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FAMILY NUMBER NON–CONSERVATION INDUCED BY THE SUPERSYMMETRIC MIXING OF SCALAR LEPTONS*

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1. Introduction

1.1 Why Supersymmetry?

Many of the original motivations for supersymmetry have long since vanished. It was recognized immediately that such a symmetry must be severely broken in the low energy world. In its early days supersymmetry was looked on principally as a hopeful solution to the gauge hierarchy problem (the question of why there should exist such a discrepancy between the two energy scales M_{weak} at 100 GeV, and M_{GUT} at 10^{15} GeV or more). In order to be useful in this respect the scale at which supersymmetry breaks cannot exceed the weak scale by more than an order of magnitude if the breaking mechanism is to be relatively simple. Thus we have replaced one symmetry-breaking hierarchy problem by another. Admittedly the *radiative* hierarchy problem is resolved but this was but a chimeral problem to begin with (presumably if we possessed the mathematical techniques, and the stamina, to solve the theory to all orders, including non-perturbative effects, we would only have to adjust the overall scales once). Early aspirations of uniting known particles into supermultiplets failed utterly with the result of an instantaneous doubling of the number of states conjectured to exist. None of these new states has yet been seen.

Despite this the supersymmetric principle has become increasingly popular. The true reason for this is the most compelling of motivations, that same æstetic belief in unification which spawned the Grand Unified Theories, and therefore the gauge hierarchy problem. Supersymmetric theories are believed to be the **only** theories capable of unifying fundamental bosonic and fermionic fields. This is undoubtedly due to our imperfect understanding of what spin *is* at a fundamental, topological, level. Surely uniting these two disparate sectors of the material world is at least as profound as considering ever-greater gauge groups to fragment.

When supersymmetry is raised to a local symmetry the Lorentz group manifests itself as a subgroup. Thus gravitation becomes incorporated into the theory in a natural way. In addition, the currently popular string models find the inclusion of supersymmetry highly desirable, if not irrecusable.

1.2 THE FAMILY PUZZLE

One of the outstanding puzzles in elementary particle physics is that of particle generations. The material constituents of the world, as opposed to the mediators of the fundamental forces, appear to be replicated in families or generations. Each seems to be a simulacrum of the previous family, but of greater mass. At this time three such generations are known. Why this should occur is a complete mystery.

This mystery is particularly profound in the leptonic, or weakly interacting, sector where each family would seem to possess an absolutely conserved quantum number, generally referred to as electron, muon and tau number or, collectively, as family number. In all reactions which have been thus far observed the amount of "electron-ness" has neither increased nor decreased. This is in stark contract to the quark (strongly-interacting) sector where mixing between the states is known to occur at a fundamental level. In the light of the "standard model" it is understood why this should be the case. In extensions beyond this it is frequently difficult to retain this symmetry in an appealing fashion.

A symmetry may be realized locally or globally. If local such a "horizontal" symmetry would have profound implications, requiring a new family-changing gauge sector. A global symmetry, if not the consequent of some local symmetry, but if imposed or maintained artificially (for instance by constraining some parameter to be small) is to be viewed as, at best, unnatural and probably untenable. Considered in this light it is interesting to ponder whether lepton family number might not be broken to some small degree. We will discuss a mechanism which might accomplish this.

1.3 SCALAR LEPTON MIXING

The most egregious aspect of (N=1) supersymmetric theories is that each particle state is accompanied by a 'super-partner', a state with identical quantum numbers save that it differs in spin by one half unit. For the leptons these are scalars and are called "sleptons", or scalar leptons. These consist of the charged sleptons (selectron, smuon, stau) and the scalar neutrinos ('sneutrinos').

We examine a model of supersymmetry with soft breaking terms in the electroweak sector. Explicit mixing among the scalar leptons results in a number of effects, principally non-conservation of lepton family number. Comparison with experiment permits us to place constraints upon the model.

Each supersymmetric family has charged sleptons associated with both the left and right-handed helicity states of the corresponding lepton. These, too, may combine (within a single generation). The effect of this will be discussed in chapter two in the context of the anomalous magnetic moment of the muon and the electron. These were selected due to the extremely precise knowledge of their experimental values. In chapter three we will examine the result of intergenerational slepton-mixing for the process $e^+e^- \rightarrow Z^0 \rightarrow \tau^\pm\mu^\mp$. Since several Z^0-factories, in particular the SLC, are expected to produce these particles copiously, this process, should it occur at a reasonable rate, could prove experimentally interesting. These matters will be discussed. In chapter four we consider the classic family number violating process, radiative muon decay: $\mu \rightarrow e\gamma$. These three chapters are followed by a string of appendices, most of which are associated with a particular chapter. These are referred to within the chapters and contain details and derivations not necessary to the cursory understanding of a given chapter.

2. Contribution to Anomalous Magnetic Moment from Slepton Mixing

2.1 INTRODUCTION

In Chapter 1 we saw that the sleptons might mix among themselves in a supersymmetric theory. In this chapter we illustrate the manner in which the anomalous magnetic moments of the electron (a_e) and muon (a_μ) are altered when the theory is made supersymmetric in a minimal way. We then permit the sleptons to mix together which will lead to new effects. We will discover, despite the fact that a_e has been determined experimentally to much greater accuracy than a_μ, that the results of the a_μ calculation tend to place the more stringent restrictions on supersymmetry parameters when compared with experiment.

We will confine ourselves to pure QED and SQED (supersymmetric QED) in this chapter. The extension to the full electroweak theory is straightforward but tedious. Exactly how this should be done will be made obvious in the later chapters and their corresponding appendices.

The calculation will be performed in a somewhat pedagogical fashion. As two-component notation is natural for supersymmetric calculations some basic notation has been introduced in Appendix A. In Appendix B we examine QED in its two and four component forms and consider the interpretation of two-component propagators. Thus armed, in Appendix C we calculate a_e in four-component form and in Appendix D we repeat the calculation in two-component form so that they may be carefully compared. In Appendix E we next compute Δa_e, the *additional* contribution from supersymmetrizing the theory (standard SQED), before the sleptons have been allowed to mix. The most important contribution from the mixing of the sleptons comes from $\widetilde{e}_L - \widetilde{e}_R$ mixing as detailed in Appendix F. If we permit intergenerational mixing then the contribution to a_e will be spread over a greater number of terms. For this reason we restrict ourselves to a single generation here. This has the added benefit of reducing 25 angles and phases to three angles (one per generation assuming three generations). The reverse of this argument will be used in the next chapter when the left-right mixing will be ignored in favor of the intergenerational mixing. The penultimate appendix (Appendix G) related to chapter two will show plots of the various results. In Appendix H we derive a few useful formulae. In the first section certain Feynman integrals, which appear frequently in these pages, are presented. In the second section we discuss generalized versions of the Gordon Decomposition, a generally useful result.

As can be readily seen most of the actual work will be presented in the appendices. This chapter is primarily a summary of the results, comparison with experiment, consequences and commentary. In Section 2.2 we review the concept of anomalous magnetic moments. In 2.3 the results of the appendix calculations are summarized and compared with experiment in 2.4.

2.2 THE MAGNETIC MOMENT

From a Hamiltonian viewpoint the only interaction between the ψ (electron) and A (electromagnetic) fields is via the term $\mathcal{H}_{int} = j^{\mu}_{em} A_{\mu}$ where $j^{\mu}_{em} = e\overline{\psi}\gamma_{\mu}\psi$. The Gordon Decomposition (Appendix H) can be used in conjunction with the Dirac equation in the presence of an A-field $(\not{D}+m)\psi = 0$ where $D_{\mu} = \partial_{\mu} + ieA_{\mu}$, to divide the current into two pieces.[2] In the non-relativistic limit the first reduces to the electric charge current density of non-relativistic quantum mechanics. It is the other term which interests us. This yields an interaction piece

$$\mathcal{H}^{mag}_{int} = j_{\mu}A^{\mu} \sim e/4m\ \overline{\psi}\, F^{\mu\nu}\sigma_{\mu\nu}\psi + \text{ total derivative.}$$

Non-relativistically

$$\overline{\psi}\, F^{\mu\nu}\sigma_{\mu\nu}\psi \approx 2\psi^{+}\underset{\sim}{\sigma}\cdot\underset{\sim}{B}\psi$$

and therefore $\mathcal{H}^{mag}_{int} \to e/2m\ \psi^{+}\underset{\sim}{\sigma}\cdot\underset{\sim}{B}\psi$. But we know that classically $\mathcal{H}^{mag}_{int} = \underset{\sim}{\mu}\cdot\underset{\sim}{B}$ where $\underset{\sim}{\mu}$ is the magnetic moment of the electron. Thus we can identify $\underset{\sim}{\mu} = e/m\ \underset{\sim}{S}$

or, in greater generality, $\underset{\sim}{\mu} = ge/2m\, \underset{\sim}{S}$ where g is the gyromagnetic ratio which here is two.* Classically g is determined by the charge current distribution of an infinitesimal current loop (magnetic dipole). $g = 2$ corresponds to a point dipole. Deviations from $g = 2$ are given by $g = 2(1 + a)$ where a is called the *anomalous magnetic moment.*

From the two-component composition of the $\overline{\psi}^e\gamma^\mu\psi^e$ and $\overline{\psi}\sigma^{\mu\nu}\psi$ vertices (see Appendix B) we may observe their helicity structure. Ignoring the mass of the electron we see that the $A_\mu\overline{\psi}^e\gamma^\mu\psi^e$ term connects e_L to e_L (equivalently e_L^- to e_R^+) while $F_{\mu\nu}\overline{\psi}^e\sigma^{\mu\nu}\psi^e$ connects e_L to e_R (or e_L^- to e_L^+). Thus photonic emission via the electric charge current leaves the helicity of the state unchanged while the magnetic dipole interaction flips the helicity.

Such observations have been made in conjunction with the lowest-order vertex (as illustrated in Fig. 2.1a). The one-loop QED vertex correction is as in the classic infundibular (triangle) diagram of Fig. 2.1b. This will contribute the interactions

$$\overline{u}(p)\Gamma_\mu(p',p)u(p) = \overline{u}(p)\left[\gamma_\mu F_1(q^2) + \frac{i}{2m}\,\sigma_{\mu\nu}q^\nu F_2(q^2)\right]u(p') \ .$$

$F_1(q^2)$ contributes to the (infinite) wavefunction renormalization while, on shell $(q^2 = 0)$, $F_2(q^2)$ contributes to the magnetic moment and thus to $a_e = (g_e - 2)/2$. Indeed, comparing the definition of a_e we see that $a_e = F_2(q^2 \to 0)$. Since QED is a renormalizable theory, and no F_2-type term appears in the QED Lagrangian, we know that the F_2 term resulting from first order contributions must be finite (since there is no bare parameter to absorb an infinity). The anomalous moment can be thought of as measuring an effective radius of the charge current distribution engendered by radiative corrections to the simple bare "point" vertex.

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* Here $\hbar = c = 1$ so that g is in units of Bohr Magnetons. Otherwise $\underset{\sim}{\mu} = ge\hbar/2mc$.

It is often the case than when we consider interactions at large energies, or have particles in internal loops whose masses are large with respect to the external lepton masses, we will find that the magnetic ($\sigma^{\mu\nu}$) contribution to the cross section will be reduced relative to that of the electric (γ^{μ}) term (by $\mathcal{O}(m^2_{lepton}/q^2, m^2_{heavy})$) and can thus often be ignored. If for some reason, such as gauge invariance, the γ^{μ} term cannot appear then the interaction must go through the convective ($\sigma^{\mu\nu}$) term. We will see this happen when we examine $\mu \to e\gamma$ on shell ($q^2 = 0$) but not for $Z^0 \to \tau^+\mu^-$ ($q^2 \neq 0$).

2.3 RESULTS

In this section the results of computing the various contributions to $a_\mu = (g_\mu - 2)/2$ and $a_e = (g_e - 2)/2$, as algebraically detailed in the first few appendices, are ingeminated. We work to one-loop (α) order. The one-loop vertex correction from pure QED (Appendices 2 and 3) yields

$$a_e = a_\mu = \frac{\alpha}{2\pi} \,. \tag{2.1}$$

When supersymmetric QED (SQED) contributions (assuming no mixing amongst the sleptonic states) are appended, as in Appendix D, we discover the additional contribution

$$\Delta a_e = \frac{\alpha}{2\pi} \left\{ I_L + I_R \right\} \tag{2.2}$$

where

$$I_{L,R} = \int_0^1 dx \int_x^1 dz \, \frac{z(1-z)}{-z^2 + \dfrac{m_e^2 - m^2_{\tilde{e}_{L,R}} + m_\lambda^2}{m_e^2} z - \dfrac{m_\lambda^2}{m_e^2}} \tag{2.3}$$

m_e is the mass of the electron.

$m_\lambda = m_{\tilde{\gamma}}$ is the mass of the photino.

$m_{\tilde{e}_{L,R}}$ are the masses of the left and right selectron.

$$\text{For } \Delta a_\mu,\ m_e \text{ is replaced by } m_\mu \text{ and } m_{\tilde{e}_{L,R}} \text{ by } m_{\tilde{\mu}_{L,R}} \ . \tag{2.4}$$

We remark on some of the limiting cases which are presented in the appendix.

A likely limit is $m_{\tilde{e}_{L,R}} \gg m_\lambda \gg m$ (equation E.24). Then

$$\Delta a_e \doteq \Delta a_a^{(m_\lambda=0)} = -\frac{\alpha}{12\pi} \left\{ \frac{m_e^2}{m_{\tilde{e}_L}^2} + \frac{m_e^2}{m_{\tilde{e}_R}^2} \right\} . \tag{2.5}$$

This is precisely Fayet's 1974 result.[3] If we relax the condition that $m_\lambda \ll m_{\tilde{e}_{L,R}}$ we find (eqn E.32) that, to the next order in $m_\lambda / m_{\tilde{e}_{L,R}}$,

$$\Delta a_e = \Delta a_e^{(m_\lambda=0)} \left[1 - 2\, \frac{m_\lambda^2}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2} \left(\frac{m_{\tilde{e}_L}^2}{m_{\tilde{e}_R}^2} + \frac{m_{\tilde{e}_R}^2}{m_{\tilde{e}_L}^2} \right) \right] . \tag{2.6}$$

In limits where supersymmetry remains unbroken $m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e$ and $m_\lambda = 0$. In this limit we find that $\Delta a_e = -\alpha/2\pi$ so that, for (2.1),

$$a_e = a_e^{QED} + \Delta a_e = 0 \ . \tag{2.7}$$

This is gratifying as we might have anticipated that there would be no anomalous magnetic moment to any order in an exactly supersymmetric theory (for the same reason that loop divergences cancel). This was first demonstrated by Ferrara and Remidi.[4]

We now consider the effect of permitting the left and right sleptons to mix. Indeed, in realistic models with Higgs fields, it is often difficult to prevent them from doing so.[5] Since these states differ in hypercharge and weak isospin we see that $SU(2) \times U(1)_Y$ is broken, although $U(1)_{\text{em}}$ is not. (This may be induced both radiatively[5] and at the tree level.[6]) Most treatments only consider the case where the photino is massless or nearly so. We shall leave m_λ as a free parameter.* This introduces a number of important new effects which would appear in a complete electroweak treatment[7] and thus represents a suitable decoction but with fewer complications.

Let us first consider muons. The mass eigenstates may be taken to be

$$\begin{aligned}\widetilde{\mu}_1 &= \cos\phi\widetilde{\mu}_L + \sin\phi\widetilde{\mu}_R \\ \widetilde{\mu}_2 &= -\sin\phi\widetilde{\mu}_L + \cos\phi\widetilde{\mu}_R \ .\end{aligned} \tag{2.8}$$

It is found (in Appendix F) that, to lowest order in $m_\mu^2/(m_{\widetilde{\mu}_2}^2 - m_\lambda^2)$,

$$\Delta a_\mu = \frac{\alpha}{2\pi}\,\frac{m_\lambda m_\mu \sin\phi\cos\phi}{(m_{\widetilde{\mu}_2}^2 - m_\lambda^2)^3}\left\{\tfrac{1}{2}\left(m_{\widetilde{\mu}_2}^4 - m_\lambda^4\right) - m_\lambda^2 m_{\widetilde{\mu}_2}^2 \ell n\,\frac{m_{\widetilde{\mu}_2}^2}{m_\lambda^2}\right\} - (m_{\widetilde{\mu}_2}^2 \rightarrow m_{\widetilde{\mu}_1}^2)\ . \tag{2.9}$$

We can write (2.9) as

$$\Delta a_\mu = C(f(X_2) - f(X_1)) = \Delta a_\mu^2 - \Delta a_\mu^1 \tag{2.10}$$

where we have let

$$\Delta a_\mu^i = C f(X_i) \tag{2.11}$$

and

$$X_i = \frac{m_{\widetilde{\mu}_i}^2}{m_\lambda^2} \tag{2.12}$$

$$f(x) = \frac{\tfrac{1}{2}(x^2 - 1) - x\ell n\,x}{(x-1)^3} \qquad f(1) = \frac{1}{6} \tag{2.13}$$

$$C = \frac{\alpha\sin\phi\cos\phi}{2\pi}\,\frac{m_\mu}{m_\lambda} \qquad f'(1) = -\frac{1}{12}\ . \tag{2.14}$$

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* There are astrophysical bounds on m_λ but these are model dependent.

Note that Δa^i_μ is perfectly regular at $m_{\tilde{\mu}_i} = m_\lambda$ and is given by

$$|\Delta a^i_\mu| = \frac{1}{6}C = \frac{\alpha}{12\pi}\frac{m_\mu}{m_\lambda}\sin\phi\cos\phi \quad [m_{\tilde{\mu}_2} = m_\lambda \gg m_\mu] \; . \tag{2.15}$$

The important point is to note that $\Delta a_\mu \sim m_\mu/m_{SUSY}$ here whereas, prior to slepton mixing, we find that $\Delta a_\mu \sim (m_\mu/m_{SUSY})^2$. The salience of this becomes apparent when we make contact with experimental limits in the next section. When the anomalous moment is linearly dependent upon the *lepton* mass we have substantially greater sensitivity to new effects. Concomitant with this statement is the assertion that we can place more stringent limits on the supersymmetry parameters of the theory. It may be noted, however, that the Feynman diagrams which are responsible for the m^2_μ terms are still present (Diagrams I, II, III, IV in Fig. F.2 of Appendix F) and there are terms of $\mathcal{O}(m^3_\mu)$ which have been dropped in (2.9) (see equation F.15) The $\mathcal{O}(m^2_\mu)$ term is (for $m_\mu \ll m_{\tilde{\mu}_{1,2}}, m_\lambda$)

$$\begin{aligned}
\Delta a_\mu^{(m^2_\mu)} &= -\frac{\alpha}{12\pi}\left\{\frac{m^2_\mu}{m^2_{\tilde{\mu}_1}}\;[1+\;F(\mathcal{R}_1)] + \frac{m^2_\mu}{m^2_{\tilde{\mu}_2}}\;[1+\;F(\mathcal{R}_2)]\right\} \\
F(x) &= x(6x^2-9x+2) + x^2(1-x)^2\ln\left(\frac{x-1}{x}\right) \qquad (2.16) \\
\mathcal{R}_{1,2} &= \frac{m^2_\lambda}{m^2_\lambda - m^2_{\tilde{\mu}_{1,2}}}
\end{aligned}$$

which can become dominant if $|\theta|$ or $|\pi-\theta| \lesssim \mathcal{O}\,(m_\mu/m_{SUSY})$. In the limit that $m_{\tilde{\mu}_{1,2}} \gg m_\lambda$ (2.16) becomes

$$\Delta a_\mu = -\frac{\alpha}{12\pi}\left\{\frac{m^2_\mu}{m^2_{\tilde{\mu}_1}} + \frac{m^2_\mu}{m^2_{\tilde{\mu}_2}}\right\} \tag{2.17}$$

which is essentially (2.5) and would give similar limits for slepton masses. For the remainder of this section we will restrict our attention to the terms which are linear in m_μ.

Appendix G contains various plots related to Δa_μ. In Fig. G.1 $\frac{1}{C}\,\Delta a^i_\mu$ is plotted against X_i (as define in (2.12)). We see that $\frac{1}{C}\,\Delta a^i_\mu$ decreases monotonically from $\frac{1}{2}$ at $X_i = 0$ to about 0.1 at $X_i = 2$ and thence asymptotically to zero. Since $\Delta a_\mu = \Delta a^2_\mu - \Delta a^1_\mu$ we have plotted $\frac{1}{C}\,\Delta a_\mu$ versus X_1 for $m_{\tilde{\mu}_2} = 1.01\ m_{\tilde{\mu}_1}$ and $m_{\tilde{\mu}_2} = 2m_{\tilde{\mu}_1}$ in figures G.2 and G.3 respectively. We note the general feature of a maximum for $|\Delta a_\mu|$, call it $|\Delta a_\mu|_{\max}$, which occurs at some $X_1 = X_{1\ \max}$. As $y \equiv m^2_{\tilde{\mu}_2}/m^2_{\tilde{\mu}_1}$ increases $X_{1\ \max}$ and $|\Delta a_\mu|_{\max}$ change. Thus $|\Delta a_\mu|_{\max} = |\Delta a_\mu|_{\max}(y)$ and $X_{1\ \max} = X_{1\ \max}(y)$. In particular for $y = 0$ ($m_{\tilde{\mu}_1} = \infty$) we find that $\Delta a^1_{\mu\ \max} = \frac{C}{2}$ and $X_{1\ \max} = \infty$. Since $\Delta a_\mu = \Delta a^2_\mu - \Delta a^1_\mu$ we see that $X_{1\ \max}(y) = (1/y)\ X_{1\ \max}(1/y)$ and $\Delta a_{\mu\ \max}(y) = -\Delta a_{\mu\ \max}(1/y)$. Thus $X_{1\ \max} = 0$ for $y = \infty$.

The value of $|\Delta a_\mu|_{\max}$ is important in that it will place the most stringent limits on the supersymmetric parameters. The maxima occur at solutions to the dimensionless transcendental equation (here $x_i = X_i$)

$$\ln x_1 = \frac{y\big[(x_1 y + 5)(x_1 y - 1) - 2(2x_1 y + 1)\ln y\big](x_1 - 1)^4 - (x_1 + 5)(x_1 - 1)(x_1 y - 1)^4}{2\big[y(2x_1 y + 1)(x_1 - 1)^4 - (2x_1 + 1)(x_1 y - 1)^4\big]} \tag{2.18}$$

(where $y \neq 1$ when $x_1 = 1$)

$$x_1 = \frac{m^2_{\tilde{\mu}_1}}{m^2_\lambda} \qquad x_2 = \frac{m^2_{\tilde{\mu}_2}}{m^2_\lambda} \qquad y = \frac{x_2}{x_1} = \frac{m^2_{\tilde{\mu}_2}}{m^2_{\tilde{\mu}_1}}\ . \tag{2.19}$$

In Fig. G.4 we have plotted $X_{1\ \max}$ as a function of y whereas $\frac{1}{C}\,\Delta a_{\mu\ \max}$ versus y appears in Fig. G.5-6. It must always be remembered that $C \sim 1/m_\lambda$. Unsurprisingly the maximal value achievable by $|\Delta a_\mu|$ is $\frac{1}{2}C$ at $y = 0$ and $y = \infty$ (at which points $X_{1\ \max} = \infty, 0$). Thus $\frac{1}{C}\,|\Delta a_\mu|$ achieves its greatest value when $m_{\tilde{\mu}_1}$ (or $m_{\tilde{\mu}_2}$) $\to 0$ while $m_{\tilde{\mu}_2}$ (or $m_{\tilde{\mu}_1}$) $\to \infty$. Since $C \sim m_\mu/m_\lambda$ we expect that $|\Delta a_\mu|$ is maximized when the supersymmetric masses are as small as possible and the mass-splitting in the sleptonic sector is maximal ($m_{\tilde{\mu}_j} \gg m_{\tilde{\mu}_{i\neq j}}$). Indeed this is a generic feature which we will find common to all processes examined in this paper. (Since we have assumed that $m_\lambda, m_{\widetilde{\mu}} \gg m_\mu$ in the above expressions, it would be improper to take both $m_{\widetilde{\mu}_i}$ and m_λ to zero). This is, however, partially an artifact from dividing (2.9) through by m_λ^6 to get (2.13). This new contribution depends fundamentally upon the photino flipping helicity, contributing a factor of m_λ which replaces one factor of m_μ, and so vanishes as $m_\lambda \to 0$. As long as the photino possesses any finite non-zero value ($m_\lambda \neq 0$) we maximize $|\Delta a_\mu|$ by taking either $m_{\widetilde{\mu}_i}$ or $m_{\widetilde{\mu}_2}$ to be as light as is consistently possible with experiment and the other to be much heavier. Since experimentally $m_{\widetilde{\mu}_i} \gtrsim 25$ GeV we might consider the case $m_\lambda \ll m_{\widetilde{\mu}_1} \ll m_{\widetilde{\mu}_2}$. In this limit (2.9) becomes

$$\Delta a_\mu = -\frac{\alpha}{4\pi} \frac{m_\lambda m_\mu}{m^2_{\widetilde{\mu}_1}} \sin\phi \cos\phi \ . \tag{2.20}$$

Although such "large mass-splitting" cases are not ruled out one generally finds, if explicit supersymmetry-breaking terms have their ultimate origin via spontaneous supersymmetry-breaking at some larger mass scale, that states tend to have meager mass-splitting (in this case $\delta m_{\widetilde{\mu}} = m_{\widetilde{\mu}_2} - m_{\widetilde{\mu}_1} \ll m_{\widetilde{\mu}_{1,2}}$). Indeed one is led in some models[8] to the conclusion that, should the spontaneous breaking arise in the gauge ("D") terms of the superpotential, then the mass-splitting is expected to be very small. In such models $\widetilde{\mu}_L$ and $\widetilde{\mu}_R$ possess equal masses and are unmixed at the tree level but develop a small mixing term at one loop via diagrams such as that in Fig.2.2.

The result is a smuon mass matrix of the form

$$(\widetilde{\mu}_L \ \ \widetilde{\mu}_R)^* \begin{pmatrix} m^2 & \delta m^2 \\ \delta m^2 & m^2 \end{pmatrix} \begin{pmatrix} \widetilde{\mu}_L \\ \widetilde{\mu}_R \end{pmatrix}$$

where

$$\delta m^2 \sim O\left(\frac{\alpha}{\pi}\right) m_\mu v$$

$$m^2 \sim O(v^2)$$

$$v \sim O(100 \ GeV)$$

and $|m^2_{\tilde\mu_1} - m^2_{\tilde\mu_2}| = 2\delta m^2 \ll m^2_{\tilde\mu_{1,2}}$. The mixing angle is near maximal.*

Should the supersymmetry-breaking terms arise in the Yukawa ("F") sector certain scenarios[8] suggest that the $\tilde\mu_L - \tilde\mu_R$ mixing angle, ϕ, would be small although the mass splitting could be larger.

When the mass-splitting is small we find (for $m_\lambda < m_{\tilde\mu}$)

$$\Delta a_\mu = -\frac{\delta m^2}{m^2_{\tilde\mu}}\frac{\alpha}{4\pi}\sin\phi\cos\phi\,\frac{m_\lambda m_\mu m^6_{\tilde\mu}}{(m^2_{\tilde\mu}-m^2_\lambda)^4}$$
$$\times\left\{1+4\,\frac{m^2_\lambda}{m^2_{\tilde\mu}}\left[1-\ln\,\frac{m^2_{\tilde\mu}}{m^2_\lambda}\right]-\frac{m^4_\lambda}{m^4_{\tilde\mu}}\left[5+2\ln\,\frac{m^2_{\tilde\mu}}{m^2_\lambda}\right]\right\} \tag{2.21}$$

where

$$\delta m^2 = m^2_{\tilde\mu_1} - m^2_{\tilde\mu_1} \qquad m_{\tilde\mu} = m_{\tilde\mu_{1,2}}$$

or

$$\Delta a_\mu = -\frac{\delta m^2}{m^2_{\tilde\mu}}\,\frac{\alpha}{4\pi}\,\sin\phi\cos\phi\,\frac{m_\mu}{m_\lambda}\,p(x)$$

where

$$x = m^2_{\tilde\mu}/m^2_\lambda$$

$$p(x) = \frac{x}{(x-1)^4}\,\{(x+5)(x-1)-2(2x+1)\ell n\, x\}$$

$$p(x\to 0)\to -4x\ell n\,x \tag{2.22}$$

$$p(x\to 1)\to \frac{1}{6}$$

$$p(x\to\infty)\to\frac{1}{x}\,.$$

(Note that this formulation is equivalent to that of reference 8 provided that one allows for the difference in the definition of α used there.)

* Since this is actually a two-loop process there are a number of other diagrammatic configurations of comparable magnitude which were not included in the original treatment. This must be done if a more precise model is required.

2.4 COMPARISON WITH EXPERIMENT

Sooner or later it behooves us to make contact with reality. Here we use the experimental limits on measurements of a_e and a_μ to place restrictions on the contributions considered in the previous subsection. It should be noted that g_e has been so well measured that a significant fraction of the uncertainty which is engendered when the empirical and theoretical values are compared may be ascribed to the error inherent in high-order calculations *within the standard model*, particularly from the hadronic contributions.

Current limits on the deviation of $\Delta a_{\mu,e}$ from predicted standard model values are:[9,10]

$$|\Delta a_\mu| \lesssim 2 \times 10^{-8} \tag{2.23}$$

$$|\Delta a_e| \lesssim 3 \times 10^{-10} \,. \tag{2.24}$$

First let us consider the implications in the case of unmixed sleptons. When $m_{\tilde{\mu}_{L,R}} \gg m_\lambda \gg m_\mu$ we obtained the result of (2.5) which, using (2.23), translates to

$$\frac{1}{m^2_{\tilde{\mu}_L}} + \frac{1}{m^2_{\tilde{\mu}_R}} \lesssim \frac{1}{(10\ GeV)^2} \,.$$

This would exclude a circle of the radius $(10\ GeV)^{-1}$ centered at the origin in the $m^{-1}_{\tilde{\mu}_L} - m^{-1}_{\tilde{\mu}_R}$ plane or, equivalently, the region indicated in Fig. 2.3.

This implies an absolute limit on the smuon masses of

$$m_{\tilde{\mu}} \gtrsim 10\ GeV \tag{2.25}$$

From (2.5) and (2.24) in the $m_{\tilde{e}} \gg m_\lambda \gg m_e$ limit we would find a corresponding constraint on the selectron mass from Δa_e of only

$$m_{\tilde{e}} \gtrsim 0.4 \; GeV \; . \tag{2.26}$$

In passing, we note that should the photino be substantially more massive than the smuon ($m_\lambda \gg m_{\tilde{\mu}} \gg m_\mu$) then (2.25) would become $m_\lambda \gtrsim 5$ GeV.

Turning now to the $\tilde{\mu}_L - \tilde{\mu}_R$ mixed case we will first consider the implications of substantial mass-splitting of the eigenstates $\tilde{\mu}_1$ and $\tilde{\mu}_2$ and then will return to the slightly split scenario. From (2.20) and (2.25) we see that when

$$m_\mu \ll m_\lambda \ll m_{\tilde{\mu}_1} \ll m_{\tilde{\mu}_2}$$

that

$$|\Delta a_\mu| = \frac{\alpha}{4\pi} \frac{m_\lambda m_\mu}{m^2_{\tilde{\mu}_1}} \sin\phi \cos\phi \lesssim 2 \times 10^{-8}$$

or, assuming maximal mixing ($\phi = \pi/4$),

$$m_{\tilde{\mu}_1} \gtrsim 39 \; GeV \; \sqrt{m_\lambda (in \; GeV)} \; . \tag{2.27}$$

We obtain a similar limit on $m_{\tilde{e}}$ under such conditions. In light of the assumption $m_{\tilde{\mu}_1} \gg m_\lambda$ this provides a constraint on $m_{\tilde{\mu}_{1,2}}$ for light photinos ($m_\lambda \ll 1$ TeV). Note that if we let $m_\lambda = m_\mu$ in (2.27) then the inequality reduces roughly to that of (2.25) in accordance with our observation in the previous section that one power of photino mass has replaced a factor of lepton mass in Δa_μ. This has its most profound effect, of course, when we consider selectrons, as in (2.26), and when the photino mass is comparable or greater than the mass of the slepton (if $m_{\tilde{\mu}} \simeq m_\lambda$ then both must exceed 0.5 TeV).

If the mass-splitting is small we might consider the case corresponding to (2.27), *i.e.*

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \ll 1 \qquad m_\mu \ll m_\lambda \ll m_{\tilde{\mu}_{1,2}}$$

Then from (2.22),

$$\Delta a_\mu \sim \frac{\delta m^2}{m_{\tilde{\mu}}^2} \frac{\alpha}{4\pi} \sin\phi \cos\phi \frac{m_\mu m_\lambda}{m_{\tilde{\mu}}^2}$$

and from (2.23)and (2.24) we obtain

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \frac{m_\mu m_\lambda}{m_{\tilde{\mu}}^2} \lesssim 7 \times 10^{-5} \tag{2.28}$$

or $\lesssim 10^{-6}$ in the $\tilde{e}$ case.

This is not a terribly useful limit. If, for instance, we were to take $m_\lambda \sim 10$ GeV, $m_{\tilde{\mu}} \sim 100$ GeV we would find

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \lesssim \mathcal{O}(1) \ .$$

We note that the above treatment differs only in detail from a full supersymmetric electroweak treatment and the results are similar. The principal difference in the latter is that there are a plethora of contributions from additional gauginos and higgsinos (which may mix amongst themselves). While we could have $m_{\tilde{G}} < m_{\tilde{\mu}}$ for a gaugino $\tilde{G}$, we might equally well have $m_{\tilde{G}'} > m_{\tilde{\mu}}$ for a second gaugino. The complication in placing bounds is evident when we point out that if $m_{\tilde{\mu}} \approx m_\lambda$ then (2.28) will become (using (2.15))

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \lesssim 4 \times 10^{-3} m_\lambda (in\ GeV) \tag{2.29}$$

which is, potentially, significantly more restrictive. We include such complications when we consider mixing among sleptons of different generations.

FIGURE CAPTIONS

1. (a) Tree-level vertex for a massless electron.

   (b) One Loop QED correction.

2. Mass-Mixing in the Model of Ref. 8.

3. Excluded region of the $m_{\tilde{\mu}_L} - m_{\tilde{\mu}_R}$ plane if $m_{\tilde{\mu}_{L,R}} \gg m_\lambda \gg m_\mu$ for case of no left-right mixing.

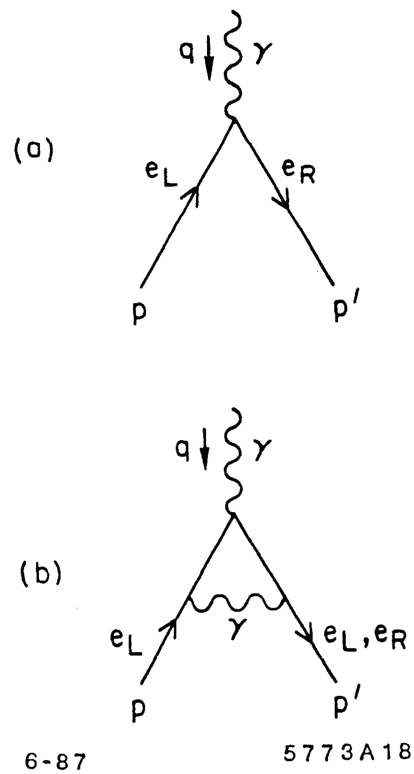


Fig. 2.1

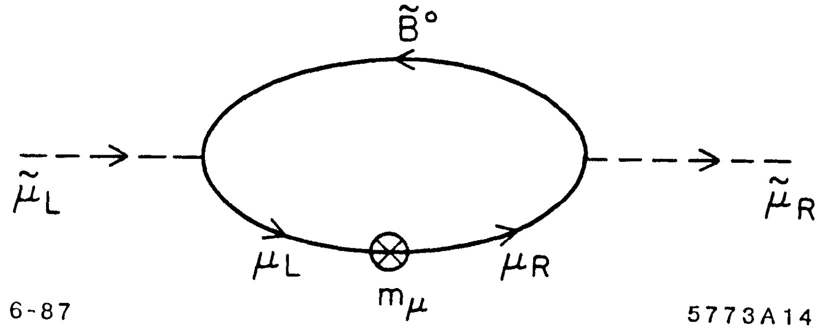


Fig. 2.2

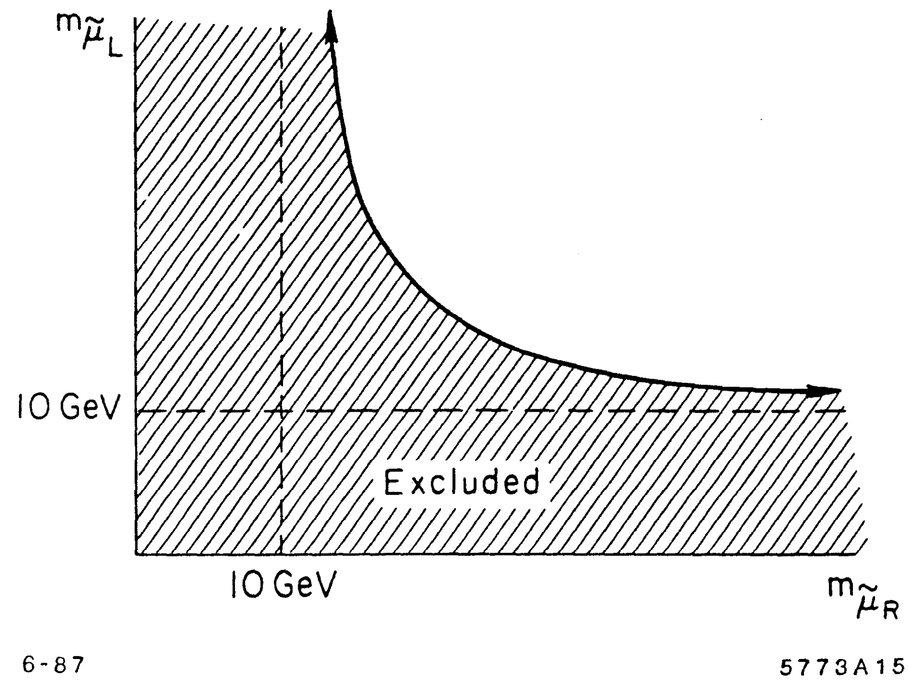


Fig. 2.3

3. $Z^0 \rightarrow \tau\mu$

3.1 INTRODUCTION

In this chapter we examine the possibility of real Z^0 decay in a way which violates lepton family number. We permit the admixture of the scalar partners of the leptons (sleptons) from different generations. This chapter duplicates, to a large extent, the work contained in a previous paper.[11] We will work in the more familiar four-component Dirac notation.[13] Our notation is based upon that of Refs. 12 and 13. Useful sample calculations using this notation may be found in several places.[12,14]

Our principal consideration will be the process $e^+e^- \rightarrow Z^0 \rightarrow \tau^\pm\mu^\mp$. Should such an interaction occur with a moderate cross-section it should be observable at the emerging generation of colliders. The choice of the final states is somewhat arbitrary. It is partially based on the possibility that the sleptonic masses might progress in the same order as their leptonic counterparts and that more massive states might have a greater tendency to mix. Neither of these assumptions is, of course, necessary, however it is highly desirable to consider that most of the mixing occurs between two generations in order to reduce the number of angular parameters present. There is also the observation that there are additional box diagrams present if one of the final states is an electron. When dealing with on-shell Z^0's such contributions are negligible and the results would be equally applicable to mixing in the $e-\mu$ or $e-\tau$ sectors. It must be noted that such computations may be, as will be seen in Chapter 4, readily extended off-shell.

As in the previous chapter the bulwark of detail will be presented in related appendices (Appendices I-N). Appendix I provides a brief overview of supersymmetry, motivates the construction of supersymmetric Lagrangia in general, establishes the notation and presents the "standard supersymmetric Lagrangian". Appendix J considers the diagonalization of the various mass matrices in different bases. In Appendix K the Feynman rules are derived and detailed examples of their use given. The computation of the matrix element is done in Appendix L, notation established and various integral functions defined. These are analyzed in detail, both analytically and numerically, in Appendix M. Finally in Appendix N the various background calculations are presented.

3.2 THE MODEL

We consider a minimal supersymmetric extension of the standard model and restrict our attention to the leptonic sector. We admit the inclusion of explicit soft supersymmetry-breaking terms, that is those which do not introduce new quadratic divergences. Thus this "supersymmetric standard model" will develop arbitrary scalar lepton masses. Presumably all such terms arise at some higher energy scale in a more fundamental way (such as supersymmetry-breaking in a manner which is not family-independent). We shall not concern ourselves here with the ultimate origin of these terms but consider the effective low-energy Lagrangian as given. In this way we may proceed in a reasonably model-independent fashion (subject to the not insubstantial restrictions of using minimal supersymmetry).

Given scalar leptons of arbitrary mass it is only lepton family number, an empirically observed global (presumably) symmetry, which would prevent them from mixing. There is some feeling that exact global symmetries are very unnatural[15] (perhaps representing the approximate remnants of some more fundamental local symmetry at larger energy scales). Thus we allow the sleptons to have arbitrary mass and mix with arbitrary angle. Family-number violation might also come about through any number of other effects in conjunction with this or independently.[11] We shall not consider this point further other than to note that specific experimental observations will generally be able to distinguish one model from another (given sufficient statistics). The same sort of analysis may be applied to the hadronic decay of the Z^0 leading to possible enhancements of flavour symmetry violation in that sector.[16,17]

We have seen that the left and right sleptons may readily mix in each of the $\widetilde{e}$, $\widetilde{\mu}$ and $\widetilde{\tau}$ sectors. We therefore anticipate mixing among six charged sleptons. In general, mixing among N states may be described by $\frac{1}{2}N(N-1)$ real angles and $\frac{1}{2}(N-1)(N-2)$ complex phases. The latter will typically result in CP violation. In the above example (three generations) we would need 15 real angles and 10 phases. In addition there are three neutral sneutrino states (six if we permit right-handed sneutrinos in the spectrum) resulting in three more angles and one additional phase. In order to reduce the burgeoning number of parameters somewhat we will consider the case where only the two 'heaviest' generations mix significantly with little left-right mixing. Thus the only mixings are between:

$$\begin{aligned}
&\tilde{\mu}_L \text{ and } \tilde{\tau}_L \text{ with angle } \theta_L \\
&\tilde{\mu}_R \text{ and } \tilde{\tau}_R \text{ with angle } \theta_R \\
&\tilde{\nu}_\mu \text{ and } \tilde{\nu}_\tau \text{ with angle } \theta_\nu
\end{aligned} \tag{3.1}$$

So that

$$\begin{aligned}
\tilde{\ell}_{L_1} &= \tilde{\mu}_L \cos\theta_L + \tilde{\tau}_L \sin\theta_L \\
\tilde{\ell}_{L_2} &= -\tilde{\mu}_L \sin\theta_L + \tilde{\tau}_L \cos\theta_L \\
\tilde{\ell}_{R_1} &= \tilde{\mu}_R \cos\theta_R + \tilde{\tau}_R \sin\theta_R \\
\tilde{\ell}_{R_2} &= -\tilde{\mu}_R \sin\theta_R + \tilde{\tau}_R \cos\theta_R \\
\tilde{\nu}_1 &= \tilde{\nu}_\mu \cos\theta_\nu + \tilde{\nu}_\tau \sin\theta_\nu \\
\tilde{\nu}_2 &= -\tilde{\nu}_\mu \sin\theta_\nu + \tilde{\nu}_\tau \cos\theta_\nu \quad .
\end{aligned}$$

with $\tilde{\ell}_{L,R_1}$, $\tilde{\ell}_{L,R_2}$ and $\tilde{\nu}_{1,2}$ the physical mass eigenstates.

The particle spectrum of the supersymmetric "standard" model is illustrated in Table I-1 in Appendix I. The superpotential of the chiral superfields must be homogeneous (no terms involving both chiral and antichiral fields as one finds in the field strength)[1,18] with the result that a minimum of two Higgs doublets are required to give the u and d quarks (and thus charged leptons) masses.[11] In this minimal model we will ignore the possible inclusion of a Higgs singlet, $\widehat{\mathrm{N}}$.

These points are discussed in Appendix I. The fields which will be needed in this chapter are presented in Table 3-1.

A major complication arises in the gaugino sector of the theory. Let us suppose that supersymmetry breaks at an energy scale above that at which $SU(2) \times U(1)$ does. At large energies we will have exact supersymmetric $SU(2) \times U(1)$. Thus we have massless $W^0, W^\pm$ $SU(2)$ gauge bosons and a massless B^0 $U(1)$ gauge boson with corresponding supersymmetric partners, all massless fermions, $\widetilde{W}^0, \widetilde{W}^\pm$ and $\widetilde{B}^0$. Similarly there are two Higgs doublets H_1 and H_2 with their superpartners $\widetilde{\psi}_{H_1}$ and $\widetilde{\psi}_{H_2}$. Note that the above are Weyl spinors (this notation differs slightly from that of Ref. 12). It is frequently convenient to write these as four-component objects. For the gauginos we shall use the same symbol.

$$\widetilde{W}^+ = \begin{pmatrix} -i\widetilde{W}^+ \\ i\overline{\widetilde{W}}^- \end{pmatrix} \qquad \widetilde{W}^0 = \begin{pmatrix} -i\widetilde{W}^0 \\ i\overline{\widetilde{W}}^0 \end{pmatrix} \qquad \widetilde{B}^0 = \begin{pmatrix} -i\widetilde{B}^0 \\ i\overline{\widetilde{B}} \end{pmatrix} \tag{3.2}$$

whereas we will distinguish the Higgs four component object as $\widetilde{H}$,

$$\widetilde{H}^+ = \begin{pmatrix} \widetilde{\psi}^+_{H_2} \\ \overline{\widetilde{\psi}}^-_{H_1} \end{pmatrix} \quad \widetilde{H}^0_1 = \begin{pmatrix} \widetilde{\psi}^0_{H_1} \\ \overline{\widetilde{\psi}}^0_{H_1} \end{pmatrix} \quad \widetilde{H}^0_2 = \begin{pmatrix} \widetilde{\psi}^0_{H_2} \\ \overline{\widetilde{\psi}}^0_{H_2} \end{pmatrix} . \tag{3.3}$$

The neutral states are Majorana. Note that $\widetilde{H}^+$ is built from both higgsinos. When supersymmetry breaks, effective low energy terms of the form

$$\begin{aligned} \mathcal{L}_{Br} = &-\tfrac{1}{2} M\tilde{W}^0\tilde{W}^0 - \tfrac{1}{2} M'\tilde{B}^0\tilde{B}^0 - \mu\tilde{\psi}^0_{H_1}\tilde{\psi}^0_{H_2} - M\tilde{W}^+\tilde{W}^- + \mu\tilde{\psi}^-_{H_1}\tilde{\psi}^+_{H_2} \\ &+ m_{ij}\tilde{\ell}^*_i\tilde{\ell}_j + \hat{m}_{ij}\tilde{\nu}_i\tilde{\nu}_j + h.c. \end{aligned} \tag{3.4}$$

develop. These are the most general bilinear soft supersymmetry -breaking terms which may develop.[19] Note that explicit diagonal Higgsino mass terms are *not* soft.

At the weak scale H_1^0 and H_2^0 develop vacuum expectation values $v_1 = \langle H_1^0 \rangle$ and $v_2 = \langle H_2^0 \rangle$. The standard electroweak VEV is replaced by

$$v = (v_1^2 + v_2^2)^{1/2} .$$

The terms $f^i H_1^0 (\ell_L)_i (\ell_R)_i^*$ $(i = e, \mu, \tau)$, $W^0 H_i^0 H_j^0$ and $W^\pm H_i^\mp H_j^0$ of the standard theory (with two Higgses) have their supersymmetric counterparts $f^i H_1^0 (\widetilde{\ell}_L)_i (\widetilde{\ell}_R)_i^*$, $\widetilde{W}^0 \widetilde{H}_i^0 H_j^0$ and $\widetilde{W}^\pm \widetilde{H}_i^\mp H_j^0$ (the general rule is to replace every *pair* of fields by their supersymmetric partners if R parity is conserved as is here assumed). Now when the neutral Higgs develop VEVs we develop more quadratic terms in $\mathcal{L}_{effective}$. The first terms gives mass to the slepton (this would be equal to the lepton mass if it weren't for the additional term in (3.4)). The others yield

$$\begin{aligned} \mathcal{L}_{Mix} &= \frac{i}{2} \left(g' \tilde{B}^0 - g \tilde{W}^0 \right) \left(v_1 \tilde{\psi}_{H_1}^0 - v_2 \tilde{\psi}_{H_2}^0 \right) \\ &- \frac{ig}{\sqrt{2}} [v_1 \tilde{W}^+ \tilde{\psi}_{H_1}^- + v_2 \tilde{W}^- \tilde{\psi}_{H_2}^+] + h.c. \end{aligned} \tag{3.5}$$

It is convenient to define

$$\tan \theta_v \equiv \frac{v_1}{v_2} \tag{3.6}$$

(this is called $\cot \beta$ in Ref. 13).

The final term in (3.5) will induce the charged gaugino and higgsino states of (3.4) to mix. The first term in (3.5) will cause $\widetilde{B}$ and $\widetilde{W}$ to mix with both $\widetilde{H}_1^0$ and $\widetilde{H}_2^0$. Since $\widetilde{H}_1^0$ and $\widetilde{H}_2^0$ are mixed by (3.5) (by virtue of possessing opposite hypercharges) all four neutral states mix. We refer to the two charged mass eigenstates as "charginos" and represent them by $\widetilde{\chi}_i^+$. The four neutral mass eigenstates are "neutralinos" represented by $\widetilde{\chi}_i^0$. If there are Higgs singlets then the number of neutralinos will be increased accordingly. It is also possible to assign a VEV to the sneutrinos: $\langle \widetilde{\nu}_i \rangle \neq 0$. This interesting possibility leads to further mixing due to terms in $\mathcal{L}$ such as $\widetilde{H}^0 \widetilde{\nu} \nu$ and have been examined elsewhere.[20] Such theories explicitly violate R-parity, lepton and family number and are therefore of a somewhat more radical nature than will be considered here.

Since higgsinos and gauginos couple quite differently, the rate of the interaction $e^+e^- \to \tau^+\nu^-$ and similar processes will depend upon the details of the mass spectrum of the physical eigenstates. These effects may be more profound away from the Z^0 resonance since there will be contributions from charginos and neutralinos in the t-channel.[21]

3.3 Mass Eigenstates

The Lagrangian may be concisely written in terms of chargino and neutralino mass matrices. The unitary matrices which diagonalize the Lagrangian are called N (4×4) for the neutralino sector and U and V (2×2) for the charginos. The details are given in Appendix J. These matrices will perforce appear in the Feynman rules as derived in appendix K. The most involved combinations will occur in the non-Abelian vertices which contain more than one gaugino or higgsino. Following Haber and Kane[12] we represent the chargino combination by matrices O' and neutralinos by O''. Since the parameters in the gaugino mass matrices may vary over a wide range of parameters one might be wary of finding that one or more mass eigenvalues have become negative. This does indeed appear to occur but this does *not* imply the existence of a tachyon. It asserts an inappropriate set of basis states were selected for parameters in this regime. These may be redefined or, equivalently, an altered diagonalization procedure used (which accomplishes the same end). Indeed in the simple 2×2 chargino system the latter approach has been adopted leading to slight modifications in the Feynman rules as this threshold is broached. It is also possible, and often desirable, to retain the negative states for computational purposes.

The chargino masses are given by the explicit expressions

$$
\begin{aligned}
m_{\tilde{\chi}_1^+} &= \tfrac{1}{2}\left\{\mathrm{M}_+ + \mathrm{M}_-\right\} \\
m_{\tilde{\chi}_2^+} &= \tfrac{1}{2}\left\{\mathrm{M}_+ - \mathrm{M}_-\right\}\,\mathrm{sign}[D] \\
\mathrm{M}_\pm &= \sqrt{(M+\mu)^2 + 2m_W^2(1 \mp \sin 2\theta_v)^2} \\
D &= M\mu - m_W^2 \sin 2\theta_v
\end{aligned}
\tag{3.7}
$$

where M, μ, θ_v are the breaking parameters described in (3.4) and (3.6).

The neutralino masses are given by Ref. 12 as the eigenvalues of the matrix

$$
Y = \begin{pmatrix}
M' & 0 & -m_Z \sin\theta_v \sin\theta_w & m_Z \cos\theta_v \sin\theta_w \\
0 & M & m_Z \sin\theta_v \cos\theta_w & -m_Z \cos\theta_v \cos\theta_w \\
-m_Z \sin\theta_v \sin\theta_w & m_Z \sin\theta_v \cos\theta_w & 0 & -\mu \\
m_Z \cos\theta_v \sin\theta_w & -m_Z \cos\theta_v \cos\theta_w & -\mu & 0
\end{pmatrix}.
\tag{3.8}
$$

The general solutions are the roots of a quartic equation. This can be solved exactly but proves to be unilluminating.[22] Furthermore, if a Higgs singlet is added Y becomes a 5×5 matrix and the corresponding quintic root equation will have no closed form solution in general.[13] Note that in specific cases, such as those discussed in appendix J, simplified forms are frequently possible.

Mass matrices for the sleptons may be similarly constructed. There are on-diagonal contributions from the lepton mass and the explicit breaking terms. The off-diagonal entries derive solely from the soft terms. Since lepton masses are expected to be much smaller than the supersymmetry-breaking terms they may effectively be ignored except in special circumstances (such as lifting a degeneracy or preventing the matrix from becoming singular). Since all of the entries are then "arbitrary", it is sufficient to merely consider the physical eigenstates and characterize them by given mixing angles and masses without reference to the unmixed precursor states.

3.4 MATRIX ELEMENTS

The diagrams contributing to $Z \to \mu\tau$ are found in Fig. 1. Details may be found in Appendix N using the Feynman rules derived in the previous appendix. The explicit expression for the leading order terms in the matrix element, $\mathcal{M}^{\omega}$, is given by

$$\mathcal{M}^\omega = \mathcal{M}_L \xi^\omega{}_L + \mathcal{M}_R \xi^\omega{}_R \tag{3.9}$$

$$\xi^\omega{}_{L,R} = \bar{u}^\mu(p_1{}')\gamma_\mp \gamma^\omega \gamma_\pm v^\tau(p_2{}') \qquad\qquad \gamma_\pm = \tfrac{1}{2}(1 \pm \gamma_5) \tag{3.9a}$$

$$\begin{aligned}
\mathcal{M}_L = \frac{\mathcal{K}_\nu}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{2} \Bigg\{ & |V_{i1}|^2 \left[\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \Delta G(T_i^+, 0)\right] \\
& + 2\sum_{j=1}^{2} V_{i1}^* V_{j1} \Big[\sqrt{S_{ij}^+}\, O_{ij}^{R\,\prime}\, \Delta\check{I}_{[1]}(T_i^+, S_{ij}^+, R_i^+) \\
& \qquad - R_i^+ O_{ij}^{L\,\prime}\, \Delta\check{I}_{[\widetilde{Z}^2 - Z^2]}(T_i^+, S_{ij}^+, R_i^+) \\
& \qquad + O_{ij}^{L\,\prime}\, \Delta\check{G}(T_i^+, S_{ij}^+, R_i^+) \Big] \Bigg\} \\
- \frac{\mathcal{K}_L}{2\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Bigg\{ & |\tan\theta_w N_{i1} + N_{i2}|^2 \cos 2\theta_w \Delta\tilde{G}(T_L{}^0{}_i, R_i^0) \\
& - 2\sum_{j=1}^{4} (\tan\theta_w N_{i1}^* + N_{i2}^*)(\tan\theta_w N_{j1} + N_{j2}) \\
& \quad \Big[\sqrt{S_{ij}^0}\, O^{R\,\prime\prime}{}_{ij}\, \Delta\check{I}_{[1]}(T_L{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad - R_i^0 O^{L\,\prime\prime}{}_{ij}\, \Delta\check{I}_{[\widetilde{Z}^2 - Z^2]}(T_L{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad + O^{L\,\prime\prime}{}_{ij}\, \Delta\check{G}(T_L{}^0{}_i, S_{ij}^0, R_i^0) \Big] \Bigg\}
\end{aligned} \tag{3.9b}$$

$$\begin{aligned}
\mathcal{M}_R = \frac{4\mathcal{K}_R}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Bigg\{ & |\tan\theta_w N_{i1}^*|^2 \sin^2\theta_w \Delta\tilde{G}(T_R{}^0{}_i, R_i^0) \\
+ & \sum_{j=1}^{4} (\tan\theta_w N_{i1})(\tan\theta_w N_{j1}^*) \\
& \quad \Big[\sqrt{S_{ij}^0}\, O^{L\,\prime\prime}{}_{ij}\, \Delta\check{I}_{[1]}(T_R{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad - R_i^0 O^{R\,\prime\prime}{}_{ij}\, \Delta\check{I}_{[\widetilde{Z}^2 - Z^2]}(T_R{}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad + O^{R\,\prime\prime}{}_{ij}\, \Delta\check{G}(T_R{}^0{}_i, S_{ij}^0, R_i^0) \Big] \Bigg\}
\end{aligned} \tag{3.9c}$$

$$\mathcal{K}_\nu = \frac{i}{16\pi^2} e^3 \sin\theta_\nu \cos\theta_\nu$$

$$\mathcal{K}_L = \frac{i}{16\pi^2} e^3 \sin\theta_L \cos\theta_L \qquad (3.9d)$$

$$\mathcal{K}_R = \frac{i}{16\pi^2} e^3 \sin\theta_R \cos\theta_R$$

R, S, and T are ratios of masses:

$$T_i^+ = \frac{m_{\tilde{\nu}_i}{}^2}{m_{\tilde{\chi}_i^+}{}^2} \qquad T_{L,R}{}^0{}_i = \frac{m_{\tilde{\ell}_{L,R_i}}{}^2}{m_{\tilde{\chi}_i^0}{}^2}$$

$$S_{ij}^+ = \frac{m_{\tilde{\chi}_j^+}{}^2}{m_{\tilde{\chi}_i^+}{}^2} \qquad S_{ij}^0 = \frac{m_{\tilde{\chi}_j^0}{}^2}{m_{\tilde{\chi}_i^0}{}^2} \qquad (3.10)$$

$$R_i^+ = \frac{q^2}{4m_{\tilde{\chi}_i^+}{}^2} \qquad R_i^0 = \frac{q^2}{4m_{\tilde{\chi}_i^0}{}^2}$$

where $m_{\tilde{\ell}_{L_1}}$ and $m_{\tilde{\ell}_{R_1}}$ are the masses of the charged slepton of the first generation mixture (see (3.1)) and $m_{\tilde{\nu}_1}$ is the mass of the first sneutrino ($\tilde{\nu}_1 = \tilde{\nu}_\mu \cos\theta_\nu + \tilde{\nu}_\tau \sin\theta_\nu$). The neutralino masses are $m_{\tilde{\chi}_i^0}$ $(i = 1,2,3,4)$ and chargino masses are $m_{\tilde{\chi}_i^+}$ $(i = 1,2)$. The energy of the Z^0 is $q^2 = m_z^2$ since we are working on-shell. The integral functions $(\Delta\check{I}_{[\mathrm{N}]}, \Delta G, \Delta\check{G}, \Delta\tilde{G})$ are defined in Appendix N and analyzed, analytically and numerically, in the subsequent appendix. The matrix element $\mathcal{M}^w$ cannot be examined in all of its complexity (there are, after all, 120 terms in Eq. (3.9)) but there are a number of special cases which prove instructive. These principally involve taking the breaking parameters to extreme values (zero or infinity) and a number of examples have been examined. We find, in accordance with the decoupling theorem,[23] that as the physical mass of a matrix becomes large its contributions to the cross-section become progressively less important, finally vanishing. The matrix element remains finite for all values of the parameters involved.*

3.5 Production Threshhold

If any of the supersymmetric particles is sufficiently light ($m < \frac{1}{2} m_Z$) then real production occurs. If such states are unstable (generally only the least massive supersymmetric state will be stable, and only if R is conserved) then final state processes with four or more particle will dominate. If the lightest supersymmetric particle is a neutral then all such decays are characterized by "missing mass" in the final state. Such signals are notoriously amenable to alternate interpretations yet remain the hallmark of experimental supersymmetric searches. Below threshold there are still contributions to closed loop processes, such as the one under consideration, however the matrix elements become complex.

Let us assume that a nearly massless sneutrino were the lightest supersymmetric particle and, furthermore, was stable. The reaction $e^+e^- \to Z^0 \to \overline{\tilde{\nu}}\tilde{\nu}$ would appear as $e^+e^- \to$ nothing. The real particle production analogues of the processes in Fig. 3.1 would be

$$
\begin{array}{l}
e^+e^- \to Z^0 \to \tilde{\chi}^- \tilde{\chi}^+ \\
\qquad \tilde{\chi}^+ \to \tilde{\nu}\tau^+ \to \tilde{\nu}\,\overline{\nu}_\mu\,\nu_\tau\,\mu^+ \\
\qquad \tilde{\chi}^- \to \overline{\tilde{\nu}}\,\mu_-
\end{array}
$$

and

$$
\begin{array}{l}
e^+e^- \to Z^0 \to \overline{\tilde{\ell}}^+ \tilde{\ell}^- \\
\qquad \tilde{\ell}^- \to \overline{\tilde{\nu}}\tau^- \to \overline{\tilde{\nu}}\,\nu_\mu\,\overline{\nu}_\tau\,\mu^- \\
\qquad \overline{\tilde{\ell}}^+ \to \tilde{\nu}\,\mu^+
\end{array}
$$

Since $\nu, \overline{\nu}, \tilde{\nu}$ and $\overline{\tilde{\nu}}$ escape the detector their reactions appear as $e^+e^- \to \mu^\pm\tau^\mp +$

---

* In the infrared limit if we take the superparticle mass to be small we must be careful to reinsert lepton mass terms which have been ignored. This is demonstrated for the off-shell $\mu e \gamma$ vertex in the appendices associated with Chapter 4.

missing energy. This six-particle final state would be difficult to distinguish from far more quotidian occurrences such as $e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\overline{\nu}_\mu\nu_\tau\overline{\nu}_\tau\nu_\mu$.

3.6 SMALL MASS SPLITTINGS

The largest cross-sections occur when the mass-splitting between $\tilde{\ell}_{L,R_1}(\tilde{\nu}_1)$ and $\tilde{\ell}_{L,R_2}(\tilde{\nu}_2)$ is large. In many models we may restrict ourselves to the case where the relative mass-splitting is small, *i.e.*

$$\delta_{\tilde{\ell}_L} \ll 1, \qquad \delta_{\tilde{\ell}_R} \ll 1, \qquad \delta_{\tilde{\nu}} \ll 1$$

where

$$\delta_{\tilde{\ell}_{L,R}} \equiv \frac{m_{\tilde{\ell}_{L,R_1}}{}^2 - m_{\tilde{\ell}_{L,R_2}}{}^2}{m_{\tilde{\ell}_{L,R_1}}{}^2} \qquad\qquad \delta_{\tilde{\nu}} \equiv \frac{m_{\tilde{\nu}_1}{}^2 - m_{\tilde{\nu}_2}{}^2}{m_{\tilde{\nu}_1}{}^2} \tag{3.11}$$

These models[24] assume (quite reasonably) that supersymmetry breaks spontaneously in a flavour-independent manner. If the breaking scale in not too large then small mass splitting follows naturally. This has not been assumed in this paper. In such scenarios the matrix elements are generally *at least* an order of magnitude smaller. In this limit we can write

$$\Delta F(T_i^+, S_{ij}^+, R_i^+) = \delta_{\tilde{\nu}} F'(T_i^+, S_{ij}^+, R_i^+) \tag{3.12}$$

$$\Delta F(T_{L,R}{}^0{}_i, S_{ij}^0, R_i^0) = \delta_{\tilde{\ell}_{L,R}} F'(T_{L,R}{}^0{}_i, S_{ij}^0, R_i^0) \tag{3.13}$$

$$\Delta F \equiv F(\tilde{\ell}_1) - F(\tilde{\ell}_2) \doteq \delta_{\tilde{\ell}} F'(\tilde{\ell}_1) \tag{3.14}$$

$$F'(T,S,R) \equiv T\frac{\partial F(T,S,R)}{\partial T} = \frac{\partial F(T,S,R)}{\partial \ln T} \tag{3.15}$$

In this case we may replace each ΔI and ΔG term in (3.9) by $\delta_{\tilde{\ell},\tilde{\nu}} I'$ and $\delta_{\tilde{\ell},\tilde{\nu}} G'$. (See Appendix N for these functional forms).

3.7 Cross-section and Special Cases

Given the matrix element, $\mathcal{M}^{\omega}$, in (3.9) we may derive

$$\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \frac{\left[|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2\right]}{8\pi\alpha(\cot^2 2\theta_w + \tan^2\theta_w)} \tag{3.16}$$

Note that if any of the supersymmetric particles has a mass of less than $\frac{1}{2}M_Z$ that real production can occur and that $\mathcal{M}_L$ and $\mathcal{M}_R$ are complex. It is the modulus which is important in (3.16).

In order to get a feeling for the range of $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ we examine (3.9) in two extreme gaugino limits. The first is the "supersymmetry" limit in which we eliminate the gaugino supersymmetry-breaking terms by letting $M', M, \mu \to 0$ and let $v_1 = v_2$. Thus the only supersymmetry-breaking terms are the explicit slepton mass terms. Then (3.9) reduces to

$$\begin{aligned}
\mathcal{M}^{\omega}{}_{SUSY} = \frac{\mathcal{K}_\nu \xi^{\omega}{}_L}{\sin^2\theta_w \sin 2\theta_w} \Big\{ & \Delta G(T_1^+, R_1^+) + \cos 2\theta_w \Delta G(T_1^+, 0) \\
& - \cos 2\theta_w \Delta \bar{I}_{[1]}(T_1^+, R_1^+) \\
& + 2\cos^2\theta_w \left[R_1^+ \Delta \bar{I}_{[\tilde{Z}^2 - Z^2]}(T_1^+, R_1^+) - \Delta\bar{G}(T_1^+, R_1^+) \right] \Big\} \\
- 2\mathcal{K}_L \xi^{\omega}{}_L \cot 2\theta_w & \left\{ \Delta\tilde{G}({T_L}^0{}_1, R_1^0) + \cot^2 2\theta_w \Delta\tilde{G}({T_L}^0{}_2, R_2^0) \right\} \\
+ 2\mathcal{K}_R \xi^{\omega}{}_R \tan\theta_w & \left\{ \Delta\tilde{G}({T_R}^0{}_1, R_1^0) + \tan^2\theta_w \Delta\tilde{G}({T_R}^0{}_2, R_2^0) \right\}
\end{aligned}$$

with

$$\Delta\bar{I}_{[\mathrm{N(Z,\tilde{Z})}]}(T,R) = \Delta\check{I}_{[\mathrm{N(Z,\tilde{Z})}]}(T, S\equiv 1, R) \qquad \bar{G}(T,R) = \Delta\check{G}(T, S\equiv 1, R) \tag{3.17}$$

where the physical masses are now

$$m_{\tilde{\chi}_i^+} = (M_W, M_W) \qquad m_{\tilde{\chi}_i^0} = (0, M_Z, M_Z, 0) \tag{3.18}$$

and the relevant mass ratios are

$$\begin{aligned} T_1^+ &= \frac{m_{\tilde{\nu}_1}{}^2}{M_W{}^2} & T_{L,R}{}^0{}_1 &= \infty & T_{L,R}{}^0{}_2 &= \frac{m_{\tilde{\ell}_{L,R_2}}{}^2}{M_Z{}^2} \\ R_1^+ &= \frac{1}{4\cos^2\theta_w} & R_1^0 &= \infty & R_2^0 &= 1/4 \end{aligned} \tag{3.19}$$

$\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ is plotted in this limit in Fig. 2. Since experiments rule out charged supersymmetry partners of mass less than about 24 GeV,[25] and in some instances much stronger limits have been placed. ASP[26] has placed limits on the selectron mass and the wino mass of approximately 60 GeV When combined with the results of other groups they find that $m_{\tilde{e}} \gtrsim 84$ GeV. While such strong limits do not, as yet, exist for $m_{\tilde{\mu}}$ or $m_{\tilde{\tau}}$, and hence for $m_{\tilde{\ell}_{1,2}}$, it would be somewhat surprising to find $m_{\tilde{\mu}} \ll m_{\tilde{e}}$. We see that for light slepton masses with large mass-splitting that $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ could be as large as 3×10^{-6}. The significance of this will be discussed shortly. In order to achieve such a value parameters conducive to a large cross-section have been selected. We have allowed the sleptons to mix maximally so that $\theta_\nu = \theta_L = \theta_R = \frac{\pi}{4}$. We have further assumed maximal mass-splitting between the two slepton sectors, *i.e.* $m_{\tilde{\nu}_1}$,$m_{\tilde{\ell}_{L_1}}$ and $m_{\tilde{\ell}_{R_1}}$ are relatively light but $m_{\tilde{\nu}_2}$, $m_{\tilde{\ell}_{L_2}}$ and $m_{\tilde{\ell}_{R_2}}$ are large and decouple.

Another extreme limit is the "unmixed" limit in which the Higgsino and Gaugino sectors have been disentangled from one another. There are several ways of achieving this. Here we consider the limiting case $M \to \infty, M' \to 0$ and $\mu \to 0$ (θ_v arbitrary). The physical mass states become

$$M_{\tilde{\chi}_i^+} = (M \to \infty, 0) \qquad\qquad M_{\tilde{\chi}_i^0} = (M \to \infty, 0, 0, 0). \tag{3.20}$$

Note that $\tilde{\chi}_1^0 = \tilde{W}_3^0$, $\tilde{\chi}_2^0 = \tilde{B}^0$ and thus are purely gaugino whereas $\tilde{\chi}_{3,4}^0$ are purely Higgsino. Thus $T^0{}_{L,R_2} \equiv \frac{m^2_{\tilde{\ell}_{L,R_1}}}{M_{\tilde{\chi}_2^0}{}^2} \to \infty$ and $R_2^0 \equiv \frac{q^2}{4M_{\tilde{\chi}_2^0}{}^2} \to \infty$. The matrix element is given by

$$\mathcal{M}^{\omega}{}_{Unmixed} = \frac{2\mathcal{K}_R \xi^{\omega}{}_R}{\cos^2\theta_w} \tan\theta_w \Delta\tilde{G}(T_R{}^0{}_2, R_2^0) - \frac{\mathcal{K}_L \xi^{\omega}{}_L}{2\cos^2\theta_w} \cot 2\theta_w \Delta\tilde{G}(T_L{}^0{}_2, R_2^0) \tag{3.21}$$

Note that this result is independent of θ_v. The corresponding cross-section is plotted in Fig. 3. Again the sleptons are assumed to have maximal mixing and mass-splitting. Note, however, that in this limiting case we have a massless chargino (a charged Higgsino). Since experimentally the mass limit for new charged particles is ~ 24 GeV, we conclude that this case is merely illustrative.

3.8 Experimental Considerations

The question arises as to whether such a process would be observable at SLC or LEP. These machines will operate as Z-factories and so are ideally suited for such a search. We see from Fig. 2 that, at best, the cross-section for $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\mu^-$ will be $\sim 3 \times 10^{-6}$ that of $e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-$. Thus we expect

$$BR(Z^0 \rightarrow \tau^+\mu^-) \lesssim \mathcal{O}(10^{-7}) \tag{3.22}$$

where we have used the relevant standard branching ratios[27]

$$BR(Z^0 \rightarrow \bar{\tau}\tau) \simeq BR(Z^0 \rightarrow \bar{\mu}\mu) \simeq BR(Z^0 \rightarrow \bar{e}e) \simeq 3\% \tag{3.23}$$

We also note that

$$BR(\tau \rightarrow \mu\bar{\nu}\nu) \simeq BR(\tau \rightarrow e\bar{\nu}\nu) \simeq 17.5\% \tag{3.24}$$

The experimental signature for such a decay would be μ and τ back-to-back with the μ having $E = \frac{M_Z}{2}$ and no missing energy. The principal background will be from $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ followed by $\tau^\pm \rightarrow \mu^\pm\bar{\nu}\nu$, with the μ having nearly all of the τ momentum. The τ will travel approximately 2.5 mm before decaying in this case and, since the τ and μ are nearly collinear, the kink in the track will be unobservable. Clearly the number of μ, $N(\Delta\varepsilon)$, produced in the energy range from $\frac{M_Z}{2} - \Delta\varepsilon$ to $\frac{M_Z}{2}$ is of paramount importance. We find that (see Appendix N)

$$N(\Delta\varepsilon) = \frac{3-\alpha}{9+\alpha} \cdot 24 \frac{\Delta\varepsilon^2}{M_Z^2} \tag{3.25}$$

where α measures the degree of polarization ($\alpha = 0$ for the unpolarized case; $\alpha = 1$ for complete polarization). At SLAC we expect $\alpha \approx 1$ and thus $N_{pol}(\Delta\varepsilon) \doteq \frac{24}{5}\frac{\Delta\varepsilon^2}{M_Z^2}$. From this and (19b) we see that we need $\Delta\varepsilon \lesssim 0.1\%(\frac{M_Z}{2})$ to achieve a background of 10^{-7} or less which would compare with (3.22). Thus for muons with $E \approx \frac{M_Z}{2}$ we need $\frac{\Delta p}{p} \approx 0.1\%$.

At the SLC the MARK II detector will have an energy resolution for e's and μ's of ~0.3%/GeV without a vertex detector and, perhaps, as low as 0.1%/GeV with the planned vertex detector.[28] For $\frac{M_Z}{2} \simeq 50$ GeV this means that $\frac{\Delta p}{p}$ ~5% and from (3.25)the background will swamp the signal by at least a factor of 200. It appears unlikely that detector momentum resolutions will be improved by the required two orders of magnitude in the near future (there remains the formidable problem of increased multiple scatterings as detector mass is added). Thus the process $e^+e^- \to Z^0 \to \tau^+\mu^-$ will not be experimentally observable at the emerging generation of machines. Furthermore, because of (3.22), when SLC achieves its eventual target luminosity of 10^6 Z^0/year, the $Z^0 \to \tau^\pm\mu^\mp$ production rate will be at best ~0.2 event/year.

Should the mixing be primarily between $\tilde{e}$ and $\tilde{\mu}$ there will be an important additional restriction to consider. Under the same conditions described above (maximal mixing; large mass-splitting; Z^0 on shell) the production rate would appear to be the same. It is found, however, that the experimental limits on the process $\mu \to e\gamma$ place severe constraints on $\tilde{e} - \tilde{\mu}$ mixing in the neutralino sector. If we assume maximal mixing and large mass splitting then (chapter 4) we find that the *lightest* slepton has $m \gtrsim 1$ TeV and so decouples. Thus the charginos provide the sole contribution to $Z \to \mu^+ e^-$ (other than in exceptional circumstances). The matrix element is then proportional to the sneutrino mass splitting. Assuming that this is also large we find, for cases of interest, that the chargino sector contribution is generally much larger than that of the neutralino sector *even before the above constraint has been imposed.* Thus the removal of the neutralino contribution, while considerably simplifying calculations, would not greatly affect the final cross-section.

For the situation illustrated in Fig. 3(b), when applied to $Z \to \mu^+ e^-$, we find that imposing the above constraints on $m_{\tilde{\ell}}$ affects the results by, at most, 15%. Furthermore, in the region of interest, this change results in an *increase* in the cross-section (because the real parts of the neutralino and chargino contributions enter with opposite signs). Note that the preceding argument (except the quoted percentage) also follows in the cases of small mass-splitting and non-maximal mixing.

When considering the process $Z^0 \to \tau^+ \mu^-$current limits from $\tau \to \mu\gamma$[27] do not impose serious constraints on the neutralino sector. In the case of large mass-splitting and maximal mixing we find only that $m_{\tilde{\ell}} \gtrsim 13$ GeV. In particular substantial effects persist in this case (arising from the neutralino sector) even when $m_{\tilde{\nu}_1} = m_{\tilde{\nu}_2}$.

We can do somewhat better on background if we assume that the principal slepton mixing occurs in the $\tilde{e} - \tilde{\mu}$ sector. The principal background will still be due to $Z^0 \to \tau^+\tau^-$with $\tau^\pm \to \mu^\pm \bar{\nu}\nu$ and $\tau^\pm \to e^\pm \bar{\nu}\nu$. The decay $Z^0 \to \bar{\mu}\mu$ will not be a problem since, at these energies, muons will not decay in the detector. Now misidentification of a $\tau^+\tau^-$ pair as a back-to-back $\mu^\pm e^\mp$ requires that both the μ and e emerge with energies near $\frac{M_Z}{2}$. Assuming complete polarization we find that $N(\Delta p)$, the number of back-to-back $e - \mu$ pairs emerging with $E = \frac{M_Z}{2}$, to within the experimental momentum resolution Δp, is given by $N(\Delta p) \doteq 1.4 \left(\frac{\Delta p}{p}\right)^4$. Thus with $\frac{\Delta p}{p} \sim 5\%$ we find $N(\Delta p) \approx 9 \times 10^{-6}$, which corresponds to roughly 0.02 misidentified events per year, assuming a sample of

10^6 Z^0. It would appear that, under the most opportune of conditions, the signal might stand well above the background. It must be cautioned, however, that such propitious circumstances are singularly unlikely. The actual $e^+e^- \to \mu^\pm e^\mp$ production rate is almost assuredly much less than the near-maximal value which we have been considering and this idealized background calculation has omitted a number of important experimental effects. This result is, none the less, encouraging particularly in light of the observation that statistics and detector sensitivities will only improve with time.

3.9 CONCLUSION

In conclusion, even in the most favourable scenarios, it appears unlikely that $e^+e^- \to \tau^+\mu^-$ will be observed at SLC within the first few years of operation if the sole contribution is from slepton family mixing. The production rate is simply insufficient and the background rate overly severe. The decay $e^+e^- \to \mu^+e^-$ is equally rare but the background problems may prove more tractable. The full parameter space of the gaugino-higgsino sector has yet to be explored. Since the cross-sections predicted in the limiting cases presented come within an order of magnitude of being experimentally interesting at the SLC, realistic symmetry-breaking parameters might exist which would increase the cross-section to observable levels. In any instance the projected luminosity available at LEP might render such searches feasible.

Table 3.1 Fields in This Chapter

Superfield	Ordinary Matter	Superpartners	Weak Isospin	y
		Vector (Gauge) Multiplets		
$\hat{\mathbf{V}}$	$W^{\pm,0}$	$\tilde{W}^{\pm,0}$	Triplet	0
$\hat{\mathbf{V}}'$	B^0	$\tilde{B}^0$	Singlet	0
		Scalar Multiplets		
$\hat{\mathbf{L}}_i$	ν_i, ${\ell_{L_i}}^-$	$\tilde{\nu}_i$, $\tilde{\ell}_{L_i}$	Doublet	-1
$\hat{\mathbf{R}}_i$	$(\ell_i^+)_L = (\ell_{R_i}^-)^*$	$(\tilde{\ell}_{R_i}^-)^*$	Singlet	2
$\hat{\mathbf{H}}_1$	${H_1}^0$, ${H_1}^-$	$\tilde{\psi}_{H_1}^0$, $\tilde{\psi}_{H_1}^-$	Doublet	-1
$\hat{\mathbf{H}}_2$	${H_2}^+$, ${H_2}^0$	$\tilde{\psi}_{H_2}^+$, $\tilde{\psi}_{H_2}^0$	Doublet	1

FIGURE CAPTIONS

1. Diagrams contributing to $Z^0 \to \tau^+\mu^-$.

   a) Chargino Diagrams.

   b) Neutralino Diagrams.

2. Ratio of cross-sections, $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$, in "supersymmetry " limit.
   $M = M' = \mu = 0.$ $\theta_v = \pi/4.$

   (a) Varying all slepton masses equally. [Ignoring $m_{\tilde{\ell}_2}$ and $m_{\tilde{\nu}_2}$].

   (b) Varying $m_{\tilde{\ell}_{L_1}}$ while holding all other mass parameters equal to the indicated values.

3. Ratio of cross-sections, $\frac{\sigma_{e^+e^-\to\tau^+\mu^-}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$, in the "unmixed" limit.
   $M = M' = \mu = 0.$ $\theta_v = \pi/4.$
   Vary $m_{\tilde{\ell}_{L_1}} = m_{\tilde{\ell}_{R_1}}$ assuming $m_{\tilde{\ell}_{L_2}} = m_{\tilde{\ell}_{R_2}} = \infty.$

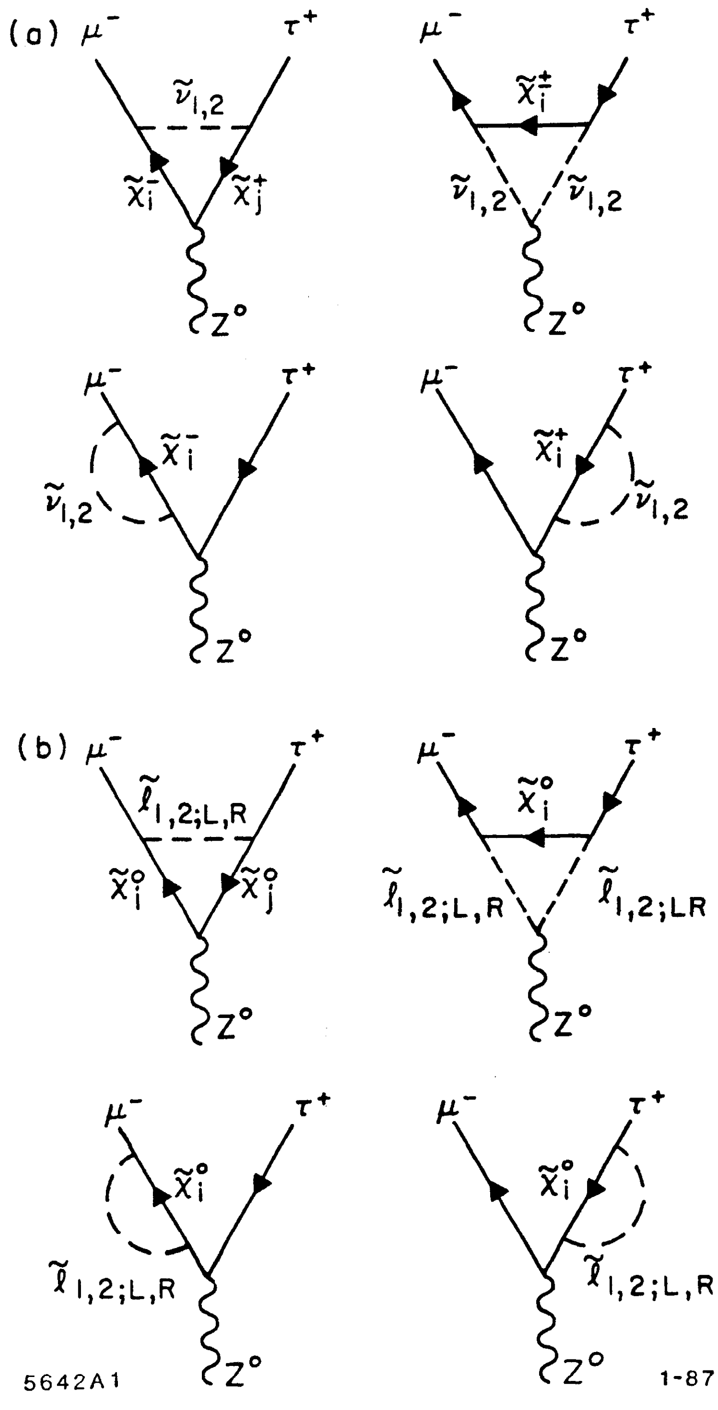
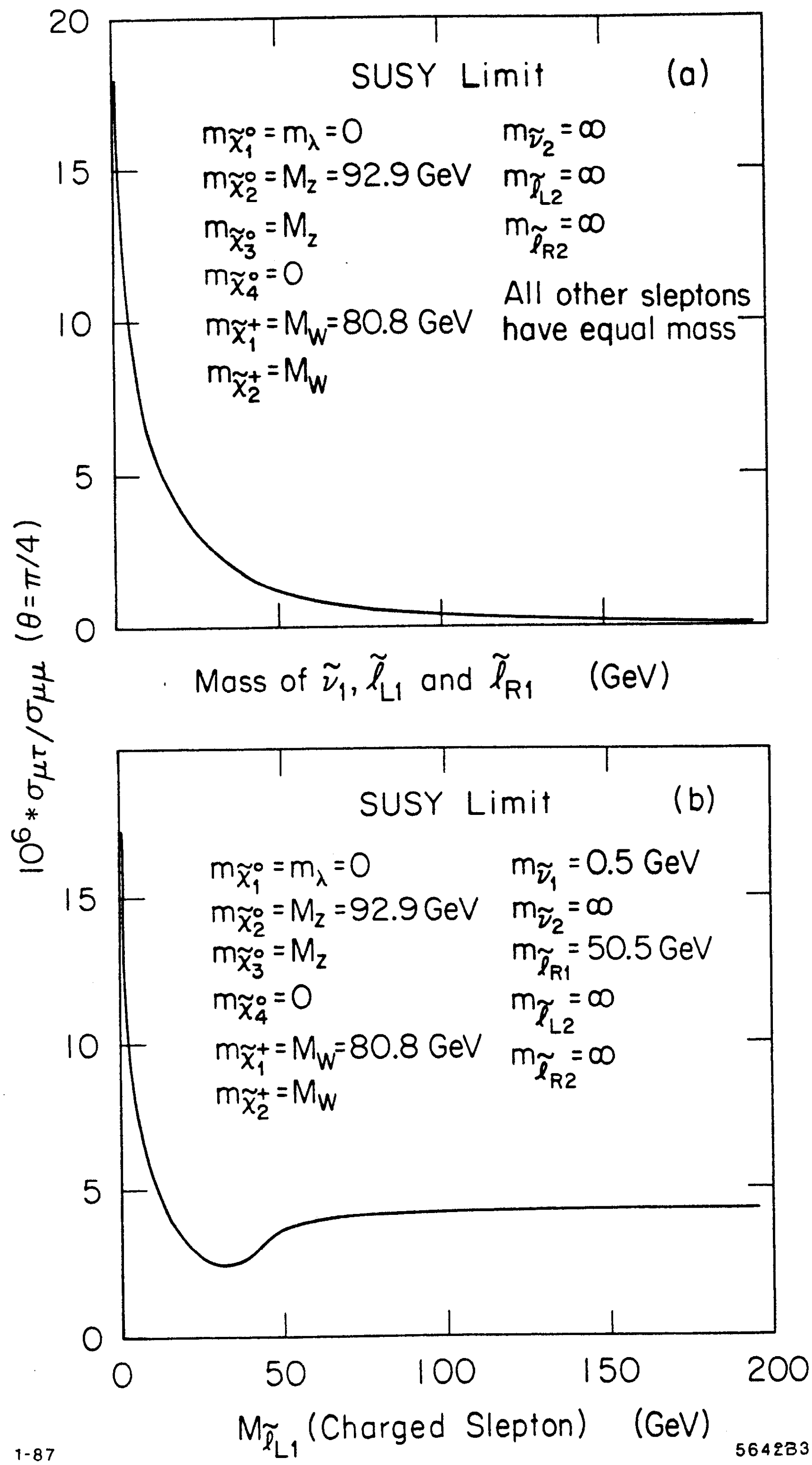
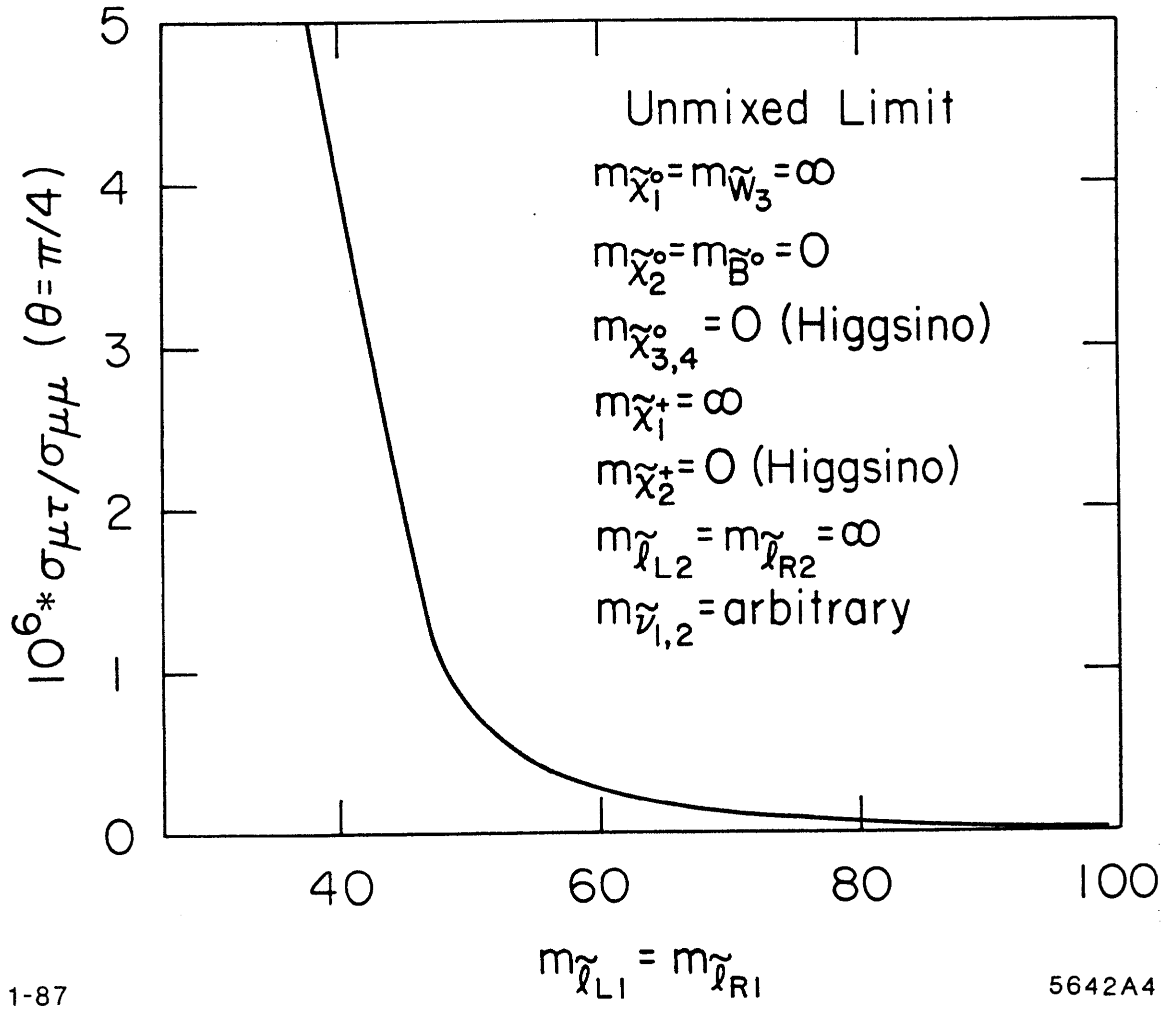


Fig. 3.1



SUSY Limit (a)
$m_{\tilde{\chi}_1^0} = m_\lambda = 0$
$m_{\tilde{\chi}_2^0} = M_Z = 92.9$ GeV
$m_{\tilde{\chi}_3^0} = M_Z$
$m_{\tilde{\chi}_4^0} = 0$
$m_{\tilde{\chi}_1^+} = M_W = 80.8$ GeV
$m_{\tilde{\chi}_2^+} = M_W$
$m_{\tilde{\nu}_2} = \infty$
$m_{\tilde{\ell}_{L2}} = \infty$
$m_{\tilde{\ell}_{R2}} = \infty$
All other sleptons have equal mass
Mass of $\tilde{\nu}_1$, $\tilde{\ell}_{L1}$ and $\tilde{\ell}_{R1}$ (GeV)
$10^6 * \sigma_{\mu\tau}/\sigma_{\mu\mu}$ $(\theta = \pi/4)$
SUSY Limit (b)
$m_{\tilde{\chi}_1^0} = m_\lambda = 0$
$m_{\tilde{\chi}_2^0} = M_Z = 92.9$ GeV
$m_{\tilde{\chi}_3^0} = M_Z$
$m_{\tilde{\chi}_4^0} = 0$
$m_{\tilde{\chi}_1^+} = M_W = 80.8$ GeV
$m_{\tilde{\chi}_2^+} = M_W$
$m_{\tilde{\nu}_1} = 0.5$ GeV
$m_{\tilde{\nu}_2} = \infty$
$m_{\tilde{\ell}_{R1}} = 50.5$ GeV
$m_{\tilde{\ell}_{L2}} = \infty$
$m_{\tilde{\ell}_{R2}} = \infty$
$M_{\tilde{\ell}_{L1}}$ (Charged Slepton) (GeV)
1-87
5642B3

Fig. 3.2



Unmixed Limit
$m_{\tilde{\chi}_1^0} = m_{\tilde{W}_3} = \infty$
$m_{\tilde{\chi}_2^0} = m_{\tilde{B}^0} = 0$
$m_{\tilde{\chi}_{3,4}^0} = 0$ (Higgsino)
$m_{\tilde{\chi}_1^+} = \infty$
$m_{\tilde{\chi}_2^+} = 0$ (Higgsino)
$m_{\tilde{\ell}_{L2}} = m_{\tilde{\ell}_{R2}} = \infty$
$m_{\tilde{\nu}_{1,2}}$ = arbitrary
$10^6 * \sigma_{\mu\tau}/\sigma_{\mu\mu}$ $(\theta = \pi/4)$
0
1
2
3
4
5
40
60
80
100
$m_{\tilde{\ell}_{L1}} = m_{\tilde{\ell}_{R1}}$
1-87
5642A4

Fig. 3.3

4. $\mu \rightarrow e\gamma$

4.1 Introduction

This is the classic lepton family number violating process. Of such processes it is the best-studied, both theoretically and experimentally, and that which has provided the keenest limits on model-building. In terms of the present model such a decay would occur due to mixing in the selectronic and smuonic sectors. We could motivate the form of the matrix elements by first considering the related decay $\tau \rightarrow \mu\gamma$ in which the mixing is principally between $\tilde{\mu}$ and $\tilde{\tau}$ (and the corresponding scalar neutrinos). We would then utilize the wisdom gleaned in the previous chapter by developing crossing relations to go from $Z^0 \rightarrow \tau^+\mu^-$ to $\tau \rightarrow \mu\gamma$ directly. This would give us the leading order "vectorial" (γ^μ) interaction term which can be shown to vanish due to the reasons discussed in section 4.2.

The lowest-order non-trivial matrix elements will be presented (Appendix O) and discussed (in section 4.3). We will consider off-shell behaviour and observe that the "vectorial" terms may indeed prove important in this case. We will not ignore the masses of the fermions in this case since, unlike the previous chapter, the lepton masses are comparable to the decay energy. Finally, comparison with experiment will be examined in section 4.4 and special cases of interest presented.

4.2 The Form of the Matrix Element

The first observation which we make is that, in the approximation of massless leptons (particularly muons and electrons), $\mu \rightarrow e\gamma$ must be principally mediated by a convective $\sigma^{\mu\nu}$ interaction. The reason is that the photon is massless and therefore transverse. Thus it has spin quantum numbers (on shell)

$$(s, s_3) = (1, \pm 1)$$

but no $s_3 = 0$ longitudinal part. Hence, denoting the 'z-component' of the spin of the (anti)muon by $s_3(\bar{\mu})$, let us consider the related process of $\gamma \rightarrow \bar{\mu}e$ depicted in Fig. 4.1. Since $s_3(\gamma) = s_3(\bar{\mu}) + s_3(e)$ and the leptons emerge back-to-back they must have spins aligned in the same direction and so have opposite chiralities. In the weak basis $W^{\pm,0}$ only couple to left-handed particles (and likewise for their supersymmetric partners) and so only $\widetilde{B}^0$ would contribute in the indicated fashion. This conclusion is true if Fig. 4.1 were rotated to illustrate $\mu \to e\gamma$. Of course, since both $\widetilde{Z}^0$ and $\widetilde{\gamma}$ contain a piece of $\widetilde{B}^0$ they would both contribute to such a process. Higgsinos couple with a strength which is proportional to the mass of the *lepton* and so they also do not contribute. In an arbitrary neutralino basis, of the forms described in the previous chapter, the $\widetilde{B}^0$ portion of each $\tilde{\chi}^0_i$ would contribute.

When electron and muon masses are included then helicity flipping becomes possible along the lepton lines and $\widetilde{W}^{\pm,0}$, $\widetilde{H}^{\pm,0}$ diagrams will contribute (suppressed by m_μ/M_{SUSY}, of course). Both terms[29] of the form $(a + b\gamma_5)\sigma^{\omega\nu}$, which flip helicity, and $(c + d\gamma_5)\gamma^\omega$, which do not, are possible. As the helicity-preserving terms are downs by $m_\ell/M_{\tilde{l},\tilde{W}}$ most papers simply ignore them and assume an interaction of the form $\sigma^{\omega\nu}q_\nu$. Since

$$
\begin{aligned}
\overline{\psi}\,\psi &= \psi^*_R\psi_L + \psi^*_L\psi_R \\
\overline{\psi}\,\gamma^\mu\psi &= \psi^*_R\sigma^\mu\psi_R + \psi^*_L\overline{\sigma}^{\,\mu}\psi_L \\
\overline{\psi}\,\sigma^{\mu\nu}q_\nu\psi &= \psi^*_R\overline{\sigma}^{\,\mu}\sigma\cdot q\psi_L + \psi^*_L\sigma^\mu\overline{\sigma}\cdot q\psi_R
\end{aligned}
\tag{4.1}
$$

we can rotate $\psi^{(i)}_L$ (i is a generation index) and $\psi^{(i)}_R$ independently when there are no mass terms present. Then $\psi^{(\mu)*}_L\sigma^\omega\psi^{(e)}_L$ may always be rotated to $\psi^{(i)*}_L\sigma^\omega\psi^{(i)}_L$. This informs us that the coefficients of the σ^ω terms must always be proportional to some power of the lepton masses.

We will use the somewhat non-standard notation for the pure SQED case:

$$
\begin{aligned}
\mathcal{M}^\mu_{\mu e\gamma} = \mathcal{K}\overline{\psi}^{\,e}(p',s')\Big\{ &\left[F_1(q^2) + F^5_1(q^2)\gamma_5\right]\frac{\not{q}\sigma^{\mu\nu}q_\nu}{m^2_\lambda} \\
&+ \left[F_2(q^2) + F^5_2(q^2)\gamma_5\right]\sigma^{\mu\nu}q_\nu\Big\}\psi^\mu(p,s)
\end{aligned}
\tag{4.2}
$$

where $\mathcal{K}$ is a constant which can be shown to be equal to

$$\mathcal{K} = -\frac{\alpha^{3/2} \sin\theta \cos\theta}{144\sqrt{\pi}} \tag{4.3}$$

and

$$q_\omega = p_\omega - p'_\omega$$

in this instance.

The $\tilde{e}-\tilde{\mu}$ mixing angle is θ (here we are tacitly taking $\theta_L = \theta_R \equiv \theta$). Weinberg and Feinberg[29] refer to these form factors as $f_{E0}(q^2)$, $f_{M0}(q^2)$, $f_{M1}(q^2)$, and $f_{E1}(q^2)$ respectively (up to factors of q^2 etc.). The $\not{q}\sigma^{\omega\nu}q_\nu$ term is actually a γ^ω-type term written in this way to more closely resemble the form of $\sigma^{\omega\nu}q_\nu$. This follows from

$$\not{q}\sigma^{\omega\nu}q_\nu = \left[\not{q}q^\omega - q^2\gamma^\omega\right] . \tag{4.4}$$

We see that the F_1, F_1^5 terms (f_{E0}, f_{M0}) will not contribute for real, free, massless photons since the cross-section, σ, goes like $\sigma \propto |\mathcal{M}^\omega \cdot \epsilon_\omega|^2$ and $q \cdot \epsilon = 0$ as well as $q^2 = 0$. For most off-shell processes, such as $\mu \to eee$ via an off-shell photon, the q^ω term will vanish in any case. With this in mind it is not surprising that the leading-order coefficients of $F_1(q^2)$ and $F_1^5(q^2)$ are found to vanish.

4.3 Decay Rate

<u>SQED Result</u>

From Appendix O we find that

$$\mathcal{M}_{\mu e\gamma} = \mathcal{K}\overline{\psi}^e(p', s')\Big\{ \left[F_1(q^2) + F_1^5(q^2)\gamma_5\right] \frac{\not{q}\sigma^{\mu\nu}q_\nu}{m_\lambda^2} + \left[F_2(q^2) + F_2^5(q^2)\gamma_5\right] \sigma^{\mu\nu}q_\nu \Big\}\psi^\mu(p, s)$$

with

$$\mathcal{K} = -\frac{\alpha^{3/2} \sin\theta \cos\theta}{144\sqrt{\pi}} \qquad q_\omega = p_\omega - p'_\omega$$

and that, when the muon decays radiatively on-shell to a photon and electron, that the F_1 and F_1^5 terms are unimportant but that

$$F_2(q^2=0) \doteq \frac{9m_\mu}{m_\lambda^2}[\Delta f_2(T_R) + \Delta f_2(T_L)]$$
$$F_2^5(q^2=0) \doteq \frac{9m_\mu}{m_\lambda^2}[\Delta f_2(T_R) - \Delta f_2(T_L)] \tag{4.5}$$

where m_λ is the photino mass, which is not necessarily small, and we are considering a purely SQED result. We have abbreviated the mass ratios (Chapter 3 and Appendix O) to:

$$T_L = \frac{m^2_{\tilde{\ell}_{L_1}}}{m_\lambda^2} \qquad T_R = \frac{m^2_{\tilde{\ell}_{R_1}}}{m_\lambda^2} \tag{4.6}$$

From the appendix

$$\Delta f_2(T_R) \equiv f_2\left(\frac{m^2_{\tilde{\ell}_{R_1}}}{m_\lambda^2}\right) - f_2\left(\frac{m^2_{\tilde{\ell}_{R_2}}}{m_\lambda^2}\right)$$
$$\Delta f_2(T_L) \equiv f_2\left(\frac{m^2_{\tilde{\ell}_{L_1}}}{m_\lambda^2}\right) - f_2\left(\frac{m^2_{\tilde{\ell}_{L_2}}}{m_\lambda^2}\right) \tag{4.7}$$

$$f_2(x) = \frac{(1-x)(x^2-5x-2) - 6x\ln x}{(1-x)^4} \qquad f_2(1) = -\frac{1}{6}\,. \tag{4.8}$$

This has been plotted in Fig. 4.2a. We find that the decay width is

$$\Gamma_{\mu e\gamma} = \Gamma_0 \sin^2\theta \cos^2\theta \left(\frac{m_\mu}{m_\lambda}\right)^4 \left(|\Delta f_2(T_L)|^2 + |\Delta f_2(T_R)|^2\right) \tag{4.9}$$

where

$$\Gamma_0 \equiv \frac{\alpha^3}{1024\pi^2}\,\frac{m_\mu c^2}{\hbar}$$
$$\simeq 6.17 \times 10^{12}\ \text{sec}^{-1}\,. \tag{4.10}$$

Small Mass-Splitting

In the case of small slepton mass-splittings, when

$$\Delta f_2(T_L) = \delta_{\tilde{\ell}_L} f_2'(T_L) \qquad \Delta f_2(T_R) = \delta_{\tilde{\ell}_R} f_2'(T_R)$$

$$\delta_{\tilde{\ell}_L} \equiv \frac{m^2_{\tilde{\ell}_{L_1}} - m^2_{\tilde{\ell}_{L_2}}}{m^2_{\tilde{\ell}_{L_1}}} \qquad f_2'(T_L) = T_L \frac{\partial f_2(T_L)}{\partial T_L} \tag{4.11}$$

$$\delta_{\tilde{\ell}_R} \equiv \frac{m^2_{\tilde{\ell}_{R_1}} - m^2_{\tilde{\ell}_{R_2}}}{m^2_{\tilde{\ell}_{R_1}}} \qquad f_2'(T_R) = T_R \frac{\partial f_2(T_R)}{\partial T_R}$$

$$\delta_{\tilde{\ell}_L} \ll 1 \qquad \delta_{\tilde{\ell}_R} \ll 1 \tag{4.12}$$

we have

$$\Gamma_{\mu e \gamma} \doteq \Gamma_0 \sin^2\theta \cos^2\theta \left(\frac{m_\mu}{m_\lambda}\right)^4 \left[\delta^2_{\tilde{\ell}_L} \left|f_2'(T_L)\right|^2 + \delta^2_{\tilde{\ell}_R} \left|f_2'(T_R)\right|^2\right] \tag{4.13}$$

Note that

$$f_2'(x) = \frac{x}{3(1-x)^5} \left\{(1-x)(x^2 - 8x - 17) - 6(6x+1)\ln x\right\} \tag{4.14}$$

$$f_2'(0) = 0 \qquad f_2'(x \to 0) \sim -\frac{1}{3} x (17 + 6 \ln x)$$

$$f_2'(1) = \frac{1}{10} \tag{4.15}$$

$$f_2'(\infty) = 0 \qquad f_2'(x \to \infty) \sim \frac{1}{3x}$$

The peak of f' is found at

$$f_2'(0.273) \simeq 0.12407 \tag{4.16}$$

This has been plotted in Fig. 4.2b. The physical meaning of this is that such a contribution will be maximal when

$$m_\lambda \simeq 2m_{\tilde{\ell}} \, .$$

Note that $f_2'(T_L) = -2I_{[\mathrm{Z(1-Z)}]}(T_L, 0)$ which is given in Table L.1.

Electroweak Calculation

As discussed in the first section, in a full supersymmetric electroweak (SGWS) calculation, within this model, the only gaugino which would contribute would be a pure $\widetilde{B}^0$, should it be a mass eigenstate.

Thus the calculation would be identical with that of the purely SQED case except that the photino λ ($\widetilde{\gamma}$) would be supplanted by a Bino. Higgsino contributions would enter into $\mathcal{M}^{\omega}_{\mu\to e\gamma}$ with a factor of m_μ^4, down even from the m_μ^2 coefficients which allowed us to ignore the F_1 and F_1^5 terms. In using a photino we have been, in effect, using only a *piece* of the full $\widetilde{B}^0$ contribution (*i.e.* if the $\widetilde{\gamma} - \widetilde{Z}^0$ were the correct mass eigenbasis we would have been neglecting the Zino terms). The Bino and photino couplings are similar except that we must make the following substitutions:

$$\begin{aligned}
&ie\sin\theta; ie\cos\theta \quad \rightarrow \quad -\frac{i}{2}g'\sin\theta; -\frac{i}{2}g'\cos\theta \quad \text{for} \quad \tilde{\ell}_L \\
&ie\sin\theta; ie\cos\theta \quad \rightarrow \quad ig'\sin\theta; ig'\cos\theta \quad \text{for} \quad \tilde{\ell}_R
\end{aligned} \tag{4.17}$$

where $g' = \dfrac{e}{\cos\theta_w}$ is the $U(1)_Y$ hypercharge coupling constant ($Ye_L = -1$; $Ye_R = 2$). Using this (4.9) becomes

$$\Gamma_{\mu e\gamma} = \Gamma_0 \frac{\sin^2\theta\cos^2\theta}{\cos^4\theta_w}\left(\frac{m_\mu}{m_{\tilde{B}^0}}\right)^4\left(\frac{|\Delta f_2(T_L)|^2}{16} + |\Delta f_2(T_R)|^2\right) \, . \tag{4.18}$$

This is exactly correct if $\widetilde{B}^0$ is indeed an eigenstate. If the photino and Zino are closer to the true eigenbases then we would use the SQED result and add in a similar $\widetilde{Z}^0$ contribution obtainable by the following substitutions from the SQED result:

$$
\begin{array}{llll}
ie\sin\theta; ie\cos\theta & \longrightarrow & -ie\cot 2\theta_w \sin\theta; -ie\cot 2\theta_w\cos\theta & \text{for} \quad \tilde{\ell}_L \\
ie\sin\theta; ie\cos\theta & \longrightarrow & ie\tan\theta_w\sin\theta; ie\tan\theta_w\cos\theta & \text{for} \quad \tilde{\ell}_R
\end{array}
\tag{4.19}
$$

which follows from the general fermion-fermion-Z^0 vertex

$$\frac{g}{cos\theta_w}\left[T_3 - Q\sin^2\theta_w\right] \tag{4.20}$$

where $g = e/\sin\theta_w$ is the $SU(2)$ coupling constant. Once again we have set the left and right mixing angles equal (to 'θ').

Pursuing this line of reasoning further the integral function terms may also be readily extended to include both photinos and Zinos:

$$
\begin{aligned}
\frac{\Delta f_2(T_L)}{m_\lambda^2} &\longrightarrow \frac{\Delta f_2(T_L^\lambda)}{m_\lambda^2} + \cot^2 2\theta_w \frac{\Delta f_2(T_L^{\tilde{Z}^0})}{m_{\tilde{Z}^0}^2} \\
\frac{\Delta f_2(T_R)}{m_\lambda^2} &\longrightarrow \frac{\Delta f_2(T_R^\lambda)}{m_\lambda^2} + \tan^2\theta_w \frac{\Delta f_2(T_R^{\tilde{Z}^0})}{m_{\tilde{Z}^0}^2}
\end{aligned}
\tag{4.21}
$$

where

$$
\begin{array}{ll}
T_L^\lambda = \dfrac{m_{\tilde{\ell}_{L_1}}^2}{m_\lambda^2} & T_R^\lambda = \dfrac{m_{\tilde{\ell}_{R_1}}^2}{m_\lambda^2} \\
T_L^{\tilde{Z}^0} = \dfrac{m_{\tilde{\ell}_{L_1}}^2}{m_{\tilde{Z}^0}^2} & T_R^{\tilde{Z}^0} = \dfrac{m_{\tilde{\ell}_{R_1}}^2}{m_{\tilde{Z}^0}^2} \quad .
\end{array}
\tag{4.22}
$$

The width then becomes

$$\Gamma_{\mu e\gamma} = \Gamma_0 \sin^2\theta\cos^2\theta m_\mu^4 \left\{ \left| \frac{\Delta f_2(T_L^\lambda)}{m_\lambda^2} + \frac{\cot^2 2\theta_w \Delta f_2(T_L^{\tilde{Z}^0})}{m_{\tilde{Z}^0}^2} \right|^2 \right. \qquad (4.23)$$
$$\left. \left| \frac{\Delta f_2(T_R^\lambda)}{m_\lambda^2} + \frac{\tan^2\theta_w \Delta f_2(T_R^{\tilde{Z}^0})}{m_{\tilde{Z}^0}^2} \right|^2 \right\}$$

It is clear how we would extend this to the instance of general neutralino mixing in the fashion used in chapter three and its associated appendices.

4.4 Comparison With Experiment

The radiative decay of the muon has been intensively searched for. Thus far no positive evidence has surfaced but improved limits are being reported on a regular basis. We will use the limit[30]

$$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} < 4.9 \times 10^{-11} \qquad (4.24)$$

and

$$\Gamma_{\mathrm{TOT}} \simeq \Gamma_{\mu\to e\nu\bar{\nu}} = 455137\ \mathrm{s}^{-1} \qquad (4.25)$$

therefore

$$\Gamma^{\mathrm{exp}}_{\mu\to e\gamma} \leq 2.2 \times 10^{-5}\ \mathrm{s}^{-1} \qquad (4.26)$$

and so

$$\Gamma^{\mathrm{exp}}_{\mu\to e\gamma}/\Gamma_0 \lesssim 3.6 \times^{-18}\ . \qquad (4.27)$$

It must be commented that (4.24) represents a remarkably accurate limit and is the culmination of much dedicated work by many people.

From (4.9) and (4.18) we may immediately obtain

$$\sin^2\theta\cos^2\theta\left(\frac{m_\mu}{m_\lambda}\right)^4\left(|\Delta f_2(T_L)|^2+|\Delta f_2(T_R)|^2\right)\leq 3.6\times 10^{-18} \tag{4.28}$$

for SQED and

$$\frac{\sin^2\theta\cos^2\theta}{\cos^4\theta_w}\left(\frac{m_\mu}{m_{\tilde{B}^0}}\right)^4\left(\frac{|\Delta f_2(T_L)|^2}{16}+|\Delta f_2(T_R)|^2\right)\leq 3.6\times 10^{-18} \tag{4.29}$$

for SGWS if the Bino were a mass eigenstate. Note that if

$$m_{\tilde{\ell}_{L_1}}\simeq m_{\tilde{\ell}_{R_1}}\qquad m_{\tilde{\ell}_{L_2}}\simeq m_{\tilde{\ell}_{R_2}}\qquad \theta=\frac{\pi}{4}$$

that (SQED)

$$\frac{|\Delta f_2(T_{L,R})|}{m_\lambda^2}\leq 2.4\times 10^{-7}GeV^{-2} \tag{4.30}$$

Small Mass-Splitting Limit

If the differences in the mass of $m_{\tilde{\ell}_{L_1}}$ and $m_{\tilde{\ell}_{L_2}}$ is small (and the same for the "right" sleptons), *i.e.*

$$\delta_{\tilde{\ell}_L}\ll 1 \qquad \delta_{\tilde{\ell}_R}\ll 1$$

as in (4.11) then we have

$$\Gamma_{\mu e\gamma}\doteq\Gamma_0\sin^2\theta\cos^2\theta\left(\frac{m_\mu}{m_\lambda}\right)^4\left[\delta^2_{\tilde{\ell}_L}\left|f_2'(T_L)\right|^2+\delta^2_{\tilde{\ell}_R}\left|f_2'(T_R)\right|^2\right].$$

Since $f_2'(0)=f_2'(\infty)=0$ we know that

$$\Gamma_{\mu\to e\gamma}=0 \text{ as } \qquad \begin{aligned}&(i)\ m_\lambda\to\infty\\ &(ii)\ m_{\tilde{\ell}}\to\infty\\ &(iii)\ m_{\tilde{\ell}_1}\to m_{\tilde{\ell}_2}\end{aligned} \tag{4.31}$$

From (4.16) $f_2' \lesssim \frac{1}{8}$. Should it be that

$$\delta_{\tilde{\ell}_L} \simeq \delta_{\tilde{\ell}_R} \simeq \delta_{\tilde{\ell}} \ll 1$$

then, for SQED,

$$\Gamma_{\mu e \gamma} \lesssim \frac{\delta_{\tilde{\ell}}^2}{32} \Gamma_0 \sin^2\theta \cos^2\theta \left(\frac{m_\mu}{m_\lambda}\right)^4$$

and since $\sin^2\theta \cos^2\theta \leq \frac{1}{4}$

$$\Gamma_{\mu e \gamma} \lesssim \frac{\delta_{\tilde{\ell}}^2}{128} \Gamma_0 \left(\frac{m_\mu}{m_\lambda}\right)^4 . \tag{4.32}$$

When we compare with experiment using (4.27) we obtain a limit on how large the relative splitting may be.

$$\delta_{\tilde{\ell}} \lesssim 1.9 \times 10^{-6} m_\lambda^2 \tag{4.33}$$

where m_λ is given in GeV. This should be compared with (4.30). Thus for $m_\lambda \sim 100GeV$ the slepton splitting would be less than 2% while for $m_\lambda \sim 1GeV$ the slepton splitting would be less than 2×10^{-4}%. These would occur near the maximal cross-section, when $m_\lambda \approx 2m_{\tilde{\ell}}$.

The condition that $m_\lambda \approx 2m_{\tilde{\ell}}$ is a rather unnatural one. A popular scenario among designers of specific models is that the neutralino state which is principally photino-like tends to be on the light side. Indeed some authors tend to assume that $m_\lambda \ll m_{\tilde{\ell}}$ almost axiomatically. Although we will not take this view it is, however, interesting to examine the results in this limit. If

$$m_\lambda \ll m_{\tilde{\ell}_{L,R}}$$

then

$$T_{L,R} \to \infty$$

and

$$f_2'(T_{L,R}) \to \frac{1}{3T_{L,R}} \,. \tag{4.34}$$

In this case (4.13) becomes

$$\Gamma_{\mu e\gamma} \doteq \Gamma_0 \sin^2\theta \cos^2\theta \left(\frac{m_\mu}{m_\lambda}\right)^4 \left(\frac{\delta^2_{\tilde{\ell}_L}}{9T_L^2} + \frac{\delta^2_{\tilde{\ell}_R}}{9T_R^2}\right) \tag{4.35}$$

and so if, in addition, $\delta_{\tilde{\ell}_L} \simeq \delta_{\tilde{\ell}_R}$ then (SQED)

$$\Gamma_{\mu e\gamma} \approx \frac{2}{9}\Gamma_0 \sin^2\theta \cos^2\theta \left(\frac{m_\mu}{m_\lambda}\right)^4 \delta^2_{\tilde{\ell}_L}$$

and if $\theta = \pi/4$ (maximal mixing)

$$\frac{\delta_{\tilde{\ell}}}{m^2_{\tilde{\ell}}} \lesssim 7.2 \times 10^{-7}\ GeV^{-2} \,. \tag{4.36}$$

If the Bino is an eigenstate and $m_{\tilde{B}^0} \ll m_{\tilde{\ell}}$ this would become ($\cos^2\theta_w = 0.77$)

$$\frac{\delta_{\tilde{\ell}}}{m^2_{\tilde{\ell}}} \lesssim 7.6 \times 10^{-7}\ GeV^{-2} \,. \tag{4.37}$$

In their classic paper[24] Ellis and Nanopoulos made the first rough estimates for such processes. Working exclusively in the small mass-splitting regime (as discussed in chapter three) they found

$$\delta_{\tilde{\ell}}/m^2_{\tilde{\ell}} < \mathcal{O}(10^{-7})\ GeV^{-2}$$

assuming maximal mixing and that $\tilde{B}^0$ is an eigenstate. At the time which that paper was written the best experimental limit was $\Gamma^{\text{exp}}_{\mu\to e\gamma}/\Gamma_{TOT} < 1.9 \times 10^{-10}$.

Using this value (4.37) would alter to

$$\frac{\delta_{\tilde{\ell}}}{m_{\tilde{\ell}}^2} \lesssim 3 \times 10^{-6}\ GeV^{-2}\ .$$

which is a factor of thirty off of their naive estimate. Thus a careful computation reveals that we are only able to place somewhat weaker limits than one might have initially hoped.

Large Mass-Splitting Limit

In this limit $m_{\tilde{\ell}_{L_2}} \gg m_{\tilde{\ell}_{L_1}}$ and $m_{\tilde{\ell}_{R_2}} \gg m_{\tilde{\ell}_{R_1}}$. Therefore

$$\Delta f_2(T_{L,R}) \rightarrow f_2(T_{L,R})\ .$$

Where $f_2(x)$ was given in (4.8). Note that

$$f_2(x \rightarrow \infty) \rightarrow -\frac{1}{3x}\ . \tag{4.38}$$

If we further consider the case where $m_{\tilde{\ell}} \gg m_{\tilde{\chi}^0}$ then (for $\tilde{\chi}^0 = \tilde{B}^0$) from (4.18)

$$\begin{aligned}\Gamma_{\mu e\gamma} &\rightarrow \Gamma_0 \frac{\sin^2\theta\cos^2\theta}{\cos^4\theta_w}\cdot\frac{1}{9}\left(\frac{m_\mu}{m_{\tilde{B}^0}}\right)^4\left(\frac{m_{\tilde{B}^0}^4}{16m_{\tilde{\ell}_{L_1}}^4}+\frac{m_{\tilde{B}^0}^4}{m_{\tilde{\ell}_{R_1}}^4}\right)\\ &= \frac{1}{9}\Gamma_0\frac{\sin^2\theta\cos^2\theta}{\cos^4\theta_w}\quad m_\mu^4\left[\frac{1}{16m_{\tilde{\ell}_{L_1}}^4}+\frac{1}{m_{\tilde{\ell}_{R_1}}^4}\right]\end{aligned} \tag{4.39}$$

For $\theta = \theta_L = \theta_R = \pi/4$ and using $\cos^2\theta_w = 0.77$ and (4.27) we find

$$\left(\frac{1}{16m_{\tilde{\ell}_{L_1}}^4}+\frac{1}{m_{\tilde{\ell}_{R_1}}^4}\right) \lesssim \frac{7.7\times10^{-17}}{m_\mu^4} \tag{4.40}$$

Further specializing to

$$m_{\tilde{\ell}_{L_1}} = m_{\tilde{\ell}_{R_1}} \equiv m_{\tilde{\ell}}$$

would result in

$$m_{\tilde{\ell}} \gtrsim 1.1 \; TeV \tag{4.41}$$

since $m_\mu = 0.105659 \; GeV$.

This result is not "viable" since it was obtained by considering the limit $m_{\tilde{\ell}_{L_2}}, m_{\tilde{\ell}_{R_2}} \gg m_{\tilde{\ell}} \approx 1 \; TeV$ and we would like the supersymmetry-breaking scale to be $\lesssim \mathcal{O}(1 \; TeV)$ if it is to alleviate the gauge hierarchy problem in a 'natural' way. If the mass eigenstates are photinos and Zinos we should anticipate a similar result in this limit. If $m_{\tilde{Z}^0} \sim m_Z$ then $m_{\tilde{\ell}} \gg m_{\lambda, \tilde{Z}^0}$ and since $T_L^\lambda = T_R^\lambda$ and $T_L^{\tilde{Z}^0} = T_R^{\tilde{Z}^0}$ we would have from (4.18)

$$\Gamma_{\mu e \gamma} \rightarrow \Gamma_0 \sin^2\theta \cos^2\theta \cdot \frac{1}{9} \left(\frac{m_\mu}{m_{\tilde{\ell}_{L_1}}} \right)^4 \left\{ \left(1 + \cot^2 2\theta_w\right)^2 + \left(1 + \tan^2\theta_w\right)^2 \right\} \; .$$

The term in the curly brackets is about 3.7. If only the photino part were included it would have been 2.0 while the corresponding term for the Bino was about 1.8. In the photino-Zino case the limit in (4.41) would become

$$m_{\tilde{\ell}} \gtrsim 1.4 \; TeV \; . \tag{4.42}$$

Such a constraint would, of course, be weakened if the angular mixing were not maximal.

Implication of ASP Results

The ASP single photon search at SLAC has placed[31] limits on $m_{\tilde{e}}$ of about 60 GeV. When combined with the results of MAC and other collaborations they find[26] (under a set of parameter assumptions) that

$$m_{\tilde{e}} \gtrsim 84 \; GeV \; . \tag{4.43}$$

The limit comes from considerations of the process

$$e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}$$

whose leading contribution arises from the diagram in Fig. 4.3. Under the assumptions that $m_{\tilde{e}_L} = m_{\tilde{e}_R}$, the photino is a mass eigenstate and is the only light neutral gaugino, and that $m_{\tilde{\gamma}} \ll m_{\tilde{e}_{L,R}}$, they quote

$$\sigma(e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}) \sim \alpha^3 \frac{s}{m_{\tilde{e}}^4} < 0.06 \; pb \; . \tag{4.44}$$

The specific search is for $e^+e^- \to \gamma$+energy where the standard model calculation of $e^+e^- \to \gamma\nu\overline{\nu}$, for three light neutrino generations, has been accounted for. Thus the search for new light species of neutrinos (as well as Higgsinos, sneutrinos and so forth) compliments the photino search, since the existence of such states would only *improve* the limit in (4.43).

The above analysis assumes that there is no intergenerational mixing. Permitting $\tilde{e} - \tilde{\mu}$ mixing, with angle θ, changes (4.44) to ($m_{\tilde{\ell}_L} = m_{\tilde{\ell}_R}$)

$$\sigma(e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}) \sim \alpha^3 s \left(\frac{\cos^2\theta}{m_{\tilde{\ell}_1}^2} - \frac{\sin^2\theta}{m_{\tilde{\ell}_2}^2} \right)^2 < 0.06 \; pb \tag{4.45}$$

so instead of

$$\frac{1}{m_{\tilde{e}}^2} < \frac{1}{(84 \; GeV)^2}$$

which follows from (4.43) we have

$$\frac{\cos^2\theta}{m_{\tilde{\ell}_1}^2} - \frac{\sin^2\theta}{m_{\tilde{\ell}_2}^2} < \frac{1}{(84 \; GeV)^2} \; . \tag{4.46}$$

Since $\sigma(\mu \to e\gamma)$ goes as $\sin^2\theta\cos^2\theta$, whereas $\sigma(ee \to \gamma\tilde{\gamma}\tilde{\gamma})$ has θ dependence as in (4.46), we see that, when these processes are compared, that the mixing angle ($\theta = \theta_L = \theta_R$) enters non-trivially.

First let us consider the case of large mass-splitting, where $m_{\tilde{\ell}_2} \gg m_{\tilde{\ell}_1}$. Then (4.46) reduces to

$$\frac{\cos^2\theta}{m^2_{\tilde{\ell}_1}} < \frac{1}{(84\ GeV)^2}$$

i.e.

$$m_{\tilde{\ell}_1} > 84\ GeV\ \cos\theta \tag{4.47}$$

which is weaker that (4.43). For $\theta = \pi/4$ (maximal mixing) this limit becomes

$$m_{\tilde{\ell}_1} > 59\ GeV\ . \tag{4.48}$$

We have found that $\sigma_{e^+e^-\to Z^0\to\mu^+e^-}$ and $\sigma_{\mu\to e\gamma}$ go as

$$\sin^2\theta\cos^2\theta\{\Delta F_1(T^0) + \Delta F_2(T^+)\}$$

where $T^0 = \frac{m^2_{\tilde{\ell}_1}}{m^2_{\lambda}}$ and $T^+ = \frac{m_{\tilde{\nu}_1}{}^2}{m^2_{\tilde{W}}}$. $\Delta F_{1,2}$ are some functions which decrease with large, increasing arguments. For $\mu \to e\gamma$ the F_2 term is absent. Thus as the limit from (4.47) becomes stronger (increasing $\cos\theta$) then $max[\Delta F_1]$ decreases. However $\sin^2\theta\cos^2\theta$ peaks at $\theta = \pi/4$. Thus as θ increases from $\theta < \pi/4$ to $\theta = \pi/4$, $max[\Delta F_1]$ decreases while $\sin^2\theta\cos^2\theta$ increases. As θ increases beyond $\theta = \pi/4$ $max[\Delta F_1]$ increases while $\sin^2\theta\cos^2\theta$ decreases. Thus there is always competition between these two effects (as far as obtaining experimental limits from $e^+e^- \to \mu^+e^-$ and $\mu \to e\gamma$ are concerned). Note that at $\cos\theta = 0, 1$ that both processes vanish.

Let us now consider the small mass-splitting case:

$$m_{\tilde{\ell}_2}^2 - m_{\tilde{\ell}_1}^2 \equiv m_{\tilde{\ell}_1}^2 \cdot \delta_{\tilde{\ell}} \ll m_{\tilde{\ell}_{1,2}}^2 \ .$$

Then

$$\frac{1}{m_{\tilde{\ell}_2}^2} \doteq \frac{1 - \delta_{\tilde{\ell}}}{m_{\tilde{\ell}_1}^2}$$

and therefore

$$\frac{\cos^2\theta}{m_{\tilde{\ell}_1}^2} - \frac{\sin^2\theta}{m_{\tilde{\ell}_2}^2} \approx \frac{\cos^2\theta - \sin^2\theta}{m_{\tilde{\ell}_1}^2} + \frac{\sin^2\theta\,\delta_{\tilde{\ell}}}{m_{\tilde{\ell}_1}^2} \ . \qquad (4.49)$$

If we are near "maximal mixing" then $\theta \doteq \pi/4$ and

$$\frac{\cos^2\theta}{m_{\tilde{\ell}_1}^2} - \frac{\sin^2\theta}{m_{\tilde{\ell}_2}^2} \approx +\frac{\delta_{\tilde{\ell}}}{2m_{\tilde{\ell}_1}^2}$$

and (4.46) becomes

$$\frac{\delta_{\tilde{\ell}}}{2m_{\tilde{\ell}_1}^2} \leq \frac{1}{(84\ GeV)^2}$$

i.e.

$$\frac{\delta_{\tilde{\ell}}}{m_{\tilde{\ell}_1}^2} \lesssim 3 \times 10^{-4}\ GeV^{-2} \qquad (4.50)$$

which is *much* weaker than the $\mu \to e\gamma$ limit

$$\frac{\delta_{\tilde{\ell}}}{m_{\tilde{\ell}_1}^2} \lesssim 7 \times 10^{-7}\ GeV^{-2}$$

which we found in (4.36).

The weakness of the limit lies in the supposition that $\theta = \pi/4$. This is due to the particular form of (4.49). If we took $\theta = \pi/3$ then the limit in (4.36) would change to

$$\frac{\delta_{\tilde{\ell}}}{m^2_{\tilde{\ell}_1}} \lesssim 10^{-6}\ GeV^{-2} \tag{4.51}$$

from $\mu \to e\gamma$ while $m_{\tilde{\ell}_1} > 42\ GeV$ from $e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}$. When θ is not near $\pi/4$ (4.46) and (4.49) imply that

$$m_{\tilde{\ell}_1} > 84\ GeV\ \cos 2\theta \tag{4.52}$$

as opposed to (4.47). Note that limits that ASP places on $m_{\tilde{W}}$ from the analogous process $e^+e^- \to \gamma\tilde{\nu}\tilde{\nu}$ are both weaker and subject to a greater number of model-dependent assumptions.

FIGURE CAPTIONS

1. The vertex $\gamma \to \bar{\mu} e$.

2. (a) The function $f_2(x)$ *vs.* x

   (b) The function $f_2'(x)$ *vs.* x.

3. Principle contribution to the process

$$e^+ e^- \to \gamma \tilde{\gamma} \tilde{\gamma}$$

   used by ASP to place limits on the selectron mass.

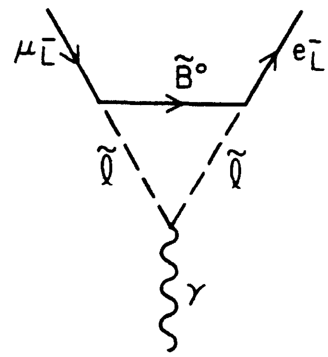


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Fig. 4.1

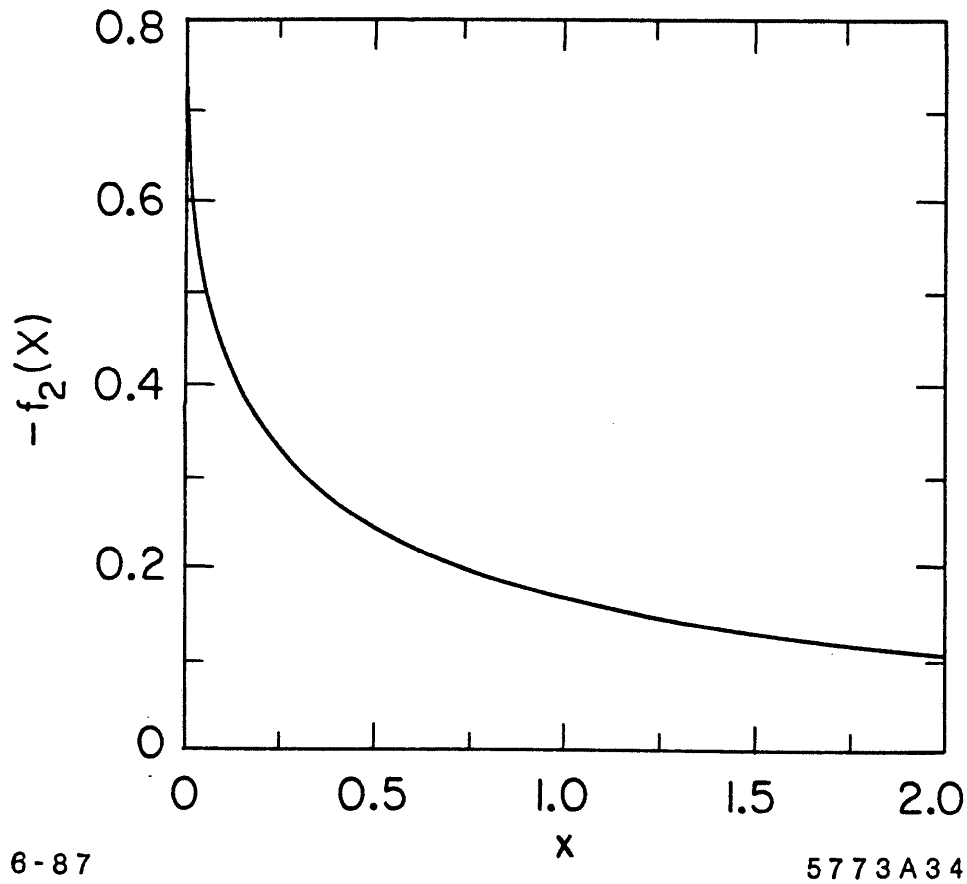


Fig. 4.2a

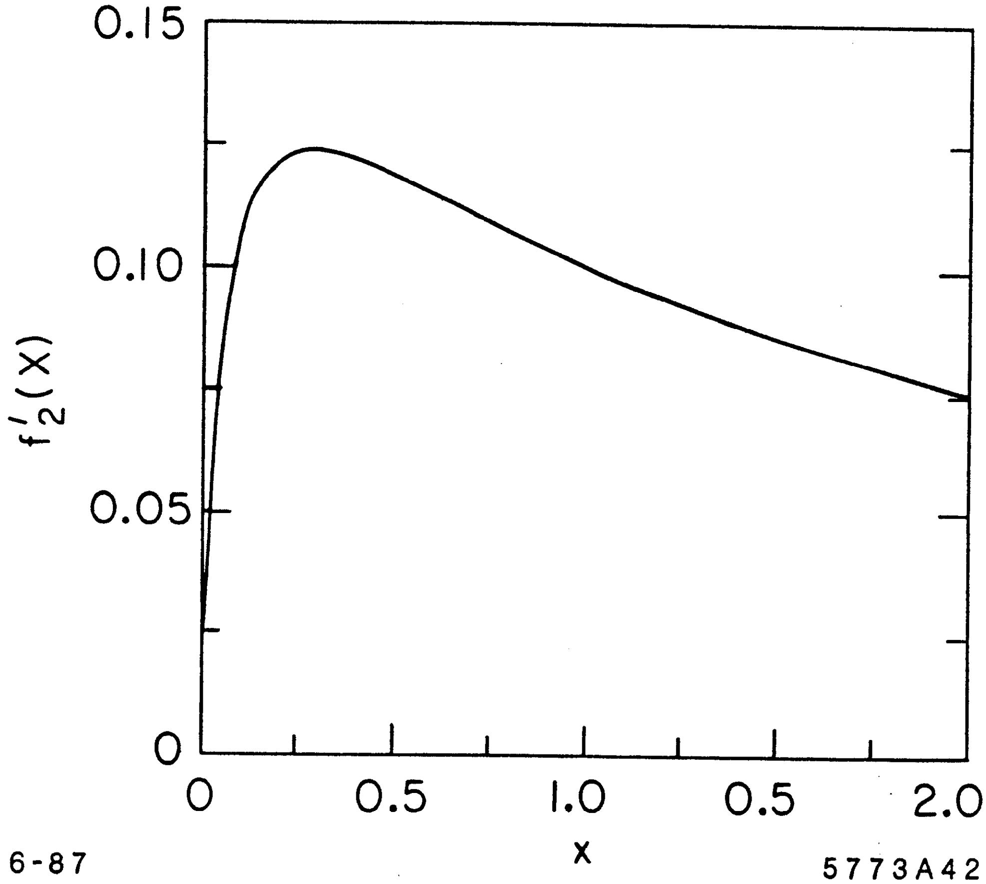


Fig. 4.2b

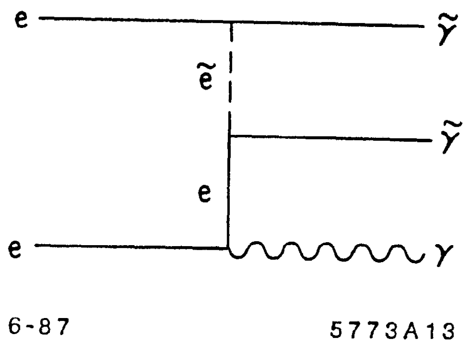


Fig. 4.3

Summary

We have seen that intergenerational mixing of sleptons may occur at experimentally acceptable rates. When compared with experiment we may begin to eliminate certain regions from an otherwise unconstrained parameter space. It is not anticipated that lepton family number violation, arising from this mechanism, will be seen at the SLC due to insufficient production rates.

The low-energy processes of chapters two and four provide more substantial constraints. If the mass-splitting between the two sectors (left-right or species one-species two) is small then reasonably strong limits on the splitting may be obtained, depending upon the specifics involved. If a large mass-splitting is assumed then some quite respectable limits of supersymmetric masses become possible, but this is again highly dependent upon specifics.

In conclusion we feel that the search for both family number non-conservation and evidence for the existence of supersymmetry is of great importance. Then interplay between these two areas can provide endless entertainment for both experimentalists and theorists alike.

APPENDIX A

Two-Component Weyl Notation

The two-component Weyl spinor formalism is a more fundamental representation of spinorial states and algebra than that of Dirac. In most theories the Dirac notation is a convenient shorthand for summarizing the actions of the different chiral states and this explains its immense utility and popularity. In supersymmetric theories the states of different chirality are dealt with independently and two-component notation is the natural language to use. Greater familiarity with the manipulation of Dirac objects will prompt us to use them in the more involved computations but the irrecusable consequence will be the proliferation of conjugation and helicity projection operators.

There are many excellent references[1,12,18] whose appendices summarize definitions (which vary from one author to the next) and properties of the Weyl algebra. We will use notation similar to that of reference 12. The most appropriate representation of the gamma matrices to use when we work in four-component notation is not the Dirac representation but the chiral representation. Note that the chiral representation used here differs from that of Itzykson and Zuber.[50]

A.1 SPINORS

We write the electron as:

$$\psi = \begin{pmatrix} \varphi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} \equiv \begin{pmatrix} \psi_{-\alpha} \\ \overline{\psi}_{+}^{\dot{\alpha}} \end{pmatrix} \qquad \begin{matrix} L.H.\ e^- \\ R.H.\ e^- \end{matrix} \tag{A.1}$$

$$\begin{matrix} \overline{\psi} = (\chi^\alpha & \overline{\varphi}_{\dot{\alpha}}) \equiv (\psi_+^\alpha & \overline{\psi}_{-\dot{\alpha}}) \\ & L.H.\ e^+ \quad R.H.\ e^+ \end{matrix} \tag{A.2}$$

where the position of the indices in conventional. Barred spinors ($\bar{\psi}$) and dotted indices denote right-handed $(0,\frac{1}{2})$ representations of the Lorentz Group. Unbarred spinors (ψ) and undotted indices denote left-handed $(\frac{1}{2},0)$ representations.

A.2 SIGMA AND GAMMA MATRICES

The metric used is[♯1] $g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

The gamma matrices, γ^μ, have the structure

$$\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{pmatrix} \quad \mu = 0,1,2,3 \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A.3}$$

where

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.4}$$

Definition:

$$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu, \gamma^\nu] \quad \text{(4-component)} \tag{A.5}$$

♯1 Note that Wess and Bagger[1] use

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \; \textit{and} \; \sigma^0 = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\bar{\sigma}^{\mu\nu} = \frac{i}{2}\left[\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}\right] \quad \text{(2-component)}$$
$$\sigma^{\mu\nu} = \frac{i}{2}\left[\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right] \quad \text{(2-component)}\ . \tag{A.6}$$

$$\sigma^{\mu\nu}[\text{4-component}] = \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} (\sigma^{\mu\nu})_{\alpha}{}^{\beta} & 0 \\ 0 & (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix} \tag{A.7}$$

$$\overline{\psi}\sigma^{\mu\nu}\psi[\text{Dirac}] = \psi_{+}\sigma^{\mu\nu}\psi_{-} + \overline{\psi}_{-}\bar{\sigma}^{\mu\nu}\overline{\psi}_{+} \tag{A.8}$$

$$\not{a}\,\not{b} = a\cdot b - i a_{\mu} b_{\nu}\sigma^{\mu\nu}$$

$$\gamma_5\,\not{a}\,\not{b} = \not{a}\,\not{b}\gamma_5 = \gamma_5[a\cdot b - i a^{\mu} b^{\nu}\sigma_{\mu\nu}]$$

$$(\sigma\cdot p)(\bar{\sigma}\cdot q) = p\cdot q - i\sigma^{\mu\nu}p_{\mu}q_{\nu} \tag{A.9}$$

$$(\bar{\sigma}\cdot p)(\sigma\cdot q) = p\cdot q - i\bar{\sigma}^{\mu\nu}p_{\mu}q_{\nu}$$

A special case is

$$(\sigma\cdot p)^2 = (\sigma^{\mu})_{\alpha\dot{\beta}}p_{\mu}(\bar{\sigma})^{\dot{\beta}\beta}p_{\nu} = p^2\delta_{\alpha}{}^{\beta} \tag{A.10}$$

A.3 INDEX GYMNASTICS

Raise and lower indices with ϵ as follows

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta} \quad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta} \quad \overline{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\overline{\psi}_{\dot{\beta}} \quad \overline{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\overline{\psi}^{\dot{\beta}} \tag{A.11}$$

$$\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \epsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \text{(in matrix form)} \tag{A.12}$$

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_{\alpha}{}^{\gamma} \qquad \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon_{\dot{\beta}\dot{\gamma}} = \delta^{\dot{\alpha}}{}_{\dot{\gamma}}\ . \tag{A.13}$$

In actuality $\bar{\sigma}^{\mu} = \sigma^{\mu}$. $(\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha}$ is just the raised index version of $(\sigma^{\mu})_{\alpha\dot{\alpha}}$ and no more. Once we have chosen the convention that (σ^{μ}) has index structure $(\sigma^{\mu})_{\alpha\dot{\alpha}}$

then $(\bar{\sigma}^{\mu})$ always refers to the operator of complementary index ordering.

$$
\begin{aligned}
(\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha} &= \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}(\sigma^{\mu})_{\beta\dot{\beta}} \\
&= \epsilon^{\dot{\alpha}\dot{\beta}}\left[\sigma^{\mu}\right]^{T}_{\dot{\beta}\beta}\left[\epsilon^{T}\right]^{\beta\alpha} \\
&= -\left(\epsilon\left[\sigma^{\mu}\right]^{T}\epsilon\right)^{\dot{\alpha}\alpha} \\
&= \left(\sigma^{2}\left[\sigma^{\mu}\right]^{T}\sigma^{2}\right)^{\dot{\alpha}\alpha}
\end{aligned}
\tag{A.14}
$$

since $\epsilon^{\alpha\beta} = i(\sigma^{2})^{\alpha\beta}$. However

$$\{\sigma^{2},(\sigma^{i})^{T}\} = -2\delta^{i2} \qquad [\sigma^{2},(\sigma^{0})^{T}] = 0 \quad i = 1,2,3$$

therefore $\sigma^{2}(\sigma^{1,3})^{T} = -\sigma^{1,3}\sigma^{2}$, $\sigma^{2}(\sigma^{2})^{T} = -1$ and $\sigma^{2}(\sigma^{0})^{T} = \sigma^{0}\sigma^{2}$ so we find that

$$(\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha} = \begin{cases} [(\sigma^{\mu})\sigma^{2}\sigma^{2}]^{\dot{\alpha}\alpha} & \mu = 0 \\ -[\sigma^{\mu}\sigma^{2}\sigma^{2}]^{\dot{\alpha}\alpha} & \mu = 1,2,3 \end{cases} = \begin{cases} (\sigma^{\mu})^{\dot{\alpha}\alpha} & \mu = 0 \\ -(\sigma^{\mu})^{\dot{\alpha}\alpha} & \mu = 1,2,3 \end{cases} \tag{A.15}$$

$$(\sigma^{\mu})_{\alpha\dot{\alpha}} = (I_{\alpha\dot{\alpha}},\sigma^{i}_{\alpha\dot{\alpha}}) \quad (\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha} = (I^{\dot{\alpha}\alpha},-(\sigma^{i})^{\dot{\alpha}\alpha})$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sigma^{0}$ is the unit matrix. Since $(\sigma^{\mu})_{\alpha\dot{\alpha}} = (1,\sigma^{i})_{\alpha\dot{\alpha}}$ and $(\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha} = (1,-\sigma^{i})^{\dot{\alpha}\alpha}$ from (A.15) then $g_{\mu\nu} = diag(1,-1,-1,-1)$ lowers the Lorentz index to give

$$g_{\mu\nu}(\sigma^{\nu})_{\alpha\dot{\alpha}} \equiv (\sigma_{\mu})_{\alpha\dot{\alpha}} = (1,-\sigma^{i})_{\alpha\dot{\alpha}} \; ; \quad g_{\mu\nu}(\bar{\sigma}^{\nu})^{\dot{\alpha}\alpha} \equiv (\bar{\sigma}_{\mu})^{\dot{\alpha}\alpha} = (1,\sigma^{i})^{\dot{\alpha}\dot{\alpha}} \; . \tag{A.16}$$

Note that when we raise one index and lower another

$$\overline{\chi}_{\dot{\alpha}}\overline{\psi}^{\dot{\alpha}} = -\overline{\chi}^{\dot{\alpha}}\overline{\psi}_{\dot{\alpha}} \tag{A.17}$$

Notation:

$$\chi^\alpha \phi_\alpha \equiv \chi\phi \tag{A.18}$$

but

$$\psi\chi = \psi^\alpha \chi_\alpha = (\epsilon^{\alpha\beta}\psi_\beta)(\epsilon_{\alpha\beta}\chi^\gamma) = \psi_\beta(\epsilon^{\alpha\beta}\epsilon_{\alpha\sigma})\chi^\gamma$$

$$= \psi_\beta(-\epsilon^{\beta\alpha}\epsilon_{\alpha\gamma})\chi^\gamma = -\psi_\beta \delta^\beta{}_\gamma \chi^\gamma \tag{A.19}$$

$$= -\psi_\beta \chi^\beta$$

$$\psi^\alpha \chi_\alpha = -\psi_\alpha \chi^\alpha = \chi^\alpha \psi_\alpha \text{ since } \chi, \psi \text{ are fermions } = \chi\psi. \text{ Thus} \tag{A.20}$$

$$\psi\chi = \chi\psi \ . \tag{A.21}$$

A.4 Conjugation

$$(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha} = \overline{\psi}_{\dot\alpha} \quad \text{(2 component spinors)}$$

$$C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta} \end{pmatrix} \quad \text{(charge conjugation operator)}$$

$$\psi^C = C\overline{\psi}^T \quad \text{in general}$$

$$\chi_\alpha = \epsilon_{\alpha\beta}(\overline{\chi})^{\dagger\beta}$$

also note

$$(\chi\psi)^\dagger = \overline{\psi}\,\overline{\chi} = \overline{\chi}\,\overline{\psi} \ .$$

A.5 USEFUL RELATIONSHIPS.

A plethora of useful relationships may be found in Wess and Bagger "Supersymmetry and Supergravity"[1] . Some of these and others of interest are (here m,n=0,1,2,3):

$$(\sigma^m\bar{\sigma}^n + \sigma^n\bar{\sigma}^m)_\alpha{}^\beta = (\sigma^m)_{\alpha\dot{\gamma}}(\bar{\sigma}^n)^{\dot{\gamma}\beta} + (\sigma^n)_{\alpha\dot{\gamma}}(\bar{\gamma}^m)^{\dot{\gamma}\beta} = 2g^m\delta_\alpha{}^\beta \qquad \text{(A.22)}$$

$$(\bar{\sigma}^m\sigma^n + \bar{\sigma}^n\sigma^m)^{\dot{\alpha}}{}_{\dot{\beta}} = 2g^{mn}\delta^{\dot{\alpha}}{}_{\dot{\beta}} \qquad \text{(A.23)}$$

$$\mathrm{tr}\,\sigma^m\bar{\sigma}^n = (\sigma^m)_{\alpha\dot{\beta}}(\bar{\sigma}^n)^{\dot{\beta}\alpha} = 2g^{mn} \qquad \text{(A.24)}$$

$$\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}_m{}^{\dot{\beta}\beta} = -2\delta_\alpha{}^\beta\delta_{\dot{\alpha}}{}^{\dot{\beta}} \qquad \text{(A.25)}$$

$$(\bar{\sigma}^m)^{\dot{\alpha}\alpha}(\sigma_m)_{\beta\dot{\beta}} = -2\delta^{\dot{\alpha}}{}_{\dot{\beta}}\delta^\alpha{}_\beta \qquad \text{(A.26)}$$

$$\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_m = -2 \qquad \text{(A.27)}$$

$$\bar{\sigma}_m\sigma^n\bar{\sigma}^m = (\bar{\sigma}_m)^{\dot{\alpha}\alpha}(\sigma^n)_{\alpha\dot{\beta}}(\bar{\sigma}^m)^{\dot{\beta}\beta} = -2(\bar{\sigma}^n)^{\dot{\alpha}\beta} \qquad \text{(A.28)}$$

$$\sigma_m\bar{\sigma}^n\sigma^m = (\sigma_m)_{\alpha\dot{\alpha}}(\bar{\sigma}^n)^{\dot{\alpha}\beta}(\sigma^m)^{\beta\dot{\beta}} = -2(\sigma^n)^{\alpha\dot{\beta}} \qquad \text{(A.29)}$$

$$\sigma^m\bar{\sigma}^n\sigma_n = -2(\sigma^n)_{\alpha\dot{\beta}} \qquad \text{(A.30)}$$

$$\bar{\sigma}^m\sigma^n\bar{\sigma}_m = -2(\bar{\sigma}^n)^{\dot{\alpha}\beta} \qquad \text{(A.31)}$$

$$(\sigma_\mu\bar{\sigma}^\nu\sigma^\eta\bar{\sigma}^\mu)_\alpha{}^\beta = 4g^{\nu\eta}\delta_\alpha{}^\beta = (\sigma^\mu\bar{\sigma}^\nu\sigma^\eta\bar{\sigma}_\mu)_\alpha{}^\beta \qquad \text{(A.32)}$$

$$(\bar{\sigma}_\mu \sigma^\nu \bar{\sigma}^\eta \sigma^\mu)^{\dot{\alpha}}{}_{\dot{\beta}} = 4g^{\nu\eta}\delta^{\dot{\alpha}}{}_{\dot{\beta}} = (\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\eta \sigma_\mu)^{\dot{\alpha}}{}_{\dot{\beta}} \tag{A.33}$$

$$(\sigma^\mu \bar{\sigma}_\nu \sigma^\eta \bar{\sigma}_\mu)_\alpha{}^\beta = 4\delta_\nu{}^\eta \delta_\alpha{}^\beta = (\sigma_\mu \bar{\sigma}_\nu \sigma^\eta \bar{\sigma}^\mu)_\alpha{}^\beta \tag{A.34}$$

$$(\sigma^\mu \bar{\sigma}^\nu \sigma_\eta \bar{\sigma}_\mu)_\alpha{}^\beta = 4\delta^\nu{}_\eta \delta_\alpha{}^\beta = (\sigma_\mu \bar{\sigma}^\nu \sigma_\eta \bar{\sigma}^\mu)_\alpha{}^\beta \tag{A.35}$$

$$(\sigma^\mu \bar{\sigma}_\nu \sigma_\eta \bar{\sigma}_\mu)_\alpha{}^\beta = 4g_{\nu\eta} \delta_\alpha{}^\beta = (\sigma_\mu \bar{\sigma}_\nu \sigma_\eta \bar{\sigma}^\mu)_\alpha{}^\beta \tag{A.36}$$

$$(\bar{\sigma}_\nu \sigma^n \bar{\sigma}^\mu \sigma^m \bar{\sigma}^\nu)^{\dot{\alpha}\beta} = -2(\bar{\sigma}^m \sigma^\mu \bar{\sigma}^n)^{\dot{\alpha}\beta} = (\bar{\sigma}^\nu \sigma^n \bar{\sigma}^\mu \sigma^m \bar{\sigma}_\nu)^{\alpha\dot{\beta}} \tag{A.37}$$

$$(\sigma_\nu \bar{\sigma}^n \sigma^\mu \bar{\sigma}^m \sigma^\nu)_{\alpha\dot{\beta}} = -2(\sigma^m \bar{\sigma}^\mu \sigma^n)_{\alpha\dot{\beta}} = (\sigma^\nu \bar{\sigma}^n \sigma^\mu \bar{\sigma}^m \sigma_\nu)_{\alpha\dot{\beta}} \tag{A.38}$$

If F is anything which has no "μ" dependence and "$\bar{F}$" means replace all σ^ν's in F by $\bar{\sigma}^\nu$'s, σ_ν's by $\bar{\sigma}_\nu$'s, $\bar{\sigma}^\nu$'s by σ^ν's, etc. then

$$\begin{aligned} \sigma^\mu F \sigma_\mu &= \bar{\sigma}^\mu \bar{F} \bar{\sigma}_\mu \\ \sigma^\mu F \bar{\sigma}_\mu &= \bar{\sigma}^\mu \bar{F} \sigma_\mu \\ \sigma_\mu F \sigma^\mu &= \bar{\sigma}_\mu \bar{F} \bar{\sigma}^\mu \\ \sigma_\mu F \bar{\sigma}^\mu &= \bar{\sigma}_\mu \bar{F} \sigma^\mu \end{aligned} \tag{A.39}$$

$$\begin{aligned} F_1 \sigma^\mu F_2 = F_3 \sigma^\mu F_4 &\Rightarrow \bar{F}_1 \bar{\sigma}^\mu \bar{F}_2 = \bar{F}_3 \bar{\sigma}^\mu \bar{F}_4 \\ F_1 \sigma^\mu F_2 = F_3 \bar{\sigma}^\mu F_2 &\Rightarrow \bar{F}_1 \bar{\sigma}^\mu F_2 = \bar{F}_3 \sigma^\mu \bar{F}_4 \ . \end{aligned} \tag{A.40}$$

Using (A.39) and (A.40) we can generate many more identities like (A.28)- (A.38).

In addition there are identities in reference 1 such as

$$\sigma^a\bar{\sigma}^b\sigma^c + \sigma^c\bar{\sigma}^b\sigma^a = 2\{g^{ab}\sigma^c + g^{bc}\sigma^a - g^{ac}\sigma^b\} \ . \tag{A.41}$$

Fierz Rearrangement Formula:

$$(\psi\phi)\overline{\chi}_{\dot{\beta}} = -\tfrac{1}{2}\,(\phi\sigma^m\overline{\chi})(\psi\sigma_m)_{\dot{\beta}} \tag{A.42}$$

Proof:

$$-\tfrac{1}{2}\,(\phi\sigma^m\overline{\chi})(\psi\sigma_m)_{\dot{\beta}} = -\tfrac{1}{2}\,(\phi^\alpha(\sigma^m)_{\alpha\dot{\alpha}}\overline{\chi}^{\dot{\alpha}})(\psi^\beta(\sigma^m)_{\beta\dot{\beta}})$$

$$= -\tfrac{1}{2}\,\{\psi^\beta\phi^\alpha\overline{\chi}^{\dot{\alpha}}(\sigma^m)_{\alpha\dot{\alpha}}(\sigma_m)_{\beta\dot{\beta}}\}$$

and from (A.26) and (A.14)

$$-\tfrac{1}{2}\,(\phi\sigma^m\overline{\chi})(\psi\sigma_m)_{\dot{\beta}} = -\tfrac{1}{2}\,\{-2\psi^\beta\phi_\beta\chi_{\dot{\beta}}\} = (\psi\phi)\overline{\chi}_{\dot{\beta}}$$

as required.

APPENDIX B

Two-Component QED

We use the two-component Weyl notation, as presented in Appendix A, to express the QED Lagrangian and Feynman rules.

B.1 Dirac Equation

$$(\gamma^\mu \overrightarrow{p}_\mu - m)\psi(p,s) = 0 = \overline{\psi}(p',s')(\gamma^\mu \overleftarrow{p}'_\mu + m') \quad \text{[4-component form]} \quad \text{(B.1)}$$

Letting $$q_\nu = p'_\nu - p_\nu$$

$$[(\sigma^\mu)_{\alpha\dot\alpha} p_\mu]\overline{\psi}{}^{\dot\alpha}_+(p,s) = m\psi_{-\alpha}(p,s)$$

$$[(\sigma^\mu)_{\alpha\dot\alpha} p'_\mu]\overline{\psi}{}^{\dot\alpha}_+(p,s) = m\psi_{-\alpha}(p,s) + [(\sigma^\mu)_{\alpha\dot\alpha} q_\mu]\overline{\psi}{}^{\dot\alpha}_+(p,s)$$

$$[(\overline{\sigma}^\mu)^{\dot\alpha\alpha} p_\mu]\psi_{-\alpha}(p,s) = m\overline{\psi}{}^{\dot\alpha}_+(p,s)$$

$$[(\overline{\sigma}^\mu)^{\dot\alpha\alpha} p'_\mu]\psi_{-\alpha}(p,s) = m\overline{\psi}{}^{\dot\alpha}_+(p,s) + [(\overline{\sigma}^\mu)^{\dot\alpha\alpha} q_\mu]\psi_{-\alpha}(p,s)$$

(B.2)

$$\overline{\psi}_{-\dot\alpha}(p',s')[(\overline{\sigma}^\nu)^{\dot\alpha\alpha} p'_\nu] = -m'\psi^\alpha_+(p',s')$$

$$\overline{\psi}_{-\dot\alpha}(p',s')[(\overline{\sigma}^\nu)^{\dot\alpha\alpha} p_\nu] = -m'\psi^\alpha_+(p',s') - \overline{\psi}_{-\dot\alpha}(p',s')[(\overline{\sigma}^\nu)^{\dot\alpha\alpha} q_\nu]$$

$$\psi^\alpha_+(p',s')[(\sigma^\nu)_{\alpha\dot\alpha} p'_\nu] = -m'\overline{\psi}_{-\dot\alpha}(p',s')$$

$$\psi^\alpha_+(p',s')[(\sigma^\nu)_{\alpha\dot\alpha} p_\nu] = -m'\overline{\psi}_{-\dot\alpha}(p',s') - \psi^\alpha_+(p',s')[(\sigma^\nu)_{\alpha\dot\alpha} q_\nu]$$

B.2 QED LAGRANGIAN

In four-component form

$$\mathcal{L}_{QED} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - e\overline{\psi}A\!\!\!/\psi - \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - m\overline{\psi}\,\psi\,. \tag{B.3}$$

Consider the two-component version of $\mathcal{L}_{KIN} = i\overline{\psi}\ \partial\!\!\!/\psi$ first.

$$\mathcal{L}_{KIN} = i\Big\{\psi_{+}^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}\partial_{\mu}\overline{\psi}_{+}^{\dot{\alpha}} + \overline{\psi}_{-\dot{\alpha}}(\overline{\sigma}^{\mu})^{\dot{\alpha}\alpha}\partial_{\mu}\psi_{-\alpha}\Big\}\,.$$

Now it is $\int d^4x\mathcal{L}$ which will be of interest in a path integral so we may integrate the second term by parts. We also anticommute the spinors in the first term to obtain (ignoring surface terms)

$$\mathcal{L}_{KIN} = i\Big\{-(\partial_{\mu}\overline{\psi}_{+}^{\dot{\alpha}})(\sigma^{\mu})_{\alpha\dot{\alpha}}\psi_{+}^{\alpha} + (\partial_{\mu}\overline{\psi}_{-\dot{\alpha}})(\overline{\sigma}^{\mu})^{\dot{\alpha}\alpha}\psi_{-\alpha}\Big\}.$$

Now we lower the spinor indices with $\epsilon_{\alpha\beta}$ and raise them on $(\sigma^{\mu})_{\dot{\alpha}\alpha}$ with $\epsilon^{\alpha\beta}$ to obtain

$$\mathcal{L}_{KIN} = -i\Big\{\partial_{\mu}\overline{\psi}_{+\,\dot{\alpha}}(\overline{\sigma}^{\mu})^{\dot{\alpha}\alpha}\psi_{+\alpha} + \partial_{\mu}\overline{\psi}_{-\dot{\alpha}}(\overline{\sigma}^{\mu})^{\dot{\alpha}\alpha}\psi_{-\alpha}\Big\} \tag{B.4}$$

which is the form often found in the literature.[1] Similarly the remainder of the Lagrangian is

$$\begin{aligned}\mathcal{L}_{QED} = \mathcal{L}_{KIN} - e\Big[\psi_{+}^{\alpha}(\sigma^{\mu})_{\alpha\dot{\alpha}}A_{\mu}\overline{\psi}_{+}^{\dot{\alpha}} + \overline{\psi}_{-\dot{\alpha}}(\overline{\sigma}^{\mu})^{\dot{\alpha}\alpha}A_{\mu}\psi_{-\alpha}\Big]\\ -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - m[\psi_{-\alpha}\psi_{+}^{\alpha} + \overline{\psi}_{+}^{\dot{\alpha}}\overline{\psi}_{-\dot{\alpha}}]\,.\end{aligned} \tag{B.5}$$

The field operators of (B.5) should be thought of as annihilating the corresponding fields. Therefore, since $\psi_{-\alpha}$ is a *left-handed electron* and ψ_{+}^{α} is a *left-handed positron*, $-m\psi_{-\alpha}\psi_{+}^{\alpha}$ destroys an $(e^-)_L$ and an $(e^+)_L$, which is the same as destroying an $(e^-)_L$ and creating an $(e^-)_R$, with strength "m".

B.3 CHIRALITY-CHANGING OPERATORS

The operators which change one chiral state to another are

Particles:

$$\underbrace{\frac{(\sigma^\mu)_{\dot\alpha\alpha}p_\mu}{m}}\;\underset{RHe^-}{\overline{\psi}{}^{\dot\alpha}_{+}(p,s)} = \underset{LHe^-}{\psi_{-\alpha}(p,s)}$$

$$\frac{(\overline{\sigma}^\mu)^{\dot\alpha\alpha}p_\mu}{m}\;\underset{LHe^-}{\psi_{-\alpha}(p,s)} = \underset{RHe^-}{\overline{\psi}{}^{\dot\alpha}_{+}(p,s)} \tag{B.6}$$

Antiparticles:

$$\underset{RHe^+}{\overline{\psi}_{-\dot\alpha}(p,s)}\;\frac{(\overline{\sigma}^\nu)^{\dot\alpha\alpha}p_\nu}{m} = \underset{LHe^+}{\psi^{\alpha}_{+}(p,s)}$$

$$\underset{LHe^+}{\psi^{\alpha}_{+}(p,s)}\;\frac{(\sigma^\nu)_{\alpha\dot\alpha}p_\nu}{m} = \underset{RHe^+}{\overline{\psi}{}^{\dot\alpha}_{-}(p,s)}$$

These relations follow directly from the Dirac equation.

B.4 PROPAGATORS

We can decompose the electron Dirac propagator into

$$\langle 0\,|T\psi_\alpha(x)\overline{\psi}_\beta(y)|\,0\rangle = \left\langle 0\middle|T\begin{pmatrix}\psi_{-\alpha}\psi^{\beta}_{+} & \psi_{-\alpha}\overline{\psi}_{-\dot\beta}\\ \overline{\psi}{}^{\dot\alpha}_{+}\psi^{\beta}_{+} & \overline{\psi}{}^{\dot\alpha}_{+}\overline{\psi}_{-\dot\beta}\end{pmatrix}\middle|0\right\rangle \tag{B.7}$$

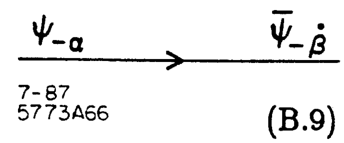
or

$$\frac{i(\not p+m)_{\alpha\beta}}{p^2-m^2} = \Delta(p)\begin{pmatrix} m\delta_\alpha{}^\beta & (\sigma^\mu)_{\alpha\dot\beta}p_\mu\\ (\overline{\sigma}^\mu)^{\dot\alpha\beta}p_\mu & m\delta^{\dot\alpha}{}_{\dot\beta}\end{pmatrix} \tag{B.8}$$

$$\text{with} \qquad \Delta(p) \equiv \frac{i}{p^2-m^2}$$

which we can interpret as

$$e_L^- \to e_L^- \ : \qquad \left\langle 0 \left| T\psi_{-\alpha} \ \overline{\psi}_{-\dot\beta} \right| 0 \right\rangle = \Delta(p)(\sigma^\mu)_{\alpha\dot\beta} p_\mu$$



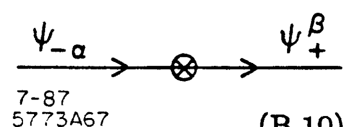
(B.9)

To get $e_L^- \to e_R^-$ we apply the helicity-changing operator $(\overline{\sigma}^\nu)^{\dot\alpha\beta}/m$ to get

$$\Delta(p)(\sigma^\mu)_{\alpha\dot\alpha} p_\mu \ \frac{(\overline{\sigma}^\nu)^{\dot\alpha\beta} p_\nu}{m} \ .$$

But since $(\sigma \cdot p)_{\alpha\dot\alpha}(\overline{\sigma} \cdot p)^{\dot\alpha\beta} = p^2 \delta_\alpha^\beta = m^2 \delta_\alpha^\beta$ (on shell) then

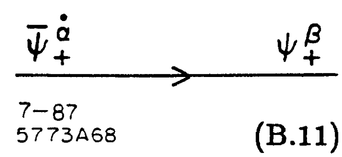
$$e_L^- \to e_R^- \ : \qquad \left\langle 0 \left| T\psi_{-\alpha} \ \psi_+^\beta \right| 0 \right\rangle = m\Delta(p)\delta_\alpha{}^\beta$$



(B.10)

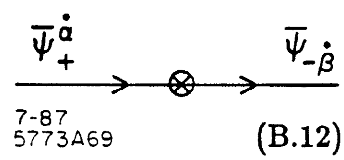
which we could have read off of (B.8) directly ($\otimes$ indicates helicity flip). Similarly

$$e_R^- \to e_R^- \ : \qquad \left\langle 0 \left| T\overline{\psi}{}_+^{\dot\alpha} \psi_+^\beta \right| 0 \right\rangle = \Delta(p)(\overline{\sigma}^\mu)^{\dot\alpha\beta} p_\mu$$



(B.11)

$$e_R^- \to e_L^- \ : \qquad \left\langle 0 \left| T\overline{\psi}{}_+^{\dot\alpha} \overline{\psi}_{-\dot\beta} \right| 0 \right\rangle = m\Delta(p)\delta^{\dot\alpha}{}_{\dot\beta}$$

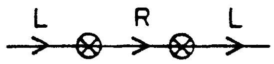


(B.12)

We may apply ϵ's to these which will raise and lower indices as desired using the rules of Appendix A.3. For example, applying $\epsilon_{\alpha\beta}$ to (B.10) results in

$$\left\langle 0 \left| \psi_{-\alpha} \psi_{+\beta} \right| 0 \right\rangle = -m\Delta(p)\epsilon_{\alpha\beta} \ .$$

In general the helicity-preserving operators are proportional to $\sigma^\mu p_\mu$ while the helicity-changing operators are proportional to m. We do not consider multiple flips since



$$= m\Delta(p)\,\frac{(\bar{\sigma}^{\nu})^{\dot{\alpha}\beta}}{m}\,p_{\nu}\cdot\frac{(\sigma^{\mu})_{\beta\dot{\beta}}p_{\mu}}{m} = m\Delta(p)\,\frac{p^2}{m^2}\,\delta^{\dot{\alpha}}{}_{\dot{\beta}} = m\Delta(p)\delta^{\dot{\alpha}}{}_{\dot{\beta}}$$

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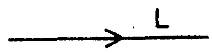
$=$ 

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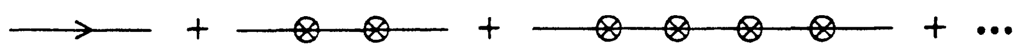
In some sense  can be thought of as the appropriately normalized sum of



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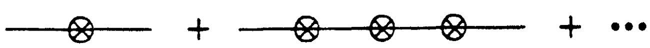
and  as the sum of



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(normalized so that "m" is the physical mass).

Note that when external massive fermions are present in a process (i.e. the diagrams or Green's functions have not been truncated) such fields may change helicity *in the legs of the diagram.*

B.5 Projection Operators

The analogue of the Dirac projection operators for positive and negative energy states

$$\sum_{\pm s} u_\alpha(p,s)\bar{u}_\beta(p,s) = \left(\frac{\not p + m}{2m}\right)_{\alpha\beta}$$

are:

$$\sum_{\pm s} \psi_{-\alpha}(p,s)\overline{\psi}_{-\dot\beta}(p,s) = \frac{(\sigma^\mu)_{\alpha\dot\beta} p_\mu}{2m}$$

$$\sum_{\pm s} \overline{\psi}{}^{\dot\alpha}_{+}(p,s)\psi^{\beta}_{+}(p,s) = \frac{(\overline{\sigma}^\mu)^{\dot\alpha\beta} p_\mu}{2m} \qquad \text{(B.13)}$$

$$\sum_{\pm s} \psi_{-\alpha}(p,s)\psi^{\beta}_{+}(p,s) = \tfrac{1}{2}\,\delta_\alpha{}^\beta$$

$$\sum_{\pm s} \overline{\psi}{}^{\dot\alpha}_{+}(p,s)\overline{\psi}_{-\dot\beta}(p,s) = \tfrac{1}{2}\,\delta^{\dot\alpha}{}_{\dot\beta}$$

In the case where spin is not summed over we project out both helicity and energy states. In Dirac notation we have

$$u_\alpha(p,s)\overline{u}_\beta(p,s) = \left[\frac{\not p + m}{2m}\,\frac{1+\gamma_5 \not s}{2}\right]_{\alpha\beta}$$

where $s^\mu = (0,\widehat{s})$ in the rest frame. Let us consider the instance of a particle moving along the $+\widehat{z}(3)$-axis very slowly ($\overrightarrow{p} \to 0$) with spin in the $+\widehat{z}$ direction for right-handed particles. Then

$$s^\mu_L = (0,0,0,1) \qquad s^\mu_R = (0,0,0,-1)\ .$$

For $s_L^\mu = (0,0,0,1)$ we find

$$\begin{aligned}
\psi_{-\alpha}(p,s)\overline{\psi}_{-\dot\beta}(p,s) &= \frac{1}{4m}\left[(\sigma^\mu)_{\alpha\dot\beta}p_\mu + m(\sigma^3)_{\alpha\dot\beta}\right] \\
\psi^{+\dot\alpha}(p,s)\psi_+^\beta(p,s) &= \frac{1}{2m}\left[(\overline{\sigma}^\mu)^{\dot\alpha\beta}p_\mu + m(\overline{\sigma}^3)^{\dot\alpha\beta}\right] \\
\psi_{-\alpha}(p,s)\psi_+^\beta(p,s) &= \frac{1}{4m}\left[m\delta_\alpha{}^\beta - (\sigma^\mu)_{\alpha\dot\alpha}p_\mu(\overline{\sigma}^3)^{\dot\alpha\beta}\right] \\
\overline{\psi}_+^{\dot\alpha}(p,s)\overline{\psi}_{-\dot\beta} &= \frac{1}{4m}\left[m\delta^{\dot\alpha}{}_{\dot\beta} + (\overline{\sigma}^\mu)^{\dot\alpha\alpha}p_\mu(\sigma^3)_{\alpha\dot\beta}\right]
\end{aligned} \qquad \text{(B.14)}$$

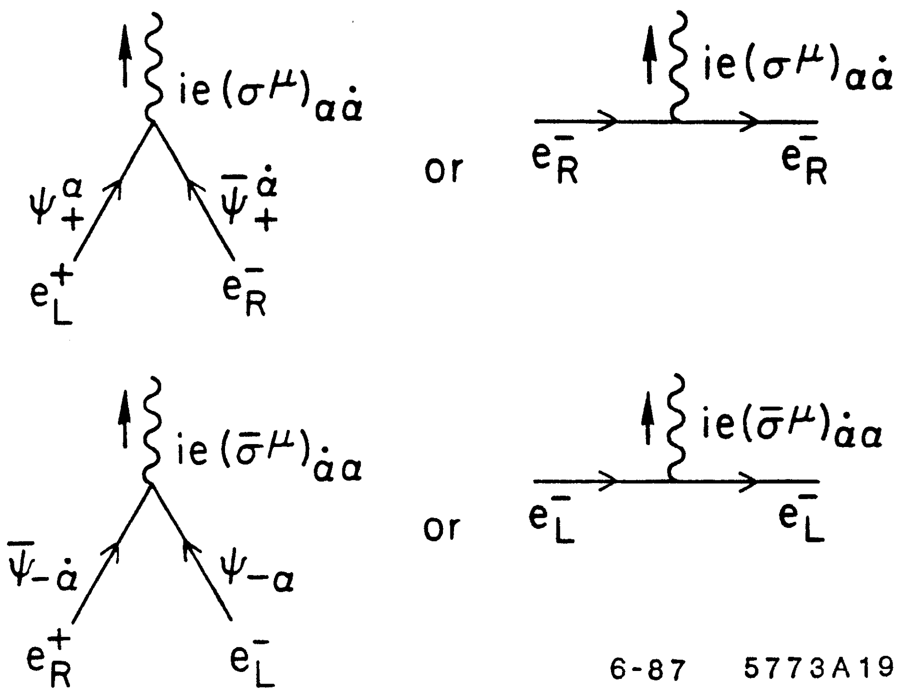
remembering that $\overline{\sigma}^3 = -\sigma^3$.

For photons

$$\sum_\lambda \epsilon_\mu(k,\lambda)\ \epsilon_\nu(k,\lambda) = -g_{\mu\nu}\ . \qquad \text{(B.15)}$$

B.6 SIMPLE VERTICES

Photon emission does not flip helicity (since the current is transverse). Thus the simple (truncated) vertices appear as


ie (σ^μ)_{αα̇}
ψ^α_+
ψ̄^α̇_+
e^+_L
e^-_R
or
ie (σ^μ)_{αα̇}
e^-_R
e^-_R
ie (σ̄^μ)_{α̇α}
ψ̄_{-α̇}
ψ_{-α}
e^+_R
e^-_L
or
ie (σ̄^μ)_{α̇α}
e^-_L
e^-_L
6-87 5773A19

APPENDIX C

Calculation of Anomalous Magnetic Moment I: Four-Component Spinors.

The standard calculation of the anomalous magnetic moment of the electron in four-component spinor notation is presented here in detail. The rationale being that this is the prototype for all processes considered in this paper. It provides a direct comparison with the two-component version which follows. We follow the approach of Ref. 2 but use the metric and spinor representations of Appendix A.

We examine the vertex correction

$$-ie\gamma^{\mu}_{\alpha\beta} \rightarrow -ie\Gamma^{\mu}_{\alpha\beta}(p,p') = -ie[\gamma^{\mu} + \Lambda^{\mu}(p,p')]_{\alpha\beta} \tag{C.1}$$

as illustrated in figure 1 and whose lowest order contribution is given by the diagram in figure 2. Since we will only be interested in the Λ^{μ} piece we will not distinguish between Γ^{μ} and Λ^{μ} but simply refer to the first order correction as $\Gamma^{\mu}(p,p')$. To guard against, *a priori*, infrared divergence problems we will let the mass of the photon be μ, which we shall let go to zero at the end. We see that

$$\begin{aligned} -ie\Gamma^{\mu}_{\alpha\beta}(p,p') = (-ie)^3 \int \frac{d^4k}{(2\pi)^4} \Bigg\{ \left(\frac{-g_{\rho\sigma} + \text{gauge terms}}{k^2 - \mu^2 + i\epsilon} \right) (\gamma^{\sigma})_{\alpha\tau} \\ \times \frac{i}{\not{p}' - \not{k} - m + i\epsilon} (\gamma^{\mu})^{\tau\eta} \frac{i}{\not{p} - \not{k} - m + i\epsilon} (\gamma^{\rho})_{\eta\beta} \Bigg\}. \end{aligned} \tag{C.2}$$

We will do away with the gauge terms in the photon by recognizing that such terms vanish in Feynman gauge when $\mu \to 0$. Hence

$$\Gamma^{\mu}(p,p')_{\alpha\beta} = (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \left\{ \left(\frac{-1}{k^2 - \mu^2 + i\epsilon} \right) \right.$$
$$\times \left. \left[\gamma^{\nu} \frac{i(\not{p}' - \not{k} + m)}{(\not{p}' - k)^2 - m^2 + i\epsilon} \gamma^{\mu} \frac{i(\not{p} - \not{k} + m)}{(p-k)^2 - m^2 + i\epsilon} \gamma_{\mu} \right]_{\alpha\beta} \right\} \tag{C.3}$$

We will evaluate this on-shell where $\not{p} = \not{p}' = m$ and then let $q^2 \to 0$, *i.e.* $p' \to p$. We ignore the other graphs which occur at this order as they only contribute to the self-energy renormalizations. In order to regulate the integral we employ the traditional Pauli-Villars[50] subtraction scheme. This introduces a "massive photon" of mass $\Lambda \to \infty$ which utilizes an incorrect metric. The "incorrect metric" in this case is the negative of the one given in section A.2. The result is that the heavy photon propagator always appears with sign opposite to that of the light ($\mu \to 0$) propagator. Thus the leading divergences cancel. Although such a propagator is not physically meaningful, as we take $\Lambda \to \infty$ such contributions vanish. Such a scheme is not overtly gauge invariant during the intermediate steps of the calculations but, of course, gauge invariance must be restored once the limit is taken. Consequently we may represent the combined photon propagators by

$$\frac{1}{k^2 - \mu^2} - \frac{1}{k^2 - \Lambda^2} = - \int_{\mu^2}^{\Lambda^2} \frac{dt}{(k^2 - t)^2} \tag{C.4}$$

resulting in

$$\Gamma^\mu(p,p') = -ie^2 \int \frac{d^4k}{(2\pi)^4} \int_{\mu^2}^{\Lambda^2} -dt \; \frac{N^\mu}{D}$$

$$N^\mu = \gamma^\nu[i(\not p' - \not k + m)]\gamma^\mu[i(\not p - \not k + m)]\gamma_\nu \tag{C.5}$$

$$D = (k^2 - t^2)^2([p' - k]^2 - m^2)([p - k]^2 - m^2] \; .$$

Now using the standard gamma matrix contraction identities of the form $\gamma^\nu[\cdots]\gamma_\nu$ which can be found in the appendices of Bjorken and Drell[34] we find that

$$N^\mu = i^2\{ -2(\not p - \not k)\gamma_\mu(\not p' - \not k) - 2m^2\gamma^\mu + 2m[(\not p' - \not k)\gamma^\mu + \gamma^\mu(\not p' + \not k)$$

$$+ (\not p - \not k)\gamma^\mu + \gamma^\mu(\not p - \not k)]\}$$

$$= 2\{(\not p - \not k)\gamma^\mu(\not p' - \not k) + m^2\gamma^\mu - m[\gamma^\mu(\not p' + \not p - 2\not k) + (\not p' + \not p - 2\not k)\gamma^\mu]\} \tag{C.6}$$

where we have used $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and therefore $2p^\mu = \not p\gamma^\mu + \gamma^\mu \not p$. Since $p^2 = m^2$ on-shell we have $(p+k)^2 - m^2 = k^2 - 2p\cdot k$ and so

$$D = (k^2 - t^2)^2(k^2 - 2p'\cdot k)(k^2 - 2p\cdot k) \; . \tag{C.7}$$

Using Feynman's trick

$$\frac{1}{a^2bc} = 6\int_0^1 dx \int_0^{1-x} dy \; \frac{1-x-y}{[a+(b-a)x+(c-a)y]^4} \tag{C.8}$$

from (H.5) to give

$$\frac{1}{D} = 6\int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \; \frac{(1-x-y)}{[k^2 - t - (2p\cdot k - t)x - (2p'\cdot k - t)y]^4} \; . \tag{C.9}$$

Now use (H.7) to evaluate this realizing that

$$p \rightarrow px + p'y, \qquad s = -t + tx + ty = -t(1 - x - y)$$

to yield

$$\int \frac{d^4k}{(2\pi)^4} \frac{N^\mu}{D} = 6 \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \; \frac{iM^\mu}{96\pi^2[-t(1-x-y) - px + p'y]^2} \tag{C.10}$$

with

$$\begin{aligned} M^\mu = {} & 2\, \not{p}\gamma^\mu \not{p}' - 2[\not{p}\gamma^\mu(\not{p}x + \not{p}'y) + (\not{p}x + \not{p}'y)\gamma^\mu \not{p}'] \\ & - 2m^2\gamma^\mu + [-t(1-x-y) - (\not{p}x + \not{p}'y)^2]\gamma^\nu\gamma^\mu\gamma_\nu \\ & - m[\gamma^\mu(\not{p}' + \not{p} - 2\,\not{p}x - 2\,\not{p}'y) + (\not{p}' + \not{p} - 2\,\not{p}x - 2\,\not{p}'y)\gamma^\mu] \\ & + 2(\not{p}x + \not{p}'y)\gamma^\mu(\not{p}x + \not{p}'y) \end{aligned} \tag{C.11}$$

and using $\gamma^\nu\gamma^\mu\gamma_\nu = -2\gamma^\mu$ we may factor out a "2". Since we are really examining $\bar{u}(p')\Gamma^\mu(p,p')u(p)$ we may use the Dirac equation

$$\bar{u}(p')(\overleftarrow{\not{p}}' - m) = (\overrightarrow{\not{p}} - m)u(p) = 0 \quad \text{with} \quad p' = p + q \tag{C.12}$$

on M^μ. Further, from (H.17 b) $(px + p'y)^2 = m^2(x+y)^2 - q^2xy$ so that

$$
\begin{aligned}
M^\mu = 2\Big\{ & (m-\not{q})\gamma^\mu(m+\not{q}) + ([m-\not{q}]x + my)\gamma^\mu(mx + [m+\not{q}]y) \\
& - \big([(m-\not{q})x + my]\gamma^\mu(m+\not{q}) + (m-\not{q})\gamma^\mu[mx + (m+\not{q})y]\big) \\
& - m\big[\gamma^\mu([m+\not{q}] + m - 2mx - 2[m+\not{q}]y) \\
& + (m + [m - q] - 2[m-\not{q}]x - 2my)\gamma^\mu\big] \\
& + [t(1-x-y) + (m^2[x+y]^2 - q^2xy)]\gamma^\mu - m^2\gamma^\mu\Big\} \\
= 2\Big\{ & \gamma^\mu[t(1-x-y) + m^2(x+y)^2 - q^2xy + m^2 - 2m^2(x+y) \\
& - 4m^2(1-x-y) + m^2(x+y)^2 - m^2] \\
& + \gamma^\mu\not{q}[m - m(x+y) - my - m + 2my + m(x+y)y] \\
& + [-m + mx + m(x+y) + m - 2mx - m(x+y)x]\not{q} \\
& + \not{q}\gamma^\mu\not{q}[-1 - x - y - xy]\Big\} .
\end{aligned}
\tag{C.13}
$$

We have exercised our gauge freedom when we chose to calculate in the Feynman gauge. This is one of the R_ξ (or Lorentz) gauges which are *covariant*, *i.e.* satisfy $q^\nu A_\nu(q^2) = 0$ (in this case $\xi = 1$). In a physical process we have terms like $\Gamma^\mu A_\mu$ and so all terms proportional to q^μ in M^μ will vanish. We explicitly impose this gauge condition: $q^\mu A_\mu(q) = 0$. Now

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

thus

$$\{\gamma^\mu, \not{q}\} = \{\gamma^\mu, \gamma^\nu q_\nu\} = 2g^{\mu\nu}q_\nu = 2q^\mu$$

which vanishes in M^μ. Therefore

$$\gamma^\mu \not{q} = - \not{q}\gamma^\mu \tag{C.14}$$

and the anti-commutation term does not contribute. Also

$$\not{q}\gamma^\mu \not{q} = -\gamma^\mu \not{q} \not{q} = -\gamma^\mu q^2.$$

Letting

$$\mathbf{A} = 2\Big\{t(1-x-y) + 2m^2([x+y]^2 - 2[1-x-y] - [x+y])$$
$$- q^2xy - q^2(1-x)(1-y)\Big\}$$

(C.13) becomes

$$M^\mu = \gamma^\mu\{\mathbf{A}\}$$
$$+ \Big\{m \not{q}\gamma^\mu[(x+y) - x(x+y) - x] - m\gamma^\mu \not{q}[(x+y) - y(x+y) - y]\Big\}$$
$$= \gamma^\mu\{\mathbf{A}\} + 2m\{\not{q}\gamma^\mu[-x^2 - xy + y] - \gamma^\mu \not{q}[-y^2 - xy + x]\}$$
$$= \gamma^\mu\{\mathbf{A}\}$$
$$+ 2m\Bigg\{ \not{q}\gamma^\mu\left[-\frac{x^2}{2} - xy + \frac{y^2}{2} + \frac{x}{2} - \frac{x^2}{2} + \left(-\frac{x^2}{2} - \frac{x}{2} + \frac{y^2}{2} + \frac{y}{2}\right)\right]\Bigg\}$$
$$- \gamma^\mu \not{q}\left[-\frac{y^2}{2} - xy + \frac{x}{2} + \frac{y}{2} - \frac{x^2}{2} + \left(-\frac{y^2}{2} - \frac{y}{2} + \frac{x^2}{2} + \frac{y}{2}\right)\right]\Bigg\}$$

but

$$\not{q}\gamma^\mu\left(-\frac{x^2}{2} - \frac{x}{2} + \frac{y^2}{2} + \frac{y}{2}\right) - \gamma^\mu \not{q}\left(-\frac{y^2}{2} - \frac{y}{2} + \frac{x^2}{2} + \frac{x}{2}\right) = 0$$

by (C.14) thus

$$M^\mu = \gamma^\mu\{\mathbf{A}\} + \frac{\not q \gamma^\mu - \gamma^\mu \not q}{2} \, [2m(x+y-x^2-y^2-2xy)]$$

$$= \gamma^\mu\{\mathbf{A}\} + \frac{[\gamma^\nu, \gamma^\mu] q_\nu}{2} \, [2m(x+y)(1-x-y)]$$

with

$$\Gamma^\mu(p,p') = 6 \int_0^1 dx \int_0^{1-x} dy \, \frac{-1}{96\pi^2} \, (ie^2) \int_{\mu^2}^{\Lambda^2} (-dt) \, \frac{i(1-x-y)M^\mu}{[t(1-x-y)+(px+py)^2]^2}.$$

Now

$$\sigma^{\mu\nu} = \frac{i}{2} \, [\gamma^\mu, \gamma^\nu]$$

and

$$\Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + \frac{i}{2m} \, \sigma^{\mu\nu} q_\nu F_2(q^2) \tag{C.15}$$

with the anomalous magnetic moment

$$a_e \equiv F_2(q^2 \to 0) \qquad a \equiv \frac{g-2}{2}. \tag{C.16}$$

Therefore

$$iF_2(q^2 \to 0) = \frac{2}{i} \cdot \frac{-6(i)^2 e^2}{-96\pi^2} \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \, \frac{(1-x-y)2m^2(x+y)(1-x-y)}{[t(1-x-y)+m^2(x-y)^2]^2}$$

$$ia_e = -i \, \frac{\alpha m^2}{\pi} \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \, \frac{(x+y)(1-x-y)^2}{[t(1-x-y)+m^2(x-y)^2]^2}$$

where $\alpha \equiv e^2/4\pi$. Performing the t-integration

$$ia_e = -i\,\frac{\alpha m^2}{\pi}\int_0^1 dx \int_0^{1-x} dy\,\frac{(x+y)(1-x-y)^2}{(1-x-y)}$$

$$\times\left\{\frac{1}{\Lambda^2(1-x-y)+m^2(x-y)^2} - \frac{1}{\mu^2(1-x-y)+m^2(x-y)^2}\right\}$$

and taking $\Lambda \to \infty$

$$ia_e = i\,\frac{\alpha}{\pi}\int_0^1 dx \int_0^{1-x} dy\,\frac{(x+y)(1-x-y)}{(\mu^2/m^2)(1-x-y)+(x-y)^2}.$$

Let $z = x+y$

$$a_e = \frac{\alpha}{\pi}\int_0^1 dx \int_x^1 dz\,\frac{z(1-z)}{(\mu^2/m^2)(1-z)+z^2}$$

and we see that this remains finite where μ^2 = mass of photon $\to 0$. Doing this

$$a_e = \frac{\alpha}{\pi}\int_0^1 dx \int_x^1 dz \frac{1-z}{z}$$

$$= \frac{\alpha}{\pi}\int_0^1 dx[\ell n\, z - z]\Big|_x^1$$

$$= \frac{\alpha}{\pi}\int_0^1 dx[-\ell n\, x - 1 + x]$$

$$= \frac{\alpha}{\pi}\,[-x\ell n\, x_x - x + \frac{x^2}{2}]\Big|_0^1$$

$$= \frac{\alpha}{2\pi}$$

Thus we have found

$$a_e = \frac{\alpha}{2\pi} \tag{C.17}$$

which is the Schwinger result.

FIGURE CAPTIONS

1. Vertex correction

$$-ie\gamma^{\mu}_{\alpha\beta} \to -ie\Gamma^{\mu}_{\alpha\beta}(p,p') = -ie[\gamma^{\mu} + \Lambda^{\mu}(p,p')]_{\alpha\beta}$$

2. Lowest order contribution to the vertex correction

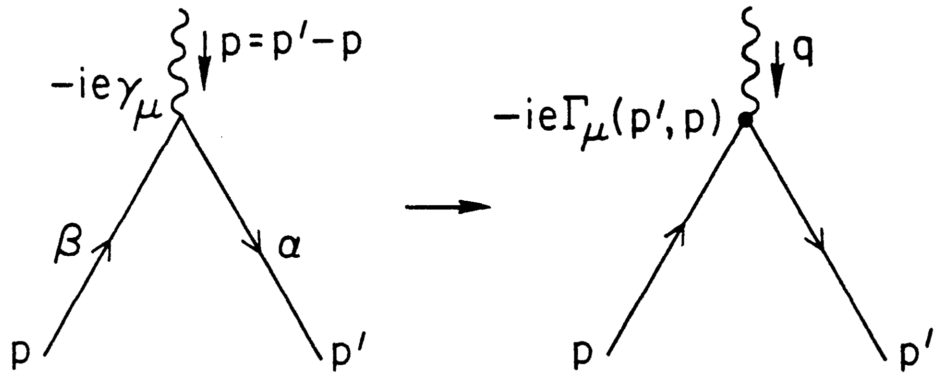


Fig. C.1

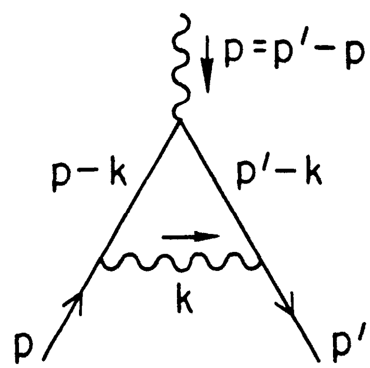


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Fig. C.2

APPENDIX D

Calculation of Anomalous Magnetic Moment II: Two-Component Spinors.

We repeat the calculation performed in Appendix C but employ two-component Weyl spinors in order to illustrate their use. We will specifically examine the vertex correction to

$$e^-_{L(R)}(p,s) + \gamma(q = p' - p) \rightarrow e^-_{R(L)}(p',s') \ . \tag{D.1}$$

Certain comments should be made. The processes $e^-_L + \gamma \rightarrow e^-_R$ and $e^-_R + \gamma \rightarrow e^-_L$ are expected to provide equivalently to the anomalous moment a. The processes $e_L \rightarrow e_L$ and $e_R \rightarrow e_R$ contribute solely to the vertex (wavefunction) renormalization. The process $\gamma \rightarrow e^-_{L(R)} e^+_{L(R)}$ is equivalent to $\gamma + e^-_{L(R)} \rightarrow e^-_{R(L)}$. Consider $e^-_L(p,s) + \gamma(q) \rightarrow e^-_R(p',s')$ as illustrated in Fig. D.1.

We would naively consider these to be the sole contributions. To see that this does not lead to the correct result we proceed to evaluate them. Remembering from Eqn. (C.1) that the vertex is given by $\mathcal{M}^\mu = -ie\psi_+\Gamma^\mu\psi_-$ we find:

Diagram I:

$$\begin{aligned}
&\psi^{\gamma}_+(p',s')\Gamma^\mu_{\mathrm{I}}(p,p')_\gamma{}^\alpha\psi_{-\alpha}(p,s) \\
&\qquad = ie^2 \int \frac{d^4k}{(2\pi)^4}\, m\Delta(p-k)\Delta(p'-k)\, \frac{1}{k^2} \\
&\qquad \times \psi^\alpha_+(\sigma^\rho)_{\gamma\dot\gamma}(\overline{\sigma}^\lambda)^{\dot\gamma\beta}(p'-k)_\lambda(\overline{\sigma}^\mu)_{\beta\dot\beta}\delta^{\dot\beta}{}_{\dot\alpha}(\overline{\sigma}^\nu)^{\dot\alpha\alpha}\psi_{-\alpha}g_{\rho\nu} \\
&\qquad = ie^2 \int \frac{d^4k}{(2\pi)^4}\, \frac{N^\mu_{\mathrm{I}}}{D}
\end{aligned} \tag{D.2}$$

where we have introduced the notation

$$\Delta(p) \equiv i/p^2 - m^2 + i\epsilon$$

we find that

$$\begin{aligned} N_{\mathrm{I}}^{\mu} &= m\psi_{+}^{\gamma}(\sigma_\nu)_{\gamma\dot\gamma}(\overline{\sigma}^{\lambda})^{\dot\gamma\beta}(\sigma^\mu)_{\beta\dot\alpha}(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}\psi_{-\alpha}(p'-k)_\lambda \\ &= m\psi_{+}^{\gamma}[4g^{\mu\lambda}(p'-k)_\lambda\delta_\gamma{}^{\alpha}]\psi_{-\alpha} \qquad \text{from (A.32)} \\ &= 4m(p'-k)^\mu\psi_{+}^{\alpha}\psi_{-\alpha} \end{aligned} \tag{D.3}$$

Diagram II:

$$\begin{aligned} &\psi_{+}^{\gamma}(p',s')\Gamma_{\mathrm{II}}^{\mu}(p,p')_\gamma{}^{\alpha}\psi_{-\alpha}(p,s) \\ &\qquad = ie^2\int\frac{d^4k}{(2\pi)^4}\, m\Delta(p-k)\Delta(p'-k)\,\frac{1}{k^2} \\ &\qquad \times \psi_{+}^{\gamma}(\overline{\sigma}^{\rho})_{\gamma\dot\gamma}\delta^{\dot\gamma}{}_{\dot\beta}(\overline{\sigma}^{\mu})^{\dot\beta\beta}(\sigma^\lambda)_{\beta\dot\alpha}(p-k)_\lambda(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}\psi_{-\alpha}g_{\rho\nu} \\ &\qquad = ie^2\int\frac{d^4k}{(2\pi)^4}\,\frac{N_{\mathrm{II}}^{\mu}}{D} \end{aligned} \tag{D.4}$$

$$\begin{aligned} N_{\mathrm{II}}^{\mu} &= m\psi_{+}^{\gamma}(\overline{\sigma}_{\nu})_{\gamma\dot\beta}(\overline{\sigma}^{\mu})^{\dot\beta\beta}(\sigma^\lambda)_{\beta\dot\alpha}(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}\psi_{-\alpha}(p-k)_\lambda \\ &= m\psi_{+}^{\gamma}[4g^{\mu\lambda}(p-k)_\lambda\delta_\gamma{}^{\alpha}]\psi_{-\alpha} \\ &= 4m(p-k)^\mu\psi_{+}^{\alpha}\psi_{-\alpha} \end{aligned} \tag{D.5}$$

and, as in Eqn. (C.7),

$$D = k^2(k^2 - 2p\cdot k)(k^2 - 2p'\cdot k)\ . \tag{D.6}$$

The total is given by

$$\psi_+^{\gamma}\Gamma^{\mu}(p,p')_{\gamma}{}^{\alpha}\psi_{-\alpha} = ie^2 \int \frac{d^4k}{(2\pi)^4}\,\frac{N^{\mu}}{D} \qquad \text{with} \tag{D.7}$$

$$N^{\mu} = N^{\mu}_{\rm I} + N^{\mu}_{\rm II} = 4m\psi_+^{\alpha}(p',s')[(p'-k)^{\mu} + (p-k)^{\mu}]\psi_{-\alpha}(p,s).$$

As in Appendix C (C.4) we could use a Pauli-Villars regulator ($\Lambda \to \infty$) and let the photon acquire a small mass ($\mu \to 0$) so that

$$\frac{1}{k^2} \to -\int_{\mu^2}^{\Lambda^2} dt\, \frac{dt}{(k^2-t^2)^2} \tag{D.8}$$

and

$$D \to (k^2-t^2)^2(k^2-2p'\cdot k)(k^2-2p\cdot k)$$

$$\int \frac{d^4k}{(2\pi)^4} \to -\int \frac{d^4k}{(2\pi)^4}\int_{\mu^2}^{\Lambda^2} dt \quad . \tag{D.9}$$

Then using (C.8)

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1\ ;\ k_{\mu}\ ;\ k_{\mu}k^{\nu}}{(k^2-2k\cdot p+s-i\epsilon)^4} = \frac{i}{96\pi^2(s-p^2)^2}\,[1\ ;\ p_{\mu}\ ;\ p_{\mu}p^{\nu} + \tfrac{1}{2}\,(s-p^2)\delta_{\mu}{}^{\nu}] \tag{D.10}$$

from (H.7) with $s = -t(1-x-y)$ and $p = px + p'y$. We have seen in Appendix C that this was unnecessary since, as we had argued from general principles, the helicity-changing part is finite. Thus we use instead

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} dy\, \frac{1}{[a+(b-a)x+(c-a)y]^3} \tag{D.11}$$

and

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1\ ;\ k_{\mu}}{(k^2-2k\cdot p+s-i\epsilon)^3} = \frac{i[1\ ;\ p_{\mu}]}{32\pi^2(s-p^2)} \tag{D.12}$$

from (H.6). Using (D.11) and (D.6) yields

$$\frac{1}{D} = 2\int_0^1 dx \int_0^{1-x} dy \; \frac{1}{[k^2 - 2p\cdot kx - 2p'\cdot ky]^3} \; . \tag{D.13}$$

Note, however, that we can equally well interchange "b" and "c" in (D.11) (as outlined in (H.20)) in which case (D.13) would have become

$$\frac{1}{D} = 2\int_0^1 dx \int_0^{1-x} dy \; \frac{1}{[k^2 - 2p'\cdot kx - 2p\cdot ky]^3} \; . \tag{D.14}$$

We see that the net result is to interchange x and y in the integrand. Using the notation of (D.12) we see, in the case of (D.13), that $p \to px + p'y$ while for (D.14) $p \to py + p'x$. In both cases $s = 0$.

Using (D.12) and (D.7) [with (D.13) being used for $1/D$] yields

$$\bar{\psi}_+\Gamma^\mu(p,p')\psi_- = 2ie^2 \int_0^1 dx \int_0^{1-x} dy \; \frac{-i}{32\pi^2} \; \frac{M^\mu}{(px+p'y)^2} \tag{D.15}$$

and, again, from (H.17 b) $(px+p'y)^2 = m^2(x+y)^2 - q^2xy$ where $q = p' - p$.

$$M^\mu = 4m\bar{\psi}_+(p')\;[p^\mu(1-2x) + p'^\mu(1-2y)]\psi_-(p) \tag{D.16}$$

thus

$$\bar{\psi}_+\Gamma^\mu(p,p')\psi_- = \frac{e^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{4m\bar{\psi}_+[p^\mu(1-2x) + p'^\mu(1-2y)]\psi_-}{m^2(x+y)^2 - q^2xy} \; . \tag{D.17}$$

Now if we had used (D.14) instead of (D.13) (i.e. let "$b \leftrightarrow c$") then we must interchange x and y *in the integrand* to get

$$\psi_+\Gamma^\mu(p,p')\psi_- \to \frac{e^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{4m\psi_+[p^\mu(1-2y)+p'^\mu(1-2x)]\psi_-}{m^2(x+y)^2-q^2xy} \; . \tag{D.18}$$

Note that the denominator is the same. We may now average these two expressions to obtain

$$\psi_+\Gamma^\mu(p,p')\psi_- = \frac{me^2}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{\psi_+(p+p')^\mu(1-x-y)\psi_-}{m^2(x+y)^2-q^2xy} \; . \tag{D.19}$$

This procedure of switching "b" and "c" in expressions like (D.11) and averaging the results after integrating out the $\int d^4k$ we shall refer to (see (H.20)) as the "$b \leftrightarrow c$ trick". It is quite general and can be also used in the regulated version presented in Appendix C. The result is generally a simple function of $x+y$ and $x-y$ (subtleties arise off-shell).

The expression in (D.19) represents one half of the anomaly (the $e_L \to e_R$ part). We may similarly compute the other ($e_R \to e_L$) part. We find that it is the same as (D.19) except that

$$\psi_+^\alpha(p')\psi_{-\alpha}(p) \to \overline{\psi}_{-\dot\alpha}(p')\overline{\psi}_+^{\dot\alpha}(p) \; . \tag{D.20}$$

This sum can be represented by

$$\frac{me^2}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \; \frac{M^\mu_{sum}}{m^2(x+y)^2-q^2xy} \tag{D.21}$$

where

$$M^\mu_{sum} = (1-x-y)[\psi_+(p')\psi_-(p) + \overline{\psi}_-(p')\overline{\psi}_+(p)](p+p')^\mu \; . \tag{D.22}$$

Now we may employ the generalized Gordon identity of Appendix H (Eqn. H.43) which states

$$[\psi_+(p')\psi_-(p) + \overline{\psi}_-(p')\overline{\psi}_+(p)](p+p')^\mu =$$

$$- i[\psi_+(p')\sigma^{\mu\nu}q_\nu\psi_-(p) + \overline{\psi}_-(p')\overline{\sigma}^{\mu\nu}q_\nu\psi_+(p)] \ . \tag{D.23}$$

$$+ 2m[\overline{\psi}_-(p')\overline{\sigma}^{\mu\nu}\overline{\psi}_+(p) + \psi_+(p')\sigma^{\mu\nu}\psi_-(p)]$$

Using this in (D.22) we may ignore the final term as it contributes solely to the vertex renormalization. Thus (D.22) may be written as

$$\Gamma^\mu(p,p') = \frac{-ime^2}{4\pi^2}\,[\psi_+(p')\sigma^{\mu\nu}q_\nu\psi_-^\mu(p) + \overline{\psi}_-(p')\overline{\sigma}^{\mu\nu}q_\nu\psi_-(p)]\, I \tag{D.24}$$

where

$$I(q^2) = \int_0^1 dx \int_0^{1-x} dy \, \frac{(1-x-y)}{m^2(x+y)^2 - q^2xy} \ .$$

From (A.8) we see that this can be written in four-component Dirac form as

$$\overline{\psi}(p)\Gamma^\mu(p,p')\psi(p) = \frac{-im\alpha I(q^2)}{\pi}\,\overline{\psi}(p')\sigma^{\mu\nu}q_\nu\psi(p) \ , \tag{D.25}$$

where we have used $\alpha = e^2/4\pi$, or simply

$$\Gamma^\mu(p,p') = \frac{-im\alpha I(q^2)}{\pi}\,\sigma^{\mu\nu}q_\nu \ . \tag{D.26}$$

From (C.15) this implies

$$F_2(q^2) = -\frac{2m^2\alpha I(q^2)}{\pi} \tag{D.27}$$

and from (C.16)

$$a_e = -\frac{2m^2\alpha I(0)}{\pi}. \tag{D.28}$$

From (D.24)

$$I(0) = \int_0^1 dx \int_0^{1-x} dy \, \frac{(1-x-y)}{m^2(x+y)^2}$$

using (H.10 c)

$$\int_0^1 dx \int_0^{1-x} dy = \tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \tag{D.29}$$

therefore

$$I(0) = \frac{1}{m^2} \int_0^1 dz \int_0^z d\tilde{z} \, \frac{1-z}{z^2} = \frac{1}{m^2} \int_0^1 dz \, \frac{1-z}{z} \, . \tag{D.30}$$

Hence

$$a_e = -\frac{2\alpha}{\pi} \int_0^1 dz \left(1 - \frac{1}{z}\right) \tag{D.31}$$

which is divergent!

The reason that the naive contributions do not reproduce the Schwinger result is that the F_1 and F_2 parts of Γ^μ are intimately tied together for massive spinors. This is manifestly evident in the Gordon decomposition (Appendix H). In (D.23) we say that we generated a " γ^μ " type term which is indicative of a helicity-preserving vertex. This is in spite of the fact that our diagrammatic representations in Fig. D.1 contained *explicit* helicity flips (indicated by $\otimes$). Similarly " $\sigma^{\mu\nu}$ " type terms may be generated from the diagrams connecting e_L to e_L and e_R to e_R. Carefully following the derivation in Appendix C we see that this is indeed the case. There we start and end with $\overline{\psi}$ and ψ which contain both left and right-handed parts. To take this over to the two-component case we would have to add the four amplitudes for $e_{L \text{ or } R} + \gamma \rightarrow e_{L \text{ or } R}$ together.

This is somehow distasteful. We would like to be able to start with a specified state, say e_L, and end with e_R without references to additional processes. One can obscure this disagreeable situation by permitting the external legs to be chirality-changing. This is not an option for truncated diagrams but we really are dealing with $\overline{\psi}\Gamma^\mu\psi$, not Γ^μ, since we use the Dirac equation and Gordon identities.

Adopting this approach we have six additional diagrams which should be added to diagrams I and II in Fig. D.1. These are found in Fig. D.2.

Following the same procedure as before but regulating the integral as in Appendix C (see (C.4)) we find

$$\mathcal{M}^\mu = ie\psi_+\Gamma^\mu(p,p')\psi_- = -ie^3 \int \frac{d^4k}{(2\pi)^4} \int_{\mu^2}^{\Lambda^2} dt\ \frac{N^\mu}{D} \tag{D.32}$$

with "D" as in (D.6) and

$$\begin{aligned} N^\mu = \psi_+(p)\Bigg\{ & \frac{\sigma\cdot p'}{m}\,\overline{\sigma}^\nu m\sigma^\mu\overline{\sigma}\cdot(p-k)\sigma_\nu\,\frac{\overline{\sigma}\cdot p}{m} \\ & + \frac{\sigma\cdot p'}{m}\,\overline{\sigma}^\nu\sigma\cdot(p'-k)m\overline{\sigma}^\mu\sigma_\nu\,\frac{\overline{\sigma}\cdot p}{m} + \frac{\sigma\cdot p'}{m}\,\overline{\sigma}^\nu m^2\sigma^\mu\overline{\sigma}_\nu \\ & + \frac{\sigma\cdot p'}{m}\,\overline{\sigma}^\nu\sigma\cdot(p'-k)\overline{\sigma}^\mu\sigma\cdot(p-k)\overline{\sigma}_\nu + \sigma^\nu\overline{\sigma}\cdot(p'-k)m\sigma^\mu\overline{\sigma}_\nu \\ & + \sigma^\nu m\overline{\sigma}^\mu\sigma\cdot(p-k)\overline{\sigma}_\nu \\ & + \sigma^\nu\overline{\sigma}\cdot(p'-k)\sigma^\mu\overline{\sigma}\cdot(p-k)\sigma_\nu\,\frac{\overline{\sigma}\cdot p}{m} + \sigma^\nu m^2\overline{\sigma}^\mu\sigma_\nu\,\frac{\overline{\sigma}\cdot p}{m}\Bigg\}\psi_-(p')\,. \end{aligned} \tag{D.33}$$

We make use of the relations of Section (A.5) to obtain

$$N^\mu = \psi_+(p)\Big\{\frac{2}{m}\,\sigma\cdot p'\big[(\overline{\sigma}^\mu\sigma\cdot(p-k)+\overline{\sigma}\cdot(p-k)\sigma^\mu)$$

$$+(\overline{\sigma}^\mu\sigma\cdot(p'-k)+\overline{\sigma}\cdot(p'+k)\sigma^\mu)\big]\overline{\sigma}\cdot p-2m[\sigma\cdot p'\overline{\sigma}^\mu+\sigma^\mu\overline{\sigma}\cdot p]$$

$$-\frac{2}{m}\,\big[\sigma\cdot p'\overline{\sigma}\cdot(p-k)\sigma^\mu\overline{\sigma}\cdot(p'-k)+\sigma\cdot(p-k)\overline{\sigma}^\mu\sigma\cdot(p'-k)\overline{\sigma}\cdot p\big]$$

$$+2m\big[\sigma\cdot(p'-k)\overline{\sigma}^\mu-\sigma^\mu\overline{\sigma}\cdot(p'-k)$$

$$+\sigma\cdot(p-k)\overline{\sigma}^\mu+\sigma^\mu\overline{\sigma}\cdot(p'-k)\big]\Big\}\psi_-(p') \tag{D.34}$$

Performing the integrals as in (C.9) (but ignoring the δ-term as it contributes solely to renormalization) and using (C.8)

$$\mathcal{M}^\mu = -ie^3\cdot 6\cdot\frac{i}{96\pi^2}\int\limits_{\lambda^2}^{\Lambda^2}dt\int\limits_0^1 dx\int\limits_0^{1-x}dy\,\frac{(1-x-y)M^\mu}{[s-p^2]^2} \tag{D.35}$$

where

$$s-p^2 = -[t(1-x-y)+(px+p'y)]^2 \tag{D.36}$$

and M^μ is an abreviation for the expression

$$M^\mu = \psi_+(p')\left\{ \frac{2}{m}\,\sigma\cdot p'\,\left[\overline{\sigma}^{\,\mu}\sigma\cdot(p+p'-2px-2p'y)\right.\right.$$

$$+\,\overline{\sigma}\,\cdot(p+p'-2px-2p'y)\sigma^\mu\big]\overline{\sigma}\,\cdot p$$

$$-\,2m[\sigma\cdot p'\overline{\sigma}^{\,\mu}+\sigma^\mu\overline{\sigma}\,\cdot p]-\frac{2}{m}\,[\sigma\cdot p'\overline{\sigma}\,\cdot p\sigma^\mu\overline{\sigma}\,\cdot p'+\sigma\cdot p\overline{\sigma}^{\,\mu}\sigma\cdot p'\overline{\sigma}\,\cdot p]$$

$$+\,2m\big[\sigma\cdot p'\overline{\sigma}\,(px+py)\sigma^\mu\overline{\sigma}\,\cdot p'+\sigma\cdot p'\overline{\sigma}\,\cdot p\sigma^\mu\overline{\sigma}\,\cdot k$$

$$+\,\sigma\cdot k\overline{\sigma}^{\,\mu}\sigma\cdot p'\overline{\sigma}\,\cdot p+\sigma\cdot p\overline{\sigma}^{\,\mu}\sigma\cdot k\overline{\sigma}\,\cdot p\big]$$

$$-\,2m\big[\sigma\cdot p'\overline{\sigma}\,\cdot(px+p'y)\sigma^\mu\overline{\sigma}\,\cdot(px+p'y)+\sigma^\mu\overline{\sigma}\,\cdot(px+p'y)$$

$$+\,\sigma\cdot(px+p'y)\overline{\sigma}^{\,\mu}\sigma\cdot(px+p'y)\overline{\sigma}\,\cdot p\big]$$

$$\left.+\,2m\big[\sigma\cdot(p+p'-2px-2py)\overline{\sigma}^{\,\mu}+\sigma^\mu\overline{\sigma}\,\cdot(p+p'-2px-2p'y)\big]\right\}\psi_-(p) \tag{D.37}$$

Now use Eqn. (B.2):

$$\overline{\psi}_+(p) = (\overline{\sigma}\,\cdot p/m)\,\psi_-(p)$$

and

$$\overline{\psi}_-(p') = \psi_+(p')(\sigma\cdot p'/m)$$

along with $q = p' - p$ to find

$$M^\mu = \overline{\psi}_-(p')\Big\{2m\big[\overline{\sigma}^\mu\sigma\cdot(p+p'-2px-2p'y)$$

$$+\overline{\sigma}\cdot(p+p'-2px-2p'y)\sigma^\mu\big]\Big\}\overline{\psi}_+(p)$$

$$-\overline{\psi}_-(p')[2m^2\overline{\sigma}^\mu]\psi_-(p) - \psi_+(p')[\sigma^\mu 2m^2]\overline{\psi}_+(p)$$

$$-2\Big\{\overline{\psi}_-(p')\overline{\sigma}\cdot p\sigma^\mu\big[\overline{\sigma}\cdot q\psi_-(p)+m\overline{\psi}_+(p)\big]$$

$$+\big[m\overline{\psi}_-(p')-\sigma\cdot q\psi(p')\big]\overline{\sigma}^\mu\sigma\cdot p'\overline{\psi}_+(p')\Big\}$$

$$+2\Big\{\overline{\psi}_-(p')\overline{\sigma}\cdot(px+p'y)\sigma^\mu\big[\overline{\sigma}\cdot q\psi_-(p)+m\overline{\psi}_+(p)\big]$$

$$+\overline{\psi}_-(p')\overline{\sigma}\cdot(p-px-p'y)\sigma^\mu\big[y\overline{\sigma}\cdot q\psi(p)+m(x+y)\overline{\psi}_+(p)\big]\Big\}$$

$$+\Big\{\big[m(x+y)\overline{\psi}_-(p)-x\psi_+(p)\sigma\cdot q\big]\overline{\sigma}^\mu\sigma\cdot(p'-px-p'y)\overline{\psi}_+(p)$$

$$+\big[m\overline{\psi}_-(p)-\psi_+(p)\sigma\cdot q\big]\overline{\sigma}^\mu\sigma\cdot(px+p'y)\overline{\psi}_+(p)\Big\}$$

$$+2m\Big\{2(1-x-y)m\overline{\psi}_-(p')\overline{\sigma}^\mu\psi_-(p)-(1-2x)\psi_+(p)\sigma\cdot q\overline{\sigma}^\mu\psi_-(p)\Big\}$$

$$+2m\psi_+(p')\sigma^\mu\Big\{2m(1-x-y)\overline{\psi}_+(p)+\overline{\psi}\cdot q(1-2y)\psi_-(p)\Big\} \tag{D.38}$$

Collecting terms as using Eqn. (B.2) again, *i.e.* $\psi_-(p) = (\sigma\cdot p/m)\,\overline{\psi}_+(p)$ and $\psi_+(p') = \overline{\psi}\,(p')\,(\overline{\sigma}\cdot p/m)$, we obtain (after doing some algebra):

$$M^\mu = 2m\overline{\psi}_-(p')\Big\{[-y^2 - xy + x]\overline{\sigma}^\mu\sigma\cdot q - [-x^2 + xy + y]\overline{\sigma}\cdot q\sigma^\mu\Big\}\overline{\psi}_+(p)$$

$$+ 2m^2\overline{\psi}_-(p')\Big\{\overline{\sigma}^\mu[-(1-x-y)^2 + (1-2x-2y)]\Big\}\psi_-(p)$$

$$+ 2m^2\psi_+(p')\Big\{\sigma^\mu[-(1-x-y)^2 + (1-2x-2y)]\Big\}\overline{\psi}_+(p)$$

$$+ 2m\psi_+(p')\Big\{(-xy - y^2 + x)\sigma^\mu\overline{\sigma}\cdot q + (x^2 + xy - y)\sigma\cdot q\overline{\sigma}^\mu\Big\}\psi_-(p) \tag{D.39}$$

We now once again use the "$b \leftrightarrow c$" trick of Eqn. (D.19). By switching what we call "b" and "c" we, in essence, switch $x \leftrightarrow y$ in the integrand of (C.8) Doing this we obtain (D.39) but with $x \leftrightarrow y$. Averaging (D.39) with this new equation we find that

$$M^\mu = 2m\overline{\psi}_-(p')\Big\{\tfrac{1}{2}\left[(x+y) - (x+y)^2\right]\left[\overline{\sigma}^\mu\sigma\cdot q - \overline{\sigma}\cdot q\sigma^\mu\right]\Big\}\overline{\psi}_+(p)$$

$$+ 2m^2[(1-x-y) - (1-x-y)^2 - (x+y)]$$

$$\times\{\overline{\psi}_-(p')\psi_-(p) + \psi_+(p)\sigma^\mu\overline{\psi}_+(p)\}$$

$$+ 2m\psi_+(p')\Big\{\tfrac{1}{2}\left[(x+y)^2 + (x+y)\right]\Big\}\Big\{-\sigma^\mu\overline{\sigma}\cdot q + \sigma\cdot q\overline{\sigma}^\mu\Big\}\psi(p)$$

$$= m(x+y)(1-x-y)\Big\{\overline{\psi}_-(p')[\overline{\sigma}^\mu\sigma\cdot q - \overline{\sigma}\cdot q\sigma^\mu]\overline{\psi}_+(p)$$

$$+ \psi_+(p')[\sigma^\mu\overline{\sigma}\cdot q - \sigma\cdot q\overline{\sigma}^\mu]\psi_-(p)\Big\}$$

$$- 2m^2(x+y)^2\Big\{\overline{\psi}_-(p')\overline{\sigma}^\mu\psi_-(p) + \psi_+(p')\sigma^\mu\overline{\psi}_+(p)\Big\} \tag{D.40}$$

and the final term is proportional to the γ^μ term (*i.e.* vertex renormalization).

Thus considering only the first term and noting that

$$\overline{\psi}\,\sigma^{\mu\nu}q_\nu\psi \text{ (Dirac) } = \psi_+ q_\nu \sigma^{\mu\nu}\psi_- + \overline{\psi}_- q_\nu \overline{\sigma}^{\,\mu\nu}\overline{\psi}_+$$

(from (A.8))

$$\overline{\psi}\,\sigma^{\mu\nu}q_\nu\psi = \psi_+\Big\{\sigma^{\mu\nu} + \frac{\sigma\cdot p'}{m}\,\overline{\sigma}^{\,\mu\nu}\frac{\overline{\sigma}\cdot p}{m}\Big\}q_\nu\psi_-$$

$$= \frac{i}{2m}\,\psi_+\Big\{m[\sigma^\mu\overline{\sigma}^{\,\nu} - \sigma^\nu\overline{\sigma}^{\,\mu}] \qquad \text{(D.41)}$$

$$+\frac{1}{m}\,[\sigma\cdot p'\overline{\sigma}^{\,\mu}\sigma^\nu\overline{\sigma}\cdot p - \sigma\cdot p'\overline{\sigma}^{\,\mu}\sigma^\nu\overline{\sigma}\cdot p]\Big\}q_\nu\psi_-$$

$$= \frac{i}{2}\Big\{\psi_+[\sigma^\mu\overline{\sigma}\cdot q - \sigma\cdot q\overline{\sigma}^{\,\mu}]\psi_- + \overline{\psi}_-(p')[\overline{\sigma}^{\,\mu}\sigma\cdot q - \overline{\sigma}\cdot q\sigma^\mu]\psi_-\Big\} \qquad \text{(D.42)}$$

From (D.42) the first term in (D.40) is

$$= m(x+y)(1-x-y)\,\frac{1}{(i/2)}\,\overline{\psi}\,\sigma^{\mu\nu}q_\nu\psi$$

(using Dirac spinor notation for short). From (D.32)

$$\mathcal{M}^\mu = -\overline{\psi}\,\sigma^{\mu\nu}q_\nu\psi\,\frac{-ie^2}{(i/2)}\,\frac{i}{16\pi^2}\int_{\mu^2}^{\Lambda^2}dt\int_0^1 dx\int_0^{1-x}dy\,\frac{m(x+y)(1-x-y)^2}{[-t(1-x-y)-m^2(x+y)^2]^2}. \qquad \text{(D.43)}$$

Since

$$\Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + \frac{i}{2m}\,\sigma^{\mu\nu}q_\nu F_2(q^2) \qquad \text{(D.44)}$$

with

$$\Delta a_e \equiv F_2(q^2\to 0) \equiv \frac{g-2}{2} \qquad \text{(D.45)}$$

we have

$$\Delta a_e = \frac{2m}{i} \cdot \frac{-ie^2}{i/2} \cdot \frac{i}{16\pi^2} \int_{\mu^2}^{\Lambda^2} dt \int_0^1 dx \int_x^1 dz \ \frac{mz(1-z)^2}{[t(1-z)+m^2z^2]} \tag{D.46}$$

and so

$$\begin{aligned}
\Delta a_e &= -\frac{m^2e^2}{4\pi^2} \int_{\mu^2}^{\Lambda^2} dt \int_0^1 dx \int_1^x dz \ \frac{z(1-z)^2}{[t(1-z)+m^2z^2]^2} \\
&= \frac{m^2\alpha}{\pi} \int_0^1 dx \int_1^x dz \ \left. \frac{(1-z)z}{t(1-z)+m^2z^2} \right|_{\mu^2 \to 0}^{\Lambda^2 \to \infty} \\
&= -\frac{m^2\alpha}{\pi} \int_0^1 dx \int_1^x dz \ \frac{z(1-z)}{m^2z^2} \\
&= -\frac{\alpha}{\pi} \int_0^1 dx \, [\ell n \, x - x + 1] \\
&= \frac{\alpha}{2\pi} \, .
\end{aligned} \tag{D.47}$$

Thus we obtain the correct result again using this more involved method.

FIGURE CAPTIONS

1. Naive contributions to $e_L + \gamma \to e_R$.

   Here $\Delta(p) \equiv i/p^2 - m^2 + i\epsilon$.

2. Additional Diagrams Contributing to $e_L + \gamma \to e_R$.

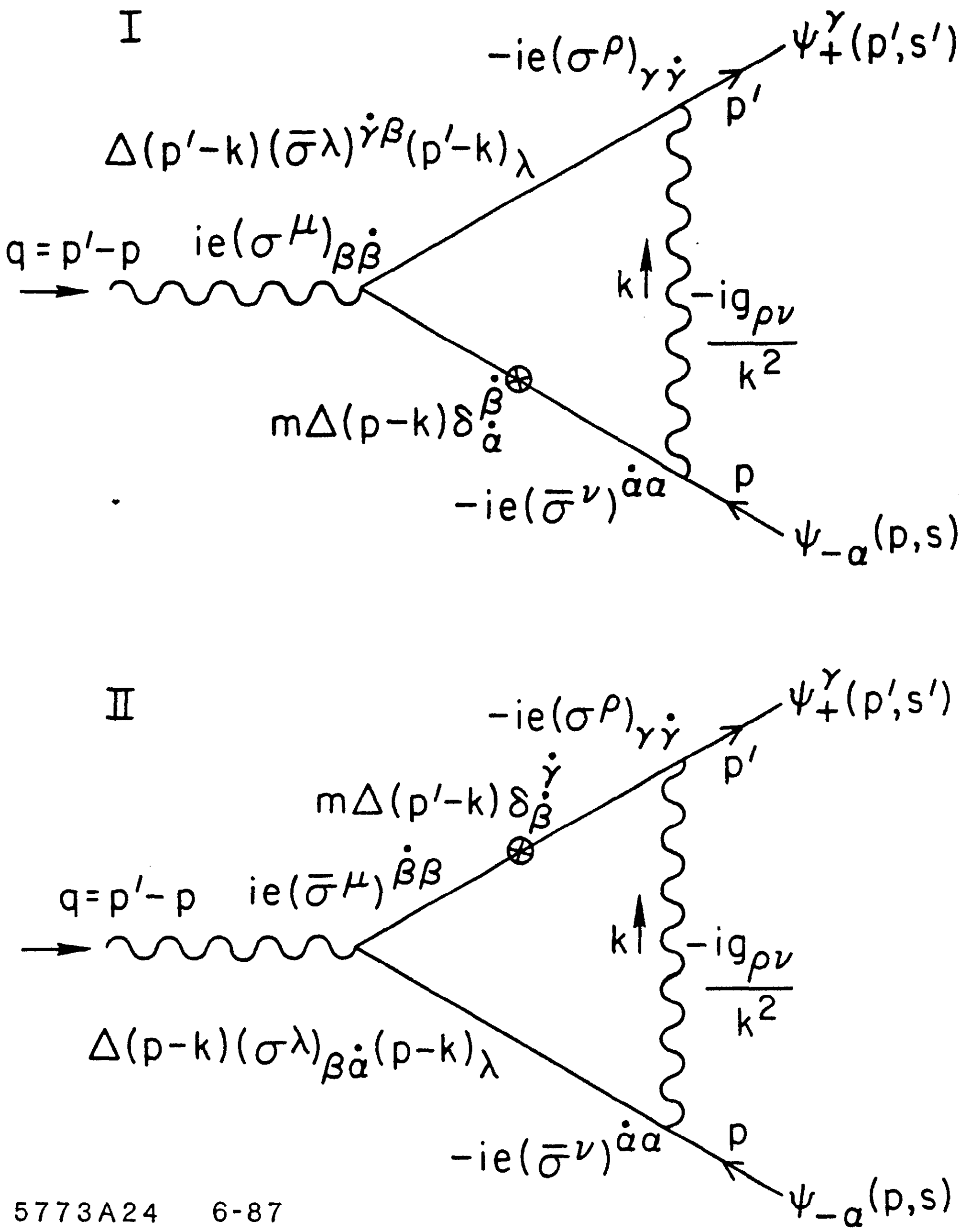
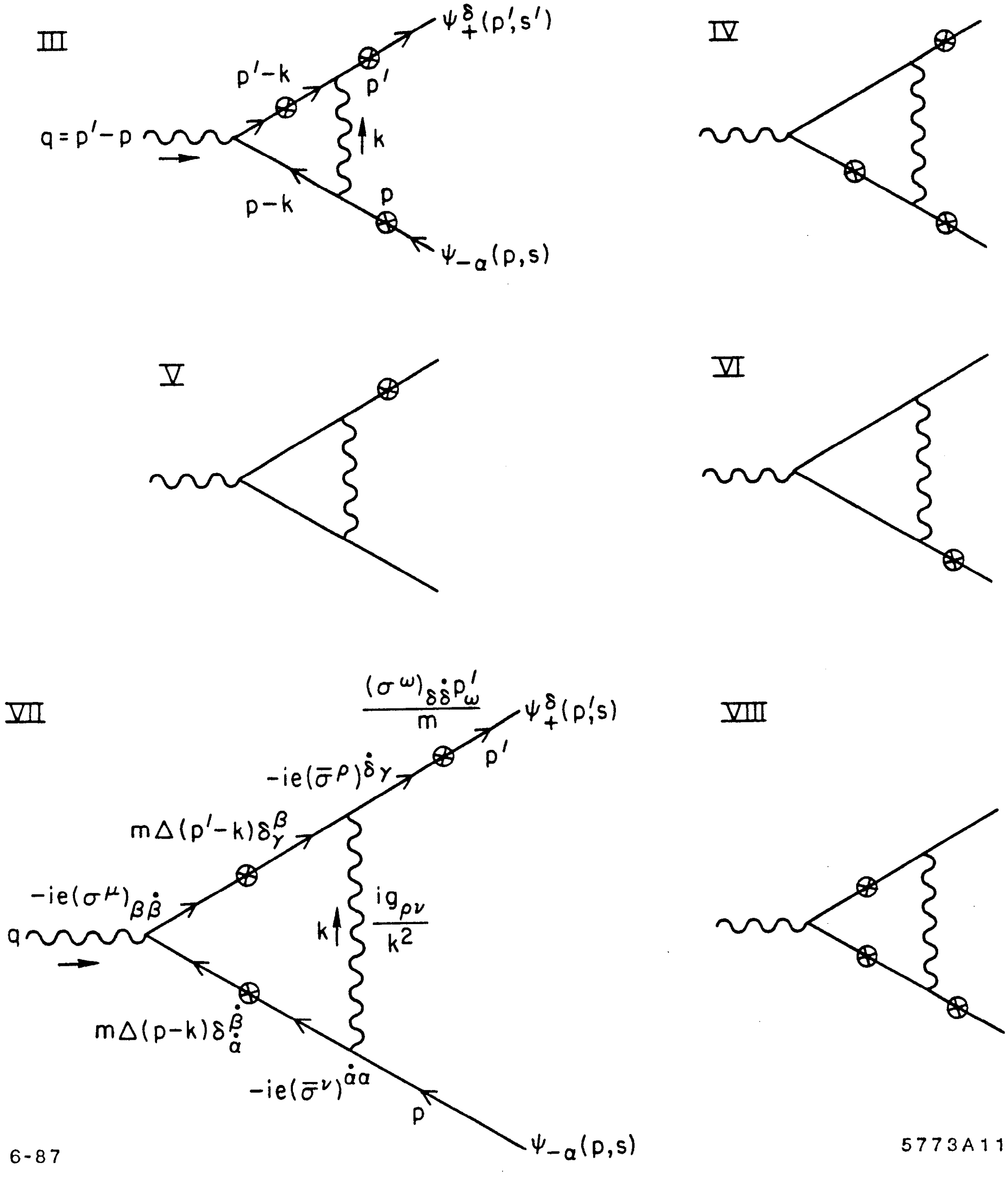

I
$-ie(\sigma^\rho)_{\gamma\dot\gamma}$
$\psi_+^\gamma(p',s')$
p'
$\Delta(p'-k)(\bar\sigma\lambda)^{\dot\gamma\beta}(p'-k)_\lambda$
$q=p'-p$
$ie(\sigma^\mu)_{\beta\dot\beta}$
k
$-ig_{\rho\nu}$
k^2
$m\Delta(p-k)\delta^{\dot\beta}_{\dot\alpha}$
$-ie(\bar\sigma^\nu)^{\dot\alpha\alpha}$
p
$\psi_{-\alpha}(p,s)$
II
$-ie(\sigma^\rho)_{\gamma\dot\gamma}$
$\psi_+^\gamma(p',s')$
p'
$m\Delta(p'-k)\delta^{\dot\gamma}_{\dot\beta}$
$q=p'-p$
$ie(\bar\sigma^\mu)^{\dot\beta\beta}$
k
$-ig_{\rho\nu}$
k^2
$\Delta(p-k)(\sigma\lambda)_{\beta\dot\alpha}(p-k)_\lambda$
$-ie(\bar\sigma^\nu)^{\dot\alpha\alpha}$
p
$\psi_{-\alpha}(p,s)$
5773A24 6-87

Fig. D.1



III
ψ₊^δ(p′,s′)
p′−k
p′
q=p′−p
k
p−k
p
ψ₋α(p,s)
IV
V
VI
VII
(σ^ω)δδ̇p′ω / m
ψ₊^δ(p′,s)
−ie(σ̄^ρ)^δ̇γ
p′
mΔ(p′−k)δγ^β
−ie(σ^μ)ββ̇
q
k
igρν / k²
mΔ(p−k)δα̇^β̇
−ie(σ̄^ν)^α̇α
p
ψ₋α(p,s)
VIII
6-87
5773A11

Fig. D.2

APPENDIX E

Supersymmetric QED Contribution to Anomalous Moment

We wish to consider Δa_e, the *additional* contribution to the anomalous magnetic moment of the electron, arising from a minimal supersymmetric extension of QED.

E.1 Lagrangian for Supersymmetric QED (SQED)

We use the following notation[11] for the superfields and their components:

Superfield	Ordinary Matter	Superpartners
$\hat{\mathbf{V}}$	γ	λ
$\hat{\mathbf{L}}_i$	${\ell_{L_i}}^-$	$\tilde{\ell}_{L_i}$
$\hat{\mathbf{R}}_i$	$(\ell_i^+)_L = (\ell_{R_i}^-)^*$	$(\tilde{\ell}_{R_i}^-)^*$

(E.1)

Note that when we consider a full electroweak theory that L_i will will be promoted to a weak doublet which will include neutrinos and their supersymmetric counterparts. The generation index "i" will now be dropped as we proceed to specialize to a single generation.

The superfield strengths are

$$W^\alpha = -\tfrac{1}{4}\overline{D}\,\overline{D}D^\alpha\widehat{V}$$
$$\overline{W}_{\dot\alpha} = -\tfrac{1}{4}\,DD\overline{D}_{\dot\alpha}\widehat{V} \tag{E.2}$$

and

$$\mathcal{L}_{SQED} = \int d^2\theta d^2\overline{\theta} \Big\{ \tfrac{1}{4}\left(WW\delta(\overline{\theta}^2) + \overline{W}\,\overline{W}\delta(\theta^2)\right)$$

$$+ \overline{L}e^{-eV}L + \overline{R}e^{eV}R + m_e\big((RL\delta(\overline{\theta}^2) + \overline{L}\,\overline{R}\delta(\theta^2)\big) \Big\} \tag{E.3}$$

where $\delta(\theta) = \theta$ and $\delta(\overline{\theta}) = \overline{\theta}$ due to the rules of Grassman integration (see Appendix I for details of this construction). Expanding this out in component fields and eliminating the auxiliary fields $F_\pm$ and D via their equations of motion yields (for electrons only)

$$\begin{aligned}
\mathcal{L}_{SQED} = &-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\overline{\lambda} + \widetilde{e}_R\Box\widetilde{e}_R{}^* + \widetilde{e}_L{}^*\Box\widetilde{e}_L \\
&+ i(\partial_\mu\overline{\psi}_+\sigma^\mu\psi_+ + \partial_\mu\overline{\psi}_-\overline{\sigma}^\mu\psi_+) \\
&+ \frac{1}{2}eA_\mu\Big[\overline{\psi}_+\sigma^\mu\psi_+ - \overline{\psi}_-\overline{\sigma}^\mu\psi_- \\
&+ i(\widetilde{e}_R\partial^\mu\widetilde{e}_R{}^* - \partial^\mu\widetilde{e}_R\widetilde{e}_R{}^* - \widetilde{e}_L{}^*\partial^\mu\widetilde{e}_L + \partial^\mu\widetilde{e}_L{}^*\widetilde{e}_L)\Big] \\
&- \frac{ie}{\sqrt{2}}\left[\widetilde{e}_R{}^*\overline{\psi}_+\overline{\lambda} - \widetilde{e}_R\psi_+\lambda - \widetilde{e}_L\overline{\psi}_-\overline{\lambda} + \widetilde{e}_L{}^*\psi_-\lambda\right] \\
&- \tfrac{1}{4}e^2A_\mu A^\mu[\widetilde{e}_R\widetilde{e}_R{}^* + \widetilde{e}_L{}^*\widetilde{e}_L] - \tfrac{1}{4}e^2[\widetilde{e}_R{}^*\widetilde{e}_R - \widetilde{e}_L{}^*\widetilde{e}_L]^2 \\
&- m[\psi_+\psi_- - \overline{\psi}_+\overline{\psi}_-] - m^2_{\widetilde{e}_R}\widetilde{e}_R{}^*\widetilde{e}_R \\
&- m^2_{\widetilde{e}_L}\widetilde{e}_L{}^*\widetilde{e}_L - m_\lambda\overline{\lambda}\lambda
\end{aligned} \tag{E.4}$$

In unbroken SQED $m_{\widetilde{e}_L} = m_{\widetilde{e}_R} = m_e$ and $m_\lambda \equiv m_{\widetilde{\gamma}} = 0$. Here explicit breaking terms have been added to give $\widetilde{e}_L$, $\widetilde{e}_R$ and $\widetilde{\gamma}$ arbitrary masses. Presumably such terms would arise in a realistic model as effective residues of some unspecified higher-energy theory. Note that $\tilde{e}_L$ and $\tilde{e}_R$ are independent fields.

E.2 FEYNMAN RULES

i) There are four types of vertices in this theory. Since QED is left-right blind ψ_e, $\tilde{e}_L$ and $\tilde{e}_R$ are subject to standard QED and scalar electrodynamics photonic coupling vertices. There are also Yukawa vertices $e_L\tilde{e}_L\tilde{\gamma}$ and $e_R\tilde{e}_R\tilde{\gamma}$ of coupling strength $\sqrt{2}\,e$.

ii) ψ_e and $\tilde{\gamma}$ have standard (Weyl) fermionic propagators ($\tilde{\gamma}$ is a Majorana particle) and $\tilde{e}_{L,R}$ have standard scalar propagators.

iii) In considering Δa_e, there are no diagrams with a chirality change in the photino propagator since that would require $\tilde{e}_L - \tilde{e}_R$ mixing in $\mathcal{L}$. Thus we find, when looking for $e_L + \gamma \to e_R$, that only the two diagrams of Fig. 1 contribute.

E.3 COMPUTATION OF DIAGRAM I

$$\begin{aligned}\mathcal{M}^\mu &\equiv -ie\psi_+^\beta(p',s')\Gamma_I^\mu(p',s)\beta^\alpha\psi_{-\alpha}(p,s)\\ &= (-ie)^3 2(i^3)\int\frac{d^4k}{(2\pi)^4}\\ &\quad\times\frac{\psi_+^\beta(p')[(\sigma^\omega)_{\beta\dot{\alpha}}p'_\omega/m](\bar{\sigma}^\nu)^{\dot{\alpha}\alpha}k_\nu\psi_{-\alpha}(p)(p+p'-2k)^\mu}{(k^2-m_\lambda^2)[(p-k)^2-m_{\tilde{e}_L}^2]\,[(p'-k)^2-m_{\tilde{e}_L}^2]}\end{aligned} \tag{E.5}$$

We shall work on shell so that $p^2 = p'^2 = m_e^2 \equiv m^2$.

Define

$$\overline{m}_L^2 \equiv m^2 - m_{\tilde{e}_L}^2 + m_\lambda^2$$
$$\overline{m}_R^2 \equiv m^2 - m_{\tilde{e}_R}^2 + m_\lambda^2 \tag{E.6}$$

from Eqn. (B.6)

$$\psi_+^\beta(p',s')\,\frac{(\sigma^\omega)_{\beta\dot\alpha}p'_\omega}{m} = \overline{\psi}_{-\dot\alpha}(p',s')$$

then (E.5) becomes

$$\psi_+\Gamma_I^\mu\psi_- = 2ie^2\int\frac{d^4k}{(2\pi)^4}\,\frac{N^\mu}{D}$$

$$N^\mu = \overline{\psi}_{-\dot\alpha}(p')(\overline{\sigma}^\nu)^{\dot\alpha\alpha}k_\nu\psi_{-\alpha}(p)(p+p'-2k)^\mu$$

$$D = (k^2-m_\lambda)^2(k^2-2p\cdot k+m^2-m_{\tilde{e}_L}^2)(k^2-2p'\cdot k+m^2-m_{\tilde{e}_L}^2) \tag{E.7}$$

We now use Eqn. (D.11) to find

$$\frac{1}{D} = 2\int_0^1 dx\int_0^{1-x} dy\big[k^2-2k\cdot(px+p'y)+\overline{m}_L^2(x+y)-m_\lambda^2\big]^{-3}\,. \tag{E.8}$$

We next use Eqn. (D.12) or (H.6) to find

$$\psi_+\Gamma_I^\mu\psi_- = 2ie^2\cdot 2\int_0^1 dx\int_0^{1-x} dy$$
$$\times\frac{i\overline{\psi}_{-\dot\alpha}(\overline{\sigma}^\nu)^{\dot\alpha\alpha}[px+p'y]_\nu\psi_{-\alpha}(p+p'-2[px+p'y])^\mu}{32\pi^2(\overline{m}_L^2[x+y]-m_\lambda^2-[px+p'y]^2)}$$

where we have ignored the $k_\nu k_\mu$ term which contributes solely to the helicity-preserving piece of the amplitude (*i.e.* $\overline{\sigma}^\mu$). Since $(px+p'y)^2 = m^2(x+y)^2-q^2xy$

as before

$$\psi_+\Gamma_I^\mu\psi_- = -\frac{e^2}{8\pi^2}\int_0^1 dx\int_0^{1-x} dy$$
$$\times\ \frac{\overline{\psi}_{-\dot\alpha}(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}[px+p'y]_\nu\psi_{-\alpha}[p(1-2x)+p'(1-2y)]^\mu}{\overline{m}_L^2(x+y)-m_\lambda^2(x+y)^2-q^2xy}\ . \tag{E.9}$$

We now perform the "$b \leftrightarrow c$" averaging trick as in (D.19) The result being that $x \leftrightarrow y$ in the integrand of (E.8), and therefore (E.9), and then we average the two expressions. Before we do this it is prudent to apply the Dirac equations of (B.2).

$$\begin{aligned}&\overline{\psi}_{-\dot\alpha}(p')(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}[px+p'y]_\nu\psi_{-\alpha}(p)\\ &\qquad=\overline{\psi}_{-\dot\alpha}\left\{mx\overline{\psi}_+^{\dot\alpha}+my\overline{\psi}_+^{\dot\alpha}+(\sigma^\nu)^{\dot\alpha\alpha}q_\nu\psi_{-\alpha}y\right\}\\ &\qquad=m(x+y)\overline{\psi}_{-\dot\alpha}\overline{\psi}_+^{\dot\alpha}+y\overline{\psi}_{-\dot\alpha}(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}q_\nu\psi_{-\alpha}\end{aligned} \tag{E.10}$$

when we now perform the $x \leftrightarrow y$ average we find

$$\psi_+\Gamma_I^\mu\psi_- = -\frac{e^2}{8\pi^2}\int_0^1 dx\int_0^{1-x} dy\ \frac{M^\mu}{-m^2(x+y)^2+\overline{m}_L^2(x+y)-m_\lambda^2-q^2xy} \tag{E.11}$$

$$\begin{aligned}M^\mu &= m(x+y)(1-x-y)(p+p')^\mu\overline{\psi}_{-\dot\alpha}\overline{\psi}_+^{\dot\alpha}\\ &\quad+\Big[y\left\{p(1-2x)+p'(1-2y)\right\}\\ &\quad+x\left\{p(1-2y)+p'(1-2x)\right\}\Big]\overline{\psi}_{-\dot\alpha}(\overline{\sigma}^{\nu})^{\dot\alpha\alpha}q_\nu\psi_{-\alpha}\end{aligned}$$

We are interested in the helicity-changing part of this (the $\overline{\psi}_{-\dot\alpha}\overline{\psi}_+^{\dot\alpha}$ term) as

$q^2 \to 0$ (on shell). This is

$$\psi_+ \Gamma^\mu_I{}_{LR}(q^2 \to 0)\psi_- = \frac{e^2}{8\pi^2}\,\frac{(p-p')^\mu}{m}\,\overline{\psi}_{-\dot\alpha}\overline{\psi}{}^{\dot\alpha}_+\,I_L \tag{E.12}$$

where

$$\begin{aligned} I_L &\equiv \int_0^1 dx \int_0^{1-x} dy\, \frac{(x+y)(1-x-y)}{-(x+y)^2 + \frac{\overline{m}_L^2}{m^2}\,(x+y) - \frac{m_\lambda^2}{m^2}} \\ &= \int_0^1 dx \int_x^1 dz\, \frac{z(1-z)}{-z^2 + \frac{\overline{m}_L^2}{m^2}\,z - \frac{m_\lambda^2}{m^2}} \end{aligned} \tag{E.13}$$

where we have let $z = x + y$.

Now in (E.10) we applied the Dirac operator to the *right.* We could equally well have had it operator to the *left.* Using Eqn. (B.2) again we would have obtained

$$\begin{aligned} &\overline{\psi}_{-\dot\alpha}(p')(\overline{\sigma}^\nu)^{\dot\alpha\alpha}[px + p'y]_\nu \psi_{-\alpha}(p) \\ &\qquad = \{[m\psi^\alpha_+ - (\overline{\sigma}^\nu)^{\dot\alpha\alpha} q_\nu \overline{\psi}_{-\alpha}]x + m\psi^\alpha_+ y\}\psi_{-\alpha} \\ &\qquad = m(x+y)\psi^\alpha_+\psi_{-\alpha} - x\overline{\psi}_{-\dot\alpha}(\overline{\sigma}^\nu)^{\dot\alpha\alpha} q_\nu \psi_{-\alpha} \end{aligned} \tag{E.14}$$

(We use $+m$ since we are considering electrons and not positrons.) Note that in (E.10) we have related the $e_L \to e_R$ term to the corresponding $e_R \to e_L$ terms whereas (E.14) expresses $e_L \to e_R$ in terms of $e_L \to e_R$. Using (E.14) instead of (E.10) would change (E.12) to

$$\psi_+ \Gamma^\mu_I{}_{LR}(q^2 \to 0)\psi_- = \frac{e^2}{8\pi^2}\,\frac{(p+p')^\mu}{m}\,\psi^\alpha_+\psi_{-\alpha} I_L\ . \tag{E.15}$$

Averaging (E.12) and (E.15) and using $\alpha = e^2/4\pi$

$$\psi_+ \Gamma^\mu_I{}_{LR}\psi_- = \frac{\alpha}{4\pi}\,\frac{(p+p')^\mu}{m}\,[\psi^\alpha_+\psi_{-\alpha} + \overline{\psi}_{-\dot\alpha}\overline{\psi}{}^{\alpha}_+]\ I_L\ . \tag{E.16}$$

From Appendix H or Eqn. (D.23) we employ the Gordon decomposition producing

$$\psi_+\Gamma^\mu_{I\,LR}\psi_- = \frac{\alpha}{4\pi m}\, I_L\Big\{ -i[\psi_+\sigma^{\mu\nu}q_\nu\psi_- + \overline{\psi}_-\overline{\sigma}^{\mu\nu}q_\nu\psi_+] + 2m[\overline{\psi}_-\overline{\sigma}^{\mu}\overline{\psi}_+ + \psi_+\sigma^\mu\psi_-]\Big\} \tag{E.17}$$

We are only interested in the $\sigma^{\mu\nu}$ and $\overline{\sigma}^{\mu\nu}$ terms. Using (A.8) these terms can be combined to yield the Dirac spinor formulation:

$$\overline{\psi}\,\Gamma^\mu_{I\,LR}\psi = \frac{-i\alpha}{4\pi m}\, I_L\overline{\psi}\,\sigma^{\mu\nu}q_\nu\psi\ . \tag{E.18}$$

From (C.15) and (C.16)

$$\Delta a_e^{(L)} = \frac{\alpha}{2\pi}\, I_L \tag{E.19}$$

which is the $\widetilde{e}_L$ contribution to a_e.

E.4 Computation of Diagram II and Total Contribution

We ingeminate the steps of each in Section E.3 beginning with

$$-ie\psi_+\Gamma^\mu_{\mathrm{II}}(p,p')\psi_- = (-ie)^2 2(i^3)\int\frac{d^4k}{(2\pi)^4} \times \frac{\psi_+^\alpha(\sigma^\nu)_{\alpha\dot\alpha}k_\nu[(\overline{\sigma}^\omega)^{\dot\alpha\beta}p_\omega/m]\psi_{-\beta}(p+p'-2k)^\mu}{[k^2-m_\lambda^2]\,[(p-k)^2-m^2_{\widetilde{e}_R}]\,[(p'-k)^2-m^2_{\widetilde{e}_R}]} \tag{E.20}$$

and conclude with

$$\Delta a_e^{(R)} = \frac{\alpha}{2\pi}\, I_R \tag{E.21}$$

where

$$I_R = \int_0^1 dx\int_x^1 dz\ \frac{z(1-z)}{-z^2+\dfrac{\overline{m}_R^2}{m^2}\,z-\dfrac{m_\lambda^2}{m^2}} \tag{E.22}$$

and $\overline{m}_R$ is given by (E.6). The total contribution to Δa_e, the incremental supersymmetric contribution to the anomalous moment of the electron a_e, is

$$\begin{aligned}
\Delta a_e &= \Delta a_e^{(L)} + \Delta a_e^{(R)} \\
&= \frac{\alpha}{2\pi}\,(I_L + I_R) \\
I_{L,R} &= \int_0^1 dx \int_x^1 dz \,\frac{z(1-z)}{-z^2 + \dfrac{m^2 + m_\lambda^2 - m_{\tilde{e}_{L,R}}^2}{m^2}\, z - \dfrac{m_\lambda^2}{m^2}}
\end{aligned} \tag{E.23}$$

E.5 SPECIFIC CASES

a) $m_\lambda^2, m_{\tilde{e}}^2 \ll m^2$. This case is excluded experimentally. In this limit

$$I_{L,R} = -\frac{1}{2}$$

$$\Delta a_e = -\frac{\alpha}{2\pi}\ .$$

This is of the same magnitude as the QED contribution but of the opposite sign.

b) $m_\lambda^2 \approx m_{\tilde{e}}^2 \gg m$. Rather unlikely in most models but not excluded.

$$I_{L,R} \approx \int_0^1 dx \int_x^1 dz\, \frac{m^2}{m_\lambda^2}\, z(z-1) = -\frac{1}{12}\,\frac{m^2}{m_\lambda^2}$$

$$\Delta a_e = -\frac{\alpha}{24\pi}\,\frac{m^2}{m_\lambda^2}$$

c) $m_{\tilde{e}}^2 \gg m_\lambda^2 \gg m^2$. The standardly presumed scenario.

$$\begin{aligned}
I_{L,R} &= \int_0^1 dx \int_x^1 dz\, \frac{m^2}{\overline{m}_{L,R}^2}\,(1-z) = -\frac{1}{6}\,\frac{m^2}{m_{\tilde{e}_{L,R}}^2} \\
\Delta a_e &= -\frac{\alpha}{12}\left\{\frac{m^2}{m_{\tilde{e}_L}^2} + \frac{m^2}{m_{\tilde{e}_R}^2}\right\} \equiv \Delta a_e^{(m_\lambda = 0)}
\end{aligned} \tag{E.24}$$

This is precisely the 1974 result of Fayet.[3]

d) $m_{\tilde{e}_L} = m_{\tilde{e}_R} = m$; $m_\lambda = 0$. Case of exact supersymmetry

$$I_{L,R} = \int_0^1 dx \int_x^1 dz \, \frac{z(1-z)}{z^2} = -\frac{1}{2}$$

$$\Delta a_e^{(SUSY)} = -\frac{\alpha}{2\pi} = -a_e^{(QED)} \, . \tag{E.25}$$

In the exact supersymmetric limit we expect that there will be no anomalous magnetic moment to any order. Therefore we should have anticipated that $\Delta a_e^{(SUSY)}$ would precisely cancel $a_e^{(QED)}$.[4]

e) $m_\lambda = 0$

$$I_{L,R} = \int_0^1 dx \int_x^1 dz \, \frac{1-z}{\rho - (1-z)} \qquad \rho \equiv \frac{m^2_{\tilde{e}_{L,R}}}{m^2}$$

$$= \rho(1-\rho)[\ell n\rho - \ell n(\rho - 1)] - \rho + \frac{1}{2}$$

f) $m_\lambda, m_{\tilde{e}} \gg m$. This is the most interesting case.

Defining

$$\mathcal{R}_{L,R} = \frac{m_\lambda^2}{\overline{m}^2_{L,R}} = \frac{m_\lambda^2}{m_\lambda^2 - m^2_{\tilde{e}_{L,R}} + m^2}$$

$$I_{L,R} = \frac{m^2}{\overline{m}^2_{L,R}} \int_0^1 dx \int_x^1 dz \, \frac{z(1-z)}{z - \mathcal{R}_{L,R}} \tag{E.26}$$

noting that

$$\int_0^1 dx \int_x^1 dz = \int_0^1 dz \int_0^z d\tilde{z}$$

where $\widetilde{z} = 2x - z$

$$I_{L,R} = \frac{m^2}{\overline{m}^2} \int_0^1 dz \, \frac{z^2 - z^3}{z - \mathcal{R}}$$

$$I = \frac{m^2}{\overline{m}^2} \left[\frac{1}{6} + \frac{1}{2}\mathcal{R} - \mathcal{R}^2 + \mathcal{R}^2(1-\mathcal{R})\ell n\left(\frac{\mathcal{R}-1}{\mathcal{R}}\right)\right] . \tag{E.27}$$

Now we note that

$$\frac{m^2}{\overline{m}^2} = -\frac{m^2}{m_{\tilde{e}}^2}\left(1 - \left[1 + \frac{m^2}{m_\lambda^2}\right]\mathcal{R}\right) \approx -\frac{m^2}{m_{\tilde{e}}^2}\,(1-\mathcal{R}) \tag{E.28}$$

if $m \ll m_\lambda$ so

$$I = -\frac{m^2}{m_{\tilde{e}}^2}\,[1-\mathcal{R}]\,h(\mathcal{R}) \tag{E.29}$$

where

$$h(\mathcal{R}) = \frac{1}{6} + \frac{1}{2}\mathcal{R} - \mathcal{R}^2 + \mathcal{R}^2(1-\mathcal{R})\ell n\left(\frac{\mathcal{R}-1}{\mathcal{R}}\right) .$$

The "1/6" term is what we found in part (c) when $m_\lambda \to 0$. This is hardly shocking since as $m_\lambda \to 0$ we have $\mathcal{R} \to 0$ and $(1-\mathcal{R})h(\mathcal{R}) \to 1/6$. Factoring this term out we can write

$$\begin{aligned} I_{L,R} &= -\frac{m^2}{6m_{\tilde{e}_{L,R}}^2}\,[1 + f(\mathcal{R}_{L,R})] \\ f(\mathcal{R}) &= 2\mathcal{R} - 9\mathcal{R}^2 + 6\mathcal{R}^3 + 6\mathcal{R}^2(1-\mathcal{R})^2\ell n\left(\frac{\mathcal{R}-1}{\mathcal{R}}\right) \end{aligned} \tag{E.30}$$

and from (E.23) and (E.24)

$$\Delta a_e = \Delta a_e^{(m_\lambda=0)}\left[1 + \frac{m_{\tilde{e}_L}^2 m_{\tilde{e}_R}^2}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2}\left(\frac{f(\mathcal{R}_L)}{m_{\tilde{e}_L}^2} + \frac{f(\mathcal{R}_R)}{m_{\tilde{e}_R}^2}\right)\right] \tag{E.31}$$

where

$$\mathcal{R}_{L,R} \doteq \frac{m_\lambda^2}{m_\lambda^2 - m_{\tilde{e}_{L,R}}^2}$$

since $m_\lambda \gg m$.

Notes:

(i) when $m_{\tilde{e}} \gg m_\lambda$ $\mathcal{R} \to 0$ and $f(\mathcal{R}) \to 0$. Then $\Delta a_e \to \Delta a_e^{(m_\lambda=0)}$ to lowest order.

(ii) The first order correction in the above [to $\mathcal{O}(m_\lambda^2/m_{\tilde{e}}^2]$ comes from noting that $f(\mathcal{R}) = 2\mathcal{R}$ as $\mathcal{R} \to 0$ where $\mathcal{R} \doteq -m_\lambda^2/m_{\tilde{e}}^2$. Thus (E.31) becomes

$$\begin{aligned}\Delta a_e &\to \Delta a_e^{(m_\lambda=0)}\left[1 + \frac{1}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2}\left\{\frac{2m_{\tilde{e}_R}^2 m_\lambda^2}{-m_{\tilde{e}_L}^2} + \frac{2m_{\tilde{e}_L}^2 m_\lambda^2}{m_{\tilde{e}_L}^2}\right\}\right] \\ &\doteq \Delta a_e^{(m_\lambda=0)}\left[1 - 2\,\frac{m_\lambda^2}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2}\left(\frac{m_{\tilde{e}_L}^2}{m_{\tilde{e}_R}^2} + \frac{m_{\tilde{e}_R}^2}{m_{\tilde{e}_L}^2}\right)\right]\end{aligned} \tag{E.32}$$

(iii) When $m_\lambda \gg m_{\tilde{e}} \gg m$ $\mathcal{R} \to 1 + m_{\tilde{e}}^2/m_\lambda^2$ and $f(\mathcal{R}) \to -1$. Using (E.29) we see that $I \to [1 - \mathcal{R}][-1/3] = -\frac{1}{3}\frac{m^2}{m_\lambda^2}$ just as in case (g) which follows.

g) $m_\lambda \gg m_e \gg m$. This case is also unlikely in most models but is not excluded.

$$I_{L,R} = \int_0^1 dz \int_0^z d\tilde{z}\, \frac{m^2 z(1-\tilde{z})}{-m^2 z^2 + (m_\lambda^2 - m_{\tilde{e}}^2 + m^2) - m_\lambda^2}$$

$$= \frac{m^2}{m_\lambda^2}\int_0^1 dz \int_0^z d\tilde{z}\, \frac{z(1-z)}{z-1} = -\frac{1}{3}\,\frac{m^2}{m_\lambda^2}$$

$$\Delta a_e = -\frac{\alpha}{3}\,\frac{m^2}{m_\lambda^2}.$$

FIGURE CAPTIONS

1. Contributions to Δa_e when $\widetilde{e}_L$ and $\widetilde{e}_R$ are unmixed.

   Here $\Delta(p) \equiv i/p^2 - m^2 + i\epsilon$.

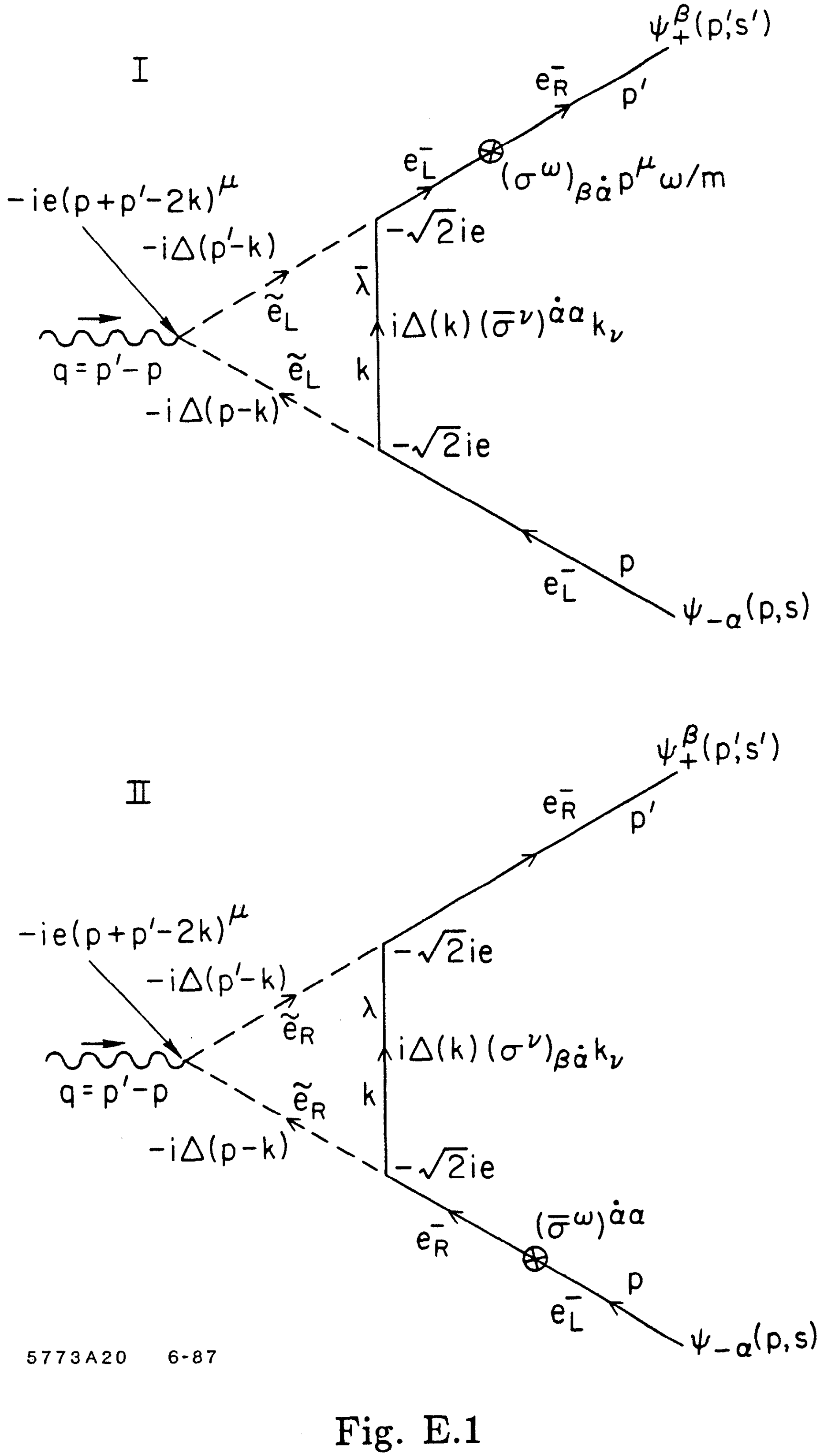

I
ψ+β(p',s')
e_R^-
p'
e_L^-
(σω)βα̇ p^μ ω/m
−ie(p+p'−2k)^μ
−iΔ(p'−k)
−√2ie
λ̄
ẽL
iΔ(k)(σ̄ν)^α̇α kν
q = p'−p
ẽL
k
−iΔ(p−k)
−√2ie
p
e_L^-
ψ−α(p,s)
II
ψ+β(p',s')
e_R^-
p'
−ie(p+p'−2k)^μ
−√2ie
−iΔ(p'−k)
λ
ẽR
iΔ(k)(σν)βα̇ kν
q = p'−p
ẽR
k
−iΔ(p−k)
−√2ie
(σ̄ω)^α̇α
e_R^-
p
e_L^-
ψ−α(p,s)
5773A20 6-87

Fig. E.1

APPENDIX F

Contribution to Anomalous Moment from Smuon Mixing

F.1 LAGRANGIAN

We permit the two smuons $\tilde{\mu}_L$ and $\tilde{\mu}_R$ to mix resulting in the mass eigenstates

$$\begin{aligned} \widetilde{\mu}_1 &= \cos\phi\widetilde{\mu}_L + \sin\phi\widetilde{\mu}_R \\ \widetilde{\mu}_2 &= -\sin\phi\widetilde{\mu}_L + \cos\phi\widetilde{\mu}_R \end{aligned} \tag{F.1}$$

or equivalently

$$\begin{aligned} \widetilde{\mu}_L &= \cos\phi\widetilde{\mu}_1 - \sin\phi\widetilde{\mu}_2 \\ \widetilde{\mu}_R &= \sin\phi\widetilde{\mu}_1 + \cos\phi\widetilde{\mu}_2 \ . \end{aligned} \tag{F.2}$$

We will confine ourselves to the muon here while recognizing that the same analysis goes through for the electron and tau. It is the muon which provides the most stringent bounds from experiment. We assume that $\lambda = (\widetilde{\gamma})$ is purely Majorana, although it may be massive (R-parity is always to be considered an inviolable symmetry).

The terms in the Lagrangian of Appendix E (E.4) are altered thusly

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} \rightarrow -\frac{ie}{\sqrt{2}} \Big[&\sin\phi\widetilde{\mu}_1{}^*\overline{\psi}_+\overline{\lambda} + \cos\phi\widetilde{\mu}_2{}^*\overline{\psi}_+\overline{\lambda} \\ &- \sin\phi\widetilde{\mu}_1\psi_+\lambda - \cos\phi\widetilde{\mu}_2\psi_+\lambda \\ &- \cos\phi\widetilde{\mu}_1\overline{\psi}_-\overline{\lambda} + \sin\phi\widetilde{\mu}_2\overline{\psi}_-\overline{\lambda} \\ &+ \cos\phi\widetilde{\mu}_1{}^*\psi_-\lambda - \sin\phi\widetilde{\mu}_2{}^*\psi_-\lambda \Big] \ . \end{aligned}$$

Where ψ represents the muon field.

$$\mathcal{L}_{\text{MASS}} \to -m^2_{\tilde{\mu}_1}\tilde{\mu}_1{}^*\tilde{\mu}_1 - m^2_{\tilde{\mu}_2}\tilde{\mu}_2{}^*\tilde{\mu}_2$$

$$\begin{aligned}\mathcal{L}_{\text{Scalar-Photon}} \to \frac{i}{2}eA_\nu\Big[&\sin^2\phi(\tilde{\mu}_1\partial^\nu\tilde{\mu}_1{}^* - \tilde{\mu}_1{}^*\partial^\nu\tilde{\mu}_1) + \cos^2\phi(\tilde{\mu}_2\partial^\nu\tilde{\mu}_2{}^* - \tilde{\mu}_2{}^*\partial^\nu\tilde{\mu}_2)\\ &+\sin\phi\cos\phi(\tilde{\mu}_1\partial^\nu\tilde{\mu}_2{}^* + \tilde{\mu}_2\partial^\nu\tilde{\mu}_1{}^* - \tilde{\mu}_2{}^*\partial^\nu\tilde{\mu}_1 - \tilde{\mu}_1{}^*\partial^\nu\tilde{\mu}_2)\\ &+\sin^2\phi(\tilde{\mu}_1\partial^\nu\tilde{\mu}_1{}^* - \tilde{\mu}_1{}^*\partial^\nu\tilde{\mu}_1) + \cos^2\phi(\tilde{\mu}_2\partial^\nu\tilde{\mu}_2{}^* - \tilde{\mu}_2{}^*\partial^\nu\tilde{\mu}_2)\\ &-\sin\phi\cos\phi(\tilde{\mu}_1\partial^\nu\tilde{\mu}_2{}^* + \tilde{\mu}_2\partial^\nu\tilde{\mu}_1{}^* - \tilde{\mu}_2{}^*\partial^\nu\tilde{\mu}_1 - \tilde{\mu}_1{}^*\partial^\nu\tilde{\mu}_2)\Big]\end{aligned}$$

$$= \frac{i}{2}eA_\nu[\tilde{\mu}_1\partial^\nu\tilde{\mu}_1{}^* - \tilde{\mu}_1{}^*\partial^\nu\tilde{\mu}_1 + \tilde{\mu}_2\partial^\nu\tilde{\mu}_2{}^* - \tilde{\mu}_2{}^*\partial^\nu\tilde{\mu}_2]$$

F.2 Feynman Rules and Diagrams I-IV

The Feynman Rules are similar to those of Section E.2 except for the changes illustrated in Fig. F.1. Note that there is no $\tilde{\mu}_1\tilde{\mu}_2\gamma$ vertex (Fig. F.1a) but the helicity-flipped photino propagator is now utilized.

In Fig. F.2 the lowest level contributions to $\mu_L+\gamma \to \mu_R$ are illustrated. Diagrams I-IV are reminiscent of the contributions of Fig. E.1. Indeed, if $m_{\tilde{\mu}_1} = m_{\tilde{\mu}_2}$, it is evident that these would sum to the unmixed result (we could rotate the mixing away all together and make diagrams V-VIII vanish recovering the previous configuration). The new features appear in diagrams V to VIII. Here the helicity flip takes place in the photino line instead of the muon.

We may evaluate the first four diagrams by direct comparison with those of the previous appendix. Comparing I and II of Fig. F.2 with I of Fig. E.1

$$\Delta a_\mu^{(\mathrm{I})} = \frac{\alpha}{2\pi}\, I_1 \cos^2\phi \qquad \Delta a_\mu^{(\mathrm{II})} = \frac{\alpha}{2\pi}\, I_2 \sin^2\phi \tag{F.3}$$

$$I_{1,2} = \int_0^1 dx \int_x^1 dz \, \frac{z(1-z)}{-z^2 + \frac{\overline{m}_{1,2}^2}{m_\mu^2}\, z - \frac{m_\lambda^2}{m_\mu^2}} \tag{F.4}$$

$$\overline{m}_{1,2}^2 = m_\lambda^2 - m_{\tilde{\mu}_{1,2}}^2 + m_\mu^2 \tag{F.5}$$

and, as before, we note that we can write this using

$$\int_0^1 dx \int_x^1 dz = \int_0^1 dz \int_0^z d\tilde{z} = \int_0^1 dz\, z \tag{F.6}$$

since there is no x dependence and $\tilde{z} = 2x - z$. Similarly diagrams III and IV give

$$\Delta a_\mu^{(\mathrm{III})} = \frac{\alpha}{2\pi}\, I_1 \sin^2\phi \qquad \Delta a_\mu^{(\mathrm{IV})} = \frac{\alpha}{2\pi}\, I_2 \cos^2\phi \tag{F.7}$$

and so the sum of the first four diagrams gives

$$\Delta a_\mu^{(\mathrm{I-IV})} = -\frac{\alpha}{2\pi}\,(I_1 + I_2)\ . \tag{F.8}$$

This in itself would place no sterner limits than did (E.23):

$$\Delta a_\mu^{(unmixed)} = \frac{\alpha}{2\pi}\,(I_L + I_R)\ .$$

In the limit $m_{\tilde{\mu}_{1,2}} \gg m_\lambda \gg m_\mu$ (F.8) becomes

$$\Delta a_\mu = -\alpha^2\pi \left\{ \frac{m_\mu^2}{m_{\tilde{\mu}_1}^2} + \frac{m_\mu^2}{m_{\tilde{\mu}_2}^2} \right\}$$

which is essentially the same as the unmixed version

$$\Delta a_\mu = -\frac{\alpha}{2\pi}\left\{\frac{m_\mu^2}{m_{\tilde{\mu}_L}^2} + \frac{m_\mu^2}{m_{\tilde{\mu}_R}^2}\right\}$$

presented in (E.24).

F.3 Evaluation of Diagrams V-VIII

Diagram V:

$$\begin{aligned}\mathcal{M}^\mu &= -ie\,\psi_+^\beta(p',s')\Gamma_{\rm V}^\mu(p',s')_\beta{}^\alpha\psi_{-\alpha}(p,s)\\ &= (-ie)^3(\sqrt{2})^2(i^3)\int\frac{d^4k}{(2\pi)^4}\\ &\quad\times\frac{\psi_+^\beta\delta_\beta{}^\alpha\psi_{-\alpha}m_\lambda(p+p'-2k)^\mu\sin\phi\cos\phi}{(k^2-m_\lambda^2)[(p-k)^2-m_{\tilde{\mu}_1}^2][(p'-k)^2-m_{\tilde{\mu}_1}^2]}\,.\end{aligned} \tag{F.9}$$

Following the steps detailed in Section E.3 we find ($\overline{m}_1^2 = m_\lambda^2 - m_{\tilde{\mu}_1}^2 + m_\mu^2$)

$$\psi_+\Gamma_{\rm V}^\mu\psi_- = \frac{ie^2}{8\pi^2}\,m_\lambda\int_0^1 dx\int_0^{1-x} dy\;\frac{i\psi_+^\alpha\psi_{-\alpha}(p+p'-2[px-p'y])^\mu\sin\phi\cos\phi}{\overline{m}_1^2(x+y)-m_\lambda^2-m_\mu^2(x+y)^2-q^2xy}\,. \tag{F.10}$$

Performing the "$b \leftrightarrow c$" trick as when we went from (E.6) to (E.11)

$$\begin{aligned}\psi_+\Gamma_{\rm V}^\mu\psi_- &= \frac{\alpha m_\lambda}{2\pi}\sin\phi\cos\phi\,\psi_+\psi_-\int_0^1 dx\int_0^{1-x} dy\\ &\quad\times\frac{(p+p')^\mu(1-x-y)}{-[m_\mu^2(x+y)^2+\overline{m}_1^2(x+y)-m_\lambda^2-q^2xy]}\,.\end{aligned} \tag{F.11}$$

From Appendix H or Eqn. (D.23), or simply following the step from (E.17) to

(E.19),we use the Gordon decomposition to find on shell ($q^2 = 0$)

$$\Delta a_\mu^{(\mathrm{V})} = \frac{\alpha}{2\pi} \frac{m_\mu m_\lambda}{m_\mu^2} \mathcal{I}_1 \tag{F.12}$$

with

$$\begin{aligned}\mathcal{I}_1 &= \int_0^1 dx \int_0^{1-x} dy \frac{1-x-y}{-(x+y)^2 + \frac{\overline{m}_1^2}{m_\mu^2}(x+y) - \frac{m_\lambda^2}{m_\mu^2}} \\ &= \frac{m_\mu^2}{\overline{m}_1^2} \int_0^1 dz \int_0^z d\widetilde{z} \frac{1-z}{-\frac{m_\mu^2}{\overline{m}_1^2} z^2 + z - \mathcal{R}_1} \\ z &= x+y \qquad \widetilde{z} = x - y \qquad \mathcal{R}_1 = m_\lambda^2/\overline{m}_1^2\end{aligned} \tag{F.13}$$

where we have used notation similar to that of (E.26).

Diagram V only provides the $\psi_+\sigma^{\mu\nu}q_\nu\psi_-$ part of this term. The $\overline{\psi}_-\overline{\sigma}^{\mu\nu}q_\nu\psi_+$ piece comes from Diagram VII.

In the most interesting limit, when $m_\lambda, m_{\tilde{\mu}_1} \gg m_\mu$ (and assuming $\overline{m}_1 \gg m_\mu$) we may expand this in powers of $m_\mu^2/\overline{m}_1^2$. The leading order term is easy to find:

$$\begin{aligned}\mathcal{I}_1 &= \frac{m_\mu^2}{\overline{m}_1^2} \int_0^1 dz \frac{z(1-z)}{z - \mathcal{R}_1} = \frac{m_\mu^2}{\overline{m}_1^2} \int_{-\mathcal{R}_1}^{1-\mathcal{R}_1} d\varsigma \frac{-\varsigma^2 + (1-2\mathcal{R}_1)\varsigma + \mathcal{R}_1(1-\mathcal{R}_1)}{\varsigma} \\ &= \frac{m_\mu^2}{\overline{m}_1^2} \left\{ \tfrac{1}{2}(1-2\mathcal{R}_1) + \mathcal{R}_1(1-\mathcal{R}_1)\ell n\left(\frac{\mathcal{R}_1 - 1}{\mathcal{R}_1}\right)\right\}\end{aligned} \tag{F.14}$$

which may be written, including the next order term in m_μ^2,

$$\mathcal{I}_1 = \frac{m_\mu^2 \left\{ -\frac{1}{2}\left(m_{\tilde{\mu}_1}^4 - m_\lambda^4\right) + m_{\tilde{\mu}_1}^2 m_\lambda^2 \ell n \frac{m_{\tilde{\mu}_1}^2}{m_\lambda^2} \right\}}{(m_{\tilde{\mu}_1}^2 - m_\lambda^2)^3} + \frac{m_\mu^4 G(m_{\tilde{\mu}_1}, m_\lambda)}{(m_{\tilde{\mu}_1}^2 - m_\lambda^2)^2} + \mathcal{O}(m_\mu^6) \tag{F.15}$$

where

$$G(m_{\tilde{\mu}_1}, m_\lambda) =$$

$$\frac{(m_{\tilde{\mu}_1}^2 + 2m_\lambda^2)(5m_{\tilde{\mu}_1}^2 + m_\lambda^2) + 6m_{\tilde{\mu}}^2 m_\lambda^2}{6(m_{\tilde{\mu}}^2 - m_\lambda^2)^2} - \frac{2m_{\tilde{\mu}_1}^2 m_\lambda^2}{(m_{\tilde{\mu}_1}^2 - m_\lambda^2)^3} \ell n \frac{m_{\tilde{\mu}_1}^2}{m_\lambda^2} \tag{F.16}$$

Eqn. (F.14) agrees with Griffols and Mendez[35] and, with some algebra, Ellis, Hagelin and Nanopoulos[8] .

The contributions of diagrams VI and VII are the same as the above except that $\tilde{\mu}_1 \to \tilde{\mu}_2$ and the overall additional factor of -1 coming from $-\sin\theta$ at one of the Yukawa vertices.

Thus the term of Δa_μ which is *linear* in m_μ is

$$\Delta a_\mu^{\text{linear}} = \frac{\alpha}{2\pi} \sum_{j=1}^{2} \frac{m_\mu m_\lambda \sin\phi\cos\phi}{(m_{\tilde{\mu}_j}^2 - m_\lambda^2)^3} \left[\tfrac{1}{2}\left(m_{\tilde{\mu}_j}^4 - m_\lambda^4\right) - m_\lambda^2 m_{\tilde{\mu}_j}^2 \, \ell n \, \frac{m_{\tilde{\mu}_j}^2}{m_\lambda^2} \right] . \tag{F.17}$$

When $m_{\tilde{\mu}_i} \approx m_\lambda$ the above formulation would appear to be invalid since $|m_{\tilde{\mu}} - m_\lambda|$ might be comparable to m_μ terms which were discarded. If $m_{\tilde{\mu}_j} \gg m_\mu$, however, then (F.17) is still valid and yields

$$\Delta a_\mu(m_{\tilde{\mu}} = m_\lambda) \to -\frac{\alpha}{2\pi}\frac{m_\mu}{m_\lambda}\sin\phi\cos\phi \, . \tag{F.18}$$

Equation (F.17) can also be written as

$$\begin{aligned} \Delta a_\mu &= C(f(X_2) - f(X_1)) \\ C &= \frac{\alpha}{2\pi}\frac{m_\mu}{m_\lambda}\sin\phi\cos\phi \\ f(x) &= \frac{\frac{1}{2}(x^2 - 1) - x\ell n\, x}{(x-1)^3} \qquad f(1) = \frac{1}{6} \\ X_i &= m_{\tilde{\mu}_i}^2 / m_\lambda^2 \, . \end{aligned} \tag{F.19}$$

F.4 A NOTE ON AXIAL COUPLINGS

Griffols and Mendez[35] originally considered the possibility of a more general vertex coupling of the form

$$\overline{\psi}\,(g_V + g_A\gamma_5)\lambda\tilde{\mu}_i\ .$$

This is more in keeping with the full electroweak theory. Since

$$\overline{\psi}\,\gamma^5\psi = \psi^{\alpha}_{+}\psi_{-\alpha} - \overline{\psi}_{-\dot{\alpha}}\overline{\psi}^{\dot{\alpha}}_{+}$$

we see that this really implies that $\overline{\psi}_{+}\overline{\lambda}$ and $\psi_{-}\lambda$ may couple with unequal coefficients. The relative Yukawa strengths of the vertices and a few of the salient diagrams are illustrated in Fig. F.3.

We note that $g_V - g_A$ comes from the $\tilde{\mu}_R$ part of $\tilde{\mu}_1$ while $g_V + g_A$ stems from the $\tilde{\mu}_L$ part of $\tilde{\mu}_1$. Thus diagrammatic contributions where the helicity flips in the photino propagator go as ($\tilde{\mu}_L$ part of $\tilde{\mu}_1$) ($\tilde{\mu}_R$ part of $\tilde{\mu}_1$) or $g_V^2 - g_A^2$, which is like $\sin\phi\cos\phi$ in the notation which we have adopted. The other two contributions are like $(g_V - g_A)^2 + (g_V + g_A)^2 \propto g_V^2 + g_A^2$. Thus in this case we would find

$$\Delta a_\mu = -\frac{(g_V^2 - g_A^2)}{\pi^2}\sum_{j=1}^{2}(-1)^j\,\frac{m_\mu m_\lambda}{(m^2_{\tilde{\mu}_j} - m^2_\lambda)^3}$$

$$\times\left[\tfrac{1}{2}\,(m^4_{\tilde{\mu}_j} - m^4_\lambda) - m^2_\lambda m^2_{\tilde{\mu}_j}\,\ell n\,\frac{m^2_{\tilde{\mu}_j}}{m^2_\lambda} + m^2_\mu(m^2_{\tilde{\mu}_i} - m^2_\lambda)G(m_{\tilde{\mu}_i}, m_\lambda)\right]$$

$$-\frac{(g_V^2 + g_A^2)^2}{6\pi^2}\sum_{j=1}^{2}\frac{m^2_\mu}{m^2_{\tilde{\mu}_j}}.$$

F.5 SMALL MASS-SPLITTING

When $m_{\tilde{\mu}_1} \approx m_{\tilde{\mu}_2}$ we may simplify the expression in (F.19).

Letting

$$\delta m^2 = m^2_{\tilde{\mu}_1} - m^2_{\tilde{\mu}_1} \qquad m_{\tilde{\mu}} = m_{\tilde{\mu}_{1,2}}$$

we find (for $m_\lambda < m_{\tilde{\mu}}$)

$$\Delta a_\mu = -\frac{\delta m^2}{m^2_{\tilde{\mu}}} \frac{\alpha}{4\pi} \sin\phi\cos\phi \, \frac{m_\lambda m_\mu m^6_{\tilde{\mu}}}{(m^2_{\tilde{\mu}} - m^2_\lambda)^4}$$
$$\times \left\{ 1 + 4\, \frac{m^2_\lambda}{m^2_{\tilde{\mu}}} \left[1 - \ln \frac{m^2_{\tilde{\mu}}}{m^2_\lambda} \right] - \frac{m^4_\lambda}{m^4_{\tilde{\mu}}} \left[5 + 2 \ln \frac{m^2_{\tilde{\mu}}}{m^2_\lambda} \right] \right\} \tag{F.20}$$

or

$$\Delta a_\mu = -\frac{\delta m^2}{m^2_{\tilde{\mu}}} \frac{\alpha}{4\pi} \sin\phi\cos\phi \, \frac{m_\mu}{m_\lambda} \, p(x) \tag{F.21}$$

with

$$x = m^2_{\tilde{\mu}}/m^2_\lambda$$

$$p(x) = \frac{x}{(x-1)^4} \, \{(x+5)(x-1) - 2(2x+1)\ell n\, x\}$$

$$p(x \to 0) \to -4x\ell n\, x$$

$$p(x \to 1) \to \frac{1}{6}$$

$$p(x \to \infty) \to \frac{1}{x} \, .$$

When we further have $m_\lambda \ll m_{\tilde{\mu}}$ (i.e. $x \gg 1$) then

$$\Delta a_\mu \doteq -\frac{\alpha}{4\pi} \sin\phi\cos\phi \, \frac{m_\mu m_\lambda}{m^2_{\tilde{\mu}}} \cdot \frac{\delta m^2}{m^2_{\tilde{\mu}}} \, . \tag{F.22}$$

F.6 Extrema of the Linear Term

In Chapter 2 we saw that it is the term linear in m_μ which places the most rigorous limits when compared with the "real world". We may divide Δa_μ into

$$\Delta a_\mu = \Delta a_\mu^2 - \Delta a_\mu^1 \qquad \Delta a_\mu^i = Cf(X_i) \tag{F.23}$$

using the notation of (F.19). The function $f(x)$ has been plotted in Fig. G.1 for $\Delta a_\mu^{\text{linear}}$. All of the plots related to the linear terms have been collected into Appendix G. We immediately see from the form of $f(x)$ that there will be an extremum in $\Delta a_\mu^{lin} = C(f(X_2) - f(X_1))$ when $X_1 \neq X_2$. Differentiating to find the maximum Δa_μ^{lin} we find that it occurs when

$$\ln x_1 = \frac{y\left[(x_1y+5)(x_1y-1) - 2(2x_1y+1)\ln y\right](x_1-1)^4 - (x_1+5)(x_1-1)(x_1y-1)^4}{2\left[y(2x_1y+1)(x_1-1)^4 - (2x_1+1)(x_1y-1)^4\right]} \tag{F.24}$$

(where $y \neq 1$ when $x_1 \neq 1$. Here $x_1 = X_1$.)

$$X_1 = \frac{m_{\tilde{\mu}_1}^2}{m_\lambda^2} \qquad X_2 = \frac{m_{\tilde{\mu}_2}^2}{m_\lambda^2} \qquad y = \frac{X_2}{X_1} = \frac{m_{\tilde{\mu}_2}^2}{m_{\tilde{\mu}_1}^2}\,. \tag{F.25}$$

This extremum, a maximum for $|\Delta a_\mu^{lin}|$, we shall call $\Delta a_{\mu\,\max}$. For a given value of y it occurs at $X_1 = X_{1\,\max}$ and so $\Delta a_{\mu\,\max} = \Delta a_{\mu\,\max}(y)$ and $X_{1\,\max} = X_{1\,\max}(y)$ while $\Delta a_{\mu\,\max} = \Delta a_\mu(X_{1\,\max})$. In he next appendix we have plotted $f(x)$, $\Delta a_\mu(X)$ for $y = 1.01$ and $y = 2$, $\Delta a_{\mu\,\max}(y)$ and $X_{1\,\max}(y)$ for a range of parameters. Discussion takes place in Chapter 2.

FIGURE CAPTIONS

1. Vertices and Propagators

   (a) Photon-Scalar Muon (smuon) Vertices.

   (b) Photino Yukawa Vertices.

   (c) Photino Propagator illustrating flipping and non-flippling propagation.

2. Lowest order contributions to Δa_{μ}.

3. Sample Diagrams from $\overline{\psi}(g_V + g_A\gamma_5)\lambda\tilde{\mu}$ Vertex.

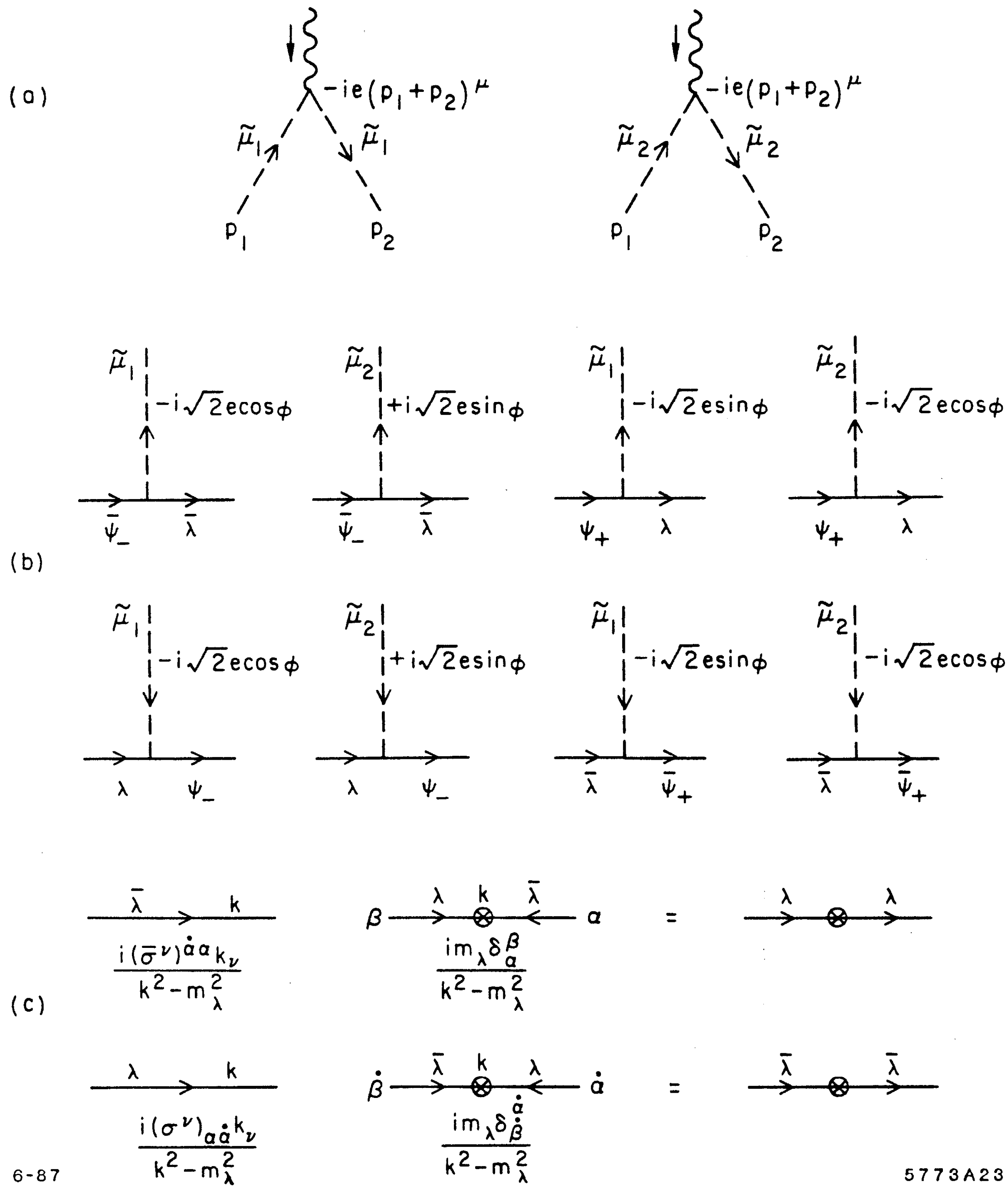

(a)
$-ie(p_1+p_2)^\mu$
$\tilde{\mu}_1$ $\tilde{\mu}_1$
p_1 p_2
$-ie(p_1+p_2)^\mu$
$\tilde{\mu}_2$ $\tilde{\mu}_2$
p_1 p_2
(b)
$\tilde{\mu}_1$ $-i\sqrt{2}e\cos\phi$ $\bar{\psi}_-$ $\bar{\lambda}$
$\tilde{\mu}_2$ $+i\sqrt{2}e\sin\phi$ $\bar{\psi}_-$ $\bar{\lambda}$
$\tilde{\mu}_1$ $-i\sqrt{2}e\sin\phi$ ψ_+ λ
$\tilde{\mu}_2$ $-i\sqrt{2}e\cos\phi$ ψ_+ λ
$\tilde{\mu}_1$ $-i\sqrt{2}e\cos\phi$ λ ψ_-
$\tilde{\mu}_2$ $+i\sqrt{2}e\sin\phi$ λ ψ_-
$\tilde{\mu}_1$ $-i\sqrt{2}e\sin\phi$ $\bar{\lambda}$ $\bar{\psi}_+$
$\tilde{\mu}_2$ $-i\sqrt{2}e\cos\phi$ $\bar{\lambda}$ $\bar{\psi}_+$
(c)
$\bar{\lambda}$ k
$\frac{i(\bar{\sigma}^\nu)^{\dot{\alpha}\alpha}k_\nu}{k^2-m_\lambda^2}$
β λ k $\bar{\lambda}$ α
$\frac{im_\lambda\delta_\alpha^\beta}{k^2-m_\lambda^2}$
=
λ λ
λ k
$\frac{i(\sigma^\nu)_{\alpha\dot{\alpha}}k_\nu}{k^2-m_\lambda^2}$
$\dot{\beta}$ $\bar{\lambda}$ k λ $\dot{\alpha}$
$\frac{im_\lambda\delta_{\dot{\beta}}^{\dot{\alpha}}}{k^2-m_\lambda^2}$
=
$\bar{\lambda}$ $\bar{\lambda}$
6-87
5773A23

Fig. F.1

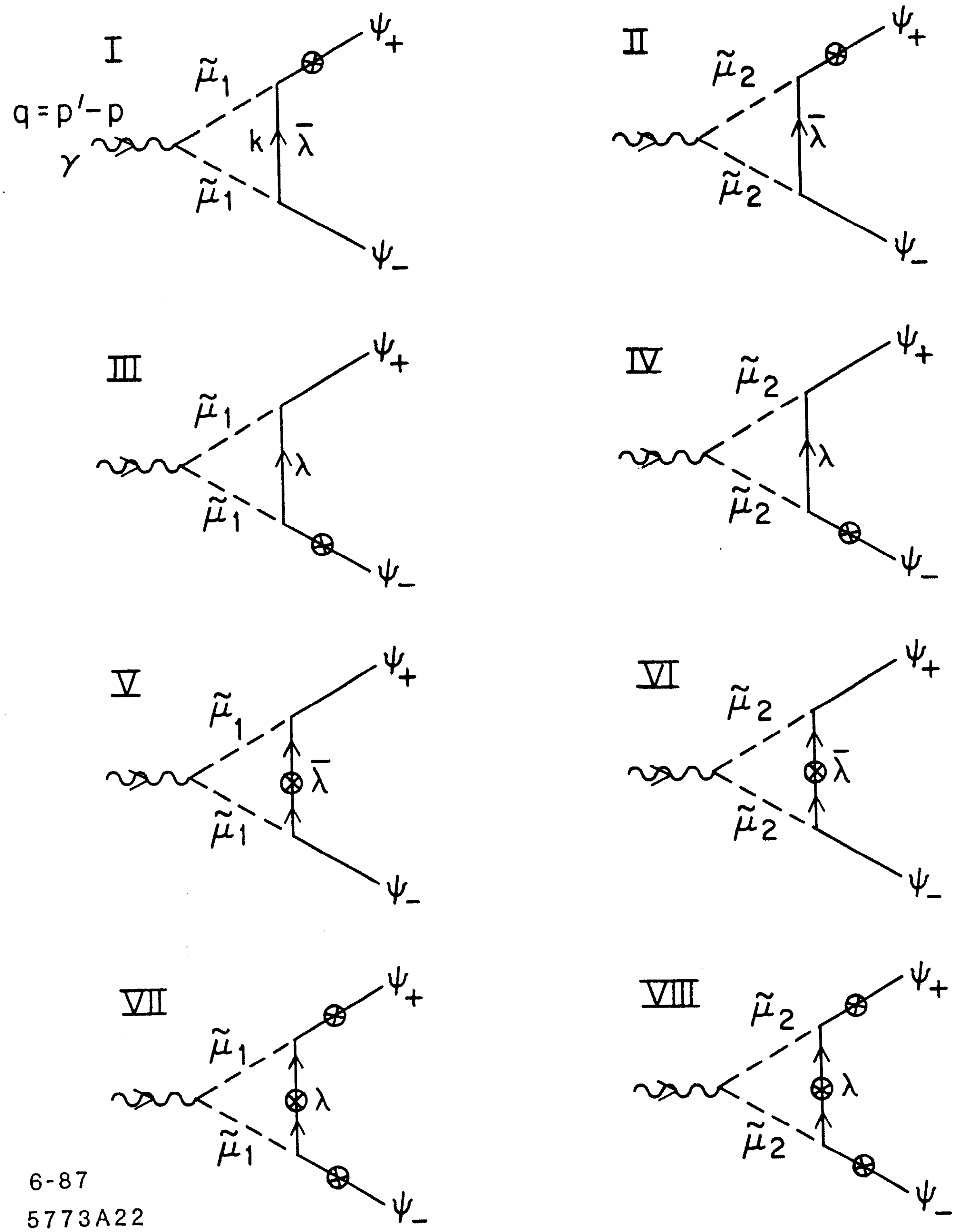
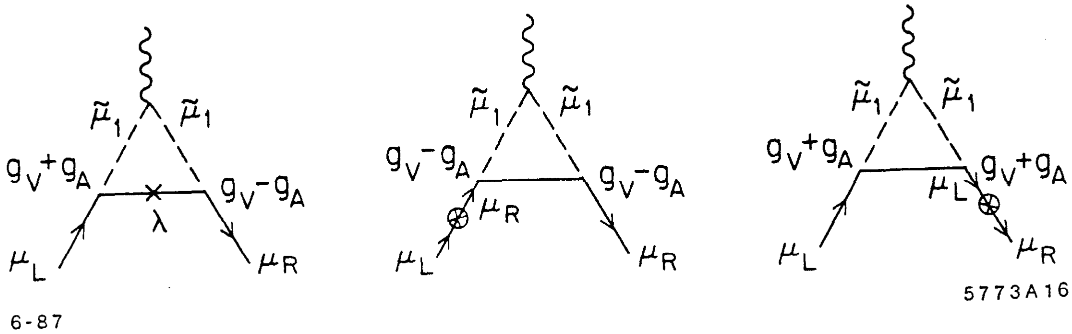

I
q = p′ − p
γ
$\tilde{\mu}_1$
$\tilde{\mu}_1$
k
$\bar{\lambda}$
ψ_+
ψ_-
II
$\tilde{\mu}_2$
$\tilde{\mu}_2$
$\bar{\lambda}$
ψ_+
ψ_-
III
$\tilde{\mu}_1$
$\tilde{\mu}_1$
λ
ψ_+
ψ_-
IV
$\tilde{\mu}_2$
$\tilde{\mu}_2$
λ
ψ_+
ψ_-
V
$\tilde{\mu}_1$
$\tilde{\mu}_1$
$\bar{\lambda}$
ψ_+
ψ_-
VI
$\tilde{\mu}_2$
$\tilde{\mu}_2$
$\bar{\lambda}$
ψ_+
ψ_-
VII
$\tilde{\mu}_1$
$\tilde{\mu}_1$
λ
ψ_+
ψ_-
VIII
$\tilde{\mu}_2$
$\tilde{\mu}_2$
λ
ψ_+
ψ_-
6-87
5773A22

Fig. F.2



$\tilde{\mu}_1$
$\tilde{\mu}_1$
g_V+g_A
g_V-g_A
λ
μ_L
μ_R
6-87
$\tilde{\mu}_1$
$\tilde{\mu}_1$
g_V-g_A
g_V-g_A
μ_R
μ_L
μ_R
$\tilde{\mu}_1$
$\tilde{\mu}_1$
g_V+g_A
g_V+g_A
μ_L
μ_L
μ_R
5773A16

Fig. F.3

APPENDIX G

Plots Related to Anomalous Moment

In this appendix we plot some of the results derived in the previous appendix. In each instance this is for the case of smuon mixing. The states $\widetilde{\mu}_L$ and $\widetilde{\mu}_R$ mix with angle ϕ to produce the eigenstates $\widetilde{\mu}_1$ and $\widetilde{\mu}_2$. Letting

$$X_i = m^2_{\widetilde{\mu}_i}/m^2_\lambda \qquad i = 1,2 \qquad y = m^2_{\widetilde{\mu}_2}/m^2_{\widetilde{\mu}_1}$$

$$C = \frac{\alpha}{2\pi}\,\frac{m_\mu}{m_\lambda}\sin\phi\cos\phi$$

we found that Δa_μ, the *additional* contribution to a_μ from supersymmetric smuon mixing, was given by

$$\Delta a_\mu = C[f(x_2) - f(x_1)]$$

where $f(x)$ is given in Eq. (F19).

FIGURE CAPTIONS

Fig. G.1

$f(x)$ versus x.

Fig. G.2

$10^4 \times \frac{1}{C}\ \Delta a_\mu$ versus X_1 for $y = 1.01$.

Fig. G.3

$\frac{1}{C}\ \Delta a_\mu$ versus X_1 for $y = 2$.

Fig. G.4

$X_{1\,\mathrm{max}}(y)$ versus y where $X_{1\,\mathrm{max}}$ is the value of X_1 where $\frac{1}{C}\ |\Delta a_\mu^{lin}|$ achieves its extremal value for a given y.

Fig. G.5

$\frac{1}{C}\ (\Delta a_\mu^{lin})_{\mathrm{max}}$ versus y for y ranging from 0 to 10 where $\frac{1}{C}\ (\Delta a_\mu^{lin})_{\mathrm{max}}$ is the extremal value of $\frac{1}{C}\ (\Delta a_\mu^{lin})$ for a given y.

Fig. G.6

$\frac{1}{C}\ (\Delta a_\mu^{lin})_{\mathrm{max}}$ in vicinity of $y = 1$.

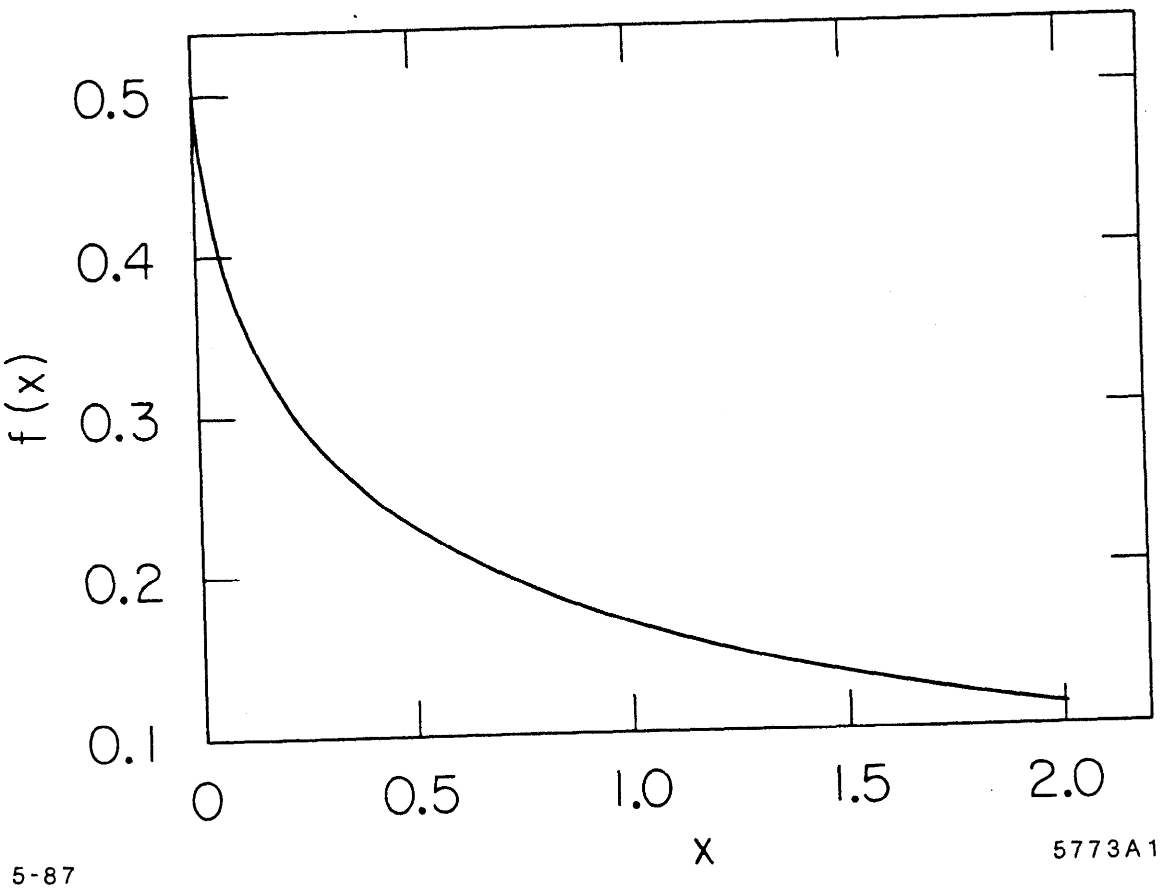
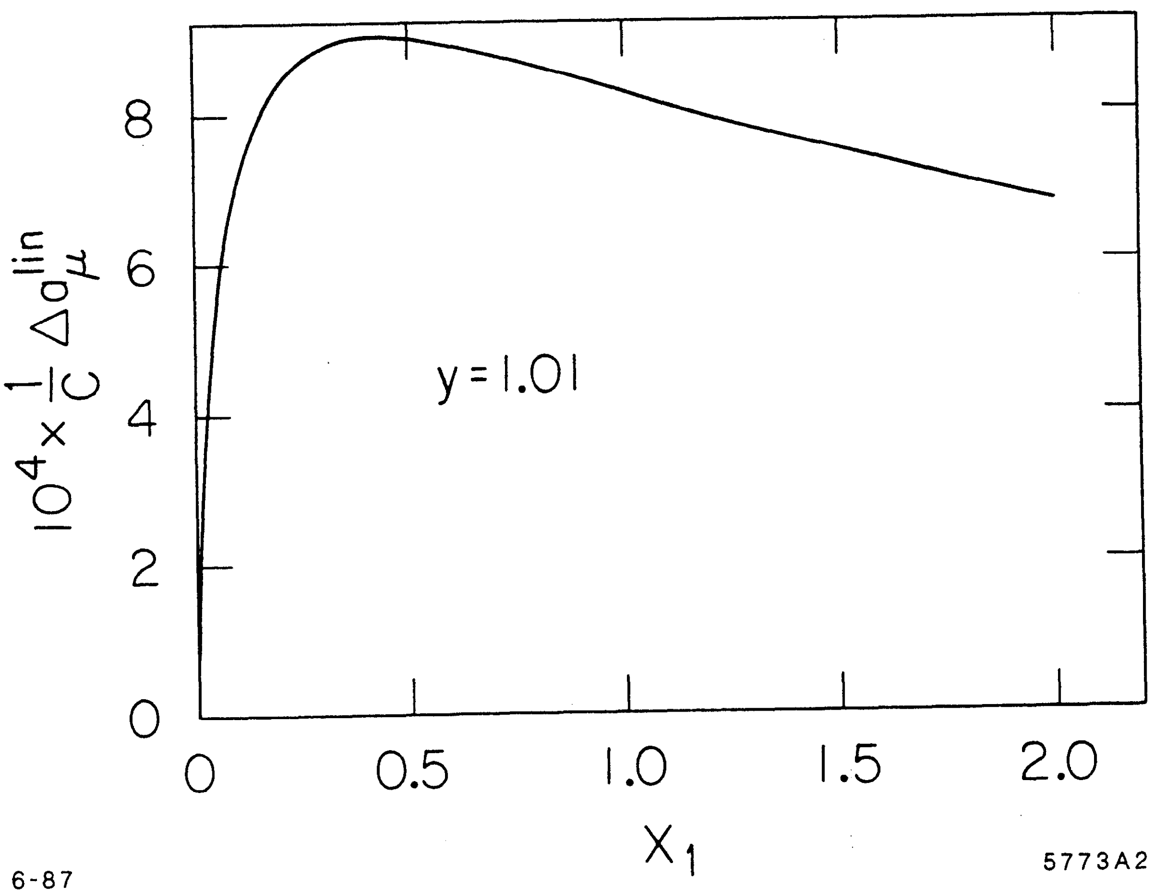


Fig. G.1



$10^4 \times \frac{1}{C} \Delta a_\mu^{lin}$
0
2
4
6
8
y = 1.01
0
0.5
1.0
1.5
2.0
X_1
6-87
5773A2

Fig. G.2

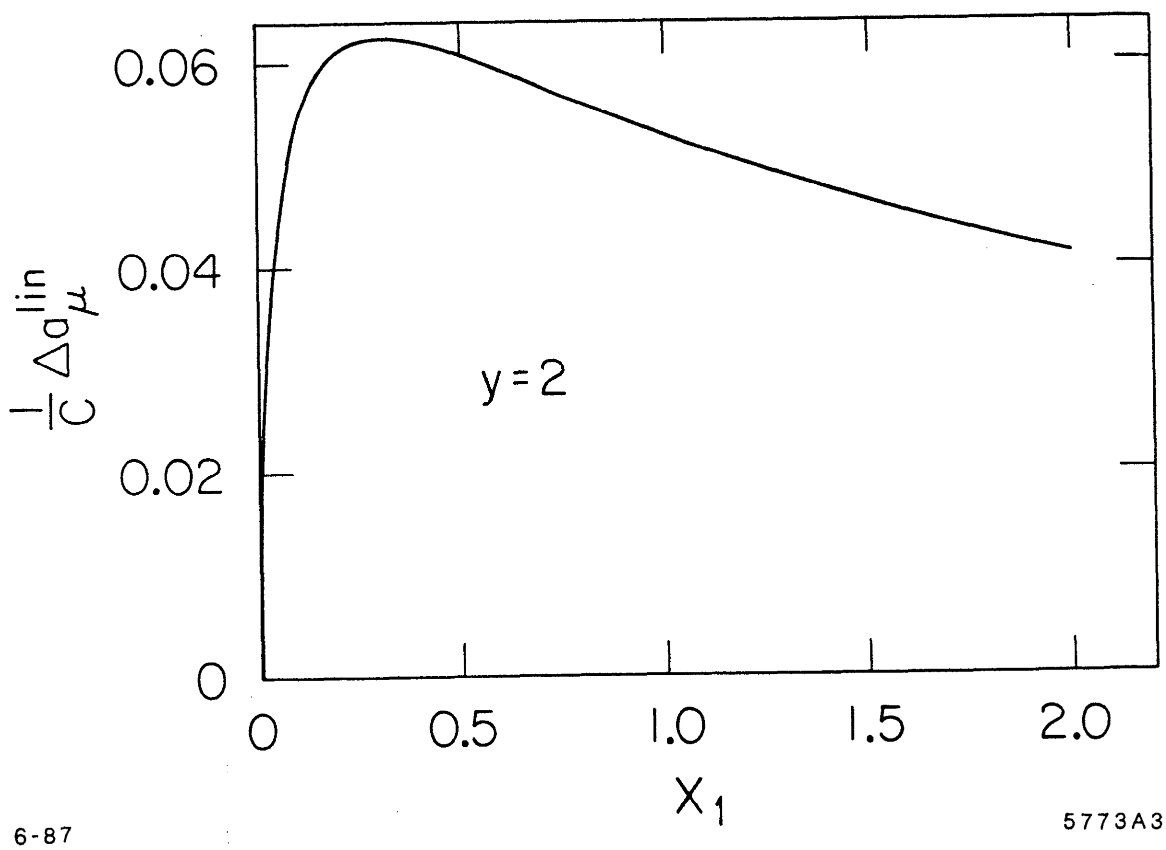


Fig. G.3

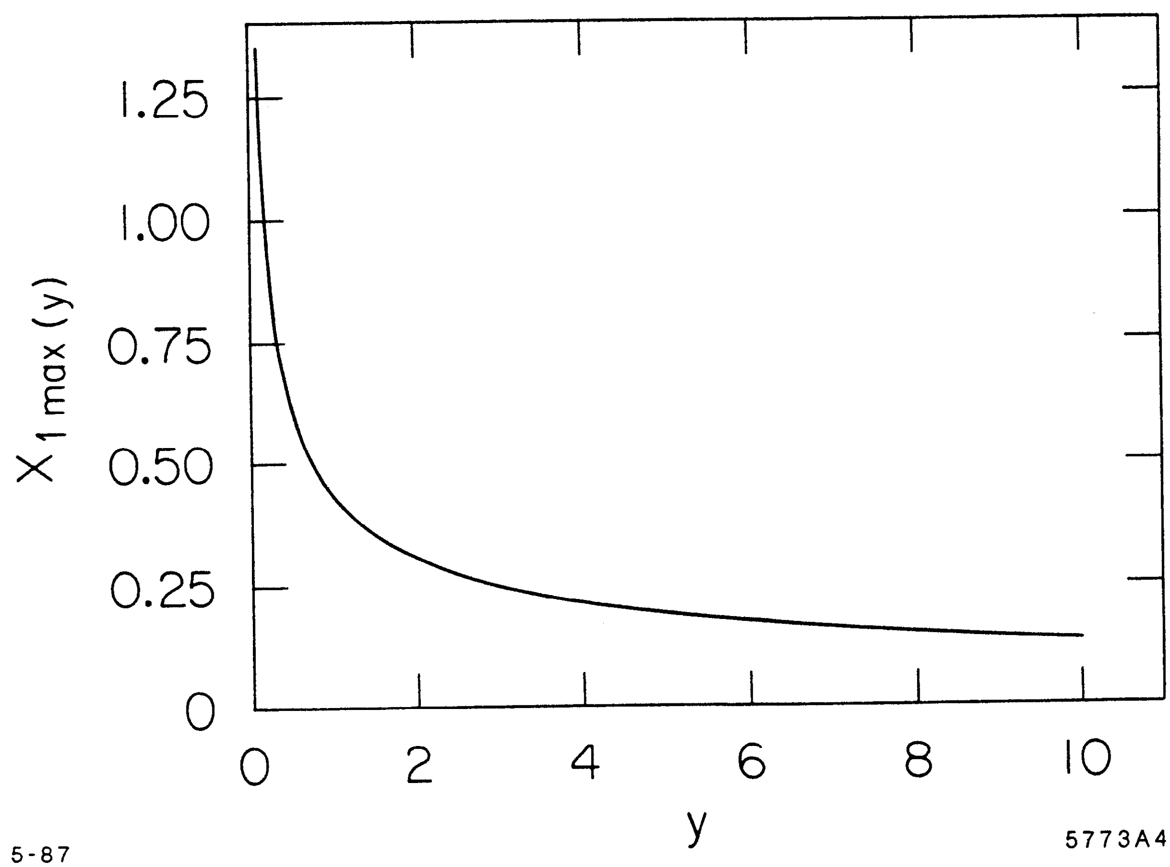


Fig. G.4

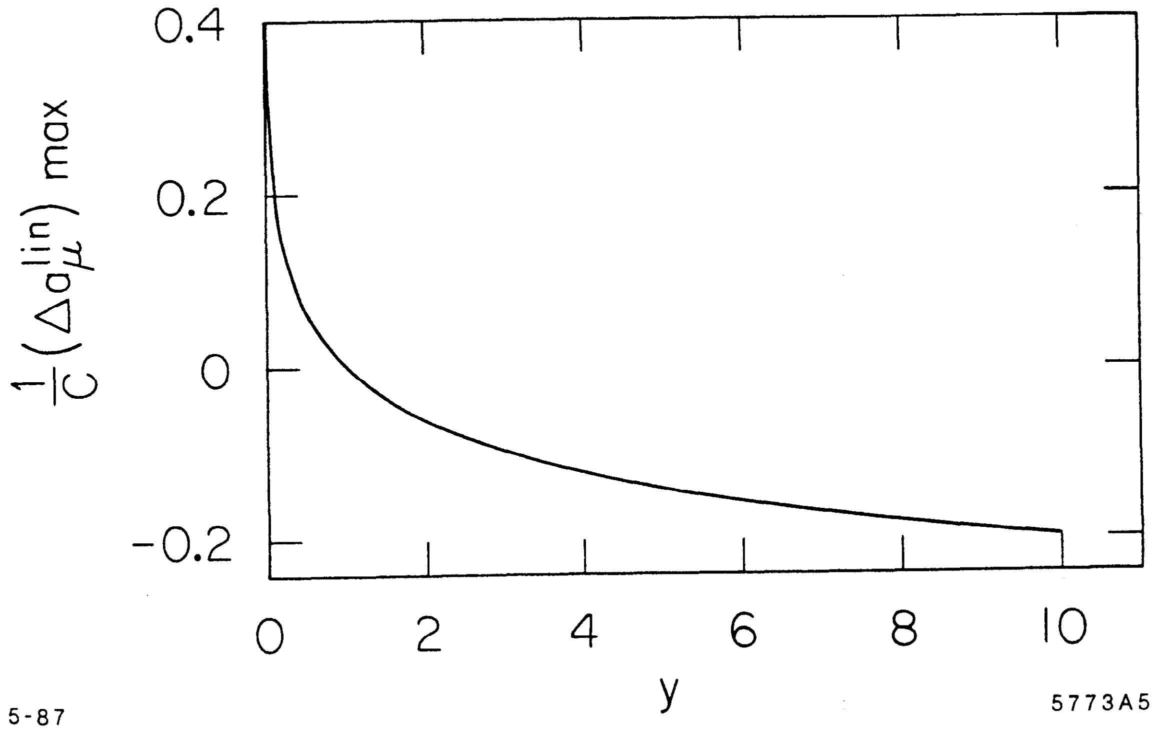


Fig. G.5

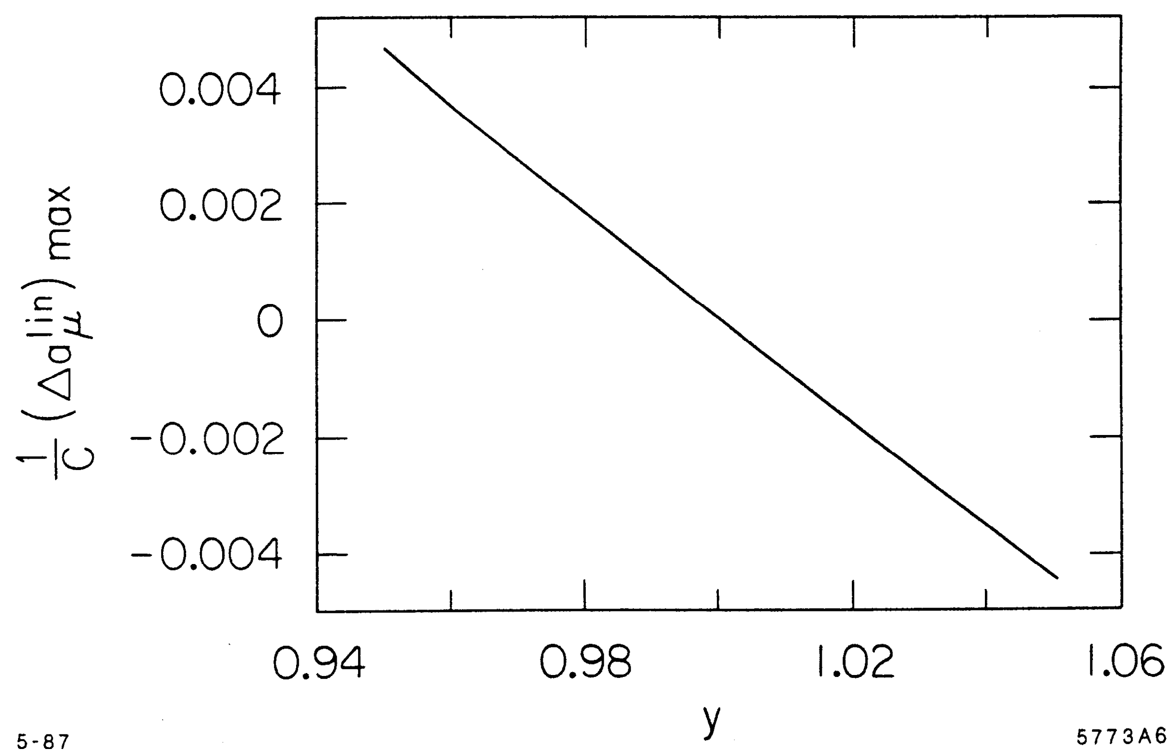


Fig. G.6

APPENDIX H

Useful Relationships

In this appendix we gather certain algebraic formulae which are frequently referred to in this paper.

H.1 FEYNMAN INTEGRALS

A number of "tricks", due to Feynman, permit us to re-parameterize the momentum integrals which frequently arise in the computation of Feynman diagrams. A general expression is[36]

$$\frac{1}{D_1^{p_1} D_2^{p_2} \cdots D_N^{p_N}} =$$

$$\frac{\Gamma(p_1+p_2+\cdots+p_N)}{\Gamma(p_1)\Gamma(p_2)\cdots\Gamma(p_N)} \int_0^1 dx_1 \cdots \int_0^1 dx_N \frac{x_1^{p_1-1} x_2^{p_2-1} \cdots x_N^{p_N-1} \delta(x_1+\cdots+x_N)}{(D_1 x_1 + \cdots + D_N x_N)^{p_1+\cdots+p_N}} \tag{H.1}$$

We will use a slightly different form of this identity.[2] The particular cases of interest are

$$\frac{1}{ab} = \int_0^1 dx \, \frac{1}{[ax+b(1-x)]^2} \tag{H.2}$$

$$\frac{1}{a^2 b} = 2\int_0^1 dx \, \frac{x}{[ax+b(1-x)]^3} \tag{H.3}$$

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} dy \, \frac{1}{[a+(b-a)x+(c-a)y]^3} \tag{H.4}$$

$$\frac{1}{a^2 bc} = 6\int_0^1 dx \int_0^{1-x} dy \, \frac{1-x-y}{[a+(b-a)x+(c-a)y]^4} \tag{H.5}$$

These are particularly useful in conjunction with

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - 2p\cdot k + s - i\epsilon)^3} = \frac{i}{32\pi^2(s-p^2)} \tag{H.6 a}$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{(k^2 - 2p\cdot k + s - i\epsilon)^3} = \frac{ip^\mu}{32\pi^2(s-p^2)} \tag{H.6 b}$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - 2p\cdot k + s - i\epsilon)^4} = \frac{i}{96\pi^2(s-p^2)^2} \tag{H.7 a}$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{(k^2 - 2p\cdot k + s - i\epsilon)^4} = \frac{ip^\mu}{96\pi^2(s-p^2)^2} \tag{H.7 b}$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k^\nu}{(k^2 - 2p\cdot k + s - i\epsilon)^4} = \frac{i[p_\mu p^\nu + \frac{1}{2}(s-p^2)\delta_\mu{}^\nu]}{96\pi^2(s-p^2)^2} \tag{H.7 c}$$

In order to improve convergence we frequently change an integral of the form (H.2) to (H.3) or (H.4) to (H.5) by introducing an additional parametric integral

$$\frac{1}{a} \to -\int_0^\infty \frac{dt}{(a-t)^2}\ . \tag{H.9 a}$$

In paricular if $a = k^2 - m^2$ we introduce the[33] Pauli-Villars regulator, $\Lambda \to \infty$:

$$\frac{1}{k^2-m^2} \to \frac{1}{k^2-m^2} - \frac{1}{k^2-\Lambda^2} = -\int_{m^2}^{\Lambda^2} \frac{dt}{(k^2-t)^2}\ . \tag{H.9 b}$$

Instead of x and y in eqn. (H.7) we frequently find it useful to introduce

$$z \equiv x + y \qquad \tilde{z} \equiv x - y \tag{H.10 a}$$

$$x = \tfrac{1}{2}(z + \tilde{z}) \qquad y = \tfrac{1}{2}(z - \tilde{z}) \tag{H.10 b}$$

whence it becomes easy to show that

$$\int_0^1 dx \int_0^{1-x} dy = \tfrac{1}{2}\int_0^1 dz \int_{-z}^{z} d\tilde{z} \tag{H.10 c}$$

and, if the integrand is even in $\tilde{z}$,

$$\int_0^1 dx \int_0^{1-x} dy = \int_0^1 dz \int_0^z d\tilde{z} \tag{H.10 d}$$

Consider a triangle loop diagram whose legs have momenta p_1, p_2 (fermions) and $q = p_1 + p_2$ with masses m_1, m_2, m_3 and internal particles whose masses are M_1, M_2 and M_3. A typical Feynman integral for such a triangle diagram would have a piece which looks like

$$\mathcal{M}^\mu \sim \int d^4k \frac{(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[k^2 - M_3^2][(p_1 - k)^2 - M_1^2][(p_2 + k)^2 - M_2^2]} \tag{H.11}$$

and from (H.9), with $\Lambda \to \infty$,

$$\mathcal{M}^\mu \sim -\int_{M_3^2}^{\Lambda^2} dt \int d^4k \frac{(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[k^2 - t]^2[(p_1 - k)^2 - M_1^2][(p_2 + k)^2 - M_2^2]} \,. \tag{H.12}$$

The denominator is of the form $\frac{1}{a^2bc}$ so we may use (H.5) to obtain

$$\mathcal{M}^\mu \sim -\int_{M_3^2}^{\Lambda^2} dt \int d^4k \; 6 \int_0^1 dx \int_0^{1-x} dy$$
$$\times \frac{(1 - x - y)(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{\left[(k^2 - t) + (t - 2p_1 \cdot k + p_1^2 - M_1^2)x + (t + 2p_2 \cdot k + p_2^2 - M_2^2)y\right]^4} \,. \tag{H.13}$$

Now, if the fermions with momenta p_1 and p_2 are on shell, then

$$p_1^2 = m_1^2 \qquad\qquad p_2^2 = m_2^2$$

and (H.13) becomes

$$\begin{aligned}
\mathcal{M}^{\mu} \sim &- \int_{M_3^2}^{\Lambda^2} dt \int d^4k \; 6 \int_0^1 dx \int_0^{1-x} dy \\
&\times \frac{(1-x-y)(\not p_1 - \not k)\gamma^{\mu}(\not p_2 + \not k)}{\left[(k^2-t)+(t-2p_1\cdot k+m_1^2-M_1^2)x+(t+2p_2\cdot k+m_2^2-M_2^2)y\right]^4} \\
= &- \int_{M_3^2}^{\Lambda^2} dt \int d^4k \; 6 \int_0^1 dx \int_0^{1-x} dy \\
&\times \frac{(1-x-y)(\not p_1 - \not k)\gamma^{\mu}(\not p_2 + \not k)}{\left[k^2-(p_1x-p_2y)\cdot k+t(x+y-1)+(m_1^2-M_1^2)x+(m_2^2-M_2^2)y\right]^4} \\
= &- \int_{M_3^2}^{\Lambda^2} dt \int d^4k \; 6 \int_0^1 dx \int_0^{1-x} dy \quad \frac{(1-x-y)(\not p_1 - \not k)\gamma^{\mu}(\not p_2 + \not k)}{\left[k^2-P\cdot k+s\right]^4}
\end{aligned}$$

where

$$\begin{aligned}
P &= p_1x - p_2y \\
s &= m_1^2x + m_2^2y - (M_1^2x + M_2^2y) - t(1-x-y)
\end{aligned} \tag{H.14}$$

We could equally we have writted (H.11) with the final two terms of the denominator in reverse order. Then, when we applied (H.5) with $b \leftrightarrow c$, instead of getting (H.12) we would have obtain the same expression but with $x \leftrightarrow y$. Thus we may always interchange x and y in the integrand. Keeping this in mind we apply (H.7) to obtain

$$\begin{aligned}
\mathcal{M}^{\mu} \sim &- \int_{M_3^2}^{\Lambda^2} dt \; 6 \int_0^1 dx \int_0^{1-x} dy \quad \frac{i}{96\pi^2} \\
&\frac{(1-x-y)[\not p_1 - (\not p_1 x - \not p_2 y)]\gamma^{\mu}[\not p_2 + (\not p_1 x - \not p_2 y)]}{(s-P^2)^2)} \\
&+ \text{ a } \delta_{\mu}{}^{\nu} \text{ term.}
\end{aligned} \tag{H.15}$$

Forgetting the δ-term for now we see that

$$P^2 = p_1^2x^2 + p_2^2y^2 - 2p_1 \cdot p_2xy$$

$$q^2 = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$

so $$2p_1 \cdot p_2 = q^2 - p_1^2 - p_2^2$$

$$= q^2 - m_1^2 - m_2^2 \tag{H.16}$$

and $$P^2 = m_1^2x^2 + m_2^2y^2 - (q^2 - m_1^2 - m_2^2)xy \tag{H.17 a}$$

$$= m^2(x+y)^2 - q^2xy \qquad \text{when } m_1 = m_2 \equiv m \tag{H.17 b}$$

$$s - P^2 = (m_1^2x + m_2^2y)(1-x-y) - (M_1^2x + M_2^2y)$$
$$- t(1-x-y) + q^2xy \tag{H.18}$$

Specializing to the case $m_1 = m_2 \equiv m$ and $M_1 = M_2 \equiv M$, for the purposes of this demonstration, we then have

$$s - P^2 = m^2(x+y)(1-x-y) - M^2(x+y) - t(1-x-y) + q^2xy \tag{H.19 a}$$

which is symmetric in x and y. Using (H.10a) we may write this as

$$s - P^2 = m^2z(1-z) - M^2z - t(1-z) + \frac{q^2}{4}(z^2 - \tilde{z}^2) \tag{H.19 b}$$

Since we are able to interchange $x \leftrightarrow y$ in the integrand and obtain and identical expression, we are similarly able to let $\tilde{z} \leftrightarrow -\tilde{z}$. We may then average these two expressions. In either case

$$x \leftrightarrow y$$
$$\tilde{z} \leftrightarrow -\tilde{z} \tag{H.20}$$

We will refer to this as the "$b \leftrightarrow c$ trick". Since the denominator, $s - P^2$, is even in of $\tilde{z}$, when we perform this averaging we will eliminate all terms in the numerator which are odd in $\tilde{z}$. Inserting (H.10 b) into (H.15) yields

$$\mathcal{M}^{\mu} \sim -\int_{M_3^2}^{\Lambda^2} dt\; 6\int_0^1 dx \int_0^{1-x} dy \quad \frac{i}{96\pi^2}\,(1-z)$$
$$\frac{\frac{1}{4}[z(\not{p}_2 - \not{p}_1) - \tilde{z}(\not{p}_1 + \not{p}_2) + 2\,\not{p}_1]\gamma^{\mu}[z(\not{p}_1 - \not{p}_2) + \tilde{z}(\not{p}_1 + \not{p}_2) - 2\,\not{p}_2]}{\left(s - P^2\right)^2} \tag{H.21}$$

Expanding this and keeping only even powers of $\tilde{z}$ gives a numerator term

$$-\frac{1}{4}\left[z^2\,\tilde{\not{q}}\gamma^{\mu}\,\tilde{\not{q}} - \tilde{z}^2\,\not{q}\gamma^{\mu}\,\not{q} + 2(\not{p}_1\gamma^{\mu}\,\not{q} - \not{q}\gamma^{\mu}\,\not{p}_2) + 4\,\not{p}_1\gamma^{\mu}\,\not{p}_2\right]$$

where here

$$\begin{aligned} q^{\mu} &= p_1^{\mu} + p_2^{\mu} \\ \tilde{q}^{\mu} &= p_1^{\mu} - p_2^{\mu}. \end{aligned} \tag{H.22}$$

The above expression will be sandwiched between $\bar{u}(p_1)$ and $v(p_2)$ so that any $\not{p}_1$ to the left of γ^{μ} will generate a factor of m_1 via the Dirac equation and any $\not{p}_2$ to the right of γ^{μ} will generate an m_2. If $m_1 = m_2 = m \ll M$ or q^2 (which appears in terms not shown) then we can ignore these terms and (H.19 b) becomes

$$s - P^2 \approx -\left[t(1-z) + M^2 z + \frac{1}{4}q^2(\tilde{z}^2 - z^2)\right] \tag{H.23}$$

which, when used in (H.21), gives us

$$\mathcal{M}^{\mu} \sim -\int_{M_3^2}^{\Lambda^2} dt\; 6\int_0^1 dx \int_0^{1-x} dy \quad \frac{i}{96\pi^2}$$
$$\frac{\frac{1}{4}(1-z)\left[z^2\,\tilde{\not{q}}\gamma^{\mu}\,\tilde{\not{q}} - \tilde{z}^2\,\not{q}\gamma^{\mu}\,\not{q}\right]}{\left(t(1-z) + M^2 z + \frac{1}{4}q^2(\tilde{z}^2 - z^2)\right)^2}\,. \tag{H.24}$$

In point of fact, since we are ignoring the external fermion mass terms,

$$\tilde{\not{q}}\gamma^{\mu}\,\tilde{\not{q}} \approx \not{q}\gamma^{\mu}\,\not{q}$$

and (H.24) may be written as

$$\mathcal{M}^{\mu} \sim -\int_{M_3^2}^{\Lambda^2} dt\; 6\int_0^1 dx \int_0^{1-x} dy \quad \frac{i}{96\pi^2} \\ -\frac{\frac{1}{4}(1-z)\left[\left(\tilde{z}^2 - z^2\right)\not{q}\gamma^{\mu}\,\not{q}\right]}{\left(t(1-z) + M^2 z + \frac{1}{4}q^2(\tilde{z}^2 - z^2)\right)^2}\,. \tag{H.25}$$

H.2 THE GORDON DECOMPOSITION

The Gordon Decomposition is a useful identity[37] relating currents of even and odd numbers of sigma (or gamma) matrices. In this sense it is merely a restatement of the Dirac equation recast into a form which includes a $\sigma^{\mu\nu}$ term. Since the statement and proof of this identity may be found in any number of standard references, it may seem strange to devote a major portion of an appendix to it. Applications of this result are to be found throughout this paper, often in generalized or atypical forms which we will proceed to derive.

Four-Component Formulations

Working with Dirac spinors the Gordon Decomposition of the Dirac vector current is generally expressed as

$$\overline{\psi}(p',s')\gamma^{\mu}\psi(p,s) = \frac{1}{2m} \left\{\overline{\psi}(p',s')\left[(p'+p)^{\mu} + i\sigma^{\mu\nu}(p'-p)_{\nu}\right]\psi(p,s)\right\} \tag{H.26}$$

or

$$\overline{\psi}(p',s')\gamma^{\mu}\psi(p,s) = \frac{1}{2m} \left\{\overline{\psi}(p',s')[\tilde{q}^{\mu} + i\sigma^{\mu\nu}q_{\nu}]\psi(p,s)\right\} \tag{H.27}$$

where, for symmetry, we have let

$$q^{\mu} = p'^{\mu} - p^{\mu} \qquad \tilde{q}^{\mu} = p'^{\mu} + p^{\mu}. \tag{H.28}$$

We use the basic identities

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu} \tag{H.29}$$

$$[\gamma^{\mu},\gamma^{\nu}] = -2i\sigma^{\mu\nu} \tag{H.30}$$

$$\not{a}\,\not{b} = a_\mu b^\nu\left(\tfrac{1}{2}\{\gamma^\mu,\gamma^\nu\} + \tfrac{1}{2}[\gamma^\mu,\gamma^\nu]\right) = \tfrac{1}{2}\,a_\mu b_\nu[g^{\mu\nu} - 2i\sigma^{\mu\nu}]$$

$$= a\cdot b - ia^\mu b^\nu\sigma_{\mu\nu}\ . \qquad \text{(H.31)}$$

The standard identity in (H.27) considers only the instance of a vector current (γ^μ) where the initial and final spinorial states are of the same type. We wish to extend this to admit the possibility of admixtures of vector and pseudo vector currents connecting states of different species. We will refer to these as ψ^i and ψ^j and consider the vertex $[A + B\gamma_5]\gamma^\mu$.

We will require a few identities involving γ_5 in addition to (H.29)-(H.31).

$$\{\gamma_5,\gamma_\mu\} = 0 \qquad [\gamma_5,\sigma^{\mu\nu}] = 0 \qquad \gamma^5 \equiv \gamma_5 \qquad \text{(H.32)}$$

$$\gamma^5\,\not{a}\,\not{b} = \gamma^5[a\cdot b - ia^\mu b^\nu\sigma_{\mu\nu}] \qquad \not{a}\,\not{b}\gamma^5 = [a\cdot b - ia^\mu b^\nu\sigma_{\mu\nu}]\gamma^5$$

First consider the case where $A = 1$ and $B = 0$. Let a^μ be any four-vector.

$$\overline{\psi}^i(p',s')(\overleftarrow{\not{p}} - m_i)\,\not{a}\psi^j(p,s) + \overline{\psi}^i(p',s')\,\not{a}(\overrightarrow{\not{p}} - m_j)\psi^j(p,s) = 0$$

by the Dirac equation. The arrows atop the momentum operators indicate in which direction we intend to operate. This is true for arbitrary a. Using (H.31) we find that this is equal to

$$-(m_i + m_j)\overline{\psi}^i(p',s')\,\not{a}\psi^j(p,s)$$

$$+\overline{\psi}^i(p',s')[\overleftarrow{p}'^\mu a_\mu + i\overleftarrow{p}'^\mu a^\nu\sigma_{\mu\nu} + a^\mu\overrightarrow{p}_\mu - ia^\mu\overrightarrow{p}^\nu\sigma_{\mu\nu}]\psi^j(p,s)$$

thus

$$(m_i+m_j)\overline{\psi}^i(p',s')\gamma^\mu a_\mu\psi^j(p,s) = \overline{\psi}^i(p',s')[(\overleftarrow{p}'^\mu+\overrightarrow{p}^\mu)a_\mu - i\sigma^{\mu\nu}(\overleftarrow{p}_\nu - \overrightarrow{p}'_\nu)]\psi^j(p,s)$$

since $\sigma^{\mu\nu} = -\sigma^{\nu\mu}$. As this is true for any a_μ we may write

$$\overline{\psi}^i(p',s')\gamma^\mu\psi^j(p,s) = \frac{1}{m_i + m_j} \left\{\overline{\psi}^i(p',s')[\widetilde{q}^\mu + i\sigma^{\mu\nu}q_\nu]\psi^j(p,s)\right\} . \qquad \text{(H.33)}$$

When $i = j$ we obtain (H.27), of course. We now consider the case where $A = 0$ and $B = 1$ and repeat the above analysis

$$\overline{\psi}^i(p',s')(\overleftarrow{\not p} - m_i)\gamma^5 \not a\psi^j(p,s) \pm \overline{\psi}^i(p',s')\gamma^5 \not a(\overrightarrow{\not p} - m_i)\psi^j(p,s) = 0 \qquad \forall\, a$$

$$= -(m_i \pm m_j)\overline{\psi}^i(p',s')\gamma^5 \not a\psi^j(p,s)$$

$$+ \overline{\psi}^i(p',s')(p',s')[-\overleftarrow{p}'^\mu a_\mu\gamma^5 + i\overleftarrow{p}'^\mu a^\nu\sigma_{\mu\nu}\gamma^5$$

$$\pm \gamma^5 a^\mu \overrightarrow{p}_\mu \mp ia^\mu\overrightarrow{p}^\nu\gamma^5\sigma_{\mu\nu}]\psi^j(p,s)$$

The two different signs give rise to the equivalent identities:

$$\overline{\psi}^i(p',s')\gamma^5\gamma^\mu\psi^j(p,s) = -\frac{1}{m_j + m_i}\,\overline{\psi}^i(p',s')\gamma^5[q^\mu + i\sigma^{\mu\nu}\widetilde{q}_\nu]\psi^j(p,s) \qquad \text{(H.34)}$$

$$\overline{\psi}^i(p',s')\gamma^5\gamma^\mu\psi^j(p,s) = +\frac{1}{m_j - m_i}\,\overline{\psi}^i(p',s')\gamma^5[\widetilde{q}^\mu + i\sigma^{\mu\nu}q_\nu]\psi^j(p,s)$$

$$(m_i \neq m_j)\ . \qquad \text{(H.35)}$$

These can now be combined with (H.33) to obtain the more general results

$$\overline{\psi}^i(p',s')[A + B\gamma^5]\psi^j(p,s)$$

$$= \frac{1}{m_i + m_j}\,\overline{\psi}^i(p',s')\{(A\widetilde{q}^\mu - Bq^\mu\gamma^5) + i(Aq_\nu - B\widetilde{q}_\nu\gamma^5)\sigma^{\mu\nu}\}\psi^j(p,s) \qquad \text{(H.36)}$$

$$\overline{\psi}^i(p',s')[A+B\gamma^5]\psi^j(p,s)$$

$$= \overline{\psi}^{\,i}(p',s')\left[\frac{A}{m_j+m_i}+\frac{B\gamma_5}{m_j-m_i}\right]\{\tilde{q}^\mu + i\sigma^{\mu\nu}q_\nu\}\,\psi^j(p,s) \qquad (m_i \neq m_j)\ . \tag{H.37}$$

In either version we recover (H.27) when $A = 1$, $B = 0$ and $m_i = m_j$. Writing

$$\gamma_\pm = \tfrac{1}{2}\,(1 \pm \gamma_5) \qquad \gamma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \gamma_- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{H.38}$$

and using (H.36) we find that

$$\begin{aligned}(m_i+m_j)\overline{\psi}^{\,i}(p')\gamma_\pm\gamma^\mu\psi^j(p) &= \\ \overline{\psi}^{\,i}(p')\{\gamma_\mp p'^\mu + \gamma_\pm p^\mu + i\sigma^{\mu\nu}&[\gamma_\mp p'^\nu - \gamma_\pm p^\nu]\}\psi^j(p)\ .\end{aligned} \tag{H.39}$$

Two-Component Formulation

As always, the Weyl spinor formulation of an identity derived for Dirac spinors is more involved. We now have a pair of spinors and their conjugates which are related via the Dirac equation. One possibility is to take the results of the previous section and expand them out in two-component form. By choosing $A+B\gamma_5$ to be $\frac{1}{2}\,(1\pm\gamma_5)$ we will project out the left- and right-pieces of the vector current.

Expanding some Dirac spinorial currents into their two-component subdivision (see Appendix A)

$$
\begin{aligned}
\overline{\psi}\gamma_+\psi &= \overline{\psi}_-\overline{\psi}_+ \\
\overline{\psi}\gamma_-\psi &= \psi_+\psi_- \\
\overline{\psi}\gamma_+\gamma^\mu\psi &= \overline{\psi}_-\overline{\sigma}^\mu\psi_- \\
\overline{\psi}\gamma_-\gamma^\mu\psi &= \psi_+\sigma^\mu\overline{\psi}_+ \\
\overline{\psi}\gamma_+\sigma^{\mu\nu}\psi &= \overline{\psi}_-\overline{\sigma}^{\mu\nu}\overline{\psi}_+ \\
\overline{\psi}\gamma_-\sigma^{\mu\nu}\psi &= \psi_+\sigma^{\mu\nu}\psi_-
\end{aligned}
\tag{H.40}
$$

Using these with (H.39) results in

$$
\psi^i_+(p')\sigma^\mu\overline{\psi}^j_+(p) =
$$

$$
\frac{1}{m_i+m_j}\quad [\psi^i_+(p')(p'^\mu + i\sigma^{\mu\nu}p'_\nu)\psi^i_-(p) + \overline{\psi}^i_-(p')(p^\mu - i\overline{\sigma}^{\mu\nu}p_\nu)\overline{\psi}^j_+(p)]
\tag{H.41}
$$

$$
\overline{\psi}^i_-(p')\overline{\sigma}^\mu\psi^j_-(p) =
$$

$$
\frac{1}{m_i+m_j}\quad [\psi^i_+(p')(p^\mu - i\sigma^{\mu\nu}p_\nu)\psi^j_-(p) + \overline{\psi}^i_-(p')(p'^\mu + i\overline{\sigma}^{\mu\nu}p'_\nu)\overline{\psi}^j_+(p)]\ .
\tag{H.42}
$$

These could equally well have been obtained by working with the Weyl spinors directly. For instance, starting with

$$
\psi^\alpha_+(p')(\sigma^\nu)_{\alpha\dot\alpha}\overleftarrow{p}_\nu(\overline{\sigma}^\mu)^{\alpha\beta}a_\mu\psi_{-\beta}(p') + 3\text{ similar terms} = 0
$$

and repeating the steps leading to (H.33) would lead us to (H.42) directly. It is often convenient to use the sum and difference of (H.41) and (H.42) which correspond to decomposing the full γ^μ and $\gamma^5\gamma^\mu$ currents.

The sum term is

$$\psi^i_+ \sigma^\mu \psi^j_+ + \overline{\psi}^i_- \overline{\sigma}^\mu \psi^j_-$$
$$= \frac{1}{m_i + m_j} \left\{ \psi^i_+ [\widetilde{q}^\mu + i\sigma^{\mu\nu} q_\nu] \psi^j_- + \overline{\psi}^i_- [\widetilde{q}^\mu + i\overline{\sigma}^{\mu\nu} q_\nu] \overline{\psi}^j_+ \right\} \tag{H.43}$$

which is, of course, merely the expansion of (H.33). We can use the Dirac equation (section B.1) to put this entirely in terms of $\overline{\psi}_-$ and ψ_-,

$$\overline{\psi}^i_-(p') \left[\frac{\overline{\sigma} \cdot p'}{m_i} + \frac{\overline{\sigma} \cdot p}{m_j} \right] \psi^j_-(p) \widetilde{q}^\mu$$
$$= -i\overline{\psi}^i_-(p') \left[\frac{\overline{\sigma} \cdot p'}{m_i} \sigma^{\mu\nu} + \overline{\sigma}^{\mu\nu} \frac{\overline{\sigma} \cdot p}{m_j} \right] q_\nu \psi^j_-(p) \tag{H.44}$$
$$+ (m_i + m_j) \overline{\psi}^i_-(p') \left[\overline{\sigma}^\mu + \frac{\overline{\sigma} \cdot p'}{m_i} \sigma^\mu \frac{\overline{\sigma} \cdot p}{m_j} \right] \psi^j_-(p).$$

A similar expression exists for ψ_+ and $\overline{\psi}_+$. (On-shell we may cancel the spinors from these expressions.)