ELECTRON SCATTERING OFF HYDROGEN AND DEUTERIUM

AT 50° AND 60° *

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ABSTRACT

The inelastic scattering of electrons for both protons and
neutrons has been measured at scattering angles of fifty and sixty
degrees and for incident energies between 6.5 GeV and 19.5 GeV.
Elastic scattering from protons was also observed. The measurements
were made at the Stanford Linear Accelerator Center using the
1.6-GeV spectrometer located in End Station A. The experimental
techniques used in making the measurements are detailed. The result-
ing cross sections are large, indicative of a charged point-like
substructure within the nucleon and match the extrapolation from
previous measurements to the new kinematic region to within forty
percent. The measurements are found to be compatible with some
existing theoretical expectations. The W_1 structure function is not
found to "scale" in previously suggested scaling variables. The
introduction of propagator scale breaking is investigated and shown
to be compatible with the measurements. An alternative scaling
variable is proposed in which W_1 is found to scale. The elastic
scattering measurements are consistent with an inverse fourth
power dependence on four-momenta transfer. The measured neutron-
proton ratio is in agreement with previous measurements.

Dedicated to my father, William Faulkner Atwood, who awakened within me an interest in the physical sciences.

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CHAPTER I
INTRODUCTION

Particle scattering experiments done by Rutherford (Ref. I-1) foreshadowed the development of particle physics. The use of this technique has grown immensely during this century and has become a major source of information about the structure of the world on the scale of atoms and nuclei. Rutherford's low energy alpha particles allowed him to "see" atomic structure, revealing the presence of a charged point-like core, the nucleus. Hofstadter and co-workers during the 1950's performed similar experiments (Ref. I-2) replacing the low energy alpha particles with a high energy electron beam.

In both experiments the fundamental interaction used to probe the target structure was electromagnetism. Due to the comparatively weak coupling of photons to charged particles (characterized by a coupling constant $\alpha = 1/137$), this type of probe can pass unaffected deep inside a target particle. The electron, believed to be a point-like charged particle, interacts with matter primarily through electromagnetism and does not participate in strong interactions (characterized by a coupling constant of approximately 15), hence the problem of strong interactions masking substructure is not present. Rutherford could not have successfully examined nuclear structure, because strong interaction between the alpha particles and the nucleus would not allow these projectiles to penetrate. The alpha particle proved adequate for examining atomic structure where the nucleus can be regarded as a point-like object on the scale of atomic dimensions.

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For scattering at a fixed angle, raising the energy of the incident particle decreases the distance of closest approach to the target particle. Thus, increasing the energy of the probe may reveal structure inside the target on a smaller distance scale. Point-like charged particles are the most effective targets to scatter incident electrons (or muons) via electromagnetic interactions. Deviation from a point-like target is discussed in terms of the form factor which has a value of one for point-like particles and is in general smaller for more tenuous charge distributions. In an impressive series of experiments, Hofstadter and his co-workers found that nuclei have a finite size roughly determined by the number of nucleons contained, and that the charge distribution inside the nucleus was nearly uniform. But, perhaps, the most remarkable observation was that the proton itself was not a simple point-like object. It, too, displayed an internal structure giving rise to an elastic form factor decreasing from unity rapidly as the momentum transfer to the proton was increased (Ref. I-3).

With the completion of SLAC in the late 1960's, electron energies up to 20 GeV became available to continue this exploration. The first experiments using electrons to probe nucleons at SLAC examined elastic scattering at higher energies. The inability of the proton to elastically scatter high energy electrons to large angles persisted with much the same dependence on momentum transfer as observed at lower energies (Ref. I-4).

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At incident electron energies in the GeV range, the proton can be excited inelastically, as well as scattering elastically, resulting in the production of many new particles such as π mesons. A big surprise came in the discovery that this inelastic cross section was very large (Ref. I-5). As the elastic cross section died away with increasing momentum transfer, the relative inelastic cross section increased. The total scattering strength at a fixed angle and incident energy of all inelastic states was quite large and suggested the existence of a substructure of charged point-like constituents within the nucleon that was responsible for the total scattering strength of this system. Models of nucleon structure based on point-like constituents suggest that the data should exhibit a "scaling" phenomenon explaining the inelastic form factor's constancy with respect to momentum transfer (Ref. I-6). Other models also offer explanations of the scaling behavior of the inelastic form factor (Ref. I-7).

Later experiments included electron-deuteron scattering as well as continuing the electron proton scattering program. The objective here was to compare the neutron to the proton in the inelastic region. The deuteron cross sections showed that the neutron inelastic form factor "scaled" like that of the proton (Ref. I-8). Again the possible presence of a substructure revealed itself. Although the neutron/proton cross section ratio scaled, it showed a nontrivial dependence on the scaling variable. In particular $N/P \lesssim 1$ everywhere. The inelastic electroproduction cross sections were not merely diffractive

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scattering of virtual photons off the target nucleon.

The present work continues the investigation of electron nucleon scattering, both elastic and inelastic, to large scattering angles and extends the kinematic range of the data near the boundary imposed by a maximum incident beam energy of 20 GeV. The experiments are in principle very simple. An electron beam of measured energy, E_o, and flux is directed onto a liquid hydrogen (or deuterium) target. Detectors are set up to count the number of electrons scattered as a function of the deflection angle, θ, and scattered energy, E'. In practice the experiment becomes increasingly difficult as the angle θ is increased, for two reasons. First (as Rutherford could have told us!) the cross section quickly becomes small. At 50° and 60° the typical size is 10^{-35} cm^2 per GeV per steradian at SLAC energies. Second, the background of particles other than scattered electrons produced by the beam striking the target increases with respect to the signal.

FIRST REMARKS ON PHENOMENOLOGY

The canonical diagram used to describe the inelastic electron scattering process is shown in Fig. I-1. The electric current $e \rightarrow e'$ is the source of a virtual photon, γ, of measured energy and momentum. The photon couples to the target (initially at rest in the laboratory) producing a recoiling final state of mass W. Due to the weakness of the electromagnetic coupling, the single photon exchange is believed to dominate as higher order processes (two or more photon exchanges) would be weaker still at each order by the coupling constant.

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FEYNMAN DIAGRAM OF SINGLE PHOTON EXCHANGE
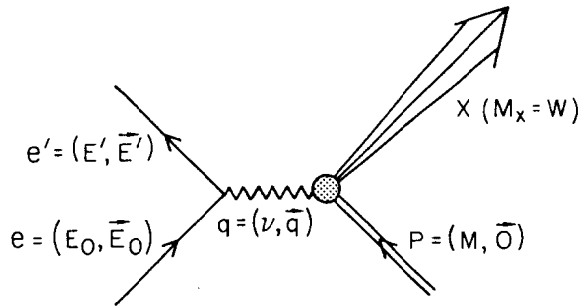
LABORATORY SCHEMATIC
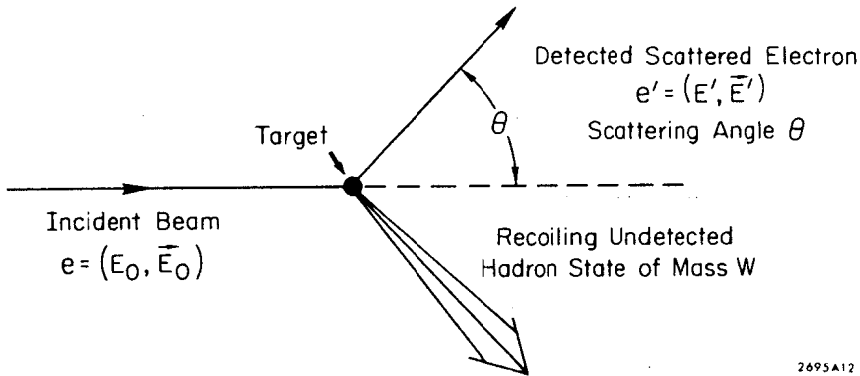
2695A12

FIG. I-1

Feynman diagram of single photon exchange
for e + p - e' + X and laboratory schematic
of the same process.

Experiments indicate that the general angular dependence of the observed cross sections is what one expects for the single photon exchange mechanism. The observation that the cross section is independent of the sign of the charge of the incident electron provides more confirmation that two photon exchange mechanisms are suppressed (Ref. I-9).

The relativistic invariants used to describe the target vertex are:

$$-q^2 = Q^2 = -(e_\mu - e_\mu')^2 = 4 \, E_o E' \sin^2 \theta/2$$

$$P \cdot q = M\nu = M(E_o - E')$$

$$W^2 = M^2 + 2M\nu - Q^2$$

Following the standard prescriptions of quantum electrodynamics the matrix element describing the one photon exchange process is

$$\mathcal{M} = i\sqrt{\alpha} \; \bar{e} \, \gamma_\mu \; e \; \frac{1}{q^2} \; J_\mu$$

where $J_\mu = \sqrt{\alpha} \; \sum_{i,f} \langle f \, |j_\mu| \, i \rangle$ is the electric current within the hadron (i, f signify initial and final states). The electric current of the incoming and outgoing electron when squared in the matrix element (averaged over initial spin states and summed over final spin states) yields the tensor:

$$k_{\mu\nu} = (e_\mu \, e_\nu' + e_\mu' \, e_\nu - \delta_{\mu\nu} e \cdot e')$$

This tensor is contracted against a $W_{\mu\nu}$ tensor describing the photon-target coupling. A general Lorentz covariant formulation for $W_{\mu\nu}$ (again only the symmetric part remains after summing over spin states) assuming one photon exchange and gauge invariance is

$$W_{\mu\nu} = -W_1 \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right)$$

The squared matrix element resulting from contracting these two tensors yields the cross section:

$$\frac{d\sigma}{d\Omega dE'} = \sigma_{Mott} \left(W_2 + 2 \tan^2 \theta/2 \ W_1 \right)$$

where

$$\sigma_{Mott} = \frac{\alpha^2 \cos^2\theta/2}{4 E_o^2 \sin^4\theta/2}$$

An analysis of the cross section in terms of photon polarization, ε_μ, parallel and perpendicular to $\vec{q}$ is instructive (Ref. I-10). The total photoabsorption cross section is proportional to $\varepsilon_\mu \varepsilon_\nu W^{\mu\nu}$. Gauge invariance requires $\varepsilon_\mu q^\mu = 0$. Choosing $\vec{q}$ along the z-direction, the two polarization cases are:

TRANSVERSE: $\varepsilon_z = \varepsilon_t = 0$. Both q and P are perpendicular to ε_μ. All terms in $W_{\mu\nu}$ containing either P or q vanish. Only W_1 remains.

$$\sigma_t = \frac{4\pi^2 \alpha}{K} W_1$$

where $K = (W^2 - M^2)/2M$

is the energy of an equivalent on mass-shell photon to produce the final mass state W.

LONGITUDINAL: $\varepsilon_z = \nu/\sqrt{Q^2}$, $\varepsilon_t = q/\sqrt{Q^2}$, only $\varepsilon_\mu q^\mu = 0$

Terms in W_1 and W_2 are both present.

$$\sigma_s = \frac{4\pi^2 \alpha}{K} \left(W_2 \left(1 + \frac{\nu^2}{Q^2} \right) - W_1 \right)$$

Often the σ_s and σ_t contributions are discussed in terms of σ_s/σ_t. Simple substitutions into the previous cross section expression gives

$$\frac{d\sigma}{d\Omega dE'} = \Gamma_t \left(\sigma_t + \varepsilon \sigma_s \right)$$

where $\varepsilon = 1/(1 + 2 \tan^2 \theta/2 (1 + \nu^2/Q^2))$

is the photon polarization parameter and

$$\Gamma_t = \frac{\alpha}{4\pi^2} \frac{2 K E'}{Q^2 E_o (1 - \varepsilon)}$$

is interpreted as the effective flux of virtual photons. Dividing this photon flux factor into the data yields a total virtual photo-absorption cross section, $\sigma_t + \varepsilon\sigma_s$. These cross sections then are plotted versus ε for fixed W and Q^2. As the range of ε is $0 \to 1$ the intercepts of a straight line fit yield σ_t and $\sigma_t + \sigma_s$ respectively. Increasing the spread of the data in ε for fixed W and Q^2 makes possible more accurate the extraction of σ_s and σ_t (or equivalently W_1 and W_2). Large angle data populates very small values of ε (50° and 60° have $\langle\varepsilon\rangle \cong .12$ for the incident energy range of 6-20 GeV). Previous electroproduction data in the deep inelastic region covered $\varepsilon = .3$ to $\varepsilon = .9$, and usually for any given W and Q^2 the spread in ε was considerably less than 0.5.

Much confidence in a model of the nucleon based on a substructure composed of point-like particles, called partons, is founded on the scaling behavior of the deep inelastic data (Ref. I-11). Scaling

means that the structure functions, W_1 and νW_2, depend only on the ratio $2M\nu /Q^2$ as ν and Q^2 both get "large." This behavior has a simple interpretation within the parton model, namely quasi elastic scattering off point-like constituents. Most "proofs" of scaling in the parton model set $W_{\mu\nu}$ for the parton (and thus for the fundamental interaction) proportional to a delta function, $\delta\left((p + q)^2 - m^2\right)$, to impose the quasi-elastic constraint. The assumption is made that the parton carries a fraction x of the nucleon's momentum. This assumption is formulated as $p = xP$, and it should become more valid as the energies in the problem get large compared to mass terms. Integrating over the parton distribution function $f(x)$ ($f(x) \, dx$ = no. of partons between x and x + dx each weighted by its $(charge)^2$) yields the following three conclusions:

1) $x = Q^2/2M\nu$ and $\omega = 1/x$

2) $\nu W_2 = xf(x)$ for both spin 0 partons and for spin ½ partons

3) $2M \, W_1 = f(x)$ for spin ½ partons

   $= 0$ for spin 0 partons

In the parton model W_1 measures the spin ½ parton distribution (assuming no spin 1 partons) as transversely polarized photons do not couple to spin 0 particles. On the other hand, νW_2 measures the distribution in x of spin ½ and spin 0 partons. Scaling is observed in x for νW_2 to about ±10% over a dynamic range of about 1000 in the size of νW_2 (Ref. I-12). The kinematic region where this phenomenon is observed to begin occurs at relatively low energies, $Q^2 > 1.5$ GeV^2 and $W > 2.5$ GeV.

The phenomenological lore of scaling starts from the variable x introduced by J. D. Bjorken (Ref. I-6). Later, Bloom and Gilman suggested a modified scaling variable $\omega' = \omega + M^2/Q^2$ (Ref. I-13). Holding a scaling variable fixed while allowing W and Q^2 to vary traces out contours in the W^2 vs. Q^2 plane. Constant x contours are straight lines all intersecting at $Q^2 = 0$ and W = M. Photo-production occurs at x = 0, and all elastic scattering lies along x = 1. The x' ($1/\omega'$) variable produces contours which are also straight lines all intersecting at $W^2 = Q^2 = 0$. Elastic scattering does not occur at fixed x' as it does in x. This allows for the connection of the inelastic to the elastic in a natural way (for $\nu W_2 \propto (1 - x')^n$ we have $W_2(el) \propto 1/(Q^2)^{n+1}$ which is the Drell-Yan-West relation (Ref. I-14)). The data when plotted in x' exhibits more "precocious" scaling than when plotted against x, extending the scaling region down to $Q^2 > 1$ GeV^2 and W > 2 GeV.

For scaling to begin at such low energies was a puzzle. Nothing in the parton theories sets an absolute energy scale. As no one has yet produced or observed a free parton in the laboratory, one might conclude that partons must be very massive objects bound extremely tightly inside the nucleon. The neglected mass terms in the traditional scaling derivation could then be important.

Recently it has been demonstrated experimentally (Ref. I-15) that the connection to photoproduction can be achieved through a "closure approximation" stolen from nuclear physics (Ref. I-16).

$$\nu \, W_2 = (1 - W_2(el)) \, F_2(\omega')$$

where

$$W_2(el) = (G_E^2 + \tau \ G_M^2)/(1 + \tau), \quad \tau = Q^2/4M^2$$

and G_E and G_M are the electric and magnetic form factors of the proton. $F_2(\omega')$ is the scaling function using the ω' variable.

The above factorization yields the original Gottfried (Ref. I-17) sum rule:

$$\int W_2 \ d\nu = (1 - W_2(el)) \sum_i \ Q_i^2$$

where the Q_i are the parton charges.

This sum rule is a formulation of the transition that a many particle system makes going from the region where elastic scattering dominates (coherent scattering) to the region where scattering off the constituents is most important (incoherent scattering). The sum rule saturates quickly with increasing Q^2 (at $Q^2 = 1$ GeV^2,$(1-W_2(el))=.925$) and sets a reasonable energy where scaling phenomena (scattering off individual partons) should become important.

Recently some theorists predicted scale breaking should become important rapidly just beyond the measured kinematic region (Ref. I-18). This scale breaking was to arise from the form factors of the partons themselves. We shall return to this question in some detail in Chapter V.

Neutron-proton comparisons provide further tests of theories. The Gottfried sum rule interprets the integral of the structure function as the sum of the squares of the charges of the nucleon's constituents at high Q^2. In a popular version of the parton model, the quark model of Gell-Mann and Zweig (Ref. I-19) there are 3 quarks

within a nucleon. The proton has two p quarks of charge +2/3 and one n quark of charge -1/3. The neutron has one p quark and two n quarks. The charge-squared sum rule evaluated over the available data (Ref. I-15) has a value of about 1.05 for the proton and .85 for the neutron giving approximately the correct ratio for this simple fractionally-charged-quark model. The N/P ratio as a function of x should not go below 1/4 as x $\rightarrow$1 for parton models. Bloom and Gilman predict N/P $\rightarrow$ ($\mu_N/ \ \mu_P)^2$ as x $\rightarrow$ 1. Unfortunately the neutron picture is complicated, particularly near x = 1, due to the neutron cross section's extraction from deuterium data. (Appendix C covers the Fermi motion problem in some detail.) The present data extend the N/P measurement to high Q^2 in the small ω region and slightly increases the measured region in ω (see Chp. V).

THE EXPERIMENT

The experiment (officially called E-89) was a double spectrometer experiment (the spectrometers were not run in coincidence). The two spectrometers were the 1.6-GeV spectrometer and the 20-GeV spectrometer. Both instruments are located in End Station A at SLAC. In the experiments, the two spectrometers shared the same beam and target with the 1.6-GeV spectrometer taking data at 50° and 60° and the 20-GeV spectrometer taking data at a variety of angles from 6° to 20.6°. This thesis is concerned with the 1.6-GeV spectrometer measurements at the large angles; the 20-GeV spectrometer experiment is described elsewhere (Ref. I-20).

Figure I-2 is a W^2 vs. Q^2 diagram depicting the kinematic range
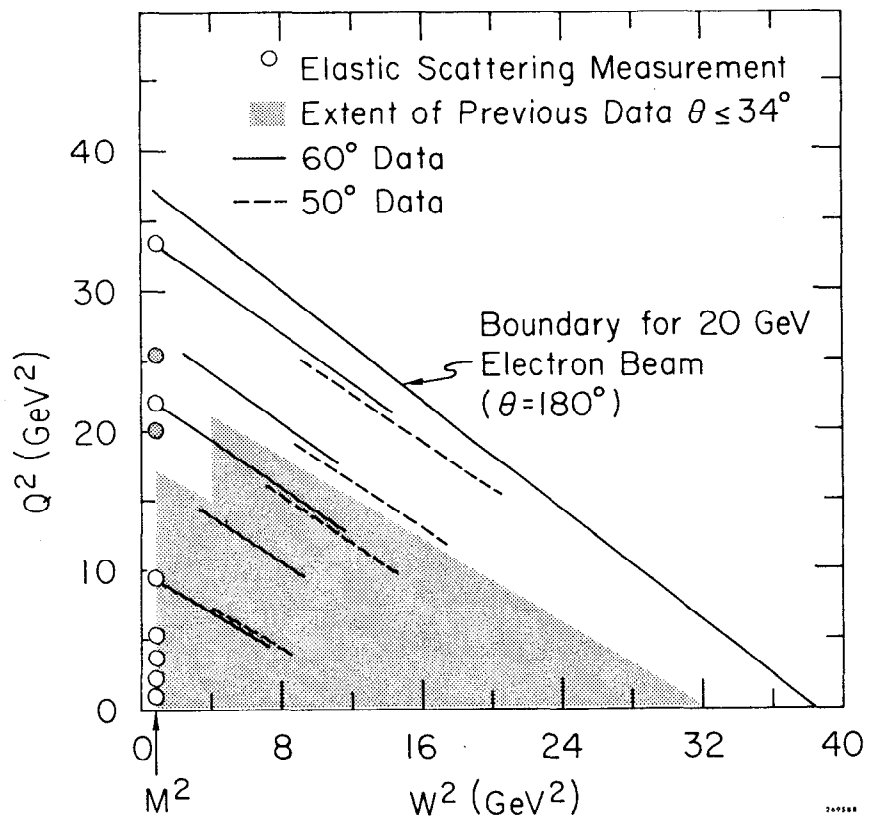
Kinematic region in W^2 and Q^2 of the
measurements at 50° and 60°.

FIG. I-2

of the 50° and 60° data. The 60° data were taken at 5 different
incident energies and the 50° data were taken at 4 energies. Data
taken at one incident energy and angle are referred to as a "line."
The low E' cut off of the data (corresponding to a W < 4.5 GeV) was
dictated by the rising experimental corrections as E' decreased.
The dots on Fig. I-2 indicate where elastic scattering data were
taken. During the experimental checkout period many low Q^2 elastic
peaks were measured mostly at 50° with incident energies from
1.5 GeV to 4.5 GeV.

The W_1 contribution to the cross section outweighs the W_2
portion by about 6:1 at 50° and 60°. In the language of total
virtual photoabsorption cross sections this means we are measuring
σ_t with very little contamination from σ_s. The ϵ parameter is
typically .12, thus the effect of σ_s when multiplied by ϵ in the
total cross section is reduced by nearly an order of magnitude. The
correction applied to the measured cross section to extract W_1 is
quite insensitive to R. In Fig. I-3 contours of constant ϵ (curve
lines) calculated for 60° and of constant ω' are shown.

The data taking proceeded as follows. A beam of electrons from
the accelerator was delivered to the End Station. The spectrometer
was set to its highest value of E' for that incident energy. Data
were taken on each of the three targets, hydrogen, deuterium, and
the empty target cell. The central momentum of the spectrometer
was then lowered, and data taking continued as for the first
setting. The spectrometer accepts particles within $\pm 5\%$ of the

Shaded Region Indicates
50° and 60° Data Region
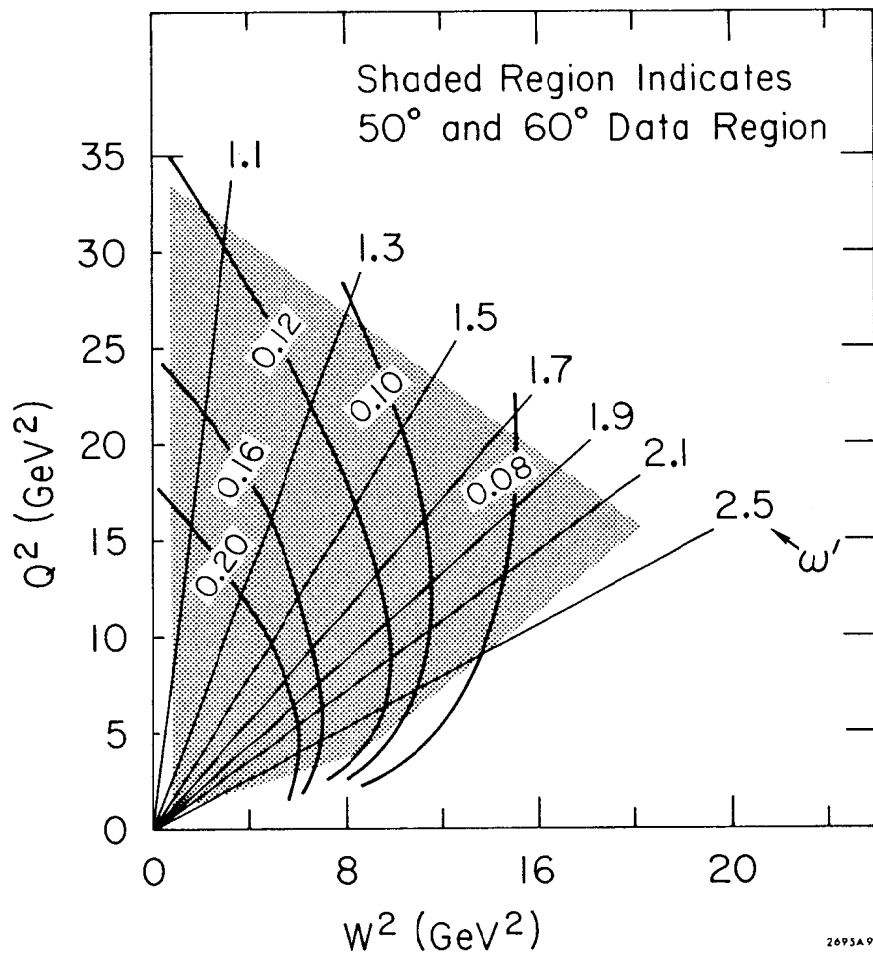
FIG. I-3

ϵ contours (curve lines) for 60° and ω'
contours overplotted on the kinematic
region of the data.

central momentum. Each new setting was lower in central momentum
by 3.33%. Thus, each cross section was measured in three separate
runs at three different locations in the detector.

The main data taking occurred in two periods. During the
first, fall of 1973, most of the 60° data were completed. In the
second cycle, winter of 1974, the 50° data were accumulated, the
60° data completed, the high Q^2 elastic points run and an interesting
comparison of e^+/e^- made to values of Q^2 as high as 15 GeV^2
(Ref. I-21). The cross section data were bracketed by "check point"
data of two types: 1) 1.5 GeV elastic scattering at 50° and
2) 6.5 GeV cross sections at 60°. These check runs tested the
reproducibility of the measurements. A complete run chronology
is given in Table I-1.

TABLE I

DATA ACQUISITION
CHRONOLOGY

DATE	RUN NO.	TO RUN NO.	E_o	θ	COMMENT
9/22/73	302	323	1.5	50°	Elastic check point
	324	364	1.5	60°	Elastic check point
	365	385	1.5	50°-60°	Elastic sweep in θ
9/26/73	386	397	2.5	50°	Elastic check
	398	417	3.5	50°	Elastic check
	418	420	4.5	50°	Elastic check
	421	424	2.5	50°	Elastic check
	425	522	19.5	53.3^o-60°	Check out data
10/9/73	523	532	1.5	50°	Elastic check point
	533	557	19.5	53.3^o-60°	Check out data

End
Check Out
– –

DATE	RUN NO.	TO RUN NO.	E_o	θ	COMMENT
10/13/73	1001	1232	6.5	60°	Data
10/22/73	1233	1297	19.5	60°	Data
11/2/73	1349	1449	13.3	60°	Data
11/8/73	1450	1557	16.0	60°	Data
11/13/73	1558	1637	10.4	60°	Data
11/16/73	1638	1647	6.5	60°	Check point data
11/16/73	1648	1664	19.5	60°	Data
11/17/73	1665	1682	1.5	50°	Elastic check point

TABLE I (continued)

DATA ACQUISITION
CHRONOLOGY

DATE	RUN NO.	TO RUN NO.	E_o	θ	COMMENT
1/10/74	2001	2011	1.5	50°	Elastic check point
1/12/74	2012	2026	6.5	60°	Check point data
1/13/74	2027	2099	7.0	50°	Data
1/14/74	2101	2191	13.5	50°	Data
1/17/74	2192	2272	16.0	50°	Data
1/20/74	2273	2474	19.5	50°	Data
1/31/74	2475	2585	13.3	50°	Data
	2586	2592	13.3	50°	Dummy target checks
2/5/74	2593	2673	16.0	60°	Data
2/10/74	2674	2720	13.9	50°	e^+/e^- comparison
2/16/74	2721	2845	19.5	60°	Data
2/26/74	2846	2857	6.5	60°	Check point data
2/28/74	2858	2861	6.5	39°	Optics test

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CHAPTER II

OVERVIEW OF THE EXPERIMENT

The experiment is conceptually very simple. The Stanford Linear Accelerator produces a beam of high energy electrons (Ref. II-1). The principal element in the LINAC is a 10000 foot disk-loaded copper waveguide. Prior to the injection of electrons from a high current electron gun, the waveguide is filled with R.F. power from up to 245 klystrons. Some of the injected electrons are caught on the crests of the travelling R.F. wave in the wave-guide and are accelerated with an energy gain of approximately 2 MeV per foot. The resulting electron beam has an R.F. structure reflecting the 2856 MHz microwave radiation used to power the wave-guide (the bunch structure has approximately 10 cm spacing). The total pulse duration is from 1.2 to 1.6 usec., and the accelerator can run at a repetition rate of up to 360 pulses per second (or a maximum duty cycle of about 1:1700). Although the pulses are short, currents over the beam spill can exceed 60 mA above 13 GeV (a single 20 GeV pulse can have almost 2000 joules of energy).

The beam from the accelerator is channeled to various experimental areas via the Beam Switch Yard (BSY) on a pulse to pulse basis. As many as 8 independent beams have been delivered in this fashion, each sharing a fraction of the available pulses. For End Station A (ESA), the BSY deflects the beam north to the "A-line." Here, a symmetric series of bending magnets and quadrupole focusing elements complete the 24.5 degree bend into ESA. High power momentum selection slits located half way through "A Bend"

determine the incident energy and resolution.

In the End Station the beam passes through a target mounted on the "pivot." Three large spectrometers sit on circular railroad tracks centered on the pivot making angle positioning of these cumbersome instruments easy. Most of the beam suffers little deflection in the target and passes out the rear of the End Station and into a beam dump buried in the hill behind the End Station.

The 1.6 GeV spectrometer is located on the north side of the beam and is the lowest momentum spectrometer of the three in ESA (Ref. II-2). It consists of a single 90 degree bending magnet weighing approximately 80 tons. Scattered electrons from the target pass through a tungsten slit at the magnet entrance which partially defines the spectrometer's acceptance. Slanted entrance and exit pole faces cause wedge focusing, yielding point to point imaging in the vertical plane, dispersing different momenta at the "p-focus," and parallel to point focusing in the horizontal plane. The line target source is focused to a point with the various allowed angles dispersed.

Particle detectors were located within a 5'(diameter) by 11' (high) enclosure shielded by over 200 tons of concrete. Particle identification was made with a threshold gas Cerenkov counter (CT) followed by a multi-segmented lead glass Total Absorption counter (TA). Particle tracking information was provided by three hodoscopes, one located in front of the Cerenkov counter and two sandwiched between the Cerenkov counter and the TA package.

A particle traversing the counters produced various signals which were transmitted to the "counting house" where they were processed by fast electronic circuits. Subsequently information from the electronics was recorded by an XDS 9300 computer on magnetic tape for each event. In addition, target information, 1.6 spectrometer status information, and beam information were recorded at regular intervals. The computer's other duties were to perform an online analysis of the experiment, to monitor the functioning of the various pieces of apparatus associated with the experiment and announce any malfunctions detected. The computer also provided experimenters (often to their annoyance) with a preprogrammed agenda. This agenda directed the data taking, calling for changes in initial and scattered energy settings, and prompting the inattentive when sufficient incident flux had been accumulated for a given kinematical setting.

THE BEAM

The beam transport system is shown in Fig. II-1. Accelerated electrons from the LINAC are collimated to a spot size determined by the high power slits CO and Cl at the entrance to the BSY. These slits were typically set at 9mm x 9mm. Pulsed bending magnets then deflect the beam 0.5 deg. into the "A-line." Eight bending magnets and five quadrupole magnets are the primary magnetic elements. These are placed symmetrically about high power momentum slits, SL10/SL11, with pairs of "quads" beginning and ending the sequence.

I.6 GeV SPECTROMETER

I.6 GeV SPECTROMETER

UNDERGROUND

TO BEAM DUMP EAST

RS2   RS3

PIVOT

GOOD SPILL COUNTER

ESA

ALCOVE

RS2

PIVOT

TOR I

TOR φ

A BEND

VSTR

RS1   VSTR

BAD SPILL COUNTER

$B_{14}B_{15}$ B_{16} B_{17}

ACCELERATOR

$C_0 C_1$ PM   .5°

1 2 3 4 5

$Q_{10}Q_{11}$ B_{10} B_{11} B_{12} B_{13}

Q_{12} SL_{10}

SL_{11}

C_{12} 12°   12°

Q_{14}

Q_{13}

A BEAM

C BEAM   12°

B BEAM

BSY

$C_0 C_1$ PRIMARY ACCELERATOR SLITS (9mm x 9mm TYP)
PM PULSE MAGNETS — .5° BEND TO A-LINE
$Q_{10}Q_{11}$ FIRST QUAD PAIR
$B_{10}-B_{13}$ FIRST 12° — BEAM DISPERSED IN MOMENTUM .09%/cm
Q_{12} SYMMETRY QUAD — MAKES ACHROMATIC SPOT AT PIVOT
SL_{10} HIGH POWER MOMENTUM SELECTION SLIT
SL_{11} SHARP EDGE LOW POWER COPPER SLIT
$B_{14}-B_{17}$ SECOND 12° BEND
C_{12} LOW POWER HALO SLIT (COPPER)
VSTR VERNIER STEERING MAGNETS
RS1,2,3 ZnS ROLLER SCREENS FOR BEAM POSITIONING
TOR φ,TOR I TOROID BEAM FLUX MONITORS

FIG. II-1

The beam transport system from the
accelerator to End Station-A.

This arrangement disperses the beam in momentum at SL10. The experiment was run with SL10 set to a width corresponding to $\pm 0.38\%$ energy resolution. The focusing properties of A-line produced an image of the slit CO at the pivot, where the beam is again achromatic (x - y position within the beam spot is not correlated with energy) due to the effect of Q12 just in front of SL10. The energy of the beam is set by measuring the magnetic field with a "flip coil" in a 9th identical bending magnet connected in series with the 8 in A-line. The absolute energy calibration for the center of SL10 is $\pm 0.1\%$ (Ref. II-3).

The uncertainty in the size of the cross sections at 60 degrees due to the energy spread in the incident beam is usually less than $\frac{1}{4}$ that of the statistical errors of the recorded data. The incident energy spread washes out missing mass structure only on the tens of MeV level. A shift in E_0 of 0.38% typically corresponds to a change in the ω' variable of 0.3% and results in a possible cross section change of at most 0.8%. The resulting shift in the missing mass for a 0.38% change in E_0 is small and quite insensitive to the value of W (for $E_0 = 16$ GeV, E' = 1.38 GeV, and W = 2.5 GeV gives $\Delta W = 6.1$ MeV). A transmission scan at the end of the full data taking period showed that for a typical running condition (19.5 GeV, 50 mA) more than 80% of the beam lay inside a $\pm .2\%$ slit opening instead of the $\pm 0.38\%$ for the full slit opening used in this experiment. The cross section measurements are more sensitive to an error in E',and this is dominated

by the hodoscope slat widths (E' was resolved with the hodoscopes to about ¼%).

The fine steering adjustments of the beam were done using four steering magnets, controlling both the pitch and the position of the beam at the pivot. Two zinc sulfide screens located 10.8' and 42.3' upstream of the target were used for this alignment. For a positioning uncertainty of 1 mm on the screens, the resulting angle misalignment would be .1 mrad (0.004 deg.). These screens were normally removed during data runs to reduce beam halo at the target. Sixteen feet downstream of the target a third screen (RS3) was left in the beam during data taking to provide continuous beam position and spot shape information. The waist of the beam was adjusted to occur near RS2. The three zinc sulfide screens were remotely monitored in the counting house by closed circuit T.V. (see Fig. II-2). On the target assembly itself an aluminum oxide screen was precisely positioned with respect to the targets. Checking the beam alignment relative to this screen after steering with the other screens assured us that the target positioning was correct.

The average beam spot size was measured using two independent techniques besides visual inspection of the roller screens. The critical parameters were the horizontal spot size and position, since the liquid target cells were long and narrow (17.7 cm x 1.9 cm). Measuring the local beam intensity by observing the secondary electron emission from a wire mounted on the target assembly showed the spot to be approximately Gaussian in intensity with a horizontal

27

Photographs of closed circuit T.V. monitors for the three zinc sulfide screen RS1, RS2 and RS3.

RS1   RS2

RS3
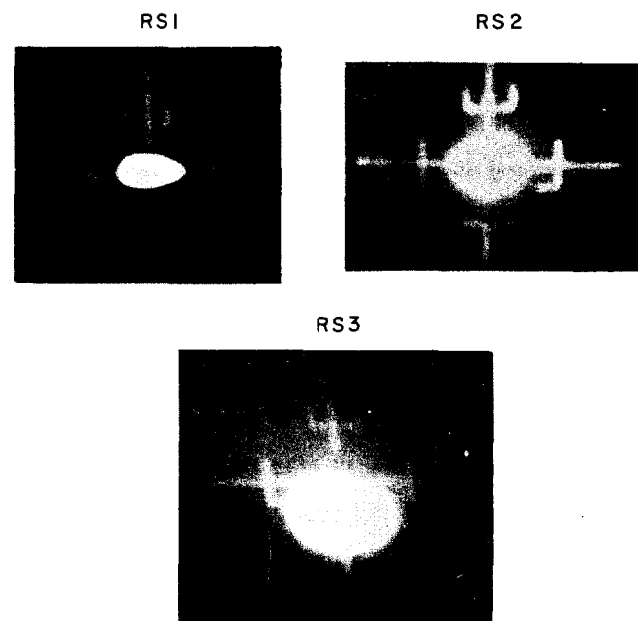
Tick marks are 1 cm spacing. Visual spots are much larger than actual spots as the ZnS screens quickly saturate.   2695A3

FIG. II-2

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sigma of 1.0 mm (this confirmed observations made of the beam burn spots

on the screens). After the experiment autoradiographs of the entry

and the exit beam burn spots on the target cells were made by

holding X-ray film against the ends of the cells. Measuring the

resulting exposures showed optical densities approximately Gaus-

sian in both horizontal and vertical directions. The measured

sigmas were: hydrogen - 1.35 mm (Hor.) x 0.99 mm (vert.); deuterium

1.36 mm (hor.) x 1.09 mm (vert.). The autoradiographs were made

approximately one week after the second data run. The principal

isotope contributing to the residual radioactivity is V^{48} (half

life of 23 days). Thus, these measurements represent a long term

average spot size including broadening due to any steering errors.

The spot positions were checked by measuring the center of the

autoradiographs, and no significant deviation ($\pm$.2 mm) from the

center of the ends of the targets was found.

The beam spill was monitored in the End Station using two

lucite Cerenkov counters (1" (dia.) x 0.5" lucite cylinders glued

to 56AVP photomultiplier tubes). One was placed downstream of the

pivot and called "good spill" (see Fig. II-1 and Fig. II-3). This

counter was exposed to the high flux of secondary particles pro-

duced by the beam striking the target. Any irregularity in

intensity or spill width was easily detectable on an oscilloscope

trace of this signal. The typical spill width was measured to be

1.5 μsec using this counter. The second spill monitor, called

"bad spill," was placed in the entry alcove to the End Station

Photograph of oscilloscope traces of the "good" spill counter (lower trace) and the "bad" spill counter (upper trace).
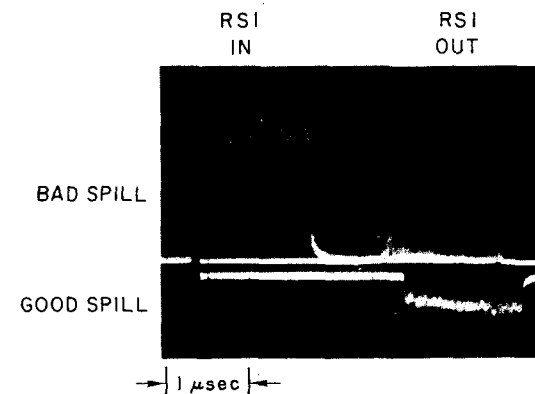
Bad spill trace is inverted. Putting RSI in the beam gave a reference signal to calibrate the bad spill.   2695A2

FIG. II-3

(see Fig. II-1) just upstream of a shield which provided about 16" of iron to attenuate the room noise from "downstream." This counter detected beam halo striking beam line elements near the counter. The counter was located a few feet downstream of RS1,and putting RS1 into the beam resulted in a very large signal from the bad spill counter (see Fig. II-3) which was used as a reference signal. In tuning the beam, we tried to minimize the bad spill signal with RS1 removed. We found this counter to be sensitive to many beam faults, some of which were: focusing in the accelerator, scraping beam on CO in front of the BSY, mis-steering in the pulsed magnets in the switch yard, any collimator scraping in the A-line, and a poor vacuum in the A-line. "Gulches" in the good spill usually corresponded to spikes in the bad spill, presumably due to beam loss on SL10 or some other collimator. It became clear early in the experiment that a well tuned beam produced a small bad spill. The bad spill was used as a primary indication of beam quality.

For the greater part of the experiment, the beam currents were run at the maximum current the accelerator could produce. In the upper half of its energy range currents over the beam spill in excess of 50 mA were usually achieved, sometimes reaching as high as 60 mA. The beam current was adjusted to keep particle counting rates in the two spectrometers at an acceptable level and was not allowed to exceed 2 per pulse for electron running (4 per pulse for positron running) in the 1.6-GeV spectrometer.

BEAM MONITORS

The incident flux of electrons for the experiment was measured by two independent, identical precision toroid charge monitors, TORO and TOR1 (see Fig. II-1 for position and Ref. II-4 for a detailed description). The detection element of the toroid system is a ferrite core toroid located upstream of the pivot. The beam passes through the toroid forming an equivalent one turn primary winding. A secondary winding is directly connected across a capacitor. The resulting LC circuit has a resonance frequency of 5.25 kHz which is slow compared with the beam pulse length. A beam pulse shock excites this LC circuit. After a delay time selected to minimize spill width effects and noise effects, the amplified wave form is sampled, digitized, and accumulated on a pulse to pulse basis in a bi-directional scaler.

The toroid monitors were frequently calibrated by "dumping" a precision capacitor charged to a precisely determined voltage through a one turn winding on the ferrite core to simulate a beam pulse. The calibration pulse shape resembles the beam pulse shape,and differences can be somewhat compensated for by appropriately adjusting the sampling delay time. Agreement, after gain shift compensation between the two toroids, was better than 0.5%, and both agreed with the total charge put through the one turn winding to the same accuracy. Although this technique is not a convincing absolute calibration, it does provide an excellent

means of monitoring drifts and malfunctions of these devices over

a long experiment.

An absolute flux calibration of the toroids was done approx-

imately once per incident beam energy change using a Faraday Cup

located about 80' downstream of the pivot. The total charge

incident on the Faraday Cup was accumulated by an integrating

digital voltmeter. The Faraday Cup has an average power limitation

of about 1 kW so the tests were made at a repetition rate of a few

per second. All gain ranges on the toroids used in E-89 were

compared to a Faraday Cup measurement. These calibrations agreed

with both toroids to $\pm 0.5\%$, and the Faraday Cup itself is believed

to have an absolute accuracy of $\pm 0.2\%$ (Ref. II-5). Beam properties

are summarized in Table II-1.

THE TARGET

To withstand the high intensity beams used in E-89, the liquid

targets for the experiment were fan driven recirculating liquid

targets developed for high power dissipation (Ref. II-6). The target

cells were the only new feature of the target assembly compared

with previous experiments. A picture of the target is shown in

Fig. II-4 and a schematic in Fig. II-5.

The long, narrow shape of the cells evolved from two consider-

ations. To maximize the rate the targets were made as long as the

1.6 could "see" with its full θ acceptance (7.00" = 17.8 cm) at

60 degrees. To minimize the production of electrons by secondary

particles, the amount of radiator (material) between the beam and

TABLE II - 1

SUMMARY OF BEAM PARAMETERS AND ERRORS

QUANTITY	VALUE	CAL. & MEAS.	RANDOM ERROR	SYSTEMATIC ERROR
E_o	1.5-19.5 GeV	A-Bend,Flip Coil, A-Bend shunt volt-age.	$\pm.38\%$ SL10 Setting	$\pm.1\%$ Absolute Cal. of A-Bend.
Spot Size(σ) and Shape	1.36mm (hor.)x 1.00mm (vert.)	Zinc-sulfide screens,SEM scan, Auto Radiographs	$\pm.5$mm(vari-ation in optics over run).	
Spill	1.5 usec (typ.) flat top	Oscilloscope trace of spill counters	$\pm.1$ usec pulse to pulse vari-ation.	
Current	1mA-60mA	Toroids,Faraday Cup	$\pm.5\%$ Fluctu-ation in Tor-oid cal.	$\pm.6\%$ Faraday Cup, Toroid cal.
Steer-ing and Beam Align-ment		Zinc-sulfide screens,SEM,Auto Radiographs,Al_2O_3 Screen on Target Assembly.	$\pm.004^{\circ}$	$\pm.002^{\circ}$ Survey Error $\pm.5$mm.
Halo	$\leqslant.5\%$ contrib. to cross section.	$\frac{1}{4}$"Fe Halo Hole Target, Bad Spill Monitor.	$\pm.2\%$	$\pm.2\%$

FIG. II-4

Photograph of target assembly

Schematic of the target showing the
circulation path and driving fan.

DIRECTION OF
LIQUID FLOW

HYDROGEN
RESERVOIR

HEAT EXCHANGER

CIRCULATION FAN

PUMP
FLOW

UPPER VAPOR
BULB

— BEAM —

LIQUID TARGET
CELLS

LOWER VAPOR
BULB

EMPTY TARGET
CELLS

HOLE TARGETS,
SOLID TARGETS

2695C18
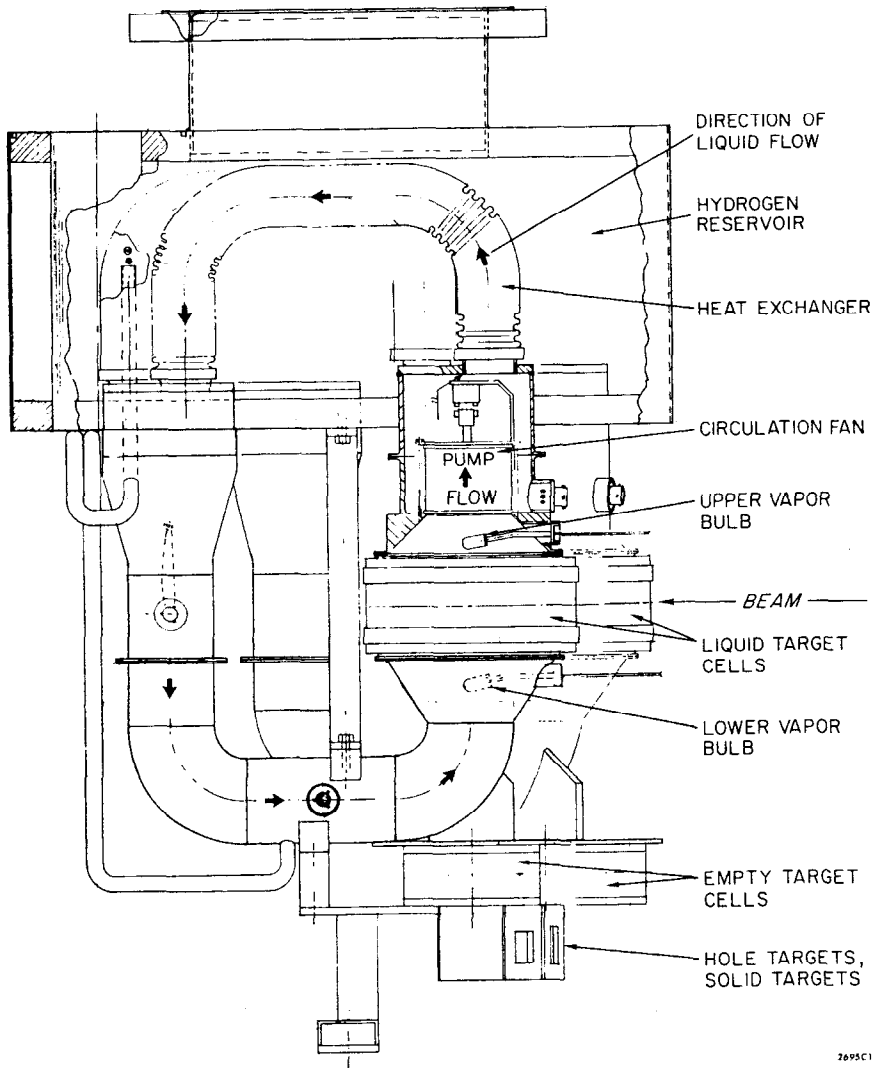
FIG. II-5

the 1.6-GeV spectrometer was made as small as possible. The target

cells were about 2 cm wide made from .0017" (.0043 cm) stainless

steel foil. The only other material between the target and the

1.6 magnet was a thin H-film (.002" plastic film) window to separate

the vacuum tank of the spectrometer from the high vacuum of the

scattering chamber in which the cells were located.

A summary of the material in the path of the electrons is

presented in Table II-2. It is parameterized in terms of the

radiation lengths (χ_o) before (t_b) and after (t_a) the center of the

target. The t_b includes ½ the radiation length of the target cell

and an anticondensation foil The t_a includes the average length of the liquid

between the beam line and the spectrometer, the cell wall, and the

H-film. This amount of material (t_a) produces about 1/3 of the total

positron signal arising from π^0 production with subsequent

conversion of the decay gammas. Neutral pion decay and other charge

symmetric processes contribute equally to e^- and e^+ rates. For

π^0's the yield is proportional to $7/9\ t_a + 1/86$ (the 1/86 comes

from Dalitz decay, $\pi^0 \rightarrow \gamma\ e^+\ e^-$).

The liquid temperatures were monitored by hydrogen vapor

bulbs placed in the liquid flow above and below each of the two

liquid target cells. Measuring the vapor pressure in these bulbs

with pressure transducers provided target temperature information.

The voltage reading from the transducer was converted to (psia)

using calibration data and then to $P(atm) = P(psia)x\ .0680457$.

TABLE II-2

SUMMARY OF RADIATORS FOR 1.6 GeV SPECTROMETER

Quantity	$t_b(X_o)$	$t_a(X_o)$	
		$50°$	$60°$
Anti-Condensation Coil	.0004		
Target Walls	.002557 (.0017" F_e)	.00328	.00295
Liquid Radiators LH_2	.00997 $\}(\frac{17.68cm}{2})$	.00166 $\}$ 1.47cm	.00147 $\}$ 1.30cm
LD_2	.01157	.00193	.00170
H-Film Vac. Isolation		.00017 (.002 Mylar)	.00017
Totals LH_2	.01293	.00511	.00459
LD_2	.01453	.00538	.00482

Summary of radiators for the 1.6 GeV
spectrometer for before and after the
center of the target.

The relation

$$\log_{10} (P(atm)) = 2.00062 - 50.09708/(T(°K) + 1.0044) + .01748495 \times T(°K)$$

taken from Ref. II-7 can be inverted to yield the target temperature. The hydrogen density was obtained using polynomial fits (Ref. II-8) to publish the hydrogen and deuterium densities (Ref. II-9). The fits used were:

$$\rho_H = .07714 + .000443 \, T - .00003866 \, T^2$$

$$\rho_D = (.1853 + .0006677 \, T - .000068727^2) \frac{2}{2.015}$$

The densities ρ_i are in atomic weights/cc, and the temperature T is in degrees Kelvin.

The target densities were calculated for every run for both the upper and the lower vapor bulb measurements. The densities were found to slightly depend on the power input into the target from the beam. The power input was parameterized as repetition rate (in pulses per second) times beam current (in mA). Scatter plots for all the data runs versus this power parameter are displayed in Fig. II-6. These densities were used to calculate the number of nuclei per cm^2 for each run.

In Table II-3, the density reduction for the average of all runs is given. The sources of systematic error are pressure transducer calibration (for the vapor bulb pressure), formula error (for the temperature-pressure relation), and the quoted accuracy

FIG. II-6

Scatter plots of hydrogen and deuterium liquid densities
versus beam current.

TABLE II - 3

A SUMMARY OF THE LIQUID TARGET PROPERTIES
(AVERAGED OVER ALL RUNS)

Quantity	LH_2		LD_2	
	Value	Error*	Value	Error*
Vapor Pressure (psia)				
Upper	16.583		16.774	
Lower	16.317		16.731	
Temperature (°K)				
Upper	20.08	±.1% (S)	20.72	±.1% (S)
Lower	20.63		20.72	
Density ($\frac{Atm.Wt.}{cc}$)				
Upper	.06973	±.3% (R)	.1684	±.3% (R)
Lower	.06982	±.3% (S)	.1684	±.6% (S)
Contamination	–		.3%(H)	±.1% (S)
Target Length (cm)	17.674	±.5% (S)**	17.631	±.5% (S)**
No. of Nuclei per cm^2	7.431×10^{23}		8.942×10^{23}	
Total Error	.6% (S)			.9% (S)
	.3% (R)			.3% (R)

Density reduction from vapor bulb pressure for the average density.

* For the errors (S) denotes Systematic Error and (R) denotes Random Error.

** Note on systematic error on target length – this ½% is mostly a guessed possible 1/32" in the optical determination of the overall length. Since the H and D cells were measured in the same way at the same time, most of the ½% is common to both and wouldn't propagate into N/P ratios very much.

of temperature versus density measurements. The random error is taken to be $\pm\frac{1}{2}$ of the full width of the typical spread of the data.

The density used in calculating cross sections was that calculated from the lower vapor bulb measurement. This should be the density of the hydrogen just before the 1.5 μsec beam pulse strikes the target. The circulation rate is supposed to move the liquid fast enough so that the beam strikes a fresh portion of liquid on each pulse, even at 360 pps. Density changes during the beam pulse are neglected.

The two liquid cells were positioned directly underneath the liquid hydrogen reservoir used to cool the recirculation loops. Beneath them were various dummy targets. Measurements of electrons and positrons off these were used to correct the full target yield for scattering from the cell walls. The principal dummy target used in the experiment was approximately 5.5 times thicker than the actual target cell walls and was constructed to have the same radiators (t_a and t_b) as the liquid cells. The number of nucleons for the thick dummy was roughly the same as the full cells, thus the counting rates were comparable.

An additional solid angle factor has to be applied to the dummy subtraction since the spectrometer didn't have full acceptance for the ends of the targets (the thick dummy was distributed in thin foils along the 17.7 cm length). This factor was calculated by Monte Carlo techniques using a model

of the spectrometer and checked experimentally by measuring the ratio of a "long" dummy (2 – .010" steel foils, 17.7 cm apart) to a "short" dummy (2 – .010" steel foils, 2.0 cm apart). The solid angle reduction factor was .78 for 60 deg. and .92 for 50 deg. The measurements agreed with these calculated ratios to within 5% and well within statistical errors.

Below the dummy targets were an $A\ell_2O_3$ screen, the SEM wire (see Chp. II, The Beam), a ¼" steel hole target (for measuring effects of beam halo), and some solid targets. The whole target assembly could be moved up and down and rotated by remote control from the counting house allowing accurate positioning ($\pm$0.1 mm (vert.) x $\pm$0.1 mm (hor.)) of any of the targets in the beam. Usually the targets were driven into position automatically under computer control.

THE 1.6 SPECTROMETER

The 1.6-GeV spectrometer is shown in Fig. II-7. A single 90 degree bend magnet transports particles scattered into its acceptance to the "counter cave." The focusing properties employ wedge focusing giving point to point imaging in the vertical plane for particles of the same momentum. This focusing effect is achieved by the different path lengths through the dipole magnetic field for different ϕ angles ($\phi = y/z =$ "vertical projected angle"). The slanted pole faces also produce magnetic fringe fields that have a non-zero y component. This produces

SLAC 1.6 GeV/c SPECTROMETER
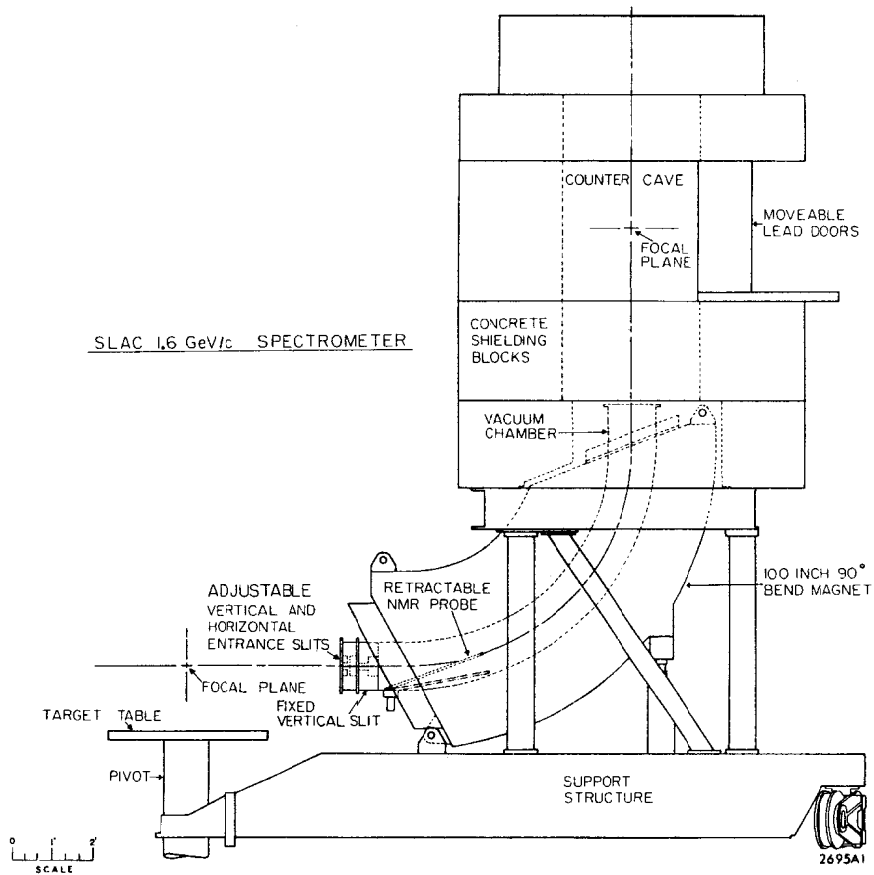
FIG. II-7

1.6 GeV Spectrometer Schematic

quadrupole focusing in the horizontal plane. The field is such that particles are first focused at the entrance then defocused at the exit. The net result is line to point focusing in the horizontal plane. This takes particles from the line source target scattered at the same angle and focuses them to a "point" at the focus in the counter cave.

The two focusing properties are obviously coupled. To make the two focal planes coincident in the cave, sextupole corrections are used. This is introduced by curved pole faces in three regions in the magnet called "beta lenses." The original idea was to be able to use a single rotatable hodoscope at the p-focus to measure missing mass spectra, but we used fixed p and θ hodoscopes in E-89.

The momentum of the 1.6 was set by measuring the magnetic field near the center of the aperture just past the first beta lens region. A nuclear magnetic resonance (NMR) probe was raised into position by a remotely operated boom to make the field measurement. The NMR would be tuned to the frequency corresponding to the desired magnetic field (and thus the momentum). Then the power supply for the magnet was adjusted to find the signal. This procedure was easily accomplished as the 1.6 followed a known current-field relation very well. The NMR frequency was recorded along with the magnet current, magnet temperature, and the voltage measured from the two Hall probes located inside the magnet at the beginning and end of every run.

An aperture defining slit was located at the entrance of the magnet. This fixed slit was accurately constructed from lead with tungsten jaws. The top and bottom tungsten pieces were angled toward the target and spaced from the center line so that rays outside of +40 mrad and -50 mrad were completely blocked. Two tungsten pieces on the sides defined the θ-target length limits and completed the frame.

Particles traversed a large aluminum vacuum tank (7 15/16" x 21 15/32") which passes through the magnet to the counter cave. The tank limits parts of the acceptance for some rays. The large θ trajectories from the ends of the targets are intercepted by the top flange of the tank. For large p and ϕ near the top of the tank the orbits grazed the aluminum. Two tungsten baffles were placed inside the tank to obstruct such rays. The limiting apertures were accurately measured during a wire float measurement of the optics of the 1.6 (see Appendix D).

The 1.6-GeV spectrometer counter cave is formed by a stack of three concrete doughnuts above the magnet. A 5' x 11' cylindrical cavity results. Two large lead filled doors located halfway up the wall away from the target provide the principal access. The whole structure is capped with a 3'thick circular concrete lid, which can be removed to load counters into the cave using the overhead crane in End Station A.

In the first checkout run, much experimenting was done to reduce background in the counter cave by adding shielding outside, particularly between the cave and the target. The extra shielding blocks reduced the singles rates at the p-focus by a factor of 2.5. Most of the remaining counting rate was coming up the vacuum tank as verified by closing a pair of movable slits located directly in front of the fixed slit. The singles rates were most severe near the magnet exit at the bottom of the cave presumably due to soft spray from particles striking the aluminum vacuum pipe. More extensive baffling might have improved this situation.

In 1970, the 1.6 suffered an unfortunate accident. The magnet was run without water cooling. Much of the epoxy insulation melted off the coil. Subsequently, in 1971, two of the 144 turns of the coil shorted. To compensate, two symmetric turns on the other side of the aperture were shorted. Measurements of the elastic proton peak showed no detectable difference from the measurements using the original coil when the remaining turns were excited to achieve the same field.

In view of this history, we decided to measure the properties of the magnet using the floating wire technique at the completion of the experiment to independently measure the optics coefficients. These results are given in Appendix D. No large differences were found from the original optical properties published in Ref. II-2.

COUNTER PACKAGE

The counters used to detect scattered electrons are shown in Fig. II-8. They were physically separated into two parts: a TA package that included all counters down to the θ hodoscope and the gas Cerenkov counter and the Y hodoscope. The TA package was hung from a rotatable mount secured at the top of the counter cave while the CT and Y hodoscope rested on the floor of the cave. Rotation of the TA package was necessary to service the equipment. Gross counter properties are summarized in Table II-4.

The counters were aligned by using the laser system designed by SLAC Group-F. On each side of the magnet an optical laser imaged a cross hair through a lens system, pointing toward the ceiling of the End Station. The cross hairs and lenses were set so that a line connecting them split the center of the exit aperture of the magnet in the direction of the p dispersion. The point half way between the cross hairs located the center of the aperture in the θ dispersion direction. Both optical systems were supported 1" off the iron return yoke of the magnet and were affixed to it. The two bright cross images described two vertical reference lines. Lucite screens attached to the various counters and TA package linked the co-ordinate systems of the spectrometer to that of the counters.

Both the p and θ hodoscopes had coarse hodoscopes behind

Counter array used to detect electrons in the 1.6 counter cave. The Pb glass hosodscope, pre-radiator, TA1 and TA2 formed the total absorption counter.

FIG.II-8a

FIG. II-8b

50
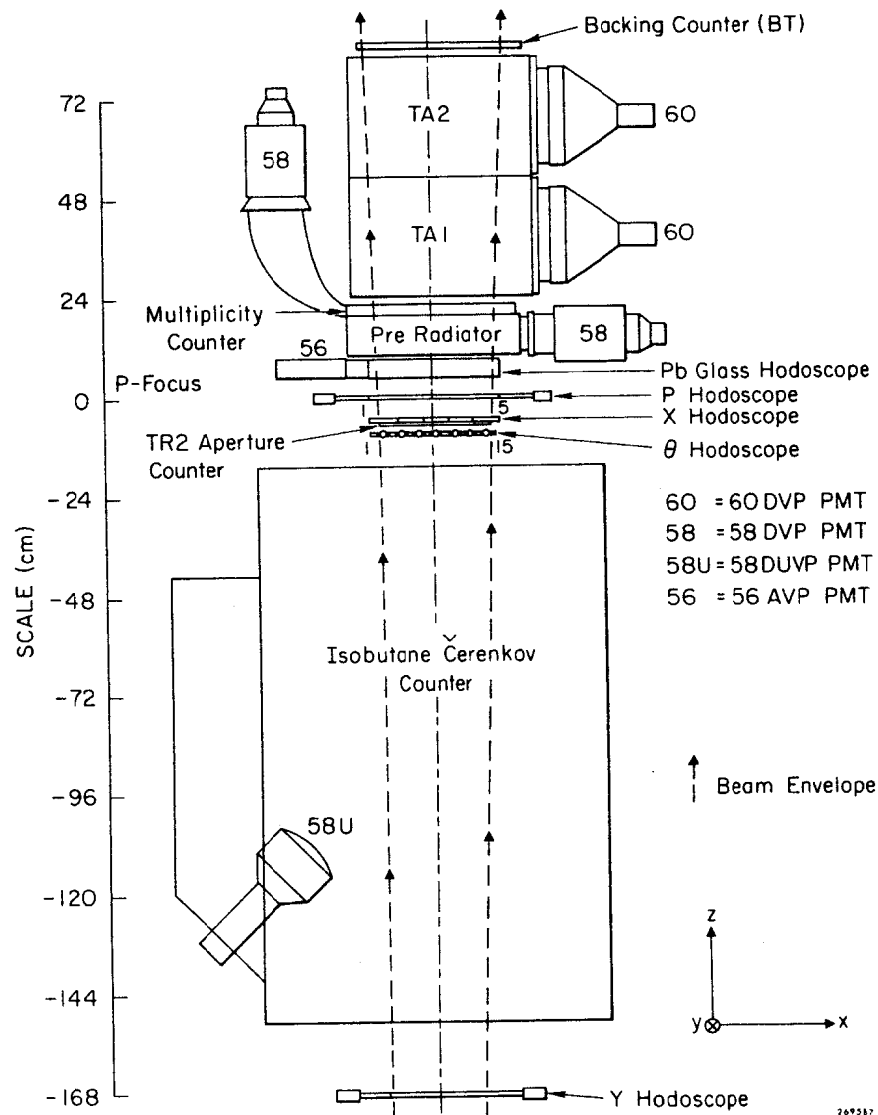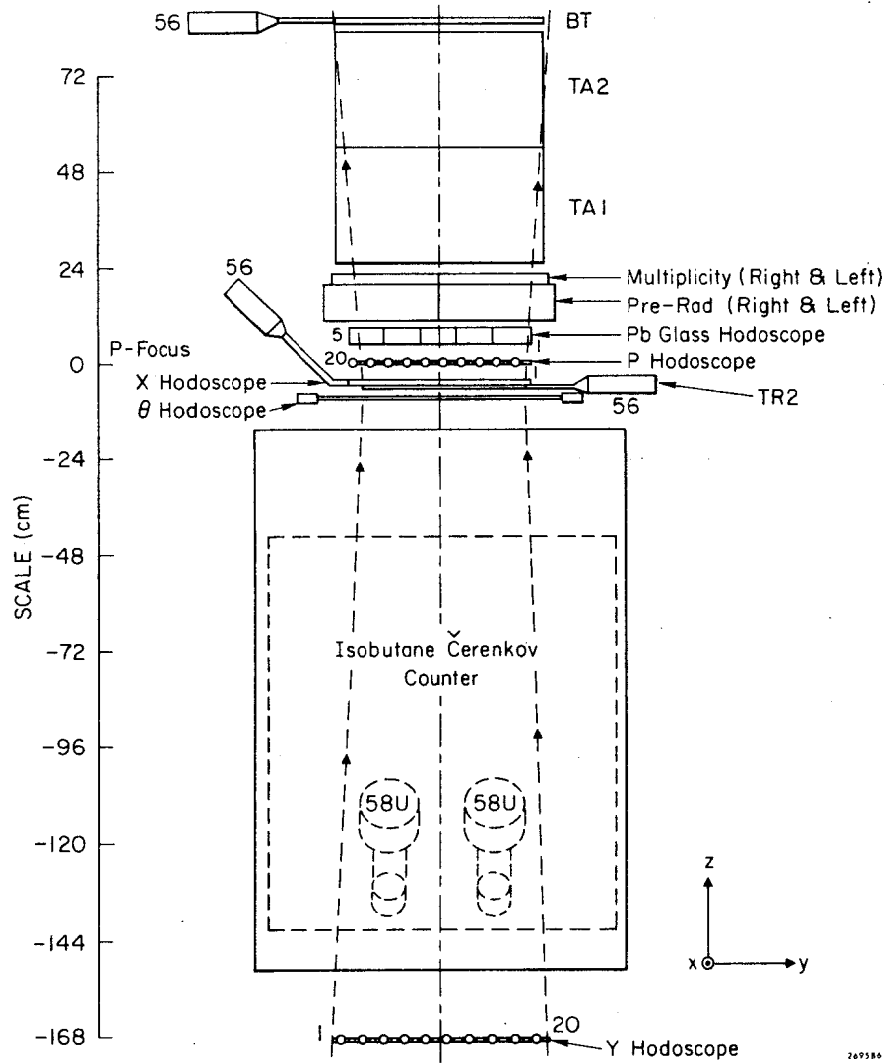
TABLE II-4

1.6 Counters

Name	X-Limits (cm)	Y-Limits (cm)	Material	gr/cm^2	$\frac{X}{X_0}$ (Rad.Len.)	Distance of Nearest Surface to Focus (cm)	Comment
Y-Hodo	±11.43	±27.19	.32cm Scint.	.33	.0074	-168.9	2.72cm Bin Width
Cerenkov	±41.9	±45.7	134.cm ISOB 2/3 cm Lucite. Two .002" Al windows.	.34 / .76 / .03	.008 / .018 / .001	- 22.8	Effective Cerenkov path length (window to mirror) 124 cm.
θ-Hodo	±15.98	±30.54	.64cm Scint.	.66	.015	- 10.3	2.13cm Bin Width
TR2	±13.89	±20.73	.64cm Scint.	.66	.015	- 3.8	
X-Hodo	±16.03	+22.00 -24.05	1.27cm Scint.	1.32	.030	- 2.5	6.39cm Bin Width
P-Hodo	±23.08	±22.90	.64cm Scint.	.66	.015	± .32	2.30cm Bin Width
G-Hodo	±15.98	±23.09	4.0cm Pb Glass	14.4	1.25	6.0	9.20cm Bin Width
Pre-Rad	±20.95	±29.00	10cm Pb Glass	36.1	3.13	10.60	Two blocks-each 29cm in Y direction.
Multiplicity	+20.00 -21.00	±27.00	2.54 cm Scint.	2.64	.060	21.3	Two pieces(LF & RT) each 27cm in Y direction.
TA1	+24.04 -20.04	±26.04	29.2cm Pb Glass	104.7	9.13	25.7	
TA2	+24.07 -20.07	±26.07	29.0cm Pb Glass	105.4	9.06	55.0	
BT	+21.27 -18.73	±26.00	1.27cm Scint.	1.32	.030	89.5	

Physical dimensions of 1.6 counters and positions within the counter cave.

them. The Pb glass hodoscope (G-HODO) was matched to the p

hodoscope with a 4:1 bin width ratio. The X-hodoscope (X-HODO,

¼" plastic scintillator) gave 3:1 binning of the θ hodoscope.

It was found that approximately 75% of the "multiple slat"

events could be resolved using the coarse bin information

(see Chap. III - Tracking and Codes).

The TR2 scintillator defined the counter package aperture.

It covered p bins 2-19 and θ bins 2-14. A signal from TR2

defined the event to be within the aperture used in calculating

the cross sections. The stacking order of the hodoscopes and

TR2 was done to minimize track reconstruction problems caused

by knock-on electrons. The p hodoscope was located at the

p-focus. TR2 was placed in front of the X-HODO to absorb

knock-on electrons produced in either the θ hodoscope or in

the Cerenkov counter's mirror.

The Cerenkov counter and the TA package made up the elec-

tron detection system. The CT had an average flight path for

particles of about 1.24 meters. The gas filling was isobutane

at atmospheric pressure. A mirror made from ¼" lucite, slump

molded and coated with approximately 2500 Å of aluminum and

overcoated with 200 Å MgF_2 imaged the Cerenkov light onto two

quartz window 5" photomultipliers. An electron traversing this

counter produced an average of 26 photoelectrons. In Fig. II-9

the distribution of the sum of the pulse heights from the two
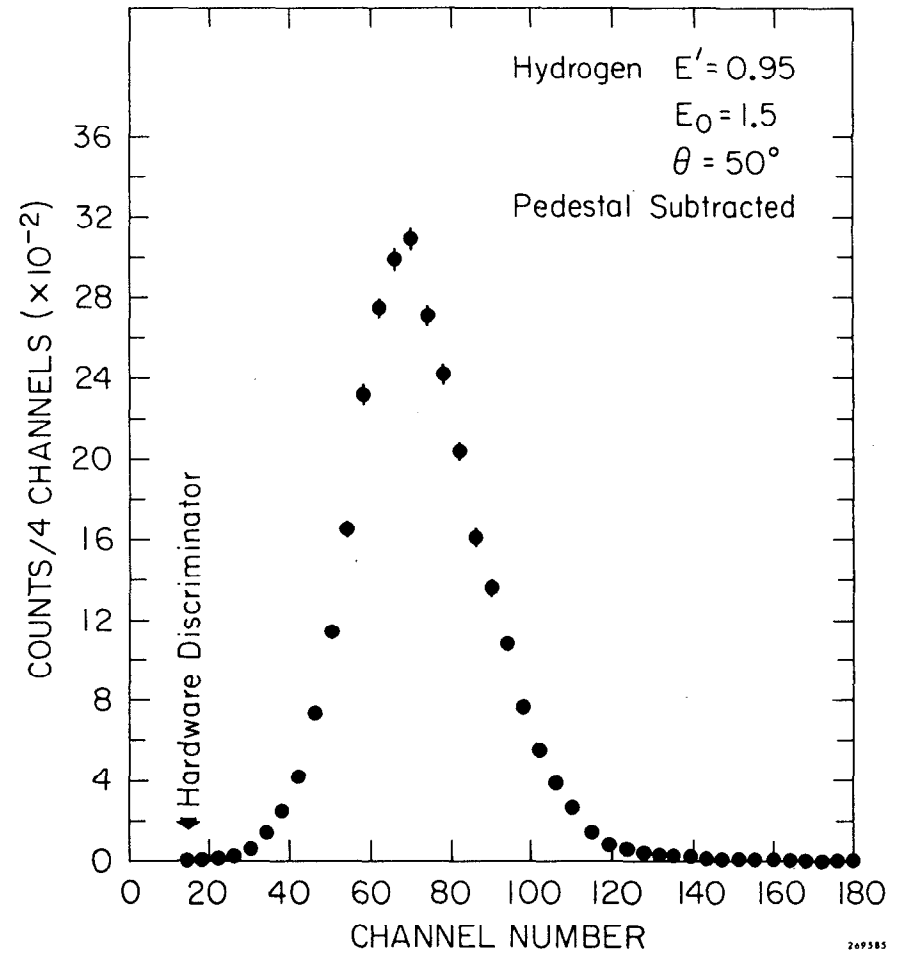
FIG. II-9

Pulse height distribution of the sum of the
two photomultiplier signals from the Cerenkov
counter. The kinematics correspond to elastic
scattering off hydrogen.

phototubes is shown (no position correction has been made). The kine-
matics were set for elastic scattering to ensure we were observing
electrons. The position of the hardware discriminator cut is indi-
cated by the arrow. This cut resulted in about 250:1 π/e rejection.

The TA package had 5 elements along the beam path (see
Table II-4). The shower material was a low density lead glass
(see Appendix B). This glass had good transmission far into the
blue end of the spectrum. The most important elements for the
total absorption signal were the PRERAD (3 rad.lengths) and TA1
(9 rad.lengths) as a 1.3-GeV showers deposited about half its
energy in each counter. Each element by itself produced a
rather broad pulse height distribution, but the correlation of
the signals produced by electrons was quite sharp (see Chap. III).
A crude hardware TA signal, SUPS, was formed using passive
adders and attenuators (see Fig. II-10). The arrow in Fig. II-10
indicates the position of the hardware discriminator threshold
just above the minimum ionizing peak which yields an approximate
3:1 π/e separation. This signal in coincidence with the CT
signal was used as the main trigger interrupt in the
experiment.

ELECTRONICS

Signals from the phototubes in the counter cave were
transmitted to the counting house on RG-58 co-axial cable.
Inverted dynode signals from the counters went directly to
8 bit pulse height analyzers. The anode signals were fed to
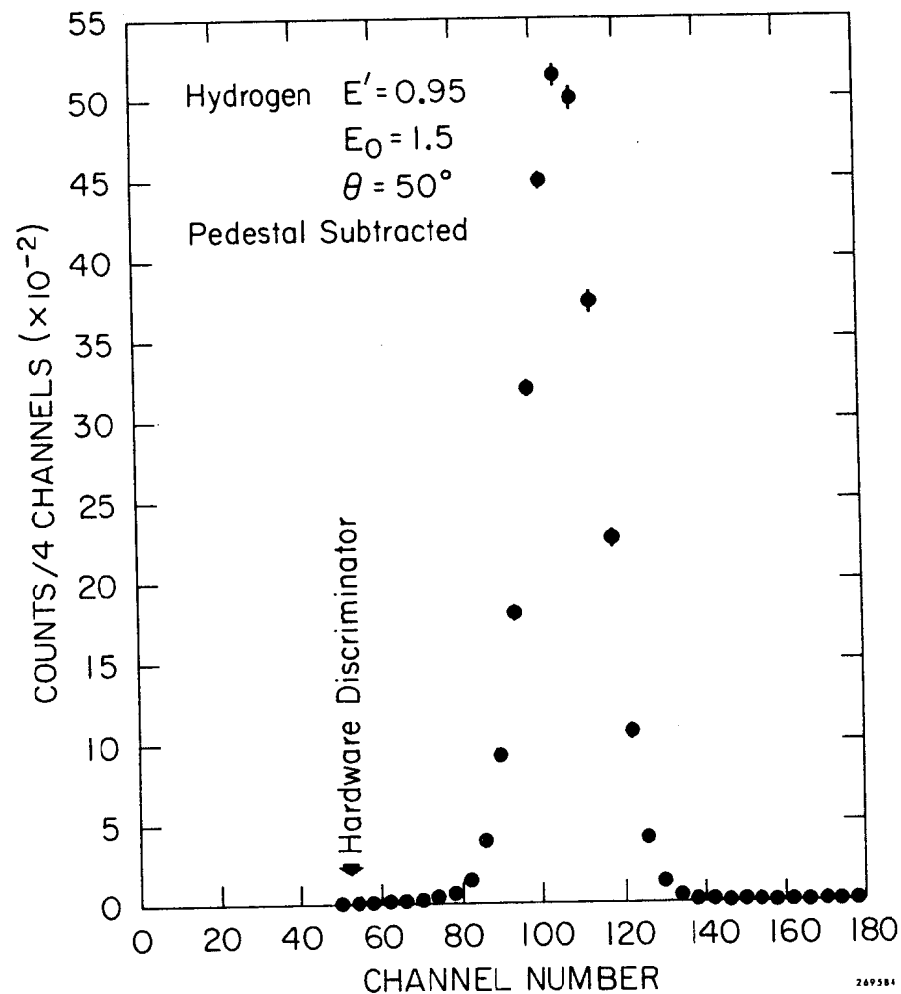
FIG. II-10

Pulse height distribution of hardware sum of
signals from TA package. The kinematics
correspond to elastic scattering off hydrogen.

the fast logic circuits. Some of the anode and dynode signals were passively added in the counter cave before transmission to the Counting House.

A schematic diagram of the electronics is shown in Fig. II-11. All discriminators were run in clipped mode. Signals from elements of the TA package were passively added to form a total energy absorption signal, SUPS (see Fig. II-10). This SUPS signal and the initial shower development signal, PRERAD, in conjunction with the Cerenkov signal (CT - see Fig. II-9) formed the various triggers for the experiment.

The main trigger ,ORT, for the experiment was a coincidence between CT and SUPS (the discriminated SUPS signal was called GL) with hardware discriminator cuts as shown in Fig. II-9, 10. The electron detection efficiency was very close to 100% as determined from runs made at the elastic peak where the particles incident on the counters are predominantly electrons. The timing was set by the CT signal. Two identical trigger systems were used as the main triggers, each monitoring the electronics of the other (e.g., CT1, CT2 - GL1, GL2, etc.). During the experiment, monitoring of scaler ratios from these discriminators and coincidence units quickly revealed malfunctions such as double pulsing and dead units.

An ORTK signal was made by latching a coincidence circuit for the duration of the beam spill once an ORT trigger was received. Thus ORTK could fire at most once per pulse. This ORTK circuit was reset at the beginning of each beam pulse. By using the ratio ORT/ORTK, an over-one correction for missed events was measured. Of course, many other over-one corrections factors can be constructed using flag and scaler information. All over-one corrections were found to be consistent except when malfunctions in one of the arms of the trigger occurred.

As the CTA1 and CTA2 rates were very low for the entire experiment, little fast electronics dead time is expected. The CT discriminators usually ran about 0.3/pulse almost independent of E' and polarity while the GL discriminator rates were dependent on E' and polarity, and these rates reached as high as 0.7/pulse. Both CT2 and GL2 were monitored for dead time by several discriminators having fixed dead times out to 180 nsec. Since most of the signals in CT2 are uncorrelated to those in GL2 (demonstrated by the small coincidence rate between them) their dead times should be added together. Straight line fits to the GL2 dead time scalers verses dead time when combined with the GL2 scaler information indicated approximately 33.4 nsec dead time for this signal. This measurement was confirmed by examining oscilloscope photographs of the pulses going into the GL2 discriminator. The CT2 dead time was found to be about 10 nsec. From these two dead times a correction to the data was made which was usually about 0.2% and always less than 2.0% for electron running.
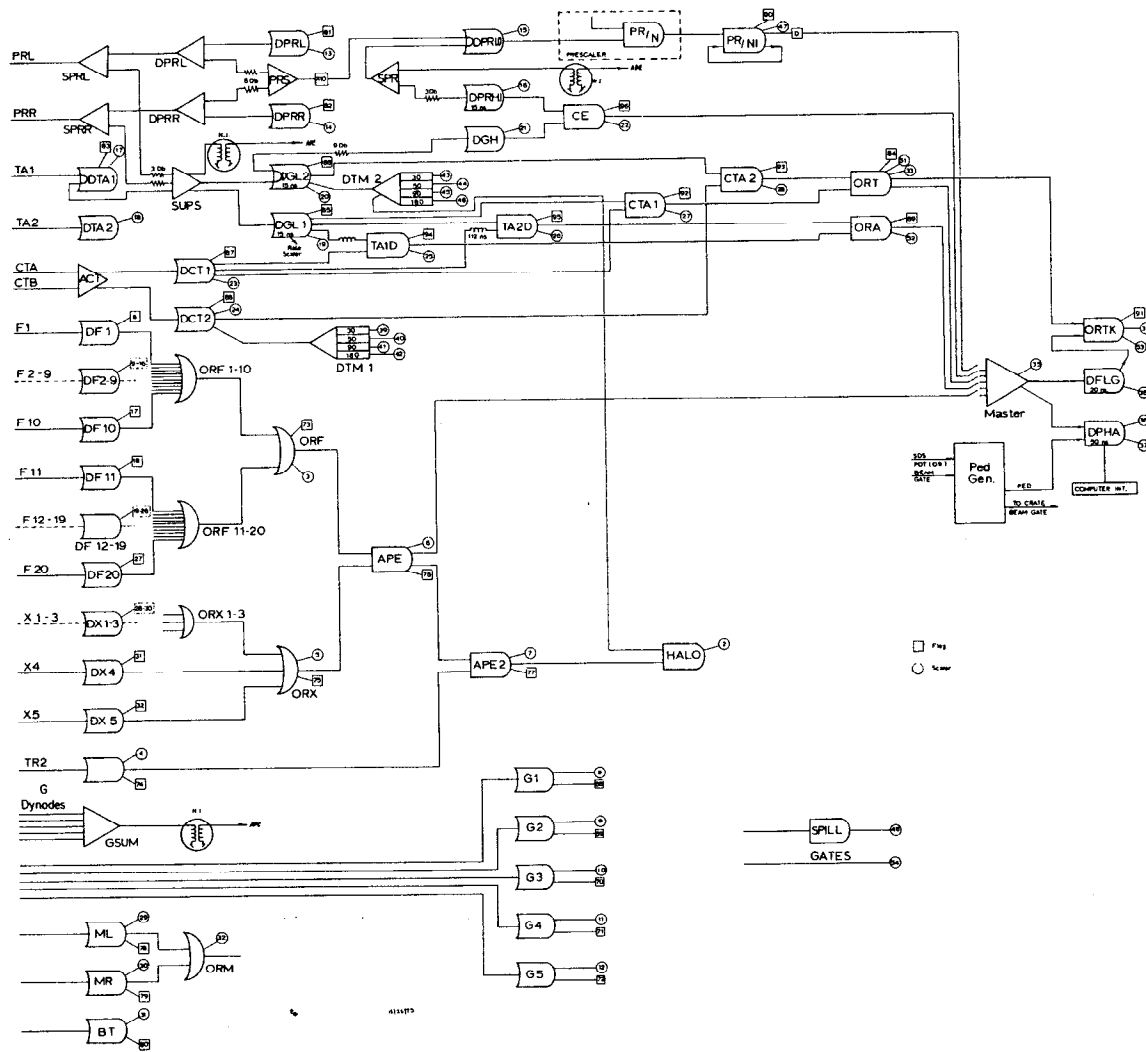
FIG.II-11

A schematic diagram of the fast elec-
tronics used for the 1.6.

In an experiment with a high background to signal ratio, measurement of chance events simulating real events is important. The accidental trigger, ORA, was made from two out-of-time coincidences, TA1D (GL1 delayed by 112 nsec with respect to CT1) and TA2D (CT1 delayed by 112 nsec with respect to GL1). The principal accidental trigger is taken to be TA2D as it strobed data into the PHA's and flag units in time with a pulse in the glass counters. As the associated CT signal is out of time by 112 nsec its PHA does not necessarily contain any information about the signal that fired its discriminator. Events gotten from TA1D were used to simulate the missing CT signals in the TA2D events.

With such an accidental scheme a correction must be made since chance coincidence can only occur 112 nsec after the beginning and before the end of each beam pulse (this correction= $((1500-224)/1500)= 1./1.176)$.

Three other triggers were also used to gather data. A CE trigger made by a coincidence between a high cut on the PRERAD and a high cut on SUPS produced data with about 80% detection efficiency for electrons, but did not require CT information. The purpose of this trigger was to monitor the CT. This data has too poor π/e discrimination to clearly monitor the Cerenkov efficiency for most of the running and is therefore useless. A "junk" trigger made by prescaling a low threshold PRERAD discriminator on beam gates (usually 1:1000) also gen-

erated data. By using this trigger in the analysis associated with the flag information, biases could be investigated in the main trigger, ORT. Finally a simple scintillator trigger made by a coincidence between ORF and ORX, called APE, was used occasionally to record pion data. This APE trigger was also used for elastic peak checkout runs so as not to bias efficiency studies of the ORT trigger.

Three LeCroy 8 bit, 8 channel ADC's digitized the 18 incoming phototube signals. Pedestal drifts were monitored every 10 minutes from the computer by pulsing the PHA gate ten times out of time with the beam pulses. Online monitoring of drifts gave quick indication of trouble (such as faulty power supplies). The gate pulse for the PHA's was set to 48 nsec by a 24 nsec clipping stub. The leading edge of the input pulses to be analyzed were set 10 nsec inside the front edge of the gate to allow for 6 nsec ADC activation time and up to 4 nsec forward time jitter in the signal.

The flag information is somewhat different from the PHA information as its time resolution was about 3 times better. The flag units were LeCroy discriminator-latch units into which the $p-\theta$ hodoscopes phototube signals were directly fed. All other signals to the latch units went through separate discriminators first. The flag gate was set to 14 nsec using a 7 nsec clipping cable. The timing was set so that the leading edge of the flag signal fell in the center of the flag gate. The front end

discriminators in these units fired off the leading edge of the

input signal and were clipped to 4 nsec internally. This, of

course, generates problems with high rates, and deadtime-less

operation of these units would have served us better.

The scalers were CAMAC "blind" scalers with 100 MHz counting

rate capacity. These were read by the computer every 10 minutes

during a "recess" which secured enough information from the

equipment to end the data run at that time. This recess concept

was used to protect against losing large amounts of data on a

long run terminated by an unexpected "crash" of some kind.

The online XDS-9300 computer wrote 7-track, 800 BPI data

tape for subsequent offline analysis. A total of 28 data tapes

were made. The computer made various pulse height histograms and

formed linear combinations of the hardware pulse heights with which to

perform an online analysis. Various counter cross sections for the

spectrometer setting as well as a missing mass spectrum for the

entire line being run were stored and updated appropriately to

help the experimenters monitor the functioning of the equipment.

In a dual experiment such as E-89 where two separate

experiments are sharing the same target and beam, generating and

maintaining a compatible run plan is a major project. The

procedure was automated by reading preprogrammed run plans into

both online computer at the start of each new incident energy

and angle. This automatic sequencing of runs made it possible to

make over 1700 runs on the 1.6-GeV spectrometer and about 1500

runs on the 20-GeV spectrometer.

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CHAPTER III

ELECTRON COUNTING AND EXPERIMENTAL CORRECTIONS

The analysis of the data proceeds in four steps referred
to as "passes." The reduction of the data in each pass involves
a loss of detailed information but results in a more compact
data set. Each pass is carefully studied to ensure that the
lost information is well understood at the time of discard.
Our objective is to retain as much information as possible at
a particular level to facilitate studies at that level, while
keeping the data set and computer analysis time within reasonable
bounds.

Briefly, the four pases are: Pass 1 launders the data and
applies some corrections to provide clean events for Pass 2 which
sorts out electrons and bins them into missing mass spectra for
Pass 3 which calculates cross sections, subtracting off the empty
target and positron contributions for Pass 4 which applies calcu-
lated corrections (radiation and Fermi motion effects) to the
data to give proton and neutron cross sections. Table III – 1 shows
the analysis scheme in more detail.

Passes 1 and 2 required the most computer time. Detailed
studies of track reconstruction and efficiencies were done in these
Passes, and the information contained in the events was processed
and used to distinguish pions from electrons. Pass 1 reduced the
data base stored on magnetic tape, and the results were stored
on magnetic disk. Event by event information was retained at
this level including 5 pulse height signals (PHS) and tracking

TABLE III - 1

ANALYSIS SCHEME

PASS	COMPUTER TIME	EVENTS	ANALYSIS SUMMARY
1	6 hours	500K	Tracking: unique P, θ, ϕ and codes assigned each event
			PHS: position corrections, combining, leaving CT,TFT,MULT,TA1 and TA2.
			Monitor: target,spectrometer, electronics.
			Output: PASS1 disk data set (event by event). Microfilm summary of each run. Paper output summarizing job.
2	1.5 hours complete. 1-4 minutes typical study run.	100K	Most detailed analysis done at this level.
			Tracking: efficiencies, combining codes into codes classes.
			PHS: cut efficiencies, combining cut criterion into PHS classes.
			Output: final run produced measured missing mass spectra including all experimental corrections. Microfilm summary of each setting. Disk file of missing mass cross sections. Paper output summarizing job.
3	1 minute	9 lines	Reduced the 6 spectra per E_o and θ to final hydrogen and deuterium cross sections, (subtract empty target and positron signals).
4	1 hour	9 lines	Radiative corrections: elastic (and quasi-elastic) radiative tails subtracted. Inelastic radiative corrections.
			Neutron extraction: proton smearing, deuteron subtraction and neutron unsmearing.
			Output: disk file with final N, P and D cross sections.

information (hodoscope binning). Once the events were on disk, many detailed studies were carried out to determine tracking efficiencies and "cut" efficiencies on the PHS's. These studies culminated in Pass 2 to produce the line by line cross sections binned in missing mass. Pass 1 was done five times as information feedback from Pass 2 affected the Pass 1 analysis. Pass 2 was repeated many times. The data searching program allowed for concatenation of any subset of all the runs defined by run number, E', θ, E_o and target type. This made for easy testing of many systematic effects at the Pass 2 level. This type of computer job usually required between 1 and 4 minutes of computer time. All of the Pass 2 sub-programs were backwards compatible with Pass 1.

The processing of the events in Pass 1 and Pass 2 will be discussed in this Chapter. Corrections, theoretical and experimental (other than electron counting), are discussed in Chapter IV.

PULSE HEIGHT ANALYSIS

The counters in the experimental setup produced 18 pulse height signals (PHS) for each event. Some of the pulse height information was complementary, that is, two phototubes viewed the same light source (as in the CT, PRERAD left, PRERAD right, TA1 and TA2). Furthermore, many of the signals were highly correlated when an electron passed through the counter stack. The first

task was to reduce the original pulse heights to a small set

of combined PHS's. The combining was done to optimize the

resolution of the system for electrons. The assumption was

made that optimizing the electron resolution would also nearly

optimize π/e separation. The PHS analysis flow chart is shown

in Fig. III-1.

All the PHA pedestals were monitored at ten minute intervals

during the experiment. This information was used to subtract the

pedestal from each PHS before further analysis. Tests on the PHA

units used assured us that they were linear to better than 2% over

an effective range of approximately 250 channels. The pedestal

usually fell in channel 5 or 6.

One can easily derive the optimization rule for linearly

combining n PHS's to minimize the resolution. Let $e_{ij} = j^{th}$

PHS of the ith event. Then for N events and n PHS's

$$S_j = \frac{1}{N} \sum_{i=1}^{N} e_{ij} \qquad \text{is the first moment of the } j^{th} \text{ PHS.}$$

$$P_{jk} = \frac{1}{N} \sum_{i=1}^{N} e_{ij} e_{ik} \qquad \text{are the second moments.}$$

We wish to find a set of C_j so events defined by

$$E_i = \sum_{j=1}^{n} C_j e_{ij} \qquad \text{have optimal resolution,}$$

R(minimum width/mean).

$$\mu = \frac{1}{N} \sum_{i=1}^{N} E_i = \sum_{j=1}^{n} C_j S_j \equiv C_j S_j \quad \text{(repeated indices are}$$

summed over)

FIG. III - 1

PULSE HEIGHT ANALYSIS

FLOW CHART

PHA NO.	NAME	PAIRS	POSITION CORRECTION	FINAL COMBINED PAS
1	CTA	CT	✓	CT
2	CTB			
3	TR2		✓	TR2
4	X-HODO		✓	X-HODO
5	PRS	not used		
6	GSUM		✓	
7	TA1A	TA1	✓	TA1
8	TA1B			
9	TA2A	TA2	✓	TA2
10	TA2B			
11	PRLA	PRL		
12	PRLB		✓	PRERAD
13	PRRA	PRR		
14	PRRB			
15	ML	MULT	✓	MULT
16	MR			
17	SUPS	not used		
18	BT		✓	BT

TFT

TASUM
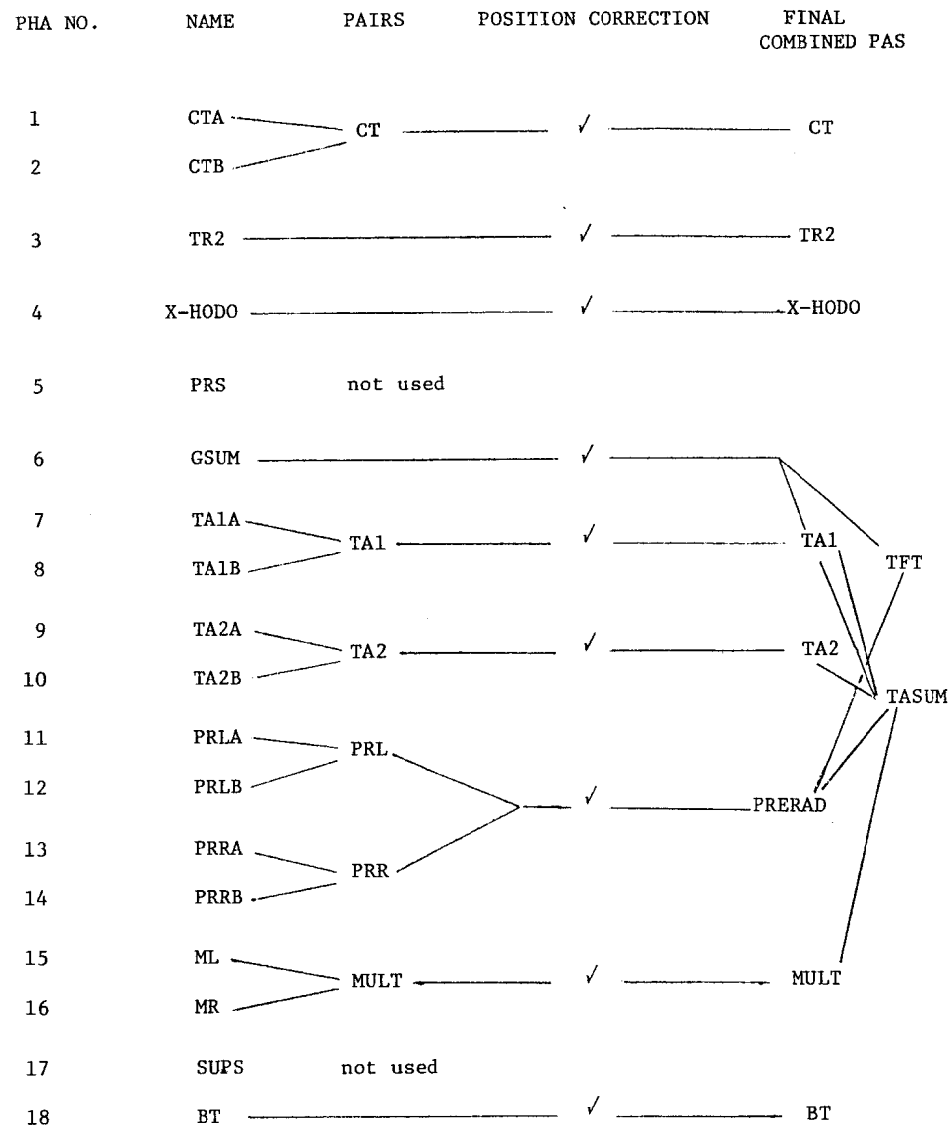
$$\sigma^2 = \frac{1}{N} (E_i - \mu)^2 = C_i C_j P_{ij} - C_i C_j S_i S_j$$
$$R = \sqrt{\sigma^2/\mu^2}$$

By differentiating R with respect to C_k one finds that the

smallest R is given by

$C_k = N_r (P_{jk})^{-1} S_j$ where $(P_{jk})^{-1}$ is inverse of the matrix of P_{jk},

where N_r is an arbitrary normalization constant, (actually

$N_r = (C_j C_n P_{jn}/C_m S_m)$, but since there are no unrepeated indices

this is just a number that can be chosen to normalize the E_i).

This rule is quite general as we have only assumed the exis-

tence of the first and second moments of the PHS's (of the e_{ij}'s)

input into the problem.

The procedure was to first combine all A,B pairs of photo-

tube signals from the CT, the left and right PRERAD, TA1, and TA2.

Optimization of each combination was performed using an almost pure

electron sample from the high statistics (about 50K events) low

E_o elastic peak runs taken near the end of the first data taking

period (see Table I-1). Events were required to have a missing

mass within 15 MeV of the proton mass, to have fired only one

hodoscope slat in each of the hodoscopes and to fall within two

hodoscope bins (about 2") of the center of the counter in question

(to avoid spurious aperture effects). Typically, each pair was

balanced with about 5K events satisfying these criteria. Also

the left and right PHS's from the PRERAD and MULT were adjusted

to have the same average pulse height and then combined to make a

single PHS at this stage.

The PHS's were then corrected for position to ensure that

electrons of the same energy gave close to identical counter

response independent of the measured X,Y, and ϕ coordinates.

The method here was to fit the means of the distributions from

different regions of the counters to polynomials in the X, Y, and

ϕ coordinates and then to divide the resultant polynomials into

the pulse height signals before further analysis. These correc-

tions never exceeded 20%. This improved the resolution across

the entire aperture and made discrimination curves insensitive

to position. Cuts on these "flattened" pulse heights minimize

spurious position dependent biases.

At this stage, most of the signals were ready for use in

the analysis except for a total energy absorption signal, TASUM.

By applying the combination formula previously derived the PHS's

of the TA stack were added together linearly to obtain a final

signal of good resolution. Choosing as a reference the minimum

ionizing signals from each segment (i.e., adjust the gain of each

signal from each section of glass to give the same average

minimum ionizing pulse height) the optimal glass addition coef-

ficients had the proportions 1:3:9 for GSUM:PRERAD:TA1. These

corresponded to their radiation lengths as expected. It was

found that small amounts of the MULT (1" plastic scintillator

"multiplicity" counter located between PRERAD and TA1) and TA2

(the 9 radiation length lead-glass counter located directly behand TA1) gave noticeably better resolution (see Appendix B). The glass hodoscope signal, GSUM, and the PRERAD were added together for Pass 2 to make a TA Front Trigger, TFT. Figure III-2 shows the various final PHS distributions used in the analysis measured at elastic peak kinematics.

Pulse height discrimination curves were fit to integrated distributions like those shown in Fig. III-2 for energies from 0.830 GeV to 1.66 GeV using the various low E_o elastic peak data at 50 and 60 degrees and were used to calculate the fraction of the electron sample eliminated by making a cut on the data requiring that the PHS be greater than the cut value. The analysis strategy was to make low cuts on many distributions, rather than high cuts on a few. The order in which cuts were made for the analysis stream was TFT, TA1, CT and then TASUM. For example, Fig. III-3a shows a TFT vs TA1 scatter plot of data taken at low E' and at an E_o of 19.5 GeV on deuterium. In Fig. III-3b, a scatter plot of low incident energy, elastic scattering data of approximately the same E' is shown for comparison. The two straight lines in Fig. III-3a indicate the cut positions of the two one-dimensional cuts used (TFT >20 and TA1 >15). These two cuts eliminate electrons from opposite ends of the correlated electron signal (diagonal blob) and are uncorrelated in detection efficiency (we are not eliminating the same electrons twice).

Further π/e discrimination is possible. Consider a CT
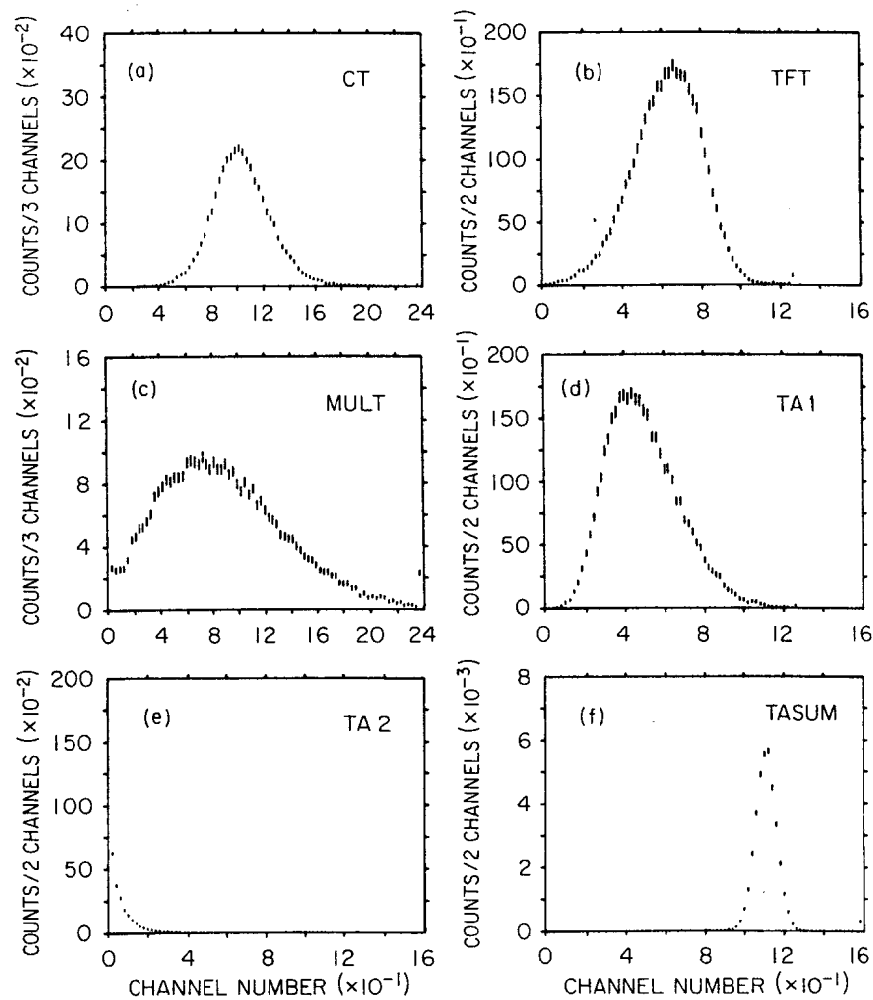
FIG. III-2

The six PHS's used in the analysis.
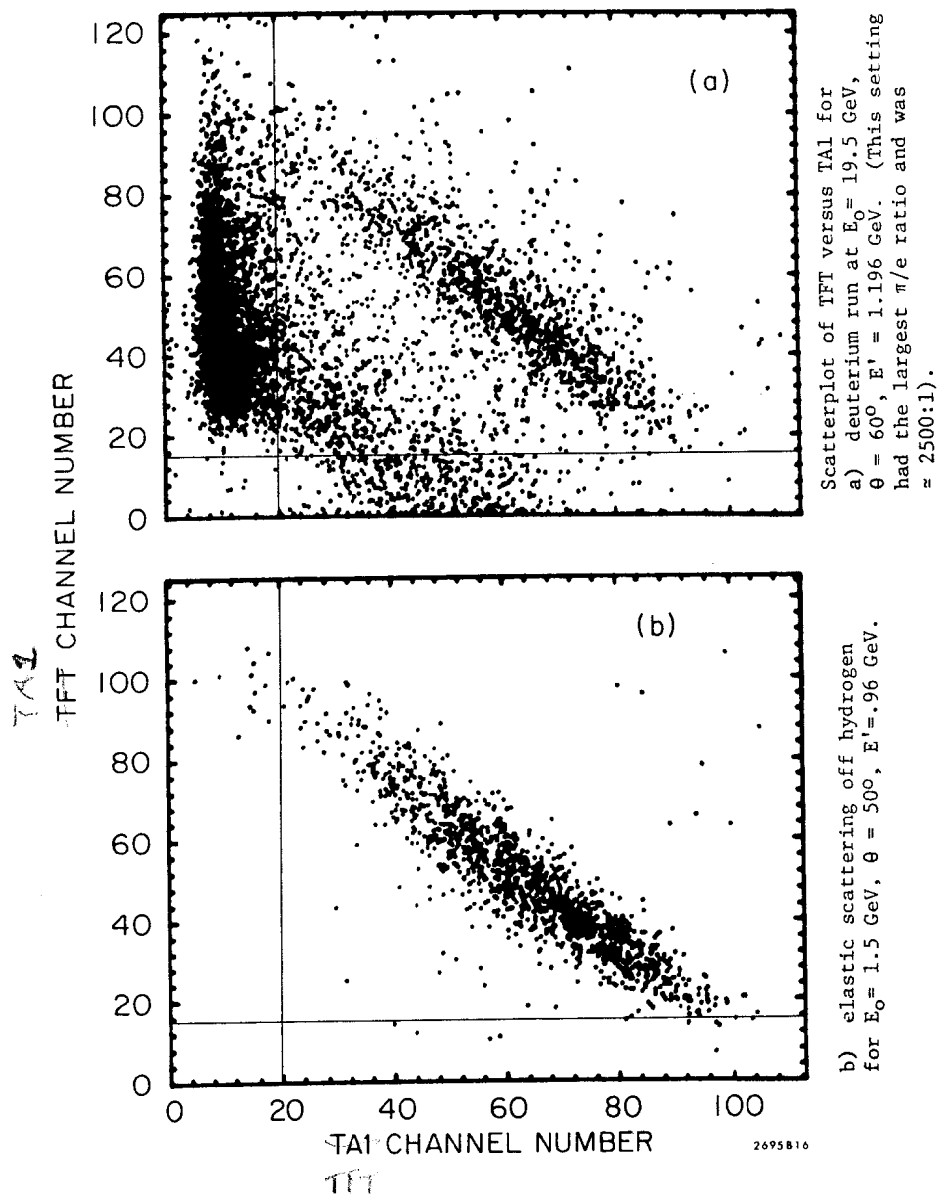
FIG. III-3

vs. TASUM scatter plot of the same data on which the previous
two cuts have been applied (see Fig. III-4a). Again, the same
elastic electron sample is shown in Fig. III-4b for comparison.
The CT cut is indicated by the line in Fig. III-4a. Notice the
electron signal from the TA package and the CT are uncorrelated.
In Fig. III-5, the TASUM distribution before and after application
of these three one-dimensional cuts is shown demonstrating that
a large fraction of the pions have been rejected while retaining
a high electron detection efficiency.

The remaining electron signal (the TASUM plot) is now cut at
x_c as shown in Fig. III-6. A small pion tail remains underneath
the electron peak. This distribution was fit to a pion shape and
an electron shape as indicated by the curves. The pion spectrum was
approximated by a falling exponential in the region of the electron
peak. The shape of this function was modeled on data that satisfied
the TFT and TA1 cut conditions but had a small or zero
CT PHS. The pion shape under the electron peak
was not very sensitive to the TFT and TA1 cut positions. The
electron distribution was fit to two shapes: A Gaussian that
accounted for most of the signal and a function similar to the
pion function at higher pulse heights than the electron peak to
account for "e + π" events. The fitted fraction of these e + π
events, whose presence is seen as a shoulder on the high pulse
height side of the electron peak in Fig. III-6, tracked well
with the APE coincidence rate. The ratio of the fitted "noise"

(a)
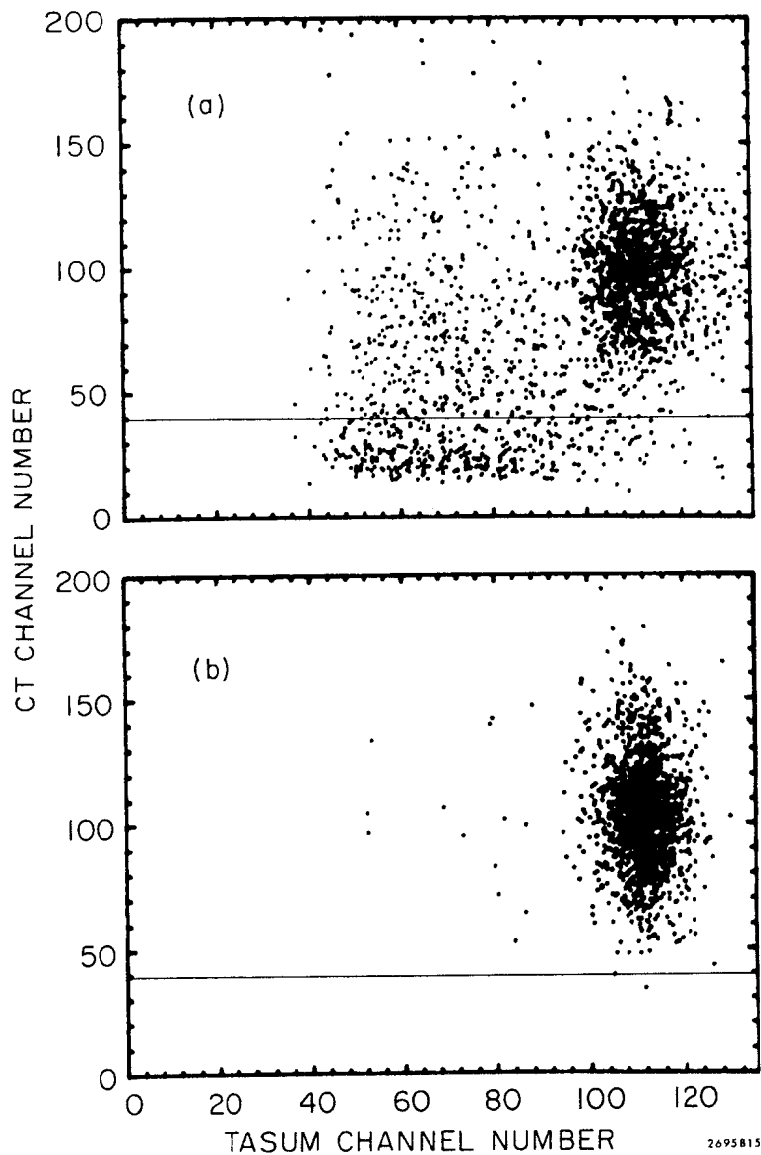
Scatterplot of CT versus TASUM for the same data shown in Fig. III-3 after the TFT and TAl cuts have been applied. Again a) is deuterium for E_0= 19.5 GeV, θ = 60° and E' = 1.196.

(b)

b) elastic scattering off hydrogen for E_0= 1.5 GeV, θ = 50° and E' = .96 GeV.

CT CHANNEL NUMBER

TASUM CHANNEL NUMBER

2695815

FIG. III-4

E_0 = 19.5 GeV
E' = 1.196
θ = 60°
Target: Deuterium

• Before PHS CUTS
○ After PHS CUTS

COUNTS/2 CHANNELS

CHANNEL NUMBER

2695814
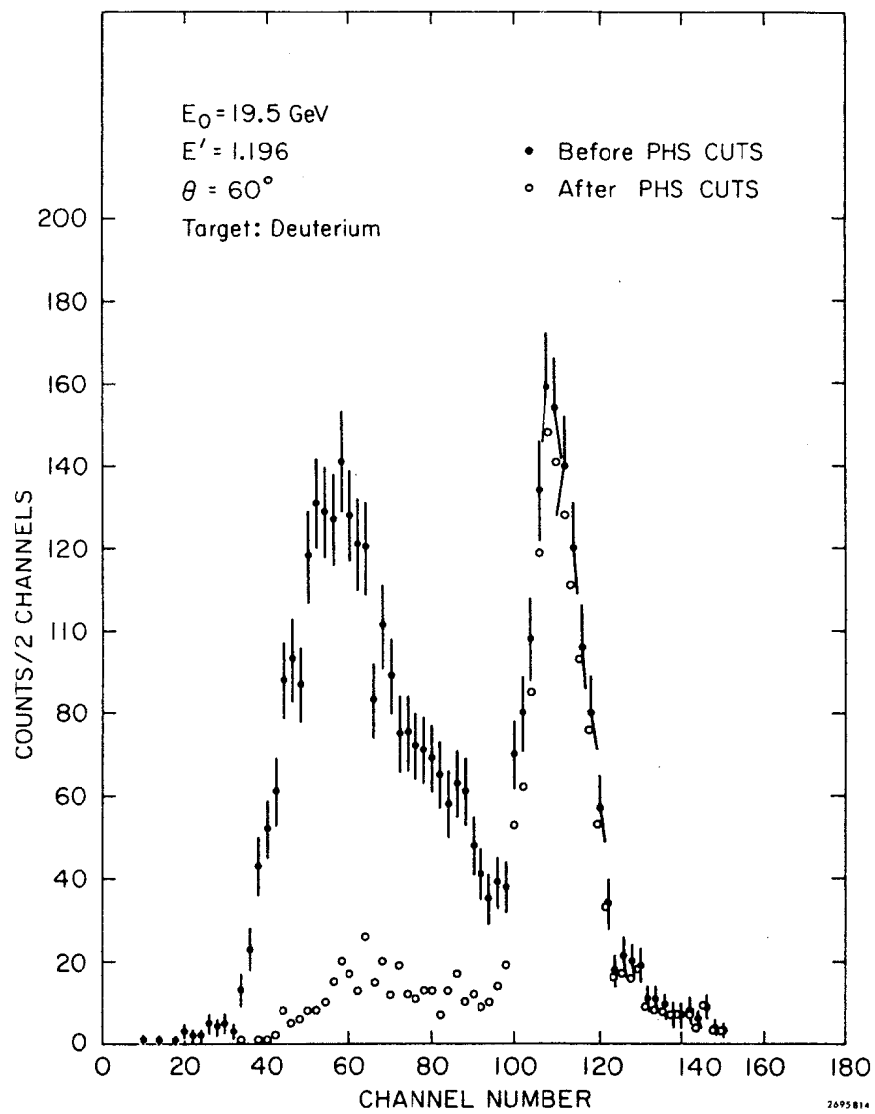
FIG. III-5

A plot of TASUM before and after the TFT, TAl and CT cuts have been applied.

Plot TASUM showing cut position, x_c, and fits used to estimate the electron detection efficiency and the pion subtraction factor.

$E_0 = 19.5$ GeV
$E' = 1.196$
$\theta = 60°$
Target: Deuterium

—— Electron Fit
– – – Pion Fit

$x_c = 95$

FIG. III-6

77
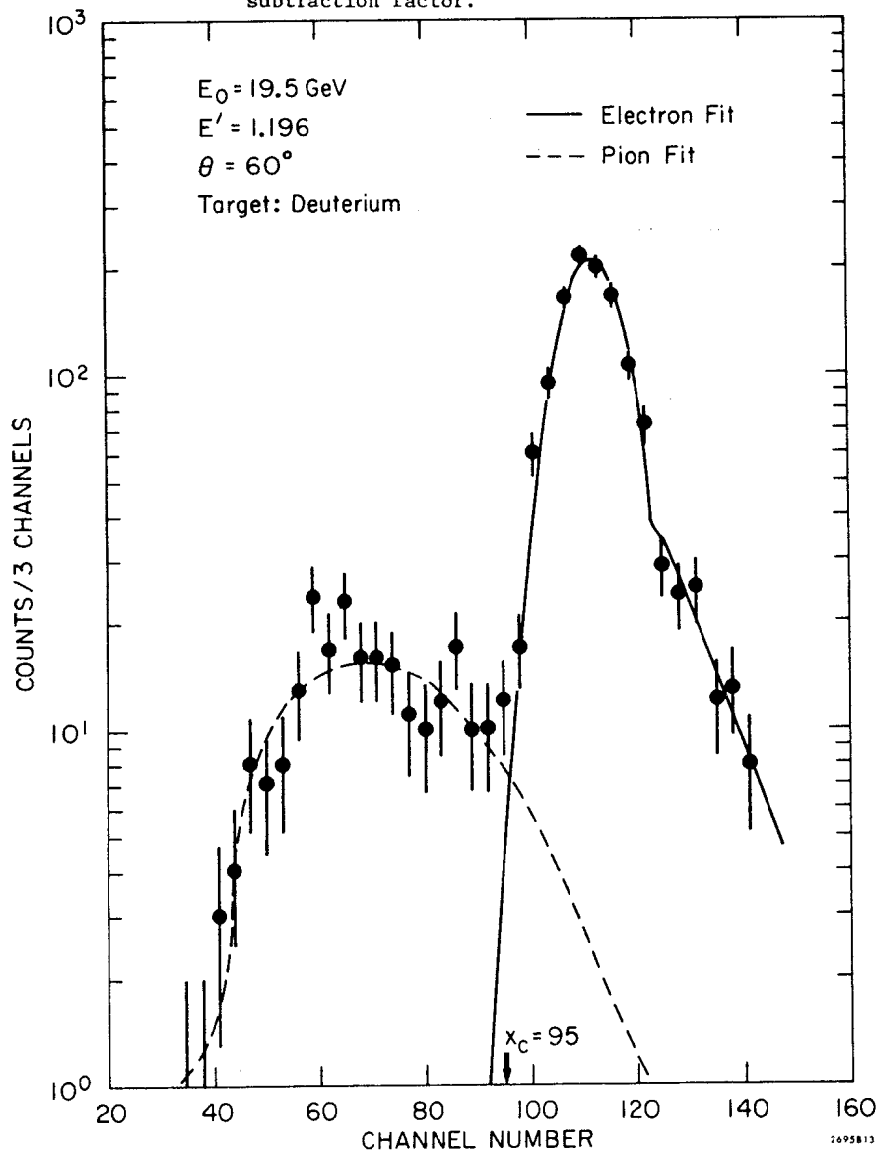
fraction $(e + \pi)/e$ to the APE rate was approximately $40/1500$ which is the PHA gate length/spill length.

A pion correction factor

$$C_\pi = \frac{\int_{x_c}^{\infty} f_e(x)\, dx}{\int_{x_c}^{\infty} (f_e(x) + f_\pi(x))\, dx}$$

was calculated from the fitted functions $f_e(x)$ (electron shape) and $f_\pi(x)$ (pion shape). Also the electron efficiency factor

$$C_{eff} = \int_{x_c}^{\infty} f_e(x)\, dx \Big/ \int_{o}^{\infty} f_e(x)\, dx$$

was calculated. Both of these factors were used to correct the electron sample.

With five separate PHS's as well as a combined TASUM PHS, a variety of cut combinations are possible. To choose an optimal set of cuts (cuts which maximize π/e separation) at a given electron detection efficiency various possibilities were systematically considered. Table III-2 summarizes the results. This is a representation of a 4 x 4 x 4 x 4 matrix with each of the 16 sub-boxes showing the 4 x 4 expansion in the TFT and CT cut effectiveness. Each of the sub-boxes has a different TA1 and/or MULT cut. The two entries for every cut condition are: 1) the calculated electron efficiency for those cuts, before a TASUM cut at $x_c = 95$. and 2) the number of counts less than 95 in TASUM divided by the number of counts greater than 95 (this gives a crude indication of π/e separation . (15% of this fraction is approximately

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TABLE III - 2

Efficiency (top) and pion contamination (bottom) for each cut combination.

		TA1 = 10				TA1 = 15				TA1 = 20				TA1 = 25			
MULT / CT		TFT 10	20	30	40	TFT 10	20	30	40	TFT 10	20	30	40	TFT 10	20	30	40
CT 30	eff	1.000	0.998	0.965	0.885	1.000	1.000	0.967	0.886	1.000	0.999	0.965	0.882	0.992	0.988	0.953	0.869
	cont	0.443	0.246	0.168	0.098	0.393	0.199	0.130	0.075	0.344	0.155	0.097	0.056	0.299	0.118	0.070	0.040
CT 40	eff	1.000	0.996	0.963	0.884	1.000	0.999	0.965	0.884	1.000	0.997	0.962	0.880	0.991	0.986	0.950	0.867
	cont	0.393	0.228	0.156	0.092	0.347	0.184	0.122	0.071	0.302	0.144	0.091	0.053	0.260	0.109	0.066	0.037
CT 50	eff	0.995	0.991	0.958	0.879	0.998	0.993	0.960	0.879	0.996	0.991	0.957	0.875	0.986	0.980	0.945	0.862
	cont	0.340	0.204	0.143	0.084	0.298	0.164	0.110	0.065	0.257	0.127	0.083	0.049	0.220	0.096	0.060	0.035
CT 60	eff	0.979	0.974	0.942	0.863	0.981	0.976	0.943	0.863	0.979	0.974	0.940	0.860	0.968	0.962	0.928	0.847
	cont	0.299	0.188	0.131	0.078	0.261	0.151	0.102	0.060	0.223	0.117	0.076	0.045	0.189	0.088	0.054	0.032
MULT 5	eff	0.996	0.994	0.962	0.883	0.998	0.996	0.964	0.883	0.997	0.995	0.961	0.879	0.986	0.983	0.949	0.866
	cont	0.409	0.225	0.156	0.092	0.367	0.184	0.122	0.071	0.324	0.144	0.092	0.053	0.283	0.111	0.066	0.038
	eff	0.995	0.993	0.960	0.881	0.997	0.994	0.962	0.881	0.996	0.992	0.959	0.877	0.985	0.981	0.947	0.864
	cont	0.362	0.208	0.145	0.086	0.323	0.170	0.114	0.067	0.283	0.134	0.086	0.050	0.245	0.102	0.062	0.036
	eff	0.991	0.988	0.955	0.876	0.993	0.989	0.956	0.876	0.991	0.987	0.953	0.872	0.980	0.975	0.941	0.859
	cont	0.312	0.186	0.132	0.079	0.277	0.151	0.103	0.062	0.241	0.119	0.078	0.047	0.207	0.090	0.056	0.033
	eff	0.974	0.970	0.939	0.861	0.976	0.971	0.940	0.861	0.974	0.969	0.937	0.857	0.963	0.957	0.924	0.844
	cont	0.273	0.171	0.121	0.074	0.241	0.139	0.095	0.057	0.208	0.109	0.071	0.043	0.177	0.082	0.051	0.031
MULT 10	eff	0.992	0.989	0.958	0.879	0.994	0.991	0.959	0.879	0.992	0.989	0.956	0.875	0.982	0.978	0.944	0.862
	cont	0.387	0.215	0.149	0.088	0.348	0.177	0.118	0.068	0.309	0.140	0.089	0.052	0.271	0.108	0.065	0.037
	eff	0.991	0.987	0.956	0.877	0.993	0.989	0.957	0.877	0.991	0.987	0.954	0.873	0.980	0.976	0.942	0.860
	cont	0.342	0.198	0.139	0.083	0.307	0.164	0.110	0.065	0.270	0.130	0.084	0.049	0.235	0.100	0.061	0.035
	eff	0.986	0.982	0.951	0.872	0.988	0.984	0.951	0.872	0.986	0.981	0.948	0.868	0.975	0.970	0.936	0.855
	cont	0.295	0.177	0.126	0.076	0.262	0.146	0.100	0.059	0.230	0.115	0.076	0.046	0.198	0.088	0.055	0.033
	eff	0.970	0.965	0.934	0.857	0.971	0.966	0.935	0.857	0.969	0.964	0.932	0.853	0.958	0.952	0.920	0.840
	cont	0.259	0.163	0.116	0.071	0.229	0.134	0.092	0.055	0.199	0.106	0.070	0.043	0.170	0.081	0.051	0.030
MULT 15	eff	0.987	0.983	0.952	0.874	0.989	0.984	0.953	0.874	0.987	0.982	0.950	0.870	0.977	0.971	0.938	0.857
	cont	0.314	0.193	0.141	0.084	0.279	0.164	0.111	0.066	0.244	0.131	0.085	0.050	0.211	0.102	0.063	0.036
	eff	0.986	0.981	0.950	0.872	0.988	0.983	0.951	0.872	0.986	0.980	0.948	0.868	0.975	0.969	0.936	0.855
	cont	0.280	0.184	0.131	0.079	0.248	0.152	0.104	0.062	0.215	0.122	0.080	0.048	0.184	0.094	0.058	0.034
	eff	0.981	0.976	0.945	0.867	0.983	0.977	0.945	0.867	0.980	0.975	0.942	0.863	0.969	0.963	0.930	0.850
	cont	0.243	0.165	0.119	0.073	0.214	0.136	0.095	0.057	0.184	0.108	0.073	0.044	0.156	0.083	0.053	0.032
	eff	0.965	0.959	0.929	0.852	0.966	0.960	0.929	0.852	0.963	0.957	0.926	0.848	0.952	0.946	0.914	0.835
	cont	0.216	0.151	0.110	0.068	0.189	0.125	0.087	0.053	0.161	0.099	0.067	0.044	0.136	0.076	0.049	0.029

A 4x4x4x4 matrix showing various cut effects for TA1, CT, MULT and TFT. For each cut combination in the two entries are 1) electron detection efficiency and 2) pion contamination.

equal to $(1 - C_\pi)$ previously defined). The boxed-in elements indicate our choices for "optimal" π/e separation points in order of decreasing electron detection efficiency.

Six PHS classes were selected and are summarized in Table III-3. A test of the analysis procedure is how invariant the cross sections are to PHS class. Over the six classes the cross sections were usually found to vary less than 4% absolute while the detection efficiency varied by over 20% from the different PHS cuts made.

TRACKING AND CODES

The first task in analyzing a hodoscope system is to assign a unique slat to every event in each hodoscope. The backup coarse hodoscope on the p-θ slats proved valuable in analyzing multiple slat events. For example, in an event consisting of a cluster of hit slats along with a single separate slat, we found that the backup hodoscope information favored assigning the event to the single slat as often as to the cluster. Events requiring the coarse hodoscope information were called "saved" events. Four hit patterns were recognized and classified: 1) a single slat, 2) two adjacent slats, 3) two slats separated by one blank slat, and 4) three slats with one imbedded blank slat or four adjacent slats. Each event had a tracking code (numbered 0-8) assigned to each hodoscope. Odd code numbers (1, 3, 5, 7) were given hodoscope patterns that didn't require saving and even numbered codes (2, 4, 6, 8) were the corresponding codes for saved patterns. The code number 0

TABLE III - 3

PHS CLASSES

CLASS	CT	TFT	MULT	TA1	TASUM	EFFICIENCY (TYP)
I	30	10	-	10	95	.998
II	40	20	-	15	95	.980
III	50	20	-	20	95	.969
IV	60	30	-	25	95	.910
V	60	40	15	25	95	.792
VI	60	40	15	25	100	.771

The Table shows the various cut conditions selected for optimal π/e separation.

All cuts pass events which have a pulse height signal greater than the value shown. Class II is our choice for the analysis class.

referred to the case where no slats in a hodoscope fired. In Fig. III-7, typical hit patterns and their code and bin assignments are shown.

As the Y-hodoscope had no backup coarse hodoscope, optics infor-mation was used to project an allowed hit range for the +40 mrad to -50 mrad slits at the spectrometer's entrance. The events resolved in this manner are the "saved" events for this hodoscope. If no Y bin showed a hit, the event was assigned a random ϕ angle that fell within the acceptance.

Different tracking criteria can have different π/e mixtures. Multiple π events can masquerade as electrons but have a high pro-bability of giving bad codes (multiple tracks). To examine this, we first selected events in the analysis class passing the TFT,TA1 and CT cuts. If the event fell above 95 in TASUM it was called an electron and below 95,a pion. Table III-4 shows p codes vs. θ codes with the upper entry being the fraction of PHS analysis class events with those codes, and the lower entry the π/e ratio.

One sees that the π/e ratio is large away from unique single tracks i.e., the (1,1) box. On the basis of Table III-4, six code classes were established of increasing track quality. These are summarized in Table III-5, which shows the code class (1-6) assigned to each (P,θ) code.

The efficiency for each code class was fit to a second order polynomial in the counting rate of the APE2 coincidence (a coincidence between the Y hodoscope, the X hodoscope and TR2), and of the TR2 counter. This tracking efficiency correction was calculated for each run and applied to the

HODOSCOPE PATTERNS

CODE 1
1 20
Single
Slat   Bin 12

CODE 2
1 20
              Bin 11

CODE 3
1 ....xx 20
Two
Adjacent   Bin 15 or 16
Slats   (Randomly Selected)

CODE 4
1 xx x xx 20
              Bin 5 or 6
              (Randomly Selected)

CODE 5
1 x x 20
Two
Slats   Bin 8
Separated
by 1

CODE 6
1 xxx   x x 20
              Bin 15

CODE 7
1 xx x 20
Three
Slats
and 1   Bin 7 - 8
Gap or   (Randomly Selected)
4 Slats

CODE 8
1 xxxx   x 20
              Bin 3
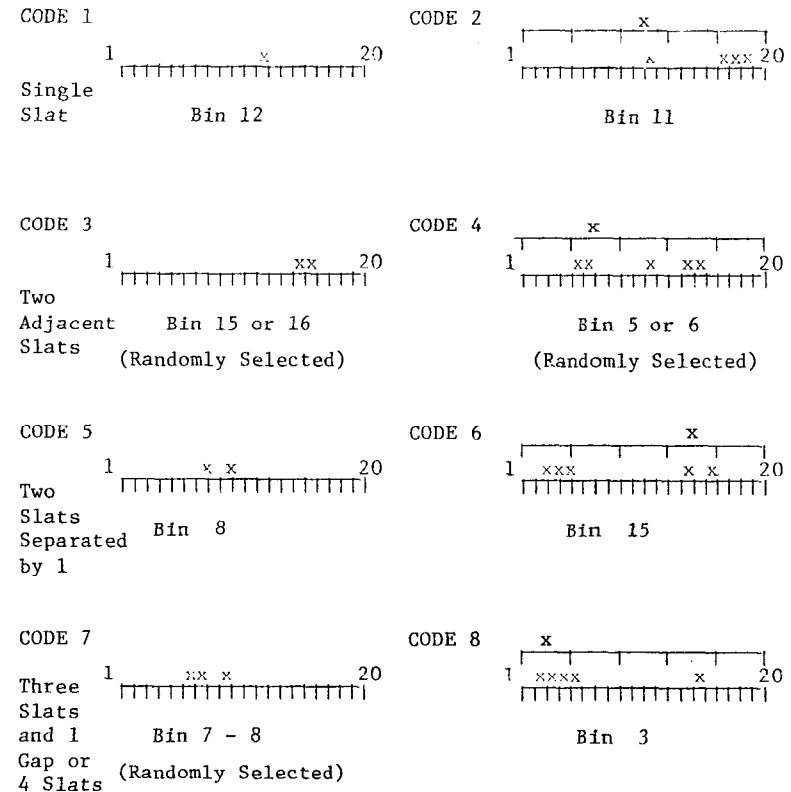
ALL OTHER PATTERNS WERE CONSIDERED UNRESOLVABLE
AND LABELED "ZOO EVENTS"

FIG. III - 7

TABLE III - 4

θ Code ⟶

		1	2	3	4	5	6	7	8
P Code ↓	1	0.8176	0.0394	0.0125	0.0007	0.0048	0.0004	0.0004	0.0080
		0.1770	0.4893	0.5532	1.6667	0.7706	1.1000	1.4444	0.4667
	2	0.0403	0.0041	0.0017	0.0002	0.0017	0.0002	0.0001	0.0003
		0.6839	1.9032	2.6154	5.7500	2.8462	5.0000	7.6667	3.1429
	3	0.0354	0.0032	0.0018	0.0002	0.0012	0.0000	0.0001	0.0004
		0.2900	1.1389	1.5000	2.0000	2.4643	8.0000	4.0000	1.2500
	4	0.0032	0.0004	0.0003	0.0	0.0001	0.0000	0.0000	0.0000
		0.7917	2.7000	1.1429	3.0000	5.0000	8.0000	4.0000	8.0000
	5	0.0083	0.0008	0.0012	0.0001	0.0006	0.0	0.0001	0.0001
		0.7433	3.1111	1.7308	4.5000	3.5000	6.0000	3.5000	2.0000
	6	0.0004	0.0001	0.0000	0.0	0.0001	0.0	0.0	0.0
		2.5556	3.0000	7.0000	1.0000	2.3333	0.0	4.0000	1.0000
	7	0.0027	0.0004	0.0004	0.0	0.0002	0.0000	0.0	0.0000
		0.8226	2.6667	1.1250	5.0000	3.6000	4.0000	9.0000	5.0000
	8	0.0050	0.0002	0.0001	0.0	0.0000	0.0	0.0	0.0001
		1.5893	3.2000	4.6667	3.0000	8.0000	0.0	3.0000	5.5000

TABLE III - 5

θ Code ⟶

		0	1	2	3	4	5	6	7	8	>8
P Code ↓	0	1	1	1	1	1	1	1	1	1	1
	1	1	6	5	5	3	4	3	3	4	1
	2	1	4	2	2	2	2	2	2	2	1
	3	1	5	3	3	2	2	2	2	2	1
	4	1	4	2	2	2	2	2	2	2	1
	5	1	4	2	2	2	2	2	2	2	1
	6	1	2	2	2	2	2	2	2	2	1
	7	1	4	2	3	2	2	2	2	2	1
	8	1	4	2	2	2	2	2	2	2	1
	>8	1	1	1	1	1	1	1	1	1	

data. The analysis stream used only events with code classes 4-6. The average efficiency for electron data runs was 98% and never lower than 94%.

ACCIDENTALS

Random master trigger coincidences were created by splicing the TA1D trigger CT signal together with the rest of the information from the TA2D trigger. These events were uniquely tagged by the trigger flags (ORT false and ORA true). Each accidental event that passed the analysis class criteria (PHS classes 4-6 and code classes 4-6) was used to decrement the missing mass histogram and final TASUM PHS plot by 1.176 (this factor is the short spill correction). The accidental rate was highest for the cross section measured off deuterium at 60 degrees, 19.5 GeV incident. For those settings the number of accidentals that was subtracted from the analysis class sample amounted to 2% of the total. For the bulk of the data the percentage contribution was much less than 1%.

OVER-ONE CORRECTION

We could accept at most one event per pulse as the electronics had only one set of PHA's and flag units. As counting rates on the main trigger, ORT, were very low (usually less than .01 per pulse) this was not the source of much data loss. Missed ORT events were corrected for by using the ORTK scaler information. Missed accidental events were corrected by using the TA2D scaler and a count of the number of TA2D events read by the computer (gotten by counting events with a TA2D flag set true). These two over-one corrections were

combined in a single correction consistent with the decrementation

scheme explained in the last section:

$$C_1 = \frac{ORT_{as} \; (\frac{ORT_{sc}}{ORTK_{sc}}) \; - \; 1.176 \quad TA2D_{as} \; (\frac{TA2D_{sc}}{TA2D_{sfc}})}{ORT_{as} \; - \; 1.176 \quad TA2D_{as}}$$

where subscripts   as = analysis class event count, sc = scaler

event count, sfc = software flag scaler event count, and the 1.176

is the short spill correction (previously mentioned).

EVENT BINNING

At a particular setting (same E_o, E', θ, and target) there are

20 p bins x 15 θ-bins into which events can fall. As a TR2 signal is

required of all events, the p-bin range is limited to 2-19, and the

θ bin range is limited to 2-14. The maximum ϕ acceptance is set by the

fixed slit at the entrance to the spectrometer. We required that

-60 mrad < ϕ < 50 mrad as reconstructed from the hodoscope informa-

tion. Typical p, θ and ϕ distributions are shown in Fig. III-8.

The solid line on the ϕ distribution is a Monte Carlo prediction

based on the 1.6 optics (see Appendix D). Similar predictions for

p and θ were not made as real events are not expected to be uniformly

distributed in p and θ. The solid angle for each of the 234 p- θ

bins was calculated using a Monte Carlo program and stored (on disk)

for both 50 and 60 degrees. Bins of constant missing mass cut across

the p-θ plane diagonally. A particular p-θ bin is included in a mass bin

if its center falls inside the boundaries of that bin.
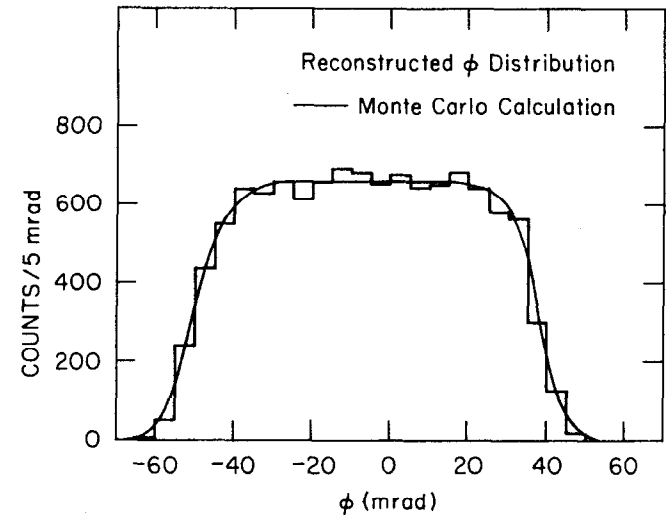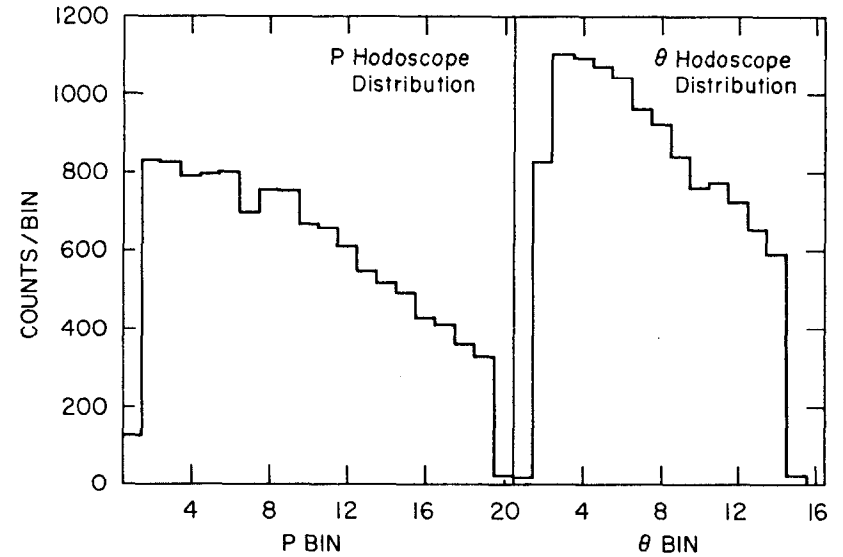
FIG. III-8

Hodoscope Distributions

The data were taken with overlapping momentum settings of the spectrometer. This "scanning" was done by lowering the momentum by one third of the total momentum acceptance for each new setting. There results a high degree of overlap among the settings comprising a line. The "counts" (number of electrons) and "weights" (the solid angle, correction factors, incident flux, etc.) for each missing mass bin were concatenated over the entire line and subsequently stored on disk.

RUN WEEDING

There were 1760 runs taken in the experiment. One hundred thirty runs were found to be unacceptable. Ninety three of these were abnormally terminated due to major equipment malfunctions or experimenter error. Twenty one runs were excluded by examining compatability with similar runs. All runs taken at the same setting were required to give reasonable χ^2 comparisons of the cross sections into the full acceptance. Runs so eliminated were found to have notes made in the experiment's log books that the beam was mis-steered or badly focused , the target had problems, or the electronics or the computer had some minor malfunction. The various run correction factors were scanned on a run-by-run basis and runs with large deviations examined carefully. The mean and width of each of the five PHS's were scanned to look for malfunctioning detectors. Large jumps in these quantities corresponded to changes in E_o, θ and sign of E'. Tracking efficiency scans unearthed a few runs with dead hodoscope slats not noted in the log book. These types of comparisons lead us to discard another sixteen runs.

TESTING THE ANALYSIS PROCEDURE

After run weeding we felt confident in concatenating runs into settings (i.e., same target, E_o, E' and θ). At this level of reduction, two tests were made. The first was designed to test tracking and PHS efficiencies, and the second was sensitive to cross section variation across the acceptance and setting to setting compatability.

The efficiency test was easily made. As described earlier in this Chapter, we derive 6 PHS classes and 6 tracking classes. We can analyze our data using 36 different criteria with PHS efficiencies running from 99.9% to 80% and tracking efficiencies ranging from 99% to 80%. The pion contamination varies from several percent (depending on running conditions) to virtually nil (<.5%) over the 36 PHS tracking classes. The total electron detection efficiency ranges from 100% to 64%. The analysis class was selected as the reference point and the 35 other cross section compared to it. Table III-6 shows a typical comparison for all the hydrogen running for E' between 1.200 GeV and 1.400 GeV for electrons at 60 degrees (the concatenation over E' was done to increase the statistics). For each line, each setting was so examined. We found no statistically significant deviation. This leads us to believe we know our inefficiencies to about 20% of their value for both the PHS efficiency and the tracking efficiency. As seen from Table III-6 there is no definite correlation between PHS efficiency and tracking efficiency.

TABLE III - 6

MINIMUM CODE CLASS vs. PULSE HEIGHT CLASS

PH CLASS ↓	1	2	3	4	5	6
1	2512.2 1.001±0.020 0.99706 0.01832	2486.4 0.997±0.020 0.99493 0.01340	2445.4 0.991±0.020 0.98830 0.00963	2434.4 0.991±0.020 0.98386 0.00942	2273.4 0.988±0.021 0.92375 0.00787	2054.5 0.989±0.022 0.83523 0.00575
2	2479.3 1.011±0.020 0.97913 0.01286	2454.3 1.006±0.020 0.97705 0.01036	2416.3 1.000±0.020 0.97056 0.00741	2405.3 1.000±0.020 0.96619 0.00696	2246.3 0.997±0.021 0.90710 0.00512	2030.3 0.997±0.022 0.82022 0.00457
				ANALYSIS CLASS		
3	2459.3 1.018±0.021 0.96892 0.00874	2434.3 1.011±0.021 0.96691 0.00683	2398.3 1.005±0.021 0.96056 0.00490	2387.3 1.005±0.021 0.95624 0.00476	2231.3 1.001±0.021 0.89777 0.00400	2016.3 1.001±0.022 0.81178 0.00373
4	2277.6 1.011±0.021 0.90972 0.00201	2255.6 1.003±0.021 0.90791 0.00178	2222.6 9.995±0.021 0.90220 0.00154	2211.6 0.995±0.021 0.89816 0.00150	2069.6 0.992±0.022 0.84328 0.00126	1866.6 0.989±0.023 0.76260 0.00110
5	2040.6 1.041±0.023 0.79210 0.00083	2021.6 1.034±0.023 0.79061 0.00080	19993.6 1.026±0.023 0.78564 0.00053	1982.6 1.025±0.023 0.78212 0.00053	1853.6 1.020±0.024 0.73476 0.00046	1670.6 1.016±0.025 0.66467 0.00040
6	1968.6 1.032±0.023 0.77104 0.00056	1951.6 1.025±0.023 0.76974 0.00054	1928.6 1.018±0.023 0.76579 0.00037	1918.6 1.017±0.023 0.76254 0.00037	1793.6 1.012±0.024 0.71637 0.00032	1614.6 1.008±0.025 0.64789 0.00028

The four numbers shown in the Table are: 1) number of electrons corrected for accidentals; 2) ratio to analysis class cross section; 3) electron detection efficiency; and 4) the pion subtraction factor. The data were taken on hydrogen with $E_0 = 19.5$, $\theta = 60^\circ$, and for 1.2 GeV $< E' < 1.4$ GeV.

Typically, we assign a 3.2% systematic error from tracking and

PHS efficiency.

The second test we made on the data was a cross section comparison. Every line with few exceptions was measured three times in each scan (some of the low E_o lines at the low E' were taken in bigger than 1/3 acceptance jumps). Except for the beginning and end of each line we have three cross section measurements of the same points which we can compare. A reference cross section is gotten by concatenating the counts and weights for the three partial cross sections. This is then compared to individual cross sections.

A χ^2 test is also made by forming the residuals

$$R_{ij} = \frac{\sigma_i - \sigma_j}{\sqrt{\Delta\sigma_i^2 + \Delta\sigma_j^2}} \quad , \quad i \text{ and } j = 1, 2, 3, i \neq j$$

A histogram of the residuals for the 50 and 60 degree data is shown in Fig. III-9. The distribution is what one expects normally distributed data to produce. We conclude that there are no serious aperture biases in the analysis.

1324 RESIDUALS
MEAN = -0.063
RMS = 0.964
CURVE = $105.6\, e^{-R^2/2}$
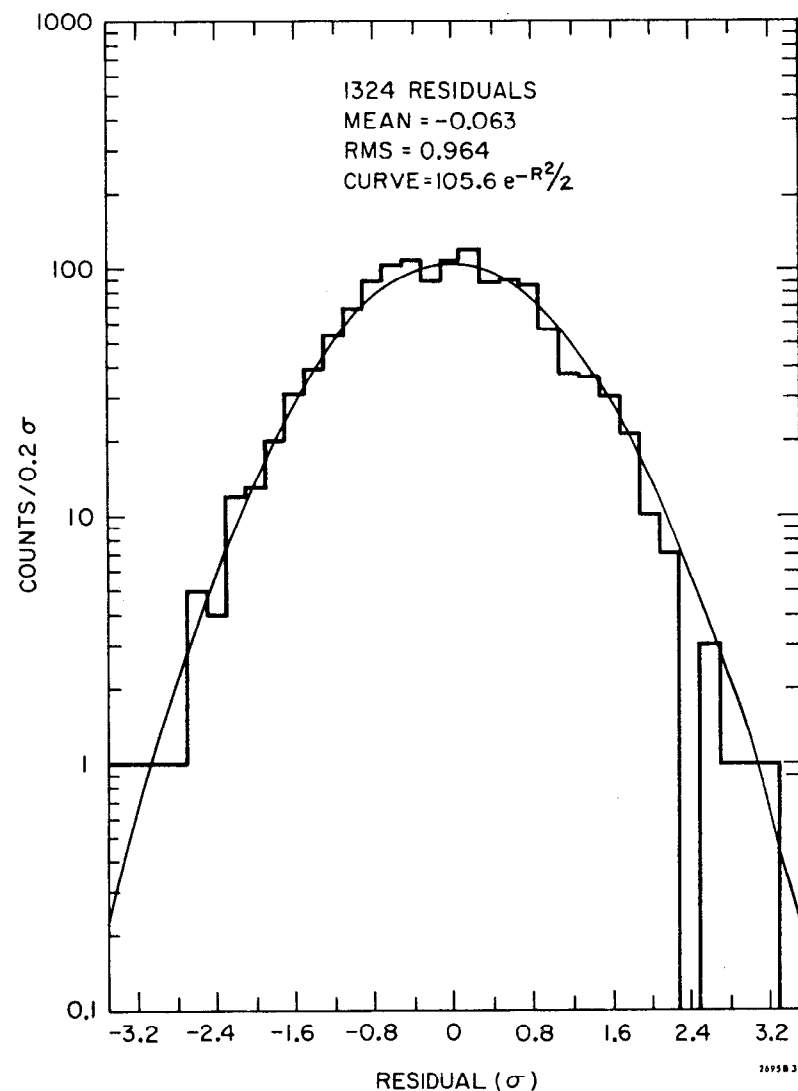
FIG. III-9

A histogram of the residuals R_{ij}. The curve is what one expects normally distributed data to give.

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CHAPTER IV

CALCULATED AND MEASURED CORRECTIONS

MEASURED CORRECTIONS

The full target yields include contributions from the target walls and from processes which produce both positrons and electrons. To remove these unwanted contributions we measured these yields besides the full target electron yields. These were the empty target cross-sections measured for both scattered electrons and positrons and the full target measured for scattered positrons. The prescription used to correct the full target cross section is given by

$$(4.1) \qquad \sigma_{COR} = \sigma_{FULL}^{-} - \sigma_{MT}^{-} - (\sigma_{FULL}^{+} - \sigma_{MT}^{+}).$$

The signal from the empty target typically amounted to about 6% of the full target signal for hydrogen and 4% for deuterium. The fraction of the measured cross section accounted for by charge symmetric processes depends on E_o, E', and θ and in general increases as E' decreases. This correction, made by subtracting the measured positron yield from the electron yield, was a primary factor in determining the lowest E' at which data were taken. Most lines were run until the ratio of positron to electron yield on the full target reached .35 $(\sigma_{FULL}^{+}/\sigma_{FULL}^{-} = .35)$.

The assumption used in Eq. 4.1 is that there are no important sources of electrons (other than scattered beam particles) which are not charge symmetric. A test of this assumption was recently made experimentally (Ref. IV-1) during the second cycle of E-89 by measuring the electron yield from the target using a positron beam

and comparing that to the charge symmetric situation with electrons incident in the same experiment. This wrong sign signal was equal for the two signs of the incident particle charge to within the accuracy of that experiment (5% at $Q^2 = 3$ GeV^2 to 10% at $Q^2 = 15$ GeV^2).

For both the empty target and positron signals we make the hypothesis that the structure in missing mass is smooth. We take smooth analytic representations of these data as our best estimate of their actual value. We are assuming that neighboring data points are not independent but contain similar physical information. Using the bulk of the data rather than the individual data points can improve estimates of these corrections.

Plots of these data show that the double differential cross sections are approximately exponential at these large angles. We find the following simple parametrization

$$(4.2) \qquad \sigma_{pos} = 10^6 (a + b\,E_o + c\,E_o^2)\, e^{-d\,E'\sin\theta} \text{(pb/GeV-sr)}$$

works well for the positron cross sections. The resulting fit parameters are given in Table IV-1 for the various E_o's, θ's and targets. To propagate the error introduced by these subtractions, we estimate that the functions thus fitted give the correct cross sections to

$$(4.3) \qquad \Delta\sigma/\sigma = MAX \ (.1, .3 - .1\ W\ (GeV))$$

This gives an error in this correction of 20% in the elastic peak regions decreasing to a minimum of 10% for W > 2 GeV. This error approximation does not significantly improve the errors over what the measured data would yield. The prescription of subtracting

TABLE IV - 1
POSITIVE SUBTRACTION

$$\sigma_{POS} = \sigma^+_{FULL} - \sigma^+_{MT} = 10^6(a + bE_0 + cE_0^2) \, e^{-d \, P_t} \, (\tfrac{pb}{GeV\text{-}sr})$$

	a	b	c	d	χ^2/N_D
60°, H	-13.3290	4.25038	-0.0484835	16.4419	85.7/112
60°, D	-13.2485	3.76622	-0.0684701	15.1213	153/130
50°, H	-19.3759	4.52011	-0.0766568	14.4748	41.1/37
50°, D	- 9.52999	3.43338	-0.0555869	13.6932	31.0/43

EMPTY TARGET SUBTRACTION

$$\sigma^-_{MT} = a \, e^{(bW + cW^2)} \, (\tfrac{pb}{GeV\text{-}sr})$$

Line	E_0	θ	a	b	c	χ^2/N_D
1	19.5	60°	0.160334E-1	-.970139	.536463	18.7/24
2	16.0		0.201885E-3	2.59807	-.0407573	15.0/17
3	13.3		0.902831E-2	-.292653	.591770	71.8/78
4	10.4		0.158756E-1	-.499663	.789576	17.4/25
5	6.5		0.250682E-2	2.89575	.179453	92.4/63
6	19.5	50°	0.195994E-4	2.62323	.0310931	13.7/9
7	16.0		0.268996E-4	3.21611	-.0468157	13.2/11
8	13.5		0.601734E-2	.237730	.445982	13.0/17
9	7.0		0.339265E-1	.986842	.521663	12.5/19

For D_2 divide result by 1.16 (empty target only).

models allows us to treat all the data in the same manner (we did not measure all the quantities in Eq. 4.1 for about 30% of the data, as these corrections were small, and we felt confident in our ability to estimate them from the measured data). These errors will be taken as part of the systematic error in the final cross sections. The χ^2's quoted in Table IV-1 were evaluated using the statistical counting errors.

An interesting side note and source of puzzlement to us was that the empty target (stainless steel) cross sections were appreciably different in character from the deuterium cross sections. Our expectation would be that iron could be well approximated by a "bag" of deuterons. Fermi motion effects would tend to be similar. But, the ratio of the iron cross sections to the LD_2 cross sections for wrong sign running is E' dependent, increasing with E'. The growth is as much as a factor of two for the highest incident energy lines and always at least a factor of two more than what would be expected. This means that the size of the empty target cross section was larger than expected from simple nucleon counting. The corrected empty target cross section, $\sigma^-_{MT} - \sigma^+_{MT}$, is the right size for scattered beam particles, but the charge symmetric part is anomalously large and increases relative to the signal as E' is increased. We doubt that this effect has been generated by a measurement or analysis error.

CALCULATED CORRECTIONS

It is traditional to correct the data for the effects of radiation in the target and in the scattering process itself. For deuterium, the motion of the bound neutron and proton provide and additional correction. We have applied three calculated corrections to this electroproduction data. First, the elastic and quasi-elastic (as appropriate to deuterium) radiative tails were subtracted. The "tail subtracted" data were then corrected to account for radiation which shifts the theoretical electron yields to higher missing mass. The deuterium cross sections are the source of the neutron data, and corrections for Fermi-motion effects were calculated and applied before comparisons with the hydrogen data were made.

ELASTIC TAIL SUBTRACTION

The elastic tail was calculated by the methods given in G. Miller's thesis (Ref. IV-2). The elastic tails come from incident electrons radiating energy through photon emission and elastically (or quasi-elastically) scattering off the target particle. The energy degradation can occur before and after the elastic scatter. Both produce scattered electrons of lower energy than elastically scattered beam particles. These lower energy electrons enhance the cross sections measured at missing masses higher than the proton mass. The materials in the beam before and after the scatter are referred to as "real radiators" and are summarized in Table II-2. In addition to the effect of this real radiator, there are the quantum

electrodynamic processes involving radiation from the electron being accelerated during the elastic scatter. This source of radiation is often discussed in terms of the "internal" or "equivalent" radiator, as such radiation is similar to that caused by the real material in the beam. An approximate expression for the equivalent radiator is

$$(4.4) \qquad t_{eq} = \frac{\alpha}{\pi} (\ln (Q^2/m_e^2) - 1)$$

For $Q^2 = 20$ GeV^2, $t_{eq} = .04$. The expressions given by Tsai (Ref. IV-3) were used to calculate this correction exactly to lowest order in α and are used in Miller's approach.

Smaller corrections also included are an estimate of multiple photon emission and target radiation from the recoiling proton (only in the case of hydrogen). The elastic tail calculation requires knowledge of the form factors G_E and G_M for Q^2 less than the effective Q^2 of the point being corrected (the effective $Q^2 = 4 E'^2 \sin^2(\theta/2)/(1 - 2 E'\sin^2(\theta/2)/M)$). We have assumed form factor scaling ($G_E = G_M/\mu_p \equiv G_E^S$) and a fit to the measured elastic scattering data for G_E^S (see Chap. V). The exact expressions used for the elastic tails (as well as the inelastic radiative corrections) are given in Ref. IV-4.

Plotted in Fig. IV-1 are the elastic tail fractions, that is the elastic radiative tails divided by the raw cross section for representative lines at 50 deg. and 60 deg. for both hydrogen and deuterium targets.

W = 2.5 GeV POINT

HYDROGEN 60° (a)

DEUTERIUM 60° (b)

W = 2.5 GeV For 7.0 GeV W = 3.5 GeV For 19.5 GeV

HYDROGEN 50° (c)

DEUTERIUM 50° (d)

TAIL FRACTION (%)

E' (GeV)

E' (GeV)

FIG. IV-1

The elastic radiative tail fraction
for representative lines at 50° and 60°.
These arrows at the top of the graph
indicate where a missing mass, W, of
2.5 GeV falls for E_O = 6.5, 13.3 and
19.5 GeV at 60° and W = 2.5 for E_O = 7.0 GeV
and W = 3.5 for E_O = 19.5 GeV at 50°.

For the deuterium quasi-elastic peak, we used the procedure
given in Ref. IV-5 (also see Appendix C) to compute the quasi-
elastic cross section which was then radiated to produce a "quasi-
elastic" tail. The Reid hard-core wave function (Ref. IV-6) for
the deuteron has been used in all the smearing and quasi-elastic
calculations.

We take the error in the elastic and quasi-elastic tails to be
the same as those estimated in Ref. IV-4 ($\pm 5\%$) plus an additional
MAX(5, Q^2(GeV^2))% to account for the high Q^2 uncertainty of the
squared form factors. This gives $\pm$ MAX(10, 5 + Q^2(GeV^2)% error
on the tail subtraction. This error usually contributes less than
2.5% to the systematic error.

INELASTIC RADIATIVE CORRECTIONS

The inelastic radiative corrections are applied to the tail-
subtracted data. This correction accounts for the radiation from
scatters occurring at low missing mass giving events at higher measured
missing mass. We have used a new technique involving an iterative
procedure with an analytic representation of all data taken at one
angle on the same target.

The exact radiation calculation (as done for the elastic
tails) would take too much computer time (as presently coded).
Instead, an equivalent radiator is used to simulate internal
bremsstrahlung. The hard radiation of this internal radiator is
strongly peaked along the incoming and the outgoing directions of
the electron, hence little angular deflection in the trajectory of

the electron occurs. Usually an "angle peaking" approximation is used which includes only radiation along these two directions. As the electron can be degraded in energy both before and after the scatter, we must, in principle, integrate over all E_o's and higher E' s that can contribute to scattering at the measured point. Examination of this double integration shows that most of the contribution to the integral comes from either radiation before or radiation after the scatter, but not both. One takes advantage of this by an "energy peaking" approximation. The two-dimensional integral in E_o and E' is well approximated by three terms: a) scattering at the measured E_o of the beam with contributions coming from all allowed higher E's; b) scattering at the measured E' with contributions coming from incident energies down to the lowest allowed E_o's (determined by one pion threshold); and c) scatters in the "near" region with soft photon emission.

The new technique uses the fact that radiating "known" cross sections involves only integrals. What is done in practice is to start with a reasonable model of the data (such as an ω' fit (Ref. IV-7)). Using the model, a radiative correction ratio $R_{rad} = \sigma_{model}/\sigma_{model\text{-}radiated}$ is calculated for each data point, and a radiatively corrected set of cross sections is formed by multiplying the data and its error point-for-point by this ratio. The model is then refit to the data. New radiative corrections are calculated and reapplied to the tail subtracted data. A flow chart of the procedure is shown in Fig. IV-2. The major part of the correction

σ_{exp} (Input)

$\sigma_{rare} = \sigma_{exp} - \sigma_{elastic\ tails}$ — Elastic Scattering Correction

$R_{rad} = \dfrac{\sigma_{model}}{\sigma_{model\text{-}rad}}$ — Model From Previous Data Gives First Estimate of R_{rad}

$\sigma_{final} = \sigma_{rare} * R_{rad}$ — Model Radiative Corrections

$\sigma_{model} = $ Fit to σ_{final} — Refit Model

$R_{rad} = \dfrac{\sigma_{model}}{\sigma_{model\text{-}radiation}}$ — Form Radiative Correction Ratio from Model

$R_{rad_{old}} \overset{?}{=} R_{rad}$ No — Check for Convergence

σ_{final}
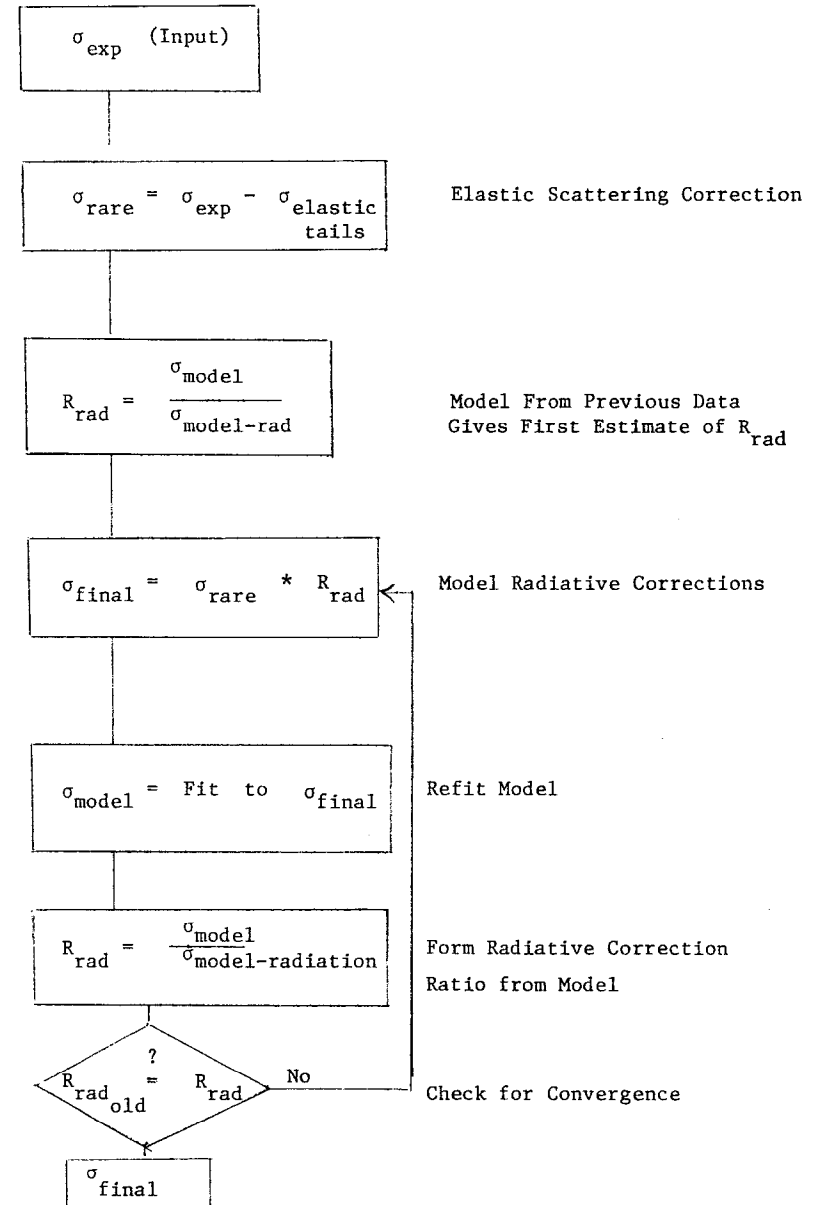
FIG. IV-2

comes from energies near the measured energies, and since the cross sections are quite smooth, convergence occurs after a few iterations. The model used has no resonance structure past a W = 2.0 GeV. Any high mass resonance (W >2.0 GeV) will not have the appropriate radiative correction enhancement. This probably is a valid assumption as no high mass resonances have been observed for W <5.7 GeV for the process e+p → e'+X (Ref. IV-8) for Q^2's of approximately 1 GeV^2. The radiative correction ratios for some of the lines are shown in Fig. IV-3. The deuterium ratios are within .02 of the plotted hydrogen ratios. The formulas used for radiating the fitted cross sections are given in Ref. IV-4.

Some questions arise as to the correctness of the peaking approximation used in the radiative correction procedure. To check its validity, the "exact" (as in the elastic case) radiative correction ratios were calculated at some W points spanning the range of the present data, again using an ω' model as the source of the W_1 and W_2 structure functions. Two independent programs agreed to better than 5% with the peaking approximations and the equivalent radiator used for W <4.5 GeV (Ref. IV-9).

We quote the systematic error given in Ref. IV-4 for the radiative correction ratio: ±3% near threshold (W ≈1.3 GeV) growing to ±5% at the lowest values of E'. This error is considered systematic and will be combined with the other sources of systematic error.

The radiative correction ratios for representative incident energies at 50° and 60°. The ratios are plotted for W > 1.75 GeV.
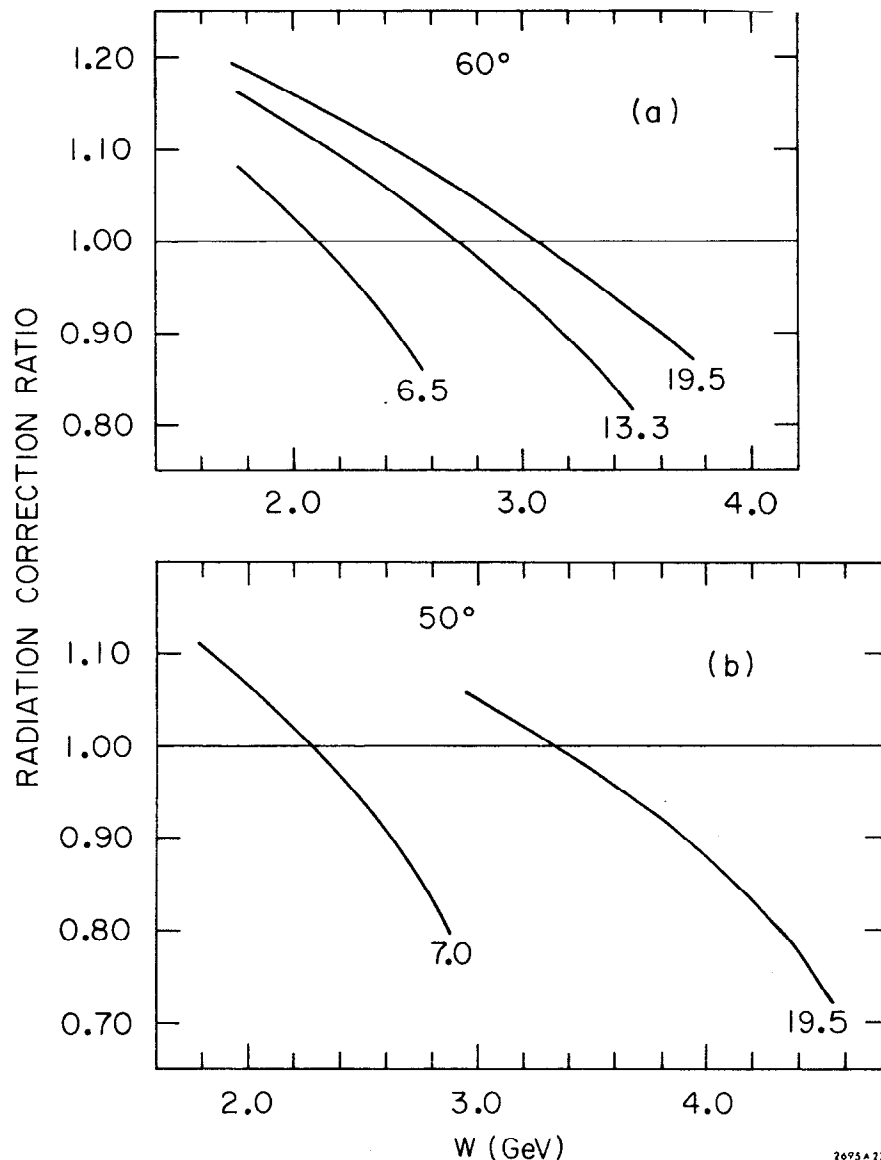
FIG. IV-3

SMEARING CORRECTIONS

The deuteron is a very loosely bound structure consisting of a neutron and a proton. A first approximation for the deuteron cross section would be the sum of the proton and the neutron cross sections. This approximation is wrong to the extent that the motion of the nucleons inside the deuteron distort the free nucleon structure functions. Structure functions so modified are referred to as "smeared" structure functions.

The smearing effect is most pronounced for quasi-elastic scattering where the electron scatters elastically off one of the nucleons inside the deuteron resulting in nuclear breakup. The narrow electron elastic peak which results from scattering off free nucleons is broadened or "smeared out" because of the target particle's motion.

The smearing correction is done within the framework of the impulse approximation. The electron is assumed to interact with only one of the nucleons, the other nucleon being "spit" off as a free "spectator." Taking the Fermi-motion into account relativistically (that is conserving energy and momentum at all times) puts the interacting nucleon off its mass-shell. So not only is the target particle not in its rest frame, but it is off its mass shell. The first effect blurs the angular resolution of the scattering and both effects shift the invariant mass of the interaction. We follow the procedure given in Appendix C to calculate smeared structure functions from unsmeared ones. The smearing

correction for the inelastic structure functions is often parameterized as a smearing ratio, $S_i = W_{1i}/W_{1i}(\text{SMEARED})$. More analysis of the smearing problem is detailed in Appendix C.

The hydrogen cross section subtraction from the deuterium data was done with a smooth analytic function representing the proton data appropriately smeared. This procedure is preferable over point-for-point data subtraction corrected by a smearing ratio. Smearing ratios are not model insensitive because of the large kinematic range of the smearing integrals (typically $\pm$ .2 units in ω'). The calculated smeared cross sections are less model sensitive because of this. Forming a smearing ratio reintroduces the model in a local manner. Furthermore, the same arguments given for the empty target subtraction and the positron subtraction by a model subtraction are valid here too. The error assigned to this procedure was $\pm5\%$ of the σ_p(model-smeared) and was considered as part of the systematic error in σ_N. The smeared neutron W_{1N} structure function can be obtained by subtracting the calculated smeared proton contribution from the deuteron:

$$(4.5) \qquad W_{1N}(\text{SMEARED}) = W_{1D} - W_{1P}(\text{SMEARED})$$

The smearing procedure like radiative corrections is not invertible. An iterative procedure, similar to that used to account for radiation processes, was used to extract the unsmeared neutron structure functions. To extract an unsmeared neutron, we have to use the smearing ratios despite their model sensitivity. The first estimate

of the neutron smearing ratios are the calculated proton smearing

ratios. This yields an "unsmeared" neutron which generates new

smearing ratios through a fitted model to that neutron data and

so on. A diagram of the logic flow is shown in Fig. IV-4. We

take the error in the smearing ratios to be similar to that cal-

culated in Ref. IV-9 and is typically less than $\pm$ 1.5%.

The principal problem that arises with smearing compared

with radiation is that smearing extends over a larger kinematic

range (see Appendix C). The principal smearing contribution comes

from approximately $\pm$ 2 units in ω' about the ω' for which the

calculation is being made. This aspect of smearing makes conver-

gence occur slowly in this iteration procedure, especially for non-

linear parameterizations of structure such as in the resonance

region.

The maximum deviation of the smearing ratios from unity occurs as

$x' \to 1$ (S_p = .62 for x' = .92, but for 90% of the data where x' <.8

the ratio lies in the range $0.90 < S_p < 1.02$). The smearing ratios

calculated for this experiment are shown in Fig. IV-5.

THE EXTRACTION OF W_1

As stated in Chapter I, the large angle cross sections are

insensitive to the value of $R \equiv \sigma_s/\sigma_t$ used in extracting W_1 from the

cross sections due to the smallness of ϵ in the measured region.

The contribution of σ_s is always less than 4.5% for R = .18. We

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UNSMEARING FLOW CHART

$$\sigma_{p\ Model} \xrightarrow{\text{Fit to}} \sigma_p\ \text{Data}$$ — Fit Proton Data

$$\sigma^{smeared}_{p\ Model} = \int d^3 P_s |\phi_{ps}|^2 \mathcal{F}\ \sigma_{p\ Model}$$ — Smear Proton Model

$$\sigma^{smeared}_N = \sigma_D - \sigma^{smeared}_{p\ model}$$ — Extract Smeared Neutron by Subtraction

$$S_N = \frac{\sigma_{p\ model}}{\sigma^{smeared}_{p\ model}}$$ — Form First Estimate of Neutron Smearing Ratio

$$\sigma_N = \sigma^{smeared}_N\ S_N$$ — Form Unsmeared Neutron

$$\sigma_{N\ model} \xrightarrow{\text{Fit to}} \sigma_N\ \text{Data}$$ — Fit Neutron Data

$$\sigma^{smear}_{N\ model} = \int d^3 P_s |\phi_{Ps}|^2 \mathcal{F}\ \sigma_{N\ model}$$ — Smear Neutron Model

$$S_N = \frac{\sigma_{N\ model}}{\sigma^{smear}_{N\ model}}$$ — Form Neutron Smearing Ratio

$$S_N^{old} = S_N^{new}$$ — Convergence

No

Yes

σ_N Finished

FIG. IV-4

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FIG. IV - 5

The smearing ratios $(\sigma_{model}/\sigma_{model\ smeared})$

for the neutron, S_N, and the proton, S_P.

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calculated W_1 by

(4.6) $W_1 = \dfrac{\sigma_{exp}}{\sigma_{mott}} \dfrac{1}{2 \tan^2(\theta/2)} \cdot \dfrac{1 - \epsilon}{1 + \epsilon R}$

We assign an error of $\pm$ .5 ϵR to this procedure, and this error
is taken as part of the total systematic error. A value of .18
was used for R.

SUMMARY OF SYSTEMATIC ERRORS

Systematic errors of two types are considered: 1) point-to-
point errors and 2) overall normalizations errors. Counter
efficiencies, tracking efficiencies, the pion subtraction factor,
and the fast electronic flag efficiencies are considered here as
sources of point-to-point fluctuations. As it is unclear what
correlations may exist between these contributions, the conservative
approach of adding them linearly together has been adopted. The
detailed contributions and the typical sizes of the estimated errors
are given in Table IV-2.

The overall systematic errors which contribute are given in
Table IV-3. In each category the detailed contributions with
typical values of the error are shown. We choose to combine these
errors by adding all contributions to a particular category together
linearly and then combining the various categories in quadrature
(see #3 at the bottom of Table IV-3). For the N/P determination
many systematic errors tend to cancel, such as those associated with
the beam, the spectrometer, the inelastic radiative corrections, and
the W_1 extraction. Others will partially cancel, such as point-to-

TABLE IV - 2

POINT-TO-POINT SYSTEMATIC ERRORS

Contribution		Size (%)	
		P	D
PHS Cuts:	CT	.05	.05
	TFT	.5	.5
	TA1	.05	.05
	TASUM	.2	.2
Codes:	ϕ Cut	.2	.2
	P_{2-19} Cut	.3	.3
	θ_{2-14} Cut	.3	.3
	Code Class 4-6	.5	.8
Flags:	ORX	.5	.8
	STR2	.05	.05
Over One:		0.0	0.0
Pion Subtraction Factor:		.5	1.0
Linear Sum:		3.2	4.3

TABLE IV - 3

OVERALL SYSTEMATIC ERRORS

Category	Contribution	Typical Size (%)		
		P	D	N
Beam	E_o	.8	.8	–
	Flux	.6	.6	–
	Halo	.2	.2	–
Target	Density	.5	.5	–
	Purity	.5	.5	–
	Length	.5	.5	–
Spectrometer	Solid Angle	1.	1.	–
Measured Subtraction	Empty Target	.7	.4	–
	Positron Yields	2.	2.	–
Radiative Corrections	Tails	1.	.8	–
	Inelastic	4.	4.	–
W_1 Extraction		1.	1.	
Neutron Extraction	P Subtraction	–	–	3
	Unsmearing	–	–	2
1) Linear Addition		11.9	12.3	17.3
2) Quadrature Addition		5.0	5.0	6.2
3) Contributions Linear, Categories quadratically		6.3	6.0	7.8
4) 3) Added in quadrature to Point-to Point Error		7.1	7.4	8.9

4) For N/P = 6.0%

point systematics, radiative tails, and the measured subtractions. For these the average of the P and D errors were used and added in quadrature with the other contributing sources of error. The resulting systematic error in the N/P ratio is approximately $\pm$ 6.0% including the point-to-point contribution ($\pm$ 4.9% excluding point-to-point errors).

These Tables include various kinematic quantities, the cross sections for P, D, and N, and W_1 for P and D. Given in the parenthesis for each P and D measurement are: 1) the statistical counting errors and 2) the total systematic error for that point. For the neutron cross sections the "counting" error includes the 5% hydrogen subtraction error added in quadrature to the statistical counting error of the deuterium cross section.

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TABLE IV - 4

19.5 GeV 50°

W (GeV)	Q^2 (GeV^2)	ϵ	x	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) PROTON	2M W_1^P
3.100	24.555	0.1428	0.738	0.719	2.973 ± (0.668, 0.135)	0.03054 ± (0.00686, 0.00138)
3.200	24.000	0.1397	0.719	0.701	3.218 ± (0.435, 0.173)	0.03319 ± (0.00449, 0.00178)
3.300	23.427	0.1365	0.701	0.683	3.752 ± (0.351, 0.220)	0.03887 ± (0.00364, 0.00228)
3.400	22.836	0.1332	0.681	0.664	4.616 ± (0.330, 0.279)	0.04803 ± (0.00344, 0.00291)
3.500	22.228	0.1298	0.662	0.645	5.931 ± (0.340, 0.353)	0.06199 ± (0.00356, 0.00369)
3.600	21.603	0.1263	0.641	0.625	7.369 ± (0.390, 0.443)	0.07737 ± (0.00410, 0.00465)
3.700	20.959	0.1227	0.621	0.605	7.652 ± (0.424, 0.557)	0.08073 ± (0.00447, 0.00588)
3.800	20.298	0.1189	0.600	0.584	10.059 ± (0.508, 0.707)	0.10664 ± (0.00539, 0.00750)
3.900	19.620	0.1151	0.578	0.563	12.063 ± (0.581, 0.899)	0.12853 ± (0.00619, 0.00958)
4.000	18.924	0.1112	0.556	0.542	13.680 ± (0.673, 1.155)	0.14652 ± (0.00721, 0.01237)
4.100	18.210	0.1071	0.533	0.520	15.486 ± (0.849, 1.511)	0.16674 ± (0.00914, 0.01627)
4.200	17.478	0.1029	0.510	0.498	19.058 ± (1.157, 2.001)	0.20630 ± (0.01253, 0.02166)
4.300	16.729	0.0986	0.487	0.475	23.916 ± (1.510, 2.759)	0.26033 ± (0.01644, 0.03003)
4.400	15.963	0.0943	0.463	0.452	29.137 ± (1.895, 3.978)	0.31896 ± (0.02075, 0.04355)
4.500	15.178	0.0897	0.439	0.428	29.083 ± (2.622, 5.990)	0.32021 ± (0.02887, 0.06595)
4.600	14.376	0.0851	0.415	0.405	39.494 ± (6.186, 9.499)	0.43740 ± (0.06851, 0.10520)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) DEUTERON	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) NEUTRON	2M W_1^D
3.100	3.90 ± (0.80, 0.22)	1.29 ± (0.80, 0.20)	0.04007 ± (0.00818, 0.00224)
3.200	3.81 ± (0.50, 0.28)	0.69 ± (0.52, 0.26)	0.03933 ± (0.00514, 0.00286)
3.300	5.10 ± (0.43, 0.35)	1.27 ± (0.47, 0.34)	0.05283 ± (0.00443, 0.00365)
3.400	6.94 ± (0.42, 0.45)	2.27 ± (0.48, 0.44)	0.07213 ± (0.00436, 0.00465)
3.500	7.93 ± (0.41, 0.57)	2.30 ± (0.50, 0.56)	0.08283 ± (0.00431, 0.00591)
3.600	10.34 ± (0.49, 0.72)	3.58 ± (0.59, 0.72)	0.10856 ± (0.00510, 0.00754)
3.700	13.02 ± (0.59, 0.92)	4.94 ± (0.72, 0.93)	0.13732 ± (0.00620, 0.00967)
3.800	16.72 ± (0.74, 1.19)	7.12 ± (0.88, 1.21)	0.17728 ± (0.00781, 0.01263)
3.900	18.20 ± (0.85, 1.56)	6.79 ± (1.03, 1.60)	0.19388 ± (0.00905, 0.01664)
4.000	24.09 ± (1.07, 2.08)	10.69 ± (1.27, 2.14)	0.25804 ± (0.01141, 0.02229)
4.100	28.50 ± (1.37, 2.63)	12.76 ± (1.59, 2.72)	0.30689 ± (0.01473, 0.02837)
4.200	30.99 ± (1.87, 3.46)	12.46 ± (2.10, 3.58)	0.33544 ± (0.02025, 0.03743)
4.300	38.90 ± (2.74, 4.80)	17.36 ± (2.96, 4.98)	0.42340 ± (0.02986, 0.05223)
4.400	51.97 ± (3.69, 6.93)	27.19 ± (3.92, 7.21)	0.56895 ± (0.04039, 0.07590)
4.500	51.20 ± (4.93, 10.49)	22.10 ± (5.16, 10.91)	0.56372 ± (0.05427, 0.11549)
4.600	77.90 ± (11.79, 16.57)	45.02 ± (11.95, 17.25)	0.86272 ± (0.13061, 0.18354)

TABLE IV – 4

16.0 GeV 50°

W (GeV)	Q^2 (GeV^2)	ϵ	x^-	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$)	$2M\,W_1^P$
					PROTON	
2.700	20.285	0.1732	0.760	0.736	3.207 ± (1.043, 0.158)	0.02128 ± (0.00692, 0.00105)
2.800	19.812	0.1695	0.740	0.716	2.819 ± (0.588, 0.208)	0.01880 ± (0.00392, 0.00139)
2.900	19.323	0.1655	0.720	0.697	4.484 ± (0.539, 0.272)	0.03037 ± (0.00361, 0.00182)
3.000	18.816	0.1614	0.699	0.676	5.967 ± (0.544, 0.352)	0.04024 ± (0.00367, 0.00238)
3.100	18.292	0.1572	0.677	0.656	8.144 ± (0.618, 0.454)	0.05524 ± (0.00419, 0.00308)
3.200	17.751	0.1527	0.655	0.634	10.711 ± (0.798, 0.581)	0.07308 ± (0.00544, 0.00396)
3.300	17.192	0.1482	0.632	0.612	10.637 ± (0.884, 0.742)	0.07303 ± (0.00607, 0.00510)
3.400	16.617	0.1435	0.609	0.590	15.507 ± (1.144, 0.951)	0.10715 ± (0.00791, 0.00657)
3.500	16.024	0.1386	0.585	0.567	19.369 ± (1.402, 1.218)	0.13471 ± (0.00975, 0.00847)
3.600	15.414	0.1335	0.561	0.543	19.451 ± (1.431, 1.562)	0.13620 ± (0.01002, 0.01094)
3.700	14.787	0.1283	0.536	0.519	26.463 ± (1.727, 2.025)	0.18658 ± (0.01218, 0.01427)
3.800	14.143	0.1229	0.511	0.495	30.870 ± (2.054, 2.672)	0.21920 ± (0.01459, 0.01897)
3.900	13.482	0.1174	0.485	0.470	36.776 ± (2.559, 3.640)	0.26304 ± (0.01831, 0.02604)
4.000	12.803	0.1117	0.459	0.445	39.231 ± (3.060, 5.199)	0.28270 ± (0.02205, 0.03747)
4.100	12.107	0.1058	0.432	0.419	46.261 ± (3.885, 7.852)	0.33591 ± (0.02821, 0.05702)
4.200	11.394	0.0998	0.405	0.392	50.718 ± (5.697, 12.562)	0.37116 ± (0.04169, 0.09193)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$)		$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$)		$2M\,W_1^D$
	DEUTERON		NEUTRON		
2.700	6.22 ± (1.56,	0.25)	2.71 ± (1.56,	0.23)	0.04130 ± (0.01035, 0.00166)
2.800	7.40 ± (1.02,	0.33)	3.10 ± (1.04,	0.30)	0.04938 ± (0.00679, 0.00218)
2.900	7.63 ± (0.77,	0.42)	2.41 ± (0.81,	0.40)	0.05116 ± (0.00518, 0.00284)
3.000	9.40 ± (0.76,	0.55)	2.95 ± (0.82,	0.53)	0.06338 ± (0.00513, 0.00370)
3.100	11.26 ± (0.83,	0.71)	3.33 ± (0.92,	0.70)	0.07639 ± (0.00565, 0.00480)
3.200	15.21 ± (1.13,	0.92)	5.48 ± (1.23,	0.92)	0.10381 ± (0.00769, 0.00628)
3.300	18.45 ± (1.39,	1.20)	6.61 ± (1.51,	1.21)	0.12667 ± (0.00951, 0.00826)
3.400	22.08 ± (1.61,	1.58)	7.74 ± (1.76,	1.61)	0.15259 ± (0.01111, 0.01091)
3.500	24.80 ± (1.87,	2.07)	7.47 ± (2.07,	2.12)	0.17249 ± (0.01304, 0.01440)
3.600	35.17 ± (2.48,	2.70)	14.52 ± (2.69,	2.78)	0.24624 ± (0.01734, 0.01888)
3.700	40.21 ± (3.38,	3.54)	15.54 ± (3.61,	3.65)	0.28352 ± (0.02385, 0.02493)
3.800	59.61 ± (4.84,	4.62)	30.75 ± (5.07,	4.79)	0.42325 ± (0.03436, 0.03282)
3.900	57.60 ± (4.92,	6.26)	23.03 ± (5.24,	6.52)	0.41196 ± (0.03522, 0.04489)
4.000	57.77 ± (4.89,	8.97)	16.69 ± (5.33,	9.33)	0.41626 ± (0.03522, 0.06462)
4.100	87.41 ± (5.56,	13.56)	40.06 ± (6.09,	14.11)	0.63457 ± (0.04037, 0.09846)
4.200	107.46 ± (7.80,	21.77)	52.40 ± (8.33,	22.66)	0.78639 ± (0.05706, 0.15930)

TABLE IV - 4

13.5 GeV 50°

W (GeV)	Q^2 (GeV²)	ε	x	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) PROTON	$2M W_1^P$
2.400	17.122	0.2030	0.778	0.748	3.869 ± (0.969, 0.240)	0.01753 ± (0.00439, 0.00109)
2.500	16.712	0.1985	0.757	0.728	4.395 ± (1.109, 0.311)	0.02004 ± (0.00505, 0.00142)
2.600	16.285	0.1938	0.735	0.707	7.810 ± (1.184, 0.406)	0.03585 ± (0.00543, 0.00186)
2.700	15.841	0.1889	0.712	0.685	10.028 ± (1.119, 0.501)	0.04635 ± (0.00517, 0.00232)
2.800	15.381	0.1838	0.688	0.662	11.449 ± (1.127, 0.664)	0.05330 ± (0.00525, 0.00309)
2.900	14.904	0.1784	0.664	0.639	14.362 ± (1.241, 0.874)	0.06736 ± (0.00582, 0.00410)
3.000	14.410	0.1729	0.640	0.616	18.431 ± (1.462, 1.090)	0.08711 ± (0.00691, 0.00515)
3.100	13.899	0.1671	0.614	0.591	20.588 ± (1.814, 1.368)	0.09808 ± (0.00864, 0.00652)
3.200	13.372	0.1611	0.588	0.566	24.944 ± (2.181, 1.757)	0.11981 ± (0.01048, 0.00844)
3.300	12.828	0.1549	0.562	0.541	33.571 ± (2.518, 2.284)	0.16262 ± (0.01220, 0.01107)
3.400	12.267	0.1485	0.535	0.515	41.059 ± (3.162, 3.000)	0.20063 ± (0.01545, 0.01466)
3.500	11.689	0.1418	0.507	0.488	47.265 ± (3.504, 3.914)	0.23304 ± (0.01727, 0.01930)
3.600	11.095	0.1349	0.479	0.461	53.889 ± (3.570, 5.329)	0.26816 ± (0.01776, 0.02652)
3.700	10.484	0.1278	0.450	0.434	67.394 ± (4.565, 7.605)	0.33854 ± (0.02293, 0.03820)
3.800	9.856	0.1204	0.421	0.406	73.099 ± (6.374, 11.205)	0.37078 ± (0.03233, 0.05684)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) DEUTERON	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) NEUTRON	$2M W_1^D$
2.400	5.80 ± (1.97, 0.85)	1.57 ± (1.97, 0.75)	0.02627 ± (0.00892, 0.00386)
2.500	7.14 ± (1.56, 0.80)	1.70 ± (1.59, 0.74)	0.03256 ± (0.00712, 0.00366)
2.600	10.40 ± (1.42, 0.59)	3.38 ± (1.47, 0.56)	0.04775 ± (0.00654, 0.00269)
2.700	11.27 ± (1.26, 0.81)	2.53 ± (1.33, 0.78)	0.05208 ± (0.00581, 0.00372)
2.800	16.17 ± (1.39, 1.02)	5.11 ± (1.50, 1.01)	0.07526 ± (0.00649, 0.00474)
2.900	21.26 ± (1.65, 1.28)	7.50 ± (1.79, 1.28)	0.09970 ± (0.00775, 0.00600)
3.000	27.65 ± (2.13, 1.65)	10.63 ± (2.30, 1.65)	0.13069 ± (0.01008, 0.00772)
3.100	29.90 ± (2.95, 2.20)	8.95 ± (3.13, 2.25)	0.14243 ± (0.01404, 0.01049)
3.200	39.31 ± (4.11, 3.10)	13.72 ± (4.32, 3.19)	0.18881 ± (0.01975, 0.01491)
3.300	50.47 ± (4.96, 3.77)	19.49 ± (5.22, 3.89)	0.24447 ± (0.02404, 0.01827)
3.400	62.25 ± (5.98, 4.96)	24.93 ± (6.29, 5.14)	0.30420 ± (0.02923, 0.02424)
3.500	72.89 ± (5.99, 6.71)	28.04 ± (6.44, 6.96)	0.35940 ± (0.02955, 0.03307)
3.600	93.15 ± (6.05, 9.19)	39.88 ± (6.66, 9.55)	0.46352 ± (0.03012, 0.04571)
3.700	101.11 ± (6.64, 13.17)	37.46 ± (7.41, 13.71)	0.50792 ± (0.03334, 0.06617)
3.800	121.10 ± (8.69, 20.24)	46.00 ± (9.54, 21.07)	0.61429 ± (0.04406, 0.10269)

TABLE IV - 4

7.0 GeV 50°

W (GeV)	Q^2 (GeV^2)	ϵ	x	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$)	$2M W_1^P$
					PROTON	
2.000	7.283	0.3058	0.700	0.645	69.54 ± (6.57, 4.32)	0.07247 ± (0.00685, 0.00450)
2.100	6.985	0.2950	0.664	0.613	82.03 ± (7.14, 5.74)	0.08698 ± (0.00758, 0.00609)
2.200	6.672	0.2835	0.628	0.580	112.96 ± (9.55, 7.55)	0.12197 ± (0.01031, 0.00815)
2.300	6.345	0.2713	0.590	0.545	155.33 ± (11.11, 10.10)	0.17095 ± (0.01223, 0.01111)
2.400	6.003	0.2583	0.552	0.510	185.54 ± (12.03, 13.33)	0.20830 ± (0.01350, 0.01496)
2.500	5.647	0.2446	0.513	0.475	237.86 ± (13.94, 17.26)	0.27263 ± (0.01598, 0.01979)
2.600	5.276	0.2300	0.473	0.438	300.61 ± (14.80, 22.73)	0.35208 ± (0.01734, 0.02662)
2.700	4.891	0.2146	0.433	0.402	387.85 ± (16.35, 30.75)	0.46455 ± (0.01959, 0.03683)
2.800	4.491	0.1984	0.392	0.364	460.71 ± (19.41, 40.38)	0.56482 ± (0.02380, 0.04950)
2.900	4.076	0.1813	0.351	0.326	536.56 ± (28.07, 56.81)	0.67385 ± (0.03526, 0.07134)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$)	$2M W_1^D$
	DEUTERON	NEUTRON	
2.000	97.33 ± (7.77, 6.34)	32.02 ± (8.82, 6.46)	0.10144 ± (0.00810, 0.00661)
2.100	139.14 ± (8.49, 8.39)	49.99 ± (9.61, 8.63)	0.14754 ± (0.00900, 0.00890)
2.200	189.78 ± (11.30, 11.56)	74.40 ± (12.77, 11.97)	0.20493 ± (0.01220, 0.01248)
2.300	224.46 ± (12.23, 16.03)	75.94 ± (14.42, 16.67)	0.24703 ± (0.01346, 0.01765)
2.400	271.43 ± (13.83, 21.61)	82.77 ± (16.87, 22.52)	0.30472 ± (0.01553, 0.02426)
2.500	368.26 ± (17.44, 29.54)	133.57 ± (21.24, 30.80)	0.42210 ± (0.01999, 0.03386)
2.600	472.68 ± (21.05, 38.00)	183.25 ± (25.85, 39.61)	0.55362 ± (0.02465, 0.04451)
2.700	558.90 ± (24.17, 48.41)	204.17 ± (30.39, 50.37)	0.66945 ± (0.02895, 0.05799)
2.800	700.62 ± (25.09, 63.74)	270.59 ± (33.62, 66.13)	0.85895 ± (0.03076, 0.07814)
2.900	763.88 ± (35.72, 83.95)	240.80 ± (45.74, 86.84)	0.95934 ± (0.04486, 0.10543)

TABLE IV - 4

19.5 GeV 60°

W (GeV)	Q² (GeV²)	ε	x"	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$)	$2M\,W_1^P$
					PROTON	
2.000	30.544	0.1151	0.907	0.884	0.053 ± (0.010, 0.004)	0.00079 ± (0.00015, 0.00006)
2.100	30.170	0.1137	0.895	0.872	0.053 ± (0.011, 0.006)	0.00080 ± (0.00017, 0.00008)
2.200	29.778	0.1123	0.883	0.860	0.131 ± (0.015, 0.008)	0.00196 ± (0.00022, 0.00011)
2.300	29.367	0.1108	0.869	0.847	0.140 ± (0.016, 0.010)	0.00210 ± (0.00024, 0.00015)
2.400	28.938	0.1093	0.856	0.834	0.244 ± (0.020, 0.014)	0.00367 ± (0.00031, 0.00021)
2.500	28.491	0.1077	0.841	0.820	0.304 ± (0.024, 0.019)	0.00458 ± (0.00037, 0.00028)
2.600	28.026	0.1060	0.827	0.806	0.379 ± (0.030, 0.025)	0.00573 ± (0.00045, 0.00038)
2.700	27.543	0.1043	0.811	0.791	0.563 ± (0.037, 0.034)	0.00852 ± (0.00056, 0.00051)
2.800	27.041	0.1025	0.795	0.775	0.716 ± (0.044, 0.045)	0.01086 ± (0.00067, 0.00068)
2.900	26.521	0.1006	0.779	0.759	0.922 ± (0.055, 0.060)	0.01402 ± (0.00084, 0.00091)
3.000	25.983	0.0986	0.762	0.743	1.039 ± (0.067, 0.079)	0.01585 ± (0.00102, 0.00121)
3.100	25.426	0.0966	0.744	0.726	1.383 ± (0.090, 0.105)	0.02114 ± (0.00138, 0.00160)
3.200	24.852	0.0945	0.726	0.708	1.825 ± (0.119, 0.138)	0.02797 ± (0.00183, 0.00211)
3.300	24.259	0.0923	0.708	0.690	2.317 ± (0.147, 0.181)	0.03562 ± (0.00226, 0.00279)
3.400	23.648	0.0901	0.689	0.672	2.997 ± (0.180, 0.238)	0.04620 ± (0.00277, 0.00368)
3.500	23.018	0.0878	0.669	0.653	3.508 ± (0.201, 0.315)	0.05423 ± (0.00311, 0.00487)
3.600	22.370	0.0854	0.649	0.633	4.730 ± (0.234, 0.418)	0.07335 ± (0.00362, 0.00648)
3.700	21.704	0.0829	0.629	0.613	5.679 ± (0.266, 0.560)	0.08835 ± (0.00414, 0.00870)
3.800	21.020	0.0804	0.608	0.593	6.348 ± (0.346, 0.760)	0.09906 ± (0.00541, 0.01186)
3.900	20.318	0.0778	0.586	0.572	7.509 ± (0.346, 1.051)	0.11758 ± (0.00542, 0.01646)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$)	$2M\,W_1^D$
	DEUTERON	NEUTRON	
2.000	0.145 ± (0.019, 0.010)	0.009 ± (0.016, 0.004)	0.00217 ± (0.00028, 0.00015)
2.100	0.176 ± (0.019, 0.013)	0.011 ± (0.016, 0.006)	0.00264 ± (0.00028, 0.00019)
2.200	0.305 ± (0.023, 0.017)	0.057 ± (0.020, 0.009)	0.00456 ± (0.00034, 0.00025)
2.300	0.336 ± (0.025, 0.022)	0.053 ± (0.027, 0.013)	0.00505 ± (0.00037, 0.00032)
2.400	0.429 ± (0.028, 0.026)	0.076 ± (0.032, 0.018)	0.00645 ± (0.00042, 0.00042)
2.500	0.484 ± (0.033, 0.037)	0.063 ± (0.038, 0.026)	0.00729 ± (0.00049, 0.00055)
2.600	0.682 ± (0.042, 0.048)	0.136 ± (0.048, 0.035)	0.01029 ± (0.00063, 0.00072)
2.700	0.826 ± (0.049, 0.062)	0.155 ± (0.058, 0.049)	0.01251 ± (0.00074, 0.00074)
2.800	1.344 ± (0.065, 0.081)	0.454 ± (0.075, 0.067)	0.02039 ± (0.00098, 0.00153)
2.900	1.342 ± (0.074, 0.106)	0.302 ± (0.088, 0.091)	0.02041 ± (0.00112, 0.00161)
3.000	1.774 ± (0.095, 0.138)	0.477 ± (0.113, 0.123)	0.02705 ± (0.00145, 0.00211)
3.100	2.306 ± (0.127, 0.182)	0.703 ± (0.148, 0.166)	0.03526 ± (0.00194, 0.00278)
3.200	2.589 ± (0.156, 0.239)	0.647 ± (0.183, 0.224)	0.03969 ± (0.00240, 0.00367)
3.300	3.620 ± (0.198, 0.317)	1.226 ± (0.229, 0.302)	0.05565 ± (0.00304, 0.00487)
3.400	4.505 ± (0.238, 0.421)	1.597 ± (0.277, 0.408)	0.06945 ± (0.00367, 0.00649)
3.500	5.672 ± (0.282, 0.560)	2.154 ± (0.331, 0.557)	0.08769 ± (0.00436, 0.00875)
3.600	7.032 ± (0.338, 0.761)	2.798 ± (0.398, 0.757)	0.10906 ± (0.00524, 0.01100)
3.700	8.611 ± (0.416, 1.031)	3.537 ± (0.488, 1.037)	0.13396 ± (0.00647, 0.01604)
3.800	9.785 ± (0.573, 1.417)	3.721 ± (0.650, 1.437)	0.15272 ± (0.00894, 0.02212)
3.900	11.749 ± (0.573, 1.983)	4.312 ± (0.650, 2.025)	0.18396 ± (0.00897, 0.03106)

TABLE IV - 4

16.0 GeV 60°

W (GeV)	Q^2 (GeV^2)	ϵ	x	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$) PROTON	$2M\ W_1^P$
2.000	24.085	0.1336	0.885	0.858	0.203 ± (0.021, 0.012)	0.00199 ± (0.00020, 0.00011)
2.100	23.718	0.1317	0.870	0.843	0.362 ± (0.027, 0.016)	0.00355 ± (0.00026, 0.00016)
2.200	23.333	0.1297	0.855	0.828	0.461 ± (0.033, 0.023)	0.00455 ± (0.00032, 0.00023)
2.300	22.930	0.1276	0.839	0.813	0.565 ± (0.043, 0.032)	0.00558 ± (0.00043, 0.00032)
2.400	22.509	0.1254	0.822	0.796	0.748 ± (0.056, 0.045)	0.00741 ± (0.00056, 0.00044)
2.500	22.071	0.1231	0.804	0.779	1.138 ± (0.075, 0.061)	0.01132 ± (0.00074, 0.00061)
2.600	21.614	0.1207	0.786	0.762	1.327 ± (0.090, 0.084)	0.01324 ± (0.00090, 0.00084)
2.700	21.140	0.1182	0.767	0.744	1.872 ± (0.120, 0.113)	0.01873 ± (0.00120, 0.00113)
2.800	20.648	0.1155	0.748	0.725	2.439 ± (0.156, 0.152)	0.02449 ± (0.00156, 0.00153)
2.900	20.137	0.1128	0.728	0.705	3.087 ± (0.196, 0.204)	0.03111 ± (0.00197, 0.00205)
3.000	19.609	0.1100	0.707	0.685	3.353 ± (0.221, 0.271)	0.03391 ± (0.00224, 0.00274)
3.100	19.063	0.1071	0.686	0.665	4.673 ± (0.262, 0.359)	0.04744 ± (0.00266, 0.00365)
3.200	18.499	0.1041	0.664	0.644	5.516 ± (0.323, 0.477)	0.05622 ± (0.00329, 0.00486)
3.300	17.918	0.1009	0.642	0.622	7.576 ± (0.480, 0.634)	0.07754 ± (0.00491, 0.00649)
3.400	17.318	0.0977	0.619	0.600	9.528 ± (0.776, 0.852)	0.09790 ± (0.00797, 0.00875)
3.500	16.700	0.0944	0.595	0.577	11.109 ± (0.776, 1.159)	0.11463 ± (0.00800, 0.01196)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$) DEUTERON	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$) NEUTRON	$2M\ W_1^D$
2.000	0.425 ± (0.032, 0.025)	0.055 ± (0.036, 0.014)	0.00416 ± (0.00032, 0.00025)
2.100	0.574 ± (0.037, 0.033)	0.094 ± (0.042, 0.021)	0.00564 ± (0.00036, 0.00033)
2.200	0.730 ± (0.046, 0.045)	0.126 ± (0.053, 0.030)	0.00720 ± (0.00045, 0.00044)
2.300	0.942 ± (0.060, 0.060)	0.174 ± (0.069, 0.043)	0.00931 ± (0.00059, 0.00059)
2.400	1.243 ± (0.080, 0.080)	0.260 ± (0.091, 0.062)	0.01232 ± (0.00079, 0.00079)
2.500	1.710 ± (0.102, 0.106)	0.442 ± (0.117, 0.087)	0.01700 ± (0.00102, 0.00106)
2.600	1.916 ± (0.116, 0.141)	0.360 ± (0.137, 0.121)	0.01911 ± (0.00116, 0.00141)
2.700	2.734 ± (0.148, 0.189)	0.732 ± (0.176, 0.168)	0.02735 ± (0.00149, 0.00189)
2.800	3.608 ± (0.185, 0.254)	1.087 ± (0.220, 0.233)	0.03622 ± (0.00185, 0.00255)
2.900	4.799 ± (0.223, 0.338)	1.641 ± (0.270, 0.318)	0.04835 ± (0.00225, 0.00340)
3.000	5.753 ± (0.265, 0.453)	1.851 ± (0.327, 0.435)	0.05818 ± (0.00268, 0.00458)
3.100	7.215 ± (0.307, 0.611)	2.393 ± (0.388, 0.597)	0.07323 ± (0.00311, 0.00620)
3.200	8.798 ± (0.360, 0.831)	2.872 ± (0.465, 0.824)	0.08966 ± (0.00367, 0.00847)
3.300	10.559 ± (0.489, 1.114)	3.307 ± (0.609, 1.117)	0.10804 ± (0.00501, 0.01140)
3.400	14.049 ± (0.832, 1.545)	5.230 ± (0.943, 1.564)	0.14436 ± (0.00854, 0.01587)
3.500	17.185 ± (0.832, 2.174)	6.289 ± (0.943, 2.218)	0.17733 ± (0.00858, 0.02243)

TABLE IV - 4

13.3 GeV 60°

W (GeV)	Q² (GeV²)	ε	x	x'	$\frac{d^2\sigma}{d\Omega dE'}(\frac{pb}{GeV\text{-}sr})$	$2M\,W_1^P$
					PROTON	
2.000	19.120	0.1523	0.860	0.827	0.658 ± (0.066, 0.043)	0.00434 ± (0.00044, 0.00028)
2.100	18.761	0.1497	0.842	0.810	0.940 ± (0.069, 0.064)	0.00622 ± (0.00046, 0.00042)
2.200	18.384	0.1469	0.823	0.792	1.342 ± (0.093, 0.128)	0.00891 ± (0.00062, 0.00085)
2.300	17.990	0.1440	0.803	0.773	1.689 ± (0.114, 0.164)	0.01126 ± (0.00076, 0.00109)
2.400	17.578	0.1409	0.783	0.753	2.455 ± (0.149, 0.173)	0.01644 ± (0.00100, 0.00116)
2.500	17.148	0.1377	0.762	0.733	3.099 ± (0.183, 0.237)	0.02085 ± (0.00123, 0.00159)
2.600	16.702	0.1343	0.740	0.712	3.792 ± (0.227, 0.385)	0.02562 ± (0.00154, 0.00260)
2.700	16.237	0.1308	0.717	0.690	4.849 ± (0.313, 0.578)	0.03292 ± (0.00212, 0.00393)
2.800	15.755	0.1271	0.694	0.668	6.611 ± (0.469, 0.801)	0.04509 ± (0.00320, 0.00546)
2.900	15.256	0.1233	0.670	0.645	8.823 ± (0.646, 0.967)	0.06048 ± (0.00443, 0.00663)
3.000	14.739	0.1194	0.645	0.621	11.855 ± (0.877, 1.017)	0.08169 ± (0.00604, 0.00701)
3.100	14.204	0.1153	0.619	0.596	16.603 ± (1.247, 1.280)	0.11641 ± (0.00864, 0.00887)
3.200	13.652	0.1110	0.593	0.571	17.300 ± (1.321, 1.846)	0.12052 ± (0.00921, 0.01286)
3.300	13.083	0.1066	0.567	0.546	24.100 ± (1.264, 2.444)	0.16886 ± (0.00886, 0.01713)
3.400	12.495	0.1020	0.539	0.519	29.057 ± (1.518, 3.161)	0.20480 ± (0.01070, 0.02228)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}(\frac{pb}{GeV\text{-}sr})$	$\frac{d^2\sigma}{d\Omega dE'}(\frac{pb}{GeV\text{-}sr})$	$2M\,W_1^D$
	DEUTERON	NEUTRON	
2.000	1.198 ± (0.069, 0.085)	0.220 ± (0.104, 0.063)	0.00790 ± (0.00059, 0.00056)
2.100	1.635 ± (0.092, 0.127)	0.364 ± (0.108, 0.100)	0.01082 ± (0.00061, 0.00084)
2.200	2.267 ± (0.121, 0.152)	0.600 ± (0.142, 0.127)	0.01506 ± (0.00081, 0.00101)
2.300	2.802 ± (0.148, 0.183)	0.690 ± (0.177, 0.161)	0.01869 ± (0.00099, 0.00122)
2.400	3.501 ± (0.182, 0.297)	0.803 ± (0.222, 0.270)	0.02345 ± (0.00122, 0.00199)
2.500	5.020 ± (0.247, 0.356)	1.510 ± (0.298, 0.334)	0.03375 ± (0.00166, 0.00240)
2.600	5.938 ± (0.325, 0.574)	1.506 ± (0.389, 0.549)	0.04012 ± (0.00220, 0.00388)
2.700	8.229 ± (0.440, 0.701)	2.573 ± (0.519, 0.685)	0.05586 ± (0.00298, 0.00476)
2.800	10.704 ± (0.561, 0.851)	3.576 ± (0.662, 0.845)	0.07301 ± (0.00382, 0.00580)
2.900	12.725 ± (0.657, 1.364)	3.805 ± (0.793, 1.369)	0.08723 ± (0.00453, 0.00935)
3.000	18.025 ± (0.854, 1.771)	6.924 ± (1.019, 1.796)	0.12421 ± (0.00588, 0.01220)
3.100	22.619 ± (1.255, 2.103)	8.909 ± (1.435, 2.151)	0.15671 ± (0.00870, 0.01457)
3.200	28.236 ± (2.183, 2.815)	11.405 ± (2.348, 2.897)	0.19671 ± (0.01521, 0.01961)
3.300	33.456 ± (4.265, 4.522)	12.854 ± (4.399, 4.673)	0.23441 ± (0.02988, 0.03168)

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TABLE IV - 4

10.4 GeV 60°

W (GeV)	Q^2 (GeV²)	ϵ	x^-	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) PROTON	$2M W_1^P$
2.000	13.890	0.1783	0.817	0.776	3.066 ± (0.242, 0.185)	0.01195 ± (0.00094, 0.00072)
2.100	13.543	0.1742	0.793	0.754	4.529 ± (0.303, 0.250)	0.01775 ± (0.00119, 0.00098)
2.200	13.179	0.1699	0.769	0.731	6.535 ± (0.407, 0.346)	0.02576 ± (0.00161, 0.00136)
2.300	12.797	0.1654	0.744	0.708	8.826 ± (0.525, 0.503)	0.03501 ± (0.00208, 0.00200)
2.400	12.399	0.1606	0.718	0.683	11.352 ± (0.654, 0.682)	0.04533 ± (0.00261, 0.00272)
2.500	11.984	0.1557	0.691	0.657	14.777 ± (0.798, 0.907)	0.05941 ± (0.00321, 0.00365)
2.600	11.552	0.1505	0.663	0.631	18.640 ± (0.918, 1.246)	0.07546 ± (0.00372, 0.00505)
2.700	11.103	0.1450	0.634	0.604	23.521 ± (1.049, 1.671)	0.09593 ± (0.00428, 0.00682)
2.800	10.637	0.1393	0.604	0.576	30.076 ± (1.285, 2.305)	0.12360 ± (0.00528, 0.00947)
2.900	10.154	0.1334	0.574	0.547	37.634 ± (1.718, 3.145)	0.15589 ± (0.00712, 0.01333)
3.000	9.655	0.1272	0.543	0.518	47.507 ± (2.438, 4.403)	0.19841 ± (0.01018, 0.01839)
3.100	9.138	0.1208	0.511	0.487	53.928 ± (4.701, 6.120)	0.22715 ± (0.01980, 0.02578)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) DEUTERON	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV\text{-}sr}$) NEUTRON	$2M W_1^D$
2.000	4.77 ± (0.31, 0.28)	1.12 ± (0.36, 0.24)	0.01860 ± (0.00121, 0.00109)
2.100	6.81 ± (0.39, 0.39)	1.95 ± (0.45, 0.35)	0.02670 ± (0.00152, 0.00154)
2.200	9.58 ± (0.50, 0.53)	3.15 ± (0.59, 0.49)	0.03776 ± (0.00199, 0.00210)
2.300	12.29 ± (0.63, 0.74)	3.94 ± (0.75, 0.70)	0.04877 ± (0.00251, 0.00293)
2.400	15.84 ± (0.80, 1.02)	5.00 ± (0.96, 0.99)	0.06324 ± (0.00319, 0.00407)
2.500	19.61 ± (0.95, 1.42)	5.66 ± (1.18, 1.41)	0.07881 ± (0.00381, 0.00572)
2.600	26.43 ± (1.16, 1.99)	8.55 ± (1.46, 2.01)	0.10700 ± (0.00469, 0.00807)
2.700	35.17 ± (1.49, 2.72)	12.50 ± (1.88, 2.77)	0.14342 ± (0.00608, 0.01111)
2.800	43.82 ± (2.02, 3.84)	15.29 ± (2.49, 3.94)	0.18009 ± (0.00831, 0.01577)
2.900	56.90 ± (2.92, 5.38)	21.32 ± (3.44, 5.55)	0.23570 ± (0.01208, 0.02229)
3.000	68.62 ± (4.34, 7.83)	24.47 ± (4.90, 8.12)	0.28660 ± (0.01815, 0.03271)
3.100	90.90 ± (9.25, 10.44)	36.80 ± (9.70, 10.85)	0.38286 ± (0.03896, 0.04398)

TABLE IV - 4

6.5 GeV 60°

W (GeV)	Q^2 (GeV²)	ε	x	x'	$\frac{d^2\sigma}{d\Omega dE'}(\frac{pb}{GeV\text{-}sr})$	$2M\,W_1^P$
					PROTON	
1.075	9.251	0.2838	0.971	0.889	-0.218 ± (0.206,-0.000)	-0.00028 ± (0.00027,-0.00000)
1.100	9.209	0.2827	0.965	0.884	0.207 ± (0.233, 0.076)	0.00027 ± (0.00030, 0.00010)
1.125	9.166	0.2816	0.960	0.879	0.542 ± (0.248, 0.082)	0.00071 ± (0.00032, 0.00011)
1.150	9.122	0.2805	0.954	0.873	0.962 ± (0.261, 0.094)	0.00126 ± (0.00034, 0.00012)
1.175	9.077	0.2794	0.948	0.868	0.442 ± (0.200, 0.074)	0.00058 ± (0.00026, 0.00010)
1.200	9.031	0.2782	0.942	0.862	1.504 ± (0.277, 0.110)	0.00198 ± (0.00036, 0.00014)
1.225	8.984	0.2770	0.935	0.857	1.879 ± (0.282, 0.124)	0.00247 ± (0.00037, 0.00016)
1.250	8.936	0.2758	0.929	0.851	2.068 ± (0.287, 0.131)	0.00273 ± (0.00038, 0.00017)
1.275	8.887	0.2745	0.923	0.845	2.484 ± (0.303, 0.147)	0.00328 ± (0.00040, 0.00019)
1.300	8.837	0.2732	0.916	0.839	3.469 ± (0.340, 0.186)	0.00462 ± (0.00045, 0.00025)
1.325	8.786	0.2719	0.909	0.833	3.000 ± (0.322, 0.170)	0.00398 ± (0.00043, 0.00023)
1.350	8.734	0.2706	0.903	0.827	4.014 ± (0.352, 0.211)	0.00534 ± (0.00047, 0.00028)
1.375	8.681	0.2692	0.896	0.821	4.442 ± (0.362, 0.231)	0.00592 ± (0.00048, 0.00031)
1.400	8.627	0.2678	0.889	0.815	5.083 ± (0.422, 0.283)	0.00759 ± (0.00056, 0.00038)
1.425	8.572	0.2664	0.882	0.808	6.479 ± (0.424, 0.319)	0.00867 ± (0.00057, 0.00043)
1.450	8.517	0.2649	0.875	0.802	6.967 ± (0.464, 0.343)	0.00935 ± (0.00062, 0.00046)
1.475	8.460	0.2634	0.867	0.795	9.519 ± (0.560, 0.451)	0.01280 ± (0.00075, 0.00061)
1.500	8.402	0.2619	0.860	0.789	13.267 ± (0.644, 0.609)	0.01788 ± (0.00087, 0.00082)
1.525	8.343	0.2604	0.852	0.782	13.895 ± (0.679, 0.643)	0.01877 ± (0.00092, 0.00087)
1.550	8.284	0.2588	0.845	0.775	11.633 ± (0.620, 0.558)	0.01575 ± (0.00084, 0.00076)
1.575	8.223	0.2572	0.837	0.768	10.733 ± (0.611, 0.530)	0.01457 ± (0.00083, 0.00072)
1.600	8.162	0.2556	0.829	0.761	11.903 ± (0.658, 0.588)	0.01620 ± (0.00089, 0.00080)
1.625	8.099	0.2539	0.821	0.754	12.925 ± (0.669, 0.641)	0.01763 ± (0.00091, 0.00087)
1.650	8.036	0.2522	0.814	0.747	17.549 ± (0.809, 0.847)	0.02400 ± (0.00111, 0.00116)
1.675	7.971	0.2505	0.805	0.740	21.931 ± (0.944, 1.046)	0.03007 ± (0.00129, 0.00143)
1.700	7.906	0.2487	0.797	0.732	25.538 ± (1.013, 1.216)	0.03511 ± (0.00139, 0.00167)
1.725	7.839	0.2469	0.789	0.725	24.795 ± (1.026, 1.202)	0.03418 ± (0.00141, 0.00166)
1.750	7.772	0.2451	0.781	0.717	28.350 ± (1.086, 1.375)	0.03919 ± (0.00150, 0.00190)
1.775	7.703	0.2432	0.772	0.710	26.288 ± (1.079, 1.308)	0.03644 ± (0.00150, 0.00181)
1.800	7.634	0.2414	0.764	0.702	27.751 ± (1.132, 1.396)	0.03858 ± (0.00157, 0.00194)
1.825	7.564	0.2394	0.755	0.694	31.461 ± (1.237, 1.585)	0.04383 ± (0.00172, 0.00221)
1.850	7.492	0.2375	0.747	0.686	32.169 ± (1.308, 1.645)	0.04498 ± (0.00183, 0.00230)
1.875	7.420	0.2355	0.738	0.679	36.085 ± (1.451, 1.852)	0.05060 ± (0.00203, 0.00260)
1.900	7.347	0.2335	0.729	0.671	37.611 ± (1.557, 1.957)	0.05290 ± (0.00219, 0.00275)
1.925	7.273	0.2314	0.720	0.662	43.562 ± (1.734, 2.266)	0.06145 ± (0.00245, 0.00320)
1.950	7.197	0.2294	0.711	0.654	45.129 ± (1.823, 2.382)	0.06385 ± (0.00258, 0.00337)
1.975	7.121	0.2272	0.702	0.640	51.844 ± (1.942, 2.738)	0.07359 ± (0.00276, 0.00389)

TABLE IV - 4

6.5 GeV 60$^{\text{o}}$

W (GeV)	Q^2 (GeV^2)	ε	x$^-$	x'	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$) PROTON	2M W$_1^P$
2.000	7.044	0.2251	0.693	0.638	55.29 ± (1.07, 3.32)	0.07873 ± (0.00152, 0.00473)
2.100	6.726	0.2161	0.656	0.604	75.04 ± (1.16, 4.63)	0.10825 ± (0.00167, 0.00669)
2.200	6.392	0.2066	0.618	0.569	97.71 ± (1.45, 6.58)	0.14291 ± (0.00213, 0.00963)
2.300	6.043	0.1965	0.578	0.533	128.22 ± (2.77, 9.29)	0.19025 ± (0.00412, 0.01378)
2.400	5.679	0.1858	0.538	0.496	157.98 ± (7.15, 13.63)	0.23798 ± (0.01077, 0.02053)
2.500	5.298	0.1744	0.497	0.459	202.57 ± (11.40, 21.27)	0.31001 ± (0.01745, 0.03255)
2.600	4.903	0.1625	0.455	0.420	288.77 ± (16.48, 25.33)	0.44928 ± (0.02563, 0.03942)

W (GeV)	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$) DEUTERON	$\frac{d^2\sigma}{d\Omega dE'}$ ($\frac{pb}{GeV-sr}$) NEUTRON	2M W$_1^D$
2.000	77.46 ± (1.74, 4.90)	25.78 ± (3.47, 4.97)	0.11030 ± (0.00248, 0.00698)
2.100	103.32 ± (2.03, 7.25)	31.97 ± (4.14, 7.44)	0.14905 ± (0.00293, 0.01046)
2.200	140.80 ± (3.50, 9.80)	47.06 ± (5.94, 10.14)	0.20592 ± (0.00512, 0.01434)
2.300	190.14 ± (6.14, 14.39)	68.91 ± (8.75, 14.96)	0.28213 ± (0.00911, 0.02135)
2.400	244.67 ± (7.42, 20.41)	89.53 ± (10.92, 21.28)	0.36857 ± (0.01118, 0.03075)
2.500	311.06 ± (7.42, 25.40)	114.83 ± (12.56, 26.50)	0.47604 ± (0.01136, 0.03888)
2.600	391.62 ± (7.42, 31.35)	146.16 ± (14.69, 32.68)	0.60930 ± (0.01155, 0.04877)

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CHAPTER V - RESULTS AND CONCLUSIONS

INTRODUCTION

The measured cross sections at 50° and 60° cover a wide range of kinematics and give new information about the nucleons for both elastic and inelastic electron scattering. A general form of the cross section

$$(5.1) \quad \frac{\partial^2 \sigma}{\partial \Omega \partial E'} = \frac{\alpha^2}{4E_o^2} \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \left(\frac{\nu W_2}{\nu} + \frac{2M W_1}{M} \tan^2 \theta/2 \right)$$

indicates that for small angles and energy loss the measurements will be dominated by the behavior of W_2. W_2 has been carefully measured in previous experiments (Ref. V-1). For the measurements reported here at large angles and energy losses the contribution of W_1 to the cross section is much larger than that of W_2.

In a composite model of the proton (with spin 1/2 and spin 0 constituents) W_1 is determined by the scattering from the particles with spin 1/2, but for W_2 particles of both spin 0 and spin 1/2 can contribute. Thus in a simple quark model W_1 directly measures scattering off the quarks, where W_2 might contain contributions from spin 0 "glue" particles.

The measurements at 50° and 60° were carried out in a region where the total mass of the recoiling hadronic state is small compared with the momentum transferred to it. (This region is referred to as the "threshold region"). Because of kinematics, the fragments all travel away from the collision with small relative momentum compared to the total momentum given to the hadronic system by the incident electron.

For the large values of momentum transfer covered in this experimer

the interaction is occurring over very short distances compared with typical nucleon dimensions (for $Q^2=20$ GeV^2 we are probing distances of approximately 1/20 of a nucleon diameter). Thus a very small fraction of the total volume of the proton is "hit." In a simple picture, the large momentum transfer must be absorbed by a small part of the nucleon. Nevertheless in the threshold region the whole nucleon and any other particles produced must share this momentum in order to recoil in a state of small mass. The cross section is therefore suppressed in this region.

Elastic scattering is the extreme case: a single recoiling particle carrying away all of the momentum transferred. If the proton is a composite particle, it is difficult for all the parts to stay together in high energy collisions. This is particularly true if all the momentum transfer is to a single constituent. We therefore expect small elastic cross sections.

The range of the interaction for inelastic scattering is also small. The large inelastic cross sections support the thesis that the nucleon can be described by small charged constituents rather than a smooth charge distribution. In the simple quark model the neutron and the proton have different quark constituents which naturally leads to a difference in the total scattering strength of these two systems. Electroproduction experiments have demonstrated that the two nucleons do have different scattering strengths and this difference is largest in the threshold region (Ref. V-2) ($W<<|\vec{q}|$). One of the goals of the

the present experiment was a measurement of the N/P ratio in this region.

This chapter proceeds as follows: Elastic scattering is discussed first and a comparison to previous data is made. The behavior of the W_1 structure function of the proton is detailed next. The extension of previous parameterizations of older data does not agree with the present data and alternate solutions are studied. The chapter concludes with the neutron to proton ratio. In the analysis only the statistical counting errors are used when plotting and when fitting functions to the new data (except where noted for elastic scattering).

ELASTIC SCATTERING

Elastic scattering from protons was measured for incident energies of 6.5 GeV, 13.3 GeV and 19.5 GeV at a scattering angle of 60°. Some lower Q^2 elastic peaks were measured at 50° and 60° with incident energies ranging from 1.5 GeV to 4.5 GeV during the experimental "check out" periods.

These low Q^2 elastic peaks are high statistics runs and clearly show the elastic radiative tail between the proton mass and one pion threshold. Empty target contributions were measured for each point and subtracted from the data. An unfolding technique was employed (Ref. V-3) to account for the radiation processes. Of the higher Q^2 data only the measurement made at Eo=6.5 GeV had sufficient statistical accuracy to allow use of the unfolding methods.

The measured data for the two highest Q^2 points are shown in

Fig. V-1. Also shown in this figure are the measured empty target contributions. The empty target cross sections were measured to be "flat" in the elastic peak region and all the empty target data for $W<1.075$ GeV were combined to reduce the error introduced by this correction.

The radiative corrections were made using the formula given by Tsai (Ref. V-4). An energy resolution equal to the missing energy between the elastic peak and one pion threshold, $\Delta E'=E'(\text{elastic})-E'(M+M_\pi)$ was used. The final values of elastic scattering cross sections are given in Table V-1.

The elastic cross section can be written in terms of the two form factors G_E and G_M as

(5.2)
$$\frac{\partial \sigma}{\partial \Omega} = \sigma_{NS} \frac{(G_E^2 + \tau G_M^2 + 2\tau \tan^2\theta/2 \; G_M^2)}{1 + \tau}$$

$$\sigma_{NS} = \frac{\alpha^2}{4E_o^2} \frac{\cos^2\theta/2}{\sin^4\theta/2} \frac{1}{1+2E_o(\sin^2\theta/2)/M} ; \text{ and } \tau = Q^2/4M^2$$

At high Q^2 and large scattering angles the G_M contribution to the cross section dominates. The assumption of form factor scaling, i.e. $G_E = G_M/\mu_p$, is often made although there is some indication that G_E falls faster than this (Ref. V-5). Form factor scaling predicts that by a Q^2 of 5 GeV^2 and $\theta=60°$, G_E contributes only 3.3% to the cross section. Thus, measurements at large angles are insensitive to G_E, if G_E is not larger than predicted by form factor scaling. G_M^{S2} is given in Table V-1 along with the measured cross sections, where G_M^{S2} is defined by

(5.3)
$$\frac{d\sigma}{d\Omega} = \sigma_{NS} (G_M^S)^2 \frac{(1/\mu_p^2 + \tau}{1 + \tau} + 2\tau \tan^2\theta/2)$$
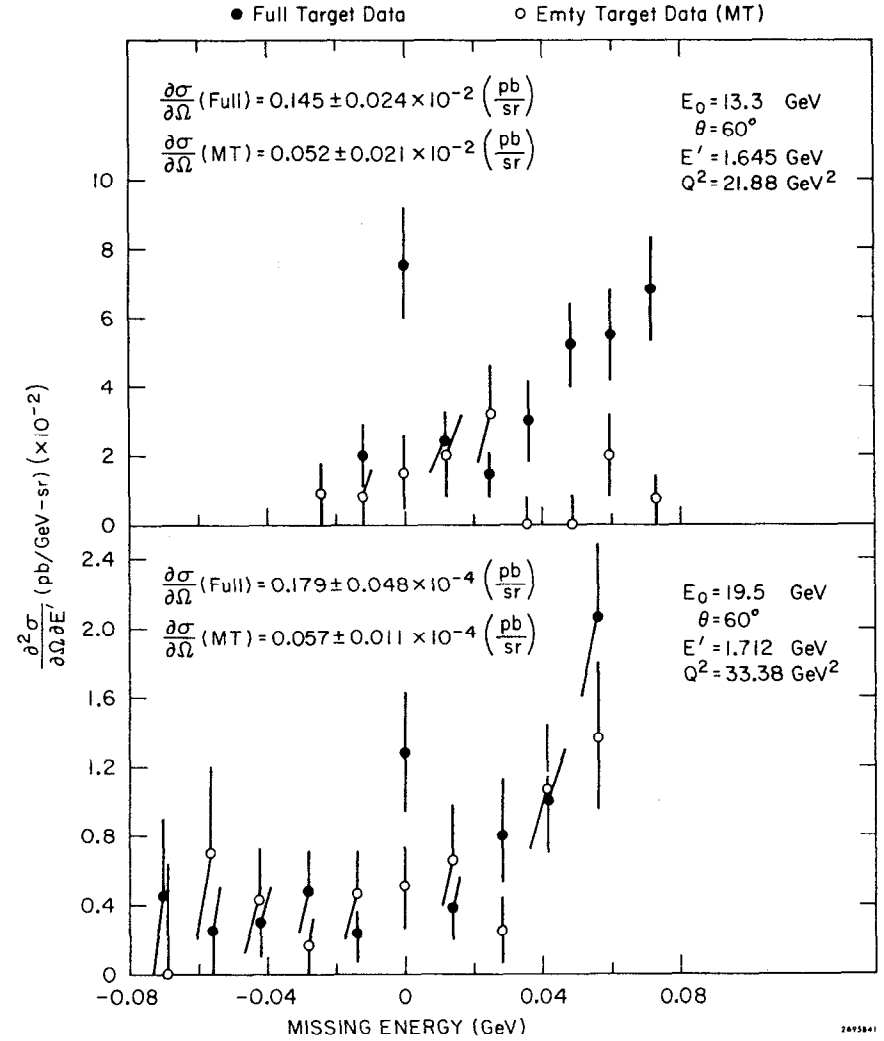
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● Full Target Data   ○ Emty Target Data (MT)

FIG. V − 1

The measured elastic peak cross sections off hydrogen and the empty target cross sections for $Q^2=$ 21.8 GeV^2 and 33.4 GeV^2.

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TABLE V – 1

ELASTIC SCATTERING CROSS SECTIONS FOR 50° AND 60°

E_o (GeV)	$\theta°$	Q^2 (GeV²)	$\dfrac{d\sigma}{d\Omega}\left(\dfrac{cm^2}{Sr}\right)$	$(G_M^S)^2$
1.501	59.87	1.249	$.132 \pm .004 \times 10^{-32}$	$.144 \pm .004 \times 10^{0}$
1.502	50.0	1.0254	$.401 \pm .003 \times 10^{-32}$	$.236 \pm .002 \times 10^{0}$
2.492	49.96	2.275	$.232 \pm .008 \times 10^{-33}$	$.280 \pm .009 \times 10^{-1}$
3.491	49.96	3.736	$.252 \pm .013 \times 10^{-34}$	$.515 \pm .026 \times 10^{-2}$
4.484	49.96	5.303	$.507 \pm .032 \times 10^{-35}$	$.158 \pm .010 \times 10^{-2}$
6.461	60.02	9.397	$.149 \pm .014 \times 10^{-36}$	$.174 \pm .016 \times 10^{-3}$
13.302	60.00	21.876	$.120 \pm .042 \times 10^{-38}$	$.550 \pm .194 \times 10^{-5}$
19.499	60.00	33.379	$.184 \pm .073 \times 10^{-40}$	$.177 \pm .070 \times 10^{-5}$

Adopting the ideas of Brodsky and Farrar (Ref. V-6), we plot $Q^4 \, G_M^S /\mu_p$ for our data and previous SLAC data in Fig. V-2 (Ref. V-3 and Ref. V-7). The data are not in conflict with an asymptotic behavior of $1/Q^4$ for the form factor.

Also plotted on this figure (dashed line) is the "dipole" formula times Q^4: $Q^4/(1+Q^2/.71)^2$. The data seem to reach the asymptotic $1/Q^4$ behavior faster than the dipole formula. The difference in the rates at which the data and the dipole reach the $1/Q^4$ behavior is the cause of the data "oscillating" about the dipole fit.

We have tried various fits to $(G_M^S / \mu_p)^2$. To include systematic errors in an approximate way between the different experiments fit, we set the minimum error in any of the measured cross sections used to be $\pm 2.5\%$ even though the statistical accuracy of much of the data is considerably better.

The functional forms, parameters, and χ^2's for various fits are given in Table V-2. The last, an ad hoc parameterization, was adjusted to have the observed $1/Q^4$ behavior at large Q^2 and the value of 1 as $Q^2 \to 0$. This form has no other physical significance but does reproduce the measured data for $.1 \text{ GeV}^2 < Q^2 < 33.4 \text{ GeV}^2$ reasonably well. It is plotted as a solid line in Fig. V-2.

THE W_1 STRUCTURE FUNCTION OF THE PROTON

A first approach to new data is usually "how well does the measurement match what we thought we would measure?" The size of W_1 can be estimated from previous measurements at smaller angles by making an

FIG. V - 2

A plot of $Q^4 G_M^S / \nu_p$ versus Q^2. The dashed line is the dipole fit and the solid line is the ad hoc fit.

TABLE V - 2

Various functions fit to the elastic data with parameters and χ^2. The minimum error was taken to be $\pm$ 2.5% of the cross section.

Functional Form	Parameters	χ^2/N_D
$\dfrac{1}{(1+C_1 Q^2)^2}$	$C_1 = 1.4062 \pm .0031$ $(1/C_1 = .711)$	$226.4/28$
$\dfrac{C_1}{(1+C_2 Q^2)^2} + \dfrac{1-C_1}{(1+C_3 Q^2)^2}$	$C_1 = 1.0013 \pm .0004$ $C_2 = 1.3699 \pm .0054$ $(1/C_2 = .730)$ $C_3 = .0432 \pm .0171$ $(1/C_3 = 23.15)$	$76.5/26$
$\dfrac{C_1 (1-e^{-Q^2/C_2})(1-e^{-Q^2/C_3})}{Q^4} + \dfrac{A}{Q^4 + C_4}$	$C_1 = .402 \pm .003$ $C_2 = 1.651 \pm .027$ $C_3 = .317 \pm .013$ $C_4 = .00795 \pm .00189$ $A = (1-C_1/C_2 C_3)\, C_4 = .00184$	$33.2/25$

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assumption about R (we choose R=.18). Shown in Fig. V-3 is the W_1 structure function extracted from the cross sections using R=.18 plotted against the scaling variable $x' = Q^2/(W^2+Q^2)$ (we have only included data with W >2.0 GeV). Over-plotted in this figure is what one would predict using

(5.4) $\nu W_2 = .6453(1-x')^3 + 1.902(1-x')^4 - 2.343(1-x')^5$

where R=.18(Ref.V-8), $2MW_1 = \omega\nu W_2(1 + Q^2/\nu^2)/(1+R)$ and $\omega = 2M\nu/Q^2$.

The W_1 structure function is seen to have a strong dependence on x' and decreases rapidly with increasing x'. It is clear that the general scaling nature of the proton data in x' works to 30% for over a factor of 1000 in the size of $2MW_1$. Of course for R constant W_1 cannot scale in x' if νW_2 scales perfectly in x', but for small values of R this is a small effect and is the reason for plotting different predicted W_1's for different incident energies.

The impression of the "goodness" of scaling changes when the data are more closely scrutinized. For both of the scaling variables $x(=Q^2/2M\nu)$ and x', W_1 falls off as Q^2 rises. To examine this we have interpolated a combined data set to fixed values of x' and plot $2M W_1$ versus Q^2 as shown in Fig. V-4. All of the data fall off with increasing Q^2. The non-scaling behavior becomes obvious when trying to fit the data, resulting in poor χ^2's for simple polynominal fits in $(1-x')$ and $(1-x)$ as shown in Table V-3.

The first question that arises is "can we fix this non-scaling behavior in W_1 by taking a different R"? The cross section as given
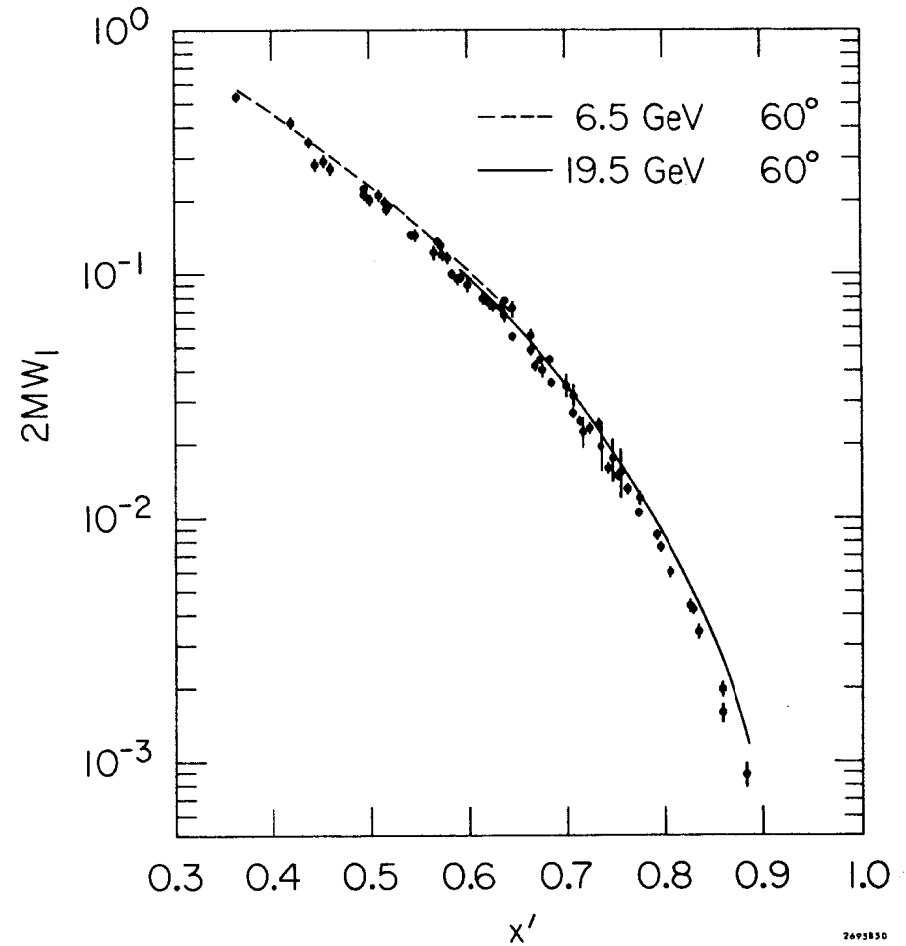
FIG. V – 3

$2MW_1$ (for R = .18) plotted versus x'. The lines indicate the predicted values of $2MW_1$ for E_o = 6.5 GeV and E_o = 19.5 GeV.
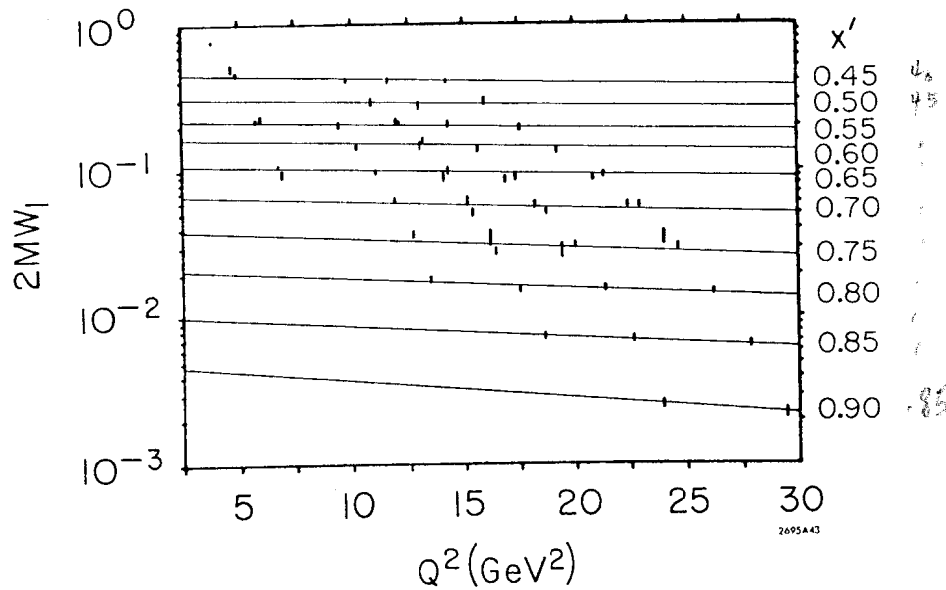
FIG. V – 4

The $2MW_1$ data interpolated to fixed values of x' (indicated on the right) plotted against Q^2.

in Eq. 4.6 shows that W_1 is proportional to $(1-\varepsilon)/(1+\varepsilon R)$ where ε is the photon polarization parameter and is less than .3 for our data. If we can take R to be increasing with increasing Q^2, the "scale breaking" of W_1 becomes larger as Q^2 increases. We can take R to be decreasing with increasing Q^2 and account for some of the observed effect. To account for the whole effect R would have to have a value near 2 in the low Q^2 region and decrease to zero in the high Q^2 region. This solution is in contradiction with the measured values of R for $Q^2 <10$ GeV^2. Thus the non-scaling behavior of the measured data cannot be successfully accounted for by modifying R within reasonable bounds.

We can accept the non-scaling behavior of W_1 and ask what kind of R values might result. To do this we will need to make some assumptions. First we assume scaling for νW_2 in the x' variable and that the value of νW_2 is correctly given by Eq. 5.4. This is an extrapolation for much of our data in Q^2 and is therefore sensitive to the scaling variable x'. Also for the large x's we must extrapolate the fit in the scaling variable (Eq. 5.4 was fit to data for which x' was mostly less than .6). The value of R for each of the data points is then calculated using

$$(5.5) \qquad R = (F - \sigma_H)/(\sigma_H - \varepsilon F)$$

where
$$F = \frac{4\pi^2\alpha}{K} \; (1+\nu^2/Q^2) \; \frac{\nu W_2}{\nu}$$

$$K = \frac{W^2 - M^2}{2M}$$

and
$$\sigma_H = \sigma_{exp}/\Gamma_t$$

$$\Gamma_t = \frac{\alpha}{4\pi^2} \; \frac{2KE'}{E_o Q^2 (1-\varepsilon)}$$

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TABLE V - 3

FITS TO THE W_1 STRUCTURE FUNCTION OF THE PROTON

Scaling Variable	Functional Form	R = .18 Parameters	$\frac{\chi^2}{N_D}$	R = .5 Parameters	χ^2	R = Q^2/ν^2 Parameters	χ^2
x	$\sum_{n=3}^{6} a_n(1-x)^n$	$a_3 = .30 \pm .16$ $a_4 = 7.69 \pm 1.51$ $a_5 = -10.29 \pm 4.47$ $a_6 = 4.52 \pm 4.11$	$\frac{1744}{122}$	$a_3 = .31 \pm .15$ $a_4 = 7.21 \pm 1.44$ $a_5 = -9.49 \pm 4.28$ $a_6 = 4.01 \pm 3.94$	$\frac{1562}{122}$	$a_3 = .31 \pm .16$ $a_4 = 7.67 \pm 1.52$ $a_5 = -10.27 \pm 4.51$ $a_6 = 4.61 \pm 4.15$	$\frac{1631}{122}$
x	$\left(\frac{1}{1+Q^2/\Lambda^2}\right)^2 \sum_{n=3}^{6} a_n(1-x)^n$	$a_3 = 2.57 \pm .35$ $a_4 = 5.44 \pm 2.99$ $a_5 = -10.48 \pm 8.34$ $a_6 = 4.60 \pm 7.29$ $(1/\Lambda^2) = .0233 \pm .00077$	$\frac{222}{121}$	$a_3 = 2.31 \pm .32$ $a_4 = 5.00 \pm 2.74$ $a_5 = -9.05 \pm 7.65$ $a_6 = 3.65 \pm 6.70$ $(1/\Lambda^2) = .0214 \pm .00074$	$\frac{221}{121}$	$a_3 = 2.50 \pm .35$ $a_4 = 5.21 \pm 2.94$ $a_5 = -9.71 \pm 8.21$ $a_6 = 4.24 \pm 7.19$ $(1/\Lambda^2) = .0221 \pm .00075$	$\frac{211}{121}$
x'	$\sum_{n=3}^{5} a_n(1-x')^n$	$a_3 = .075 \pm .131$ $a_4 = 4.114 \pm 1.212$ $a_5 = .279 \pm 3.437$ $a_6 = -2.941 \pm 3.039$	$\frac{348}{122}$	$a_3 = .066 \pm .126$ $a_4 = 4.091 \pm 1.162$ $a_5 = -.356 \pm 3.289$ $a_6 = -2.226 \pm 2.904$	$\frac{282}{122}$	$a_3 = .063 \pm .132$ $a_4 = 4.340 \pm 1.223$ $a_5 = -.532 \pm 3.468$ $a_6 = -2.061 \pm 3.067$	$\frac{303}{122}$
x'	$\left(\frac{1}{1+Q^2/\Lambda^2}\right)^2 \sum_{n=3}^{5} a_n(1-x')^n$	$a_3 = .139 \pm .179$ $a_4 = 6.838 \pm 1.593$ $a_5 = -7.261 \pm 4.417$ $a_6 = 2.866 \pm 3.814$ $(1/\Lambda^2) = .00775 \pm .00059$	$\frac{134}{121}$	$a_3 = .118 \pm .160$ $a_4 = 6.192 \pm 1.457$ $a_5 = -6.227 \pm 4.052$ $a_6 = 2.330 \pm 3.506$ $(1/\Lambda^2) = .00630 \pm .00056$	$\frac{135}{121}$	$a_3 = .121 \pm .172$ $a_4 = 6.776 \pm 1.563$ $a_5 = -7.348 \pm 4.346$ $a_6 = 3.260 \pm 3.762$ $(1/\Lambda^2) = .00687 \pm .00057$	$\frac{130}{121}$
x_s	$\sum_{n=4}^{5} a_n(1-x_s)^n$ $x_s = \dfrac{1}{\dfrac{2M\nu}{Q^2} + \dfrac{M_s^2}{Q^2}}$	$a_4 = 3.429 \pm .112$ $a_5 = -.509 \pm .189$ $M_s^2 = 1.473 \pm .042$	$\frac{129}{123}$	$a_4 = 3.543 \pm .113$ $a_5 = -.896 \pm .191$ $M_s^2 = 1.368 \pm .042$	$\frac{130}{123}$	$a_4 = 3.599 \pm .116$ $a_5 = -.715 \pm .197$ $M_s^2 = 1.405 \pm .042$	$\frac{126}{123}$

The resulting values of νR so calculated are plotted in Fig. V-5

versus Q^2. This clearly results in large scale breaking of νR and it would

be hard for the standard parton models to accommodate such behavior.

Intuitively one would not expect the longitudinal virtual photoabsorption

cross section corresponding to large values of R to dominate in the thres-

hold region. This leads us to suspect the assumption of scaling in x'

and/or the extrapolation of the fit given in Eq. 5.4 to the region of

large x'. We now leave the question of R and focus our attention on

the behavior of W_1 itself.

Many theorists have speculated about the structure functions

eventually not scaling as the range of the data was extended to high

Q^2. One such model was put forward by Drell and Chanowitz (Ref. V-9).

In this model, the structure functions fall off at high Q^2 (compared

to the scaling prediction) due to the form factors of the constituents.

G. B. West (Ref. V-10) considered the possibility that the quarks in

the model could have anomalous magnetic moments as well. Then one can

construct a fortuitous cancellation of terms so that νW_2 continues to

scale for high Q^2, but no such cancellation occurs for W_1 and it should

fall as $1/(1+Q^2/\Lambda^2)^2$ with respect to its "scaling" value. We tried fits

to our data of the form "scaling function" times a propagator for the

scaling variables x and x'; the parameters and χ^2's are given in Table V-3.

The data is not in contradiction with this notion for the scaling varia-

ble x'. But a scaling function in the variable x is incompatible with
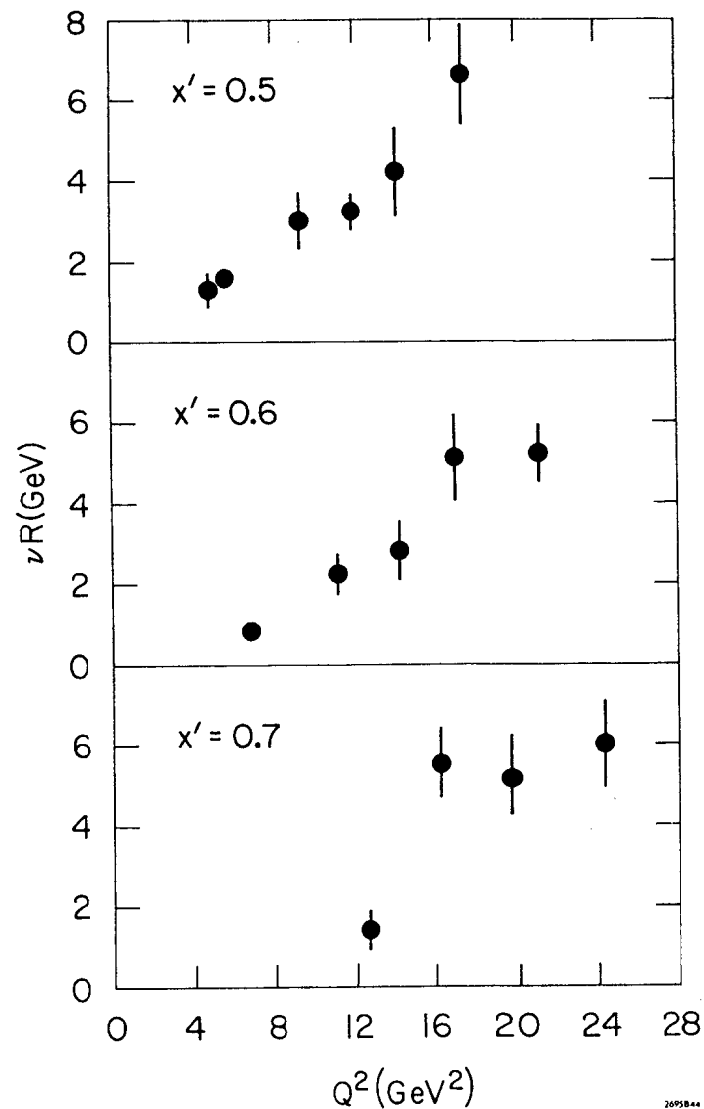
the data even with propagator scale breaking.

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FIG. V - 5

νR extracted from the data (assuming
νW_2 given by Eq. 5.4) plotted against
Q^2 for various values of x'.

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The undesirable features of this interpretation are: 1) we have had to introduce more free parameters into the theory (namely two - the propagator mass and the anomalous quark magnetic moment) and 2) the same theory would have the elastic scattering form factor fall off faster than $1/Q^4$ due to the parton's form factor. The second point is in mild contradiction with the measured data presented in the last section. The ideas of propagator scale breaking factors were originally motivated by the rising ratio of $e^+ - e^-$ annilation cross sections into hadrons compared to muon production by the same process. Better data has converted the smooth rise into a step indicative of some new threshold and the propagator ideas no longer seem to be necessary to explain that data.

Another alternative to "explain" the W_1 data is to change the definition of the scaling variable. Theoretically the approach of the scaling variable to the asymptotic form, ν/Q^2, is not understood. The introduction of terms like M^2/Q^2 into the definition of the scaling variable has had some degree of success, and ω' has supplanted ω as a scaling variable which describes the data better. Consider the variable

(5.5) $\omega_s = 2M\nu/Q^2 + M_s^2/Q^2$

where we will let the data determine the best value of M_s^2.

The scaling variables ω and ω' are the special cases of M_s^2 equal to zero and M^2. We fitted the W_1 structure function in ω_s allowing the mass parameter to vary, with W_1 extracted from the data using various values for R. The results are given at the bottom of Table V-3. The χ^2's for these fits are the best of those given in Table V-3 and are

very close to one per degree of freedom (recall that only statistical counting errors have been used in the data to which the fits were made).

The leading power of $(1-x')$ or $(1-x_s)$ is of interest for it determines the fall off of the structure functions at high Q^2 for fixed W. W_1 is found to be mostly accounted for by $(1-x_s)^4$. The third power is not required at all and the best χ^2's were obtained by including a small amount of the fifth power. To illustrate the power law dependence we have plotted $2M W_1/(1-x_s)^4$ in Fig. V-6a versus Q^2 and in Fig. V-6b versus x_s. The curves in Fig. V-6b show what the third and the fifth power would look like.

This power law is different from what was expected. The $1/Q^4$ behavior of G_M^S and the Drell-Yan-West relation (Ref. V-12) would predict that W_1 should fall as $1/Q^6$ for fixed W and not as $1/Q^8$ (from the fourth power of $(1-x_s)$). The data on W_1 show little dependence on the third power for fits using $(1-x')$ or $(1-x_s)$.

The simple parton model given in Chapter I tells us that $\nu W_2 = x f(x)$ and $2M W_1 = f(x)$. The new data gives $f(x_s) = c(1-x_s)^4$. M. Mestayer has pointed out (Ref. V-11) that fits in x' such as that given in Eq. 5.4 can be refactored into a form surprisingly close to $x'(1-x')^4$. For the region of x' between .3 and .7 where most of the data falls, the factor $x'(1-x')$ is quite flat so that $x'(1-x')^4$ and $(1-x')^3$ have nearly the same shape in this region. This suggests that the previous data on νW_2 should be refit in the form of $\nu W_2/x$ to powers in $(1-x')$ or $(1-x_s)$ to see if it is compatible with the $f(x_s)$ obtained from fitting $2M W_1$.

Axis labels for figure a):
$2MW_1/(1-x_s)^4$ vs Q^2 (GeV^2)

a) $2MW_1/(1-x_s)^4$ plotted against Q^2.

$1.26/(1-x_s)$

$7.88 \ (1-x_s)$

Axis labels for figure b):
$2MW_1/(1-x_s)^4$ vs x_s

b) $2MW_1/(1-x_s)^4$ plotted versus x_s. The lines indicated what the third (solid line) and the fifth powers (dashed line) would look like.
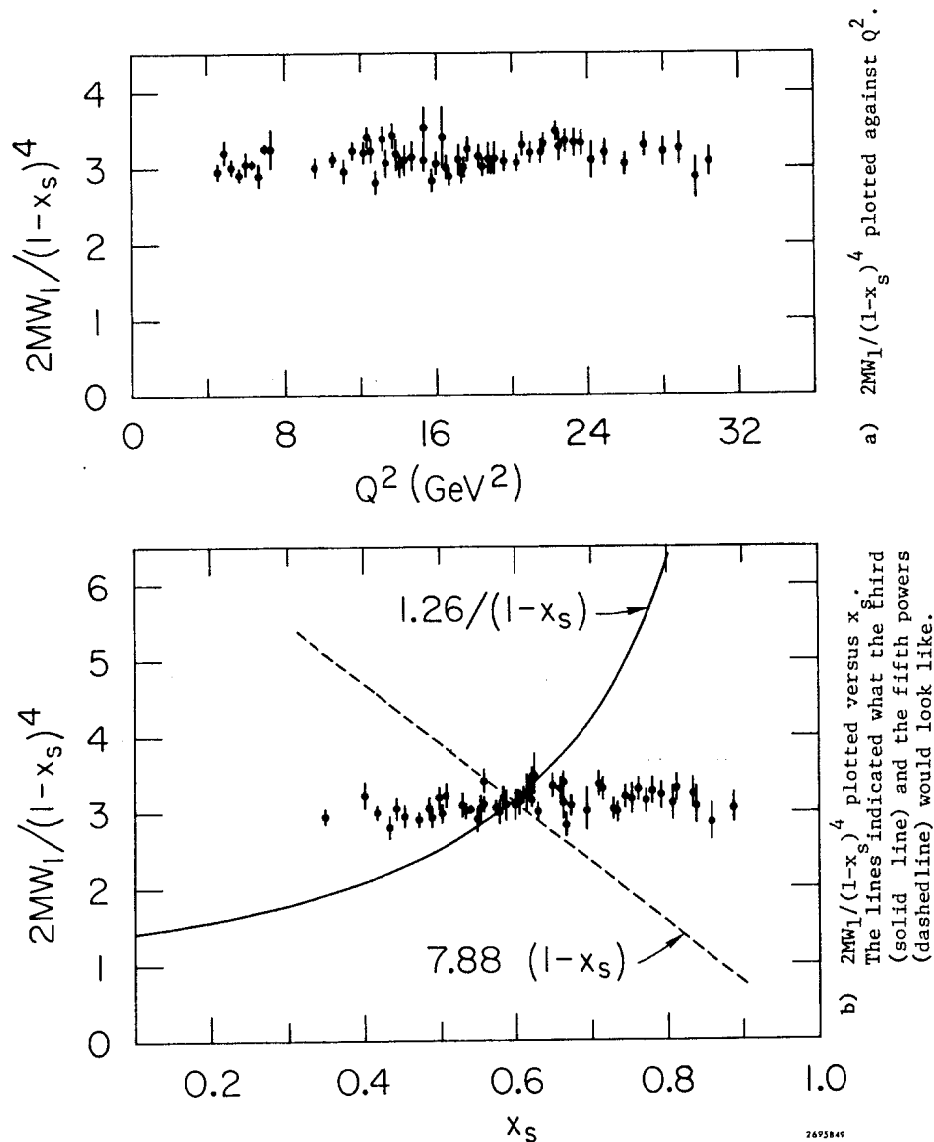
FIG. V - 6

One way of showing the effect of the scaling variable on the Q^2 dependence is to plot the slope in Q^2 obtained by fitting data at constant x_s versus x_s for various values of M_s^2. The results are shown in Fig. V-7 for $M_s^2=0$, M^2, 1.42 GeV^2. Clearly W_1 scales well in x_s. We note that the variable ω_s is very similar to ω_W introduced by Rittenberg and Rubinstein (Ref. V-14), and that the value of $M_s^2 = 1.42$ is close to that found for the equivalent ω_W parameter, M_W (Ref. V-15).

THE PROTON CROSS SECTIONS FOR W <2.0 GeV

There is some interest in whether or not the nucleon resonances excited in electroproduction persist as Q^2 is increased. The data taken with 6.5 GeV incident energy at 60° were scanned with high statistics through the mass region M<W< 3.0 GeV. The cross sections plotted against W are shown in Fig. V-8. Also shown on the top scale of the figure are the corresponding values of Q^2. The resonance enhancements located at W=1512 MeV and W=1700 MeV are large with respect to the background but the first resonance located at 1238 MeV is small suggesting that its form factor falls with increasing Q^2 faster than the background under it. The statistical accuracy of the data is too poor to afford much comment on the fourth resonance enhancement around W=1950 MeV.

NEUTRON/PROTON COMPARISON

We have combined our N/P data into bins of .025 in x'. We have used the model subtraction techniques described in Chapter IV, but instead of assigning ±5% error on a point-to-point basis, we used the error of the measured hydrogen cross section for that point. The calculated N/P values are given in Table V-4 and are plotted in

The slope in Q^2 for fixed values of the scaling variable for values of $M_S^2 = 0$, M^2, and 1.42. The poor χ^2 for $M_S^2 = 0$ and M^2 indicate that the Q^2 dependence is not well parameterized by a straight line.

FIG. V - 7

Q^2 Slopes for x
$\chi^2 \simeq 2.2 / D.F.$

$\langle S_x \rangle = -.0168$

Q^2 Slopes for x'
$\chi^2 \simeq 1.9 / D.F.$

$\langle S_{x'} \rangle = -.0083$

Q^2 Slopes for x_s
$\chi^2 \simeq 1.2 / D.F.$

$\langle S_{x_s} \rangle = 0.000$

SCALING VARIABLE

$E_0 = 6.5$ GeV
$\theta = 60°$
TARGET: HYDROGEN
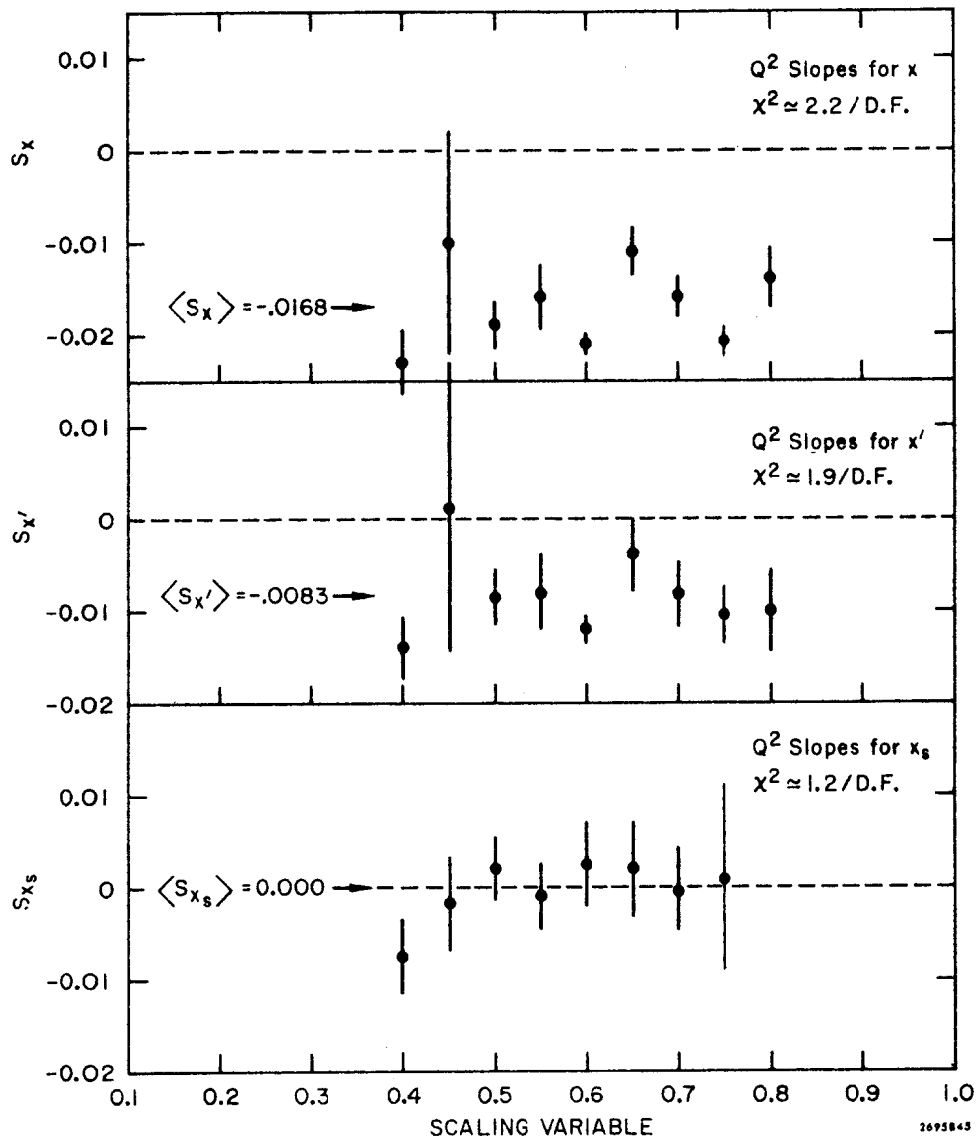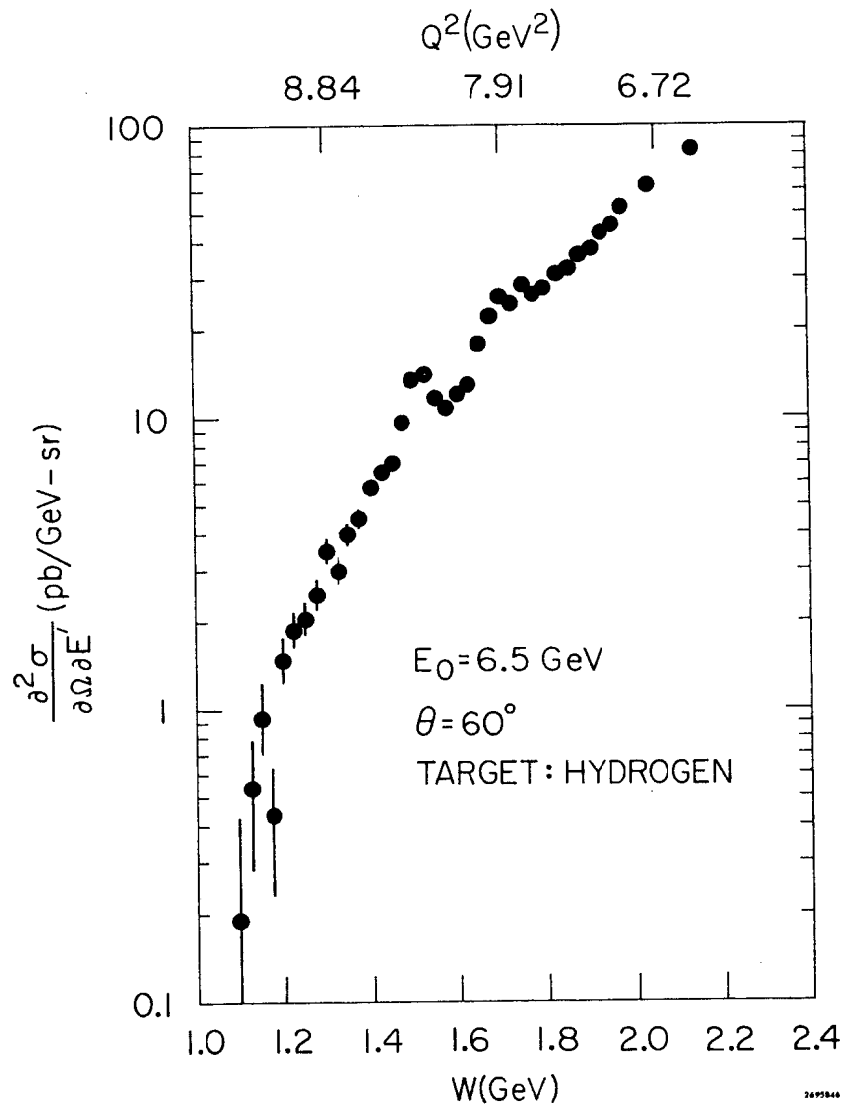
FIG. V - 8

Resonance region data for
6.9 GeV$^2 < Q^2 < 9.2$ GeV^2.

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TABLE V - 4

x'	x' Range of Smearing	N/P	Systematic Error
.325	.31 - .35	.525 ± .085	± .031
.375	.35 - .41	.606 ± .100	± .031
.400	.37 - .44	.508 ± .054	± .031
.425	.39 - .46	.599 ± .123	± .031
.450	.41 - .50	.602 ± .059	± .031
.475	.43 - .53	.598 ± .064	± .031
.500	.46 - .56	.602 ± .099	± .031
.525	.48 - .59	.553 ± .060	± .031
.550	.50 - .62	.572 ± .049	± .031
.575	.52 - .65	.506 ± .054	± .031
.600	.54 - .68	.470 ± .031	± .031
.625	.56 - .71	.490 ± .030	± .031
.650	.58 - .75	.414 ± .036	± .031
.675	.60 - .78	.435 ± .038	± .031
.700	.61 - .81	.430 ± .044	± .031
.725	.63 - .85	.455 ± .055	± .031
.750	.65 - .88	.349 ± .049	± .033
.775	.67 - .92	.409 ± .062	± .034
.800	.69 - .95	.360 ± .057	± .034
.825	.71 - .99	.284 ± .057	± .035
.850	.73 - 1.00	.271 ± .068	± .036
.875	.74 - 1.00	.251 ± .163	± .038

The measure N/P ratios for 50° and
60°. The systematic errors include
target uncertainties and smearing
errors.

Fig. V-9 together with the data from Ref. V-7 and from Ref. V-3.
The new data is in fair agreement with the previous measurements.
The N/P ratio near x'equal to 1 should be considered very cautiously
as the neutron information really represents an average over the
cross section for values of x'from 1 to .7 with the bulk of the contribu-
tion coming from the lower x's(see Appendix C). The ratio N/P is ex-
pected to be in the range of $1/4 < N/P < 4$ on rather general grounds,
and we have no evidence that the lower limit is violated.

We have also considered the neutron data separately and performed
a similar analysis of scale breaking. We found that the neutron data
favored simple scaling in x'with no propagator scale breaking. Going
to the x_s variable increases the χ^2 of the fit by a factor of two.
This suggests the possibility that N/P does not scale, but the statis-
tical accuracy of the data and uncertainties in the neutron extraction
do not allow firm conclusions to be made.

A SUMMARY OF THE PHYSICS RESULTS

The elastic scattering measurements made on hydrogen are consistent
with a $1/Q^4$ fall off of the form factor for values of Q^2 as high as
33.4 GeV^2 as predicted by Brodsky and Farrar.

The behavior of the inelastic W_1 structure function is roughly what
one would have expected but a close examination reveals inconsistencies.
W_1 does not scale in x' and this situation cannot be repaired by choosing
various values of R consistent with previous data. W_1 does not scale
in x. Propagator scale breaking is consistent with the measurements when

σ_n/σ_p plot with x axis x'

× 4° Data
○ 15°–34° Data
● 50°, 60° Data
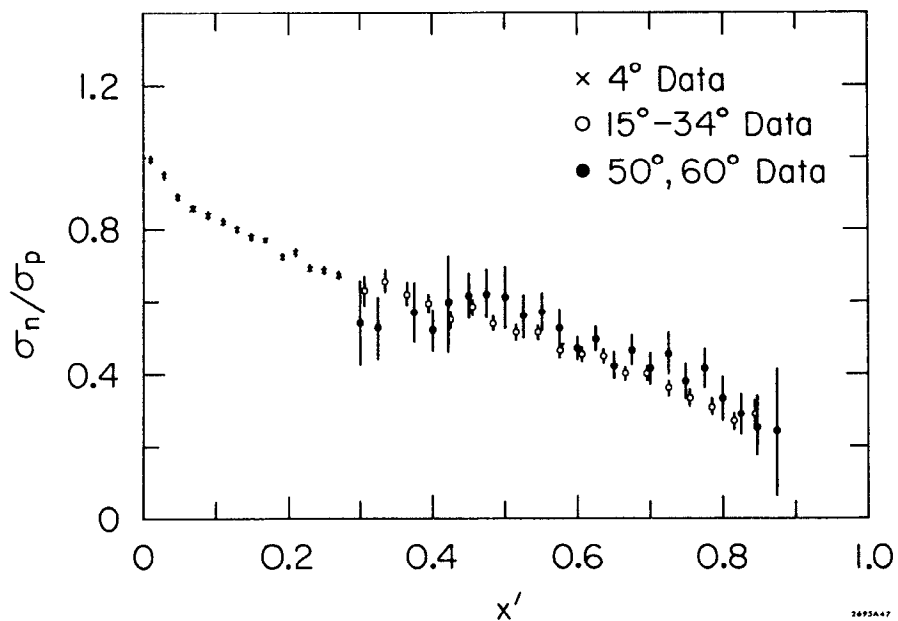
FIG. V - 9

The N/P ratio plotted versus x'.

the x'scaling variable is used. Also changing the scaling variable to $\omega_s = \omega + M_s^2/Q^2$ with $M_s^2 = 1.42$ results in W_1 scaling with a simple $(1-x_s)^4$ power law dependence. This power law for W_1 and the $1/Q^4$ behavior of G_M are in conflict with the Drell–Yan–West relation.

The various proton resonances at high Q^2 have different dependences on Q^2. The "1238" is not seen in the data for $Q^2 = 9$ GeV^2 while the second and third resonance regions remain comparable to the background.

The N/P ratio is consistent with previous measurements.

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APPENDIX A - THE CERENKOV COUNTER

A major component of the electron detection/pion rejection
system was a large aperture, threshold gas Cerenkov counter. This
appendix presents specific details about its design and its func-
tioning in the experiment.

As shown in Fig. A-1, an aluminum frame supported the mirror
assembly and phototubes. Two beam windows of .002" aluminum foil
and 1/16" aluminum walls attached to the frame formed the gas enclo-
sure. The beam windows were made as thin as possible to reduce
knock-on electron production. The mirror blank was formed from a
¼" sheet of plexiglas slump molded on a spherical carbon mold with a
radius of 138 cm. After cutting the blank to 26"x 26", the mirror
was coated by vapor depositing aluminum to a thickness of 2500 Å.
A 200 Å over-coating of MgF_2 was deposited on the front surface of the
mirror immediately after the aluminum was deposited to protect the
aluminum from oxidation. The backside of the mirror was coated with
aluminum to trap Cerenkov light produced inside the plexiglass.

The phototube assembly consisted of two Amperex 58 DUVP photo-
multipliers placed side by side. A double conical aluminized light
collector helped to reflect the Cerenkov light onto the photocathodes.
This assembly as well as the mirror was aligned in the Cerenkov
counter using a small optical laser. The focusing properties of
the mirror were optimized and checked using a conical plexiglass
lens system to simulate the Cerenkov light radiation pattern. These
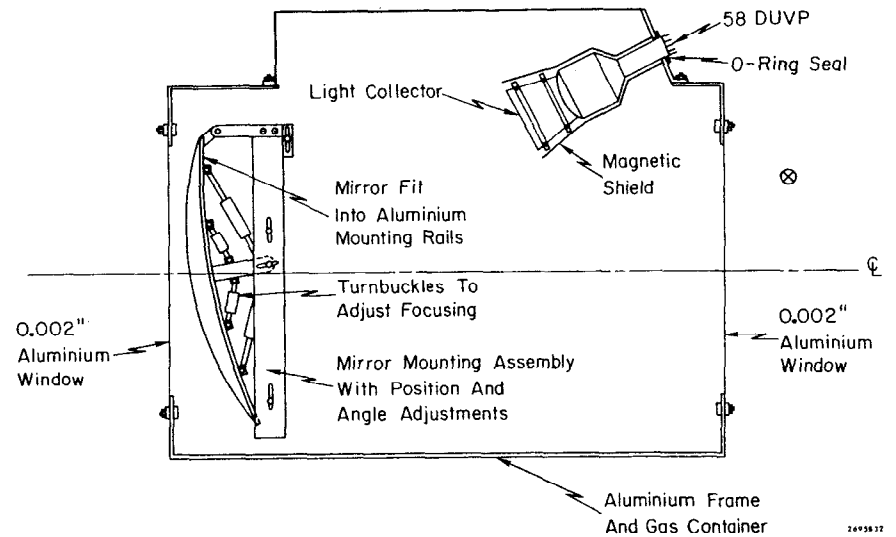adjustments were made by warping the mirror using the turnbuckles.

FIG. A-1

A layout of the isobutane threshold gas
Cerenkov counter used in E-89.

mounted on the mirror support frame (see Fig. A-1).

In Table A-1 some physical properties of several gases are shown. The data on the various gases were compiled from Ref. A-1. All densities and indexes of refraction are quoted for 26.5° at 1 atm. pressure. The number of Cerenkov photons, N_g was calculated using

$$(A.1) \qquad \frac{dN_g}{d\ell\,d\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2} \right) \frac{1}{\lambda^2}$$

where n is the index of refraction, $\beta = P/E$ ($\beta = 1$ for energetic electrons), and $\alpha = 1/137$ (the fine structure constant). The limits of integration were 3000 Å to 6000 Å, and n was assumed to be constant over this interval. In column 7 the $\gamma_t = 1/\sqrt{1 - 1/n^2}$ is given. The next two columns give the threshold energies for pions, π_{th}, and electrons, E_{th}, to count. In column 10 is a calculation of the maximum knock-on production per meter of gas from 1.5 GeV pions using (Ref. A-2).

$$(A.2) \cdot \quad \eta = 2\left(\frac{.15\,Z}{A}\right) \frac{m_e}{\beta_\pi E_{th}} \left(1 - \frac{E_{th}}{E_{max}} \left(1 + \beta_\pi^2 \ln(E_{max}/E_{th})\right)\right)$$

where Z = atomic number, A = atomic weight, $E_{max} = m_e\left(\frac{1+\beta_{cm}^2}{1-\beta_{cm}^2}\right)$

($\beta_{cm} = P_\pi/(E_\pi + m_e)$), and E_{th} = threshold energy for electrons to count.

Despite the obvious drawback of high flamability, isobutane was selected for the following three reasons: 1) large Cerenkov light output per unit path length, 2) compared to the Cerenkov light output it has a low knock-on electron efficiency, and 3)

PHYSICAL PROPERTIES OF GASES USED IN THRESHOLD CERENKOV COUNTERS

PION MOMENTUM OF 1500.0 MeV GIVES KNOCK-ON E_{MAX} = 112.22 MeV

Gas	ρ(gr/cc)	Z	M	$(n-1) \times 10^4$	γ_t	E_{th} (MeV)	π_{th} (GeV)	N_g (γ's/m)	η(knock-ons/m)
N_2	0.00114	7	14	2.699	43.05	21.998	5.941	41.231	0.1944E-03
C_3H_4	0.00166	22	40	7.401	26.01	13.290	3.589	112.942	0.6674E-03
C_3H_8	0.00184	26	44	9.203	23.32	11.919	3.219	140.399	0.9235E-03
$CH_3C\ell$	0.00210	26	50	15.202	18.16	9.278	2.506	231.778	0.1292E-02
C_4H_{10}	0.00246	34	58	13.704	19.55	9.991	2.698	199.317	0.1550E-02
C_4H_{10}(ISO)	0.00257	34	58	13.103	19.12	9.771	2.639	208.954	0.1667E-02
CF_4	0.00370	42	88	4.196	34.53	17.643	4.765	64.082	0.8542E-03
$CC\ell F_3$	0.00430	58	104	7.296	26.19	13.384	3.614	111.343	0.1496E-02
$CC\ell_2F_2$	0.00510	58	120	7.500	26.84	13.614	3.678	160.217	0.2288E-02
$CC\ell_3F$	0.00540	70	146	7.200	26.36	13.473	3.638	109.890	0.2201E-02
SF_6	0.00510	70	146	13.160	13.473	3.014	2.658	205.907	0.3836E-02
$C_2C\ell_2F_2$	0.00730	82	171	13.504	19.26	9.843	2.658	205.907	0.3836E-02
$C_2C\ell_2F_4$	0.00730	82	171	13.504	19.26	9.843	2.658	205.907	0.3836E-02

it has good light transmission far into the ultraviolet (the light transmission cuts off at approximately 2200 Å). Isobutane is considerably superior to the Freons in all of these categories.

Data on elastically scattered electrons using a 1.5 GeV incident beam were taken at the beginning and end of the first data run and at the beginning of the second data run (see Table I - 1). At the beginning and end of each data taking period, parts of the 6.5 GeV incident energy line at 60° were taken to monitor any equipment changes. These test runs revealed a small gradual improvement in the Cerenkov counter over the course of the experiment probably due to the continuously flowing isobutane purging the counter of impurities.

Threshold gas Cerenkov counters fail to give 100% particle discrimination primarily through knock-on electron production in front of the counter as well as in the gas itself. Sometimes other mechanisms such as Cerenkov light produced in the mirror or scintillation of the gas can spoil a counter's discrimination capability. The raw rejection of pions to electrons for this counter in the environment of the 1.6 GeV Spectrometer counter cave varied from 350:1 to a low of 200:1, depending on beam and target conditions. This rejection corresponded to our on-line hardware discriminator cut place in PHA channel 15 (see Fig. II - 9). This cut had no measureable inefficiency for electron detection.

There are numerous ways in which pions can count in this counter: 1) electrons produced by particles hitting the end of the 1.6 aluminum vacuum tank (referred to as "spray"); 2) knock-on electrons produced in the exit beam window of the spectrometer or Y-hodoscope; 3) particles striking the phototubes directly; 4) Cerenkov light produced in the mirror and light collection cones; and 5) scintillation of the gas. Of these, only the first three are real problems, as 4) and 5) probably contribute little because of the counter's design.

Knock-on electrons produced near the top of the 1.6 Spectrometer vacuum tank and near the bottom of the counter cave were quickly found to be the most serious problem for this Cerenkov counter. During the first checkout runs in June and July of 1973, before the installation of the Y-hodoscope, a remotely controlled moveable two-piece counter made of 1/8" plastic scintillator (TR1) was positioned in front of the Cerenkov counter. It was determined that the "noise" rate in the Cerenkov counter was less when this counter covered the magnet's exit aperture. Presumably some of the soft spray coming up the vacuum tank was absorbed by this counter. We verified that the "spray" was coming up the vacuum tank rather than penetrating the shielding by closing the moveable slits in front of the spectrometer and observing a much reduced counting rate. It was decided that the knock-on electrons produced in the material of the hodoscope (equivalently TR1) were less

damaging to the pion rejection capability of the counter than this

spray. The Y-hodoscope replaced the moveable TR1 for the rest of

the experiment.

This Cerenkov counter gave approximately 44 photoelectrons

(using a factor of 1.3 for first dynode fluctuations(Ref. A-3))

in the phototubes per passage of an electron. The value calculated

from Table A-1 for 1.24 meters of Isobutane is 259 photons incident

on the photocathodes. This would indicate an average quantum

efficiency of 17% for these Amperex DUVP photomultipliers weighted

with a Cerenkov light spectrum. The knock-on efficiency is cal-

culated to be 1/484 for the gas and 1/154 for the hodoscope in

front of the counter. As the calculation is the maximum number of

knock-ons produced, a reduction of a factor of two is reasonable.

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APPENDIX B - DESIGN CONSIDERATIONS FOR LEAD GLASS SHOWER COUNTERS

High energy electrons and photons incident on matter form electromagnetic cascade showers. Rossi (Ref. B-1) discusses shower theory in detail. We shall only sketch the shower process here. Upon entering matter, high energy electrons or photons $(E_{e,\gamma} > 100 \text{ MeV})$ start to lose energy through the emission of energetic photons or by pair production. The created particles then continue the process, thus producing still more particles. The resulting avalanche continues to build up until the typical energy of the photons has dropped to less than about 10 MeV. Below this energy, Compton scattering dominates over pair production. When the electrons (positrons) have less energy than the critical energy, E_{cr} (typical values are 8 MeV for Pb and 15 MeV for lead glass) ionization losses dominate over bremsstrahlung losses.

The characteristic distance within matter for an electron (positron) bremsstrahlung is approximately equal to the distance for pair production by a photon at high energies. This electromagnetic interaction length is referred to as the radiation length, χ_o, and depends on the specific atomic properties (principally the atomic number and the density) of the shower medium. Shower characteristics can be discussed in units of radiation lengths and be somewhat independent of the specific material.

A crude picture of a shower is as follows. An incident high energy electron starts an electromagnetic cascade shower which builds up exponentially with a doubling length of approximately a

radiation length. This build-up ceases when the typical energy of the shower photons reach an energy of about 10 MeV (i.e., where pair production and Compton scattering are equal). At this "shower maximum" point given by (Ref. B-1)

$\chi_{o \text{ max}} = 1.01 \ (\ln (E_{inc}/E_{cr}) - 1)$ where E_{inc} is the incident particles' energy.

The shower has a charge multiplicity of approximately

$$\Pi_{max} = \frac{.31}{(\ln(E_{inc}/E_{cr}) - .37)^{\frac{1}{2}}} \ \frac{E_{inc}}{E_{cr}}$$

The shower now starts to fade, less rapidly than its build-up, through ionization loss. Processes such as Moller scattering, Bhabha scattering, and Compton scattering disperse the shower. A simple picture of the transverse shower development caused by a high energy incident particle is a narrow "core" of energetic components which have little deflection from the incident particle's trajectory, surrounded by a "fuzz" of soft Compton and Moller scattered electrons. The radial spread is typically $2 \chi_o - 3 \chi_o$ and occurs after shower maximum. Shower maximum occurs at $3-6 \chi_o$ for energies of 1-10 GeV and varies logarithmically with energy. The shower tail (after shower maximum) can be as long as $14 \chi_o$ long and is insensitive to incident energy. For an incident photon the same picture is approximately correct except that one should add a radiation length for the photon to convert into an electron-positron pair, but then the shower begins with a multiplicity of two.

The shower loses energy to the material in which it occurs, mainly through ionization losses. To the extent that ionization

loss is independent of the particle's energy, the total charged particle path length, τ, will be proportional to the energy of the particle initiating the shower. In lead glass particle counters, charged particles produce light via Cerenkov radiation. The total Cerenkov light output for shower particles with velocities near the speed of light is proportional to the charged particle path length.

The fluctuations in short counters are due primarily to the initial shower development. Typically the point of the first interaction is a radiation length into the material, but on a shower-to-shower basis this point will fluctuate by approximately a radiation length, thus shifting the whole shower back and forth by a radiation length. This can cause more or less leakage from the back of the counter.

We now examine shower development in lead glass. The principal physical properties of several commercially available lead glasses are shown in Table B-1 (Ref. B-2). The light transmission of these glasses is shown in Fig. B-1 for a length of 11 χ_o of glass (Ref. B-2). Also plotted in Fig. B-1 are the approximate quantum efficiencies of AVP and DVP photocathodes.

Using a crude shower model that has all the shower particles going in the same direction with approximately the right buildup and decay in multiplicity, we can fold in the glass transmission and photocathode response to estimate the total photoelectron yield. Figure B-2a shows the photon number spectrum per $\overset{o}{A}$ for a 1 GeV shower in a 16 χ_o counter that would be transmitted by the glass to

TABLE B - I

LEAD GLASS PROPERTIES

Some physical properties of various commercial lead glasses.

No.	Schott	Ohara	Ref.Index	Density (gr/cc)	% PbO	Rad.Len. (cm)	Int.Len. (cm)	I.L./R.L.
1a	F2	PEMG2A	1.6200	3.63	45.0	3.22	28.0	8.7
1b	-	PEMG2	1.6200	3.63	45.0	3.22	28.0	8.7
2	SF2	PEMG1	1.6477	3.85	50.0	2.84	27.3	9.6
3	SF5	PEMG4	1.6727	4.08	55.0	2.54	25.6	10.1
4	SF1	PEMG5	1.7174	4.47	61.0	2.17	24.9	11.5
5a	SF6	PEMG3A	1.8052	5.20	71.0	1.68	22.7	13.5
5b	-	PEMG3	1.8052	5.20	71.0	1.68	22.7	13.5

FIG. B-1

Transmission curves for lead glass types
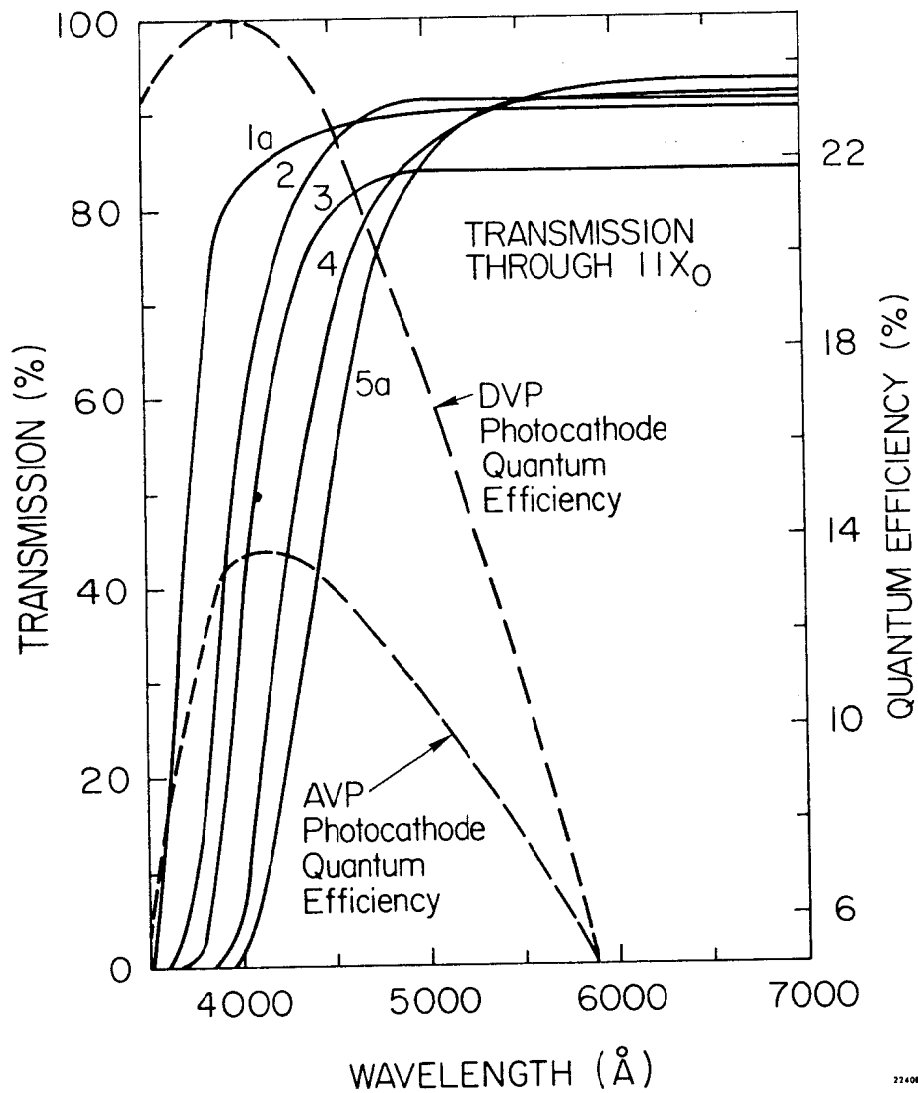1a, 2, 3, 4, 5a (see Table B-1). Dash lines
and right-hand scale show AVP and DVP photo-
cathode response.

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Number of photons produced from a 1 GeV
electron shower and transmitted through
a 16 X_0 long counter.

Number of photoelectrons expected for a
1 GeV shower for various glass types. No
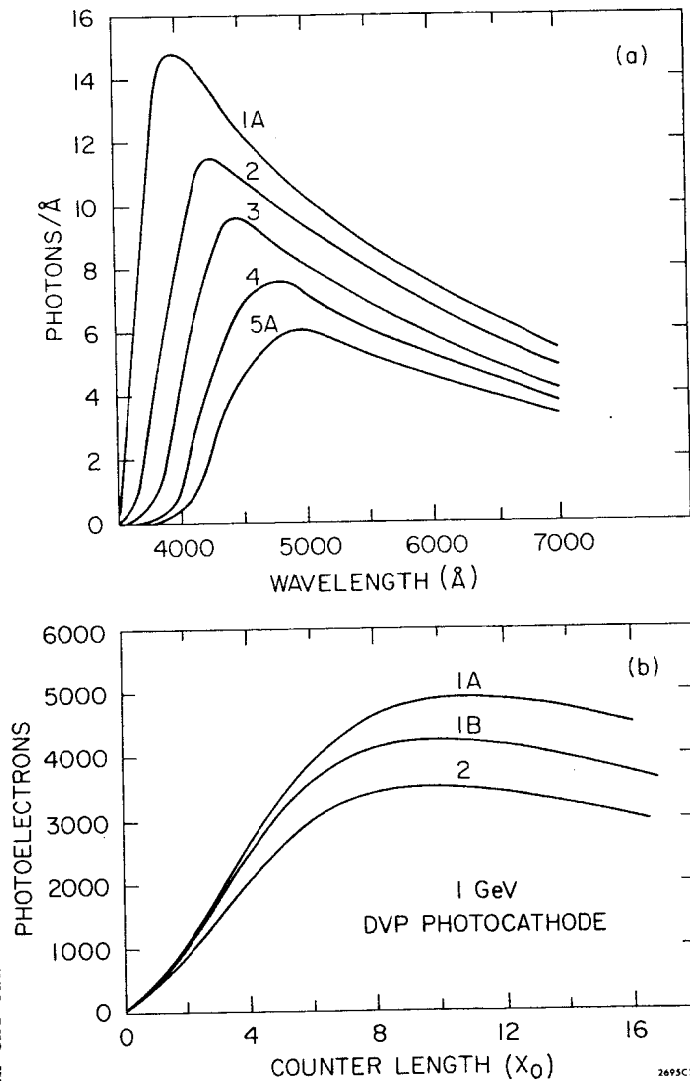light collection efficiency has been included
in the calculation.

FIG. B-2

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the photocathodes located at the opposite end of the counter from the entering particle. The curves in the red end of the spectrum are separated by approximately the ratio of their radiation lengths, while the short wavelength cutoff is determined by the transmission properties of the particular glass type. The least dense glass (longest χ_o) gives the largest number of photons for both of the above reasons. This photon number spectrum was then folded against the photocathode response of a DVP bialkali and the calculated number of photoelectrons versus counter length is shown in Fig. B-2b for 1.0 GeV showers. No light collection efficiency has been included in the calculation, so these numbers corrected for collection efficiency will depend on the particular geometry, and, in our case, were perhaps a factor of 5 too large.

Shower fluctuations in spatial development and charged particle path lengths were studies using a shower program initially written by Nagel (Ref. B-3) and modified by R. Ford (Ref. B-4). For lead glass Cerenkov shower counters, an interesting quantity to calculate is the charged particle path length adjusted for decreasing Cerenkov light intensity as β falls away from unity.

$$\tilde{\tau} = \tau \frac{(1 - \frac{1}{n^2 \beta^2})}{(1 - \frac{1}{n^2})}$$

This effective charge particle pathlength, $\tilde{\tau}$, cuts off the low energy shower particles as they do not radiate Cerenkov light as effectively. By generating Monte Carlo showers using the program the spread in various shower parameters can be estimated. The

medium in which showers were generated corresponded to the mixture of elements found in Schott F2 glass. The shower tails were followed until particles had less than 1.0 MeV of energy ($\beta^2 = .739$ for electrons). The showers developed in 40 χ_o (dia.) by 41 χ_o (long) counter geometries. The shower development was binned in radial and longitudinal bins of 1 χ_o width. The incident particles were normal to the front surface and centered on it.

In Tables B-2a through B-2c, we present the results of such Monte Carlo shower simulations for 1, 5 and 20 GeV incident electrons. Several interesting quantities may be plotted from such tables. In Fig. B-3, the fraction of the shower contained versus the length and the radius of the counter is plotted. A spread in the lines plotted for different energies indicates non-linear energy responses for that geometry. The conclusions are: 1) the radial containment is insensitive to incident energy and thus does not affect the linearity; and, 2) for energies in the 1-20 GeV range, counters appreciably shorter than 18 χ_o will not have a linear energy response.

Shower resolution is a more sensitive parameter to both depth and radius. Fig. B-4a shows that the RMS shower fluctuations in $\tilde{\tau}$ improve approximately exponentially, finally reaching a minimum at 18 χ_o for 1.0 GeV and 26 χ_o for 20 GeV. The resolution sensitivity to radial size is less dramatic and tends to become less dependent on the radius as the energy increases (see Fig. B-4b).

In Fig. B-5, we show the calculated counter resolution versus energy for a 22 χ_o by a 4 χ_o radius counter. We have used 5000

TABLE B - 2a

FRACTIONAL SHOWER PATH LENGTHS FOR 1.000 GEV ELECTRONS, AVERAGE MAX. PATH LENGTH= 49.81 R.L., 500 SHOWERS

DEPTH	RADIAL 1	2	3	4	5	6	7
0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0
1	0.04118+-0.4831	0.04129+-0.4833	0.04134+-0.4830	0.04142+-0.4839	0.04146+-0.4841	0.04147+-0.4841	0.04148+-0.4841
2	0.14116+-0.4381	0.14165+-0.4391	0.14184+-0.4392	0.14202+-0.4394	0.14210+-0.4393	0.14214+-0.4394	0.14215+-0.4394
3	0.28667+-0.3653	0.28844+-0.3665	0.28898+-0.3669	0.28933+-0.3670	0.28947+-0.3670	0.28954+-0.3671	0.28961+-0.3671
4	0.43519+-0.2958	0.44043+-0.2977	0.44180+-0.2977	0.44251+-0.2978	0.44283+-0.2977	0.44296+-0.2977	0.44307+-0.2978
5	0.56476+-0.2414	0.57639+-0.2429	0.57910+-0.2430	0.58026+-0.2433	0.58088+-0.2433	0.58111+-0.2433	0.58127+-0.2434
6	0.66327+-0.1975	0.68342+-0.1987	0.68839+-0.1989	0.69033+-0.1992	0.69130+-0.1993	0.69169+-0.1994	0.69193+-0.1994
7	0.73354+-0.1587	0.76306+-0.1602	0.77023+-0.1607	0.77301+-0.1609	0.77436+-0.1611	0.77498+-0.1611	0.77534+-0.1611
8	0.78528+-0.1251	0.82436+-0.1256	0.83404+-0.1264	0.83776+-0.1265	0.83957+-0.1267	0.84041+-0.1267	0.84088+-0.1267
9	0.81876+-0.0988	0.86585+-0.0981	0.87839+-0.0987	0.88324+-0.0989	0.88566+-0.0990	0.88686+-0.0989	0.88749+-0.0989
10	0.83934+-0.0793	0.89272+-0.0768	0.90759+-0.0772	0.91356+-0.0774	0.91641+-0.0776	0.91784+-0.0775	0.91862+-0.0774
11	0.85319+-0.0647	0.91210+-0.0600	0.92916+-0.0596	0.93624+-0.0597	0.93958+-0.0599	0.94123+-0.0597	0.94211+-0.0596
12	0.86185+-0.0557	0.92522+-0.0477	0.94414+-0.0461	0.95224+-0.0457	0.95613+-0.0458	0.95794+-0.0455	0.95893+-0.0454
13	0.86722+-0.0505	0.93397+-0.0398	0.95445+-0.0370	0.96343+-0.0362	0.96775+-0.0362	0.96980+-0.0358	0.97096+-0.0357
14	0.87107+-0.0467	0.94023+-0.0336	0.96202+-0.0291	0.97177+-0.0280	0.97640+-0.0277	0.97867+-0.0271	0.97991+-0.0271
15	0.87369+-0.0447	0.94437+-0.0298	0.96702+-0.0239	0.97730+-0.0221	0.98218+-0.0214	0.98459+-0.0208	0.98594+-0.0208
16	0.87490+-0.0440	0.94677+-0.0282	0.97015+-0.0215	0.98077+-0.0190	0.98589+-0.0180	0.98840+-0.0174	0.98987+-0.0172
17	0.87535+-0.0438	0.94789+-0.0277	0.97180+-0.0204	0.98265+-0.0177	0.98787+-0.0167	0.99042+-0.0160	0.99198+-0.0155
18	0.87576+-0.0436	0.94862+-0.0272	0.97275+-0.0197	0.98390+-0.0167	0.98926+-0.0156	0.99186+-0.0150	0.99347+-0.0144
19	0.87601+-0.0435	0.94925+-0.0270	0.97358+-0.0193	0.98488+-0.0161	0.99037+-0.0149	0.99305+-0.0141	0.99467+-0.0135
20	0.87619+-0.0435	0.94963+-0.0269	0.97413+-0.0191	0.98554+-0.0158	0.99106+-0.0145	0.99377+-0.0136	0.99542+-0.0130
21	0.87632+-0.0435	0.94991+-0.0270	0.97453+-0.0190	0.98598+-0.0157	0.99151+-0.0142	0.99426+-0.0134	0.99592+-0.0127
22	0.87643+-0.0434	0.95012+-0.0270	0.97480+-0.0190	0.98634+-0.0156	0.99188+-0.0142	0.99465+-0.0133	0.99633+-0.0126
23	0.87650+-0.0434	0.95025+-0.0269	0.97500+-0.0189	0.98658+-0.0155	0.99214+-0.0141	0.99497+-0.0132	0.99667+-0.0124
24	0.87650+-0.0434	0.95029+-0.0269	0.97508+-0.0189	0.98666+-0.0155	0.99225+-0.0142	0.99511+-0.0131	0.99681+-0.0124
25	0.87651+-0.0434	0.95031+-0.0270	0.97514+-0.0189	0.98673+-0.0155	0.99233+-0.0142	0.99519+-0.0131	0.99689+-0.0124
26	0.87655+-0.0434	0.95035+-0.0269	0.97518+-0.0189	0.98678+-0.0155	0.99238+-0.0141	0.99525+-0.0131	0.99696+-0.0124
27	0.87655+-0.0434	0.95035+-0.0269	0.97518+-0.0189	0.98679+-0.0155	0.99243+-0.0141	0.99532+-0.0131	0.99702+-0.0124
28	0.87655+-0.0434	0.95035+-0.0269	0.97519+-0.0189	0.98679+-0.0155	0.99244+-0.0141	0.99532+-0.0131	0.99703+-0.0124
29	0.87655+-0.0434	0.95035+-0.0270	0.97519+-0.0189	0.98680+-0.0155	0.99244+-0.0142	0.99533+-0.0131	0.99705+-0.0124
30	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
31	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
32	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
33	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
34	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
35	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
36	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
37	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
38	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
39	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
40	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124
41	0.87655+-0.0434	0.95036+-0.0269	0.97520+-0.0189	0.98681+-0.0155	0.99246+-0.0141	0.99534+-0.0131	0.99707+-0.0124

FRACTIONAL SHOWER PATH LENGTHS FOR   5.000 GEV ELECTRONS, AVERAGE MAX. PATH LENGTH= 248.85 R.L.,   117 SHOWERS

RADIAL

DEPTH	1	2	3	4	5	6	7
0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0
1	0.01151+-0.5184	0.01153+-0.5186	0.01155+-0.5191	0.01157+-0.5190	0.01158+-0.5183	0.01159+-0.5186	0.01161+-0.5168
2	0.05123+-0.4560	0.05134+-0.4562	0.05141+-0.4564	0.05146+-0.4564	0.05151+-0.4568	0.05154+-0.4568	0.05158+-0.4562
3	0.12900+-0.3881	0.12963+-0.3890	0.12988+-0.3892	0.12999+-0.3893	0.13010+-0.3892	0.13016+-0.3893	0.13025+-0.3890
4	0.23636+-0.3248	0.23858+-0.3260	0.23921+-0.3260	0.23948+-0.3261	0.23965+-0.3262	0.23975+-0.3263	0.23986+-0.3262
5	0.35806+-0.2645	0.36342+-0.2663	0.36467+-0.2667	0.36528+-0.2666	0.36560+-0.2668	0.36579+-0.2668	0.36595+-0.2668
6	0.47931+-0.2124	0.49002+-0.2143	0.49255+-0.2151	0.49359+-0.2152	0.49414+-0.2154	0.49441+-0.2154	0.49462+-0.2155
7	0.58437+-0.1755	0.60176+-0.1771	0.60584+-0.1774	0.60753+-0.1776	0.60835+-0.1778	0.60878+-0.1779	0.60906+-0.1779
8	0.66887+-0.1449	0.69449+-0.1468	0.70076+-0.1470	0.70330+-0.1472	0.70444+-0.1474	0.70504+-0.1476	0.70537+-0.1476
9	0.73326+-0.1169	0.76731+-0.1191	0.77591+-0.1198	0.77945+-0.1200	0.79105+-0.1203	0.78186+-0.1204	0.78231+-0.1205
10	0.78041+-0.0917	0.82251+-0.0945	0.83361+-0.0954	0.83818+-0.0959	0.84031+-0.0962	0.84138+-0.0964	0.84196+-0.0965
11	0.81340+-0.0719	0.86210+-0.0742	0.87562+-0.0752	0.88128+-0.0755	0.88392+-0.0758	0.88526+-0.0760	0.88598+-0.0761
12	0.83516+-0.0582	0.88971+-0.0599	0.90528+-0.0609	0.91204+-0.0612	0.91511+-0.0615	0.91669+-0.0617	0.91758+-0.0618
13	0.85207+-0.0440	0.91183+-0.0452	0.92935+-0.0460	0.93702+-0.0463	0.94050+-0.0465	0.94240+-0.0467	0.94347+-0.0468
14	0.86150+-0.0317	0.92457+-0.0322	0.94355+-0.0329	0.95194+-0.0332	0.95586+-0.0335	0.95794+-0.0337	0.95912+-0.0338
15	0.86785+-0.0249	0.93355+-0.0238	0.95372+-0.0241	0.96253+-0.0243	0.96681+-0.0246	0.96910+-0.0248	0.97043+-0.0248
16	0.87189+-0.0224	0.93980+-0.0196	0.96100+-0.0194	0.97026+-0.0196	0.97483+-0.0199	0.97729+-0.0200	0.97867+-0.0200
17	0.87447+-0.0201	0.94392+-0.0154	0.96587+-0.0145	0.97584+-0.0147	0.98070+-0.0148	0.98331+-0.0149	0.98475+-0.0148
18	0.87564+-0.0188	0.94604+-0.0132	0.96865+-0.0118	0.97901+-0.0118	0.98409+-0.0117	0.98686+-0.0116	0.98840+-0.0115
19	0.87648+-0.0183	0.94771+-0.0118	0.97075+-0.0098	0.98132+-0.0098	0.98650+-0.0097	0.98934+-0.0094	0.99096+-0.0094
20	0.87718+-0.0180	0.94889+-0.0112	0.97214+-0.0085	0.98286+-0.0084	0.98817+-0.0081	0.99113+-0.0078	0.99286+-0.0078
21	0.87760+-0.0179	0.94953+-0.0113	0.97299+-0.0081	0.98390+-0.0077	0.98932+-0.0073	0.99237+-0.0069	0.99411+-0.0069
22	0.87778+-0.0179	0.94986+-0.0112	0.97345+-0.0080	0.98443+-0.0073	0.98993+-0.0070	0.99305+-0.0065	0.99483+-0.0064
23	0.87782+-0.0179	0.95000+-0.0111	0.97370+-0.0080	0.98475+-0.0074	0.99030+-0.0070	0.99345+-0.0064	0.99530+-0.0063
24	0.87789+-0.0179	0.95021+-0.0111	0.97396+-0.0079	0.98507+-0.0073	0.99065+-0.0068	0.99383+-0.0062	0.99569+-0.0062
25	0.87796+-0.0179	0.95043+-0.0112	0.97424+-0.0080	0.98538+-0.0073	0.99096+-0.0068	0.99420+-0.0062	0.99607+-0.0061
26	0.87799+-0.0180	0.95053+-0.0112	0.97436+-0.0080	0.98557+-0.0072	0.99116+-0.0066	0.99440+-0.0060	0.99628+-0.0060
27	0.87801+-0.0179	0.95060+-0.0112	0.97446+-0.0080	0.98569+-0.0071	0.99130+-0.0066	0.99454+-0.0060	0.99642+-0.0060
28	0.87801+-0.0179	0.95061+-0.0113	0.97448+-0.0080	0.98573+-0.0072	0.99133+-0.0065	0.99459+-0.0060	0.99647+-0.0059
29	0.87801+-0.0179	0.95061+-0.0113	0.97448+-0.0080	0.98574+-0.0071	0.99135+-0.0065	0.99460+-0.0060	0.99649+-0.0059
30	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
31	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
32	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
33	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
34	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
35	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
36	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
37	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
38	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
39	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
40	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059
41	0.87801+-0.0179	0.95062+-0.0113	0.97449+-0.0080	0.98575+-0.0071	0.99136+-0.0065	0.99463+-0.0059	0.99652+-0.0059

FRACTIONAL SHOWER PATH LENGTHS FOR 20.000 GEV ELECTRONS, AVERAGE MAX. PATH LENGTH= 995.03 R.L., 28 SHOWERS

| DEPTH	RADIAL 1	2	3	4	5	6	7
0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0	0.0 +-0.0
1	0.00292+-0.5683	0.00293+-0.5671	0.00294+-0.5612	0.00295+-0.5603	0.00296+-0.5586	0.00298+-0.5590	0.00299+-0.5579
2	0.01926+-0.5245	0.01932+-0.5228	0.01936+-0.5216	0.01938+-0.5215	0.01940+-0.5214	0.01944+-0.5210	0.01945+-0.5207
3	0.06031+-0.4240	0.06060+-0.4238	0.06073+-0.4227	0.06081+-0.4223	0.06084+-0.4227	0.06089+-0.4226	0.06091+-0.4225
4	0.12865+-0.3574	0.12949+-0.3576	0.12979+-0.3575	0.12998+-0.3574	0.13006+-0.3575	0.13012+-0.3575	0.13015+-0.3574
5	0.22492+-0.2982	0.22748+-0.2994	0.22914+-0.2992	0.22850+-0.2990	0.22865+-0.2990	0.22878+-0.2990	0.22884+-0.2989
6	0.33653+-0.2422	0.34196+-0.2433	0.34331+-0.2433	0.34391+-0.2432	0.34418+-0.2432	0.34437+-0.2432	0.34446+-0.2432
7	0.44624+-0.1933	0.45663+-0.1963	0.45930+-0.1964	0.46011+-0.1963	0.46059+-0.1963	0.46090+-0.1962	0.46106+-0.1962
8	0.54422+-0.1594	0.56141+-0.1626	0.56548+-0.1630	0.56683+-0.1631	0.56757+-0.1631	0.56806+-0.1631	0.56832+-0.1631
9	0.62704+-0.1354	0.65176+-0.1379	0.65793+-0.1385	0.66007+-0.1385	0.66109+-0.1385	0.66178+-0.1384	0.66211+-0.1385
10	0.69322+-0.1213	0.72550+-0.1238	0.73399+-0.1242	0.73691+-0.1242	0.73833+-0.1242	0.73925+-0.1242	0.73966+-0.1242
11	0.74307+-0.1087	0.78319+-0.1109	0.79424+-0.1111	0.79800+-0.1110	0.79983+-0.1111	0.80095+-0.1111	0.80146+-0.1111
12	0.78108+-0.0955	0.82813+-0.0978	0.84115+-0.0980	0.84608+-0.0979	0.84836+-0.0983	0.84975+-0.0980	0.85035+-0.0980
13	0.80970+-0.0802	0.86280+-0.0828	0.87800+-0.0830	0.88402+-0.0830	0.88671+-0.0831	0.88834+-0.0831	0.88907+-0.0831
14	0.82972+-0.0640	0.88812+-0.0669	0.90545+-0.0674	0.91244+-0.0673	0.91556+-0.0675	0.91742+-0.0675	0.91833+-0.0676
15	0.84433+-0.0503	0.90650+-0.0531	0.92538+-0.0538	0.93324+-0.0536	0.93697+-0.0536	0.93899+-0.0536	0.94012+-0.0536
16	0.85425+-0.0384	0.91935+-0.0407	0.93951+-0.0416	0.94824+-0.0414	0.95232+-0.0415	0.95459+-0.0413	0.95588+-0.0414
17	0.86134+-0.0280	0.92983+-0.0302	0.94999+-0.0310	0.95930+-0.0308	0.96374+-0.0309	0.96619+-0.0308	0.96756+-0.0308
18	0.86648+-0.0200	0.93574+-0.0232	0.95776+-0.0230	0.96762+-0.0226	0.97224+-0.0228	0.97482+-0.0226	0.97627+-0.0227
19	0.86989+-0.0148	0.94071+-0.0169	0.96349+-0.0175	0.97375+-0.0170	0.97868+-0.0171	0.98140+-0.0169	0.98291+-0.0170
20	0.87207+-0.0113	0.94390+-0.0124	0.96737+-0.0128	0.97795+-0.0123	0.98312+-0.0123	0.98598+-0.0121	0.98760+-0.0122
21	0.87332+-0.0096	0.94576+-0.0096	0.96970+-0.0097	0.98047+-0.0091	0.98569+-0.0091	0.98866+-0.0088	0.99035+-0.0090
22	0.87437+-0.0097	0.94731+-0.0078	0.97173+-0.0078	0.98272+-0.0071	0.98805+-0.0069	0.99108+-0.0067	0.99281+-0.0067
23	0.87491+-0.0082	0.94815+-0.0066	0.97282+-0.0063	0.98393+-0.0056	0.98940+-0.0052	0.99248+-0.0049	0.99422+-0.0051
24	0.87520+-0.0079	0.94858+-0.0061	0.97339+-0.0056	0.98463+-0.0050	0.99016+-0.0047	0.99330+-0.0042	0.99509+-0.0041
25	0.87558+-0.0077	0.94905+-0.0056	0.97405+-0.0049	0.98541+-0.0042	0.99095+-0.0040	0.99410+-0.0031	0.99592+-0.0035
26	0.87568+-0.0077	0.94921+-0.0054	0.97428+-0.0044	0.98565+-0.0039	0.99123+-0.0038	0.99441+-0.0032	0.99624+-0.0033
27	0.87571+-0.0077	0.94935+-0.0054	0.97457+-0.0041	0.98598+-0.0038	0.99157+-0.0036	0.99477+-0.0027	0.99660+-0.0031
28	0.87573+-0.0077	0.94944+-0.0053	0.97469+-0.0040	0.98613+-0.0037	0.99175+-0.0035	0.99496+-0.0028	0.99681+-0.0031
29	0.87574+-0.0077	0.94947+-0.0053	0.97472+-0.0039	0.98619+-0.0037	0.99184+-0.0034	0.99506+-0.0029	0.99692+-0.0028
30	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
31	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
32	0.87574+-0.0077	0.94947+-0.0053	0.97472+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
33	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
34	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
35	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
36	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
37	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
38	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
39	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
40	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
41	0.87574+-0.0077	0.94947+-0.0053	0.97473+-0.0040	0.98620+-0.0037	0.99185+-0.0033	0.99507+-0.0028	0.99693+-0.0028
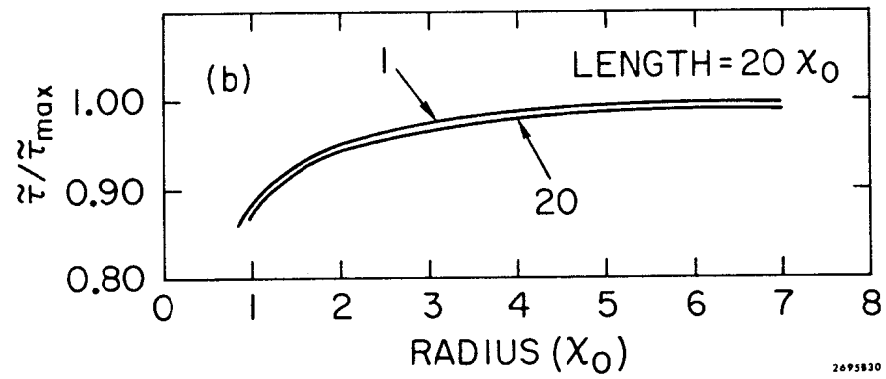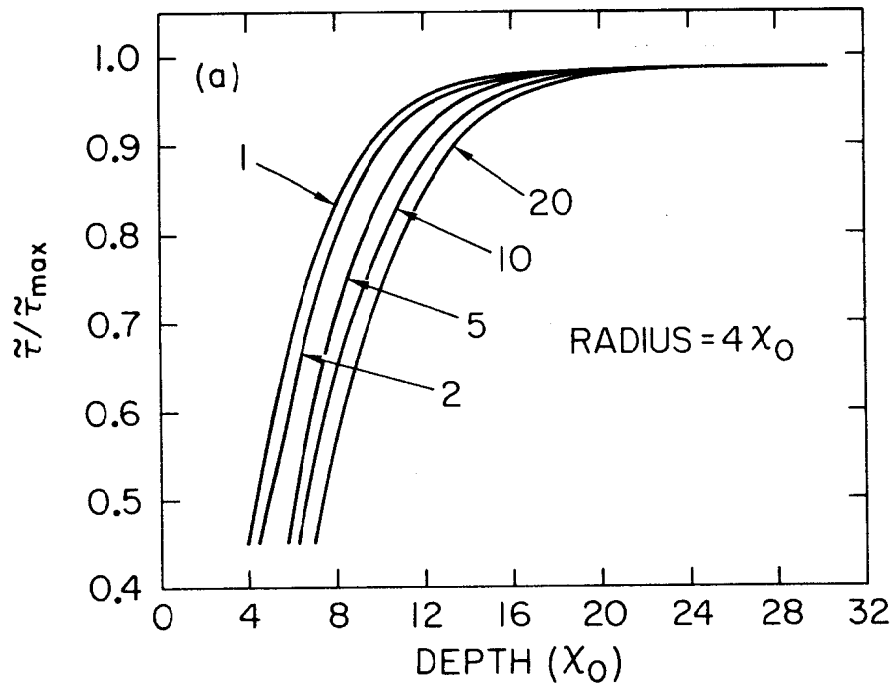
FIG. B-3

Fractional containment of showers for
1, 2, 5, 10 and 20 GeV incident electrons
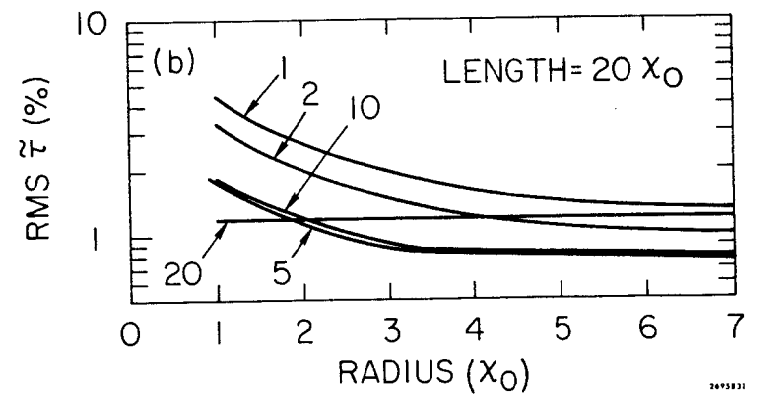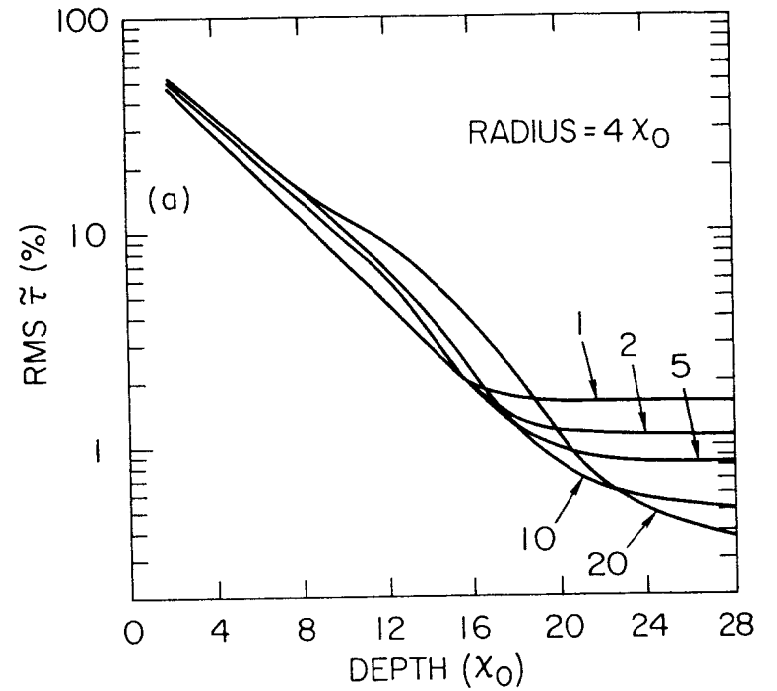versus a) counter length and b) counter
radius.

172

FIG. B-4

Shows fluctuations for 1, 2, 5, 10 and
20 GeV incident electrons versus a) counter
length and b) counter width.

173

RMS RESOLUTION (%) vs E_{inc} (GeV)

Shower Fluctuations and Photo Electron Statistics
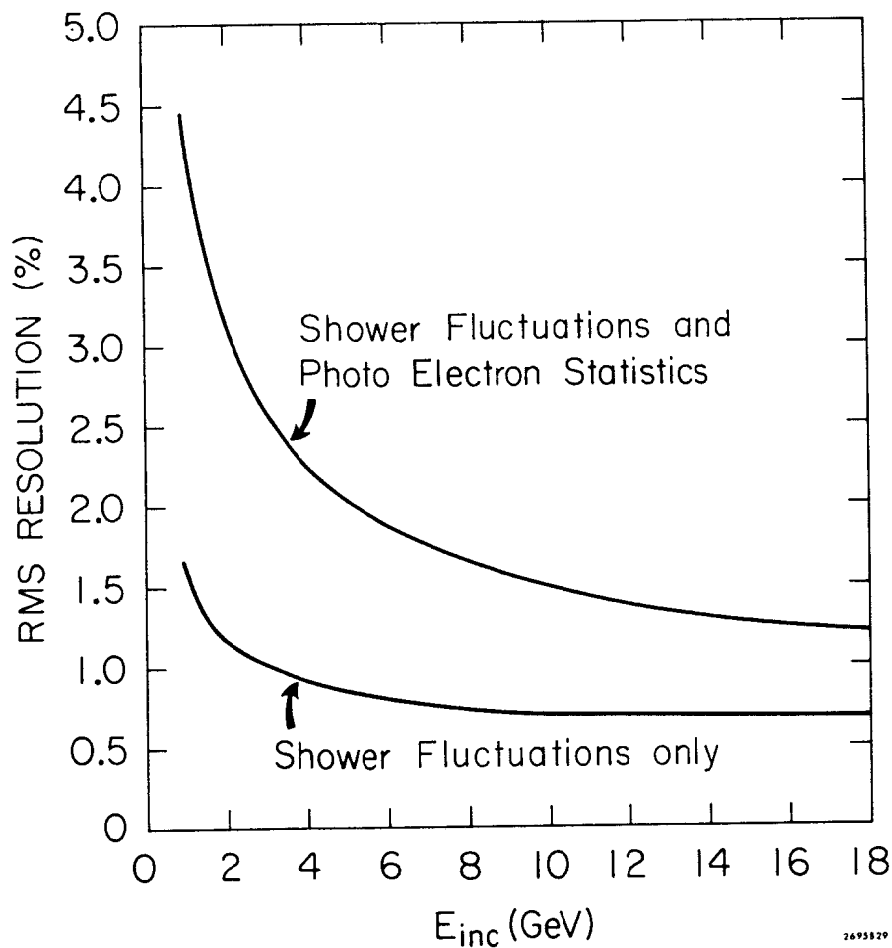
Shower Fluctuations only

FIG. B-5

Calculated RMS resolution for a lead glass counter (see text) using 20% light collection efficiency.

photoelectrons for a 1 GeV shower times a 20% light collection efficiency (so chosen to match the E-89 counter). The photoelectron fluctuations were increased by 30% to account for amplification fluctuations in the first few dynodes of the phototubes. The resultant resolution was folded in quadrature with the shower fluctuations calculated by the shower program. The important features of Fig. B-5 are: 1) for the 1-10 GeV energy region, lead glass counter resolution is dominated by photoelectron statistics and improves approximately as $1/\sqrt{E_{inc}}$; and 2) at high energies ($E_{inc} \gtrsim 10$ GeV) the resolution should improve more slowly approaching a $1/E_{inc}^{\frac{1}{4}}$ response similar to that of NaI(Tℓ).

As photoelectron statistics are the dominant contribution to the resolution, care should be taken to obtain optimum light collection. To examine the Cerenkov radiation pattern in glass counters, we have plotted $\tilde{\tau}$ versus $\cos(\theta_i)$ where θ_i is the angle of the path length with respect to the incident direction in Fig. B-6. The forward peaking is a factor of about 2 over the backwards contribution. This gives a Cerenkov radiation pattern somewhat peaked at the Cerenkov angle($\theta_c = 51.9$ deg. for F2 glass) with respect to the incident direction and a large flat background of more or less isotropically distributed light. One probably optimizes light collection by having the phototubes "look" into the shower direction. The E-89 counter was not constructed in this way.

A proper index match from the glass to the phototube is also important. For the counter constructed for E-89, oil light pipes
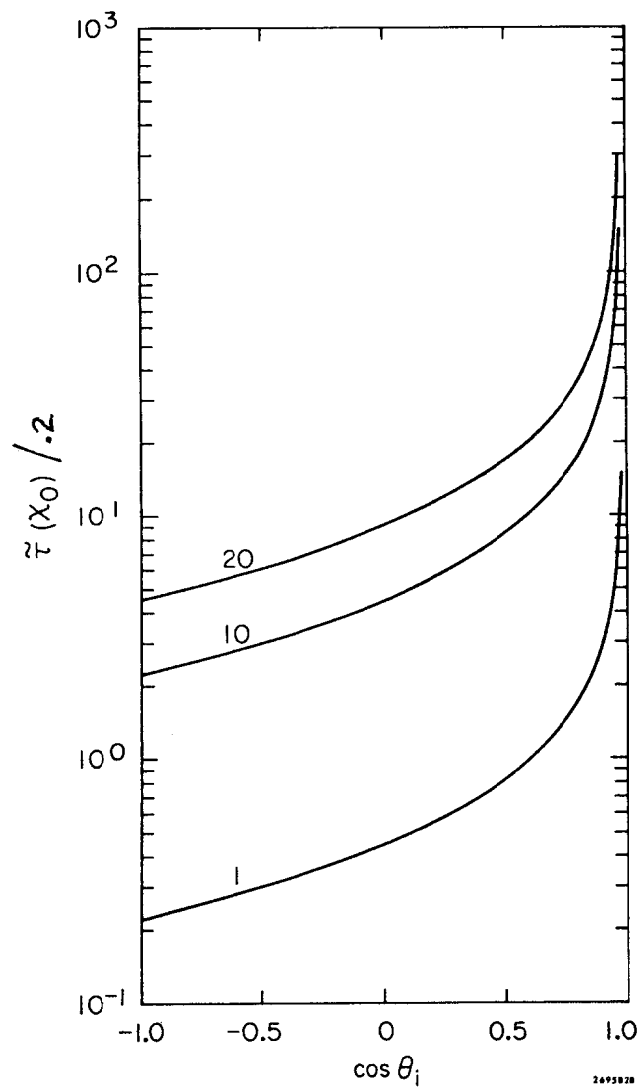
FIG. B-6

Plot of $\tilde{\tau}$ versus cosine of its angle from particle's direction for 1, 10 and 20 GeV electrons.

filled with high index fluid (n = 1.58, immersion liquid) were used. The light pipes were aluminum frames glued to the glass and the phototubes with RTV to make the seals. The system continues to work well after two years of service.

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5. L. P. Cargille Laboratory, Cedar Grove, New Jersey.

APPENDIX C - A QUASI-ELASTIC APPROACH TO SMEARING

In this Appendix, smearing is reviewed, and a unified method
of handling quasi-elastic scattering and inelastic smearing is
developed. The term smearing refers to the effect of the Fermi
motion of the nucleons within a nucleus on measured particle cross
sections. In electroproduction data the effect is perhaps most
graphically seen in quasi-elastic scattering where the narrow
electron-nucleon elastic scattering peak is broadened out by this
Fermi-motion effect. First, the quasi-elastic formalism as put
forward by G. B. West (Ref. C-1) is reviewed for the case of
arbitrary target, spectator, and produced hadronic masses. This
formalism is then taken over to the inelastic case. The kinematic
regions over which smearing is important are reviewed.

In electroproduction, we assume an impulse approximation for
the interaction of the electron and nucleon within the target nucleus
(see Fig. C-1). The notation used in this Appendix is given in
Table C-1. Each nucleon is considered to be "free" and interacts
independently of the rest of the nucleus (no interference terms) but
with shifted kinematics resulting from its motion. This interacting
nucleon leaves behind a spectator particle of equal but opposite
momentum. Due to conservation of momentum at the breakup of the
target particles, the probability of finding an interacting nucleon
of a given momentum is equally well given by that of finding a
spectator of opposite momentum.

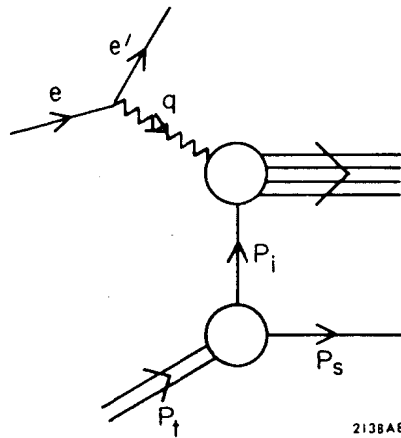These ideas are expressed in the following equations relating

FIG. C-1

Feynman diagram of electron-deuterium
scattering in the impulse approximation.

TABLE C - 1

Quantity	Definition
M	Mass of the proton
M_S	Mass of the free spectator particle
M_i	Mass of the interacting particle were it on its mass shell
M_t	Mass of the target particle
P_s	Magnitude of the 3 momentum of the spectator particle
$E_s = \sqrt{P_s^2 + M_s^2}$	Energy of the spectator particle
$P_i = (P_o, P_x, P_y, P_z)$	Four vector of the interacting particle (note $P_i^2 \neq M_i^2$)
$q = (\nu, \vec{q})$	Four vector of the virtual photon (q is the magnitude of $\vec{q}$ and $q^2 = -Q^2$)
$P_t = (M_t, \vec{0})$	Four vector of the target particle in the laboratory

Definition of the variables used in the equations of Appendix C.

the nucleon structure functions to that of the target particle

(we consider here the deuteron case):

(C-1) $\quad W_{\mu\nu}^{D} = \int \frac{d^3P_s}{E_s/M_s} \mid \phi \, (P_s) \mid^2 \{ W_{\mu\nu}^{P} + W_{\mu\nu}^{N} \}$

where $\quad W_{\mu\nu}^{k} = -W_1^{k} (q_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{W_2^{k}}{M^2} (P_\mu^{k} - \frac{P^{k} \cdot q}{q^2} q_\mu)(P_\nu^{k} - \frac{P^{k} \cdot q}{q^2} q_\nu)$

$\mid \phi(P_s) \mid^2$ is the square of the relativistic momentum wave function

of the spectator particle.

Defining $\vec{q}$ along the z-axis, examination of the various

tensor components of equation C-1 yields the following two equations

for the smearing of the W_1 and the W_2 structure functions:

(C-2a) $\quad W_1^{s}(\nu,Q^2) = \int \frac{d^3P_s}{E_s/M_s} \mid \phi(P_s) \mid^2 \{ W_1(W^2,Q^2) + \frac{P_x^2}{M^2} W_2(W^2,Q^2) \}$

(C-2b) $\quad W_2^{s}(\nu,Q^2) = \int \frac{d^3P_s}{E_s/M_s} \mid \phi(P_s) \mid^2 \mathcal{F}(P_i,\nu,q) \; W_2(W^2,Q^2)$

and

$\mathcal{F}(P_i,\nu,q) = \frac{1}{M^2} \left[(P_o - \frac{\nu}{q} P_z)^2 - \frac{q^2}{q^2} P_x^2 \right]$

Consider the general case of $(P_i + q)^2 = W^2$. In the case of quasi-

elastic scattering, the condition that the interacting nucleon be

put onto its mass-shell, $(P_i + q)^2 = M^2$, in the final state is imposed.

We set

(C-3a) $\quad W_1(W^2,Q^2) \equiv G_1(Q^2,S) \; \delta \, (S-W^2)$

(C-3b) $\quad W_2(W^2,Q^2) \equiv G_2(Q^2,S) \; \delta \, (S-W^2)$

(for quasi-elastic scattering $S = M_i^2$)

Transforming the invariant phase-space volume element to the

C.M. system (denoted by a tilde) of the photon-target particle

system, the delta function is used against the dP_s integration.

This leaves only the angle integration d $\cos(\tilde\theta s)$ to be performed.

Specifically, we have

(C-4a) $\quad W_1^{SQ} (W^2,Q^2) = G_1(Q^2,W^2) \; I_1 + G_2(Q^2,W^2) \; I_2$

(C-4b) $\quad W_2^{SQ} (W^2,Q^2) = G_2(Q^2,W^2) \; I_3$

where

(C-5a) $\quad I_1 = \pi M \; \frac{\tilde{P}_s}{\sqrt{S_t}} \int_{-1}^{+1} d \cos \tilde\theta_s \mid \phi(P_s) \mid^2$

(C-5b) $\quad I_2 = \pi M \; \frac{\tilde{P}_s}{\sqrt{S_t}} \int_{-1}^{+1} d \cos \tilde\theta_s \mid \phi(\theta_s) \mid^2 \; \frac{P_x^2}{M^2}$

(C-5c) $\quad I_3 = \pi M \; \frac{\tilde{P}_s}{\sqrt{S_t}} \int_{-1}^{+1} d \cos \tilde\theta_s \mid \phi(P_s) \mid^2 \; \mathcal{F}(P_i,\nu,q)$

and

$S_t = (P_t + q)^2 = M_t^2 + 2M_t\nu - Q^2$

$\tilde{P}_s^2 = \frac{S_t}{4} - \frac{1}{2} (W^2 + M^2) + \frac{1}{4S_t} (W^2 - M^2)^2$

$\tilde{E}_s = \sqrt{\tilde{P}_s^2 + M_s^2}$

$P_x^2 = \frac{1}{2} (1 - \cos^2 \tilde\theta_s) \; \tilde{P}_s^2$

$P_z = \frac{1}{\sqrt{S_t}} ((M_t + \nu) \; \tilde{P}_s \cos \tilde\theta_s - q \; \tilde{E}_s)$

$P_s = \sqrt{2P_x^2 + P_z^2}$

The quasi-elastic case is the special one where $S = M_i^2$ and

$G_1 = 2M\tau G_M^2$, $G_2 = 2M(G_E^2 + \tau G_M^2)/(1 + \tau)$, $\tau = Q^2/4M^2$. The inelastic

case is gotten by using the identity $\quad W_1(W^2,Q^2) = \int_{W_T^2}^{W_M^2} dS\ W_1(S,Q^2)\delta(S-W^2)$

where we have substituted W_1 for G_1, and the limits of integration

contain W^2. We then write

$$(C\text{-}7a) \quad W_1^s(\nu,Q^2) = \int_{W_T^2}^{W_M^2} d\,W^2\ \{W_1(W^2,Q^2)\,I_1 + W_2(W^2,Q^2)\,I_2\}$$

$$(C\text{-}7b) \quad W_2^s(\nu,Q^2) = \int_{W_T^2}^{W_M^2} d\,W^2\ W_2(W^2,Q^2)\,I_3$$

where

$$W^2 = (P_i + q)^2 = P_i^2 + 2\,P_i\cdot q - Q^2$$

$$W_T^2 = (M + M\pi)^2 = \text{ one pion threshold}$$

$$W_M^2 = (\sqrt{S_t} - M_s)^2 = \text{maximum producible hadronic mass}$$

off the interacting nucleon.

We now have an identical formalism for handling quasi-elastic

scattering and inelastic smearing. The convolution function I_1,

I_2 and I_3 are responsible for the quasi-elastic peak shape. One

might find this formalism more attractive than previous methods

since: 1) the limits of integration of the $d\cos(\tilde{\theta}s)$ integral

and of the dW^2 are physically simple to understand; and 2) the

structure function is not buried inside two integral signs, and

the smearing formalism now appears as a simple convolution integral

over quasi-elastic peak shapes (of course if one chooses a mixed

variable approach (Ref. C-2) you lose this simplicity).

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We have chosen what A. Bodek (Ref. C-2) calls the "off shell

kinematics" – "on shell dynamics" formalism. Stated plainly we use

only W^2 and Q^2 to describe the photon-nucleon vertex. As the

interacting nucleon is off its mass-shell, the structure functions

could, in principle, depend on ν of the photon as well. Using such

a prescription that has an additional ν dependence might suggest

that the elastic form factors for off mass-shell nucleons would also

have a ν dependence thus resulting in a different Q^2 dependence from

free nucleons (the quasi-elastic case is fixed by requiring $W^2 = M^2$).

Experimentally this is probably impossible to resolve as any Q^2

variation of the quasi-elastic cross section would be interpreted as

information about $G_E(N)$ and $G_M(N)$. There are simply too many unde-

termined functions. In any case, for the present, we adopt the pre-

scription in which only W^2 and Q^2 figure in the dynamics at the nucleon

photon vertex.

We can easily transform Eq. C-2a,b to a scaling form by defining

$$\omega \equiv 2M\nu/Q^2$$

$$\omega_W \equiv \frac{W^2 - M^2}{Q^2} + 1$$

Then, for example, Eq. C-7a becomes

$$(C\text{-}8) \quad W_1^s(\nu,Q^2) = \int_{\omega_T}^{\omega_M} d\,\omega_W\ Q^2\ \{W_1(W^2,Q^2)\,I_1 + W_2(W^2,Q^2)\,I_2\}$$

where

$$\omega_{T,M} = \frac{W_{M,T}^2 - M^2}{Q^2} + 1$$

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The principal contribution to W_1 smearing comes from the

I_1 term in C-8. The quantity

$$(C-9) \quad \Gamma_\omega = Q^2 I_1 (Q^2, \omega, \omega_W)$$

is shown by Fig. C-2 for various values of ω versus the integration

variable ω_W. This quantity scales remarkably well (i.e., it

is insensitive to Q^2). ~~It is also quite independent of ω in its~~

~~shape -- all the curves in Fig. C-2 have approximately the same width~~

~~independent of ω.~~ The insensitivity to Q^2 of the convolution func-

The width of the convolution function (Γ) has been found to scale in ω and is approximately 1ω.

tion reveals the "scaling" nature of smearing. Perhaps "scaling"

always results when the impulse approximation is used. The convo-

lution function Γ may also be expressed in terms of the scaling

variable ω'. A plot of $\Gamma_\omega = Q^2 I_1(Q^2, \omega', \omega'_W)$ versus $\omega'_W = W^2/Q^2 + 1$

for fixed $\omega' = 2M\nu/Q^2 + M^2/Q^2$ shows that Γ_ω scales even better in ω'

than it does in ω (see Fig. C-2b, 2c and 2d).

In Fig. C-3, we show the convolution function expressed in

terms of missing mass variables

$$\Gamma_W = 2W I_1 (Q^2, W_i, W)$$

for various values of the input missing mass $W_i^2 = M^2 + 2M\nu - Q^2$

versus W. These curves show the range in W and size of the con-

volution function over which the structure functions contribute.

These curves are of course equivalent to the ω curves in Fig. C-2

and dramatically show that ± 0.2 units of ω can mean very different
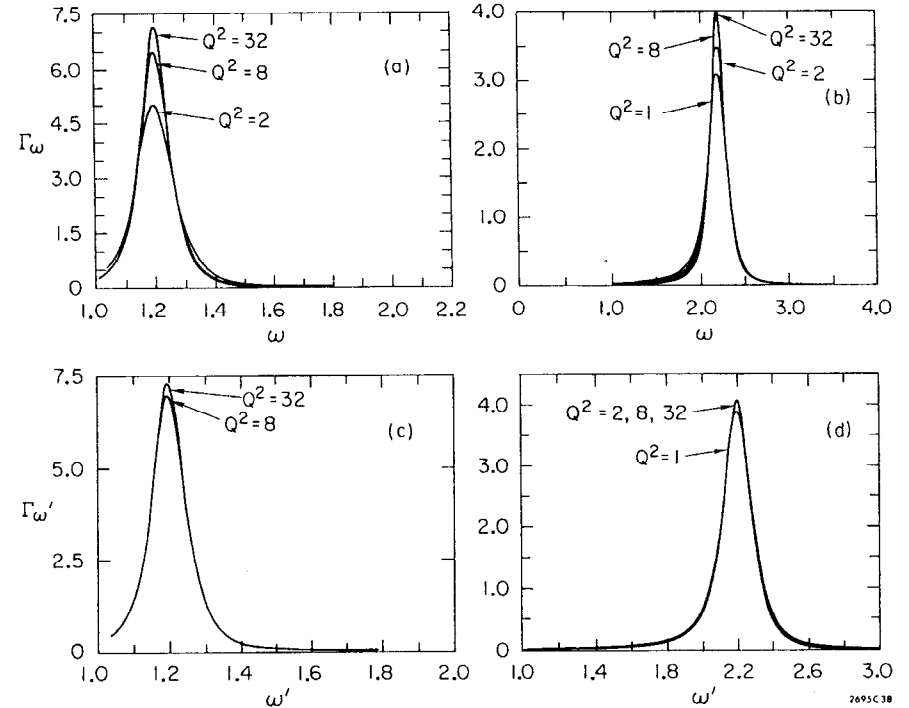
ranges in W depending on Q^2.

FIG. C-2

Graphs of the convolution function Γ_ω
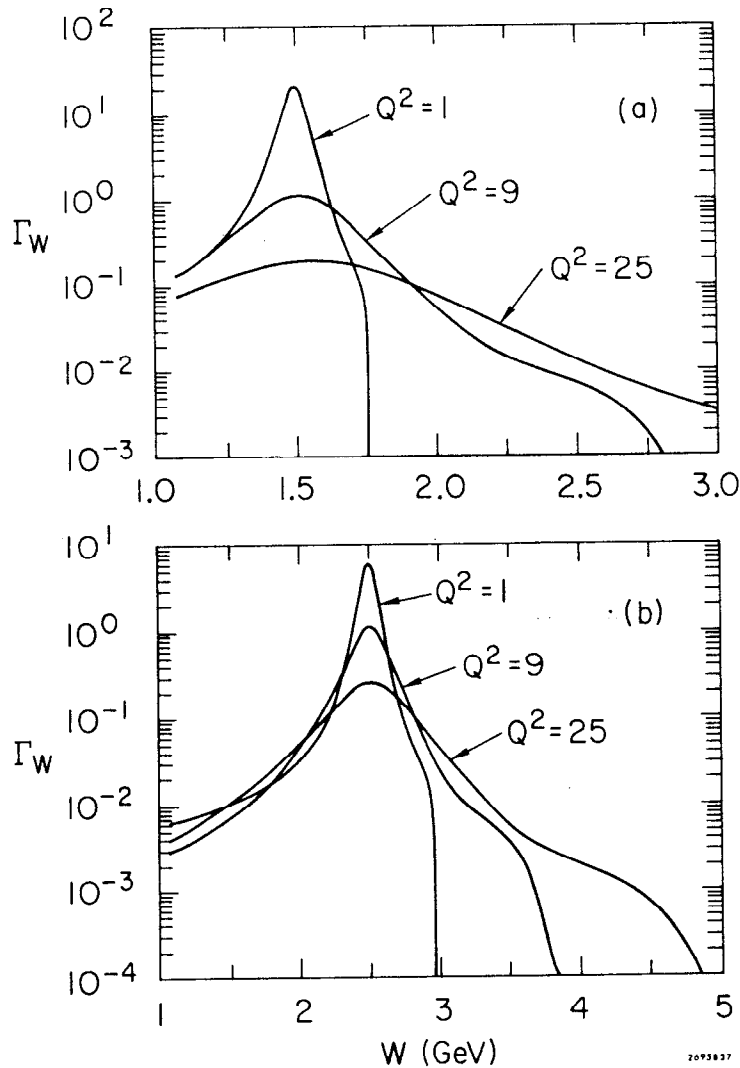for input (ω')'s of a) 1.2 and b) 2.2
for various values of Q^2.

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FIG. C-3

The convolution function Γ_W for input W's of a) 1.5 GeV and b) 2.5 GeV for various values of Q^2.

The Γ_W curves also demonstrate another interesting feature of smearing. As Q^2 increases for fixed W any missing mass structure is more and more washed out. For example at $Q^2 = 4$ GeV^2 the resonances should be broader by a factor of approximately 2.

We have used the Reid hard core wave function to describe the spectator momentum within the deuteron. It is of some interest to find the region in $|\vec{P}_s|$ which contributes to the smearing integrals. To do this, we revert back to the previous formalism (Ref. C-1) and define a Γ_{ps} by

$$W_1^s (\nu, Q^2) = \int_0^\infty dP_s \ \Gamma_{ps} (\nu, Q^2, P_s)$$

Γ_{ps} involves the structure functions (as well as $|\phi(\vec{P}s)|^2$ for which we have used

$$\nu W_2 = (1 - x')^3 (.6453 + 1.902 (1 - x') - 2.343 (1 - x')^2)$$

$$2M \ W_1 = \frac{\omega}{1+R} (1 + Q^2/\nu^2) \ \nu W_2$$

$$x' = \frac{1}{\omega'} = \frac{Q^2}{Q^2 + W^2}$$

and $R = \sigma_s/\sigma_t = .18$.

Fig. C-4 shows Γ_{ps} versus P_s for $\omega = 2.2$ and Q^2 values from 1-25 GeV^2. The rapid fall off is caused by the wave function. The smearing functions at best linearly enhance the high P_s regions over which $|\phi (P_s)|^2$ can contribute to smearing and these functions eventually fall to zero as physical final states can no longer be produced off such

FIG. C-4

The function Γ_{P_S} shows .where the contri-
butions to smearing come from in the
squared momentum wave function.

virtual nucleons. This figure shows why the smearing formalism

is so insensitive to various deuteron wave functions. Where

the wave functions exhibit large differences (at large p_s), little
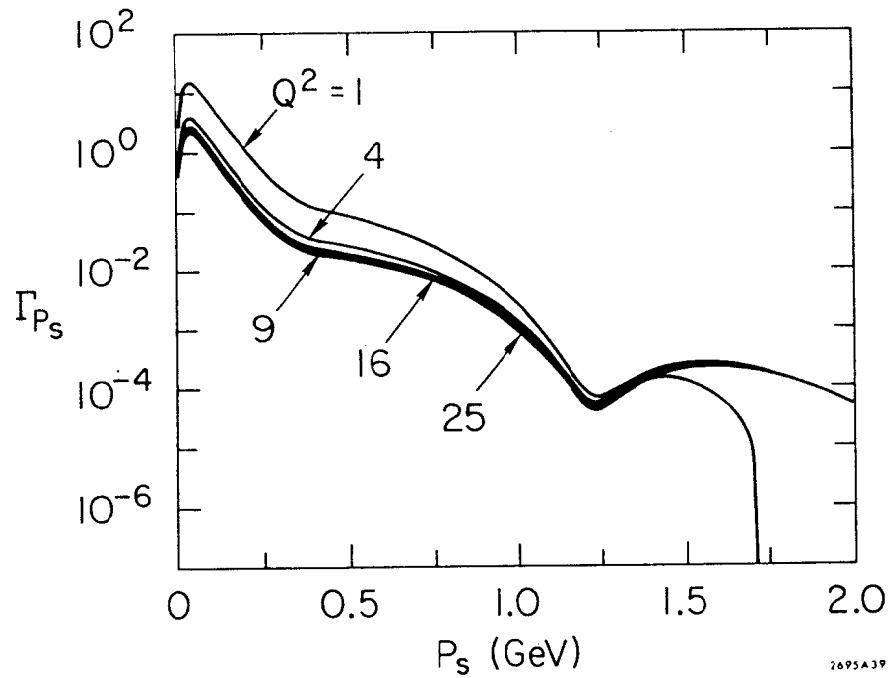
contribution to smearing occurs.

APPENDIX C – REFERENCES

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   Technical Report No. COO-3069-116.

APPENDIX D – WIRE ORBIT OF THE 1.6 GeV SPECTROMETER

The optical properties of the 1.6 GeV spectrometer were originally calculated using a Transport model (Ref. D-1) of the magnet. Optical coefficients were then measured by Group-F at SLAC using two techniques. Before the magnet was raised into its carriage and was lying horizontally on the End Station floor, the momentum dispersion and ϕ angle focusing properties were measured using wire orbit techniques. With the magnet in place, the optics were again measured using the primary electron beam positioned and steered at the magnet entrance by magnets placed in the beam line of ESA. Both sets of measurements agreed with the values calculated from Transport to within their measured accuracy (Ref. D-2).

In 1970, the 1.6 suffered an accident. The magnet was left on without cooling water circulating through the coil. Subsequently, in 1971, two of the 144 turns in the coil shorted due to the poor condition of epoxy insulation. To "temporarily" fix the situation the two symmetric turns on the other half of the aperture were shorted. Remeasurements of elastic proton peaks showed no appreciable change in the optics.

At the end of data taking for E-89, Group-A decided to remeasure the optics coefficient and the limiting apertures in the 1.6 GeV spectrometer to confirm the values of the optics coefficient and solid angle of this instrument. To accomplish this measurement, we used wire orbit techniques.

An elementary analysis shows that a current carrying wire under tension will follow the trajectory of a charged particle through a magnetic field. The relationship between tension (resulting from a weight of mass M (kg) hung on the wire), current I (amps), and equivalent momentum pc(GeV) is

$$pc(GeV) = \frac{gc}{10^9} \frac{M(kg)}{I(amps)}$$

(The classical current flow is in the same direction as the momentum of a negatively charged particle). The measurements of the magnet were carried out by passing the current carrying wire through the magnet field and measuring the input and output trajectories of this wire "orbit." To achieve stable orbits in a magnet system that is focusing, the wire cannot be held at both foci but must be held inside at least one of the foci. The weight of the wire makes the tension in the wire depend on the mass of the wire as well as the weight, M, hung on it. This effect must be measured experimentally to make absolute calibrations between the magnet field strength and the momentum.

The coordinate systems used are shown in Fig. D-1. The laboratory coordinates (subscript ℓ) are related to the input spectrometer coordinates (subscript i) by a 60° rotation about the Y_ℓ axis. The counter cave coordinates are rotated by 90° about the X_i axis with respect to the input coordinates. This makes the Z axis roughly the direction of the flight path of particles as they traverse the magnet. The coordinates used by Group-F are
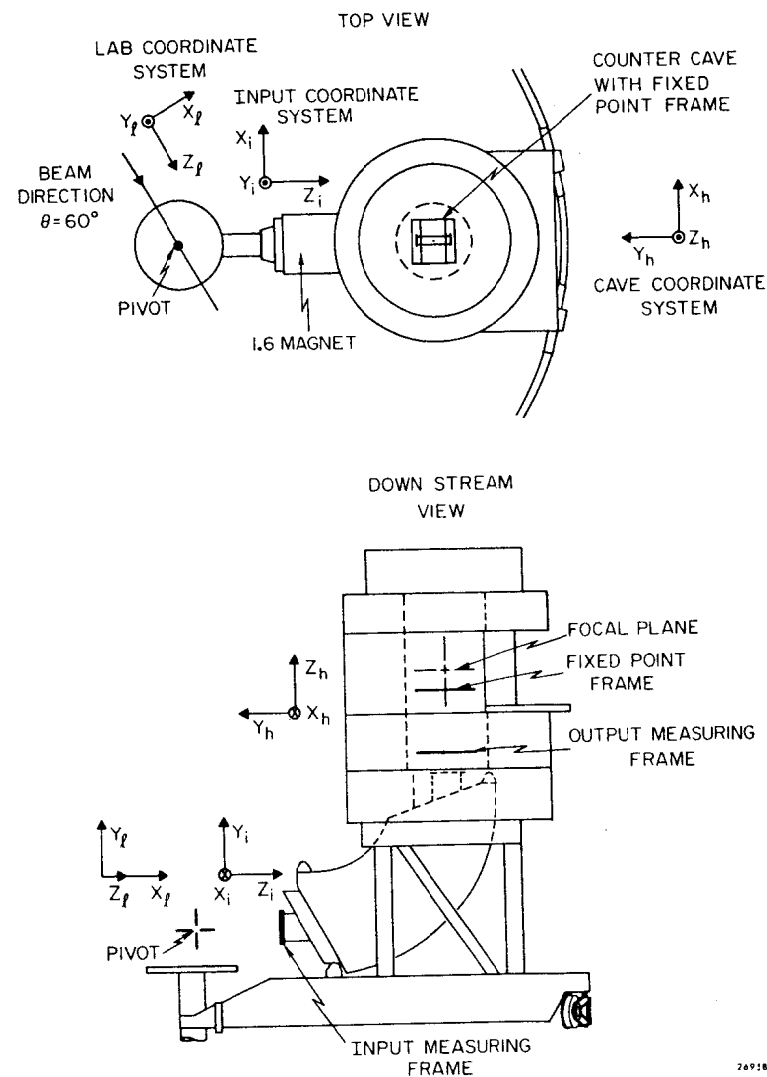
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FIG. D-1

Definitions of the various co-ordinate systems used to describe the 1.6 GeV spectrometer optics.

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different from the ones used here, and the correspondences are given in Table D-1.

The experimental setup used had an air bearing pulley positioned at the pivot on a travelling screw table allowing for positioning along the beam direction (Z_ℓ). The spectrometer was positioned at 60° with respect to the beam line. A phosphor-bronze wire (.010" dia.) was draped over the pulley with a weight hung on the side away from the spectrometer. The input trajectory of the wire was measured using an aluminum frame with sensing devices mounted on moveable arms. This frame was mounted on the magnet (see Fig. D-1) and was surveyed in place to an accuracy of $\pm$ .010" with respect to the pivot.

Inside the counter cave, two more similar frames were mounted; one to hold the wire and the other to measure its position near the exit of the magnet. Both of these frames were aligned using the laser system (See Chapter II) and leveled.

The power supply for the wire was a precision current regulated supply, and the current in the wire was obtained by measuring the voltage drop across a .9835($\pm$ .05%) ohm resistor (calibrated with a precision current source) with a digital voltmeter (DVM).

A four man team performed the measurements; two located at the pivot and two in the counter cave. The measuring frames had photo diode sensors to measure the wire position (without touching the wire) and were accurate to approximately $\frac{1}{2}$ the wire diameter (.005"). When the wire triggered the photodiode sensor, an indicator lamp was

TABLE   D - 1

COORDINATE SYSTEM CONVERSIONS FOR INPUT AND OUTPUT TRAJECTORIES

No.	Group-A	Group-F
1	-Y	X
2	-ϕ	θ
3	X	Y
4	θ	ϕ
5	Z	Z
6	δ	δ

Group-F input coordinates were subscripted with o, e.g., $(-Y_i \mid_A = X_o \mid_F)$ and the output coordinates had no subscript e.g., $(\phi_h \mid_A = \theta \mid_F)$.

lit on each measuring frame, one in the counter cave and one at the pivot. This made it possible to measure both the input and the output trajectories simultaneously (we required that both indicator lamps be lit at the time of the measurement) without touching the wire at the measuring points. As the sensing devices only allowed a measurement in one direction, two such systems were made for measuring the X and the Y coordinates. Simultaneously with the Y coordinate measurement (corresponding to momentum and ϕ) the DVM reading of the shunt voltage for the current in the wire was recorded. The overall accuracy of this system was $\pm$ .010" for all the coordinates and $\pm$ .001 volts for the DVM. The magnetic field strength was measured using an NMR (see Chapter II). A total of 167 orbits were measured at 1.00 GeV and 159 orbits at 1.68 GeV.

Different orbits were achieved by varying the current in the wire, the position of the air bearing pulley at the pivot, and the X and Y positions of the fixed point in the counter cave, a total of four variables. Measurements were made over the entire aperture and covered 7.0" of travel at the pivot, $\pm$ 8% changes in the current in the wire, and fixed positions in the counter cave corresponding to changes in θ_i of $\pm$ 20 mrad and changes in ϕ_i of $\pm$ 80 mrad.

The first task in the analysis was to establish a central orbit. To do this, the measured current, I, was fit to a function of X_i, θ_i, ϕ_i, X_h and Y_h. We then selected $X_i = Y_i = X_h = Y_h = 0$ and $\theta_i = \phi_i = 0$ as defining the central orbit. From the fit we calculated the current in the wire, I_o, for this orbit. This was done for both values of magnet field for

which orbits were measured. For all the measured orbits, the deviation from the central momentum, δ, was then calculated by

$$\delta = (P_o - P)/P_o = \frac{I - I_o}{I}$$

The absolute value of the tension in the wire doesn't enter into the determination of δ if the tension remains constant for all orbits (different orbits can have different path lengths with different lengths of wire, thus changing the tension slightly due to the finite wire mass). The resistance of the shunt and the values of g and c also cancel in the determination of δ.

Optics coefficients were determined by fitting coordinates of the input trajectories as functions of the coordinates of the output trajectories and vice-versa. Since we didn't vary the coordinates transverse to the beam direction, Y_ℓ and X_ℓ , the optics coefficients that depend on these coordinates are not determined. We assumed all optics coefficients involving these coordinates to be given by the calculated numbers in Ref. D-2. We found no significant differences in the optics coefficients between the high and the low momentum measurements, so we fit the optics coefficients to the combined data set. The optics coefficients through second order are given in Table D-2 for transforming input trajectories to output trajectories (i.e., lab. coordinates to hut coordinates). In Table D-3 the inverse fits are given using the three counter cave coordinates measured in E-89, namely X_h, Y_h and ϕ_h. One difference between these coefficients and Transport coefficients should be noted. As we have

OPTICS COEFFICIENTS FOR THE 1.6 GeV SPECTROMETER

OFFSETS

```
1  CORRECTED OFFSET=   0.76731E-01
2  CORRECTED OFFSET=   0.11941E 02
3  CORRECTED OFFSET=   0.14010E 00
4  CORRECTED OFFSET=   0.40560E 00
```

FIRST ORDER COEFFICIENTS
```
1 1 -0.66100E 00   1 2 -0.54541E-02   1 3 -0.30558E-01   1 4 -0.13383E-01   1 5 0.0        1 6 0.41156E 01
2 1 -0.46100E 01   2 2 -0.15295E 01   2 3 -0.35858E-01   2 4 -0.11458E-01   2 5 0.0        2 6 0.65220E 01
3 1 0.0            3 2 -0.55978E-02   3 3 0.23392E-01    3 4 0.81439E 00    3 5 0.0        3 6 0.87493E-02
4 1 0.0            4 2 -0.97021E-02   4 3 -0.11853E 01   4 4 0.13401E 01    4 5 0.0        4 6 0.15554E-01
5 1 -0.15090E 01   5 2 -0.63500E 00   5 3 0.0            5 4 0.0            5 5 0.10000E 01 5 6 -0.14600E 01
6 1 0.0            6 2 0.0            6 3 0.0            6 4 0.0            6 5 0.0        6 6 0.10000E 01
```

SECOND ORDER COEFFICIENTS
```
1 11 -0.64100E-03
1 12 0.14110E-03   1 22 -0.21389E-03
1 13 0.0           1 23 0.19620E-03   1 33 -0.47878E-02
1 14 0.0           1 24 0.59720E-04   1 34 -0.93230E-03   1 44 -0.24579E-03
1 15 0.0           1 25 0.0           1 35 0.0            1 45 0.0            1 55 0.0
1 16 0.11410E-01   1 26 -0.42836E-03  1 36 -0.19655E-02   1 46 -0.17375E-02   1 56 0.0   1 66 -0.58299E-01

2 11 0.18000E-01
2 12 0.41300E-02   2 22 -0.10913E-02
2 13 0.0           2 23 0.11622E-02   2 33 -0.35632E-01
2 14 0.0           2 24 0.19549E-03   2 34 -0.99902E-02   2 44 0.15055E-02
2 15 -0.21290E-01  2 25 1.0           2 35 0.0            2 45 0.0            2 55 0.0
                   2 26 -0.32404E-01  2 36 -0.30677E-04   2 46 -0.25473E-02   2 56 0.0   2 66 -0.19424E 00

3 11 0.0
3 12 0.0           3 22 -0.40653E-04
3 13 0.29733E-01   3 23 0.22451E-02   3 33 -0.23377E-02
3 14 0.53020E-02   3 24 0.37625E-03   3 34 -0.16642E-02   3 44 -0.38987E-03
3 15 0.0           3 25 0.0           3 35 0.0            3 45 0.0            3 55 0.0
3 16 0.0           3 26 -0.36688E-03  3 36 0.51843E-02    3 46 0.11624E-01    3 56 0.0   3 66 0.36486E-02

4 11 0.0
4 12 0.0           4 22 -0.14931E-03
4 13 0.57700E-01   4 23 0.37198E-02   4 33 -0.51896E-02
4 14 0.79200E-02   4 24 0.46000E-03   4 34 -0.37111E-02   4 44 -0.85559E-03
4 15 0.0           4 25 0.0           4 35 0.0            4 45 0.0            4 55 0.0
4 16 0.0           4 26 -0.87454E-03  4 36 0.31917E-01    4 46 0.47414E-01    4 56 0.0   4 66 0.81508E-02

5 11 0.29600E-02
5 12 -0.17400E-03  5 22 -0.58000E-03
5 13 0.0           5 23 0.0           5 33 -0.10300E-01
5 14 0.0           5 24 0.0           5 34 -0.37600E-02   5 44 -0.85440E-03
5 15 0.0           5 25 0.0           5 35 0.0            5 45 0.0            5 55 0.0
5 16 0.41100E-02   5 26 -0.39670E-03  5 36 0.0            5 46 0.0            5 56 0.0   5 66 -0.23930E-02

6 11 0.0
6 12 0.0           6 22 0.0
6 13 0.0           6 23 0.0           6 33 0.0
6 14 0.0           6 24 0.0           6 34 0.0            6 44 0.0
6 15 0.0           6 25 0.0           6 35 0.0            6 45 0.0            6 55 0.0
6 16 0.0           6 26 0.0           6 36 0.0            6 46 0.0            6 56 0.0   6 66 0.0
```

$$x_{hi} = x^o_i + \sum_{j=1}^{6} C^1_{ij} x_{\ell j} + \sum_{\substack{j,k=1 \\ k \geqslant j}}^{6} C^2_{ijk} x_{\ell j} x_{\ell k}$$

where

x_{hi} are the hut coordinates

$x_{\ell i}$ are the laboratory (input) coordinates

x^o_i are the offsets

C^1_{ij} are the first order coefficients

and C^2_{ijk} are the second order coefficients

INPUT COORDINATES AS FUNCTIONS OF OUTPUT
COORDINATES X_h, Y_h AND ϕ_h

OFFSETS

```
2      7.3070E00
4     -.25683E-01
6      .28116E-01
```

FIRST ORDER
```
2 1  .99721E00      4 1  .80756E-02      6 1  .24608E00
2 2  -.65255E00     4 2  -.45699E-02     6 2  -.12124E-02
2 3  .32008E-02     4 3  1.2193E00       6 3  .27510E-02
```

SECOND ORDER
```
2 11  -.95704E-02
2 12  .38169E-02    2 22  -.21912E-03
2 13  -.82572E-04   2 23  -.15397E-03    2 33  .22228E-02

4 11  -.45948E-04
4 12  -.94906E-04   4 22  .27021E-04
4 13  -.43377E-02   4 23  .23560E-03     4 33  -.38951E-03

6 11  .84475E-03
6 12  -.71581E-04   6 22  .20106E-04
6 13  .10472E-03    6 23  .56203E-05     6 33  .12579E-03
```

$$x_{\ell i} = x^o_{hi} + \sum_{j=1}^{3} \tilde{C}^1_{ij} x_{hj} + \sum_{\substack{k,j=1 \\ k \geqslant j}}^{3} \tilde{C}^2_{ijk} x_{hj} x_{hk}$$

where

$x_{\ell i}$ are the laboratory (input) coordinates

x_{hi} are the counter cave coordinates

x^o_{hi} are the offsets

$\tilde{C}^1_{ij}$ are the first order coefficients

$\tilde{C}^2_{ij}$ are the second order coefficients

made a physical measurement on a real magnet, we have chosen a

specific, fixed coordinate system. This procedure can introduce

non-zero constants (offsets) into the fits.

To estimate the error in our numbers, we randomized the

measurements within the measured accuracy of $\pm$ .010" for the

coordinates and $\pm$ .001 volts in the DVM and repeated the fitting

procedure many times. The RMS deviations calculated from 40

such trials for the important first order coefficients are given in

Table D-4. The residuals of the fits for some of the important first

order coefficients are histogrammed in Fig. D-2. Also shown in

Fig. D-2 are the hodoscope bin widths used in E-89.

The fits were calculated only through second order in the input

(output) coordinates. We tested the self consistency and the reso-

lution of this system (the hodoscope bin widths and the forward and

backward spectrometer optical models) by Monte Carlo techniques.

In Fig. D-3 we show the difference between the reconstructed

coordinates and their starting values. We calculate from these the

following resolutions (RMS deviations): $\phi \pm 5.08$ mrad, $\theta \pm 1.16$ mrad,

and $\delta \pm .23\%$. These are comparable to the RMS deviations of the

residuals shown in Fig. D-2. Since the coefficients are those of

finite Taylor expansions about our chosen central ray, second order coefficients

will fail to describe significant third or higher order effects

away from the central orbit. The spread in the residuals in Fig. D-2

is thought not to be caused by measurement error but by the use of

TABLE D - 4

COEFFICIENT ERROR ESTIMATES FOR 40 TRIALS FOR

OFFSETS AND FIRST ORDER COEFFICIENTS

OUTPUT AS FUNCTIONS OF INPUT

OFFSETS

| 1 | .0191 | 2 | .0543 | 3 | .0137 | 4 | .0468 |

FIRST ORDER

1	2	.431E-3	2	2	.126E-2	3	2	.257E-3	4	2	.861E-3
1	3	.176E-2	2	3	.417E-2	3	3	.126E-2	4	3	.463E-2
1	4	.839E-3	2	4	.234E-2	3	4	.523E-3	4	4	.172E-2
1	6	.300E-2	2	6	.645E-2	3	6	.196E-2	4	6	.618E-2

INPUT AS FUNCTIONS OF OUTPUT

OFFSETS

| 2 | .230E-1 | 4 | .135E-1 | 6 | .475E-2 |

FIRST ORDER

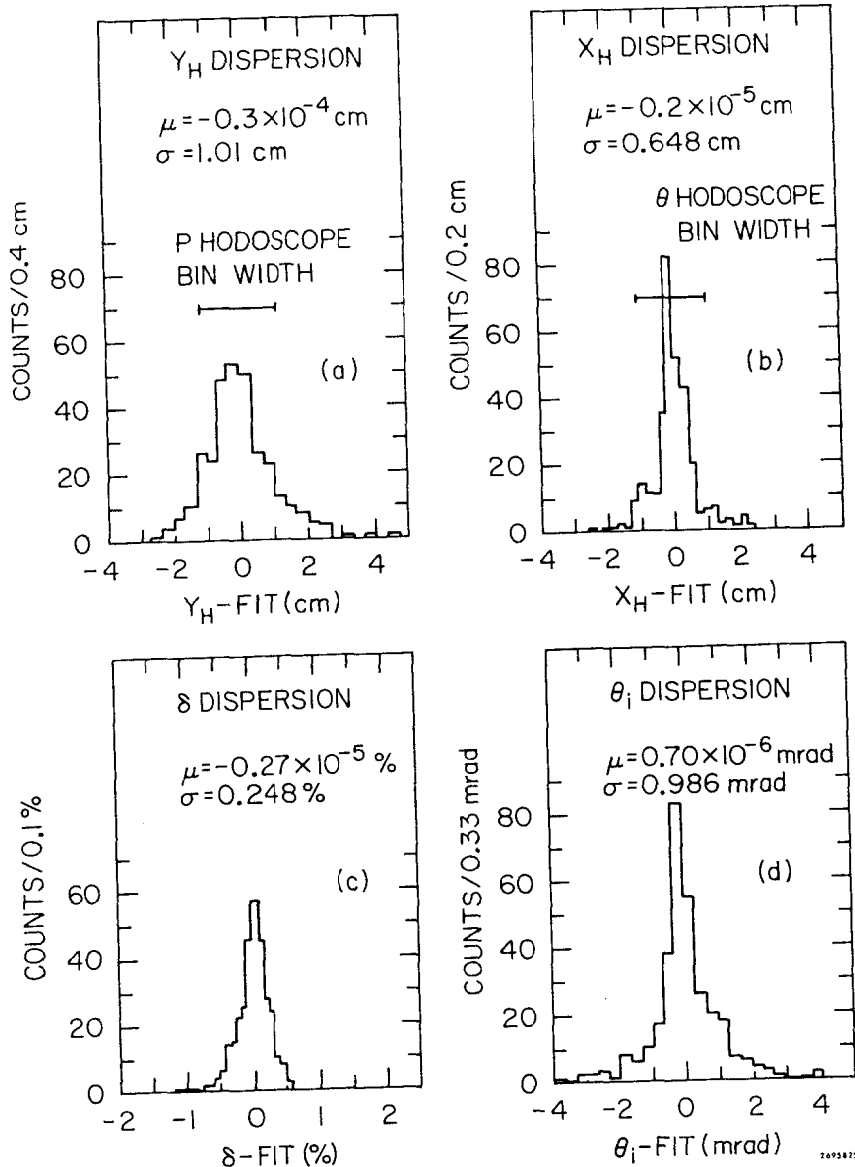| 2 | 1 | .618E-3 | 4 | 1 | .693E-3 | 6 | 1 | .195E-3 |
| 2 | 2 | .243E-3 | 4 | 2 | .175E-3 | 6 | 2 | .563E-4 |
| 2 | 3 | .959E-3 | 4 | 3 | .633E-3 | 6 | 3 | .218E-3 |

FIG. D-2

Histograms of the residuals for the fits
to the optics coefficients. The mean, μ,
and RMS deviations, σ, of each histo-
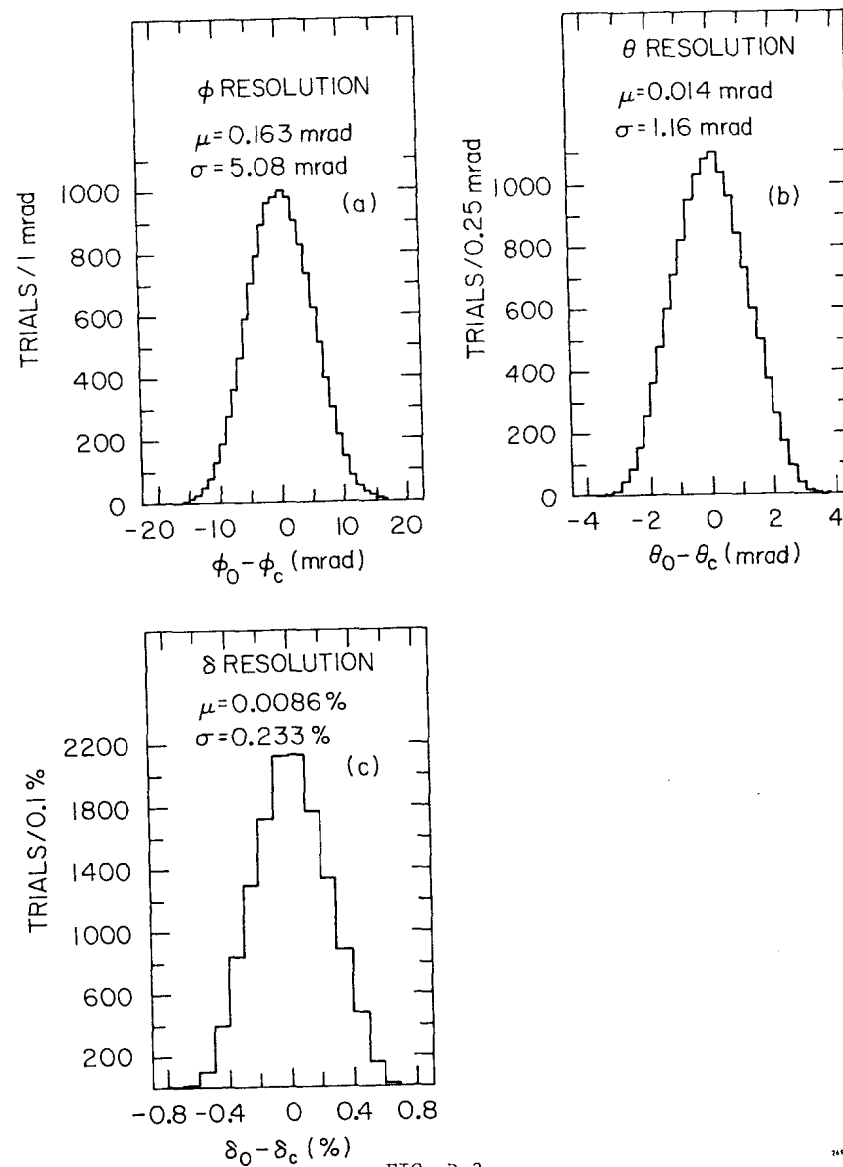gram is shown.

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FIG. D-3

Calculated resolutions of the optics for
the 1.6 using the fitted optics coeffi-
cients and hodoscope bin widths.

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a finite Taylor expansion to approximate the optical properties of

the 1.6-GeV spectrometer.

The improvement made in the resolution of the 1.6 is marginal

using our new fits over that obtained from the previous measurements.

The only place where differences were noticed was for the elastic

peaks. The resolution in missing mass improved about 10% over that

obtained using the simple first order prescription of the original

Group-F coefficients. The resolutions of the hodoscope bin widths

used for E-89 are comparable to the resolution gotten with the

second order optics coefficients. Further refinements of the optics

(such as third order coefficients) are unnecessary for our present

purposes.

To test the absolute quality of the measurements,we changed

the weight and current in the wire while keeping the ratio fixed

for several orbits with the spectrometer set for approximately 1 GeV.

The weights used ranged from .401 kg to .772 kg. For all these orbits

the ratio of the current in the wire to the weight on it was held

approximately constant. From the measured orbits we calculated

δ using the coordinates measured in the cave and corrected each orbit

for slight missettings of the ratio, M/I. The central momentum was

calculated using

$$P_o c = \frac{gc}{10^9} \quad \frac{M(kg)}{I(amps)} \quad \frac{1}{(1+\delta)}$$

(the range of δ was $-.0067 < \delta < -.0052$)

In Fig. D-4 we have plotted the calculated $P_o c$ (GeV) versus

the reciprocal of the tension (1/Mg). We have used the values of

.98062 m/sec^2 for g and 2.9979 x 10^9 m/sec for the speed of light,

c. The error bars indicate the full spread of the data measured

using a particular weight. A straight line extrapolated to 1/Mg = 0

gives an absolute calibration of magnetic field strength in momentum

for the central ray. We find this to be .9984 GeV. The NMR fre-

quency, measured with a deuterium probe, was 8.6905 MHz corresponding

to a field strength of 13.30 kgauss. This calibration was compared

to that obtained from the position of the elastic peaks in missing

mass. The elastic peak calibration predicts a momentum of .9981 GeV

for this NMR frequency ($P_o c$ (MeV) = 17.63 x 6.51436 x f_{NMR} (MHZ)).

This fortuitous agreement between the wire float calibration and the

elastic peak calibration should be taken with a grain of salt as the

magnet power supply current regulation was only $\pm.03\%$.

The limiting apertures in the 1.6-GeV spectrometer were

measured using the wire orbits. The volume of phase space accepted

by the spectrometer relevant to E-89 is shown in Fig. D-5 with the

various limiting apertures denoted on the drawing. The limiting

aperture measurements showed the largest differences between the

high and the low momentum data,and the data are plotted in Fig. D-5.

The solid line is the Monte Carloed results for the optics coefficients

fit to the combined data set. The difference in the solid angle

between the high and the low momentum measurements was calculated to

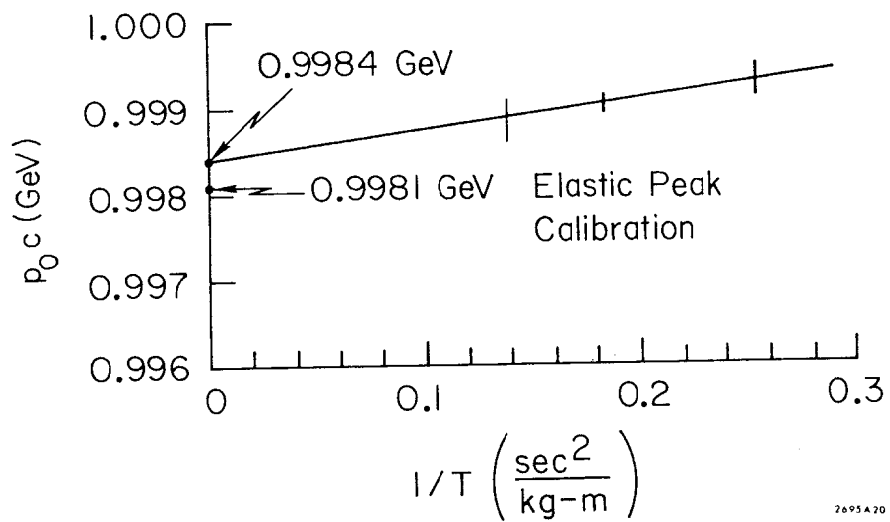be approximately .5%. We take the uncertainty in our solid angle

FIG. D-4

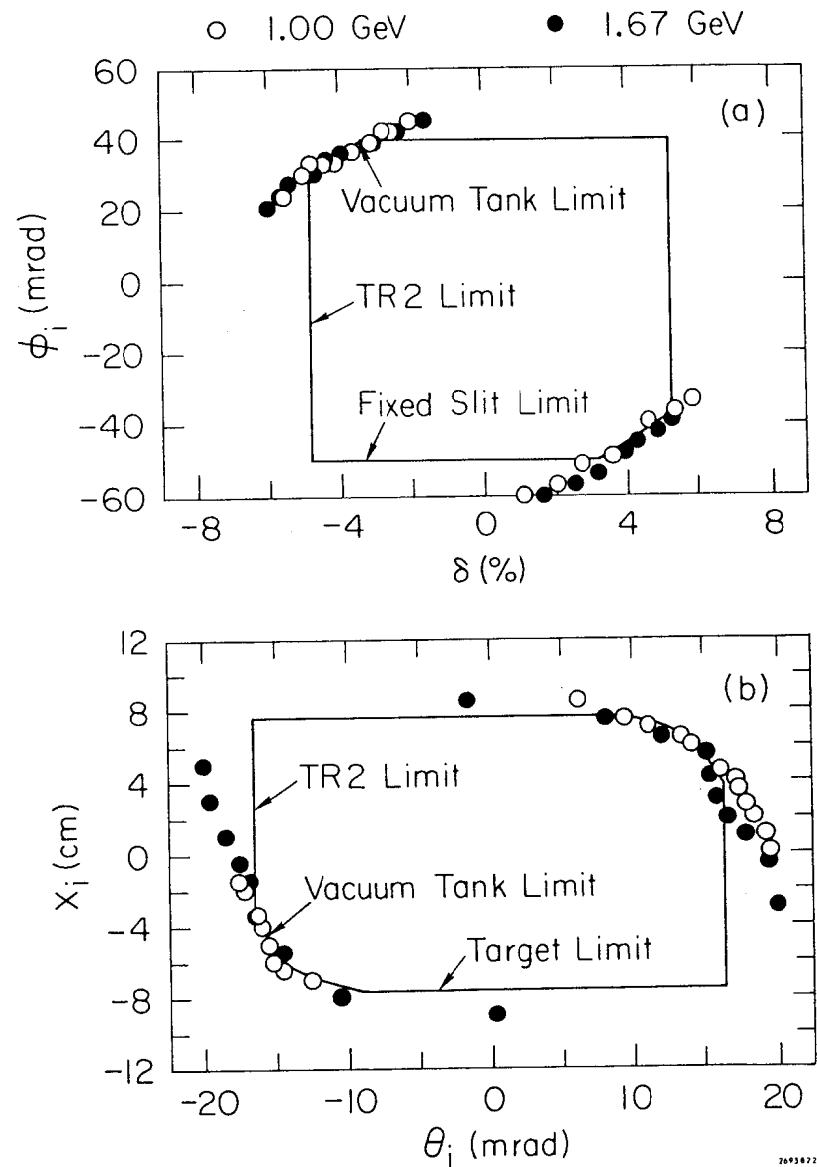Calculated values of $p_o c$ for various
values of (1/tension).

FIG. D-5

The limiting apertures of the 1.6 GeV spectrometer for a) ϕ_i
(vertical projected scattering angle) versus δ (deviation in
momentum from the central ray), b) X_i (target length) versus θ_i
(horizontal projected scattering angle).

determination to be $\pm$ 1.0%. The solid angle subtended by our counter package was limited by the TR2 counter. Averaged over a 17.7 cm target the solid angle was calculated by Monte Carlo techniques to be 2.904 msr $-$ 10% ($\Delta p/p$) at 60° and 2.955 msr $-$ 10% ($\Delta p/p$) at 50°. The contribution to the TR2 solid angle from each p-θ bin was calculated, and these were applied to the data on a bin-by-bin basis.

APPENDIX D - REFERENCES

1. K. Brown, SLAC Report No. 75, 1967.

2. R. Anderson et al., Nuclear Instr. and Methods <u>66</u>, 328 (1968).