to be set by initial conditions in baryon-number conserving theories. Baryon-number violation alone, however, is not sufficient to generate a net baryon asymmetry. As first pointed out by Sakharov[1], there are two more necessary conditions. The theory must have C and CP violation, otherwise equal numbers of baryons and antibaryons are produced, giving no net increase. There must also be a departure from thermal equilibrium, otherwise the dynamics drives the system to equal mixtures of baryons and antibaryons. GUTs naturally violate baron number and CP. The phase transitions are typically strongly first-order, so a departure from thermal equilibrium is easily achieved. One can usually produce the observed ratio of baryon number to entropy, $\eta \equiv n_B/s \sim 10^{-11}$. This is one of the attractive features of GUTs. There is, however, one problem that cannot be overlooked: the proton has not yet been observed to decay. This completely rules out the minimal $SU(5)$ theory. That theory has other problems, the most notable being the hierarchy problem. The minimal supersymmetric extension solves the hierarchy problem and gives a proton life time consistent with observation. However, supersymmetric GUTs have their own set of problems. Furthermore, many non-minimal extensions soon become rather contrived, and the number of free parameters becomes so large, explaining one ratio at such a price is unsatisfying. In addition, the temperature range between the GUT and the weak scale is large enough that baryon-number violation proceeding through the electroweak anomaly will wash out any asymmetry, unless the initial $B - L$ is non-zero. While grand unification is a very beautiful idea, these nontrivial problems are motivation for an alternative method of baryogenesis.

The weak scale in minimal extensions of the standard model turns out to be a promising place to look. The standard model naturally possesses two of the three necessary conditions for baryogenesis: baryon number is not conserved (due to the axial vector anomaly) and CP violation is automatic. If the weak phase transition is not too weakly first-order, then a large enough departure from thermal equilibrium can be achieved. This is the case if the Higgs mass is not too large. It is clear, however, that CP violation will be too small in the minimal standard model to produce any thing like the observed asymmetry. But there are many extensions of the standard model, such as multi-Higgs theories, supersymmetric theories, technicolor theories, or the like, with ample CP violation. It then becomes a quantitative question as to whether the actual baryon-number violating rates themselves are large enough.

In this thesis, I examine the necessary conditions under which the baryon asymmetry may be generated at the weak scale. The key point is that a time-dependent Higgs field biases the baryon production and generates an asymmetry, the sign of which is determined by the sign of the CP violating parameter. The expansion of bubbles of true vacuum during a first order phase transition can generate this time dependence, as well as a sufficient departure from thermal equilibrium. If the baryon-number violating rates are not too small, the observed asymmetry can be produced this way. This is the subject of chapter 3. In chapter 4 I illustrate the general techniques with a simple $1 + 1$ dimensional model. I also show why a two-Higgs model will not yield anything like the observed asymmetry. In chapter

5 I find an upper bound on the Higgs mass by requiring that once the asymmetry is produced, the baryon-number violating rates turn off fast enough so as not to erase it. This is a relevant bound for minimal extensions of the standard model.

I will end this chapter with a brief review of baryon-number violation in the standard model. This is standard material and a nice review can be found in Ref. [3]. Due to the presence of axial couplings, baryon and lepton number are not conserved. The baryon-number current satisfies

$$\partial \cdot \mathcal{J}_B = \frac{g^2 N_f}{16\pi^2} \operatorname{tr}(F\tilde{F}), \tag{1.1}$$

where N_f is the number of flavors, and the dual field strength is defined as $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$.* Integrating (1.1) and discarding the surface terms of the baryonic current gives a change in the baryon number

$$\begin{aligned} \Delta B =& \frac{g^2 N_f}{16\pi^2} \int d^4x \operatorname{tr}(F\tilde{F}) && (1.2)\\ =& N_f \int d^4x\ \partial_\mu K^\mu && (1.3)\\ =& N_f \oint d\sigma_\mu K^\mu, && (1.4) \end{aligned}$$

where the last surface integral is taken over a large three-sphere, S_3, of infinite radius, and the Chern-Simons current is given by

$$K^\mu = \frac{g^2}{8\pi^2}\ \epsilon^{\mu\nu\alpha\beta} \operatorname{tr}[A_\nu(\partial_\alpha A_\beta - \frac{2}{3} ig\ A_\alpha A_\beta)]. \tag{1.5}$$

I will work in the static $A^0 = 0$ gauge, in which the only non-zero component of the Chern-Simons current is K^0. For finite-action gauge field configurations, $F = 0$ at

* I use the conventions: $\epsilon_{0123} = +1$; also, elements of the gauge group G are parametrized by $u(x) = \exp[ig\alpha(x) \cdot \mathbf{t}]$, the covariant derivative is $D_\mu = \partial_\mu - igA_\mu^a t^a$, and a general gauge transformation on the gauge fields is given by $A_\mu \to uA_\mu u^{-1} - \frac{i}{g}\partial_\mu u\ u^{-1}$.

spatial infinity. Using this restriction in (1.4) gives

$$\Delta B = N_f \; [N_{cs}(t=+\infty) - N_{cs}(t=-\infty)] = N_f \; \Delta N_{cs}, \tag{1.6}$$

where the Chern-Simons number for a field configuration $\mathbf{A}(\mathbf{x})$ is defined by

$$N_{cs}[\mathbf{A}] = \frac{ig^3}{24\pi^2} \int d^3\mathbf{x} \; \epsilon^{ijk} \; \mathrm{tr}(A^i \; A^j \; A^k). \tag{1.7}$$

Vacuum configurations $\mathbf{A}_{vac} = \frac{i}{g} \; \nabla u \; u^{-1}$ define a natural map $S_3 \rightarrow G$ when restricted to $\alpha(\mathbf{x}) \rightarrow 0$ as $|\mathbf{x}| \rightarrow \infty$. With this restriction, points in space can be thought of as lying on a three-sphere, and the induced vacuum map is simply $\mathbf{x} \mapsto u(\mathbf{x}) \in G$. The Chern-Simons number for these configurations is just the homotopy or winding number of this induced map. For semi-simple groups, such vacuum configurations can then be labeled by an integer, and the true quantum mechanical vacuum state is a linear superposition of the corresponding perturbative wave functionals, which each have support only over a definite winding number. The above restriction to classical vacuum configurations in which $u \rightarrow 1$ at spatial infinity can be justified *a posteriori*, since tunneling only mixes such states among themselves[4].

't Hooft[5] first calculated the tunneling rate between adjacent perturbative vacua in the standard model to be $\sim e^{-4\pi/\alpha_W} \sim 10^{-164}$ – which is to say, it never happens. This small number can be understood in terms of the very large potential barrier, of height $\sim M_W/\alpha_W \sim 10$ TeV, separating the perturbative vacua of definite winding number. In Ref. [9], it was shown that there exist static, unstable solutions to the field equations with one negative mode. These solutions are called sphalerons and represent saddle points of the potential-energy functional in field space. Fig. 1 illustrates the basic vacuum structure of a pure gauge theory. This interpretation of the sphaleron is further justified since it has a Chern-Simons number half way between that of the successive perturbative vacua flanking the sphaleron.

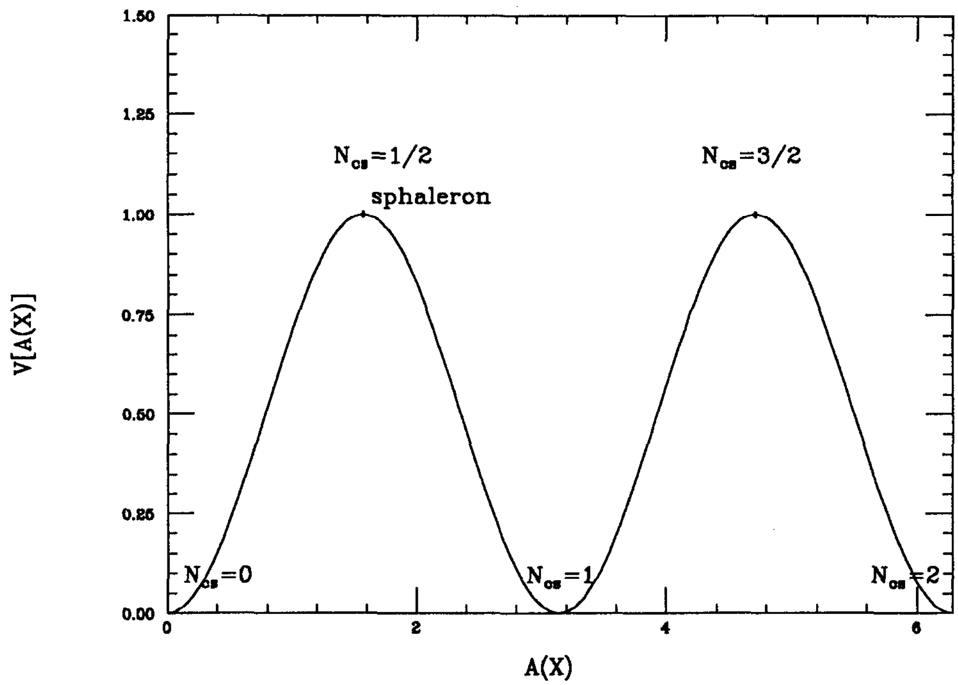

Figure 1. Vacuum structure of a gauge theory. The maxima represent sphaleron configurations.

In chapter 3 I will discuss the high temperature limit. It is quite probable that under these extreme conditions, gauge field configurations can simply sail over the barrier rather than tunnel through it, and then baryon-number violation becomes unsuppressed. The question is then whether this erases any previously generated asymmetry, or creates an asymmetry of its own.

2. Finite Temperature Field Theory

Because the calculations which follow involve field theory at finite temperatures, I will briefly review relevant aspects of the subject. The aim is to establish finite temperature Feynman rules and to use them to investigate symmetry restoration. In particular I will calculate the free energy to one loop. Finally, I will give a quick review of bubble formation in first order phase transitions, since this is a crucial ingredient of chapter 3. The results of this chapter may be found in Ref. [6].

Consider a field theory with a Lagrangian $\mathcal{L}(\partial\phi, \phi)$, where ϕ includes all the fields in the theory, both fermionic and bosonic. The statistical average of an operator $\mathcal{O}$, at temperature β^{-1}, is defined as

$$< \mathcal{O} >_\beta \equiv \frac{\mathrm{Tr}[T\ e^{-\beta H}\mathcal{O}]}{\mathrm{Tr}[e^{-\beta H}]}. \tag{2.1}$$

The usual time ordering is performed, and $\mathcal{O}(x_1, x_2, ..., x_n)$ is understood to be in the Heisenberg representation. This means that, formally, the factor $e^{-\beta H}$ acts as an imaginary-time development operator. In effect, it translates the system by $-i\beta$ units of imaginary time. It is not hard to show that in the in the imaginary-time direction, the statistical average is periodic (with period β) for boson fields and antiperiodic for fermion fields. For simplicity I will only consider $\mathcal{O}(x) = \phi(x)\phi(0)$, where ϕ may be either a boson or a fermion. First analytically continue to Euclidean space by defining $\phi_E(\tau) = \phi_H(t)|_{t\to -i\tau}$. Take $\tau < \beta$, and since the

Euclidean time development is given by $\phi_E(\tau) = e^{\tau H}\phi(0)e^{-\tau H}$,

$$\begin{aligned}
< \mathcal{O}_E(\tau) >_\beta \ \mathrm{Tr}[e^{-\beta H}] \equiv & \ \ \mathrm{Tr}[e^{-\beta H} \ \phi_E(\tau)\phi(0)] \\
= & \ \ \mathrm{Tr}[e^{-\beta H} \ e^{H\tau}\phi(0)e^{-H\tau} \ \phi(0)] \\
= & \pm\mathrm{Tr}[\phi(0) \ e^{(\tau-\beta)H}\phi(0)e^{-\tau H}] \\
= & \pm\mathrm{Tr}[\phi(0)\phi(\tau-\beta) \ e^{-\beta H}] \\
= & \pm\mathrm{Tr}[e^{-\beta H} \ T_\tau\phi(\tau-\beta)\phi(0)] \\
= & \pm < \mathcal{O}_H(\tau-\beta) >_\beta \ \mathrm{Tr}[e^{-\beta H}],
\end{aligned} \tag{2.2}$$

where T_τ is the Euclidean time-ordering operator, and the plus sign is for bosons and the minus sign for fermions. The Feynman rules take on a particularly simple form for operators that are analytically continued to imaginary time, which I denote by $\mathcal{O}_E(\mathrm{x},\tau)$. If real-time correlation functions are needed, analytic continuation may be performed back into Minkowski space. However, this procedure is delicate since the Euclidean Greens functions are only calculated approximately. The Feynman rules are most easily derived from the path integral approach, in which

$$< \mathcal{O}_E >_\beta = \frac{\int \mathcal{D}\phi \ e^{-S_E} \ \mathcal{O}_E}{\int \mathcal{D}\phi \ e^{-S_E}} \ , \tag{2.3}$$

where the Euclidean action is given by

$$S_E \ = \ \int_0^\beta d\tau \int d^3x \ \mathcal{L}_E \ = \ -i \int_0^{-i\beta} dt \int d^3x \ \mathcal{L}|_{t=-i\tau}, \tag{2.4}$$

and the bose(fermi) fields are taken to be periodic(antiperiodic) with period β. The finite temperature Feynman rules in Euclidean space are formally similar to the usual zero temperature ones. For simplicity I will consider $\lambda\phi^4$ theory: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - U(\phi)$ with,

$$U(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4. \tag{2.5}$$

Each four-point interaction vertex has an associated factor of $-\lambda$. Each internal propagator of momentum $p_n = (\omega_n, \mathbf{p})$ and mass m takes the form

$$\Delta_\beta(p_n) = \frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2}\ , \tag{2.6}$$

where $\omega_n = 2n\pi T$ for bosons and $(2n+1)\pi T$ for fermions. The appropriate spin structure must also be included in more general propagators. For example, in a fermion propagator, there is an additional factor of $\not{p}_n + m$, where the gamma matrices are now Euclidean, i.e. $\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$. For each internal loop of momentum $p_n = (\omega_n, \mathbf{p})$, there is an integral-sum of the form

$$T \sum_{n=-\infty}^{\infty} \int \frac{d^3\mathbf{p}}{(2\pi)^3}\ , \tag{2.7}$$

and at each vertex there is an energy-momentum conserving delta function of the form $\beta(2\pi)^3\delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2)\delta_{\omega_1,\omega_2}$. By convention, an overall delta function $\beta(2\pi)^3$ $\delta^{(3)}(\mathbf{p}_{in} - \mathbf{p}_{out})$ is factored out of the momentum-space Greens functions. These rules are easily obtained from the generating functional

$$Z[J] = N \int \mathcal{D}\phi\ e^{-\int_\beta \mathcal{L}_E + \int_\beta J\phi}\ , \tag{2.8}$$

where $\int_\beta = \int_0^\beta d\tau \int d^3\mathbf{x}$, and the normalization is chosen so that $Z[0] = 1$. The connected Greens functional is defined by $W[J] = \ln Z[J]$. To investigate symmetry restoration it is useful to define the *classical field*

$$\begin{aligned}\phi_{cl}(x) &\equiv <\phi(x)>_\beta \\ &= \frac{\delta W}{\delta J(x)}.\end{aligned} \tag{2.9}$$

In calculating (2.9), the Euclidean action is given by $S_E \rightarrow S_E - \int_\beta J\phi$, and ϕ_{cl} is then a functional of $J(x)$. If this classical field is nonzero for $J = 0$, then spontaneous symmetry breaking persists at finite temperature. To investigate this further, it is helpful to introduce the effective action defined by the functional Legendre transform of $W[J]$:

$$\Gamma[\phi_{cl}] = W[J] - \int_\beta J\phi_{cl}. \tag{2.10}$$

In the above, equation (2.9) is to be inverted to give $J(x) = J[\phi_{cl}; x]$. That is to say, J is a *functional* of ϕ_{cl} as well as a function of x. It is easy to show that

$$\frac{\delta\Gamma}{\delta\phi_{cl}(x)} = -J(x). \tag{2.11}$$

Therefore, nonzero solutions to $\delta\Gamma/\delta\phi_{cl} = 0$ signal spontaneous symmetry breaking at finite temperature. The one-point-irreducible (1PI) Greens functions are defined by

$$\Gamma[\phi_{cl}] = \sum_m \frac{1}{m!}\int_{\beta_1} \cdots \int_{\beta_m} \Gamma^{(m)}(x_1 \ldots x_n)\ \phi_{cl}(x_1) \ldots \phi_{cl}(x_n), \tag{2.12}$$

where $\int_{\beta_i} = \int_0^\beta d\tau_i \int d^3\mathbf{x}_i$. It is also useful to expand Γ in powers of derivatives:

$$\Gamma[\phi_{cl}] = \int_0^\beta d\tau \int d^3\mathbf{x}\big[-\Omega(\phi_{cl}) + Z(\phi_{cl})(\partial_\mu\phi_{cl})^2 + \cdots\big]. \tag{2.13}$$

Ω is called the effective potential, and at the minima, it is the thermodynamic potential density of the system. For zero chemical potential, however, this is just the free-energy density. For a translation invariant ground state, $\phi_{cl}(x) = const$, and it is sufficient that there exists a nonzero solution to $d\Omega/d\phi_{cl} = 0$ for the persistence of symmetry breaking. The momentum space 1PI Greens functions $\tilde{\Gamma}$ are defined by

$$\Gamma(x_1 \ldots x_m) = T\sum_{n_1}\int\frac{d^3\mathbf{p}_1}{(2\pi)^3}\cdots T\sum_{n_m}\int\frac{d^3\mathbf{p}_m}{(2\pi)^3}\; e^{i\sum p_i\cdot x_i}\;\beta(2\pi)^3\delta^{(3)}(\textstyle\sum \mathbf{p}_i)\;\delta_\omega\,\tilde{\Gamma}^{(m)}(p_1 \ldots p_m)\,, \tag{2.14}$$

where $x_i = (\tau_i, \mathbf{x}_i)$, $\omega = \sum \omega_i$, and $\delta_\omega = 1$ if $\omega = 0$ and vanishes otherwise. All inner products are with a Euclidean metric. From here on I will drop the subscript from ϕ_{cl} when no confusion will arise. Expanding $\tilde{\Gamma}^{(m)}(p_1 \ldots p_m)$ in a momentum power series and writing $\beta(2\pi)^3\delta^{(3)}(\sum \mathbf{p}_i)\;\delta_\omega = \int_\beta e^{i\sum p_i\cdot x}$, gives

$$\Omega = -\sum_m \frac{1}{m!}\tilde{\Gamma}^{(m)}(0)[\phi(x)]^m. \tag{2.15}$$

For nonzero m, the 1PI Greens functions in $\lambda\phi^4$ theory at zero momenta are given by

$$\tilde{\Gamma}^{(2m)}(0) = S_m\; T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[\frac{-\lambda}{p_n^2+m^2}\right]^m, \tag{2.16}$$

where $S_m = (2m)!/2^m(2m)$ is the symmetry factor associated with the number of ways of leaving the graph fixed upon interchanging external legs. In finite temperature field theory there is one more graph to consider: the noninteracting closed loop (m=0). At zero temperature this graph contributes an infinite constant, but at finite temperature it also gives a temperature dependent correction. This loop may be calculated by first calculating the noninteracting partition function and then using the relation $\Omega = -T \ln Z/V$,

$$\begin{aligned} \ln Z_{free} &= \ln \det{}_\beta(\partial^2 + m^2) \\ &= -\frac{1}{2} V \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln[\beta^2(p_n^2 + m^2)]. \end{aligned} \tag{2.17}$$

So, up to one loop, the free energy takes the form

$$\begin{aligned} \Omega &= \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{T}{2} \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \Big[\ln[\beta^2(p_n^2 + m^2)] + \\ &\qquad \ln[1 + \frac{\lambda\phi^2/2}{p_n^2 + m^2}] \Big] \\ &= U + \frac{T}{2} \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln[\beta^2(p_n^2 + m^2 + \lambda\phi^2/2)]. \end{aligned} \tag{2.18}$$

Define a ϕ-dependent frequency and mass by $\omega_p^2(\phi) = \mathbf{p}^2 + m^2 + \lambda\phi^2/2 = \mathbf{p}^2 + m^2(\phi)$. It will be useful in performing the frequency sum to get rid of the logarithm using the identity (Kapusta in Ref. [6])

$$\ln[\beta^2(\omega_n^2 + \omega_p^2)] = \int_1^{(\beta\omega_p)^2} \frac{d\theta^2}{\theta^2 + (2n\pi)^2} + \ln(1 + (2n\pi)^2). \tag{2.19}$$

The second term in (2.19) just gives a temperature independent contribution to the partition function and may be dropped. The frequency sum may be performed using the following relation (Kapusta in Ref. [6]):

$$\sum_{n=-\infty}^{\infty} \frac{1}{(n-x)(n-y)} = \frac{\pi[\cot \pi x - \cot \pi y]}{y - x}. \tag{2.20}$$

After some algebra, the one-loop correction becomes

$$\frac{T}{2} \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int_1^{(\beta\omega_p)^2} \frac{d\theta^2}{\theta^2 + (2n\pi)^2} = T \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\frac{\beta\omega_p}{2} + \ln(1 - e^{-\beta\omega_p}) \right]. \tag{2.21}$$

It is convenient to split the free energy as $\Omega = V_0 + V_T$, where:

$$V_0 = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2}\omega_p \tag{2.22}$$

$$V_T = \frac{T^4}{2\pi^2}\int_0^\infty dx x^2 \ln(1 - e^{-\epsilon}), \tag{2.23}$$

with $\epsilon = (x^2 + y^2)^{1/2}$, $x = \beta p$ and $y = \beta m(\phi)$. Notice that V_0 is just the zero temperature effective potential to one loop. This is apparent since up to an infinite ϕ-independent constant

$$\int \frac{d\omega}{2\pi} \ln\left[1 + \frac{\lambda\phi^2/2}{\omega^2 + \mathbf{p}^2 + m^2}\right] = \frac{1}{2}\omega_p. \tag{2.24}$$

Therefore,

$$V_0 = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \int \frac{d^4p_E}{(2\pi)^4} \ln\left[1 + \frac{\lambda\phi^2/2}{p_E^2 + m^2}\right]. \tag{2.25}$$

This is just the usual one-loop effective potential for $\lambda\phi^4$ theory. Since V_T is finite, $\Omega(T)$ is renormalized with the same counter terms as the zero temperature effective potential. This is a general feature of finite temperature Greens functions. If there were fermions coupled to the scalars, then appropriate sign changes must be performed. For a general number of bosons and fermions, the finite temperature contribution to the effective potential is

$$V_T = \frac{T^4}{2\pi^2}\left[\sum_b g_b I_-(y_b) + \sum_f g_f I_+(y_f)\right], \tag{2.26}$$

where g_b (g_f) is the number of degrees of freedom associated with a given boson (fermion) type, and $y_b(y_f)$ is the associated ϕ-dependent rescaled mass, and $I_\mp$ is defined by

$$I_{\mp}(y) = \pm \int_0^\infty dx\ x^2 \ln(1 \mp e^{-\epsilon}), \tag{2.27}$$

with $\epsilon = (x^2 + y^2)^{1/2}$. If the zero temperature potential V_0 is chosen to give spontaneous symmetry breaking, i.e. a minimum at a nonzero ϕ, then it can be arranged that finite temperature effects restore the symmetry above some critical temperature T_c.

This illustrates the main technical points. For most of this thesis, however, I will be concerned with the standard model, or minimal extensions. I will work in a gauge in which there is one real component of the Higgs field that takes a vacuum expectation value (VEV) of $v \approx$248 GeV. Including heavy fermion effects, the zero temperature effective potential to one loop is

$$V_0 = -\frac{\mu^2}{2}(1 - \frac{4B}{\lambda})\phi^2 + \frac{\lambda}{4}(1 - \frac{6B}{\lambda})\phi^4 + B\phi^4 \ln\left(\frac{\phi^2}{v^2}\right), \tag{2.28}$$

where

$$B = \frac{3}{64\pi^2}\left[2\left(\frac{M_W}{v}\right)^4 + \left(\frac{M_Z}{v}\right)^4 - 4\left(\frac{M_t}{v}\right)^4\right], \tag{2.29}$$

with $\alpha_W = g^2/4\pi \approx 1/30$. The Higgs mass is related to the VEV by $M_H^2 = 2\lambda v^2$, where $v = \mu/\sqrt{\lambda}$. In equation (2.29), g is the $SU(2)$ gauge coupling constant and M_t is the top quark mass. The W and Z gauge boson masses are related by $M_W = \frac{1}{2}gv$ and $M_Z = M_W/\cos\theta_W$. For a top mass $M_t \sim$ 100 GeV and a Higgs mass $M_H \sim$ 50 GeV, $B \sim 0.001$ and $\lambda \sim 0.02$. Quantum corrections are then some what important since $4B/\lambda \sim 0.2$. The renormalization conditions are*

$$\frac{dV_0}{d\phi}(v) = 0 \tag{2.30}$$

$$\frac{d^2V_0}{d\phi^2}(v) = M_H^2. \tag{2.31}$$

The finite temperature potential now becomes

$$V_T = \frac{T^4}{2\pi^2}\left[6I_-(y_w) + 3I_-(y_z) + 12I_+(y_t)\right], \tag{2.32}$$

where $y_i = M_i\phi/vT$. This is a general expression (up to one loop), valid for any temperature at which non-standard-model physics is unimportant. It is often useful to make a high temperature (small y) expansion of (2.32). The first two terms of the Taylor series expansion in y^2 are easy to find. However, there is a subtlety in the y^4 term. The functions $I_\pm$ are not analytic at $y = 0$, and the Taylor series expansion breaks down in the third term. There is a simple pole in $I_-''(y)$ and a logarithmic singularity in both $I_\pm''(y)$. This suggests there are cubic and logarithmic terms in the expansion. Dolan and Jackiw of Ref. [6] use a clever trick for dealing with this non-analyticity. They calculate $I_\pm''(y)$ at a non-zero but small argument and then integrate twice, using the known values of $I_\pm(0)$ and $I_\pm'(0)$ to fix the integration constants. The result of this calculation is

$$I_-(y) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}y^2 - \frac{\pi}{6}y^3 - \frac{1}{32}y^4\ln y^2 + \cdots \tag{2.33}$$

$$I_+(y) = -\frac{7}{8}\frac{\pi^4}{45} + \frac{\pi^2}{24}y^2 + \frac{1}{16}y^4\ln y^2 + \cdots . \tag{2.34}$$

* This convention differs some what from Shaposhnikov in Ref. [15] in which the self coupling is redefined to absorb the quantum corrections.

Using the above expansions in (2.32) gives an effective potential

$$\Omega = -\frac{T^4}{90}g^* + \gamma(T^2 - T_0^2)\phi^2 - \delta T\phi^3 + \frac{\lambda}{4}(1 - \frac{6B}{\lambda})\,\phi^4$$
$$+ B\phi^4 \ln(\frac{\phi^2}{v^2}) - \frac{3}{64\pi^2}\phi^4\left[2\left(\frac{M_W}{v}\right)^4 \ln\left(\frac{M_W\phi}{vT}\right)^2 + \right. \tag{2.35}$$
$$\left. \left(2\frac{M_Z}{v}\right)^4 \ln\left(\frac{M_Z\phi}{vT}\right)^2 - 8\left(\frac{M_t}{v}\right)^4 \ln\left(\frac{M_t\phi}{vT}\right)^2\right],$$

where $g^* = N_b + \frac{7}{8}N_f$, and

$$\gamma = \frac{1}{8}\left[2\left(\frac{M_W}{v}\right)^2 + \left(\frac{M_Z}{v}\right)^2 + 2\left(\frac{M_t}{v}\right)^2\right] \tag{2.36}$$

$$\delta = \frac{1}{4\pi}\left[2\left(\frac{M_W}{v}\right)^3 + \left(\frac{M_Z}{v}\right)^3\right] \tag{2.37}$$

$$T_0^2 = \frac{M_H^2}{4\gamma}\left(1 - \frac{4B}{\lambda}\right). \tag{2.38}$$

Above a critical temperature T_c, which I will soon determine, the minimum occurs at zero Higgs field, and symmetry is restored. This is a natural initial condition in a hot Big Bang scenario. As the universe expands and the temperature drops, a relative minimum appears, and at temperature T_c, it becomes degenerate with the original. Then at some temperature, given by T_0 above, the potential develops a relative maximum at zero and is qualitatively similar to the usual zero temperature Mexican hat potential. Since zero Higgs field is a relative minimum down to temperature T_0, and there is a potential barrier between the false and true vacua, the phase transition is not instantaneous. Instead, it proceeds via bubble nucleation due to quantum tunneling and thermal fluctuations of the Higgs field in small regions of space. If a bubble of true vacuum appears with a radius larger than

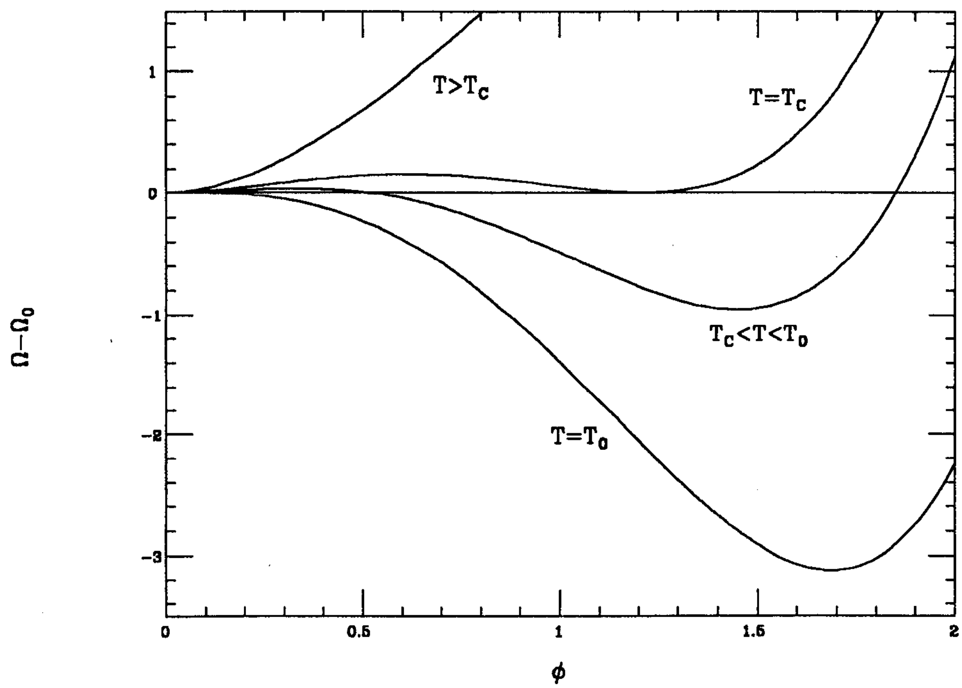

Figure 2. A typical temperature evolution (in arbitrary units) for a first order transition. The free energy at zero field has been subtracted off.

some critical value, the bubble of Higgs field expands. Its evolution is determined classically, and at some temperature T_b, typically greater than T_0, all the bubbles collide filling up space. This, in effect, produces one large bubble of Higgs field at the minimum of the potential. During bubble expansion, the change in the Higgs field is much faster than the expansion of the universe. However, after T_b, the Higgs field changes with the minimum of the potential, which is of order the Hubble parameter and hence quite small. This point is very important for weak scale baryogenesis and will be elaborated on in the next chapter. In any event, phase transitions of this type are called first order, and a typical temperature evolution of the potential is shown in Fig. 2.

I will now find the critical temperature T_c, neglecting all logarithmic terms. If greater accuracy is needed, then numerical techniques can be used and the logarithms kept. From (2.35), there is a non-zero minimum at

$$v(T) = \frac{12\delta T}{\lambda(1-6B/\lambda)}\left[1 \; + \; \left[1 - \frac{2\gamma\lambda}{9\delta}(1-\frac{T_0^2}{T^2})(1-\frac{6B}{\lambda})\right]^{1/2}\right]. \tag{2.39}$$

The critical temperature is where this minimum disappears:

$$T_c^2 = T_0^2\left[1 \; - \; \frac{9\delta}{2\gamma\lambda(1-6B/\lambda)}\right]^{-1}. \tag{2.40}$$

Notice that $T_c \sim T_0 \sim v/h$, where h is the larger of the gauge coupling g or the top Yukawa coupling h_t. This means that the high temperature expansion near the critical temperature is only valid in the weak coupling limit. In particular, for heavy top $\sim$ 100 GeV, the expansions (2.33) and (2.34) are unreliable, and (2.32) must be used directly.

Finally, I give a brief sketch of bubble formation. Again, this is standard material and can be found in Ref. [8]. The general theory of vacuum instability at zero temperature was developed by Callan and Coleman in the previous reference. Given a metastable state, such as the false vacuum, its energy develops an imaginary part which can be calculated using instanton methods. The decay rate is then proportional to e^{-S_4}, where S_4 is a four dimensional Euclidean action associated with the so-called "bounce" solution. Similar to an instanton, the bounce is a classical solution to the Euclidean equations of motion. While an instanton interpolates between different perturbative vacua, the bounce connects the metastable false vacuum and the true vacuum and then bounces back to the metastable state once again. It is quite reasonable that bounce states of least action are $O(4)$ symmetric. This means the equations of motion are really ordinary differential equations with boundary conditions, a problem well suited to numerical techniques. Linde applied these arguments to finite temperature field theory[8]. As was shown in (2.2), finite temperature field theory is equivalent to Euclidean field theory on a cylinder of circumference β in the time direction. If the temperature is increased sufficiently, the cylinder radius becomes smaller than a typical $O(4)$ bubble. This means one may take the bounce solutions to be constant in Euclidean time and $O(3)$ symmetric instead. The vacuum tunneling rate is then proportional to $e^{-\beta S_3}$. This three dimensional action has the interpretation of free energy, and this formalism then agrees with the theory of boiling[7,8]. There is also a prefactor in the bubble formation rate that involves a determinant of small fluctuations about the bounce solution. This is typically very difficult to calculate, but fortunately it may be estimated using dimensional analysis. The complete expression for the bubble formation rate per unit volume was found by Linde in Ref. [8] to be:

$$\Gamma_{bub}(T) = T(\frac{S_3}{2\pi T})^{3/2}\left[\frac{\det'(-\nabla^2 + \Omega''|_\phi)}{\det(-\nabla^2 + \Omega''|_0)}\right]^{-1/2} e^{-\beta S_3}, \tag{2.41}$$

where $\det'$ means that the zero modes have been removed, and the three dimensional action is defined by

$$S_3 = \int \frac{d^3\mathbf{x}}{(2\pi)^3}\,[\frac{1}{2}(\nabla\phi)^2 + \Omega(\phi, T)]. \tag{2.42}$$

The $O(3)$ symmetric bounce solution must be used in the above action:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \Omega'(\phi, T), \tag{2.43}$$

where the boundary conditions are $\phi \to 0$ as $r \to \infty$, and $\phi' \to 0$ as $r \to 0$. Since the temperature sets the scale, the prefactor in (2.41) must be roughly proportional to T^4 by dimensional analysis. This means one can use the approximation

$$\Gamma_{bub} \sim T^4 (\frac{S_3}{2\pi T})^{3/2} \ e^{-\beta S_3}. \tag{2.44}$$

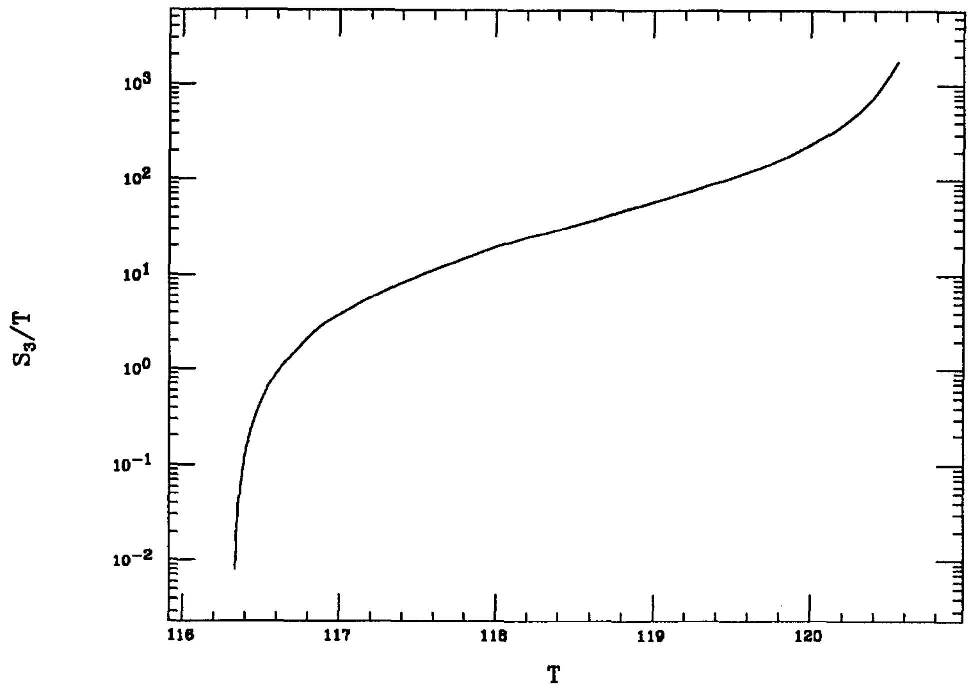

Figure 3. A typical temperature evolution of S_3/T.

For a given potential $\Omega(\phi, T)$, a numerical solution to (2.43) satisfying the appropriate boundary conditions can be found, the three dimensional action calculated, and the bubble production rate found. Fig. 3 illustrates a typical temperature evolution of the bounce action. It was produced by numerically solving for the bounce with a Higgs mass $M_H \sim 50$ GeV and with a light top. Right after the phase transition, the high potential barrier separating the false and true vacua (see Fig. 2) produces a very large action, but as the universe cools and the barrier drops, the action decreases exponentially. This means the bubble production increases quickly on a macroscopic scale, and the false vacuum is gone soon after the first bubbles appear. Roughly speaking, when the rate within a volume of T^{-3} becomes comparable to the Hubble parameter, all of space fills up with bubbles of true vacuum. To be more precise, bubble formation is a Poisson process, and when the bubbles form they begin to expand with some speed v_b. Eventually the bubbles collide filling all of space. Guth, in Ref. [8], derives the following expression for the fraction of false vacuum left at time t:

$$f(t) = \exp\left[-\frac{4\pi}{3} \int_{t_c}^{t} dt' \; \Gamma_{bub}(T') \; V(t;t') \right], \tag{2.45}$$

where t_c is the critical time associated with T_c, and

$$V(t,t') \doteq \left[v_b R(t) \int_{t'}^{t} \frac{dt''}{R(t'')} \right]^3 . \tag{2.46}$$

This last expression is the volume that a bubble produced at time t' occupies at a later time t due to both the expansion of the universe and the bubble expansion itself. In the Standard Model, by matching the pressure gradients across the bubble wall, it can be show that the bubble expands at non-relativistic speeds: $v_b^2 \sim \alpha_W^2/\lambda$. I define the bubble temperature, T_b, as the temperature at which $\ln f = -1$, at which point the false vacuum is mostly gone. Since f changes so abruptly, the bubble temperature is very insensitive to the wall velocity v_b. Even for non-relativistic velocities, since the Hubble parameter $H \sim 10^{-14}$ GeV at the weak scale, the bubble expansion rate is much larger than the expansion rate of the universe. As will be shown in the next chapter, such a rapidly changing Higgs field biases the baryon-number violation in a given direction. As the bubbles expand, baryons are produced in the outer walls, and with any luck the observed ratio $n_B/n_\gamma \sim 10^{-10}$ will be produced. As it turns out, if the CP violating phases are not small, this can just barely be done. This marginal production places some stringent constraints on the theory. In particular, the Higgs boson cannot be too heavy. After the bubbles collide, the rate of change of the Higgs field is set by the Hubble parameter. Baryon-number violation is then no longer biased in a particular direction, and unless the Higgs mass is small enough, any previously produced baryons get wiped out. This is the subject of Chapter 5. In the following chapter I will examine baryon production on the bubble surfaces in more detail and derive a somewhat general bayron rate equation in the presence of a changing Higgs field.

3. Baryon Production from Higgs Biasing

In this chapter I will discuss the adiabatic production of baryons in a rather general manner. I will consider an arbitrary field theory with CP violation and an anomalous baryon current. This could be the standard model or some extension of it, the one most relevant for this discussion being the minimal supersymmetric extension. This is because CP violation in the standard model is too small to reproduce the correct baryon asymmetry, but minimal extensions can have much larger CP violating phases and thereby stand a chance of producing the observed baryon asymmetry.

It has long been known that baryon and lepton number are not conserved in the standard model, as a consequence of anomalies[5]. States of different baryon number are smoothly connected to one another through different configurations of the gauge and Higgs fields, but they are separated by a very large energy barrier of order $M_W/\alpha_W \sim 10$ TeV, which makes zero temperature tunneling an extremely unlikely process. In the last few years, however, it has become clear that baryon number is badly violated at temperatures much above M_W (with $B-L$ being conserved)[9-12]. The full proof of this is quite involved[13], but the following heuristic argument provides some insight into the situation. At high temperatures, the system is well described by classical statistical mechanics. At temperatures below the weak phase transition, the lowest energy barrier separating baryon-number states is called the sphaleron – this is a static, unstable solution to the field equations. It has one negative mode and represents a saddle point in the field space[9]. The rate for barrier penetration is essentially the Boltzmann factor associated with forming a sphaleron:

$$\Gamma \sim e^{-BM_W/\alpha_W T}, \tag{3.1}$$

where B is a number which depends rather weakly on the Higgs mass, varying between about 3 to 6. Above the weak phase transition, the situation is equivalent to a three dimensional field theory with no small dimensionless parameter. On dimensional grounds, however, the rate must be given by

$$\Gamma \sim \kappa(\alpha_W T)^4. \tag{3.2}$$

A recent simulation gives $\kappa = 0.01 - 1$[14]. It is very reasonable that the baryon-number violation rate becomes unsuppressed, since gauge configurations may easily pass over the barrier.

While no single classical configuration dominates this rate, a heuristic description in terms of instanton trajectories can be given. It is generally believed that the three dimensional field theory has a mass gap, $a\alpha_W T$, where a is a number of order unity. Correspondingly, the correlation length of the high temperature theory (the so-called magnetic screening length) is $\xi = (a\alpha_W T)^{-1}$. Consider now instantons in the high temperature theory. These will exist with arbitrary scale size, from $\rho = 0$ to $\rho \sim \xi$. The instanton represents a particular tunneling trajectory through configuration space. The barrier height associated with such a trajectory is necessarily of the form $E_\rho = c/\alpha_W \rho$, where $c \sim 1$. Clearly, then, the smallest barriers are associated with the largest possible values of ρ, i.e. $\rho \sim \xi$. Such configurations have a Boltzmann factor of order unity, while the prefactor is of order ξ^{-4}.

The large rate of baryon-number violation has important implications for any baryon number produced at very early times. For example, if no net $B-L$ is produced at early times, the baryon (and lepton) numbers will completely disappear. It also raises the intriguing possibility that the observed baryon number could arise at temperatures of order the scale of weak interactions. This could have significant implications for our understanding of cosmology. In particular, in inflationary models, one usually requires significant reheating after inflation in order to produce baryons. This would not be necessary if baryons could be produced at such low temperatures.

The possibility that the baryon asymmetry might be produced at the weak phase transition was first discussed by Kuzmin, Rubakov and Shaposhnikov,[10] and has been most extensively explored in subsequent papers of Shaposhnikov and collaborators[15]. Other important works on the subject are those of McLerran[16], Turok and Zadrozny[17], and of Cohen, Kaplan and Nelson[18]. The main point is that if the phase transition in the Weinberg-Salam model is at least mildly first order, then the three conditions enumerated by Sakharov[1] necessary to obtain a net asymmetry are satisfied. Baryon-number violation is provided by the $SU(2)$ gauge interactions themselves. CP violation is already present in the standard model, and extensions of the standard model, such as multi-Higgs systems, supersymmetry or technicolor tend to yield larger violations of CP. Deviations from equilibrium will automatically arise if the transition is first order.

Many of the specific proposals which have been made for the origin of the baryon asymmetry at the weak phase transition are based on the minimal standard model. It is clear from the start, however, that unless the dynamics of the

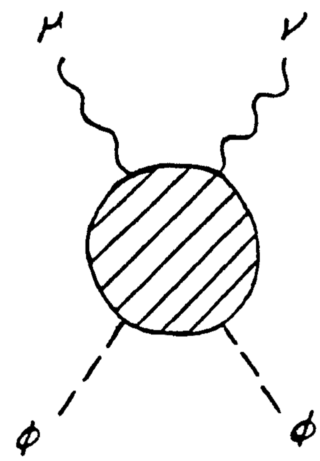

Figure 4. General loop diagram contributing to the CP violating dimension six operator considered in the text.

high temperature theory exhibits certain bizarre features[15], CP violation in this theory is simply too small to yield anything like the observed asymmetry, whatever the details of the phase transition might be. Moreover, as recently stressed in Ref. [19], there is another strong constraint on any such picture of baryon-number production, which almost rules out the minimal standard model. Once the phase transition is completed, the Higgs field will have some expectation value $v(T)$. The corresponding sphaleron (free-) energy is proportional to $v(T)$. If this VEV is too small, the rate of sphaleron-induced B-violating transitions (commonly called the "sphaleron rate") will be larger than the expansion rate and any baryon-number produced during the phase transition will be washed out. This almost certainly requires that the Higgs boson be so light that it would have shown up in recent LEP experiments. I will have more to say about this in chapter 5.

Since there are numerous possible extensions of the standard model, it is necessary to make a few simplifying assumptions. The assumptions I make here are not essential, and the analysis is easily extended to a wide variety of situations, including supersymmetry, technicolor, and multi-Higgs theories. In particular, I will assume in the discussion which follows that the new physics responsible for CP violation is associated with energy scales large compared to T_c, the transition temperature, and that the effective theory at T_c contains the usual quarks and leptons, and a Higgs doublet, ϕ. For reasons which will become clear shortly, I will also allow for the possibility of an additional scalar singlet, s. In the effective lagrangian, CP will be broken not only by the usual phase in the KM matrix, but also by various non-renormalizable operators. I will focus on the dimension-six operator

$$\mathcal{O} = \frac{1}{M^2}\frac{g^2}{32\pi^2}|\phi|^2 F\tilde{F} \;=\; -\frac{1}{3M^2}\partial_\mu|\phi^2|\mathcal{J}_B^\mu. \tag{3.3}$$

Here $\mathcal{J}_B^\mu$ is the baryon current, and I have used the anomaly equation (1.1) and integrated by parts. In theories with singlets, I will consider the dimension-5 operator

$$\mathcal{O}' = -\frac{1}{3M'}\partial_\mu s \mathcal{J}_B^\mu. \tag{3.4}$$

In the minimal supersymmetric standard model, for example, $\mathcal{O}$ would be generated at one loop by a diagram with gauginos and higgsinos in the intermediate state, as illustrated in Fig. 4. The coefficient $1/M^2$ would thus be of order some combination of CP violating phases, δ, divided by some typical supersymmetry breaking mass-squared. There are no strong limits on δ. In a non-minimal supersymmetric model with a complex gauge singlet field, S, s could be some component of this field. It could possess tree-level, CP violating couplings to the higgsino fields. The coefficient $1/M'$ would be of order δ divided by a supersymmetry breaking mass.

Already, the potential for baryon-number creation is present. I will consider two extreme cases. First I will examine a slowly changing Higgs field, so that the system can respond adiabatically, in the sense that at each instant the baryon-number violation rate, $\Gamma(\phi, T)$, is that appropriate to the value of the temperature and Higgs field at that moment. Then I close this chapter by examining rapidly changing Higgs fields. Surprisingly, this does not yield a substantial increase in the baryon number. Since the dominant processes are associated with gauge boson wavelengths of order ξ, rapid change means change on a time scale much shorter than ξ.* A simple model of the baryon-number violation rate is†

$$\Gamma(\phi, T) = \begin{cases} \kappa(\alpha_W T)^4; & T > T_B \\ 0 \qquad\quad ; & T < T_B \end{cases} \tag{3.5}$$

where the cut-off temperature T_B is given by $g\phi(T_B) \sim \alpha_W T_B$. At this temperature, the Boltzmann factor for sphaleron-like configurations becomes of order unity: $E_{sp} \sim M_W(T_B)/\alpha_W T_B \sim 1$. For temperatures less than T_B, the rate is Boltzmann suppressed, so I approximate it by zero. The rate in this region may still be much larger than the Hubble parameter, in which case any baryon asymmetry gets washed out, which places an upper bound on the Higgs mass. There is another justification for this simple model. Place the system in a box of length $\sim \xi$. For

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$\star$ This may be seen as follows. The scattering cross section for particles of such momenta on one another is of order $\alpha_W \xi^2$. However, the number density of such particles is of order their energy density times ξ; the product is of order ξ^{-1}.

† For large ϕ, the rate has been computed in Ref. [30]. For $m_H \sim m_W$, and small ϕ, their result is similar to the $\phi = 0$ result with $\kappa \sim 1$.

large temperatures the system is classical, and since there is a mass gap of order ξ^{-1}, the gauge fields of wave length ξ^{-1} obey an equation of the form

$$[\partial_t^2 + \xi^{-2} + (g\phi)^2]A_\mu(\mathbf{k}) \;=\; c_1\xi^{-5/3}A^2 + c_2\xi^{-3}A^3, \tag{3.6}$$

where c_1 and c_2 are of order one, and the left hand side has an implicit an integral over Fourier modes. For $\phi = 0$, the system becomes non-linear for $A \sim \xi^{1/2}$. However, for $g\phi > \alpha_W T$ the equation becomes linear. Sphaleron-like configurations that pass over the barrier are associated with non-linearities of the field equations, so when (3.6) becomes linear, the baryon-number violation rate turns off. No further passage over the barrier can occur; the barrier has simply "grown" and there is not enough energy available in these modes. Thus the process turns off both for slow and rapid changes in ϕ at about the same value of ϕ. In each case, the relevant value of the Higgs field is very small. For $T_B \sim 100$ GeV, for example, the rate turns off when $\phi \sim 5$ GeV.

I will now derive a baryon-number violating rate equation for the adiabatic limit. Let Γ_+ be the sphaleron rate per unit volume of increase in the Chern-Simons number N_{cs}, and correspondingly let Γ_- be the rate of decrease in N_{cs}. In Fig. 1, the bottom axis may represent Chern-Simons number, and then $\Gamma_{+,-}$ is the sphaleron rate over the barrier to the left and right, respectively. Exactly how the system approaches equilibrium depends upon the initial configuration. For simplicity, I will assume there is initially no baryon or lepton number. In the standard model, there are 24 types of fermions which I denote by $i = e,\ \nu,\ 3u,\ 3d$, and their corresponding number densities by n_i. Due to sphaleron transitions and the axial vector anomaly,

$$\frac{dn_i}{dt} = \frac{1}{2}(\Gamma_+ - \Gamma_-). \tag{3.7}$$

Left handed doublets of each family must be involved in a single sphaleron transition. This is because the corresponding zero modes must be eaten in the path integral for the amplitude of the process. This is why the rate above is family independent. But even though each doublet must be produced, there is only a fifty percent chance that the fermion has a specific isospin. This gives the factor of one half. Actually, there is an isospin preference in sphaleron transitions, but since I am only interested in order-of-magnitudes, for simplicity I assume $I_3 = \pm 1/2$ are equally likely. I should also consider constraints such as electric charge and isospin conservation, but this only complicates things, and (3.7) contains the essential physics. Given the initial conditions $n_i(0) = 0$, all the fermion densities are equal at subsequent times: $n_i(t) = n(t)$. Taking N_f flavors, the lepton and baryon-number densities are defined by

$$\begin{aligned} n_L &= \sum_f \left[n_{ef} + n_{\nu f}\right] = 2N_f\, n \\ n_B &= \frac{1}{3}\sum_{fc} \left[n_{ufc} + n_{dfc}\right] = 2N_f\, n, \end{aligned} \tag{3.8}$$

where the sum is over flavors and/or colors, and I have used a short hand notation n_{ef} to represent the density of isospin $-1/2$ fermions of flavor f, and a similar notation for other fermions. I will now use detailed balance to constrain the rates $\Gamma_\pm$ in (3.7)[21]. In equilibrium, $\Gamma_\pm$ satisfy

$$\Gamma_+ \prod_i \mathcal{P}(\frac{N_i}{V}) \;=\; \Gamma_- \prod_i \mathcal{P}(\frac{N_i + 1/2}{V}), \tag{3.9}$$

where the probability of producing N fermions is

$$\mathcal{P}(\frac{N}{V}) = const\ e^{-\beta V F_f}, \tag{3.10}$$

with F_f being the fermionic free energy. Since the system is adiabatic and not too far from equilibrium, I will apply this equilibrium constraint to the right hand side of rate equation (3.7). Letting $\Gamma = \Gamma_+ + \Gamma_-$, and after some algebra, to first order in the number densities,

$$\frac{dn_i}{dt} = -\frac{N_f}{8}\,\Gamma\beta \sum_j \frac{\partial F_f}{\partial n_j}. \tag{3.11}$$

For a free fermi gas of number density n, $F_f = -\kappa\ T^4 + 3n^2/T^2$, where κ is a constant depending on the number of light particle species. The CP violating operator $\mathcal{O}$ produces a shift in the minimum of the free energy:

$$F_f = \sum_i F_i + \mathcal{O} = \sum_i \frac{3(n_i - n_i^0)^2}{T^2}. \tag{3.12}$$

The minimum must be found according to the constraint $n_B = 2N_f\ n$. Defining $n_B^0 = 2N_f\ n_i^0$, and using the form of the CP violating operator (3.3) gives

$$n_B^0 = \frac{T^2}{12M^2}\partial_0|\phi^2| \qquad\qquad n_B^0 = \frac{T^2}{12M'}\partial_0 s \tag{3.13}$$

for the doublet or singlet case, respectively. I have dropped the spatial gradient of the Higgs field. This is justified since I am working in the adiabatic limit where gradients of the Higgs field on the expanding bubble walls is not too steep. Using

(3.12) in (3.11) gives the baryon-number violation rate equation

$$\frac{dn_B}{dt} = -\frac{6N_f}{T^3}\,\Gamma(n_B - n_B^0). \tag{3.14}$$

Because of the four powers of α_W appearing in Γ, n_B can be neglected relative to n_B^0, on the right hand side of this equation, provided dn_B^0/dt is large enough. I will shortly demonstrate that this is the case for a broad range of model parameters. Substituting the expression for n_B^0, and using the simple model (3.5) for Γ yields the baryon number

$$n_B \sim \kappa\frac{3\alpha_W^6}{2g^2M^2}T^5 \qquad\qquad n_B \sim \kappa\frac{3\alpha_W^5}{2gM'}T^4 \tag{3.15}$$

for the doublet and singlet respectively. Here, in the singlet case, I have assumed that $gS \sim \alpha_W T$ when baryon-number violation turns off.* These numbers need not be so small. In the singlet case, if the CP violating phase is of order one, and $M' \sim T$, then the baryon to photon ratio is of order 10^{-8}! In models with only doublets, this result is suppressed by an additional power of α_W. These estimates are rather rough. It is already clear, though, that potentially one can obtain a baryon asymmetry as large as that which is observed.

If n_B^0 is changing much more slowly in time, $n_B(t) \approx n_B^0(t)$ until Γ becomes exponentially small. In this case, one obtains a result suppressed by more powers of α_W, due to the time derivative in n_B^0. The extreme case of this type arises if the transition is second order. Then the asymmetry is suppressed by the Hubble constant[10].

⋆ Whether or not this is the case depends on the details of the phase transition. One can easily imagine that $S \sim |\phi|^2$, for example.

Before describing the case where the transition occurs suddenly, it is helpful to understand these results in another way. Consider the operator $\mathcal{O}$ written in the form containing $F\tilde{F}$. As in the heuristic discussion above, consider a single instanton trajectory, and treat the usual instanton time, τ, as parameterizing a path in configuration space. $\tau = 0$ corresponds to the top of the barrier. If the gauge field in the lagrangian is replaced by its classical value as a function of τ, then the lagrangian for τ (for small τ) is of the form

$$\mathcal{L}(\tau, \dot{\tau}) = c_1 \frac{4\pi}{g^2 \rho} \dot{\tau}^2 + \frac{4\pi b_1}{g^2 \rho^3} \tau^2, \tag{3.16}$$

where $\rho \sim \xi$ is the instanton scale size, and c_1 and b_1 are coefficients of order unity.† For small τ, $\mathcal{O}$ has the form

$$\mathcal{O} = c_2 \frac{|\phi|^2}{M^2} \frac{\dot{\tau}}{4\pi\rho} \tag{3.17}$$

and similarly for $\mathcal{O}'$. In the adiabatic limit, where the field ϕ is essentially constant, τ and $\dot{\tau}$ will be Boltzmann distributed at each instant. The canonical momentum receives a ϕ-dependent contribution from $\mathcal{O}$, eqn. (3.17). This has the effect of skewing the velocity distribution, giving rise to an excess flux over the barrier in one direction. Because of the anomaly, this corresponds to a net production of baryons or antibaryons, depending on the sign of δ. Proceeding in this way one obtains a rate equation of the form eqn. (3.7). In particular, this heuristic argument gives the correct dependence on α_W.

This picture is readily adapted to the case where the field ϕ changes suddenly. Despite the fact that this corresponds to a more violent departure from equilibrium,

† There is some arbitrariness in these definitions, since the result depends on the gauge choice for the instanton. Here I have indicated the factor of 4π coming from the angular integration.

it does not in general lead to a much larger production of baryons. Before the transition, one has a Boltzmann distribution for τ and $\dot{\tau}$, and this distribution remains essentially unchanged as ϕ changes. However, the system receives a "kick" from the sudden change in ϕ. In the time ϕ changes from 0 to ϕ_0, the value at which baryon-number violation turns off, the velocity changes by an amount:

$$\Delta\dot{\tau} = \int dt\ \frac{d}{dt}\left(\frac{c_2 g^2}{c_1 16\pi^2}\frac{|\phi|^2}{M^2}\right) = \frac{c_2 g^2}{c_1 16\pi^2}\frac{\phi_0^2}{M^2} \tag{3.18}$$

$\Delta\dot{\tau}$ has a definite sign. If it is large compared to the initial velocity, it will send the system over the barrier in the direction corresponding to the production of (say) baryons rather than antibaryons. If it is small compared to this velocity, it will have no effect on the baryon number. The fraction of the distribution with velocities $\dot{\tau} < \Delta\dot{\tau}$ is simply of order $\Delta\dot{\tau}$. If Δt is the time it takes for the Higgs field to rise to ϕ_0 over a correlation volume, ξ^{-3}, the final baryon number is of order the product of this fraction, Δt, and Γ:

$$n_B \sim \frac{\kappa}{4\pi}\frac{|\phi_0|^2}{M^2}\alpha_W^5 \Delta t T^4. \tag{3.19}$$

Here I have attempted to keep track of g's and 4π's, but not (unknown) coefficients of order unity. Since $g\phi_0 \sim \alpha_W T$, this result is comparable to that obtained in the "adiabatic" case only if $\Delta t \sim \xi$. A similar expression holds in the case of the operator $\mathcal{O}'$. The picture described here is close to that described in Ref. [17], where the behavior of particular field configurations is considered.

All the ingredients to estimate the baryon asymmetry are in place, once the behavior of the Higgs field is known as a function of time. In a first order phase transition, baryon number will be produced near the bubble walls, where the Higgs field is changing in time. In order to compute the asymmetry, it is thus necessary to know about the shape and velocity of the walls. Here I simply illustrate some of the possible behaviors by considering the minimal standard model[20], even though this cannot be a realistic model of baryon generation. For Higgs masses smaller than M_W, the transition is first order. Ignoring the heavy top contribution for the moment, for small self-coupling λ and setting $\sin^2\theta_W = 0$ to simplify the writing, the effective potential for the ϕ field as a function of temperature is given by

$$V(\phi, T) = M^2(T)\phi^2 - \frac{3g^3}{32\pi}T\phi^3 + \frac{\lambda}{4}\phi^4, \tag{3.20}$$

where $M^2(T) = \frac{3g^2T^2}{32} - m^2$. The discussion to follow is only meant to give a qualitative flavor, so (3.20) will suffice for now. This potential should be contrasted with the more complicated forms (5.2) and (5.6)–(5.9), which I use for a quantitative analysis. When the phase transition occurs, the coefficient of the quadratic term is extremely small, $M^2(T) \sim \alpha_W^3 T^2/\lambda$; otherwise the potential has only a minimum at the origin. I can make a crude estimate of the bubble wall velocity and size (well after the bubble forms) by requiring that in the rest frame of the wall, the pressure is constant. This pressure receives an extra contribution from the motion of the gas in this frame. The momentum change of a particle passing through the wall can be estimated by assuming that the particle's energy is conserved, while its mass changes due to the change in ϕ. This gives $v_b^2 \sim \Delta P/\Delta E$, where ΔP and ΔE are the changes in pressure and internal energy across the wall. From eqn. (3.20), $v_b^2 \sim \alpha_W^2/\lambda$. The shape of the wall can be inferred from similar considerations. For small ϕ, one finds $\phi \sim e^{M(x-vt)}$, where $M \sim (\alpha_W^3/\lambda)^{\frac{1}{2}}T$. As a result, if λ is not too small, the scalar field is changing rather slowly in space and time and the system is in the adiabatic regime described earlier. For such a field, n_B^0 is changing quickly enough that the approximations leading to eqn. (3.15) are valid. As one increases λ, and the transition becomes more second order, the amount of baryon number is reduced; decreasing λ brings the system to the "sudden" regime. Considerations of this type apply as well to the minimal supersymmetric standard model, where the quartic couplings are of order g^2, and the scalar masses are of order M_W.

In other models, the transition might be strongly first order, with bubbles expanding at nearly the speed of light, and with a wall of microscopic dimensions. This is the regime of rapid change of the Higgs field. Here, what is needed is an estimate of the time Δt, appearing in eqn. (3.19), required for the zero momentum mode of the field in a correlation volume, ξ^3, to reach ϕ_0. In a multi-Higgs model, one might expect this time to be of order α_W times some microscopic (mass) parameter in the lagrangian. Since the characteristic time for baryon-number violation is rather long (ξ), this may be a source of additional suppression.

In summary, it is possible to think that the baryon number of the universe was created at the electroweak phase transition, in some modest extension of the standard model. However, there are uncertainties in the calculations described here, particularly in the actual calculation of the rate Γ. Detailed studies of the phase transition in particular models are also essential, including not only the structure of the bubble wall but also flow of baryon number across the wall. One should also reconsider models such as that of Ref. [18], in which there are other sources of lepton number violation in the theory, but in which the mechanisms described here may also operate efficiently.

4. 1+1 Dimensional and Two Higgs Models

The 1 + 1 dimensional Abelian Higgs model coupled to fermions has been widely studied as a model for four dimensional baryon-number violation. Indeed, many features of this model are similar to the standard model. There are anomalies, instantons, and sphalerons, and "baryon-number violation" is enhanced at high temperatures. As will now be seen, this model, and variations on it, provide an extremely simple illustration of the issues in weak scale baryogenesis. I first consider the case where the theory contains, in addition to the gauge boson and Higgs boson, ϕ, of charge e, two Dirac fermions of charge e, ψ and χ, and a pseudoscalar, a. The Lagrangian contains gauge invariant kinetic terms and the couplings

$$\mathcal{L} = M\overline{\psi}\psi + \lambda ia\overline{\psi}\gamma_5\psi. \tag{4.1}$$

Note that the field ψ is massive, with mass M, while χ is massless. The standard anomaly argument, or a simple one loop calculation, leads to a coupling of the "axion", a, to the "photon", in the effective action at scales below M,

$$\frac{\lambda e}{4\pi M}\ a\epsilon_{\mu\nu}F^{\mu\nu}. \tag{4.2}$$

In this model, the current $j_5^\mu = \overline{\chi}\gamma^\mu\gamma^5\chi$ plays the role of the baryon current. Choosing the $A^0 = 0$ gauge, it is easy to see that a constant background A_1 field is equivalent, up to a factor of e, to a chemical potential for the corresponding charge. (A convenient choice for the gamma matrices in this model is $\gamma^0 = \sigma_1$, $\gamma^1 = -i\sigma_2$, and $\gamma_5 = \sigma_3$; with this choice the connection is obvious). It is helpful, here, to recall some well-known facts about the vacuum structure of this theory. As shown in the introduction, the classical vacua are labeled by an integer n representing the winding number of the field configuration. At the classical level, these states are separated from one another by a barrier, and are degenerate in energy. Examining the Dirac equation for the field χ in such an A_1 field, it is easy to see that changing n by one unit changes the "baryon number," $n_5 = j_5^0$ by two units.* Quantum mechanically, states with different values of A_1 differ in energy. This is not surprising, since they contain different numbers of baryons (a gauge-invariant notion). This energy difference may be computed, either at zero or finite temperature, either by calculating the contribution at zero momentum of the field χ to the A_1 two-point function, or equivalently by introducing a chemical potential for n_5 and calculating the free energy in textbook fashion.

Suppose now there is a slowly varying background a field, at a temperature $T \ll M$.† This leads to a baryon number which can be computed in either of two ways. If the scalar field changes slowly enough, the system will respond adiabatically. At temperatures well below M, the minimum of the free energy may be found by using the anomaly equation to make the replacement

$$\frac{e}{2\pi}\epsilon_{\mu\nu}F^{\mu\nu} = \partial_\mu j_5^\mu. \tag{4.3}$$

If a is constant in space, integrating by parts gives a term in the effective action

$$-\frac{\lambda}{2M}\partial_0 a \; n_5. \tag{4.4}$$

⋆ For a nice review, see Ref. [22].

† Such a field violates P; to be cosmologically relevant, this presupposes P violation either in the fundamental lagrangian or in the choice of ground state. Otherwise, different regions of the universe, as will be clear below, would acquire different signs of the baryon number, and the baryon-number averaged over several horizon lengths would be zero.

Note that a factor of $e/2\pi$ has disappeared. With the adiabatic assumption, it is simply necessary to minimize the free energy with this term. This yields for the "baryon asymmetry,"

$$n_5^0 = \frac{2\lambda}{\pi M}\partial_0 a. \tag{4.5}$$

This result may be understood in a different, yet equivalent way. Instead of using the anomaly equation, the coupling of eqn. (4.2) can be viewed as a source for A^1. Integrating by parts gives a coupling

$$-\frac{e}{2\pi M}\partial_0 a A_1. \tag{4.6}$$

At high temperatures, the potential for A_1 is quadratic. A one-loop calculation of the polarization yields

$$V(A) = \frac{e^2}{2\pi}A_1^2. \tag{4.7}$$

The coupling of eqn. (4.6) shifts the minimum of the A_1 potential. A_1 quickly settles to the minimum of this potential; how quickly depends on the coupling of A_1 to the thermal bath. (For example, by choosing the charge and mass of the scalar field appropriately, it is possible to arrange that A_1 is underdamped or overdamped.) This corresponds to the appearance of a chemical potential, or equivalently to a non-zero baryon density. The coefficient of the term linear in the chemical potential is $\frac{\lambda}{2\pi M}\partial_0 a\mu$.

This focus on A_1 may not appear to be gauge invariant. However, a completely gauge invariant calculation may be formulated by computing the term in the free

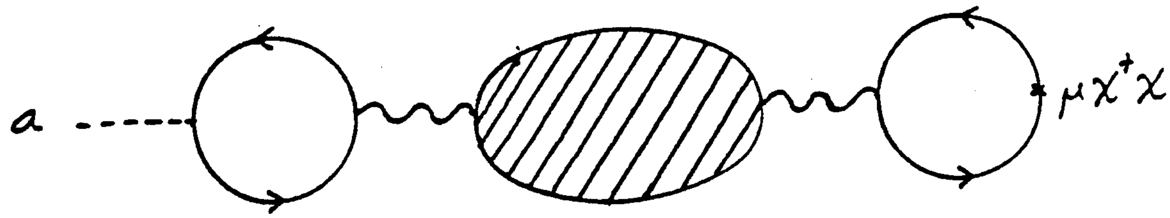

Figure 5. Diagram leading to chemical potential for the field χ. The blob denotes the full dressed propagator for the gauge boson.

energy linear in the chemical potential for χ. The corresponding Feynman diagram is drawn in Fig. 5.

The blob in the figure denotes the full propagator for the field A_1, evaluated at zero momentum. Up to a factor of e, this just cancels the χ loop indicated explicitly in the figure. Thus a term in the free energy linear in the chemical potential is directly obtained, precisely as above. In either case, an elementary calculation gives a result in agreement with eqn. (4.5) for the density at the minimum of the free energy.

In this model, it is not too difficult to determine what happens as the mass M is decreased. I am interested here in a problem in real time. The imaginary-time formalism, however, provides a clue as to how to proceed. Parity violating couplings of the gauge fields to the scalar fields are of interest. Since, in both two and four dimensions, the theory without fermions preserves parity, fermions must play a crucial role. In the imaginary-time approach, if one is considering phenomenon at low momentum and zero frequency, the effects of fermions may be represented by local operators. Since I am now interested in a real time-dependent problem at finite temperature, I use the real-time formalism[23]. In general, this formalism is rather complicated. There is no simple Feynman diagram expansion, and it is not immediately obvious what the role of the effective action is. In the real-time approach, the linear response of a system at equilibrium to a perturbation is typically calculated. In the present context, for example, one might ask the value of A_1 as a function of time in the presence of a time-varying a. In the textbook treatments of this subject[23], the required Green's functions are obtained by first evaluating them for imaginary frequency and then analytically continuing. Now consider some complicated Feynman diagram, containing fermion loops. If one is interested in continuing to a region where the external frequencies and momenta are small, then the analytic continuation of the fermion loops is trivial, since possible cuts are far away from the momenta. The frequencies are simply replaced by their small (real) values. The fermion propagators may now be expanded in powers of the external frequencies and momenta, and thus the fermion loops may still be replaced by local operators. The resulting effective action should be gauge invariant. These remarks apply equally to two or four dimensions.

Now consider the two-dimensional model. Suppose $M, e\phi \ll T$. The effect of the fermion, ψ, can, by the arguments above, be absorbed into a gauge invariant, local operator. The lowest dimension operator allowed by the surviving symmetries is simply $\mathcal{O} = aF^{01}$.

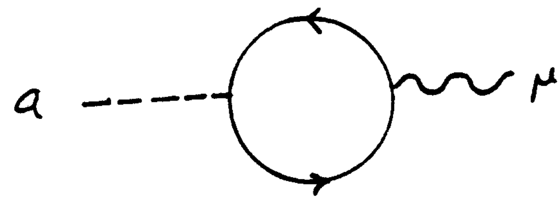

Figure 6. One loop diagram yielding coupling of the field a to the gauge boson.

In the present framework, the computation of the coefficient is elementary. The one-loop diagram of Fig. 6 is calculated using the usual (Euclidean) finite temperature Feynman rules, but with the external lines carrying a small imaginary frequency, q_0. It is simplest to do the integral over spatial momenta, followed by the discrete frequency sums. The result is

$$\frac{7}{8}\frac{\zeta(3)}{\pi}\,\frac{i\lambda eM}{\pi^2T^2}\,q^\alpha\epsilon^{\alpha\mu}. \tag{4.8}$$

Now the thermodynamics of the system is simply that of a model with the operator $\mathcal{O}$ in the Hamiltonian. In particular, the equilibrium configuration can be found by precisely the arguments previously given for the case of large M. This gives the minimum of the free energy at

$$n_B^0 = \frac{7}{4}\frac{\zeta(3)}{\pi}\frac{\lambda M}{\pi^2 T^2}\,\partial_0 a. \tag{4.9}$$

As an analog for the standard model, the model described so far is not completely satisfactory in a number of respects. Most important, in the standard model, the same field is responsible for the breaking of $SU(2) \times U(1)$ and for giving mass to fermions. This is important for the generation of the baryon asymmetry at the phase transition. This limitation is easily remedied. Consider a theory with gauge group $U(1)$ and with a single scalar, ϕ, of unit charge. Suppose also the theory contains a left moving fermion, ψ_L, of charge q, a right moving fermion, ψ_R, of charge $-(q+1)$, and another left moving fermion, χ, with $Q_\chi^2 = 1 - 2q$. With these charge assignments, the theory is anomaly free. The potential for ϕ is chosen so that ϕ has a non-zero expectation value. This breaks the gauge symmetry. It is now possible to write a Yukawa coupling,

$$\mathcal{L}_Y = \lambda\phi\psi_L\psi_R + cc. \tag{4.10}$$

The VEV for ϕ leads to a mass, $M = \lambda < \phi >$ for ψ_L and ψ_R. Thus it is natural to combine them into a two-component field, ψ, and rewrite the Yukawa coupling as

$$\mathcal{L}_Y = \lambda(\rho\overline{\psi}\psi + ia\overline{\psi}\gamma_5\psi), \tag{4.11}$$

where $\phi = \rho + ia$.

By varying λ and e, we can vary the masses of the fermion and the gauge boson. It is interesting to consider various limits. We will be interested in the case where the gauge boson mass is much smaller than the temperature. Suppose, first, that the fermion mass, M, is much larger than the temperature. Then the fermions can be integrated out, giving a Lorentz invariant effective lagrangian for the remaining fields. The one loop diagram of Fig. 6 yields a coupling

$$\mathcal{L}_{an} = \frac{\lambda e(2q+1)}{8\pi|M|}\, a\epsilon_{\mu\nu}F^{\mu\nu}. \tag{4.12}$$

This coupling is not gauge invariant, since a transforms non-linearly under a gauge transformation. However, at scales below M, the effective theory contains only the fermion χ, and appears to be anomalous. The coupling of eqn. (4.12) is precisely what is needed to cancel the anomaly, and render the complete theory gauge invariant. To make the analogy with the standard model complete, imagine for some period the fields ρ and a are changing in time. In this limit, the above analysis can easily be repeated, or equivalently that of Ref. [26], to compute the resulting asymmetry. In particular, the minimum of the free energy at a given instant can be obtained by any of the following methods: using the anomaly equation to replace $\epsilon_{\mu\nu}F^{\mu\nu}$ by the "baryon current" (in this case the χ-number current), and reading off the linear term in the baryon density; by determining the value of A_1 resulting from the coupling in eqn. (4.12); or by computing directly the linear term in the chemical potential, from a diagram analogous to that of Fig. 5. Again, each of these calculations yields the same result.

Now consider the case that $M \ll T$. As discussed above, in this limit the effects of the fermions ψ can be described by a local operator. In the present case, as stressed by the authors of Ref. [28], the possible operators are not restricted by the requirement of Lorentz invariance. In order to create a baryon asymmetry in this model, I am interested in operators which involve $\partial_0 a$ and the Chern-Simons density A_1. The diagram of Fig. 6 indeed yields such a coupling:

$$\mathcal{L}_{a-A} = \frac{7}{16}(2q+1)\zeta(3)\frac{Me\lambda}{\pi^2 T^2}\partial_0 a\ A_1. \tag{4.13}$$

(Here $M = \lambda|\phi|$.) Unlike the large mass case, I cannot appeal to any anomaly argument here to explain away any non-gauge invariance in this effective action. Instead it must be possible to write this result in a gauge invariant fashion. Indeed, it is easy to see that this coupling is one term which would arise from a coupling

$$\mathcal{L}_{inv} = \mathcal{L}_{a-A} = \frac{7}{32}(2q+1)\zeta(3)\frac{Me\lambda}{\pi^2 T^2}(D_1\phi^\dagger D_0\phi + hc), \tag{4.14}$$

where D_0 and D_1 are the usual gauge covariant derivatives. It is straightforward to check that the other couplings implied by this term in the effective lagrangian are indeed generated. For example, the diagram of Fig. 6 gives the required $A_0 A_1|\phi|^2$ coupling.

I can no longer use the anomaly to replace the operator appearing in eqn. (4.14). On the other hand, using the various techniques described up to now, it is easy to determine the baryon-number created in a time-varying a field (or, stated in a gauge invariant way, in a field configuration for which $D_0\phi \neq 0$). One can, as before, either determine, in a fixed gauge such as Coulomb gauge, the minimum of the A_1 potential, or one can compute the term in the free energy linear in μ_χ. Again, both calculations are elementary and yield the same result:

$$n_\chi = \frac{7}{8}\sqrt{2q+1}\ \zeta(3)\ \frac{M\lambda}{\pi T}\ \partial_0 a. \tag{4.15}$$

Note the final result for low mass is suppressed, not by T, but by πT. This lowers, by an order of magnitude, some of the estimates presented in Ref. [26].

I will close this chapter by returning to four dimensions and considering the problem of producing the baryon asymmetry in multi-Higgs models, where the only new sources of CP violation are the terms in the Higgs potential. The simplest such model, studied in Refs. [28] and [17], is the two Higgs doublet model. However, it is easy to see that such models can not yield a large enough asymmetry. Consider first the quadratic terms in the Higgs potential. Calling the two Higgs fields H_1 and H_2, these take the form

$$V_{quad} = m_1^2|H_1|^2 + m_2^2|H_2|^2 + (\mu^2 H_1 H_2 + cc). \tag{4.16}$$

By a field redefinition, μ can always be taken real. Thus, ignoring KM phases and quartic couplings of the scalars, there is no CP violation.

What does this mean for the baryon asymmetry? As previously stressed, the baryon-number violating processes essentially turn off once $\phi \sim \alpha_W T$. But for such small ϕ, the quartic terms in the potential can be neglected, to a good approximation, in considering the (essentially classical) evolution of the Higgs field. This means that any CP violation in this evolution is suppressed by at least two powers of α_W. As discussed in Ref. [28], the operator relevant to baryon-number creation in the two Higgs model is

$$\mathcal{O}_{\mathcal{H}} = \epsilon_{ijk}\phi^* \tau^a D_0 D_i \phi F^{ajk}. \tag{4.17}$$

Using the equations of motion, these authors indeed find that this operator is of order ϕ^4. As a result, the asymmetry is of order $\alpha_W^8 \times \delta$, where δ is again some measure of CP violation; four powers of α_W come from the rate, four from the four powers of ϕ. Again, the coefficient can be computed, as suggested by the two dimensional model, from thermodynamic arguments. The result obtained is in qualitative agreement with Ref. [28], written in terms of the scalar field ϕ. However, ϕ must be understood as being of order $\alpha_W T$, rather than as the value of the scalar field after the phase transition. As a result, the asymmetry in such models is unacceptably small, no matter how large the CP violation.

The situation can be improved by considering models with larger number of Higgs particles. Once there are three or more Higgs, the quadratic terms in the potential do violate CP. Of course, if multi-Higgs models are to be taken seriously, flavor changing neutral currents must be suppressed. I will not explore here the question of simultaneously obtaining a large baryon asymmetry and satisfying this condition.

5. Baryon Persistence and Higgs Mass Bounds

In an important series of papers, Shaposhnikov and collaborators have pointed out that there is an important constraint on any scheme to produce the observed asymmetry at the electroweak transition (at least any scheme with zero $B-L$)[24]. Immediately after the phase transition, the baryon number violation rate due to sphaleron transitions may be large compared to the expansion rate of the universe. If this is so, any asymmetry produced during the transition will be quickly wiped out. The demand that the transition rate be low enough that this not occur places constraints on models. In the minimal standard model, the authors of Ref. [24] argue that a Higgs mass of about 42 GeV cannot be exceeded. The basic idea is quite simple. One computes the sphaleron energy as a function of Higgs mass and temperature, and from this the transition rate. As the Higgs mass increases, the transition becomes more and more weakly first order, so the Higgs field after the transition is smaller, as is the sphaleron energy. Since the expansion rate at these times is quite small in microscopic terms, the sphaleron rate quickly becomes large compared to the expansion rate.

The limit obtained in Ref. [24] is particularly striking when compared with the recent limits on Higgs particles reported from LEP[25]($M_H \geq 48$ GeV). Moreover, this limit is relevant to models other than the minimal standard model. First, as noted in Refs. [26] and [28], unless some rather exotic physics is operative[29], there is no hope for producing a large enough asymmetry in the minimal model. However, even in a model with a single doublet, new physics could provide new sources of CP violation. Moreover, even in models with multiple doublets, the effective theory at the phase transition often involves only a single doublet.

In fact, in the supersymmetric standard model, the phase transition is similar to that in the minimal standard model. As will now be shown, requiring the Higgs expectation value to be large after the phase transition forces one into a narrow range of parameters in which the model at zero temperature contains one light and one heavy doublet. This is certainly a theory with additional sources of CP violation; it also has two doublets. The potential for the doublets is highly constrained. In particular, the quartic couplings are completely fixed. The full zero-temperature potential has the form

$$\begin{aligned} V_{susy} =& m_1^2|H_1|^2 + m_2^2|H_2|^2 + \mu^2(H_1H_2 + cc) + \\ & \frac{g_2^2}{8}(H_1^*\tau_a H_1 - H_2\tau_a H_2^*)^2 + \frac{g_1^2}{8}(|H_1|^2 - |H_2|^2)^2. \end{aligned} \tag{5.1}$$

As is well known, this potential is subject to various constraints. Either m_1^2 or m_2^2 must be positive. Requiring the energy be bounded below gives

$$m_1^2 + m_2^2 - 2\mu^2 > 0, \tag{5.2}$$

while if both m_1^2 and m_2^2 are greater than zero, demanding that the Higgs mass matrix possesses a negative eigenvalue yields

$$m_1^2 m_2^2 < \mu^4. \tag{5.3}$$

In this model, the phase transition occurs near the point where the temperature-dependent effective mass of one of the doublets nearly vanishes. At this point, the second doublet is generically much heavier, and to first approximation can be ignored. The corresponding effective theory is then that of a single doublet with a quartic potential (plus temperature dependent corrections). In order to make the Higgs VEV after the phase transition as large as possible, the quartic coupling must be as small as possible. To determine this coupling, return first to the zero temperature potential, eqn. (5.1). Ignoring quark Yukawa couplings, and μ^2 itself, μ^2 does not receive finite temperature corrections while m_1^2 and m_2^2 do. The temperature dependent mass matrix has a zero eigenvalue at the point where the condition of eqn. (5.3) is an equality. At this point, it is straightforward to find the effective quartic coupling of the massless field. It is given by

$$V = \frac{(g_1^2 + g_2^2)}{8} \frac{(\mu^4 - m_1^4)^2}{(m_1^4 + \mu^4)^2} |\phi|^4, \tag{5.4}$$

where ϕ is the light field, and the masses appearing in this equation are the temperature dependent ones. In order to have a large Higgs VEV after the phase transition, this quartic coupling must be as small as possible, i.e. one requires $m_1^2 \approx \mu^2$. This in turn means that $m_1^2 \approx m_2^2$. Combined with the conditions on the zero temperature masses above, and recalling that there are no finite temperature corrections to μ^2, it is easy to see that one is forced into a situation in which the zero-temperature theory also has a single very light Higgs and one massive Higgs. But this is precisely the situation under consideration.

Because the transition rate depends exponentially on the sphaleron energy, small errors in the energy density can lead to large changes in the rate. Thus I wish to examine carefully the analysis of Ref. [24] to determine whether or not there still exists a window of allowed Higgs masses for which it might be possible to obtain the observed baryon-asymmetry. There are several sources of uncertainty which I examine here. First, it is important to include all finite temperature effects in the effective action. This includes both terms in the effective potential, as well as derivative terms. Corrections to the effective potential are relatively easy to incorporate. One can simply compute the sphaleron solution appropriate to the corrected potential, and calculate its energy. Derivative terms turn out to be more complicated. To compute these requires summation of an infinite set of diagrams, and I know of no general way of accomplishing this. Study of some particular diagrams suggests that the resulting corrections to the sphaleron energy are of order $10-20\%$ for the parameter range of interest. Again, since this uncertainty is exponentiated, this is an important effect. Previous analyses have also not taken into account the likely large value of the top quark mass. Including this effect (i.e. the large Yukawa coupling of the top quark to the Higgs) also tends to yield substantial corrections.

The sphaleron transition rate is proportional to $e^{-E_{sph}/T}$. Determining the proportionality constant requires evaluation of a certain determinant in the three dimensional field theory which describes the classical thermodynamic limit[30]. This prefactor also introduces significant uncertainties. For certain values of the quartic coupling, this prefactor has been evaluated numerically in Ref. [30]. There are, however, a number of problems with using these results. These authors noted a drastic dependence on the Higgs self-coupling λ, and realized that this could be explained, at least in part, by the need to use a corrected sphaleron solution. In the range of parameters which will be important to us, these corrections cannot be treated perturbatively. I will deal with this problem by making a simple estimate of the determinant which is at least consistent with the results of Ref. [30]. To be more specific, when $M_W < T < M_W/\alpha_W$, the baryon-number violation rate takes the form[30]:

$$\Gamma = 4\pi g\kappa\omega\mathcal{N}_{rot}\mathcal{N}_{tran}\left[\frac{v(T)}{T}\right]^7 T^4\, e^{-E_{sph}/T}, \tag{5.5}$$

where $v(T)$ is the minimum of the effective potential, $\mathcal{N}_{tr}$ and $\mathcal{N}_{rot}$ are factors associated with translational and rotational zero modes respectively, ω is the frequency of the unstable mode of the sphaleron in units of $gv(T)$, and κ is the functional determinant associated with fluctuations about the sphaleron. It is the factor κ that involves the large uncertainties mentioned above. In this work, I continue to use the numerical values for the zero mode and frequency factors found in Ref. [30]. This is justified since the primary uncertainty lies in κ. At the high end, I estimate $\kappa \sim 10^{-1}$. For a lower bound, absorbing the uncertainties of the derivative contributions to the effective action into this factor, $\kappa \sim 10^{-1}\, e^{-0.2E_{sph}/T} \sim 10^{-4}$.

Finally, there is another important effect which must be taken into account. In Ref. [24], it was assumed that the phase transition occurs at the temperature, T_0, where the effective Higgs mass vanishes. However, the transition actually occurs at a higher temperature, and as a result the Higgs expectation value is somewhat smaller after the transition. This tends to increase the sphaleron rate. Once all of these effects are taken into account, and allowance is made for the various uncertainties, I find that indeed a small window of Higgs mass remains; the Higgs can possibly be as heavy as 55 GeV, without leading to a significant reduction of the baryon asymmetry.

Now I consider each of the points mentioned above in greater detail. First, the form of the effective potential, and some aspects of the phase transition are discussed. I am interested in relatively weak Higgs coupling; however, account must be taken of the relatively large value of the top quark mass. In the standard model, the phase transition occurs near the temperature where the mass of the Higgs doublet vanishes[20]. If this temperature is sufficiently high, then for small values of the Higgs field, the potential has the form

$$V(\phi,T) = \gamma(T^2 - T_0^2)\phi^2 \; - \; \delta T\phi^3 \; + \; \frac{\lambda}{4}\phi^4, \tag{5.6}$$

where

$$\gamma = \frac{1}{8}\left[2\left(\frac{M_W}{v}\right)^2 + \left(\frac{M_Z}{v}\right)^2 + 2\left(\frac{M_t}{v}\right)^2\right] \tag{5.7}$$

$$\delta = \frac{1}{4\pi}\left[2\left(\frac{M_W}{v}\right)^3 + \left(\frac{M_Z}{v}\right)^3\right], \tag{5.8}$$

$$T_0^2 = \frac{M_H^2}{4\gamma}. \tag{5.9}$$

The temperature T_0 is where the curvature at $\phi = 0$ changes sign. Without the cubic term, T_0 is simply the critical temperature, and the phase transition is second order. However, even for a small cubic term, the transition is at least weakly first order. This means that for temperatures slightly larger than T_0, $\phi = 0$ becomes a relative minimum. At a temperature T_c, this relative minimum becomes degenerate with the true minimum, and at temperatures greater than T_c it becomes the true minimum.

In practice, these statements require some modifications. First, the top quark is likely to be quite heavy. Typically the minimum of the potential occurs at values of ϕ for which the effective top quark mass, $h_t\phi$, is of order T_0 or larger. (Here h_t is the top quark Yukawa coupling). As a result, one cannot make the approximation of small ϕ for the top quark contribution to the free energy, and it is necessary to use the exact result. The one-loop finite temperature correction from gauge bosons and top quarks is

$$V_T = \frac{T^4}{2\pi^2}\Big[6I_-(y_W) + 3I_-(y_Z) + 12I_+(y_t)\Big], \tag{5.10}$$

where $y_i = M_i\phi/vT$, and

$$I_\mp(y) = \pm\int_0^\infty dx\; x^2 \ln(1 \mp e^{-\sqrt{x^2+y^2}}\,). \tag{5.11}$$

For the numerical work, I fit the above integrals to a tenth order polynomial in y. Fits to the first and second derivatives were also performed. These are essential in solving the sphaleron rate equations. The second derivative is needed for numerical programs that solve the rate equation using relaxation methods in which a solution is initially guessed and then relaxed to an approximate solution. These fits are given in the appendix.

It is also necessary to include the first order quantum corrections to the zero temperature potential. These turn out to be as important as the top quark finite-temperature corrections. I write the effective potential as $V = V_0 + V_T$, where V_0 is the zero temperature potential, and V_T is the finite temperature correction. The zero temperature potential takes the form

$$V_0 = -\frac{\mu^2}{2}(1 - \frac{4B}{\lambda})\phi^2 + \frac{\lambda}{4}(1 - \frac{6B}{\lambda})\phi^4 + B\phi^4 \ln\big(\frac{\phi^2}{v^2}\big), \tag{5.12}$$

where

$$B = \frac{3}{64\pi^2}\left[2\left(\frac{M_W}{v}\right)^4 + \left(\frac{M_Z}{v}\right)^4 - 4\left(\frac{M_t}{v}\right)^4\right]. \tag{5.13}$$

The simpler form of the potential, (5.6), still gives the qualitatively correct behavior to the more exact expressions (5.10)–(5.13). The temperature at which the curvature vanishes is now given by $T_0^2 = (M_H^2/4\gamma)(1-4B/\lambda)$. Between the critical temperatures T_c and T_0, there is a potential barrier separating the true and false vacua. Thermal fluctuations of the Higgs field produce bubbles of true vacuum, which then expand and collide to fill space. The rate of bubble nucleation can be computed using the methods of Ref. [31]. One calculates the action of a three dimensional bounce; the nucleation rate is then roughly the exponential of the bounce action, i.e. $\Gamma_{bub} \sim T^4\ (S_3/2\pi T)^{3/2}\ e^{-S_3/T}$. The bubbles then expand with a certain velocity. This velocity can be estimated by requiring that the pressure difference between the inside of the bubble and the outside be compensated by the force exerted by the bubble on the particles just outside. This gives a velocity, $v_b^2 \sim \alpha_W^2/\lambda$. The fraction of false vacuum left at time t, $f(t)$, is given by (2.45) and (2.46). As previously stated, the bubble temperature, T_b, is defined to be the temperature for which $\ln f = -1$, at which point the false vacuum is mostly gone. Since the barrier separation between the true and false vacua is rather large at first, bubble formation is suppressed until the temperature drops sufficiently low. At such a point, due to the exponential behavior of (2.45), space fills up with bubbles rather quickly on a macroscopic scale. This means that the bubble temperature is in fact rather insensitive to the wall velocity v_b. In integrating (2.45), it is convenient to change variables to $\delta = 1 - T/T_c$ using $T^2 = m_{pl}/2ht$, where $h = (4\pi^3 g_*/45)^{1/2}$ (I take $g_* = 100$). I also take $R \sim t^{1/2}$, the scale factor in a flat, hot Robertson-Walker universe. I have numerically determined the bounce action and then integrated (2.45) to determine the bubble temperature for several values of the Higgs mass. This is illustrated in Fig. 7. This curve is actually a fit to ten Higgs mass points, each of which were numerically determined in the above manner:

$$\delta_b = 0.0404 - 0.00111\ M_H + 0.00000799\ M_H^2, \tag{5.14}$$

where M_H is given in GeV.

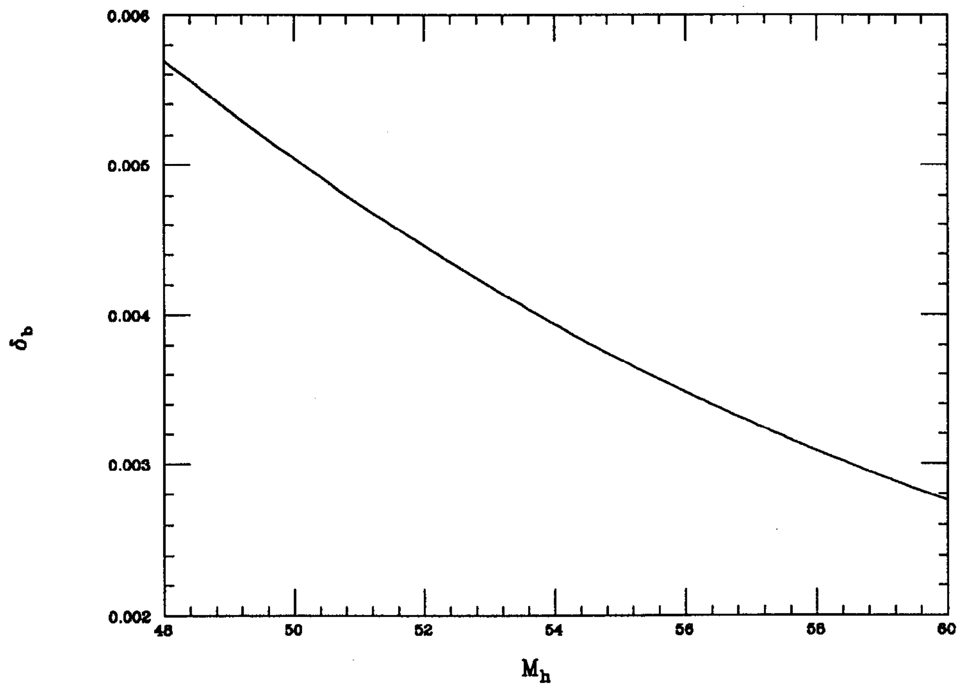

Figure 7. The temperature, expressed as $\delta_b = 1 - T_b/T_c$, at which space fills up with bubbles of true vacuum as a function of Higgs mass for $M_t = 120$ GeV.

Typically, T_b is only slightly below T_c. The reason for this is easy to understand. At the time of the phase transition, the Hubble constant is of order $H \sim 10^{-14}$ GeV. This corresponds to a very long time in microscopic terms. A typical bubble expands for a finite fraction of the Hubble time. Thus an extremely low bubble nucleation rate, of order H^3T, is sufficient to fill the universe with bubbles. The fact that the temperature is higher than T_0 tends to decrease the upper limit on the Higgs mass over that in Ref. [24], since the expectation value of the scalar field is correspondingly smaller, as is the sphaleron energy. However, as will be seen, there are a variety of effects which work in the other direction.

In actually computing the sphaleron energy, one should also use the full effective potential. This fact has already been discussed in Ref. [30]. These authors computed the determinant numerically beginning with a solution of the temperature-dependent potential including only the quadratic and quartic pieces. They note that their result contained a severe λ dependence, and that this could be at least partially accounted for by treating as a perturbation the ϕ^3 term in the potential displayed in eqn. (5.6) above. Indeed, the determinant calculation of these authors can only yield a good approximation to the correct answer if this cubic term *can* be treated perturbatively. This is certainly not the case in the range of temperatures of interest here, where the effective mass of the Higgs field is very small, and where the ϕ^3 terms is at least as important.

In view of this fact, I have obtained the sphaleron solution for the full effective potential, including all effects to one loop, particularly the top quark. The net effect of this is to increase the sphaleron energy as a function of λ. Starting from the bubble temperature, I have integrated the baryon-number rate equation to determine the suppression factor, the fraction of baryon number finally left, as a function of the Higgs mass.

The arguments of Ref. [9] are easily generalized to include the finite temperature effects. For simplicity I will set the Weinberg angle to zero and consider an $SU(2)$ gauge theory with a Higgs doublet Φ. Apart from this, everything else is identical to the standard model. I will work in the $A^0 = 0$ gauge, and it is convenient to make the following rescalings for the position vector, the gauge fields and the Higgs field:

$$\begin{aligned}\mathbf{r} &\rightarrow \xi/gv(T)\\ \mathbf{A} &\rightarrow v(T)\mathbf{A}\\ \Phi &\rightarrow v(T)\Phi\end{aligned} \tag{5.15}$$

where $v(T)/\sqrt{2}$ is the minimum of the Higgs field Φ. It is also useful to work with the real scalar $\phi = \sqrt{2}\,|\Phi|$, which simply takes the value $v(T)$ at the minimum. As pointed out in Ref. [9], there is an unstable, static solution to the classical field equations given by the parameterization

$$\begin{aligned}\mathbf{A} =& 2\frac{f(\xi)}{\xi}\,\hat{\xi}\times\tau,\\ \Phi =& \sqrt{2}\,h(\xi)\,\hat{\xi}\cdot\tau\,u_{1/2},\end{aligned} \tag{5.16}$$

where $u_{1/2} = (0,1)$ and $\tau^a = \sigma^a/2$. When the ansatz (5.16) is substituted into the effective finite temperature action, the energy functional becomes

$$\begin{aligned}E = \frac{gv(T)}{\alpha_W}\int_0^\infty d\xi\Big[4(\frac{df}{d\xi})^2 + \frac{8}{\xi^2}[f(1-f)]^2 \;+\; \frac{1}{2}\xi^2(\frac{dh}{d\xi})^2 +\\ [h(1-f)]^2 + \xi^2\tilde{\Omega}(h,T)\Big],\end{aligned} \tag{5.17}$$

where the rescaled free energy is defined by $\tilde{\Omega}(h) = \Omega(\Phi)/g^2v^4(T)$. This is a rather long, but straightforward exercise. Static solutions extremize the energy, so the sphaleron field equations simply become

$$\begin{aligned}\frac{d^2h}{d\xi^2} =& -\frac{2}{\xi}\frac{dh}{d\xi} + \frac{2}{\xi^2}h(1-f)^2 + \frac{d\tilde{\Omega}}{dh}\\ \frac{d^2f}{d\xi^2} =& \frac{2}{\xi^2}f(1-f)(1-2f) - \frac{1}{4}h^2(1-f).\end{aligned} \tag{5.18}$$

The boundary conditions are taken to be $f,h \rightarrow 0$ as $\xi \rightarrow 0$ and $f,h \rightarrow 1$ as

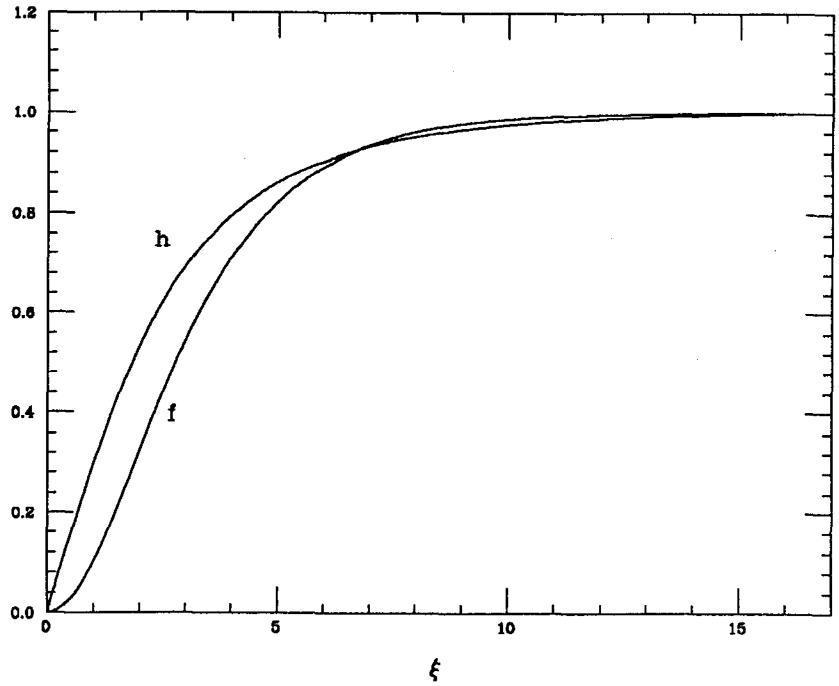

Figure 8. The sphaleron solution for the one-loop finite temperature potential for $M_t = 120$ GeV and $M_H = 52$ GeV.

$\xi \to \infty$. Fig. 8 shows a typical solution to this boundary value problem.

Now, in the adiabatic regime, the baryon number satisfies an equation of the form

$$\frac{dn_B}{dt} = -c\frac{\Gamma}{T^3}\left[n_B - n_B^0\right], \tag{5.19}$$

where $c \sim 10$. The exact value of this constant depends upon the initial mixtures of baryon and lepton number. It is not crucial since uncertainties in Γ are far more important (I take c=10 for definiteness). The term $n_B^0 \sim \partial_0|\phi|^2$, and it reflects a bias in the free energy generated by a changing Higgs field. On the boundary of the expanding bubble walls, where the Higgs field is rapidly changing, the second term in (5.19) dominates, and baryons are produced. However, when the bubbles finish colliding near temperature T_b, the Higgs field changes with the

Hubble rate, and now the second term must be dropped. Unless the baryon-number violation rate falls quickly below the Hubble expansion rate, any previously produced baryon number will be eaten. The sphaleron energy was computed as a function of temperature for about ten values of the Higgs mass. Equation (5.19) was then integrated, and the asymptotic form of $n_B(t_{large})/n_B(0) \equiv S$ was determined. The fit to this suppression factor is shown in Fig. 9 for $\kappa = 10^{-1}$ and $\kappa = 10^{-4}$ and is given by

$$- \ln S = \frac{\kappa}{1000} \exp\big[- 110 + 3.42\ M_H - 0.0249\ M_H^2 \,\big], \tag{5.20}$$

where M_H is in GeV. It is apparent that a Higgs mass greater than about 55 GeV cannot be tolerated.

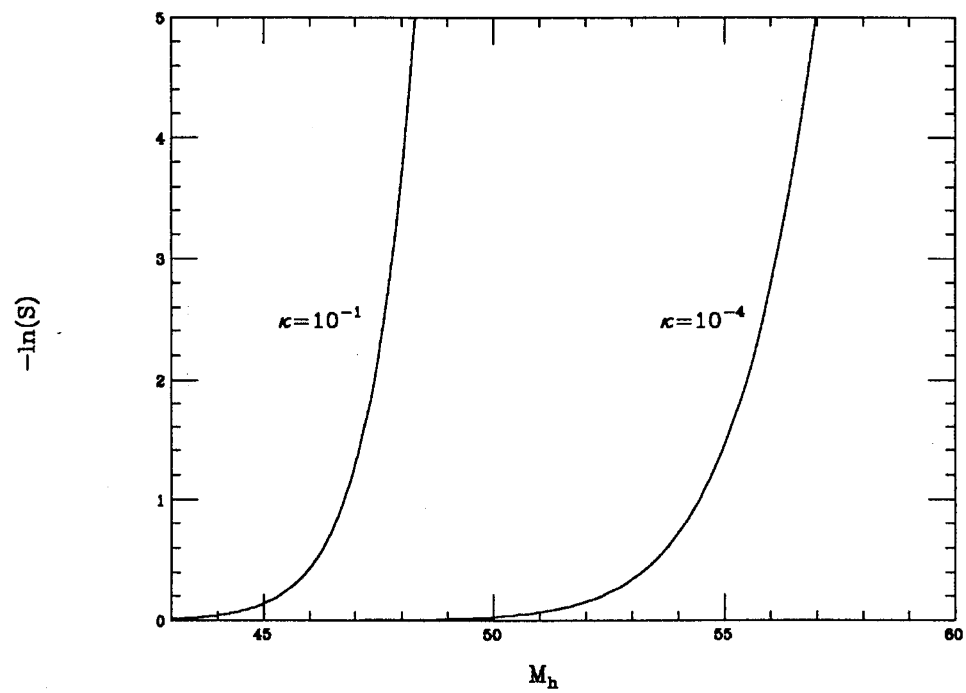

Figure 9. The suppression factor versus Higgs mass.

APPENDIX A

In this appendix I briefly sketch for completeness the method of performing the frequency sums found in the main body of the text. A nice feature of this method is that the zero temperature contribution is easy to locate. I wish to perform the following two fermion sums,

$$I_1^f(a) = \sum_{n=-\infty}^{\infty} \frac{1}{[\tilde{\omega}_n^2 + a^2]^2} \tag{A.1}$$

$$I_2^f(a) = \sum_{n=-\infty}^{\infty} \frac{\tilde{\omega}_n^2}{[\tilde{\omega}_n^2 + a^2]^2}, \tag{A.2}$$

where $\tilde{\omega}_n = (2n+1)\pi$. The basic trick is to write the sum as a contour integral of a function with poles at $\tilde{\omega}_n$. I will first concentrate on the more general expression $\sum f(\tilde{\omega}_n)$, where $f(z)$ is a function with no poles or branch cuts along the real axis. A simple calculation gives

$$\sum_n f(\tilde{\omega}_n) = -\sum_n \oint_{C_n} \frac{dz}{4\pi i} \tan\frac{z}{2}\, f(z), \tag{A.3}$$

where the contour C_n is a small circle centered at $\tilde{\omega}_n$, with orientation shown in Fig. 10. The small circles may be joined to form two lines, one above and one below the real axis, and then these contours may be closed, as shown in Fig. 10. If the function $f(z)$ has poles in the complex plane, the residue theorem may be used to evaluate the integral. For the sums (A .1) and (A .2), there are double poles at $\pm ia$, which give two equal contributions. After some algebra,

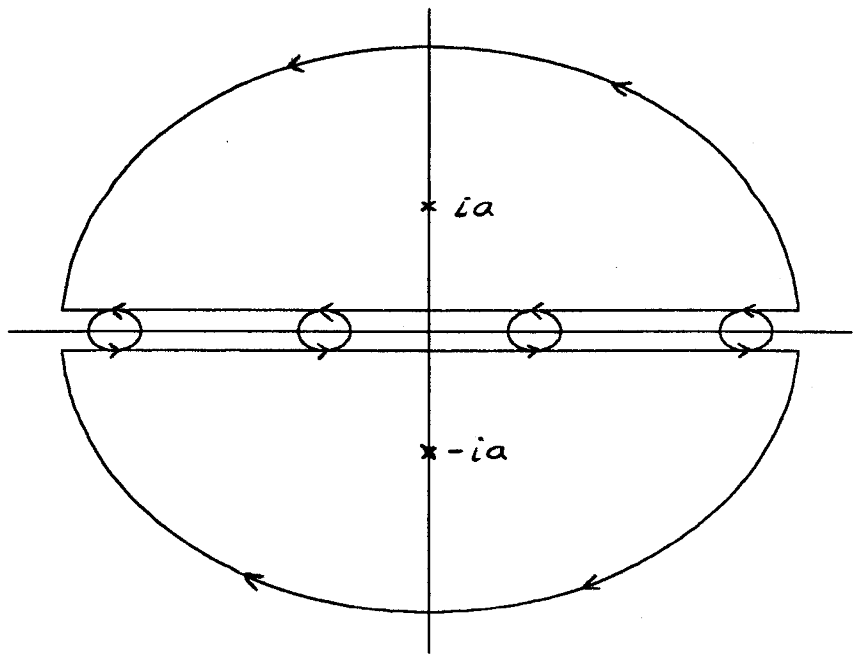

Figure 10. Contour deformation for the integral (A .3).

$$I_1^f(a) = \frac{1}{4a^3}\tanh\frac{a}{2} - \frac{1}{8a^2}\operatorname{sech}^2\frac{a}{2} \tag{A.4}$$

$$I_2^f(a) = \frac{1}{4a}\tanh\frac{a}{2} + \frac{1}{8}\operatorname{sech}^2\frac{a}{2}. \tag{A.5}$$

Boson loops have even frequency sums, $\tilde{\omega}_n = 2n\pi$. Denoting the boson sums corresponding to (A .1) and (A .2) by I_1^b and I_2^b, it is easy to show that

$$I_1^f(a) = \frac{1}{4a^3}\coth\frac{a}{2} + \frac{1}{8a^2}\operatorname{csch}^2\frac{a}{2} \tag{A.6}$$

$$I_2^f(a) = \frac{1}{4a}\coth\frac{a}{2} - \frac{1}{8}\operatorname{csch}^2\frac{a}{2}. \tag{A.7}$$

APPENDIX B

In this appendix I present the fits used in the text. A high temperature (small y) expansion of (5.11) was performed by Dolan and Jackiw, of ref. [6]. This is what is usually done, and it leads to a potential like (5.6). However, a heavy top brings the critical temperature down too low for such an expansion to remain valid. I am then forced to evaluate $I_{\pm}(y)$ with numerical techniques. I do this at 100 points in the interval $y = 0$ to $y = 3$, and then fit to a tenth order polynomial. Fits to the first and second derivatives with respect to y were also performed. The first derivative fits were needed in solving the sphaleron field equation, and the second derivatives were needed in the differential equations program. I list them here for completeness.

$$
\begin{aligned}
I_{-}(y) = &-2.165 + 0.0001952\, y + 0.8193\, y^2 - 0.4958\, y^3 + 0.2017\, y^4 \\
&- 0.08131\, y^5 + 0.03264\, y^6 - 0.01062\, y^7 + 0.002372\, y^8 \\
&- 0.0003133\, y^9 + 0.00001831\, y^{10} \\
I_{+}(y) = &-1.894 - 0.0001938^{y} + 0.4144\, y^2 - 0.02774\, y^3 - 0.1152\, y^4 \\
&+ 0.08152\, y^5 - 0.03468\, y^6 + 0.01065\, y^7 - 0.002267\, y^8 \\
&+ 0.0002928\, y^9 - 0.00001699\, y^{10}.
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
I'_{-}(y) = {} & 0.00004204 \; + \; 1.642\, y \; - \; 1.512\, y^2 \; + \; 0.8868\, y^3 \; - \; 0.5530\, y^4 \\
& + \; 0.3582\, y^5 \; - \; 0.1878\, y^6 \; + \; 0.06916\, y^7 \; - \; 0.01646\, y^8 \\
& + \; 0.002261\, y^9 \; - \; 0.0001358\, y^{10}
\end{aligned}
$$

$$
\begin{aligned}
I'_{+}(y) = {} & - 0.00004214 \; + \; 0.8254\, y \; - \; 0.05914\, y^2 \; - \; 0.5399\, y^3 \; + \; 0.5522\, y^4 \\
& - \; 0.3683\, y^5 \; + \; 0.1864\, y^6 \; - \; 0.06761\, y^7 \; + \; 0.01608\, y^8 \\
& - \; 0.002217\, y^9 \; + \; 0.0001337\, y^{10}.
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
I''_{-}(y) = {} & 1.644 \; - \; 3.074\, y \; + \; 3.001\, y^2 \; - \; 3.298\, y^3 \; + \; 3.733\, y^4 \\
& - \; 3.239\, y^5 \; + \; 1.935\, y^6 \; - \; 0.7645\, y^7 \; + \; 0.1901\, y^8 \\
& - \; 0.02692\, y^9 \; + \; 0.001652\, y^{10}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
I''_{+}(y) = {} & 0.8229 \; - \; 0.06742\, y \; - \; 1.961\, y^2 \; + \; 3.296\, y^3 \; - \; 3.787\, y^4 \\
& + \; 3.235\, y^5 \; - \; 1.927\, y^6 \; + \; 0.7628\, y^7 \; - \; 0.1901\, y^8 \\
& + \; 0.0270\, y^9 \; - \; 0.001656\, y^{10}.
\end{aligned}
$$

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