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STRONG EFFECTS IN WEAK NONLEPTONIC DECAYS*

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ABSTRACT

In this report the weak nonleptonic decays of kaons and hyperons are examined with the hope of gaining insight into a recently proposed mechanism for the $\Delta I = 1/2$ rule. The effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays and that for $K^0 - \bar{K}^0$ mixing are calculated in the six-quark model using the leading logarithmic approximation. These are used to examine the CP violation parameters of the kaon system. It is found that if Penguin-type diagrams make important contributions to $K \to \pi\pi$ decay amplitudes then upcoming experiments may be able to distinguish the six-quark model for CP violation from the superweak model. The weak radiative decays of hyperons are discussed with an emphasis on what they can teach us about hyperon nonleptonic decays and the $\Delta I = 1/2$ rule.

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CHAPTER I

INTRODUCTION

One of the prominent features of the nonleptonic weak decays of kaons and hyperons is the $\Delta I = 1/2$ rule. The effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays can be written as the sum of isospin 1/2 and 3/2 pieces. Experimentally it is observed that those decays which proceed through the isospin 1/2 part of the effective Hamiltonian are enhanced by roughly a factor of 20 in amplitude over those which proceed through the isospin 3/2 part of the effective Hamiltonian. This is known as the $\Delta I = 1/2$ rule. As an example consider kaon decay into two pions. The decay $K^+ \to \pi^+\pi^o$ proceeds only through the $I = 3/2$ part of the effective Hamiltonian since the two-pion state is charged and therefore must have $I = 2$. The decay $K_S^o \to \pi^+\pi^-$, on the other hand, can proceed through both the $I = 1/2$ and $I = 3/2$ parts of the effective Hamiltonian. Experimentally[1]

$$\frac{\Gamma\left(K_S^o \to \pi^+\pi^-\right)}{\Gamma\left(K^+ \to \pi^+\pi^o\right)} \approx 450 \quad . \tag{1.1}$$

In the standard 4-quark Weinberg-Salam[2] model for weak and electromagnetic interactions the quarks are assigned to right-handed singlets

$$(u)_R \quad ; \quad (c)_R \quad ; \quad (d)_R \quad ; \quad (s)_R \tag{1.2a}$$

and left-handed doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad ; \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad . \tag{1.2b}$$

The fields d_L' and s_L' are weak eigenstates and related to the mass eigenstates by a unitary transformation. With an appropriate choice for the quark field phases this transformation can be written in the following form

$$\begin{pmatrix} d' \\ s' \end{pmatrix}_L = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L \quad . \tag{1.3}$$

θ_c is called the Cabibbo angle. In the absence of strong interactions an effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays can be derived in this model by treating the W-boson mass as very large and neglecting the momentum transfer in the W-boson propagator. This is illustrated in Fig. 1. The resulting effective Hamiltonian is the familiar local four-fermion (V-A) ⊗ (V-A) current-current interaction

$$\mathscr{H}_{\text{eff}}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \left(\bar{s}_\alpha \gamma^\mu (1-\gamma_5) u_\alpha\right)\left(\bar{u}_\beta \gamma_\mu (1-\gamma_5) d_\beta\right) - [u \to c] + \text{h.c.} \quad , \tag{1.4}$$

where G_F is the Fermi constant and α and β are color indices which are summed over {1,2,3} when repeated. It is convenient to decompose this Hamiltonian into a sum of color symmetric and color antisymmetric pieces in the following manner

$$\begin{aligned} \mathscr{H}_{\text{eff}}^{|\Delta S|=1} = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \Big[\Big\{ &\left(\bar{s}_\alpha \gamma^\mu (1-\gamma_5) u_\alpha\right)\left(\bar{u}_\beta \gamma_\mu (1-\gamma_5) d_\beta\right) \\ &+ \left(\bar{s}_\alpha \gamma^\mu (1-\gamma_5) u_\beta\right)\left(\bar{u}_\beta \gamma_\mu (1-\gamma_5) d_\alpha\right) \Big\} + \Big\{ \left(\bar{s}_\alpha \gamma^\mu (1-\gamma_5) u_\alpha\right)\left(\bar{u}_\beta \gamma_\mu (1-\gamma_5) d_\beta\right) \\ &- \left(\bar{s}_\alpha \gamma^\mu (1-\gamma_5) u_\beta\right)\left(\bar{u}_\beta \gamma_\mu (1-\gamma_5) d_\alpha\right) \Big\} \Big] - [u \to c] + \text{h.c.} \quad . \end{aligned} \tag{1.5}$$

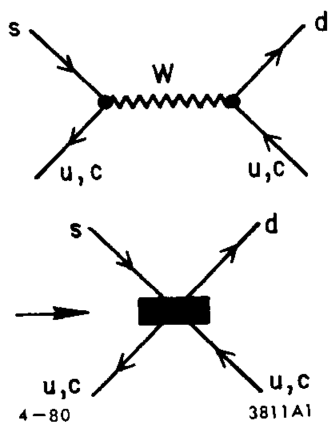

Fig. 1. Tree level diagram which gives rise to effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays in the absence of strong interactions.

The first set of brace brackets contains a piece which is symmetrized on the color indices of the u and d fields while the second set of brace brackets contains a color antisymmetrized piece. The color antisymmetric piece is pure I = 1/2 while the color symmetric piece has an I = 1/2 portion and an I = 3/2 portion. Of course all the terms with charm and anti-charm quark fields are I = 1/2 since the charm quark has no isospin.

It was originally conjectured by Wilson[3] that strong interaction corrections would enhance the I = 1/2 portion of the effective Hamiltonian thus providing an explanation for the $\Delta I = 1/2$ rule. With the advent of Quantum Chromodynamics (QCD) as a theory for the strong interactions[4] such corrections became calculable. Consider, for example, the correction in Fig. 2a. If the momentum transfer in the W-boson propagator could be neglected this diagram would just give the order g^2 (where g is the strong coupling) correction, shown in Fig. 2b, to the matrix elements of the local 4-quark operators in the effective Hamiltonian of Eq.(1.5). However, since Fig. 2b is ultravioletly divergent the convergence of the loop integral in Fig. 2a is not good enough for such an approximation. If one differentiates the amplitude, represented by Fig. 2a, with respect to an external momentum the ultraviolet convergence is improved enough so that the momentum transfer can be neglected in the W-boson propagator. This means that Figs. 2a and 2b differ (to leading order in the large W-boson mass) by a constant independent of the external momenta which is thus proportional to the tree approximation for the matrix elements of a local 4-quark operator. The constant of proportionality $A(M_W/\mu, g)$ is also independent of the

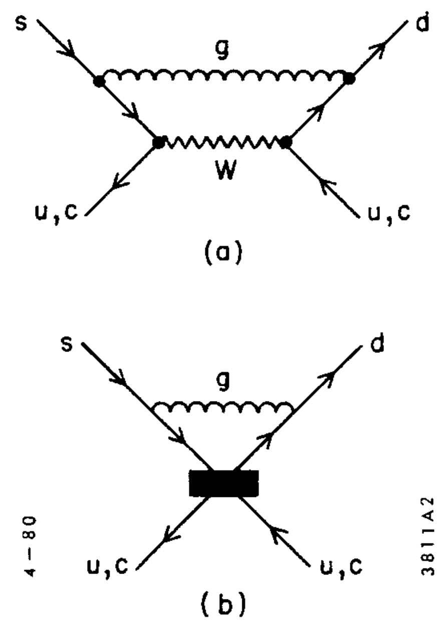

Fig. 2. (a) Higher order diagram contributing to weak $\Delta S = 1$ nonleptonic decays.
(b) Higher order diagram contributing to the matrix elements of local four-quark operators in the effective Hamiltonian for $\Delta S = 1$ nonleptonic weak decays.

quark masses since the above argument can be repeated differentiating now with respect to quark masses. Other diagrams can be accommodated in a similar fashion the net result being that the effective Hamiltonian is a sum of color symmetric and color antisymmetric pieces, but with coefficients differing from their free quark values. That is

$$\mathcal{H}_{\text{eff}}^{|\Delta S|=1} = \frac{G_F}{-2\sqrt{2}} \sin\theta_c\cos\theta_c\Big[A_+(M_W/\mu,g)\Big\{\big(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)u_\alpha\big)\big(\bar{u}_\beta\gamma_\mu(1-\gamma_5)d_\beta\big)$$
$$+\big(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)u_\beta\big)\big(\bar{u}_\beta\gamma_\mu(1-\gamma_5)d_\alpha\big)\Big\} + A_-(M_W/\mu,g)\Big\{\big(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)u_\alpha\big)$$
$$\times\big(\bar{u}_\beta\gamma_\mu(1-\gamma_5)d_\beta\big) - \big(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)u_\beta\big)\big(\bar{u}_\beta\gamma_\mu(1-\gamma_5)d_\alpha\big)\Big\}\Big] - [u\to c] + \text{h.c.} \quad . \tag{1.6}$$

The functions $A_\pm(M_W/\mu,g)$ depend on the renormalization scheme. Of course, renormalization scheme dependence in the matrix elements of the operators must cancel this so that physical processes do not depend on choice of regularization scheme. μ is the renormalization point mass and dependence of the Wilson coefficients $A_\pm$ on it is likewise cancelled by the dependence of the matrix elements on μ. The coefficients A_+ and A_- have been calculated in the leading logarithmic approximation by Gaillard and Lee[5] and Altarelli and Maiani.[6] They found, for typical values of the QCD parameters, that A_- was enhanced (compared with its free quark value) by roughly a factor of 2 and A_- was reduced (compared with its free quark value) by roughly the factor .7. While this result is in the correct direction to explain the $\Delta I = 1/2$ rule it is much too small in magnitude.

The W-boson mass is not the only large mass scale in the problem. The charm quark mass is also "large" when compared with typical light hadronic masses. Thus one can imagine treating the charm quark as a very heavy particle and removing its field from explicitly appearing in the theory. When this is done local 4-quark operators with a chiral structure (V-A) $\otimes$ (V+A) will enter the Hamiltonian. Consider, for example, the diagram shown in Fig. 3 (sometimes called a Penguin diagram). Calculation reveals that in the approximation of treating the W-boson and charm quark as very heavy the loop integral gives a factor of k^2 which cancels the pole in the gluon propagator. As a result the amplitude corresponding to Fig. 3 can be reproduced by the tree approximation to the matrix elements of a local 4-quark operator involving only light u, d and s quark fields. It is natural to wonder whether this local four-fermion result is an artifact of the lowest order calculation or will persist to higher orders.[7] In Fig. 4a the factor of k^2 from the upper loop integral cancels the gluon propagator (when the masses of the light strange and down quarks are zero). This, however, does not lead to a local 4-quark structure but instead to a structure shown schematically in Fig. 4b. Another class of diagrams that might seem to show that the local four-fermion result of Fig. 3 is an artifact is shown in Fig. 5. Again diagrams of this type do not admit an interpretation in terms of a local four-fermion structure. Moreover they are no smaller than the lowest-order diagram even in the limit of large charm quark and W-boson masses. The diagrams of Fig. 5 would, taken by themselves, ruin the lowest-order local four-fermion result. However, when the contributions of Figs. 4a and 5 are added together a cancellation of soft-gluon effects occurs between these diagrams so that their sum is included in the matrix elements of a local four-fermion operator

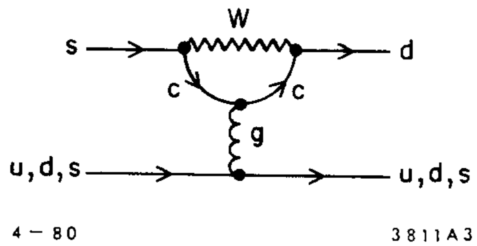

Fig. 3. Lowest order Penguin-type diagram.

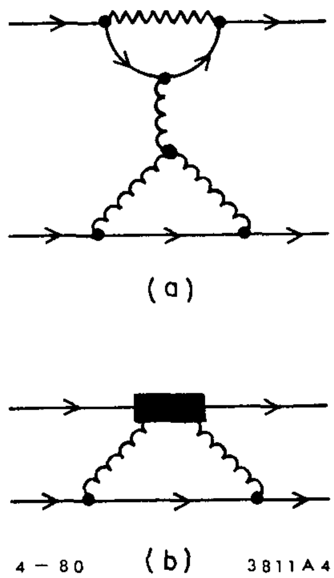

Fig. 4. (a) A two-loop Penguin-type diagram.
(b) Symbolic representation of (a) illustrating the cancellation of a gluon propagator by the upper loop integration.

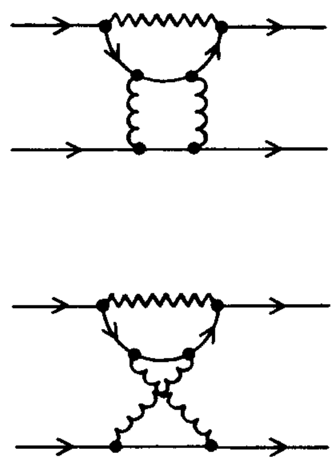

Fig. 5. Two-loop Penguin-type diagrams which are not included in the matrix elements of a local four-fermion operator composed only of light u, d and s quark fields.

constructed out of light-quark fields. This cancellation is a result of gauge invariance. It occurs because (for soft gluons) the amplitudes corresponding to the diagrams in Fig. 6 which contribute to the process $s \to d$ + gluons are reproduced (apart from constant pieces which will cancel by the GIM mechanism) by the tree approximation to the matrix elements of the operator

$$\mathcal{O}^{\text{Penguin}} = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \frac{g}{12\pi^2} \ln\left(\frac{m_c^2}{\mu^2}\right) \times \left(\bar{s}_\alpha \gamma^\nu (1-\gamma_5) T^a_{\alpha\beta} d_\beta\right) \left[D^\mu F_{\mu\nu}\right]^a + \text{h.c.} \quad , \tag{1.7}$$

to leading order in the large masses. In Eq. (1.7) g is the strong coupling, T^a, $a \in \{1,2,\ldots,8\}$, are SU(3) color matrices normalized by $\mathrm{Tr}(T^a T^b) = \delta^{ab}/2$, $F^a_{\mu\nu}$ is the gluon field strength tensor and D_μ denotes a covariant derivative. Diagrams with more than three gluons attached to the quark loop are not important since they cannot produce a large logarithm in the c-quark mass. Using the equations of motion for QCD

$$\left(D^\mu F_{\mu\nu}\right)^a = J^a_\nu = g\left(\bar{u}_\alpha \gamma_\nu T^a_{\alpha\beta} u_\beta + \bar{d}_\alpha \gamma_\nu T^a_{\alpha\beta} d_\beta + \bar{s}_\alpha \gamma_\nu T^a_{\alpha\beta} s_\beta\right) \tag{1.8}$$

$\mathcal{O}^{\text{Penguin}}$ becomes a local four-fermion operator. From this discussion it is clear that cancellations similar to that between the diagrams in Figs. 4a and 5 will occur between other higher order diagrams so that the local four-fermion structure of the lowest order Penguin diagram in Fig. 3 will be preserved in the sense that the sum of all Penguin-type diagrams, with arbitrary gluon insertions, equals a sum of Wilson coefficients times matrix elements of local 4-quark operators. Some of the operators induced by the Penguin-type diagrams will have a chiral

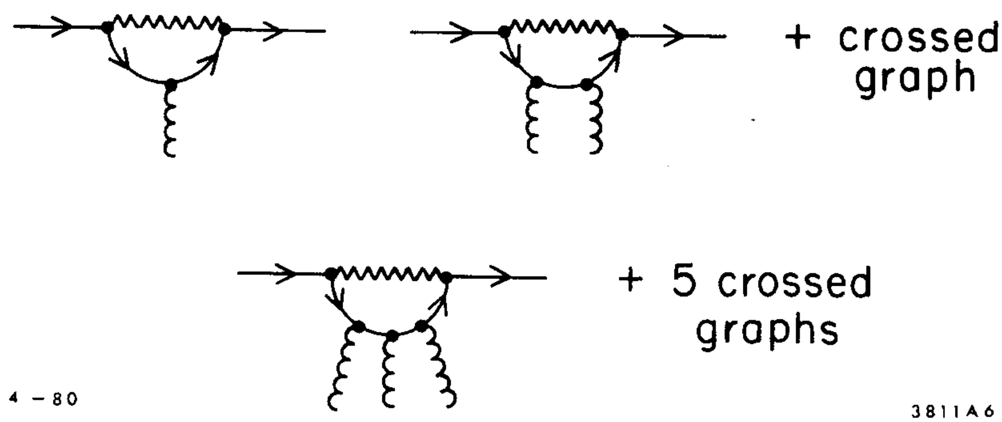

Fig. 6. One particle-irreducible diagrams contributing to the transition s → d + gluons at the one-loop level. In the absence of strong interactions there is a one-loop diagram that gives a s → d transition however to leading order in the W-boson mass it can be absorbed into mass renormalization.

structure (V-A) $\otimes$ (V+A) due to the vector coupling of the gluons to quarks. Note also that all the Penguin-type diagrams are pure I = 1/2 since the gluon carries no isospin.

The effective Hamiltonian for weak $\Delta S = 1$ nonleptonic decays, which results from successively treating the W-boson and charm quark as heavy fields and removing them from explicitly appearing in the theory, has been calculated in the leading logarithmic approximation by Shifman, Vainshtein and Zakharov.[8] They found that the operators induced by the Penguin-type diagrams have small Wilson coefficients and at first glance appear to make only an insignificant contribution to the weak nonleptonic decays of kaons and hyperons. However, the matrix elements of these operators with a (V-A) $\otimes$ (V+A) chiral structure may be greatly enhanced over those of operators with the usual (V-A) $\otimes$ (V-A) chiral structure.[8] Such an enhancement occurs, for example, when the matrix elements are evaluated by saturating the matrix element of a product of quark bilinears with the vacuum intermediate state. Since the (V-A) $\otimes$ (V+A) operators are pure I = 1/2, combining the enhancement of their matrix elements with the enhancement of the Wilson coefficients of the I = 1/2 combination of the familiar (V-A) $\otimes$ (V-A) operators may provide a qualitative explanation for the $\Delta I = 1/2$ rule. Much of this report will be devoted to examining the consequences of this possible mechanism for the $\Delta I = 1/2$ rule and to testing its validity.

The nonleptonic weak decays of the neutral kaons have another feature which is even more striking that the $\Delta I = 1/2$ rule. They violate CP invariance. If CP was conserved the physical neutral kaon eigenstates K_S^o and K_L^o would be the CP eigenstates

$$|K_1\rangle = \frac{|K^o\rangle + |\bar{K}^o\rangle}{\sqrt{2}} \qquad (1.9a)$$

and

$$|K_2\rangle = \frac{|K^o\rangle - |\bar{K}^o\rangle}{\sqrt{2}} \quad , \qquad (1.9b)$$

with CP = +1 and -1 respectively. Since a neutral two pion s-wave state has even CP a K_2 cannot decay into two pions when CP is conserved. However, experimentally it is observed that[1]

$$|\eta_{+-}| \equiv \left| \frac{\langle\pi^+\pi^-|H_{eff}^{|\Delta S|=1}|K_L^o\rangle}{\langle\pi^+\pi^-|H_{eff}^{|\Delta S|=1}|K_S^o\rangle} \right| = (2.274 \pm .022) \times 10^{-3} \qquad (1.10a)$$

and

$$|\eta_{00}| \equiv \left| \frac{\langle\pi^o\pi^o|H_{eff}^{|\Delta S|=1}|K_L^o\rangle}{\langle\pi^o\pi^o|H_{eff}^{|\Delta S|=1}|K_S^o\rangle} \right| = (2.32 \pm .09) \times 10^{-3} \quad . \qquad (1.10b)$$

The difference of η_{+-} and η_{00} from zero is a measure of CP violation.

In the 4-quark Weinberg-Salam model with the minimal Higgs sector (i.e., one Higgs doublet) CP is conserved.[9] However, as was pointed out by Kobayashi and Maskawa,[10] in the six quark model with right handed singlets

$$(u)_R \ ; \ (c)_R \ ; \ (t)_R \ ; \ (d)_R \ ; \ (s)_R \ ; \ (b)_R \qquad (1.11a)$$

and left handed doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \ ; \ \begin{pmatrix} c \\ s' \end{pmatrix}_L \ ; \ \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad (1.11b)$$

there is enough freedom for CP violation to occur. This model has become popular because of the discovery of a fifth lepton,[11] τ, and a fifth quark, b.[12] A sixth quark t is expected,[13] being necessary for the (generalized) GIM mechanism[14] as well as the cancellation of anomalies.[15] The primed fields in Eq. (1.11b) are not mass eigenstates but are related to the mass eigenstates s_L, d_L and b_L by a unitary transformation, U, which for the standard choice of phases for the quark fields is[10]

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} -c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L , \qquad (1.12)$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. The signs of the quark fields may be adjusted so that the three Cabibbo-type angles θ_j, $j \in \{1,2,3\}$, all lie in the first quadrant.[16] Then the quadrant of δ has physical significance and cannot be chosen by convention.

In this model weak interactions involving the charged hadronic current follow from the interaction term in the Hamiltonian density

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}} J^{\mu^+} W_\mu^- + \text{h.c.} \qquad (1.13)$$

where W_μ^- is the charged W-boson field, J_μ^+ is the charged weak current defined by

$$J_\mu^+ = \bar{u}_\alpha\gamma_\mu(1-\gamma_5)d'_\alpha + \bar{c}_\alpha\gamma_\mu(1-\gamma_5)s'_\alpha + \bar{t}_\alpha\gamma_\mu(1-\gamma_5)b'_\alpha \qquad (1.14)$$

and g is the gauge coupling of the weak SU(2) subgroup. Since the CP operator takes a (mass eigenstate) quark field into an antiquark field and a $W^\pm$ boson into a $W^\mp$ boson CP will be violated by this interaction Hamiltonian if the phase δ is nonzero. Actually there are arbitrary phases in the definition of the CP operator corresponding to the arbitrariness of the choice of phases for the quark fields. The correct statement is that there will be CP violation if it is impossible by readjusting the phases of the quark fields to find a parametrization of the unitary matrix U (that relates weak and mass eigenstates) which is purely real. In the 4-quark model where U is a 2×2 unitary matrix it is possible by readjusting the phases of the quark fields to find the real parametrization given in Eq. (1.3). However, in the six-quark model readjusting the phases of the quark fields will just move the phase δ from one place in U to another, but it can never be completely removed from appearing in Eq. (1.12).

The phenomenological consequences of the Kobayashi-Maskawa six-quark model for CP violation have been worked out by Ellis, Gaillard and Nanopoulos,[17] with strong interactions neglected, and were found to be consistent with experimental data on K decays. Part of this report is devoted to a study of the effects that strong interactions have on the predictions which the six-quark model makes for various CP violation parameters. In particular, the Penguin-type diagrams, with heavy c and t quarks in the loop have an imaginary CP violating part and their CP violating contributions to $K \to \pi\pi$ decay amplitudes are discussed.

In Chapter II the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays is computed in the six-quark model using the leading logarithmic approximation. In the following chapter the effects of QCD corrections on the $K^o - \bar{K}^o$ mass matrix are calculated. Chapter IV uses the results of these two calculations to make predictions for the CP violation parameters η_{+-} and η_{00} and in particular for the deviation of η_{+-}/η_{00} from unity. It is shown that this deviation may be measurable if the Penguin-type diagrams make important contributions to the nonleptonic decays of kaons. Chapter V contains a brief discussion of weak radiative hyperon decays. The weak radiative decays of the negatively charged hyperons Ω^- and Ξ^- are particularly interesting since they may proceed mostly through Penguin-type diagrams. Finally, Chapter VI contains a brief summary of results and some general conclusions are drawn.

CHAPTER II

EFFECTIVE HAMILTONIAN FOR $\Delta S = 1$ WEAK NONLEPTONIC DECAYS IN THE SIX-QUARK MODEL[18]

In the standard six-quark model with charge +2/3 quarks u,c, and t and charge -1/3 quarks d,s, and b the left-handed quarks are assigned to weak isospin doublets and the right-handed quarks to weak isospin singlets of the $SU(2) \otimes U(1)$ gauge group of weak and electromagnetic interactions. The mixing between quarks in doublets characterized, say, by their charge +2/3 members, is describable by three Cabibbo-like angles θ_1, θ_2, and θ_3, and by a single phase, δ, which results in CP violation. The nonleptonic weak interaction that can result in a net change in quark flavors is given to lowest order in weak interactions, and zeroeth order in strong interactions, by the product of a weak current of left-handed quarks, a charged W-boson propagator, and another weak current of left-handed quarks. Neglecting the momentum transfer dependence of the W-boson propagator, one has the usual local $(V-A) \otimes (V-A)$ structure of a current-current weak nonleptonic Hamiltonian.

With the introduction of strong interactions, in the form of quantum chromodynamics (QCD), things become more complicated. Consider, for example, that part of the nonleptonic Hamiltonian responsible for decay of kaons and hyperons which we write in terms of the "light" quarks u,d, and s. As the strong interactions are turned on, not only is the lowest order $(V-A) \otimes (V-A)$ term involving u,d, and s quarks modified by gluon exchanges between the quarks, but there are diagrams involving virtual "heavy" quarks in loops which contribute to the strangeness changing nonleptonic Hamiltonian. These alter the strength of the

(V-A) $\otimes$ (V-A) terms and introduce new terms with different chiral structure, e.g., (V-A) $\otimes$ (V+A).

It is the purpose of this chapter to calculate the effective nonleptonic Hamiltonian for strangeness changing decays in the six-quark model. The W-boson, t-quark, b-quark, and c-quark are successively considered as very heavy, and renormalization group techniques used to calculate (in the leading logarithmic approximation) the resulting effective Hamiltonian remaining at each stage.

The basic techniques for carrying out such calculations have been laid out previously.[5,6,8,19,20] They were even applied in the four-quark model to get the effective Hamiltonian for strangeness changing decays with the charm quark (and W-boson) taken as heavy.[8] However, there is only one Cabibbo angle in the four-quark model and no CP violating phase. It is the CP violating pieces of the effective nonleptonic Hamiltonian which are of special interest in this chapter.

In the next section the method by which the effective Hamiltonian for nonleptonic strangeness changing decays is to be calculated in the six-quark model is described. The approach is pedagogical and emphasizes the underlying assumptions and the conditions necessary for the validity of the leading log approximation. In Section 2, numerical results are given. As expected, CP violating terms appear in the resulting effective Hamiltonian, both in the old terms of (V-A) $\otimes$ (V-A) form and in the new "Penguin"-type terms. In the former they are quite small, but in the latter are large. Many of the details concerning the matrices of anomalous dimensions and their eigenvectors and eigenvalues are relegated to an appendix.

1. Derivation of the Effective Nonleptonic Weak Hamiltonian

Recall that in the standard model[2,10] where the gauge group of weak and electromagnetic interactions is SU(2) ⊗ U(1), the six quarks, u,c, and t with charge +2/3 and d,s, and b, with charge -1/3, are assigned to left-handed doublets and right-handed singlets:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L ; \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \begin{pmatrix} t \\ b' \end{pmatrix}_L ; \quad (u)_R ; (d)_R ; (c)_R ; (s)_R ; (t)_R ; (b)_R .$$

As was mentioned in Chapter I, the standard choice of quark fields is such that

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L , \quad (2.1)$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $i \in \{1,2,3\}$. Equation (2.1) defines the three Cabibbo-like mixing angles θ_i and the CP violating phase, δ.

Weak interactions involving the charged hadronic current follow from the interaction term in the Hamiltonian density

$$\mathscr{H}_I(x) = \frac{g}{2\sqrt{2}} J^{\mu^+}(x)\, W^-_\mu(x) + \text{h.c.} \qquad , \qquad (2.2)$$

where W^-_μ is the charge W boson field, J^+_μ the charged weak current defined by

$$J^+_\mu(0) = \bar{u}(0)\gamma_\mu(1-\gamma_5)d'(0) + \bar{c}(0)\gamma_\mu(1-\gamma_5)s'(0) + \bar{t}(0)\gamma_\mu(1-\gamma_5)b'(0)$$

$$= (\bar{u}d')_{V-A} + (\bar{c}s')_{V-A} + (\bar{t}b')_{V-A} \qquad , \qquad (2.3)$$

and g is the gauge coupling constant of the weak SU(2) subgroup. With no strong interactions the lowest order weak current-current interaction at zero momentum transfer is described by the effective Hamiltonian density

$$\mathscr{H}_{eff}(0) = \frac{g^2}{8M_W^2} J^{\mu^+}(0)\, J_\mu^-(0) + h.c. \qquad , \qquad (2.4)$$

so that the Fermi coupling $G_F/\sqrt{2} = g^2/(8M_W^2)$. In particular the strangeness changing piece of Eq. (2.4) is

$$\begin{aligned}\mathscr{H}_{eff}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} \Big\{ & -c_1 s_1 c_3 (\bar{s}_\alpha u_\alpha)_{V-A}\, (\bar{u}_\beta d_\beta)_{V-A} \\ & + s_1 c_2 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta}) (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta d_\beta)_{V-A} \\ & + s_1 s_2 (c_1 s_2 c_3 + c_2 s_3 e^{-i\delta}) (\bar{s}_\alpha t_\alpha)_{V-A} (\bar{t}_\beta d_\beta)_{V-A} \Big\} \quad , \qquad (2.5)\end{aligned}$$

where the color indices α and β on the quarks (which when repeated are summed from 1 to 3) have been made explicit in preparation for the inclusion of the strong interactions. It is convenient to rewrite Eq. (2.5) as

$$\mathscr{H}_{eff}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}} \Big\{ A_c\big(O_c^{(+)} + O_c^{(-)}\big) + A_t\big(O_t^{(+)} + O_t^{(-)}\big) \Big\} \quad , \qquad (2.6)$$

where

$$O_q^{(\pm)} = \Big[(\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \pm (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A}\Big] - [u \to q], \quad (2.7)$$

and

$$A_c = s_1 c_2 \big(c_1 c_2 c_3 - s_2 s_3 e^{-i\delta}\big) \qquad (2.8a)$$

$$A_t = s_1 s_2 \big(c_1 s_2 c_3 + c_2 s_3 e^{-i\delta}\big) \quad . \qquad (2.8b)$$

Normal ordering of the four-fermion operators is understood. The space-time coordinates of all operators are suppressed.

Now introduce the strong interactions in the form of quantum chromodynamics (QCD), the gauge theory based on the color SU(3) gauge group involving vector gluons interacting with quarks.[4] The strong interactions modify the lowest order weak effective Hamiltonian from the form in Eqs. (2.4) and (2.5). We now proceed to derive in leading logarithmic approximation the form of the effective weak Hamiltonian in the presence of strong interactions with heavy W-bosons and heavy t,b, and c quarks.

First, the W-boson is taken as much heavier than any other mass scale in the problem and the S-matrix elements of the weak interaction between low momentum hadron states composed of light quarks and differing in strangeness by one unit are considered. This is just the calculation performed in Refs. 5 and 6. Using the operator product expansion[3] (noting that the operators $O_c^{(\pm)}$ and $O_t^{(\pm)}$ are multiplicatively renormalized and do not mix with other operators at the one loop level) it follows that to leading order in the heavy W-boson mass

$$\left(-\frac{i}{2}\right)\int d^4x \langle|T(\mathcal{H}_I(x),\mathcal{H}_I(0))|\rangle = -\frac{G_F}{2\sqrt{2}}\left\{A_c^{(+)}\left(\frac{M_W}{\mu},g\right)\langle|O_c^{(+)}(0)|\rangle + A_c^{(-)}\left(\frac{M_W}{\mu},g\right)\langle|O_c^{(-)}(0)|\rangle + A_t^{(+)}\left(\frac{M_W}{\mu},g\right)\langle|O_t^{(+)}(0)|\rangle + A_t^{(-)}\left(\frac{M_W}{\mu},g\right)\langle|O_t^{(-)}(0)|\rangle\right\}, \quad (2.9)$$

where μ is the renormalization point of the strong interactions. The matrix elements of the right-hand side are to be evaluated to all orders in the strong interactions (since perturbation theory is probably not valid) and to zeroeth order in the weak interactions.

The Wilson coefficients $A_t^{(\pm)}(M_W/\mu,g)$ and $A_c^{(\pm)}(M_W/\mu,g)$ depend on the choice of renormalization scheme. Of course, matrix elements of the renormalized operators $O_t^{(\pm)}$ and $O_c^{(\pm)}$ also depend on the renormalization scheme in such a way that physical quantities are rendered scheme independent. We use the mass independent $\overline{\mathrm{MS}}$ subtraction scheme[21] where the renormalization group equations[22] are

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma^{(\pm)}(g)\right) A_q^{(\pm)}\left(\frac{M_W}{\mu}, g\right) = 0 \quad . \qquad (2.10)$$

The $\gamma^{(\pm)}$ characterize the anomalous dimension of the operators $O_q^{(\pm)}$ with $q = c$ or t. The function $\beta(g)$ has the perturbation expansion:[23]

$$\beta(g) = -(33-2N_f)\frac{g^3}{48\pi^2} + \mathcal{O}(g^5) \quad , \qquad (2.11)$$

where N_f (which equals 6 here) is the number of quark flavors. A standard one loop calculation[5,6] shows that $\gamma^{(\pm)}(g)$ has the perturbation expansion:

$$\gamma^{(+)}(g) = \frac{g^2}{4\pi^2} + \mathcal{O}(g^4) \qquad (2.12a)$$

$$\gamma^{(-)}(g) = -\frac{g^2}{2\pi^2} + \mathcal{O}(g^4) \quad . \qquad (2.12b)$$

With the running coupling constant $\bar{g}(y,g)$ defined by

$$\ln y = \int_g^{\bar{g}(y,g)} \frac{dx}{\beta(x)} \qquad (2.13)$$

and $\bar{g}(1,g) = g$, Eq. (2.10) has the solution

$$A_q^{(\pm)}\left(\frac{M_W}{\mu}, g\right) = \left[\exp \int_g^{\bar{g}(M_W/\mu,g)} - \frac{\gamma^{(\pm)}(x)}{\beta(x)}\, dx\right] A_q^{(\pm)}\left(1, \bar{g}\left(\frac{M_W}{\mu}, g\right)\right). \tag{2.14}$$

In a leading log calculation the coefficients $A_q^{(\pm)}\left(1, \bar{g}(M_W/\mu, g)\right)$ can be replaced by their free field values A_q given in Eq. (2.8) because the running fine structure constant $\alpha = \bar{g}^2/4\pi$ is small at the mass scale of the W and because the value of their first dependent variable being unity implies no other large logarithms can be generated by higher order strong interactions. Using Eqs. (2.11) and (2.12)

$$-\frac{\gamma^{(\pm)}(x)}{\beta(x)} = \frac{2a^{(\pm)}}{x} + \text{terms finite at } x = 0 \quad , \tag{2.15}$$

with

$$a^{(+)} = \frac{6}{33 - 2N_f} \tag{2.16a}$$

$$a^{(-)} = \frac{-12}{33 - 2N_f} \quad . \tag{2.16b}$$

Choosing μ above the onset of scaling, Eq. (2.15) may be substituted back into Eq. (2.14) to obtain the result:[24]

$$\begin{aligned} A_q^{(\pm)}\left(\frac{M_W}{\mu}, g\right) &= \left[\frac{\bar{g}^2\left(\frac{M_W}{\mu}, g\right)}{\bar{g}^2(1, g)}\right]^{a^{(\pm)}} A_q \\ &= \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(\pm)}} A_q \quad . \end{aligned} \tag{2.17}$$

At this stage our effective weak Hamiltonian density is

$$\mathscr{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}}\left\{\left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(+)}}\left(A_c O_c^{(+)} + A_t O_t^{(+)}\right)\right.$$

$$\left. + \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(-)}}\left(A_c O_c^{(-)} + A_t O_t^{(-)}\right)\right\} \quad . \tag{2.18}$$

The matrix elements of the above effective weak Hamiltonian density are to be evaluated to all orders in the strong interactions and to zeroeth order in the weak interactions. Note that $\mathscr{H}_{\text{eff}}$ does not explicitly involve the W boson field. We want to derive an effective Hamiltonian without explicit dependence on the heavy W-boson, t-quark, b-quark and c-quark fields. Equation (2.18) is the first step towards this goal.

The next step is to consider the t-quark as very heavy and eliminate it from explicitly appearing in the effective weak Hamiltonian for strangeness changing processes. What happens to the operator $O_c^{(\pm)}$ and $O_t^{(\pm)}$ is different, and the more complicated case of $O_t^{(\pm)}$ is considered first.

We assume that m_t is much greater than all other quark masses, the momenta of the external states, and the renormalization point mass, μ. The work of Appelquist and Carrazone[25] implies that to order $1/m_t^2$ all the dependence of amplitudes on the heavy t-quark mass can be absorbed into renormalization effects and hence into a redefinition of the coupling constant, mass parameters, and scale of operators. This suggests the following factorization:

$$\langle|O_t^{(\pm)}|\rangle = \sum_i B_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right)\langle|O_i|\rangle' + \mathscr{O}\left(\frac{1}{m_t^2}\right) \quad , \tag{2.19}$$

where the primed matrix elements are evaluated to all orders in an effective theory of strong interactions[26] with 5-quark flavors, coupling $g'(m_t/\mu, g)$ and mass parameters $m_u', m_d', \ldots, m_b'$. Thus,

$$\langle|O_i|\rangle' = \langle|O_i|\rangle\left(g', \mu, m_u', \ldots, m_b'\right) \quad .$$

To carry out the expansion of Eq. (2.19) in leading log approximation six linearly independent operators O_i are sufficient. They are choosen as follows:

$$\begin{aligned}
O_1 &= (\bar{s}_\alpha d_\alpha)_{V-A} \, (\bar{u}_\beta u_\beta)_{V-A} \\
O_2 &= (\bar{s}_\alpha d_\beta)_{V-A} \, (\bar{u}_\beta u_\alpha)_{V-A} \\
O_3 &= (\bar{s}_\alpha d_\alpha)_{V-A} \left[(\bar{u}_\beta u_\beta)_{V-A} + \ldots + (\bar{b}_\beta b_\beta)_{V-A}\right] \\
O_4 &= (\bar{s}_\alpha d_\beta)_{V-A} \left[(\bar{u}_\beta u_\alpha)_{V-A} + \ldots + (\bar{b}_\beta b_\alpha)_{V-A}\right] \\
O_5 &= (\bar{s}_\alpha d_\alpha)_{V-A} \left[(\bar{u}_\beta u_\beta)_{V+A} + \ldots + (\bar{b}_\beta b_\beta)_{V+A}\right] \\
O_6 &= (\bar{s}_\alpha d_\beta)_{V-A} \left[(\bar{u}_\beta u_\alpha)_{V+A} + \ldots + (\bar{b}_\beta b_\alpha)_{V+A}\right] \quad .
\end{aligned} \tag{2.20}$$

These operators are sufficient since they close under renormalization at the one loop level. The operators O_1 and O_2 already occur to zeroeth order in strong interactions: it follows from Eq. (2.7) that

$$\begin{aligned}
B_1^{(\pm)} &\equiv B_1^{(\pm)}(1,0) = \pm 1 \\
B_2^{(\pm)} &\equiv B_2^{(\pm)}(1,0) = +1 \quad .
\end{aligned} \tag{2.21a}$$

The operators O_3, O_4, O_5, and O_6 are generated by the strong interactions through "Penguin"-type diagrams, so that in free field theory

$$B_3^{(\pm)} = B_4^{(\pm)} = B_5^{(\pm)} = B_6^{(\pm)} = 0 \quad . \tag{2.21b}$$

However, the operators O_i are not multiplicatively renormalized at the one loop level, i.e., they mix among themselves. As shown in the appendix, the renormalization group equation their coefficients $B_i^{(\pm)}(m_t/\mu,g)$ satisfy is

$$\sum_j \left[\left(\mu \frac{\partial}{\partial\mu} + \bar{\beta}(g) \frac{\partial}{\partial g} + \gamma_t(g)\, m_t \frac{\partial}{\partial m_t} + \gamma^{(\pm)}(g)\right)\delta_{ij} - \gamma'^{T}_{ij}(g')\right] \cdot B_j^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = 0 \quad . \tag{2.22}$$

Here γ'^T is the transpose of the anomalous dimension matrix of the operators O_i in the effective theory of strong interactions with 5 quarks and coupling g'. It is the eigenvectors of γ'^T that correspond to operators which are multiplicatively renormalized. The coefficient functions $\tilde{B}_i^{(\pm)}(m_t/\mu,g)$ of these multiplicatively renormalized operators are written as

$$\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \sum_j V^{-1}_{ij}\, B_j^{(\pm)}\left(\frac{m_t}{\mu}, g\right) \quad , \tag{2.23}$$

and the eigenvalues of γ'^T are denoted by γ'_i. The matrix γ' is found in Appendix A along with its eigenvalues and the matrix V. For the $\tilde{B}_i^{(\pm)}(m_t/\mu,g)$, the renormalization group equation corresponding to Eq. (2.22) is

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_t(g)\, m_t \frac{\partial}{\partial m_t} + \gamma^{(\pm)}(g) - \gamma'_i(g')\right)\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = 0 \; . \tag{2.24}$$

The solution to this equation may be found with the aid of the running coupling constant $\bar{g}(y,g)$ defined by[19]

$$\ell n\, y = \int_{g}^{\bar{g}(y,g)} \left[\frac{1-\gamma_t(x)}{\beta(x)}\right] dx \quad , \tag{2.25}$$

with $\bar{g}(1,g)=g$. Note that this is not the usual definition of the running coupling constant (Eq. (2.13)), but the integrand in Eq. (2.25) for small x has the same leading behavior given by $1/\beta(x)$ as the integrand in Eq. (2.13). Setting $y = m_t/\mu$, it is now easily shown that the solution of Eq. (2.24) is

$$\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \left[\exp \int_{g}^{\bar{g}(m_t/\mu,g)} \frac{\gamma^{(\pm)}(x)}{\beta(x)}\, dx\right]\left[\exp \int_{g'(m_t/\mu,g)}^{g'(1,\bar{g})} \frac{-\gamma_i'(x)}{\beta'(x)}\, dx\right]$$

$$\cdot \; \tilde{B}_i^{(\pm)}(1,\bar{g}) \quad . \tag{2.26}$$

β' is the beta function in the effective theory with 5-quarks and coupling g'. This beta function has the perturbation expansion

$$\beta'(g') = -(33-2N_f)\,\frac{g'^3}{48\pi^2} + \mathcal{O}(g'^5) \tag{2.27}$$

with $N_f = 5$, and we write

$$-\frac{\gamma_i'(x)}{\beta'(x)} = \frac{2a_i'}{x} + \text{finite terms at } x=0 \quad . \tag{2.28}$$

Choosing μ as before, above the onset of scaling, Eqs. (2.15) and (2.28) may be used to get

$$\tilde{B}_i^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \left[\frac{\alpha(m_t^2)}{\alpha(\mu^2)}\right]^{-a^{(\pm)}} \cdot \left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a_i'} \tilde{B}_i^{(\pm)}(1,\bar{g}) \quad . \tag{2.29}$$

We have used $g'(1,\bar{g}) \approx \bar{g}(m_t/\mu,g)$, which is valid in a leading log calculation since the running fine structure constant is small at the t-quark mass. Finally, using the linear relationship between the eigenvectors $\tilde{B}_i$ and the B_i

$$B_k^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \left[\frac{\alpha(m_t^2)}{\alpha(\mu^2)}\right]^{-a^{(\pm)}} \sum_{i,j} V_{kj}\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a_j'} V_{ji}^{-1}\, B_i^{(\pm)}(1,\bar{g}) \ . \tag{2.30}$$

Notice that the factor $\left[\alpha(m_t^2)/\alpha(\mu^2)\right]^{-a^{(\pm)}}$ out in front of the summation in Eq. (2.30) combines with the earlier factor $\left[\alpha(M_W^2)/\alpha(\mu^2)\right]^{a^{(\pm)}}$ in Eq. (2.16) to give $\left[\alpha(M_W^2)/\alpha(m_t^2)\right]^{a^{(\pm)}}$. In leading log approximation the coefficients $B_i^{(\pm)}(1,\bar{g})$ can be replaced by their free field values as given in Eq. (2.21), since no large logarithms can be generated from QCD loop integrals with the first argument of $B_i^{(\pm)}(m_t/\mu,g)$ set equal to unity and because we assume the running fine structure constant is small at the t-quark mass.

The case of the operators $O_c^{(\pm)}$ is much simpler. The charm quark field which appears explicitly in these operators is of course not directly affected at this stage of considering the t-quark as very heavy and the $O_c^{(\pm)}$ are just multiplicatively renormalized:

$$\langle|O_c^{(\pm)}|\rangle = B^{(\pm)}\left(\frac{m_t}{\mu}, g\right)\langle|O_c^{(\pm)}|\rangle' \quad . \tag{2.31}$$

Note that the matrix elements on the right-hand side are again to be evaluated in the effective five-quark theory with coupling $g'(m_t/\mu,g)$. The coefficients $B^{(\pm)}(m_t/\mu,g)$ satisfy

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_t(g)\, m_t \frac{\partial}{\partial m_t} + \gamma^{(\pm)}(g) - \gamma'^{(\pm)}(g')\right) B^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = 0. \tag{2.32}$$

The anomalous dimension $\gamma'^{(\pm)}(g')$ is that of $O_c^{(\pm)}$ and is a function of the coupling g' in the effective five-quark theory, while $\gamma^{(\pm)}(g)$ depends on g, the coupling in the six-quark theory.

Solving Eq. (2.32) in the same manner as Eq. (2.24), gives

$$B^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \left[\exp \int_g^{\bar{g}(m_t/\mu,g)} \frac{\gamma^{(\pm)}(x)}{\beta(x)}\, dx\right]\left[\exp \int_{g'}^{g'(1,\bar{g})} -\frac{\gamma'^{(\pm)}(x)}{\beta'(x)}\, dx\right] B^{(\pm)}(1,\bar{g})$$

$$= \left[\frac{\alpha(m_t^2)}{\alpha(\mu^2)}\right]^{-a^{(\pm)}} \left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a'^{(\pm)}} B^{(\pm)}(1,\bar{g}) \quad . \tag{2.33}$$

In leading log approximation $B^{(\pm)}\left(1,\bar{g}(m_t/\mu,g)\right)$ can be replaced by its free field value of +1.

The effective weak Hamiltonian density is now free of explicit dependence on the heavy t-quark field and has the form:

$$\mathscr{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}}\left\{\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a'^{(+)}}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(+)}} A_c\, O_c^{(+)}\right.$$

$$+\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a'^{(-)}}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(-)}} A_c\, O_c^{(-)} \tag{2.34}$$

$$+\sum_k\left(\sum_{i,j} V_{kj}\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a_j'} V_{ji}^{-1} B_i^{(+)}\right)\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(+)}} A_t\, O_k$$

$$\left.+\sum_k\left(\sum_{i,j} V_{kj}\left[\frac{\alpha(m_t^2)}{\alpha'(\mu^2)}\right]^{a_j'} V_{ji}^{-1} B_i^{(-)}\right)\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(-)}} A_t\, O_k\right\}$$

All operators on the right-hand side are to have their matrix elements evaluated in the effective theory with five quarks, coupling $g'(m_t/\mu, g)$ and masses $m_u', m_d', \ldots, m_b'$.

The next step of considering the b-quark as very heavy is similar to what was just accomplished for the t-quark, with the addition of some indices. This time the matrix elements of the operators O_i of Eq. (2.20) evaluated in the effective five-quark theory are to be expressed in terms of matrix elements of

$$P_1 = (\bar{s}_\alpha d_\alpha)_{V-A}\,(\bar{u}_\beta u_\beta)_{V-A}$$

$$P_2 = (\bar{s}_\alpha d_\beta)_{V-A}\,(\bar{u}_\beta u_\alpha)_{V-A}$$

$$P_3 = (\bar{s}_\alpha d_\alpha)_{V-A}\left[(\bar{u}_\beta u_\beta)_{V-A} + \ldots + (\bar{c}_\beta c_\beta)_{V-A}\right]$$

$$P_4 = (\bar{s}_\alpha d_\beta)_{V-A}\left[(\bar{u}_\beta u_\alpha)_{V-A} + \ldots + (\bar{c}_\beta c_\alpha)_{V-A}\right]$$

$$P_5 = (\bar{s}_\alpha d_\alpha)_{V-A} \left[(\bar{u}_\beta u_\beta)_{V+A} + \dots + (\bar{c}_\beta c_\beta)_{V+A}\right]$$

$$P_6 = (\bar{s}_\alpha d_\beta)_{V-A} \left[(\bar{u}_\beta u_\alpha)_{V+A} + \dots + (\bar{c}_\beta c_\alpha)_{V+A}\right] \qquad (2.35)$$

evaluated in an effective theory with four quark flavors (u,d,s, and c). The coupling and masses in the effective four-quark theory are denoted by $g''(m_b'/\mu, g')$ and $m_u'', \dots, m_c''$, respectively. To leading order in the b-quark mass

$$\langle|O_k|\rangle' = \sum_n C_k^n \left(\frac{m_b'}{\mu}, g'\right) \langle P_n\rangle'' \quad , \qquad (2.36)$$

where the prime (double prime) denotes evaluation in the effective five (four) quark theory. The $C_k^n(m_b'/\mu, g')$ can be shown to obey an equation of the form

$$\sum_{k,n} \left[\left(\mu \frac{\partial}{\partial\mu} + \beta'(g') \frac{\partial}{\partial g'} + \gamma_b' m_b' \frac{\partial}{\partial m_b'}\right) \delta_{jk}\, \delta_{mn} \right.$$

$$\left. + \gamma_{jk}'(g')\, \delta_{mn} - \delta_{jk}\, \gamma_{mn}''^{T}(g'') \right] C_k^n \left(\frac{m_b'}{\mu}, g'\right) = 0 \quad , \qquad (2.37)$$

with γ' and γ'' being anomalous dimension matrices of the operators $O_1, \dots, O_6$ and $P_1, \dots, P_6$, respectively.

Defining the linear combinations of coefficient functions

$$\tilde{C}_k^n \left(\frac{m_b'}{\mu}, g'\right) = \sum_\ell W_{n\ell}^{-1}\, C_k^\ell \left(\frac{m_b'}{\mu}, g'\right) \qquad (2.38)$$

as corresponding to operators which are multiplicatively renormalized, i.e., do not mix with other operators, the renormalization group equations diagonalize into the form

$$\left(\mu \frac{\partial}{\partial\mu} + \beta(g') \frac{\partial}{\partial g'} + \gamma_b' m_b' \frac{\partial}{\partial m_b'} + \gamma_k'(g') - \gamma_n''(g'')\right)$$

$$\cdot \sum_j \tilde{C}_j^n \left(\frac{m_b'}{\mu}, g'\right) V_{jk} = 0 \quad . \tag{2.39}$$

The matrices W and γ'' together with the eigenvalues of the latter are found in Appendix A.

With the aid of a new running coupling defined by

$$\ell n\ y = \int_{g'}^{\bar{g}'(y,g')} \frac{1 - \gamma_b'(x)}{\beta'(x)} dx \quad , \tag{2.40}$$

these equations may be solved very analogously to Eq. (2.24). Leaving out some of the details, the solution in the leading logarithmic approximation is

$$C_k^n\left(\frac{m_b'}{\mu}, g'\right) = \sum_{i,\ell}\left(\sum_j V_{ij}\left[\frac{\alpha'(m_b'^2)}{\alpha'(\mu^2)}\right]^{-a_j'} V_{jk}^{-1}\right)$$

$$\cdot \left(\sum_m W_{nm}\left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{a_m''} W_{m\ell}^{-1}\right) C_i^\ell\left(1, \bar{g}'\left(\frac{m_b'}{\mu}, g'\right)\right) \quad . \tag{2.41}$$

For reasons stated before, in a leading log calculation the coefficients $C_i^\ell(1,\bar{g}')$ can be replaced by their free field values:

$$C_i^\ell \equiv C_i^\ell(1,0) = \delta_{i\ell} \quad . \tag{2.42}$$

The operators $O_c^{(\pm)}$ are multiplicatively renormalized and the expansion of their matrix elements gives results like those in Eq. (2.33) with appropriate changes.

The effective Hamiltonian now takes the following form at the four-quark level:

$$
\begin{aligned}
\mathscr{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}} \Bigg\{ & \left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{a''(+)} \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{a'(+)} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(+)}} A_c\, O_c^{(+)} \\
& + \left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{a''(-)} \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{a'(-)} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(-)}} A_c\, O_c^{(-)} \\
& + \sum_{k,n} \left(\sum_{\ell,m} W_{nm} \left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{a''_m} W^{-1}_{m\ell}\, C^{\ell}_{k} \right) \left(\sum_{i,j} V_{kj} \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{a'_j} V^{-1}_{ji} \right. \\
& \cdot \left. \left(B_i^{(+)} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(+)}} A_t + B_i^{(-)} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(-)}} A_t \right) \right) P_n \Bigg\}
\end{aligned}
\qquad (2.43)
$$

The final step of considering the charm quark as very heavy is more questionable from the phenomenological viewpoint. It also involves a technical point which is easy to miss. When the matrix elements of the operators $P_1, \ldots, P_6$ evaluated in the effective four-quark theory are expanded in terms of matrix elements of operators evaluated in an effective three-quark theory, it is natural to define

$$Q_1 = (\bar{s}_\alpha d_\alpha)_{V-A}\, (\bar{u}_\beta u_\beta)_{V-A}$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_{V-A}\, (\bar{u}_\beta u_\alpha)_{V-A}$$

$$Q_3 = (\bar{s}_\alpha d_\alpha)_{V-A} \left[(\bar{u}_\beta u_\beta)_{V-A} + (\bar{d}_\beta d_\beta)_{V-A} + (\bar{s}_\beta s_\beta)_{V-A}\right]$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \left[(\bar{u}_\beta u_\alpha)_{V-A} + (\bar{d}_\beta d_\alpha)_{V-A} + (\bar{s}_\beta s_\alpha)_{V-A}\right]$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_{V-A} \left[(\bar{u}_\beta u_\beta)_{V+A} + (\bar{d}_\beta d_\beta)_{V+A} + (\bar{s}_\beta s_\beta)_{V+A} \right]$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \left[(\bar{u}_\beta u_\alpha)_{V+A} + (\bar{d}_\beta d_\alpha)_{V+A} + (\bar{s}_\beta s_\alpha)_{V+A} \right] \qquad (2.44)$$

These operators close under renormalization at the one-loop level, but they are linearly dependent:

$$Q_4 = -Q_1 + Q_2 + Q_3 \qquad . \qquad (2.45)$$

Hence only the 5 operators Q_1, Q_2, Q_3, Q_5 and Q_6 are necessary.

Expressing matrix elements of the operators evaluated in the effective four-quark theory in terms of matrix elements of operators evaluated in the effective three-quark theory,

$$\langle|P_n|\rangle'' = \sum_{r=1,2,3,5,6} D_n^r\left(\frac{m_c''}{\mu}, g''\right)\langle|Q_r|\rangle''' + \mathcal{O}\left(\frac{1}{m_c^2}\right) , \qquad (2.46)$$

with g''' and m_u''', m_d''', m_s''' representing the coupling constant and quark masses in the effective three quark theory. The linear combinations

$$\tilde{D}_n^r\left(\frac{m_c''}{\mu}, g''\right) = \sum_s X_{rs}^{-1} D_n^s\left(\frac{m_c''}{\mu}, g''\right) \qquad (2.47)$$

are the coefficients of multiplicatively renormalized operators. The diagonalized renormalization group equations are

$$\left[\mu \frac{\partial}{\partial\mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c'' m_c'' \frac{\partial}{\partial m_c''} + \gamma_n''(g'') - \gamma_r'''(g''')\right]$$

$$\cdot \sum_m \tilde{D}_m^r\left(\frac{m_c''}{\mu}, g''\right) W_{mn} = 0 \qquad , \qquad (2.48)$$

and have the solution in leading logarithmic approximation after re-expressing the $\tilde{D}$'s in terms of D's,

$$D_\ell^r\left(\frac{m_c''}{\mu}, g''\right) = \sum_{n,p}\left(\sum_m W_{nm}\left[\frac{\alpha''(m_c''^2)}{\alpha''(\mu^2)}\right]^{-a_m''} W_{m\ell}^{-1}\right)$$

$$\cdot\left(\sum_q X_{rq}\left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{a_q'''} X_{qp}^{-1}\right) D_n^p(1,\bar{g}'') \quad . \qquad (2.49)$$

In leading log approximation the $D_n^p(1,\bar{g}'')$ can be replaced by their free field values, D_n^p. These are δ_{np}, except when $n=4$, in which case $D_4^1=-1$, $D_4^2=1$, $D_4^3=1$, and $D_4^5=D_4^6=0$.

Because the charm quark is being considered as heavy, the operators $O_c^{(\pm)}$ are no longer just multiplicatively renormalized at the one loop level. It is also necessary to expand

$$\langle|O_c^{(\pm)}|\rangle'' = \sum_r D_r^{(\pm)}\left(\frac{m_c''}{\mu}, g''\right)\langle|Q_r|\rangle''' \quad . \qquad (2.50)$$

The renormalization group equations obeyed by the $D_r^{(\pm)}(m_c''/\mu, g'')$ are

$$\sum_r\left[\left(\mu\frac{\partial}{\partial\mu} + \beta''(g'')\frac{\partial}{\partial g''} + \gamma_c''(g'')\, m_c''\frac{\partial}{\partial m_c''} + \gamma''^{(\pm)}(g'')\right)\delta_{pr} - \gamma_{pr}'''^{T}(g''')\right]$$

$$\cdot\, D_r^{(\pm)}\left(\frac{m_c''}{\mu}, g''\right) = 0 \quad . \qquad (2.51)$$

The coefficients corresponding to multiplicatively renormalized operators are just as in Eq. (2.47), and the solution to Eq. (2.51) with the usual approximations is

$$D_r^{(\pm)}\left(\frac{m_c''}{\mu}, g''\right) = \left[\frac{\alpha''(m_c''^2)}{\alpha''(\mu^2)}\right]^{-a''^{(\pm)}} \sum_{p,q} X_{rq}\left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{a_q'''} X_{qp}^{-1}\, D_p^{(\pm)}(1,\bar{g}'') \quad . \qquad (2.52)$$

The free field values, $D_p^{(\pm)} \equiv D_p^{(\pm)}(1,0)$, are $D_1^{(\pm)} = 1$, $D_2^{(\pm)} = +1$, and all others zero.

Finally, collecting all the results the previously advertised effective Hamiltonian in the "light" three-quark sector can be written. It is the following sum of Wilson coefficients times local four-fermion operators which do not explicitly involve the heavy W-boson, top, bottom, and charm quark fields:

$$\begin{aligned}
\mathscr{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{2\sqrt{2}} \Bigg\{ & \sum_r \Bigg(\sum_{p,q} X_{rq} \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{a_q'''} X_{qp}^{-1} D_p^{(+)} \Bigg) \\
& \cdot \left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{a''^{(+)}} \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{a'^{(+)}} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(+)}} A_c\, Q_r \\
& + \sum_r \Bigg(\sum_{p,q} X_{rq} \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{a_q'''} X_{qp}^{-1} D_p^{(+)} \Bigg) \\
& \cdot \left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{a''^{(-)}} \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{a'^{(-)}} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(-)}} A_c\, Q_r \\
& + \sum_{k,n,r} \Bigg(\sum_{p,q} X_{rq} \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{a_q'''} X_{qp}^{-1} D_n^p \Bigg) \\
& \cdot \Bigg(\sum_{\ell,m} W_{nm} \left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{a_m''} W_{m\ell}^{-1} C_k^{\ell} \Bigg) \Bigg(\sum_{i,j} V_{kj} \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{a_j'} V_{ji}^{-1} \\
& \cdot \Bigg(B_i^{(+)} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(+)}} A_t + B_i^{(-)} \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{a^{(-)}} A_t \Bigg) \Bigg) Q_r \Bigg\} .
\end{aligned} \tag{2.53}$$

All summations are from 1 through 6, except those over p, q and r which run through 1,2,3,5 and 6.

2. Numerical Results for the Effective Nonleptonic Hamiltonian

It is now possible to perform the arithmetic operations made explicit in Eq. (2.53) and to examine the resulting Wilson coefficients of the operators Q_1,Q_2,Q_3,Q_5 and Q_6 in the effective Hamiltonian for nonleptonic, strangeness changing interactions. Since the matrices, V,W and X, as given in the appendix, are composed of irrational numbers and since various fractional powers of $\alpha(M^2)$ with $M^2 = M_W^2$, m_t^2, etc. are rampant, quantitatively rather little is transparent about these coefficients in general. We then are forced to proceed by choosing a parametrization for $\alpha(M^2)$ and values for the W and quark masses, substituting in Eq. (2.53), and reading off the coefficients of the Q_i for that particular set of choices.

Moreover, the outlook is basically qualitative. The QCD effects have been calculated in the leading log approximation. While we have some confidence that at the first step M_W is a large enough mass for this to be a credible procedure, by the last step of considering m_c a heavy mass this approximation has been used beyond the region where it can be reasonably justified.

On the positive side, what is carried out here is well defined and systematic. The degree of accuracy is obviously no worse than any of the earlier calculations[8] which involve only the "heavy" charm quark (and W boson) in leading log approximation. Not only is the accuracy of the calculation expected to be better for the b and t-quarks, but their effect was not taken into account previously. With regard to CP violation they play a dominant role.

To investigate the effective nonleptonic Hamiltonian numerically we first of all need to decide on the running QCD fine structure constant $\alpha(Q^2)$, the values of the heavy quark masses, and μ^2 or alternately $\alpha(\mu^2)$. In leading log approximation

$$\alpha(Q^2) = \frac{12\pi}{33-2N_f} \frac{1}{\ln(Q^2/\Lambda^2)} , \qquad (2.54)$$

where we take $\Lambda^2 = 0.1$ GeV^2 and $\Lambda^2 = 0.01$ GeV^2, values consistent with recent data when QCD is used to parametrize the breakdown of scale invariance in deep inelastic neutrino scattering.[27] When the leading log approximation is valid, the calculation is insensitive to the precise value of Λ and the difference between Λ's in the various effective field theories can be neglected. The number of quark flavors is $N_f = 6$ for the fine structure constant we have called $\alpha(Q^2)$, while $\alpha'(Q^2)$, $\alpha''(Q^2)$, and $\alpha'''(Q^2)$ have $N_f = 5,4$ and 3 respectively, as they pertain to effective theories with those corresponding numbers of quark flavors.

m_c is taken to be 1.5 GeV and m_b to be 4.5 GeV on the basis of ψ and Υ spectroscopy.[28] The t-quark mass is unknown at this time, and values of 15 GeV and 30 GeV are used to get an idea of the sensitivity of the results to this quantity. For M_W the value 85 GeV is taken. In evaluating Eq. (2.53), m_b' and m_b, m_c'' and m_c, are not distinguished between, again consistent with the leading log approximation philosophy.

Finally a value is required for $\alpha(\mu^2)$ (or more exactly $\alpha'''(\mu^2)$). We want to choose μ to be a typical "light" hadron mass scale or inverse size, where $\alpha(\mu^2)$ is of order unity. We let $\alpha(\mu^2) = 0.75$, 1.0 and 1.25

to check the variation of the resulting effective nonleptonic Hamiltonian to this choice. In fact, the values of S-matrix elements of the weak interaction cannot depend on the choice of the renormalization point μ, or equivalently $\alpha(\mu^2)$. The matrix elements of the four-fermion operators, Q_i, also have an implicit μ dependence which exactly compensates that of their coefficients (at least when the coefficients are computed exactly). We are left to make a choice of μ, hopefully close to the typical light hadron mass scale of the problem, so that "hard" gluon effects are contained as much as possible in the Wilson coefficients and not the matrix elements of Q_i, but high enough that their calculation in leading log approximation makes some sense.[29]

In terms of the operators, Q_1, Q_2, Q_3, Q_5 and Q_6 defined previously in Eq. (2.44), the nonleptonic Hamiltonian involving u,d and s quark fields has the form:

$$\begin{aligned}\mathcal{H}_{\text{eff}}^{(\Delta S=1)} = -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \Big\{ & (-0.87 + 0.036\tau)Q_1 \\ & + (1.51 - 0.036\tau)Q_2 \\ & + (-0.021 - 0.012\tau)\, Q_3 \\ & + (0.011 + 0.007\tau)Q_5 \\ & + (-0.047 - 0.072\tau)Q_6 \Big\} \quad , \qquad (2.55)\end{aligned}$$

when $m_t = 15$ GeV and $\alpha(\mu^2) = 1$ and where

$$\tau = s_2^2 + s_2 c_2 s_3 e^{-i\delta}/c_1 c_3 \quad , \qquad (2.56)$$

along with the other masses specified previously. Values of the coefficients for all six cases corresponding to $\alpha(\mu^2) = 0.75$, 1.0 and 1.25

and $m_t = 15$ GeV and 30 GeV are found in Table I for $\Lambda^2 = .1$ GeV^2 and Table II for $\Lambda^2 = .01$ GeV^2.

Referring back to Eq. (2.5), it is apparent that before accounting for the effects of QCD, the coefficients of the usual four-fermion operator Q_1, as well as the "Penguin" induced operators Q_3, Q_5 and Q_6 were all zero. In the sector involving u,d and s quarks the strangeness changing weak Hamiltonian then just involves Q_2 with unit coefficient. Thus the presence of strong interaction QCD corrections has brought in the operators Q_1, Q_3, Q_5 and Q_6, changed the coefficient of Q_2, and given all coefficients an imaginary (CP violating) part through the quantity τ, which enters through "Penguin"-type diagrams involving a heavy quark loop.

The portion of the nonleptonic Hamiltonian involving only the operators Q_1 and Q_2 is the traditionally calculated $(V-A)\otimes(V-A)$ four-fermion piece with neglect of all "Penguin" effects. The sum of coefficients of Q_1 and Q_2 is proportional to the coefficient of an operator transforming purely as $I = 3/2$, which cannot mix under strong interaction renormalization with "Penguin" contributions which are pure $I = 1/2$. As a consequence, one simple check of the calculation is to note that the quantity τ, arising from "Penguin" contributions, always has the same magnitude and opposite sign in its contribution to the coefficients of Q_1 and Q_2.

The combination of operators $Q_2 - Q_1$ transforms purely as $I = 1/2$, while the combination $Q_1 + Q_2$ has an $I = 3/2$ piece. The ratio of coefficients of $Q_2 - Q_1$ and $Q_2 + Q_1$ is a measure of $\Delta I = 1/2$ or octet enhancement by QCD, as first calculated in Refs. 5 and 6. The inclusion

TABLE I

Coefficients of the operators Q_1, Q_2, Q_3, Q_5 and Q_6 defined in Eq. (46) in the effective Hamiltonian, $\mathcal{H}_{eff} = (-G_F\, s_1 c_1 c_3/\sqrt{2})(\sum_i C_i Q_i)$, for strangeness changing, nonleptonic weak decays. $\tau \equiv s_2^2 + s_2 c_2 s_3 e^{-i\delta}/c_1 c_3$. $\Lambda^2 = .1\ GeV^2$.

Parameters	C_1	C_2	C_3	C_5	C_6
$\alpha(\mu^2) = 0.75$ $m_t = 15$ GeV	−0.72 +0.035τ	+1.40 −0.035τ	−0.013 −0.015τ	+0.007 +0.008τ	−0.025 −0.059τ
$\alpha(\mu^2) = 1.00$ $m_t = 15$ GeV	−0.87 +0.036τ	+1.51 −0.036τ	−0.021 −0.012τ	+0.011 +0.007τ	−0.047 −0.072τ
$\alpha(\mu^2) = 1.25$ $m_t = 15$ GeV	−1.00 +0.036τ	+1.61 −0.036τ	−0.028 −0.010τ	+0.015 +0.006τ	−0.069 −0.085τ
$\alpha(\mu^2) = 0.75$ $m_t = 30$ GeV	−0.71 +0.042τ	+1.39 −0.042τ	−0.013 −0.017τ	+0.007 +0.009τ	−0.025 −0.076τ
$\alpha(\mu^2) = 1.00$ $m_t = 30$ GeV	−0.86 +0.043τ	+1.50 −0.043τ	−0.021 −0.013τ	+0.011 +0.008τ	−0.047 −0.093τ
$\alpha(\mu^2) = 1.25$ $m_t = 30$ GeV	−0.99 +0.043τ	+1.60 −0.043τ	−0.027 −0.011τ	+0.014 +0.007τ	−0.068 −0.109τ

Table II

Same as Table I but with $\Lambda^2 = 0.01\ GeV^2$

Parameters	C_1	C_2	C_3	C_5	C_6
$\alpha(\mu^2) = 0.75$ $m_t = 15$ GeV	$- 0.77$ $+ 0.021\tau$	$+ 1.43$ $- 0.021\tau$	$- 0.026$ $- 0.006\tau$	$+ 0.013$ $+ 0.004\tau$	$- 0.065$ $- 0.045\tau$
$\alpha(\mu^2) = 1.00$ $m_t = 15$ GeV	$- 0.93$ $+ 0.021\tau$	$+ 1.55$ $- 0.021\tau$	$- 0.032$ $- 0.005\tau$	$+ 0.017$ $+ 0.003\tau$	$- 0.097$ $- 0.055\tau$
$\alpha(\mu^2) = 1.25$ $m_t = 15$ GeV	$- 1.06$ $+ 0.021\tau$	$+ 1.65$ $- 0.021\tau$	$- 0.037$ $- 0.003\tau$	$+ 0.020$ $+ 0.002\tau$	$- 0.128$ $- 0.065\tau$
$\alpha(\mu^2) = 0.75$ $m_t = 30$ GeV	$- 0.76$ $+ 0.026\tau$	$+ 1.42$ $- 0.026\tau$	$- 0.025$ $- 0.008\tau$	$+ 0.013$ $+ 0.005\tau$	$- 0.065$ $- 0.060\tau$
$\alpha(\mu^2) = 1.00$ $m_t = 30$ GeV	$- 0.92$ $+ 0.027\tau$	$+ 1.54$ $- 0.027\tau$	$- 0.032$ $- 0.006\tau$	$+ 0.017$ $+ 0.004\tau$	$- 0.097$ $- 0.075\tau$
$\alpha(\mu^2) = 1.25$ $m_t = 30$ GeV	$- 1.05$ $= 0.027\tau$	$+ 1.65$ $- 0.027\tau$	$- 0.037$ $- 0.004\tau$	$+ 0.020$ $+ 0.003\tau$	$- 0.127$ $- 0.088\tau$

of "Penguin" operators and their mixing makes little numerical difference for the coefficients of Q_1 and Q_2. Slightly more important in comparison with earlier work is taking into account not only the heavy W-boson, but each heavy quark successively in computing the leading log QCD effects. As a result the earlier $[\alpha(M_W^2)/\alpha(\mu^2)]^{a^{(\pm)}}$ is replaced by

$$[\alpha(M_W^2)/\alpha(m_t^2)]^{a^{(\pm)}} \; [\alpha(m_t^2)/\alpha'(m_b'^2)]^{a'^{(\pm)}} [\alpha'(m_b'^2)/\alpha''(m_c''^2)]^{a''^{(\pm)}}$$

$$\cdot \; [\alpha''(m_c''^2)/\alpha'''(\mu^2)]^{a'''^{(\pm)}} \qquad ,$$

even if all "Penguin" effects are neglected. Numerically the coefficient of $Q_2 - Q_1$ is enhanced by a factor of 2 to 3 and that of $Q_2 + Q_1$ suppressed by 0.6 to 0.7 for our choice of masses. In agreement with all earlier results this is in the correct direction, but much too small to explain the high degree of accuracy of the $\Delta I = 1/2$ rule in nonleptonic decays of strange particles.

The "Penguin" terms Q_3, Q_5 and Q_6 transform as purely $I = 1/2$ on the other hand. Tables I and II indicate that their coefficients are smaller than those of Q_1 and Q_2, typically by an order of magnitude for Q_6. However, arguments can be made that the $(V-A)\otimes(V+A)$ structure of Q_6 may lead to enhanced matrix elements,[30] by one order of magnitude or more, for the nonleptonic decays of kaons and hyperons.

As already noted, through strong interaction effects each operator in the effective Hamiltonian has a coefficient with an imaginary as well as real part. This imaginary part, which in each case enters through $\mathrm{Im}\,\tau$ and is then proportional to $s_2c_2s_3\sin\delta$, leads to CP violation in decay amplitudes.

When $s_2c_2s_3\sin\delta \neq 0$ and CP is violated, an inspection of the coefficients of the operators Q_1 and Q_2 immediately shows that the ratio of their imaginary to real parts is $\sim 10^{-2}s_2c_2s_3\sin\delta$. This is not true for the Penguin-type operators Q_3, Q_5 and Q_6 where the corresponding ratio is $\sim s_2c_2s_3\sin\delta$. If these later operators contribute at all significantly to K^o decay, clearly they will yield the largest CP violating effects in these amplitudes.[16] Recall in particular that the matrix elements of Q_6 are supposed to be especially large and important in decays like $K^o \to \pi\pi$. This is in addition to CP violating effects which occur in the kaon mass matrix in the six-quark model. These latter CP violating effects are considered in the following chapter.

CHAPTER III

EFFECTIVE HAMILTONIAN FOR K^o-$\bar{K}^o$ MIXING IN THE SIX-QUARK MODEL[31]

The K^o-$\bar{K}^o$ mass matrix has played an important role in particle physics over the past decade. The small value of the real part of the off diagonal elements found an explanation in the GIM mechanism[14] which conjectured the existence of a fourth quark flavor (charm). Later calculations of the magnitude of these matrix elements led to a quantitative estimate for the charm quark mass.[32] While these four-quark model computations were originally done without strong interaction corrections, with the development of quantum chromodynamics (QCD) the short distance effects due to strong interaction were soon computed[33,34] and found to change the answer rather little.

With the standard phase conventions (see Chapter IV) an imagninary part of the off diagonal kaon mass matrix elements is an expression of CP violation and leads to the kaon eigenstates K_L^o and K_S^o not being CP eigenstates. With four quark flavors there is no imaginary part[9] but, as was mentioned in the introduction, the six-quark model has a phase in the heavy quark couplings to the weak vector bosons which leads to CP violation and an imaginary part in the mass matrix. In this chapter the QCD corrections to the K^o-$\bar{K}^o$ mass matrix are calculated in the six-quark Kobayashi-Maskawa[10] model.

1. Derivation of the Effective Hamiltonian for K^o-$\bar{K}^o$ Mixing

Using the trigonometric identities

$$c_3^2c_1^2 = c_2^2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)^2 + s_2^2\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right)^2$$
$$+ 2s_2c_2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right) \quad , \quad (3.1a)$$

$$c_1c_2c_3\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right) = c_2^2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)^2$$
$$+ s_2c_2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right) \quad , \quad (3.1b)$$

and

$$c_1s_2c_3\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right) = s_2^2\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right)^2$$
$$+ s_2c_2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right) \quad , \quad (3.1c)$$

the effective Hamiltonian density, which contributes to K^o-$\bar{K}^o$ mixing in the six-quark model, can be written uniquely as

$$\mathcal{H}_{eff}^{|\Delta S|=2} = s_1^2c_2^2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)^2\mathcal{H}_1$$
$$+ s_1^2s_2^2\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right)^2\mathcal{H}_2 \quad (3.2)$$
$$+ 2s_1^2s_2c_2\left(c_1c_2c_3 - s_2s_3e^{-i\delta}\right)\left(c_1s_2c_3 + c_2s_3e^{-i\delta}\right)\mathcal{H}_3 + \text{h.c.} \quad .$$

In Eqs. (3.1) and (3.2), $s_i = \sin\theta_i$, $c_i = \cos\theta_i$, $i \in \{1,2,3\}$. The Cabibbo-type angles θ_1, θ_2, θ_3 and the phase δ are defined in Eq. (1.12) of Chapter I as well as in Eqs. (2.1) of Chapter II. The components, $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ of the complete Hamiltonian have relatively complicated expressions in terms of time ordered products of four weak charged currents contracted with W-boson fields corresponding in the free-quark model to forming the box diagram, shown in Fig. 7, with virtual W-bosons and quarks in the loop. In the absence of strong interactions treating the W-boson as very heavy and keeping only leading contribution in $1/M_W^2$

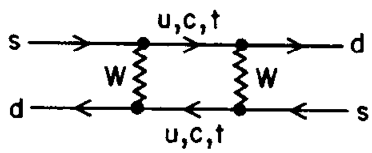

Fig. 7. Box diagram giving a K^o-$\bar{K}^o$ transition.

yields the following expressions for $\mathscr{H}_1$, $\mathscr{H}_2$, and $\mathscr{H}_3$:

$$\mathscr{H}_1(0) = \frac{iG_F^2}{4}\int d^4x \Big[T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)u_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big)\Big\} - 2T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times \big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)c_\lambda(0)\big)\big(\bar{c}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\}$$

$$+ T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)c_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)c_\lambda(0)\big)$$

$$\times \big(\bar{c}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\} - 2T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)c_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big)\Big\}\Big] \quad , \qquad (3.3a)$$

$$\mathscr{H}_2(0) = \frac{iG_F^2}{4}\int d^4x \Big[T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)u_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big)\Big\} - 2T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times \big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)\big(\bar{t}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\}$$

$$+ T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)t_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)$$

$$\times \big(\bar{t}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\} - 2T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\}\Big] \quad , \qquad (3.3b)$$

and

$$\mathscr{H}_3(0) = \frac{iG_F^2}{4}\int d^4x \Big[T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)u_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\} - T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times \big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)c_\lambda(0)\big)\big(\bar{c}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\}$$

$$- T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{u}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)$$

$$\times \big(\bar{t}_\delta(0)\gamma^\nu(1-\gamma_5)\bar{d}_\delta(0)\big)\Big\} - T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)c_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big)\Big\} - T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times \big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\}$$

$$+ T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)c_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)\big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)t_\lambda(0)\big)$$

$$\times \big(\bar{c}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big)\Big\} + T\Big\{\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)c_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\times \big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)\big(\bar{t}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)\Big\}\Big] \quad . \tag{3.3c}$$

α, β, λ and δ are color indices which are summed over $\{1,2,3\}$ when repeated. Normal ordering of the local 4-quark operators is understood. It is convenient to decompose these operators into pieces that will not mix under renormalization when the strong interactions are introduced. We write for $j \in \{1,2,3\}$ (in the absence of strong interactions)

$$\mathscr{H}_j = \frac{G_F^2}{16}\Big[\mathscr{H}_j^{(++)} + \mathscr{H}_j^{(+-)} + \mathscr{H}_j^{(-+)} + \mathscr{H}_j^{(--)}\Big] \quad , \tag{3.4}$$

where

$$\mathscr{H}_1^{(\pm\pm)}(0) \;\equiv\; i\int d^4x \Big[T\Big\{O_c^{(\pm)}(x)O_c^{(\pm)}(0)\Big\} - 2T\Big\{\Big(\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big) \pm \big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)d_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)u_\beta(x)\big)\Big)$$

$$\times\Big(\big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)c_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big)$$

$$\pm\big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)d_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)c_\delta(0)\big)\Big)\Big\}\Big] \quad , \qquad (3.5a)$$

$$\mathscr{H}_2^{(\pm\pm)}(0) \;\equiv\; i\int d^4x \Big[T\Big\{O_t^{(\pm)}(x)O_t^{(\pm)}(0)\Big\} - 2T\Big\{\Big(\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big) \pm \big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)d_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)u_\beta(x)\big)\Big)$$

$$\times\Big(\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big)$$

$$\pm\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)d_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)t_\delta(0)\big)\Big)\Big\}\Big] \quad , \qquad (3.5b)$$

and

$$\mathscr{H}_3^{(\pm\pm)}(0) \;\equiv\; i\int d^4x \Big[T\Big\{O_c^{(\pm)}(x)O_t^{(\pm)}(0)\Big\} - T\Big\{\Big(\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)$$

$$\times\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big) \pm \big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)d_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)u_\beta(x)\big)\Big)$$

$$\times\Big(\big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)c_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)d_\delta(0)\big) \pm \big(\bar{s}_\lambda(0)\gamma^\nu(1-\gamma_5)d_\lambda(0)\big)$$

$$\times\big(\bar{u}_\delta(0)\gamma_\nu(1-\gamma_5)c_\delta(0)\big)\Big)\Big\} - T\Big\{\Big(\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big)$$

$$\pm\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)d_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)u_\beta(x)\big)\Big)\Big(\big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\big)$$

$$\times\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\big) \pm \big(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)d_\lambda(0)\big)\big(\bar{u}_\delta(0)\gamma^\nu(1-\gamma_5)t_\delta(0)\big)\Big)\Big\}$$

$$+\;T\Big\{\Big(\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)c_\alpha(x)\big)\big(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big) \pm \big(\bar{s}_\alpha(x)\gamma^\nu(1-\gamma_5)d_\alpha(x)\big)$$

$$\times \left(\bar{t}_\beta(x)\gamma^\mu(1-\gamma_5)c_\beta(x)\right)\Big(\left(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)t_\lambda(0)\right)\left(\bar{c}_\delta(0)\gamma^\nu(1-\gamma_5)d_\delta(0)\right)$$

$$\pm \left(\bar{s}_\lambda(0)\gamma_\nu(1-\gamma_5)d_\lambda(0)\right)\left(\bar{c}_\delta(0)\gamma^\nu(1-\gamma_5)t_\delta(0)\right)\Big)\Big\}\Big] \quad . \tag{3.5c}$$

$O_c^{(\pm)}$ and $O_t^{(\pm)}$ are defined in Eq. (2.7) of Chapter II.

Now introduce strong interactions in the form of quantum chromodynamics (QCD). The pieces of the effective Hamiltonian $\mathscr{H}_j$ defined by Eq. (3.2) are modified from this free field expressions given in Eqs. (3.3) and (3.4). Treating the W-boson as heavy in the presence of strong interactions yields the following expression

$$\mathscr{H}_j = \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{2a^{(+)}} \mathscr{H}_j^{(++)} + \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(+)}+a^{(-)}} \mathscr{H}_j^{(+-)}$$
$$+ \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{a^{(-)}+a^{(+)}} \mathscr{H}_j^{(-+)} + \left[\frac{\alpha(M_W^2)}{\alpha(\mu^2)}\right]^{2a^{(-)}} \mathscr{H}_j^{(--)} \quad , \tag{3.6}$$

in the leading logarithmic approximation. α is the strong interaction fine structure constant, and μ the renormalization point. The matrix elements of the $\mathscr{H}_j$ are to be evaluated to all orders (since perturbation theory is probably not valid) in the six-quark theory of strong interactions using the $\overline{MS}$ subtraction scheme. Finally $a^{(+)} = 6/21$ and $a^{(-)} = -12/21$. The derivation of Eq. (3.6) is very similar to the removal of the W-boson field from the effective Hamiltonian for $\Delta S = 1$ weak non-leptonic decays discussed in Chapter II.

The next step is to successively treat the t-quark, b-quark and c-quark as heavy and remove their fields from explicitly appearing in the theory. For $\mathscr{H}_1$ this is particularly simple since the t and b-quark fields do not appear explicitly in it. The effect of removing the t and b-quark fields from the theory of strong interactions is to change the strong coupling g and masses $m_u,\ldots,m_t$ in the six-quark theory to a coupling g', and masses $m_u',\ldots,m_b'$ in an effective 5-quark theory and then to a coupling g'' and masses $m_u'',\ldots,m_c''$ in an effective 4-quark theory of the strong interactions. Also the exponents $a^{(+)}$ ($a^{(-)}$) change from 6/21 (-12/21) to 6/23 (-12/23) and then to 6/25 (-12/25) as one goes from the six-quark theory to the effective 5-quark theory and then to the effective 4-quark theory of strong interactions. Thus the effective Hamiltonian density $\mathcal{H}_1$ becomes

$$\begin{aligned}
\mathcal{H}_1 = & \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{12/21}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{12/23}\left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{12/25}\mathcal{H}_1^{(++)} \\
& + \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-6/21}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-6/23}\left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{-6/25}\mathcal{H}_2^{(+-)} \\
& + \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-6/21}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-6/23}\left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{-6/25}\mathcal{H}_1^{(-+)} \\
& + \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-24/21}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-24/23}\left[\frac{\alpha'(m_b'^2)}{\alpha''(\mu^2)}\right]^{-24/25}\mathcal{H}_1^{(--)} \, . \qquad (3.7)
\end{aligned}$$

The matrix elements of the effective Hamiltonian density $\mathcal{H}_1$ are now to be evaluated in an effective 4-quark theory of strong interactions. It only remains to treat the charm quark as heavy and remove it from explicitly appearing in $\mathcal{H}_1$. To leading order in the c-quark mass the matrix elements of $\mathcal{H}_1^{(\pm\pm)}$ can be expanded in the following fashion

$$\langle|\mathscr{H}_1^{(\pm\pm)}|\rangle'' = L^{(\pm\pm)}\left(\frac{m_c''}{\mu}, g''\right) m_c''^2 \langle|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|\rangle''' . \quad (3.8)$$

The double primed matrix elements are evaluated in an effective four-quark theory of strong interactions while the triple primed matrix elements are to be evaluated in an effective three-quark theory of strong interactions with coupling g''' and masses m_u''' , m_d''' and m_s''' . The operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ is a color symmetric four fermion operator with the usual anomalous dimension

$$\gamma'''^{(+)}(g''') = \frac{g'''^2}{4\pi^2} + \mathcal{O}(g'''^4) \quad . \quad (3.9)$$

The mass parameter m_c'' depends on the renormalization point μ and its anomalous dimension is

$$\gamma_c''(g'') = \frac{g''^2}{2\pi^2} + \mathcal{O}(g''^4) \quad . \quad (3.10)$$

The components $\mathscr{H}_1^{(++)}$, $\mathscr{H}_1^{(+-)}$, $\mathscr{H}_1^{(-+)}$ and $\mathscr{H}_1^{(--)}$ are composed of a sum of time ordered products of two local four-quark operators with color indices respectively symmetrized in both operators, symmetrized in the first operator and antisymmetrized in the second operator, antisymmetrized in the first operator and symmetrized in the second operator and finally antisymmetrized in both operators. They have the familiar anomalous dimensions,[5,6] $g''^2/2\pi^2 + \mathcal{O}(g''^4)$, $-g''^2/4\pi^2 + \mathcal{O}(g''^4)$, $-g''^2/4\pi^2 + \mathcal{O}(g''^4)$, and $-g''^2/\pi^2 + \mathcal{O}(g''^4)$ respectively. It follows that the Wilson coefficients $L^{(\pm\pm)}(m_c''/\mu, g'')$ obey the renormalization group equations

$$\left(\mu \frac{\partial}{\partial\mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c''(g'')m_c'' \frac{\partial}{\partial m_c''} - \frac{g''^2}{2\pi^2} - \frac{g'''^2}{4\pi^2}\right)L^{(++)}\left(\frac{m_c''}{\mu}, g''\right) = 0 \quad , \tag{3.11a}$$

$$\left(\mu \frac{\partial}{\partial\mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c''(g'')m_c'' \frac{\partial}{\partial m_c''} - \frac{5g''^2}{4\pi^2} - \frac{g'''^2}{4\pi^2}\right)L^{(+-)}\left(\frac{m_c''}{\mu}, g''\right) = 0 \quad , \tag{3.11b}$$

$$\left(\mu \frac{\partial}{\partial\mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c''(g'')m_c'' \frac{\partial}{\partial m_c''} - \frac{5g''^2}{4\pi^2} - \frac{g'''^2}{4\pi^2}\right)L^{(-+)}\left(\frac{m_c''}{\mu}, g''\right) = 0 \quad , \tag{3.11c}$$

$$\left(\mu \frac{\partial}{\partial\mu} + \beta''(g'') \frac{\partial}{\partial g''} + \gamma_c''(g'')m_c'' \frac{\partial}{\partial m_c''} - \frac{2g''^2}{\pi^2} - \frac{g'''^2}{4\pi^2}\right)L^{(--)}\left(\frac{m_c''}{\mu}, g''\right) = 0 \quad . \tag{3.11d}$$

These may be solved in the standard fashion, introducing a running coupling constant $\overline{g''}(y,g'')$ defined by

$$\ell n\; y = \int_{g''}^{\overline{g''}(y,g'')} dx \frac{1-\gamma_c''(x)}{\beta''(x)} \quad , \quad \overline{g''}(1,g'') = g'' \quad , \tag{3.12}$$

and noting that the coefficients $L^{(\pm\pm)}\left(1,\overline{g''}(m_c/\mu,g'')\right)$ may be replaced by their free field values $L^{(\pm\pm)}(1,0)$ since the running fine structure constant is taken as small at the scale of the charm quark mass and because no large logarithms can be generated from higher order QCD loop integrals when $m_c''/\mu = 1$. A straightforward computation yields

$$L^{(++)}(1,0) = -\frac{1}{\pi^2}\left[\frac{3}{2}\right] \quad , \tag{3.13a}$$

$$L^{(+-)}(1,0) = L^{(-+)}(1,0) = -\frac{1}{\pi^2}\left[-\frac{1}{2}\right] , \tag{3.13b}$$

and

$$L^{(--)}(1,0) = -\frac{1}{\pi^2}\left[\frac{1}{2}\right] \quad . \tag{3.13c}$$

The factors in the square brackets stem from color summations. Solving the renormalization group Eqs. (3.11) using the leading logarithmic approximation then gives

$$L^{(++)}\left(\frac{m_c''}{\mu}, g\right) = -\frac{1}{\pi^2}\left[\frac{\alpha''(\mu^2)}{\alpha''(m_c''^2)}\right]^{12/25}\left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{6/27}\left[\frac{\alpha''(m''^2)}{\alpha''(\mu^2)}\right]^{24/25}\left[\frac{3}{2}\right], \tag{3.14a}$$

$$L^{(+-)}\left(\frac{m_c''}{\mu}, g\right) = L^{(-+)}\left(\frac{m_c''}{\mu}, g\right)$$

$$= -\frac{1}{\pi^2}\left[\frac{\alpha''(\mu^2)}{\alpha''(m_c''^2)}\right]^{-6/25}\left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{6/27}\left[\frac{\alpha''(m_c''^2)}{\alpha''(\mu^2)}\right]^{24/25}\left[-\frac{1}{2}\right], \tag{3.14b}$$

and

$$L^{(--)}\left(\frac{m_c''}{\mu}, g\right) = -\frac{1}{\pi^2}\left[\frac{\alpha''(\mu^2)}{\alpha''(m_c''^2)}\right]^{-24/25}\left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{6/27}\left[\frac{\alpha''(m_c''^2)}{\alpha''(\mu^2)}\right]^{24/25}\left[\frac{1}{2}\right].$$

Using these results the effective Hamiltonian density $\mathcal{H}_1$ becomes

$$\mathcal{H}_1 = -\frac{G_F^2}{16\pi^2} m_c^{*2}\left(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)d_\alpha\right)\left(\bar{s}_\beta\gamma_\mu(1-\gamma_5)d_\beta\right)$$

$$\times\left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{6/27}\left(\frac{3}{2}\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{12/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{12/23}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{12/21}\right.$$

$$-\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{-6/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-6/23}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-6/21}$$

$$\left.+\frac{1}{2}\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{-24/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-24/23}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-24/21}\right) \tag{3.15}$$

where m_c^* is the running charm quark mass evaluated at $m_c''^2$, i.e.,

$$m_c^* = m_c'' \left[\alpha_s''(m_c''^2) \,/\, \alpha_s''(\mu^2)\right]^{12/25} \quad .$$

The Hamiltonian $\mathcal{H}_1$ already occurs in the four-quark model and our results agree with some of the previous results[33] for the QCD corrected $\mathcal{H}_1$, when the appropriate simplifications are made.

The deviation of the effective Hamiltonian density $\mathcal{H}_2$ proceeds along similar lines except that already at the step of removing the t-quark field from explicitly appearing each of the $\mathcal{H}_2^{(++)}$, $\mathcal{H}_2^{(+-)}$, $\mathcal{H}_2^{(-+)}$ and $\mathcal{H}_2^{(--)}$ collapses to a Wilson coefficient times $m_t^2\left(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\right) \times \left(\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta\right)$ to leading order in the t-quark mass. From that point on the successive steps are marked by renormalization of this latter color index symmetric four-fermion operator. The final result is

$$\mathcal{H}_2 = -\frac{G_F^2 m_t^{*2}}{16\pi^2}\left(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\right)\left(\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta\right)$$

$$\times \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{6/27}\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{6/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{6/23} \tag{3.16}$$

$$\times \left(\frac{3}{2}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{12/21} - \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-6/21} + \frac{1}{2}\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-24/21}\right),$$

where m_t^* is the running t-quark mass evaluated at m_t^2, i.e., $m_t^* = m_t\left[\alpha(m_t^2) \,/\, \alpha(\mu^2)\right]^{12/21}$.

The computation of the effective Hamiltonian density $\mathcal{H}_3$ in the presence of strong interactions is somewhat more complex. At the step of removing the t-quark from $\mathscr{H}_3^{(\pm\pm)}$ eight operators are generated even with the condition of keeping only those whose matrix elements can yield a contribution of order $m_c''^2$ or mix under renormalization with operators whose matrix elements can. Expanding the matrix elements of $\mathscr{H}_3^{(\pm\pm)}$ in terms of matrix elements of these operators gives

$$\langle|\mathscr{H}_3^{(\pm\pm)}|\rangle = \sum_{j=1}^{7} L_j^{(\pm\pm)}\langle|O_j^{(\pm\pm)}|\rangle' + L_8^{(\pm\pm)}\langle|O_8|\rangle' \tag{3.17}$$

to leading order in the t-quark mass. The primed matrix elements are evaluated in an effective 5-quark theory with strong coupling g'. Six of the operators

$$O_1^{(\pm\pm)} = i\int d^4x\; T\left\{O_c^{(\pm)}(x)(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}\right\} \quad , \tag{3.18a}$$

$$O_2^{(\pm\pm)} = i\int d^4x\; T\left\{O_c^{(\pm)}(x)(\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}\right\} \quad , \tag{3.18b}$$

$$O_3^{(\pm\pm)} = i\int d^4x\; T\left\{O_c^{(\pm)}(x)(\bar{s}_\alpha d_\alpha)_{V-A}\left[(\bar{u}_\beta u_\beta)_{V-A} + \cdots + (\bar{b}_\beta b_\beta)_{V-A}\right]\right\} , \tag{3.18c}$$

$$O_4^{(\pm\pm)} = i\int d^4x\; T\left\{O_c^{(\pm)}(x)(\bar{s}_\alpha d_\beta)_{V-A}\left[(\bar{u}_\beta u_\alpha)_{V-A} + \cdots + (\bar{b}_\beta b_\alpha)_{V-A}\right]\right\} , \tag{3.18d}$$

$$O_5^{(\pm\pm)} = i\int d^4x\; T\left\{O_c^{(\pm)}(x)(\bar{s}_\alpha d_\alpha)_{V-A}\left[(\bar{u}_\beta u_\beta)_{V+A} + \cdots + (\bar{b}_\beta b_\beta)_{V+A}\right]\right\} , \tag{3.18e}$$

$$O_6^{(\pm\pm)} = i\int d^4x\; T\left\{O_c^{(\pm)}(x)(\bar{s}_\alpha d_\beta)_{V-A}\left[(\bar{u}_\beta u_\alpha)_{V+A} + \cdots + (\bar{b}_\beta b_\alpha)_{V+A}\right]\right\} , \tag{3.18f}$$

originate from the portion of $\mathscr{H}_3^{(\pm\pm)}$,

$$i\int d^4x\; T\left\{O_c^{(\pm)}(x)\; O_t^{(\pm)}\right\} \quad , \tag{3.19}$$

which is an integral of a time ordered product of two pieces of the effective $\Delta S = 1$ weak nonleptonic Hamiltonian, one containing a t-quark and the other a c-quark. Note that $O_j^{(\pm\pm)} = O_j^{(\pm\mp)}$ for $j \varepsilon \{1,\ldots,6\}$. The two additional operators needed are

$$\begin{aligned} O_7^{(\pm\pm)} = i\int d^4x\, T\Big\{\Big[&\big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)u_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)d_\beta(x)\big) \\ &\pm \big(\bar{s}_\alpha(x)\gamma_\mu(1-\gamma_5)d_\alpha(x)\big)\big(\bar{c}_\beta(x)\gamma^\mu(1-\gamma_5)u_\beta(x)\big)\Big] \\ &\times\Big[\big(\bar{s}_\lambda\gamma_\nu(1-\gamma_5)c_\lambda\big)\big(\bar{u}_\delta\gamma^\nu(1-\gamma_5)d_\delta\big) \\ &\pm \big(\bar{s}_\lambda\gamma_\nu(1-\gamma_5)d_\lambda\big)\big(\bar{u}_\delta\gamma^\nu(1-\gamma_5)c_\delta\big)\Big]\Big\} \end{aligned} \tag{3.20}$$

and

$$O_8 = \frac{m_c'^2}{g'^2}\big(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)d_\alpha\big)\big(\bar{s}_\beta\gamma_\mu(1-\gamma_5)d_\beta\big) \quad . \tag{3.21}$$

The factor of $1/g'^2$ is inserted into the definition of O_8 so that to lowest order the anomalous dimension matrix $\gamma_{ij}'^{(\pm\pm)}(g')$ has all its entries proportional to g'^2. If O_8 did not contain the factor of $1/g'^2$ then the elements $\gamma_{i8}'^{(\pm\pm)}(g')$ would be (to lowest order) constants independent of g' for $i \varepsilon \{1,\ldots,7\}$. Then in solving the renormalization group equations $L_8^{(\pm\pm)}$ would have to be treated in a different fashion from the $L_j^{(\pm\pm)}$, $j \varepsilon \{1,\ldots,7\}$. On the other hand, with our definition[35] of O_8 it can be treated on the same footing as all the other operators. Of course in calculating its renormalization we must now be careful to include the coupling constant renormalization. The matrix elements of the operators $O_1^{(\pm\pm)}$ and $O_2^{(\pm\pm)}$ cannot produce a factor of $m_c''^2$, however, they must in principle be included since under renormalization they mix with the operators $O_3^{(\pm\pm)}$, $O_4^{(\pm\pm)}$, etc. whose matrix elements can produce a factor of $m_c''^2$. The anomalous dimension matrices $\gamma_{ij}'^{(\pm\pm)}(g')$ for these eight operators are given in Appendix B. The coefficients $L_j^{(\pm\pm)}(m_t/\mu,g)$ satisfy renormalization group equations which can be solved in the standard way. In this solution values are needed for the coefficients $L_j^{(\pm\pm)}\left(1,\bar{g}(m_t/\mu,g)\right)$, where $\bar{g}$ is the running coupling in the six-quark theory defined in Eq. (2.13) of Chapter II. These are found by noting that in the leading logarithmic approximation the $L_j^{(\pm\pm)}\left(1,\bar{g}(m_t/\mu,g)\right)$ can be replaced by their free field values $L_j^{(\pm)}(1,0)$ for $j \in \{1,\ldots,7\}$.

$$L_1^{(\pm\pm)}(1,0) = \pm 1 \quad , \tag{3.22a}$$

$$L_2^{(\pm\pm)}(1,0) = 1 \quad , \tag{3.22b}$$

$$L_3^{(\pm\pm)}(1,0) = L_4^{(\pm\pm)}(1,0) = L_5^{(\pm\pm)}(1,0) = L_6^{(\pm\pm)}(1,0) = 0 \quad , \tag{3.22c}$$

and

$$L_7^{(\pm\pm)}(1,0) = -1 \quad . \tag{3.22d}$$

For the coefficient $L_8^{(\pm\pm)}\left(1,\bar{g}(m_t/\mu,g)\right)$ the situation is somewhat more subtle since the operator O_8 contains a factor of $1/g'^2$. Explicit calculation gives that in the $\overline{MS}$ regularization scheme

$$L_7^{(\pm\pm)}\left(m_t/\mu = 1,\bar{g}\right) \propto \bar{g}^2 \, \ell n\left(m_t^2/\mu^2\right)\Big|_{\mu=m_t} = 0 \quad . \tag{3.23}$$

The last step follows, not because the factor of $\bar{g}^2$ is small, but rather because the logarithm vanishes at $\mu = m_t$. The final aim is to derive an effective Hamiltonian independent of the heavy W-boson, t-quark, b-quark and c-quark fields. To do this the b-quark and c-quark must still be considered as heavy and removed from explicitly appearing in the theory. Removal of the b-quark is similar to the previous step. There are still eight operators whose renormalization is characterized by the anomalous dimension matrices $\gamma''^{(\pm\pm)}(g'')$ given in Appendix B. Finally at the step of removing the charm quark only one operator $m_c''^2\left(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\right) \times \left(\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta\right)$ appears and its anomalous dimension follows from mass renormalization and the renormalization of the color symmetric local four-fermion operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$. This program for deriving the effective Hamiltonian $\mathscr{H}_3$ in the presence of strong interactions is a straightforward generalization of that used in Chapter II to derive the effective Hamiltonian for weak nonleptonic decays. Its complexity is such that, unlike the case of $\mathscr{H}_1$ and $\mathscr{H}_3$, we cannot write a simple analytic expression for $\mathscr{H}_3$. However there are some further approximations, beyond the leading logarithmic approximation, which make the derivation of a simple analytic expression for $\mathscr{H}_3$ possible. As can be seen from Eqs. (3.22) the operators $O_3^{(\pm)},\ldots,O_6^{(\pm)}$ are induced through strong interactions and thus their contribution is less important than $O_7^{(\pm\pm)}$ which has a non-zero coefficient even in the absence of the strong interactions. It follows since O_1 and O_2 do not mix directly with O_7 and O_8 that to a good approximation, at the stage of removing the t-quark, the set of eight operators can be truncated to the two operators $O_7^{(\pm\pm)}$ and O_8. These two operators then have the 2×2 anomalous dimension matrices

$$\gamma'^{(++)}(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} 4 & -24 \\ 0 & 7/3 \end{pmatrix} + \mathscr{O}(g'^4) \quad , \qquad (3.24a)$$

$$\gamma'^{(+-)}(g') = \gamma'^{(-+)}(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} -2 & 8 \\ 0 & 7/3 \end{pmatrix} + \mathscr{O}(g'^4) \quad , \qquad (3.24b)$$

and

$$\gamma'^{(--)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -8 & -8 \\ 0 & 7/3 \end{pmatrix} + \mathcal{O}(g'^4) \quad . \qquad (3.24c)$$

On removing the b-quark there are again two operators which enter. They have the same form as $O_7^{(\pm\pm)}$ and O_8 defined in Eqs. (3.20) and (3.21) except that in O_8 the factor of $m_c'^2/g'^2$ is replaced by $m_c''^2/g''^2$. The corresponding anomālous dimension matrices for these operators are

$$\gamma''^{(++)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} 4 & -24 \\ 0 & 5/3 \end{pmatrix} + \mathcal{O}(g''^4) \quad , \qquad (3.25a)$$

$$\gamma''^{(+-)}(g'') = \gamma''^{(-+)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -2 & 8 \\ 0 & 5/3 \end{pmatrix} + \mathcal{O}(g''^4) \quad , \qquad (3.25b)$$

and

$$\gamma''^{(--)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -8 & -8 \\ 0 & 5/3 \end{pmatrix} + \mathcal{O}(g''^4) \quad . \qquad (3.25c)$$

These are the same matrices as in Eqs. (3.24) except that the 8-8 entry has changed from 7/3 to 5/3 corresponding to the change of the number of flavors from 5 to 4 in the coupling constant renormalization (i.e., β-function). Finally on removing the charm quark only an operator proportional to O_8 appears. Carrying through the steps of successively treating the t, b and c quarks as heavy and removing them from explicitly appearing in the theory using the 2 × 2 anomalous dimension matrices above yields the following analytic approximation for $\mathcal{H}_3$:

$$\mathcal{H}_3 = \frac{G_F^2 m_c^{*2}}{64\pi\alpha''(m_c''^2)} \left(\bar{s}_\alpha\gamma^\mu(1-\gamma_5)d_\alpha\right)\left(\bar{s}_\beta\gamma_\mu(1-\gamma_5)d_\beta\right)$$

$$\times \left[\frac{\alpha''(m_c''^2)}{\alpha'''(\mu^2)}\right]^{6/27} \left\{ \frac{72}{35}\left(5\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{12/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{12/23}\right.\right.$$

$$+ 2\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{5/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{12/23} - 7\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{5/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{7/23}\Bigg)$$

$$\times \left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{12/21} + \frac{48}{143}\left(13\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{-6/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-6/23} - 2\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{5/25}\right.$$

$$\times \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-6/23} - 11\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{5/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{7/23}\Bigg)\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-6/21}$$

$$+ \frac{24}{899}\left(-31\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{-24/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-24/23} + 2\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{5/25}\right.$$

$$\left.\left.\times \left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{-24/23} + 29\left[\frac{\alpha'(m_b'^2)}{\alpha''(m_c''^2)}\right]^{5/25}\left[\frac{\alpha(m_t^2)}{\alpha'(m_b'^2)}\right]^{7/23}\right)\left[\frac{\alpha(M_W^2)}{\alpha(m_t^2)}\right]^{-24/21}\right\} . \tag{3.26}$$

The matrix elements of the three parts of the effective Hamiltonian for K^o-$\bar{K}^o$ mixing in Eqs. (3.15), (3.16) and (3.26) are to be evaluated using the mass independent $\overline{\text{MS}}$ subtraction scheme in an effective theory of strong interactions with three light quark flavors u, d and s. The effects of QCD can be ascertained by comparing $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}_3$ given by Eqs. (3.15), (3.16) and (3.26) with their free quark values

$$\mathcal{H}_1 = -\frac{G_F^2 m_c^2}{16\pi^2}\left(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\right)\left(\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta\right) \quad , \tag{3.27}$$

$$\mathcal{H}_2 = -\frac{G_F^2 m_t^2}{16\pi^2}\left(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\right)\left(\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta\right) \quad , \tag{3.28}$$

and

$$\mathcal{H}_3 = -\frac{G_F^2 m_c^2}{16\pi^2}\,\ell n\left(\frac{m_t^2}{m_c^2}\right)\left(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha\right)\left(\bar{s}_\beta\gamma^\mu(1-\gamma_5)d_\beta\right) \quad . \tag{3.29}$$

These are derived by integrating the heavy t and c-quark fields out of the expressions given in Eqs. (3.3) and keeping only the leading contribution in the large quark masses m_t and m_c.

2. Numerical Results

It is now possible to calculate (for given values of the parameters) the coefficient of $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ in the pieces $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}_3$ of the effective $\Delta S = 2$ Hamiltonian for $K^o-\bar{K}^o$ mixing and determine the magnitude of the QCD effects by comparing these results with their free quark values. Unlike the case of the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays, we have simple analytic expressions for the pieces $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}_3$ of the effective Hamiltonian. In order to derive an analytic expression for $\mathcal{H}_3$ new approximations beyond the leading logarithmic approximation were introduced. However these are not expected to significantly alter the numerical results. (The skeptical reader can verify this by using the results given in Appendix B to perform the calculation keeping all eight operators.)

Again the outlook is basically qualitative. The QCD effects have been computed in the leading logarithmic approximation and the c-quark mass was treated as a large quantity. For example, dispersive pieces which arise when the two u quarks in the loop of the box diagram shown in Fig. 7 bind to form a low mass hadronic state, have been neglected in comparison with pieces that contain explicit factors of the heavy c-quark mass. This is certainly a crude approximation, but it has the advantage of being a systematic expansion and other contributions are not expected to be larger than the ones computed.

The effective Hamiltonian for K^o-$\bar{K}^o$ mixing differs from that for the $\Delta S = 1$ weak nonleptonic decays in that at the final stage only one operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ appears. Since any renormalization point dependence in the Wilson coefficients is cancelled by renormalization point dependence of the matrix elements of this operator (at least if the Wilson coefficients are computed exactly) the Wilson coefficients of this operator in $\mathscr{H}_1$, $\mathscr{H}_2$ and $\mathscr{H}_3$ all have the same μ dependence. Unfortunately, the matrix elements cannot be calculated exactly so that some final predictions may not appear renormalization point independent. However the quantity $\mathrm{Im}\,\langle K^o|H_{eff}^{|\Delta S|=2}|\bar{K}^o\rangle \,/\, \mathrm{Re}\,\langle K^o|H_{eff}^{|\Delta S|=2}|\bar{K}^o\rangle$, which will be of interest, is independent of the matrix elements of $(\bar{d}s)_{V-A}(\bar{d}s)_{V-A}$ and so predictions for it will also be free of renormalization point dependence.

To investigate the effective Hamiltonian for K^o-$\bar{K}^o$ mixing values for the QCD running fine structure constant $\alpha(Q^2)$, the values of the heavy masses and μ^2, or alternatively $\alpha'''(\mu^2)$, are required. For $\alpha(Q^2)$

we again use

$$\alpha(Q^2) = \frac{12\pi}{33 - 2N_f} \frac{1}{\log (Q^2/\Lambda^2)} \qquad (3.30)$$

and take $\Lambda^2 = 0.1$ GeV^2 and $\Lambda^2 = 0.01$ GeV^2. The number of quark flavors is $N_f = 6$ for the fine structure constant in the six-quark theory $\alpha(Q^2)$, while $\alpha'(Q^2)$, $\alpha''(Q^2)$ and $\alpha'''(Q^2)$ have $N_f = 5$, 4 and 3 respectively, as they pertain to effective theories of strong interactions with those corresponding number of quark flavors.

The quantity m_c^*, unlike m_c'', is free of renormalization point dependence and hence more appropriately associated with the mass scale characterized by charmonium spectroscopy than m_c'' is. Thus for m_c^* the value 1.5 GeV is taken on the basis of ψ spectroscopy.[28] The difference between m_c'' and m_c^* can be neglected in the argument of the running fine structure constant in the leading logarithmic approximation but this is not the case for the explicit factor of the heavy c-quark mass squared which appears multiplying the operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ in $\mathscr{H}_1$ and $\mathscr{H}_3$. Similar remarks hold for the bottom and top quark masses. For m_b' the value 4.5 GeV is taken on the basis of Υ spectroscopy.[28] Again for m_t^* values of 15 GeV and 30 GeV are used since the t-quark mass is unknown at the present time. $M_W = 85$ GeV. Since we shall be primarily concerned with the quantity $\mathrm{Im}\,\langle K^o|H_{eff}^{|\Delta S|=2}|\bar{K}^o\rangle \,/\, \mathrm{Re}\,\langle K^o|H_{eff}^{|\Delta S|=2}|\bar{K}^o\rangle$ which is independent of μ, only the value $\alpha'''(\mu^2) = 1$ is used. Values for the quantities η_1, η_2 and η_3, which are defined respectively as the ratios of the coefficients of $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ in $\mathscr{H}_1$, $\mathscr{H}_2$ and $\mathscr{H}_3$ with strong interactions included, to those in the free quark model, are presented for the above choices of parameters in Table III. The free quark values of the coefficients were determined from Eqs. (3.27), (3.28) and (3.29) with $m_c = 1.5$ GeV, and $m_t = 15$ GeV and 30 GeV. Note that the QCD corrections tend to reduce the magnitude of $\mathscr{H}_1$, $\mathscr{H}_2$ and $\mathscr{H}_3$, $\mathscr{H}_1$ being effected the least and $\mathscr{H}_3$ the most. The QCD corrections to $\mathscr{H}_2$ and $\mathscr{H}_3$ are quite stable against variations of the parameters m_t and Λ. However $\mathscr{H}_1$ changes by roughly a factor of 0.6 on going from $\Lambda^2 = 0.1$ GeV^2 to $\Lambda^2 = 0.01$ GeV^2.

The results of this chapter can be combined with those of the previous one to make predictions for CP violation parameters in the kaon system. This is done in the following chapter.

Table III

QCD correction factors η_1, η_2, and η_3 to the pieces $\mathscr{H}_1$, $\mathscr{H}_2$, and $\mathscr{H}_3$ of the effective Hamiltonian for $K^o - \bar{K}^o$ mixing.

Parameters	η_1	η_2	η_3
$\Lambda^2 = 0.1\ GeV^2$ $m_t = 15\ GeV$	0.93	0.61	0.37
$\Lambda^2 = 0.1\ GeV^2$ $m_t = 30\ GeV$	0.92	0.62	0.34
$\Lambda^2 = 0.01\ GeV^2$ $m_t = 15\ GeV$	0.67	0.59	0.33
$\Lambda^2 = 0.01\ GeV^2$ $m_t = 30\ GeV$	0.67	0.60	0.33

CHAPTER IV

CP VIOLATION PARAMETERS OF THE K^o-$\bar{K}^o$ SYSTEM

The K^o-$\bar{K}^o$ system may be treated as a closed two-state system. Since the kaons decay, probability is not conserved for this system and the time development is described by a 2 × 2 Hamiltonian matrix which is the sum of mass and width matrices,

$$H = M - \frac{i\Gamma}{2} \quad . \tag{4.1}$$

In the K^o-$\bar{K}^o$ basis H is given by

$$H = \begin{bmatrix} M - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M - i\Gamma/2 \end{bmatrix} \tag{4.2}$$

where

$$M_{12} = \langle K^o | H_{eff}^{|\Delta S|=2} | \bar{K}^o \rangle + \ldots \tag{4.3}$$

and

$$\Gamma_{12} = 2\pi \sum_F \rho_F \langle K^o | H_{eff}^{|\Delta S|=1} | F \rangle \langle F | H_{eff}^{|\Delta S|=1} | \bar{K}^o \rangle + \ldots \tag{4.4}$$

with ρ_F the density of final states F. The effective Hamiltonians $H_{eff}^{|\Delta S|=1}$ and $H_{eff}^{|\Delta S|=2}$ were calculated in Chapters II and III treating the strong interactions in the leading logarithmic approximation. The physical eigenstates for the system are

$$K_S^o = \frac{1}{\left[2(1+|\varepsilon|^2)\right]^{1/2}} \left[(1+\varepsilon)K^o + (1-\varepsilon)\bar{K}^o\right] \qquad (4.5a)$$

and

$$K_L^o = \frac{1}{\left[2(1+|\varepsilon|^2)\right]^{1/2}} \left[(1+\varepsilon)K^o - (1-\varepsilon)\bar{K}^o\right] \qquad (4.5b)$$

with mass and width eigenvalues

$$M_S - \frac{i\Gamma_S}{2} = M - \frac{i\Gamma}{2} + \left[\left(M_{12}^* - \frac{i\Gamma_{12}^*}{2}\right)\left(M_{12} - \frac{i\Gamma_{12}}{2}\right)\right]^{1/2} \qquad (4.6a)$$

and

$$M_L - \frac{i\Gamma_L}{2} = M - \frac{i\Gamma}{2} - \left[\left(M_{12}^* - \frac{i\Gamma_{12}^*}{2}\right)\left(M_{12} - \frac{i\Gamma_{12}}{2}\right)\right]^{1/2} . \qquad (4.6b)$$

The quantity ε is given by

$$\varepsilon = \frac{i\mathrm{Im}M_{12} + \mathrm{Im}\Gamma_{12}/2}{\left[\left(M_{12}^* - i\Gamma_{12}^*/2\right)\left(M_{12} - i\Gamma_{12}/2\right)\right]^{1/2} + \mathrm{Re}M_{12} - i\mathrm{Re}\Gamma_{12}/2}$$

$$= \frac{\left[\left(M_{12}^* - i\Gamma_{12}^*/2\right)\left(M_{12} - i\Gamma_{12}/2\right)\right]^{1/2} - \mathrm{Re}M_{12} + i\mathrm{Re}\Gamma_{12}/2}{-i\mathrm{Im}M_{12} - \mathrm{Im}\Gamma_{12}/2} \qquad (4.7)$$

Since CP $K^o = \bar{K}^o$ and CP $\bar{K}^o = K^o$, $\mathrm{Im}\Gamma_{12}$ and $\mathrm{Im}M_{12}$ are zero when CP is conserved and hence ε is also zero. Note that the states K_S^o and K_L^o are not in general orthogonal since probability is not conserved in the kaon system. In fact

$$\langle K_L|K_S\rangle = \frac{2\mathrm{Re}\varepsilon}{(1+|\varepsilon|^2)} \qquad (4.8)$$

so that ε is pure imaginary when the width matrix vanishes. ε is not a physical quantity and its value depends on the phase convention one adopts for the kaon states or equivalently for the quark fields. The standard phase convention is to have the $K^o \to \pi\pi(I=0)$ amplitude, A_0, real. A_0 is defined by

$$\langle\pi\pi(I=0)|H_{eff}^{|\Delta S|=1}|K^o\rangle = A_o e^{i\delta_0} \qquad (4.9)$$

where δ_0 is the $\pi\pi$ isospin zero phase shift. If CP, or equivalently T, is conserved then A_0 is automatically real since then

$$\begin{aligned} A_0 e^{i\delta_0} &= {}_{out}\langle\pi\pi(I=0)|H_{eff}^{|\Delta S|=1}|K^o\rangle_{in} \\ &= {}_{out}\langle\pi\pi(I=0)|T^{-1}H^{|\Delta S|=1}T|K^o\rangle_{in} \\ &= {}_{out}\langle K^o|H_{eff}^{|\Delta S|=1}|\pi\pi(I=0)\rangle_{in} \quad . \end{aligned} \qquad (4.10)$$

The kaon in state equals its out state[36] (here in and out refer to strong interactions) while for the two pion state below inelastic threshold isospin conservation of the strong interactions implies that

$$\begin{aligned} |\pi\pi(I=0)\rangle_{in} &= |\pi\pi(I=0)\rangle_{out}\ {}_{out}\langle\pi\pi(I=0)|\pi\pi(I=0)\rangle_{in} \\ &= e^{2i\delta_0}|\pi\pi(I=0)\rangle_{out} \quad , \end{aligned} \qquad (4.11)$$

which when put into Eq. (4.10) gives

$$A_0 e^{i\delta_0} = {}_{\text{in}}\langle K^o | H_{eff}^{|\Delta S|=1} | \pi\pi(I=0)\rangle_{\text{out}} \, e^{2i\delta_0}$$

$$= \left(A_0 e^{i\delta_0}\right)^* e^{2i\delta_0} = A_0^* e^{i\delta_0} \quad . \tag{4.12}$$

In the six-quark Kobayashi-Maskawa model, where CP is violated, one can always make A_0 real by judiciously choosing how the phase δ enters the matrix U (which relates mass and weak eigenstates). The choice of U we have made puts the phase only in the couplings of the heavy quarks (see Eq. (1.12) of Chapter I and Eq. (2.1) of Chapter II). Thus the CP violating couplings enter the effective Hamiltonian $H_{eff}^{\Delta S=1}$ only through Penguin-type diagrams which are pure I = 1/2. Therefore the phase convention defined by the choice of quark fields in Eq. (1.12) of Chapter I and Eq. (2.1) of Chapter II corresponds to making the isospin two amplitude A_2, defined by

$$\langle \pi\pi(I=2) | H_{eff}^{|\Delta S|=1} | K^o\rangle = A_2 e^{i\delta_2} \quad , \tag{4.13}$$

real. δ_2 is the $\pi\pi(I=2)$ phase shift. It will therefore be necessary to transform results calculated on the basis of this form of the weak couplings to that which corresponds to making A_0 real.

As was mentioned in the introduction, non-zero values for the physical quantities η_{+-} and η_{00} defined by

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|H_{eff}^{|\Delta S|=1}|K_L^o\rangle}{\langle\pi^+\pi^-|H_{eff}^{|\Delta S|=1}|K_S^o\rangle} \tag{4.14a}$$

and

$$\eta_{00} = \frac{\langle\pi^o\pi^o|H_{eff}^{|\Delta S|=1}|K_L^o\rangle}{\langle\pi^o\pi^o|H_{eff}^{|\Delta S|=1}|K_S^o\rangle} \tag{4.14b}$$

is a measure of CP violation. The quantities η_{+-} and η_{00} can be expressed in terms of the isospin amplitudes A_0 and A_2 using the following decompositions for s-wave two-pion states:

$$|\pi^+\pi^-\rangle = \frac{1}{\sqrt{3}}\,|\pi\pi(I=2)\rangle + \sqrt{\frac{2}{3}}\,|\pi\pi(I=0)\rangle \ , \tag{4.15a}$$

$$|\pi^o\pi^o\rangle = \sqrt{\frac{2}{3}}\,|\pi\pi(I=2)\rangle - \frac{1}{\sqrt{3}}\,|\pi\pi(I=0)\rangle \quad . \tag{4.15b}$$

Since the experimental values of η_{+-} and η_{00} are small (see Eq. (1.10) of Chapter I) we will drop terms like $\varepsilon\mathrm{Im}A_2$ and $\varepsilon\mathrm{Im}A_0$ which are doubly CP violating. To leading order in CP violating quantities

$$\eta_{+-} \simeq \left[\frac{\sqrt{2}\ \varepsilon\mathrm{Re}A_o + i\sqrt{2}\ \mathrm{Im}A_0 + \varepsilon\mathrm{Re}A_2 e^{i(\delta_2-\delta_0)} + i\mathrm{Im}A_2 e^{i(\delta_2-\delta_0)}}{\sqrt{2}\ \mathrm{Re}A_0 + \mathrm{Re}A_2 e^{i(\delta_2-\delta_0)}}\right] \tag{4.16a}$$

and

$$\eta_{00} \simeq \left[\frac{\sqrt{2}\ i\mathrm{Im}A_2 e^{i(\delta_2-\delta_0)} + \varepsilon\sqrt{2}\ \mathrm{Re}A_2 e^{i(\delta_2-\delta_0)} - i\mathrm{Im}A_0 - \varepsilon\mathrm{Re}A_0}{\sqrt{2}\ \mathrm{Re}A_2 e^{i(\delta_2-\delta_0)} - \mathrm{Re}A_0}\right] \ . \tag{4.16b}$$

Within the convention A_0 real these simplify to

$$\eta_{+-} \simeq \epsilon + \epsilon' \tag{4.17a}$$

$$\eta_{00} \simeq \epsilon - 2\epsilon' \tag{4.17b}$$

when terms of order $\epsilon'(\mathrm{Re}A_2/A_0)$ are dropped (experimentally $\mathrm{Re}A_2/A_0 \simeq +1/20$). The quantity ϵ' is defined by

$$\epsilon' = \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_0)}\, \frac{\mathrm{Im}A_2}{A_0} \quad , \tag{4.18}$$

and the experimental values of η_{+-} and η_{00} imply that

$$\epsilon'/\epsilon = -0.003 \pm 0.014 \quad . \tag{4.19}$$

To leading non-trivial order in CP violating quantities Eqs. (4.6) and (4.7) become

$$M_S - \frac{i\Gamma_S}{2} = M - \frac{i\Gamma}{2} + \mathrm{Re}M_{12} - \frac{i\mathrm{Re}\Gamma_{12}}{2} \tag{4.20a}$$

$$M_L - \frac{i\Gamma_L}{2} = M - \frac{i\Gamma}{2} - \mathrm{Re}M_{12} + \frac{i\mathrm{Re}\Gamma_{12}}{2} \tag{4.20b}$$

and

$$\epsilon = \frac{i(\mathrm{Im}\Gamma_{12} + i\mathrm{Im}M_{12})}{\frac{1}{2}(\Gamma_S - \Gamma_L) + i(M_S - M_L)} \quad . \tag{4.21}$$

Since experimentally[1] $-(M_S - M_L) \simeq (\Gamma_S - \Gamma_L)/2$ and within the convention[37] A_0 real $(\mathrm{Im}\Gamma_{12}/\mathrm{Im}M_{12}) \lesssim 1/10$, it follows that when A_0 is chosen real

$$\epsilon \simeq \frac{1}{2\sqrt{2}}\, e^{i\pi/4}\, \frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}} \quad . \tag{4.22}$$

1. Predictions for ε'/ε

In Chapter II it was noted that if the matrix elements of the operator Q_6, in the effective Hamiltonian for $\Delta S=1$ weak nonleptonic decays, contribute significantly in $K\to\pi\pi$ decays then they will give the largest imaginary CP violating parts to the $K\to\pi\pi$ amplitude.[16] Recall also that Q_6 arose from Penguin-type diagrams and has a $(V-A)\otimes(V+A)$ chiral structure which may lead to enhanced matrix elements. Let f be the fraction of the $K\to\pi\pi(I=0)$ amplitude due to matrix elements of Q_6 when the CP violating phase δ is set to zero. It is important to realize that the value of f is strongly renormalization point dependent. In fact, in the leading logarithmic approximation, f would be almost zero if the renormalization point was equal to the charm quark mass. This may seem somewhat paradoxical since the Penguin-type diagrams are supposed to be the source of the $\Delta I=1/2$ rule which is a physical effect independent of μ. However, one should keep in mind that a given diagram in perturbation theory contributes, in general, to the Wilson coefficients and matrix elements of many of the operators in the effective Hamiltonian. Consider for example the lowest order Penguin-type diagram with u and c quarks in the loop. This diagram not only gives a contribution to the Wilson coefficient of Q_6, it also gives a higher order contribution to the matrix elements of Q_1 and Q_2. How much goes into matrix elements of Q_1 and Q_2 depends (within the $\overline{MS}$ regularization scheme) on the value of the renormalization point mass. In order to make predictions one chooses μ to be at the typical light hadronic mass scale for the problem. It is then hoped that enough of the features of the strong interactions have been included in the Wilson coefficients so that a simple estimate (for example using the naive quark model or bag model) of the matrix elements will lead to a qualitative understanding of the problem. We shall assume that there exists some μ (near the typical light hadronic mass scale) where the fraction f is large. Then at this renormalization point the total amplitude for $K \to \pi\pi(I=0)$, A_0, is given by

$$A_0 \simeq A_0^{(\delta=0)} + ifA_0^{(\delta=0)} \mathrm{Im}C_6/\mathrm{Re}C_6 \tag{4.23}$$

where $A_0^{(\delta=0)}$ is the $K^o \to \pi\pi(I=0)$ amplitude when the CP violation parameter δ is set to zero. That is

$$\langle 2\pi(I=0)|H_{eff}^{|\Delta S|=1}(\delta=0)|K^o\rangle = A_0^{(\delta=0)} e^{i\delta_0} \quad . \tag{4.24}$$

As was remarked previously, $A_0^{(\delta=0)}$ is real. C_6 denotes the Wilson coefficient of the operator Q_6 in the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays. C_6 was computed in Chapter III and values are presented in Tables I and II for various choices of QCD parameters.

In addition there is CP violation in the kaon mass matrix. From Chapter III it follows that

$$\varepsilon_m \equiv \frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}} = 2s_2c_2s_3\sin\delta\Big[\eta_1 m_c^2\Big(-c_1c_2^2c_3 + s_2c_2s_3\cos\delta\Big)$$

$$+ \eta_2 m_t^2\Big(c_1s_2^2c_3 + s_2c_2s_3\cos\delta\Big) + \eta_3 m_c^2 \ln\Big(m_t^2/m_c^2\Big)$$

$$\times\Big(c_1c_2^2c_3 - c_1s_2^2c_3 - 2s_2c_2s_3\cos\delta\Big)\Big]\Big[\eta_1 m_c^2 c_2^2\Big\{\Big(c_1c_2c_3 - s_2s_3\cos\delta\Big)^2$$

$$- s_2^2s_3^2\sin^2\delta\Big\} + \eta_2 m_t^2 s_2^2\Big\{\Big(c_1s_2c_3 + c_2s_3\cos\delta\Big)^2 - c_2^2s_3^2\sin^2\delta\Big\}$$

$$+ 2\eta_3 m_c^2 \ln\Big(m_t^2/m_c^2\Big)c_2s_2\Big\{\Big(c_1c_2c_3 - s_2s_3\cos\delta\Big)\Big(c_1s_2c_3 + c_2s_3\cos\delta\Big)$$

$$+ c_2s_2s_3^2\sin^2\delta\Big\}\Big]^{-1} . \qquad (4.25)$$

This expression is quite complicated; however, in the limit where s_1 and s_3 are treated as small quantities it simplifies to

$$\varepsilon_m = 2s_2c_2s_3\sin\delta\left\{\frac{-c_2^2m_c^2\eta_1 + s_2^2m_t^2\eta_2 + \left(c_2^2 - s_2^2\right)m_c^2\ln\left(\frac{m_t^2}{m_c^2}\right)\eta_3}{c_2^4m_c^2\eta_1 + s_2^4m_t^2\eta_2 + 2s_2^2c_2^2m_c^2\ln\left(\frac{m_t^2}{m_c^2}\right)\eta_3}\right\} . \qquad (4.26)$$

η_1, η_2, and η_3 are the QCD corrections to the three portions of the effective $\Delta S = 2$ Hamiltonian $\mathscr{H}_1$, $\mathscr{H}_2$, and $\mathscr{H}_3$ respectively. The quantities η_j, $j\varepsilon\{1,2,3\}$, were computed in Chapter III and values of η_1, η_2, and η_3 for typical QCD parameters are presented in Table III. Note that all renormalization point dependence drops out of the expression for ε_m. If f were zero then the amplitude A_0 given in Eq. (4.23) would be real and it follows from Eq. (4.22) that ε_m would then be proportional to the CP violation parameter ε. However, we are interested in the case where f is a large fraction. Then

$$A_0 \simeq A_0^{(\delta=0)} e^{i\xi} \tag{4.27}$$

where $\xi = f \mathrm{Im} C_6 / \mathrm{Re} C_6$. The standard phase convention, A_0 real, may be accomplished by readjusting the phase of the strange quark field

$$s \to e^{i\xi} s \tag{4.28}$$

so that

$$|K^o\rangle \to e^{-i\xi} |K^o\rangle \quad , \tag{4.29a}$$

$$|\bar{K}^o\rangle \to e^{+i\xi} |\bar{K}^o\rangle \quad , \tag{4.29b}$$

At the same time

$$\frac{\mathrm{Im} M_{12}}{\mathrm{Re} M_{12}} \to (\varepsilon_m + 2\xi) \quad , \tag{4.30}$$

where ε_m is given by Eq. (4.25). It follows from Eq. (4.22) that the CP violation parameter ε is given by

$$\varepsilon = \frac{1}{2\sqrt{2}} e^{i\pi/4} (\varepsilon_m + 2\xi) \quad . \tag{4.31}$$

ε_m and 2ξ give, in general, comparable contributions to ε. The phase angle of $\pi/4$ originates from the K_L^o and K_S^o mass and width values and has the precise value of $43.8^o \pm 0.2^o$ just as in the superweak model. In general no prediction can be made for ε since the angles θ_2, θ_3, and δ can be adjusted to fit the experimental value of ε. The other

CP violation parameter is ε' defined in Eq. (4.18). CP violation from the Penguin-type operator Q_6 (with $I = 1/2$) cannot enter the A_2 amplitude which involves a $\Delta I = 3/2$ transition. However through the readjustment of the kaon phases to make A_0 real A_2 picks up an imaginary part proportional to ξ and

$$\varepsilon' \approx \frac{1}{20\sqrt{2}}\, e^{i\pi/4}(-\xi) \tag{4.32}$$

where the experimental value of the $\pi\pi$ phase shifts δ_0 and δ_2 together with $\mathrm{Re}A_2/A_0 \simeq +1/20$ have been used. The experimental value of the phase angle which we have approximated by $\pi/4$ is $37^\circ \pm 6^\circ$. Combining Eqs. (4.31) and (4.32) gives

$$\varepsilon'/\varepsilon \approx \frac{1}{20}\left(\frac{-2\xi}{\varepsilon_m + 2\xi}\right) \quad . \tag{4.33}$$

In general ε'/ε like ε_m is a complicated function of the Cabibbo type angles θ_1, θ_2, θ_3 and the phase δ. Examination of this function using results presented in Tables I, II, and III reveals that values of ε'/ε at the fraction of a percent level are typical.[38] For example, when s_1 and s_3 are treated as small quantities both ε_m and ξ are proportional to $s_2c_2s_3\sin\delta$ and all dependence on θ_3, and δ drops out of Eq. (4.33). Values of the quantities $\xi/s_2c_2s_3\sin\delta$, $\varepsilon_m/s_2c_2s_3\sin\delta$, and ε'/ε for this case are listed in Tables IV and V. $\theta_2 = 15^\circ$ and $f = 0.75$ were used for the tables. Inspection of the results indicate that values from 3×10^{-3} to 3×10^{-2} are typical for ε'/ε when s_3 is a small quantity. Smaller values of Λ or f can give smaller values for ε'/ε. The quadrant of the phase δ can be adjusted to fit the

Table IV

Values of the quantity ξ, which leads to CP violation in decay amplitudes; ε_m, the contribution to CP violation from the kaon mass matrix and the resulting ratio of CP violation parameters ε'/ε. These are calculated with s_3, s_1 treated as small quantities, $\theta_2 = 15^\circ$, $f = .75$, and $\Lambda^2 = 0.1\ \mathrm{GeV}^2$.

Parameters	$\xi/fs_2c_2s_3\sin\delta$	$\varepsilon_m/s_2c_2s_3\sin\delta$	ε'/ε
$\alpha(\mu^2) = 0.75$, $m_t = 15$ GeV	$-\left(0.42 + s_2^2\right)^{-1}$	7.2	1/27
$\alpha(\mu^2) = 1.00$, $m_t = 15$ GeV	$-\left(0.65 + s_2^2\right)^{-1}$	7.2	1/49
$\alpha(\mu^2) = 1.25$, $m_t = 15$ GeV	$-\left(0.81 + s_2^2\right)^{-1}$	7.2	1/64
$\alpha(\mu^2) = 0.75$, $m_t = 30$ GeV	$-\left(0.33 + s_2^2\right)^{-1}$	16	1/65
$\alpha(\mu^2) = 1.00$, $m_t = 30$ GeV	$-\left(0.51 + s_2^2\right)^{-1}$	16	1/103
$\alpha(\mu^2) = 1.25$, $m_t = 30$ GeV	$-\left(0.62 + s_2^2\right)^{-1}$	16	1/127

Table V

Same as Table IV but with $\Lambda^2 = 0.01\ \mathrm{GeV}^2$

Parameters	$\xi/fs_2c_2s_3\sin\delta$	$\varepsilon_m/s_2c_2s_3\sin\delta$	ε'/ε
$\alpha(\mu^2) = 0.75,\ m_t = 15$ GeV	$-\left(1.46 + s_2^2\right)^{-1}$	8.9	1/161
$\alpha(\mu^2) = 1.00,\ m_t = 15$ GeV	$-\left(1.76 + s_2^2\right)^{-1}$	8.9	1/197
$\alpha(\mu^2) = 1.25,\ m_t = 15$ GeV	$-\left(1.96 + s_2^2\right)^{-1}$	8.9	1/220
$\alpha(\mu^2) = 0.75,\ m_t = 30$ GeV	$-\left(1.08 + s_2^2\right)^{-1}$	18	1/255
$\alpha(\mu^2) = 1.00,\ m_t = 30$ GeV	$-\left(1.30 + s_2^2\right)^{-1}$	18	1/308
$\alpha(\mu^2) = 1.25,\ m_t = 30$ GeV	$-\left(1.44 + s_2^2\right)^{-1}$	18	1/342

measured phase of ε. When s_1 and s_2 are treated as small quantities, we find that δ should be in the upper half plane.[16] Then ε'/ε is almost real and positive. For some larger values of s_3 it is possible to fit the measured phase of ε with δ in the lower half plane[39] and in this case ε'/ε is almost real and negative. The predictions for ε'/ε presented in Table IV are renormalization point dependent. As was mentioned before, our approach is to assume that a value of μ exists for which f is large. Since we do not know exactly what μ this is, ε'/ε is calculated for several different choices of $\alpha'''(\mu^2)$. Several authors have adopted a different approach.[40] Since the real part of the Wilson coefficient C_6 depends on integrations over virtual momenta primarily in the range $\mu^2 \lesssim p^2 \lesssim m_c^2$ whereas the imaginary part of C_6 depends on integrations over virtual momenta primarily in the range $m_c^2 \lesssim p^2 \lesssim m_t^2$ a leading log calculation of the real part of the Wilson coefficient for Q_6 is more uncertain than that of the imaginary part. Thus to calculate ξ they take the real part of the $K \to \pi\pi$ $(I = 0)$ amplitude from the experimental width and rely on either a vacuum insertion or bag model estimate for the matrix elements of Q_6 to calculate the imaginary part of A_0. This approach also involves an implicit choice of μ, namely that which makes the matrix element computation correct, and tends to give somewhat smaller values for the ratio ε'/ε.

The present experimental value is $\varepsilon'/\varepsilon = -0.003 \pm 0.014$ but experiments are now planned[41] which should be capable of measuring ε'/ε to the fraction of a percent level. As such they might be capable of distinguishing the six-quark model, with important contributions to $K \to 2\pi$ decay from Penguin-type diagrams, from the superweak model[42] where all CP violation originates from the kaon mass matrix and $\varepsilon' = 0$.

CHAPTER V

WEAK RADIATIVE HYPERON DECAYS[43]

The strangeness changing radiative decays of hyperons have received considerable attention by theorists.[44,45] Many of the recent theoretical analyses have attempted to view these decays as arising from a local $s \to d\gamma$ magnetic moment type transition.[45] Then the effective Hamiltonian for weak radiative decays is

$$\mathscr{H}_{eff} = ieG_F\bar{s}\sigma_{\mu\nu}(a+b\gamma_5)dF^{\mu\nu} + h.c. \quad , \tag{5.1}$$

where G_F is the Fermi constant, e the electromagnetic charge of the electron, s and d are strange and down quark fields, and $F^{\mu\nu}$ is the electromagnetic field strength tensor.

The matrix elements of the effective Hamiltonian in Eq. (5.1) can be calculated reliably in the SU(6) quark model.[46] The decays $B_1 \to B_2\gamma$ where B_1 and B_2 are baryons differing in strangeness by one unit are conveniently described in terms of helicity amplitudes[47] $g_{\lambda_2,\lambda_\gamma}$ labeled by the helicities of the outgoing baryon and photon. $g_{\lambda_2,\lambda_\gamma}$ is just the Feynman amplitude in the situation where the initial baryon has spin component $\lambda_1 = \lambda_2 - \lambda_\gamma$ along the direction of the final baryon three-momentum. When B_1 has spin component λ_1 along a given axis the resulting decay angular distribution is

$$\frac{d\Gamma}{d\cos\theta} = \frac{M_2|\vec{q}|}{4\pi M_1} \sum_{\lambda_2,\lambda_\gamma} |g_{\lambda_2,\lambda_\gamma}|^2 |d^{J_1}_{\lambda_1,\lambda_2-\lambda_\gamma}(\theta)|^2 \tag{5.2}$$

so that

$$\Gamma(B_1 \to B_2\gamma) = \frac{|\vec{q}|M_2}{2\pi(2J_1+1)M_1} \sum_{\lambda_2,\lambda_\gamma} |g_{\lambda_2,\lambda_\gamma}|^2 \quad , \qquad (5.3)$$

where θ is the angle between the given axis and the direction of the out-going baryon. The helicity amplitudes $g_{\lambda_2,\lambda_\gamma}$ are easily calculated from the effective Hamiltonian in Eq. (5.1) using SU(6) wave functions for the initial and final baryons. The helicity amplitude contains several factors: first, a function which depends on the overlap of the initial and final wave functions (when they are "separated" in momentum space by the photons momentum $\vec{q}$) $F(\vec{q})$; second, a spin dependent factor $C_{\lambda_2,\lambda_\gamma}$ which is essentially a Clebsch-Gordon coefficient arising from the spin part of the baryon wave functions; and third, a factor linear in the constants a and b of Eq. (5.1). This last factor is proportional to $G_F e(a-b)|\vec{q}|$ when $\lambda_\gamma = +1$ (in which case the initial s quark spin is parallel to the photon three momentum) and proportional to $G_F e(a+b)|\vec{q}|$ when $\lambda_\gamma = -1$ (in which case the initial quark spin is antiparallel to the photon three momentum). Therefore

$$g_{\lambda_2,+1} = 2\sqrt{2}\, G_F e|\vec{q}|(a-b)F(\vec{q})C_{\lambda_2,+1} \qquad (5.4a)$$

$$g_{\lambda_2,-1} = 2\sqrt{2}\, G_F e|\vec{q}|(a+b)F(\vec{q})C_{\lambda_2,-1} \qquad (5.4b)$$

The spin dependent factor from the quark model wave functions of the baryons is the same when all helicities are reversed in sign, i.e.,

$C_{\lambda_2,+1} = C_{-\lambda_2,-1}$. The overlap function $F(\vec{q})$ is normalized so in the nonrelativistic quark model $F(\vec{0}) = 1$. Inserting Eqs. (5.4) into Eq. (5.3) gives

$$\Gamma(B_1 \to B_2\gamma) = \frac{8G_F^2 e^2 |\vec{q}|^3 M_2}{\pi(2J_1+1)M_1} |F(\vec{q})|^2 \left(|a|^2 + |b|^2\right) \sum_{\lambda_2} |C_{\lambda_2,+1}|^2 \tag{5.5}$$

The only observed radiative hyperon decay is $\Sigma^+ \to p\gamma$ with a branching ratio[1] of $(1.24 \pm 0.18) \times 10^{-3}$. For the other weak radiative hyperon decays only upper limits exist at the present time. Normalizing the observed $\Sigma^+ \to p\gamma$ width, predictions for the other baryon decays can be made provided $F(\vec{q})$ is slowly varying with $\vec{q}$. In this case the factor $(|a|^2 + |b|^2)|F(\vec{q})|^2$ is determined from the observed $\Sigma^+ \to p\gamma$ width and the branching ratios for the other hyperon decays follow from this and the values of $|\vec{q}|^3 M_2/[(2J_1+1)M_1]$ and $C_{\lambda_2,\lambda_\gamma}$. Predictions for the weak radiative hyperon decays are presented in Table VI. There is a large disagreement between the predicted rates for $\Xi^- \to \Sigma^-\gamma$ and $\Omega^- \to \Xi^-\gamma$ and the experimental upper limits on these decay modes.[48] Thus it appears that the weak radiative decays of all the hyperons cannot arise from a local magnetic moment type transition.

So far no dynamical assumptions on the origin of the effective Hamiltonian in Eq. (5.1) have been made. In the 4-quark Weinberg-Salam model the coefficients a and b can be calculated since diagrams like Fig. 8, with gluon corrections, are short distance dominated to the extent that the W-boson and the charm quark are very massive.

Table VI

Predictions for weak radiative decays of hyperons based on the local magnetic moment transition.

Decay	$\lvert\vec{q}\rvert$ MeV	$C_{\lambda_2,\lambda_\gamma}$	Predicted branching ratio	Measured branching ratio (a)
$\Sigma^+ \to p\gamma$	225	$C_{\frac{1}{2},1} = 1/3$	1.24×10^{-3} (input)	$(1.24 \pm 0.18) \times 10^{-3}$
$\Lambda \to n\gamma$	162	$C_{\frac{1}{2},1} = \sqrt{6}/2$	2.2×10^{-2}	
$\Xi^0 \to \Sigma^0\gamma$	117	$C_{\frac{1}{2},1} = 5\sqrt{2}/6$	9.1×10^{-3}	$< 7 \times 10^{-2}$
$\Xi^0 \to \Lambda\gamma$	184	$C_{\frac{1}{2},1} = -1/\sqrt{6}$	4×10^{-3}	$(5 \pm 5) \times 10^{-3}$
$\Xi^- \to \Sigma^-\gamma$	118	$C_{\frac{1}{2},1} = 5/3$	1.1×10^{-2}	$< 1.2 \times 10^{-3}$
$\Omega^- \to \Xi^-\gamma$	314	$C_{\frac{1}{2},1} = -\sqrt{6}/3$	4.1×10^{-2}	$< 3.1 \times 10^{-3}$
$\Omega^- \to \Xi^{-*}\gamma$	132	$C_{-\frac{1}{2},1} = \sqrt{2}$ $C_{\frac{3}{2},1} = 1$ $C_{\frac{1}{2},1} = 2/\sqrt{3}$ $C_{-\frac{1}{2},1} = 1$	4.5×10^{-3}	

(a) The branching ratio for $\Xi^0 \to \Lambda^0\gamma$ is given as $(2.3 \pm 0.7) \times 10^{-3}$ in Ref. 37.

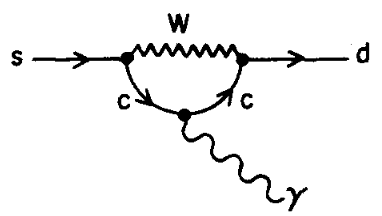

Fig. 8. Diagram which gives rise to a local magnetic moment transition in the absence of strong interactions.

Coefficients a and b which are not suppressed by a factor of $\left(\frac{m_c}{M_W}\right)^2$ first arise from diagrams at the two loop level and hence the values of a and b are quite small. These coefficients have been calculated in the standard 4-quark model by Shifman, Vainshtein and Zakhavov.[49] They found

$$a = \frac{\sin\theta_c \cos\theta_c}{\sqrt{2}\, 16\pi^2}\, \ell(m_s + m_d) \qquad (5.6a)$$

$$b = \frac{\sin\theta_c \cos\theta_c}{\sqrt{2}\, 16\pi^2}\, \ell(m_d - m_s) \qquad (5.6b)$$

where

$$\ell = -\left[\frac{\alpha(m_c^2)}{\alpha'(\mu^2)}\right]^{16/27} \left\{ 2/7\left(\left[\frac{\alpha(m_c^2)}{\alpha'(\mu^2)}\right]^{-28/27} - 1\right)\left[\frac{\alpha(M_W^2)}{\alpha(m_c^2)}\right]^{-12/25} + \frac{4}{5}\left(\left[\frac{\alpha(m_c^2)}{\alpha'(\mu^2)}\right]^{-10/27} - 1\right)\left[\frac{\alpha(M_W^2)}{\alpha(m_c^2)}\right]^{6/25} \right\} . \qquad (5.7)$$

Putting this into Eq. (5.5) yields predictions for branching ratios of weak radiative hyperon decays of order 10^{-5} (or less).

If the local magnetic moment transition given by the effective Hamiltonian in Eq. (5.1) is not the mechanism for weak radiative decays then what is? Other possible contributions come from the matrix elements of the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays evaluated to order e in the electromagnetic interactions. Within the context of the pole model, where the weak radiative decays are viewed as a weak nonleptonic transition followed by the radiation of a photon or vice versa, the rates for weak radiative decays are related to those of the nonleptonic decays. It is noteworthy that when the local magnetic moment transition is neglected the weak radiative decays of negatively charged baryons can only proceed through Penguin-type diagrams (with a photon radiated off one of the quark legs in the Penguin diagram or off a "spectator" quark leg). A measurement of branching ratios for the decays $\Xi^- \to \Sigma^- \gamma$ and $\Omega^- \to \Xi^- \gamma$ of order 10^{-3} would be strong evidence that the Penguin-type diagrams are important in the weak radiative decays of hyperons and qualitative evidence that Penguin-type diagrams, which have been proposed as an explanation of the $\Delta I = 1/2$ rule, are important in the weak nonleptonic decays of hyperons.

The special role of Penguin-type diagrams (with a photon radiated off one of the quark legs in the Penguin diagram or off a "spectator" quark leg) in the weak radiative decays of negatively charged hyperons leads one to believe that a similar effect should exist for the nonleptonic hyperon decays. This is examined in the next section.

1. The f/d Ratio in Nonleptonic Weak Hyperon Decays

Let $H_{eff}^{\Delta S=1,8}$ be the portion of the effective Hamiltonian for weak nonleptonic decays that transforms like the sixth component of an octet under SU(3) flavor. Let B_i, $i \in \{1,\ldots 8\}$ denote the spin 1/2 positive parity baryon states in the octet. The s-wave nonleptonic hyperon decay amplitudes are related through PCAC[50] to the matrix elements of the parity conserving part of the effective nonleptonic Hamiltonian between baryon states differing in strangeness by one unit.

The p-wave nonleptonic baryon decay amplitudes are not directly related through PCAC to matrix elements of the parity violating part of the effective nonleptonic Hamiltonian due to the presence of pole terms. Hence we shall focus our attention on the s-wave amplitudes. Define

$$\langle B_i | H_{\text{eff parity conserving}}^{\Delta S=1,8} | B_j \rangle = \bar{u}_i \sigma_{p.c.}^{ij} u_j \tag{5.8}$$

where u_i and u_j are Dirac spinors for the spin 1/2 baryon states B_i and B_j (see Appendix C). Assuming that SU(3) is a good symmetry the above matrix elements can be characterized by two reduced matrix elements[51] f and d and Clebsch-Gordon coefficients f_{ijk} and d_{ijk} in the following manner

$$\sigma_{p.c.}^{ij} = -if_{6ij}f + d_{6ij}d \quad , \tag{5.9}$$

where f_{ijk} and d_{ijk} are defined by the commutators and anticommutators of the SU(3) Gell Mann matrices λ_i (normalized by $Tr(\lambda_i\lambda_j) = 2\delta_{ij}$):

$$[\lambda_i, \lambda_j] = 2if_{ijk}\,\lambda_k \quad , \tag{5.10a}$$

and

$$\{\lambda_i, \lambda_j\} = 2d_{ijk}\,\lambda_k + 4/3\;\delta_{ij} \quad . \tag{5.10b}$$

Since the negatively charged hyperon transition matrix element can only proceed through Penguin-type diagrams (W exchange cannot occur between all Q = - 1/3 quarks) and the contribution of the part of the

Hamiltonian that is not octet (i.e., the 27) is known from experiment to be small,[52] $\sigma^{\Sigma^-\Xi^-}$ must approximately vanish in the absence of Penguin-type diagrams. Using the results in Appendix C and the values of the coefficients f_{ijk} and d_{ijk}, given in Table VII,

$$\sigma^{\Sigma^-\Xi^-} = \frac{1}{2}[f+d] \quad . \tag{5.12}$$

Thus we conclude that, in the absence of Penguin-type contributions to the baryon-baryon transition matrix element $f/d \approx -1$. This is not to say that if the coefficients of the operators $Q_3,\ldots,Q_6$ in the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays were set to zero then f/d is necessarily equal to minus one. As was mentioned previously the Penguin-type diagrams also give higher order contributions to the matrix elements of Q_1 and Q_2. However, it is usually assumed that these are small when μ is chosen to be at the typical light hadronic mass scale which characterizes the decay. Experimentally a good fit to s-wave hyperon decay amplitudes occurs for $f \approx -2d$. This is evidence that Penguin-type diagrams do play a significant role in weak nonleptonic hyperon decays.

Table VII

Non zero f_{ijk} and d_{ijk}. Non zero elements not listed in the table below can be derived by noting that f_{ijk} is completely antisymmetric and d_{ijk} is completely symmetric

ijk	f_{ijk}	ijk	d_{ijk}
123	1	118	$1/\sqrt{3}$
147	1/2	146	1/2
156	$-1/2$	157	1/2
246	1/2	228	$1/\sqrt{3}$
257	1/2	247	$-1/2$
345	1/2	256	1/2
367	$-1/2$	338	$1/\sqrt{3}$
458	$\sqrt{3}/2$	344	1/2
678	$\sqrt{3}/2$	366	$-1/2$
		377	$-1/2$
		448	$-1/2\sqrt{3}$
		558	$-1/2\sqrt{3}$
		668	$-1/2\sqrt{3}$
		778	$-1/2\sqrt{3}$
		888	$-1/\sqrt{3}$

CHAPTER VI

CONCLUDING REMARKS

In this report some of the implications of a possible mechanism for the $\Delta I = 1/2$ rule, based on a prominent role for Penguin-type diagrams, were discussed. It was found that within the six-quark model for CP violation an important role for Penguin-type diagrams in $K \to \pi\pi$ decays leads to values of ε'/ε of order a fraction of a percent. Thus the Kobayashi-Maskawa six-quark model for CP violation may be distinguishable from the superweak model where $\varepsilon' = 0$. In addition to gaining insight into the relationship between CP violation and the $\Delta I = 1/2$ rule it was found that weak radiative hyperon decays may play a role in ascertaining the significance of Penguin-type diagrams in the weak nonleptonic decays of hyperons.

In order to calculate ε'/ε the effective Hamiltonians for $\Delta S = 1$ weak nonleptonic decays and $\Delta S = 2$ $K^o - \bar{K}^o$ mixing were derived by successively treating the W-boson, t-quark, b-quark, and c-quark as heavy and removing their fields from explicitly appearing the theory. Strong interaction effects were taken into account in the leading logarithmic approximation. It is hoped that the method for performing such calculations may prove useful to workers in other areas (e.g., deep inelastic scattering[19,53] and grand unified theories[54]) where the effective field theory formalism can be applied.

Most of the results derived in this report, while quantitative in principle, have been qualitative in their applciation to strange particle decays. This is partly because the treatment of the charm quark mass as large and using it as an expansion parameter is suspect.[55] However, the greatest limitation on our ability to make quantitative predictions comes from difficulties in calculating the matrix elements of the operators which enter the effective Hamiltonian for $\Delta S = 1$ weak nonleptonic decays. These are renormalized local four-quark operators and any serious attempt to calculate their matrix elements must deal with the dependence of these matrix elements on the renormalization point mass. In the vacuum insertion approximation, which is commonly used to evaluate these matrix elements, a renormalized local four-quark operator is split into a product of renormalized quark bilinears. This completely destroys the renormalization point dependence of the matrix elements (note that $\langle|(\psi_1\psi_2\psi_3\psi_4)^R|\rangle \neq \sum_n \langle|(\psi_1\psi_2)^R|n\rangle \times \langle n|(\psi_3\psi_4)^R|\rangle$), and therefore if this approximation is ever valid it can only be at one particular value of the renormalization point mass. This value is usually taken to be the typical light hadronic mass scale which characterizes the decay. Similar remarks hold for bag-model estimates of the matrix elements.[56] Much further work is needed on this problem before it can be claimed that we have a quantitative understanding of the weak nonleptonic decays of kaons and hyperons.

APPENDIX A

In this appendix we outline the derivation of the equations and give numerical results for the quantities which appear in Section 1 of Chapter II. In Section 1 of Chapter II a rather fundamental role was played by the renormalization group Eqs. (2.22), (2.32), (2.37) and (2.48). To get Eq. (2.22), for example, one merely applies $\mu \frac{d}{d\mu}$ to both sides of Eq. (2.18) using

$$\mu \frac{d}{d\mu} \langle | O_t^{(\pm)} | \rangle = \left(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} + \sum_q \gamma_q(g)\, m_q \frac{\partial}{\partial m_q}\right) \langle | O_t^{(\pm)} | \rangle$$

$$= -\gamma^{(\pm)}(g) \langle | O_t^{(\pm)} | \rangle \quad , \qquad \text{(A.1)}$$

$$\mu \frac{d}{d\mu} \langle | O_i | \rangle' = \left(\mu \frac{\partial}{\partial\mu} + \beta'(g') \frac{\partial}{\partial g'} + \sum_q \gamma_q'(g')\, m_q' \frac{\partial}{\partial m_q'}\right) \langle | O_i | \rangle'$$

$$= -\sum_j \gamma_{ij}'(g') \langle | O_j | \rangle' \quad , \qquad \text{(A.2)}$$

and

$$\mu \frac{d}{d\mu} B_j^{(\pm)}\left(\frac{m_t}{\mu}, g\right) = \left(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_t(g)\, m_t \frac{\partial}{\partial m_t}\right) B_j^{(\pm)}\left(\frac{m_t}{\mu}, g\right) \; . \qquad \text{(A.3)}$$

In Eqs. (A.1) and (A.3) the partial derivative with respect to μ is at constant g and m_q, where $q \in \{u,d,\ldots,t\}$, while in Eq. (A.2) it is at constant g' and m_q', where $q \in \{u,d,\ldots,b\}$.

The $\gamma^{(\pm)}(g)$ and the matrix $\gamma_{kj}'(g')$ arise because the operators $O_t^{(\pm)}$ and O_i are local four-fermion operators and require renormalization. The renormalization of the operators $O_q^{(\pm)}$ at the one-loop level was considered in Refs. 5 and 6 where it was shown that the $\gamma^{(\pm)}(g)$ are given by Eq. (2.12). From Eq. (2.16), with $N_f = 6$, it follows that

$$a^{(+)} = \frac{6}{21} \quad , \qquad a^{(-)} = -\frac{12}{21} \quad . \tag{A.4}$$

At the one-loop level the operators O_j undergo a renormalization

$$O_j^o = \sum_k Z_{jk}\, O_k \tag{A.5}$$

where a superscript "o" denotes a bare unrenormalized quantity. Z_{jk} is the matrix renormalization which arises because of the composite nature of the local four-fermion operators O_j. The renormalized operators are defined so that their matrix elements are finite. The matrix $\gamma'_{ij}(g')$ is defined by

$$\gamma'_{ij}(g') = \sum_k Z^{-1}_{ik}\, \mu \frac{d}{d\mu} Z_{kj} \quad . \tag{A.6}$$

Note that the Z_{jk} are a function of the coupling g' since the renoramlization of the operators O_j is calculated in the effective 5 quark theory with that coupling. A straightforward calculation of the "infinite part" of the one-particle-irreducible diagrams in Fig. 9, using Landau gauge, gives

$$\gamma'_{ij}(g') = \frac{g'^2}{8\pi^2} \begin{pmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 \\ 0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 \\ 0 & 0 & 22/9 & 2/3 & -5/9 & 5/3 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & -5/9 & 5/3 & -5/9 & -19/3 \end{pmatrix} + \mathcal{O}(g'^4) \quad . \tag{A.7}$$

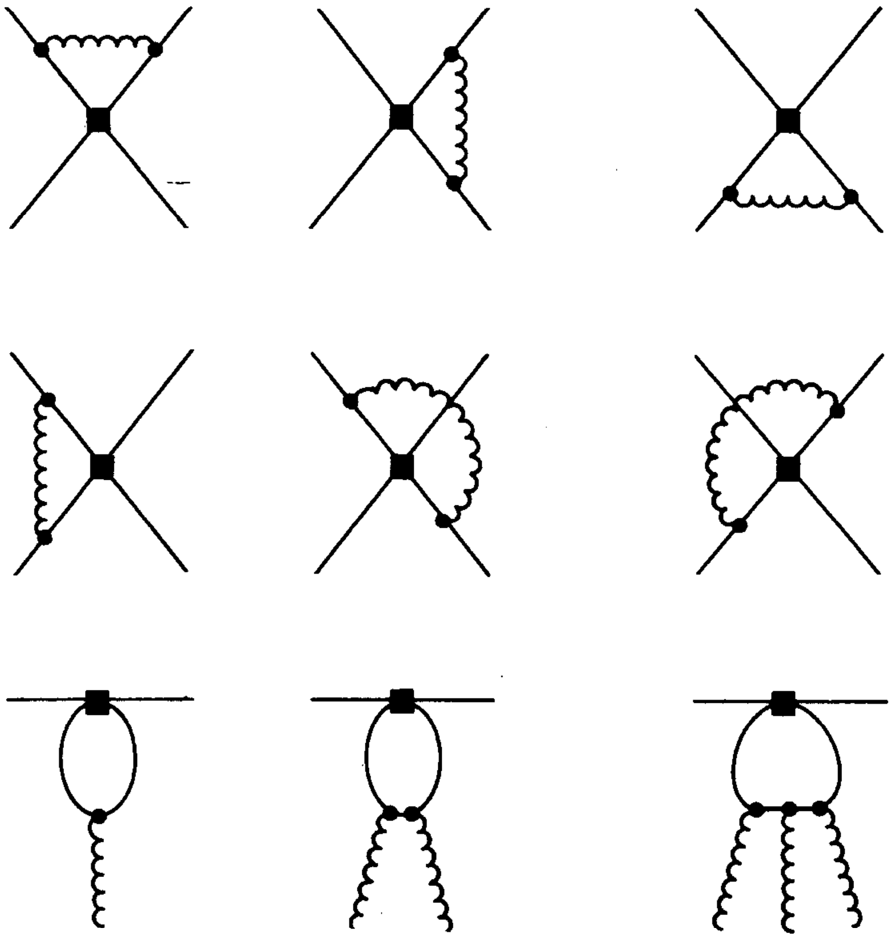

Fig. 9. Diagrams entering the calculation of the renormalization of the local four-fermion operators (represented by the black box) through QCD effects.

In the calculation of the renormalization of the local four-fermion operators, O_j, the masses of the light up, down, and strange quarks was set to zero. If this was not done the operators O_j would close under renormalization at the one-loop level but at the two loop level a transition color magnetic moment term must be added. However, the presence of such an operator does not alter the Wilson coefficients of the local four-fermion operators, O_j, from their value calculated with the light quark masses set to zero. The transition color magnetic moment operator itself is explicitly proportional to a light quark mass yielding small matrix elements. Also the Wilson coefficient of the magnetic moment operator is expected to be small. These facts justify our approximation of setting the u,d, and s quark masses to zero.

The matrix $\gamma'^T_{ij}(g')$ can be diagonalized by the transformation

$$\sum_{k,\ell} V^{-1}_{i\ell}\, \gamma'^T_{\ell k}(g')\, V_{kj} = \delta_{ij}\, \gamma'_j(g') \tag{A.8}$$

where

$$V_{kj} = \begin{pmatrix} 0 & -.69483 & 0 & 0 & .70576 & 0 \\ 0 & .69483 & 0 & 0 & .70576 & 0 \\ .15042 & .23161 & -1.253 & .16684 & -.10082 & .42681 \\ -.2089 & -.23161 & 1.0843 & .081196 & -.10082 & .82414 \\ .032942 & 0 & .10426 & .93924 & 0 & -.3322 \\ .61688 & 0 & .21323 & -.34513 & 0 & .28045 \end{pmatrix} \tag{A.9}$$

and

$$\gamma'_j(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} -6.8954 \\ -4 \\ -3.2429 \\ 1.1166 \\ 2 \\ 3.1327 \end{pmatrix} + \mathcal{O}(g'^4) \quad . \tag{A.10}$$

Combining (A.10) with the perturbative expansion of $\beta'(g')$ in Eq. (27) yields the a'_j of Eq. (28):

$$a'_j = \begin{pmatrix} -.8994 \\ -12/23 \\ -.42299 \\ .14564 \\ 6/23 \\ .40861 \end{pmatrix} \quad . \tag{A.11}$$

Note that $a'_2 = a'^{(-)}$ and $a'_4 = a'^{(+)}$ where

$$-\frac{\gamma'^{(\pm)}(x)}{\beta'(x)} = \frac{2a'^{(\pm)}}{x} + \text{terms finite at } x=0 \tag{A.12}$$

and

$$\gamma'^{(+)}(g') = \frac{g'^2}{4\pi^2} + \mathcal{O}(g'^4) \tag{A.13a}$$

$$\gamma'^{(-)}(g') = -\frac{g'^2}{2\pi^2} + \mathcal{O}(g'^4) \quad . \tag{A.13b}$$

The case where the bottom quark is treated as very heavy is similar to the above and we simply state results:

$$\gamma''_{mn}(g'') = \frac{g''^2}{8\pi^2} \begin{pmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 \\ 0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 \\ 0 & 0 & 23/9 & 1/3 & -4/9 & 4/3 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & -4/9 & 4/3 & -4/9 & -20/3 \end{pmatrix} + \mathcal{O}(g''^4) \; .$$

(A.14)

$\gamma''^{T}_{mn}(g'')$ is diagonalized by the transformation

$$\sum_{\ell,k} W^{-1}_{n\ell}\, \gamma''^{T}_{\ell k}(g'')\, W_{km} = \delta_{nm}\, \gamma''_n(g'') \tag{A.15}$$

where

$$W_{km} = \begin{pmatrix} 0 & .67552 & 0 & 0 & .70598 & 0 \\ 0 & -.67552 & 0 & 0 & .70598 & 0 \\ -.13011 & -.33776 & -1.2092 & .14075 & -.11766 & .47246 \\ .18274 & .33776 & 1.1043 & .067129 & -.11766 & .80199 \\ -.02959 & 0 & .064119 & .96326 & 0 & -.30023 \\ -.65316 & 0 & .14969 & -.34859 & 0 & .23908 \end{pmatrix} ,$$

(A.16)

and

$$\gamma''_n(g'') = \frac{g''^2}{8\pi^2} \begin{pmatrix} -7.0428 \\ -4 \\ -3.501 \\ 1.0974 \\ 2 \\ 2.8909 \end{pmatrix} + \mathcal{O}(g''^4) \; . \tag{A.17}$$

It follows from (A.17) and the perturbative expansion of $\beta''(g'')$ that

$$a''_n = \begin{pmatrix} -.84514 \\ -\ 12/25 \\ -.42012 \\ .13169 \\ 6/25 \\ .34691 \end{pmatrix} \quad . \qquad (A.18)$$

Again $a''_2 = a''^{(-)}$ and $a''_4 = a''^{(+)}$.

When the heavy charm quark expansion is performed only the five operators Q_1, Q_2, Q_3, Q_5, and Q_6 defined in Eq. (44) are required. We find that

$$\gamma'''_{pr}(g''') = \frac{g'''^2}{8\pi^2} \begin{pmatrix} -1 & 3 & 0 & 0 & 0 \\ 8/3 & -2/3 & 2/9 & -1/9 & 1/3 \\ -11/3 & 11/3 & 22/9 & -2/9 & 2/3 \\ 0 & 0 & 0 & 1 & -3 \\ -1 & 1 & 2/3 & -1/3 & -7 \end{pmatrix} + \mathcal{O}(g'''^4) \qquad (A.19)$$

The matrix $\gamma'''^T_{pr}(g''')$ is diagonalized by the transformation

$$\sum_{p,r} X^{-1}_{np}\, \gamma'''^T_{pr}(g''')\, X_{rq} = \delta_{nq}\, \gamma'''_q(g''') \quad , \qquad (A.20)$$

where

$$X_{np} = \begin{pmatrix} .16866 & -.71436 & .052633 & .84853 & .69088 \\ -.16866 & .71436 & -\ .052633 & .56569 & -\ .69088 \\ -.050165 & -.030949 & -\ .16552 & -.28284 & -1.1481 \\ .028133 & .018728 & -1.0044 & 0 & .23229 \\ .78361 & .049722 & .35726 & 0 & -\ .17486 \end{pmatrix} , \qquad (A.21)$$

and

$$\gamma_q'''(g''') = \frac{g'''^2}{8\pi^2}\begin{pmatrix} -7.2221 \\ -3.7559 \\ 1.0761 \\ 2 \\ 2.6797 \end{pmatrix} + \mathcal{O}(g'''^4) \quad . \tag{A.22}$$

Note that these eigenvalues check with those of Ref. 8 where the effective Hamiltonian for strangeness changing nonleptonic decays was calculated in the four-quark model using a different operator basis. The fourth eigenvalue corresponds to the multiplicatively renormalized SU(3) 27 operator $3Q_1 + 2Q_2 - Q_3$. This operator has both $I = 1/2$ and $I = 3/2$ pieces.

$$a_q''' = \begin{pmatrix} -.80246 \\ -.41732 \\ .11957 \\ 6/27 \\ .29774 \end{pmatrix} \quad . \tag{A.23}$$

The octet operators used in Ref. 8 were

$$Q_1' = (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A} - (\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} \tag{A.24a}$$

$$Q_2' = (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A} + (\bar{s}_\alpha u_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A} + 2(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{d}_\beta d_\beta)_{V-A} + 2(\bar{s}_\alpha d_\alpha)_{V-A}(\bar{s}_\beta s_\beta)_{V-A} \tag{A.24b}$$

$$Q_5' = 4(\bar{s}_\alpha T^a_{\alpha\beta} d_\beta)_{V-A}\left[(\bar{u}_\alpha T^a_{\alpha\beta} u_\beta)_{V+A} + (\bar{d}_\alpha T^a_{\alpha\beta} d_\beta)_{V+A} + (\bar{s}_\alpha T^a_{\alpha\beta} s_\beta)_{V+A}\right] \tag{A.24c}$$

$$Q_6' = (\bar{s}_\alpha d_\alpha)_{V-A}\left[(\bar{u}_\beta u_\beta)_{V+A} + (\bar{d}_\beta d_\beta)_{V+A} + (\bar{s}_\beta s_\beta)_{V+A}\right] \tag{A.24d}$$

where T^a $a \varepsilon \{1,...,8\}$ are the SU(3) color matrices normalized to $Tr(T^aT^b) = \delta^{ab}/2$. These operators can be written in terms of the operators $Q_1,...,Q_6$ of Chapter II in the following manner

$$Q_1' = -Q_2 + Q_1 \tag{A.25a}$$

$$Q_2' = -Q_1 + Q_2 + 2Q_3 \tag{A.25b}$$

$$Q_5' = -\frac{2}{3}Q_5 + 2Q_6 \tag{A.25c}$$

$$Q_6' = Q_5 \tag{A.25d}$$

Using these relations it can be shown that the anomalous dimension matrix in Eq. (A.19) agrees with that used in Ref. 8.

APPENDIX B

In this appendix values are given for the various quantities which enter the computation of $\mathscr{H}_3$ (a portion of the effective Hamiltonian for $K^o - \bar{K}^o$ mixing) when all eight operators $O_j^{(\pm\pm)}$ $j \in \{1, \ldots, 7\}$ and O_8 are kept. These operators (defined in Eqs. 18, 20 and 21 of Chapter III) close under renormalization of the one loop level and their renormalization is characterized by the anomalous dimension matrices

$$\gamma_{ij}^{\prime(++)}(g') = \frac{g'^2}{8\pi^2}\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{7}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 32 \\ 0 & 0 & \frac{22}{9} & \frac{8}{3} & -\frac{5}{9} & \frac{5}{3} & 0 & 16 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 & -32 \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{13}{3} & 0 & -16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & -24 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{3} \end{bmatrix} + \mathscr{O}(g'^4) \qquad \text{(B.1)}$$

$$\gamma_{ij}^{\prime(--)}(g') = \frac{g'^2}{8\pi^2}\begin{bmatrix} -5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -5 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{47}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & -16 \\ 0 & 0 & \frac{22}{9} & -\frac{10}{3} & -\frac{5}{9} & \frac{5}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & -3 & 0 & 16 \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{31}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & -8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{3} \end{bmatrix} + \mathscr{O}(g'^4) \qquad \text{(B.2)}$$

$$\gamma'^{(+-)}_{ij}(g') = \frac{g'^2}{8\pi^2}\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{7}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 32 \\ 0 & 0 & \frac{22}{9} & \frac{8}{3} & -\frac{5}{9} & \frac{5}{3} & 0 & 16 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 & -32 \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{13}{3} & 0 & -16 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{3} \end{bmatrix} + \mathcal{O}(g'^4) \qquad \text{(B.3)}$$

$$\gamma'^{(-+)}_{ij}(g') = \frac{g'^2}{8\pi^2}\begin{bmatrix} -5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -5 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{47}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & -16 \\ 0 & 0 & \frac{22}{9} & -\frac{10}{3} & -\frac{5}{9} & \frac{5}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & -3 & 0 & 16 \\ 0 & 0 & -\frac{5}{9} & \frac{5}{3} & -\frac{5}{9} & -\frac{31}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{3} \end{bmatrix} + \mathcal{O}(g'^4) \qquad \text{(B.4)}$$

The matrices $\gamma'^{(\pm\pm)^T}_{ij}(g')$ can be diagonalized by the transformations

$$\sum_{k,\ell} V^{(\pm\pm)^{-1}}_{i\ell}\ \gamma'^{(\pm\pm)^T}_{\ell k}(g')\ V^{(\pm\pm)}_{kj} = \delta_{ij}\ \gamma'^{(\pm\pm)}_{j}(g') \qquad \text{(B.5)}$$

where

$$V_{kj}^{(++)} = \begin{bmatrix} 0 & .69589 & 0 & 0 & 0 & -.70658 & 0 & 0 \\ 0 & -.69589 & 0 & 0 & 0 & -.70658 & 0 & 0 \\ -.20236 & -.23196 & .95985 & 0 & .17132 & .10094 & 0 & -.40226 \\ .28103 & .23196 & -.83058 & 0 & .083375 & .10094 & 0 & -.77672 \\ -.044316 & 0 & -.079869 & 0 & .96445 & 0 & 0 & .31309 \\ -.82989 & 0 & -.16334 & 0 & -.35439 & 0 & 0 & -.26431 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1.7593 & .85647 & -6.3181 & 1 & -23.46 & 2.9071 & -14.4 & -11.106 \end{bmatrix} \tag{B.6}$$

$$V_{kj}^{(--)} = \begin{bmatrix} 0 & 0 & .69589 & 0 & 0 & -.70658 & 0 & 0 \\ 0 & 0 & -.69589 & 0 & 0 & -.70658 & 0 & 0 \\ -.20236 & 0 & -.23196 & .95985 & .17132 & .10094 & -.40226 & 0 \\ .28103 & 0 & .23196 & -.83058 & .083375 & .10094 & -.77672 & 0 \\ -.044316 & 0 & 0 & -.079869 & .96445 & 0 & .31309 & 0 \\ -.82989 & 0 & 0 & -.16334 & -.35439 & 0 & -.26431 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -.19115 & .77419 & -.35917 & 1.7372 & -2.4326 & .3727 & -3.5761 & 1 \end{bmatrix} \tag{B.7}$$

$$V_{kj}^{(+-)} = \begin{bmatrix} 0 & 0 & .69589 & 0 & 0 & 0 & -.70658 & 0 \\ 0 & 0 & -.69589 & 0 & 0 & 0 & -.70658 & 0 \\ -.20236 & 0 & -.23196 & .95985 & 0 & .17132 & .10094 & -.40226 \\ .28103 & 0 & .23196 & -.83058 & 0 & .083375 & .10094 & -.77672 \\ -.044316 & 0 & 0 & -.079869 & 0 & .96445 & 0 & .31309 \\ -.82989 & 0 & 0 & -.16334 & 0 & -.35439 & 0 & -.26431 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.7593 & -1.8462 & .85647 & -6.3181 & 1 & -23.46 & 2.9071 & -11.106 \end{bmatrix}$$

(B.8)

$$V_{kj}^{(-+)} = \begin{bmatrix} 0 & .69589 & 0 & 0 & 0 & -.70658 & 0 & 0 \\ 0 & -.69589 & 0 & 0 & 0 & -.70658 & 0 & 0 \\ -.20236 & -.23196 & .95985 & .17132 & 0 & .10094 & -.40226 & 0 \\ .28103 & .23196 & -.83058 & .083375 & 0 & .10094 & -.77672 & 0 \\ -.044316 & 0 & -.079869 & .96445 & 0 & 0 & .31309 & 0 \\ -.82989 & 0 & -.16334 & -.35439 & 0 & 0 & -.26431 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -.19115 & -.35917 & 1.7372 & -2.4326 & -1.8462 & .3727 & -3.5761 & 1 \end{bmatrix}$$

(B.9)

and

$$\gamma_j^{'(++)}(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} -4.8954 \\ -2 \\ -1.2429 \\ 2.3333 \\ 3.1166 \\ 4 \\ 4 \\ 5.1327 \end{pmatrix} + \mathcal{O}(g'^4) \qquad \text{(B.10)}$$

$$\gamma_j^{'(--)}(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} -10.895 \\ -8 \\ -8 \\ -7.2429 \\ -2.8834 \\ -2 \\ -.86725 \\ 2.3333 \end{pmatrix} + \mathcal{O}(g'^4) \qquad \text{(B.11)}$$

$$\gamma_j^{'(+-)}(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} -4.8954 \\ -2 \\ -2 \\ -1.2429 \\ 2.3333 \\ 3.1166 \\ 4 \\ 5.1327 \end{pmatrix} + \mathcal{O}(g'^4) \qquad \text{(B.12)}$$

$$\gamma_j^{'(-+)}(g') = \frac{g'^2}{8\pi^2}\begin{pmatrix} -10.895 \\ -8 \\ -7.2429 \\ -2.8834 \\ -2 \\ -2 \\ -.86725 \\ 2.3333 \end{pmatrix} + \mathcal{O}(g'^4) \qquad \text{(B.13)}$$

At the stage of removing the b-quark the same operators enter except that the factor $m_c^{\prime 2}/g^{\prime 2}$ in the definition of O_8 is replaced by $m_c^{\prime\prime 2}/g^{\prime\prime 2}$ and the b-quark field terms in O_3-O_6 are absent. Again these operators close under strong interaction renormalization. The calculation of the anomalous dimension matrix is the same as when the t-quark was removed except it is calculated in an effective 4-quark theory (instead of an effective 5-quark theory) of strong interactions with coupling g". The resulting anomalous dimension matrices are:

$$\gamma_{ij}^{\prime\prime(++)}(g'') = \frac{g''^2}{8\pi^2}\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{7}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 32 \\ 0 & 0 & \frac{23}{9} & \frac{7}{3} & -\frac{4}{9} & \frac{4}{3} & 0 & 16 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 & -32 \\ 0 & 0 & -\frac{4}{9} & \frac{4}{3} & -\frac{4}{9} & -\frac{14}{3} & 0 & -16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & -24 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} + \mathcal{O}(g''^4) \qquad \text{(B.14)}$$

$$\gamma_{ij}^{\prime\prime(--)}(g'') = \frac{g''^2}{8\pi^2}\begin{bmatrix} -5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -5 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{47}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & -16 \\ 0 & 0 & \frac{23}{9} & -\frac{11}{3} & -\frac{4}{9} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & -3 & 0 & 16 \\ 0 & 0 & -\frac{4}{9} & \frac{4}{3} & -\frac{4}{9} & -\frac{32}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & -8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} + \mathcal{O}(g''^4) \qquad \text{(B.15)}$$

$$\gamma_{ij}^{''(+-)}(g'') = \frac{g''^2}{8\pi^2}\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{7}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 32 \\ 0 & 0 & \frac{23}{9} & \frac{7}{3} & -\frac{4}{9} & \frac{4}{3} & 0 & 16 \\ 0 & 0 & 0 & 0 & 3 & -3 & 0 & -32 \\ 0 & 0 & -\frac{4}{9} & \frac{4}{3} & -\frac{4}{9} & -\frac{14}{3} & 0 & -16 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} + \mathcal{O}(g''^4) \qquad \text{(B.16)}$$

$$\gamma_{ij}^{''(-+)}(g'') = \frac{g''^2}{8\pi^2}\begin{bmatrix} -5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & -5 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{47}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & -16 \\ 0 & 0 & \frac{23}{9} & -\frac{11}{3} & -\frac{4}{9} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & -3 & 0 & 16 \\ 0 & 0 & -\frac{4}{9} & \frac{4}{3} & -\frac{4}{9} & -\frac{32}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} + \mathcal{O}(g''^4) \qquad \text{(B.17)}$$

The matrices $\gamma_{ij}^{(\pm\pm)^T}(g'')$ can be diagonalized by the transformations

$$\sum_{k,\ell} W_{i\ell}^{(\pm\pm)^{-1}} \gamma_{\ell k}^{''(\pm\pm)}(g'')^T W_{kj}^{(\pm\pm)} = \delta_{ij}\gamma_j''(g'') \qquad \text{(B.18)}$$

where

$$W_{kj}^{(++)} = \begin{bmatrix} 0 & .6558 & 0 & 0 & 0 & .70643 & 0 & 0 \\ 0 & -.6558 & 0 & 0 & 0 & .70643 & 0 & 0 \\ .14452 & -.3279 & -.78005 & 0 & .1414 & -.11774 & 0 & -.67561 \\ -.20298 & .3279 & .71236 & 0 & .067442 & -.11774 & 0 & -1.1468 \\ .032867 & 0 & .041364 & 0 & .96775 & 0 & 0 & .42931 \\ .72549 & 0 & .096564 & 0 & -.35021 & 0 & 0 & -.34187 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1.6816 & 1.4308 & 5.1876 & 1 & -13.812 & -2.4221 & -10.286 & -14.961 \end{bmatrix} \qquad (B.19)$$

$$W_{kj}^{(--)} = \begin{bmatrix} 0 & 0 & .6558 & 0 & 0 & .70643 & 0 & 0 \\ 0 & 0 & -.6558 & 0 & 0 & .70643 & 0 & 0 \\ .14452 & 0 & -.3279 & -.78005 & .1414 & -.11774 & -.67561 & 0 \\ -.20298 & 0 & .3279 & .71236 & .067442 & -.11774 & -1.1468 & 0 \\ .032867 & 0 & 0 & .041364 & .96775 & 0 & .42931 & 0 \\ .72549 & 0 & 0 & .096564 & -.35021 & 0 & -.34187 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ .14055 & .82759 & -.54273 & -1.4336 & -2.8936 & -.51377 & -6.3689 & 1 \end{bmatrix} \qquad (B.20)$$

$$W_{kj}^{(+-)} = \begin{bmatrix} 0 & 0 & .6558 & 0 & 0 & 0 & .70643 & 0 \\ 0 & 0 & -.6558 & 0 & 0 & 0 & .70643 & 0 \\ .14452 & 0 & -.3279 & -.78005 & 0 & .1414 & -.11774 & -.67561 \\ -.20298 & 0 & .3279 & .71236 & 0 & .067442 & -.11774 & -1.1468 \\ .032867 & 0 & 0 & .041364 & 0 & .96775 & 0 & .42931 \\ .72549 & 0 & 0 & .096564 & 0 & -.35021 & 0 & -.34187 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.6816 & -2.1818 & 1.4308 & 5.1876 & 1 & -13.812 & -2.4221 & -14.961 \end{bmatrix} \tag{B.21}$$

$$W_{kj}^{(-+)} = \begin{bmatrix} 0 & .6558 & 0 & 0 & 0 & .70643 & 0 & 0 \\ 0 & -.6558 & 0 & 0 & 0 & .70643 & 0 & 0 \\ .14452 & -.3279 & -.78005 & .1414 & 0 & -.11774 & -.67561 & 0 \\ -.20298 & .3279 & .71236 & .067442 & 0 & -.11774 & -1.1468 & 0 \\ .032867 & 0 & .041364 & .96775 & 0 & 0 & .42931 & 0 \\ .72549 & 0 & .096564 & -.35021 & 0 & 0 & -.34187 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ .14055 & -.54273 & -1.4336 & -2.8936 & -2.1818 & -.51377 & -6.3689 & 1 \end{bmatrix} \tag{B.22}$$

and

$$\gamma_j''^{(++)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -5.0428 \\ -2 \\ -1.501 \\ 1.6667 \\ 3.0974 \\ 4 \\ 4 \\ 4.8909 \end{pmatrix} + \mathcal{O}(g''^4) \tag{B.23}$$

$$\gamma_j''^{(--)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -11.043 \\ -8 \\ -8 \\ -7.501 \\ -2.9026 \\ -2 \\ -1.1091 \\ 1.6667 \end{pmatrix} + \mathcal{O}(g''^4) \tag{B.24}$$

$$\gamma_j''^{(+-)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -5.0428 \\ -2 \\ -2 \\ -1.501 \\ 1.6667 \\ 3.0974 \\ 4 \\ 4.8909 \end{pmatrix} + \mathcal{O}(g''^4) \tag{B.25}$$

$$\gamma_j''^{(-+)}(g'') = \frac{g''^2}{8\pi^2}\begin{pmatrix} -11.043 \\ -8 \\ -7.501 \\ -2.9026 \\ -2 \\ -2 \\ -1.1091 \\ 1.6667 \end{pmatrix} + \mathcal{O}(g''^4) \tag{B.26}$$

At the stage of removing the c-quark only the operator $m_c''^2 (\bar{s}d)_{V-A} \cdot (\bar{s}d)_{V-A}$ appears. It is multiplicatively renormalized (i.e., does not mix with other operators) and has the anomalous dimension $g''^2/\pi^2 + g'''^2/4\pi^2$ to leading order in the strong coupling.

APPENDIX C

Here some useful SU(3) relations are given. The Gell Mann matrices λ_i, $i\epsilon\{1,...8\}$ are

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

These matrices form the generators of SU(3). In addition they transform as a basis for the adjoint octet representation. The spin 1/2 ground state positive parity baryons also transform as a basis for the adjoint representation of SU(3). Let $|B_k\rangle$ denote the baryon state with the same SU(3) quantum numbers as λ_k. Then

$$|\Sigma^+\rangle = \frac{1}{\sqrt{2}} |B_1 + iB_2\rangle$$

$$|\Sigma^-\rangle = \frac{1}{\sqrt{2}} |B_1 - iB_2\rangle$$

$$|\Sigma^o\rangle = |B_3\rangle$$

$$|p\ \rangle = \frac{1}{\sqrt{2}} |B_4 + iB_5\rangle$$

$$|n\rangle = \frac{1}{\sqrt{2}}\,|B_6 + iB_7\rangle$$

$$|\Xi^o\rangle = \frac{1}{\sqrt{2}}\,|B_6 - iB_7\rangle$$

$$|\Xi^-\rangle = \frac{1}{\sqrt{2}}\,|B_4 - iB_5\rangle$$

$$|\Lambda^o\rangle = |B_8\rangle$$

The matrix element for an operator O_k which transforms like the k'th component of an octet (i.e., like λ_k) under SU(3) is given by

$$\langle B_i|O_k|B_j\rangle = d_{kij}d - if_{kij}f$$

The non-zero d_{ijk} and f_{ijk} are listed in Table VII.

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