

THEMI
MONTE CARLO GENERATION OF ELECTROMAGNETIC EVENTS

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I. INTRODUCTION

In designing or analyzing experiments at high-energy electron accelerators, it is often useful to be able to simulate the effects of the dominant interactions of photons in a realistic way. For example, in an experiment designed to study these interactions, a simulation by Monte-Carlo is useful in choosing design parameters. In other experiments, where these interactions may constitute a severe background; this background may be made manageable by Monte-Carlo studies. For this purpose, a set of FORTRAN subroutines, THEMI, has been written, which provides (a) total cross sections for hydrogen and (b) random "events" whose kinematics are distributed according to theoretical models which have been essentially confirmed by numerous experiments.

The usefulness of this set of routines is illustrated by its application in a real experiment. The experiment is a photoproduction experiment using the SLAC 2-M streamer chamber.¹ The target is a 1/2-inch-diameter thin-walled tube filled with hydrogen gas, and a well-collimated photon beam is introduced along the axis of the tube. Vertices of events are invisible within this tube. Four distinct but related questions have been investigated.²

1. We wish to determine the spectrum of the incident bremsstrahlung beam by measuring electron pairs. However, certain necessary cuts, such as on fiducial volume, distort the spectrum of the measured pair events. What is the nature of this distortion?

2. Because more than one electron pair may be present in a picture, their tracks may be inadvertently interchanged because the measurer cannot see the vertices of either event. If this happens, how is the measured spectrum altered?

3. A track of a Compton electron may be misidentified as a member of an electron-positron pair. How does this affect the measured pair spectrum?

4. Because of the difficulties 1 - 3, it was decided that the absolute determination of photon flux not be done by normalizing to the number of electron pairs measured in a sample of film. Instead, having determined the proper photon spectrum taking into account the effects 1 - 3, we relate the absolute photon flux to the total number of positrons (regardless of energy) found in a certain fiducial region. Again, because of the invisibility of the vertices, the number of positrons counted is not identical to the number of electron pairs (and triplets) produced. What is the correct ratio of positrons counted to equivalent quanta incident?

THEMI has been used to study these particular problems, but other applications readily suggest themselves: in counter experiments one could use THEMI to determine the rate of e^+ or e^- from a hydrogen target into a particular counter arrangement. With appropriate modification for materials other than hydrogen, these subroutines could be used in a program to study EM shower development. Shielding studies, and collimator edge effects are examples of applications.

II. GENERAL DESCRIPTION

The processes provided at this writing include:

- a) Compton effect (Klein-Nishima).³
- b) Pair production with Wheeler-Lamb screening on hydrogen.⁴
- c) Triplet production with Wheeler-Lamb screening on hydrogen.⁴
- d) Bremsstrahlung spectrum modified by a beam hardener.⁵

The main program is written by the user to do Monte-Carlo studies of these processes. A first application is to study the distributions of these events in the SLAC 2.2-meter streamer chamber, with arbitrary selections on the tracks of the events.

In this application, the following operations are typically performed:

1. Call PHOTON to choose a random photon energy EGAM in the region 40-18,000 MeV.
2. Determine the total cross sections for pair, triplet and Compton processes for this EGAM by calling PAIRWL, TRIPL, and COMPT, respectively.
3. Determine randomly which, if any, of these processes occur.
4. If a process occurs, determine the kinematics (partition of energy, production angle, etc.) of the event chosen by again calling PAIRWL, TRIPL, or COMPT, depending on which process is chosen. A flag IGEN = 1 tells the subroutine to generate the kinematics on this call.
5. The subroutine POSN is now called to choose randomly the coordinates of the event in the beam.
6. We then call a routine TUBE for each track to determine the coordinates of the point where the track becomes visible in the chamber.
7. We make various selections corresponding to scanning criteria, such as on fiducial volume, minimum accepted momenta, etc.
8. At the conclusion of a run we plot various distributions, such as the photon energy of the accepted events, the distribution of the position along the beam, and the momentum of e^+ or e^- .

III. RANDOM SELECTION OF KINEMATICS

Extensive use is made of the random number generator RANDOM, which supplies numbers uniformly distributed between 0.0 and 1.0. We use the following well-known procedures:

1. Suppose the physical distribution of a quantity E is not uniform over the range of interest E_1 to E_2 ; the physical distribution has a form $p(E)$. If $p(E)$ is integrable in terms of simple functions, we may easily program a method of choosing a value E_{chosen} by the simple recipe:

$$\int_{E_1}^{E_{\text{chosen}}} p(E) dE = R \cdot \int_{E_1}^{E_2} p(E) dE$$

where E_{chosen} is here considered to be a function of R (the random number) alone.

A "rectangular" bremsstrahlung spectrum is an example. We have

$$p(E) = Q/E ,$$

where Q is a constant. A trivial integration gives

$$E_{\text{chosen}} = E_2^R E_1^{1-R} .$$

If the $p(E)$ is not easily integrable, a tabular form of the integral may be used.

This method is 100% efficient, in the sense that a usable value E_{chosen} results from each call of the random-number generator.

2. If $p(E)$ is not easily integrable, another method is often used: A function $P(E)$ is chosen which is easily integrable, and always greater than or equal to $p(E)$ in the range of interest E_1 to E_2 . Then the random number R_1 is used in the procedure just described to choose an E_{chosen} according to $P(E)$. A second random number R_2 is then used in the following test:

If $(p(E_{\text{chosen}}) \text{ is greater than } R_2 \cdot P(E_{\text{chosen}}))$, the value E_{chosen} is retained; otherwise it is discarded and we begin again with a new R_1 .

This method is well illustrated in the descriptions of the subroutines in the appendices.

The method is not 100% efficient, in the sense of method 1, but can be nearly so, if $P(E)$ closely follows $p(E)$.

IV. SUBROUTINES

Major subroutines which may be of interest in themselves, separate from THEMI, are described in separate appendices.

1. PHOTON - See paragraph A, Appendix I.
2. BREMS - See paragraph B, Appendix I.
3. PAIRWL - See paragraph C, Appendix I.
4. WELAMB - See paragraph D, Appendix I.
5. TRIPL - See paragraph E, Appendix I.
6. SCREEN - See paragraph F, Appendix I.
7. COMPT - See paragraph G, Appendix I.
8. POSN - (R, X, Y) - Assumes that the origins of the EM events are uniformly distributed within a circle of radius R (for a uniform cylindrical beam). Call POSN with R equal to the assumed radius (say 0.15 cm), and POSN returns choices of X and Y in the same units as R. We first choose a radius of the origin by the use of random number RANI: $RAD = R * \sqrt{RANI}$. Then a random angle THETA is chosen, and RAD, THETA are converted to X and Y. See Section III.
9. TUBE (X0, Y0, Z0, P, XL, YL, ZL, LEAVE) - Given a track with origin (X0, Y0, Z0) within the streamer chamber hydrogen target (the 'tube') and with momentum P (MeV/c), TUBE calculates the coordinates (XL, YL, ZL) where it leaves the tube, assuming it begins travelling exactly in the (+Z) direction. We have the coordinate system X=0 and Y=0 at the center of the tube and the (+Z) axis identical with the axis of the tube, going in the beam direction. The charge convention is such that if TUBE is called with positive P, the track will be bent towards the left. Negative P causes the track to be bent to the right towards +X assuming it is initially travelling in the +Z direction. When the momentum, P, of the track is so small that it cannot leave the tube, the routine sets LEAVE=1. Otherwise LEAVE=0.

Two versions of TUBE are available. One assumes a constant magnetic field in the Y-direction.

The second uses the track integration routine TRACK by D.E.C. Fries (Ref. 6). Values of the magnetic field are supplied by subroutines CONSET, FLDBZ and FLDBR. Note that the coordinate system defined just above is not the standard SYBIL system used by Group D.⁷ Therefore the second version of

TUBE makes the necessary transformation to enable it to utilize TRACK and its subroutines. Of further interest are the constants RTUBE (cm), the radius of the tube (now 0.635 cm), and H (Kgauss), the magnetic field (now 10.43 in Version I of TUBE).

10. PRFUNC - Refer to paragraph D, Appendix I.

11. TPFUNC - Refer to paragraph F, Appendix I.

12. RANDOM - Taken from the IBM Scientific Subroutine Package. Refer to the manual 360A-CM-03X, p. 47.

13. TZRULE (BL, UL, NSTEP, FUNCT) - Integrates the function FUNCT using the trapezoidal rule. FUNCT is the name of an arbitrary function subroutine, provided via an EXTERNAL statement. BL and UL are the lower and upper limits of x, respectively, and NSTEP is the number of intervals (set at 20 in THEM1).

14. PTINT (XVAL, KDIM, X, Y, YVAL, PNAME, *) - This subroutine performs a second-order interpolation of a tabulated function. X and Y, dimensioned by KDIM, are the tables of 'x' and 'y' values and YVAL is the interpolated 'y' value evaluated at XVAL.

PNAME contains the six-character (maximum) name of the calling program, used for identification purposes in the error message, and '*' provides a secondary return for alternate branching if XVAL is not in the range of X. Except for YVAL, all other parameters are supplied by the calling program.

15. LTINT - Performs linear interpolation of a tabulated function. The arguments have the same definitions as for PTINT.

16. CDATE (NYR, NDY, NMNTH, NDAY) - Converts the day count (NDY) to the day (NDAY) and month (NMNTH) of the year (NYR).

17. CTIME (ITIME, IHR, IMN, ISC) - converts the time (ITIME) in seconds to hours (IHR), minutes (IMN), and seconds (ISC).

18. TDATE (NYR, NDY) - An assembly routine which provides the run date (year, day count) of a job from the system clock.

19. QTIME (K) - An assembly routine which provides the current time of day (K) as a fixed point number in hundredths of seconds.

20. HGLOT (DATA, WEIGHT, DMIN, DMAX, N) - Refer to paragraph H, Appendix I.

V. INPUT/OUTPUT

No tapes or data cards are required.

VI. TIMING

About two minutes for a run generating approximately 24,000 random photon energies, and 1,500 EM events, using an IBM360/91.

PROGRAM: THEMI

APPENDIX I: SUBROUTINES

A. PHOTON (E0, EL, EU, EGAM)

This routine computes a random photon energy EGAM between limits EL and EU for a bremsstrahlung spectrum with peak energy E0 .

It calls the function program BREMS (paragraph B) which provides the spectrum shape $kN(k)$, where $k = EGAM$ is the photon energy.

It first chooses a trial value for EGAM randomly, by the equation

$$EGAM = EL^{R1} * EU^{1.0-R1}$$

It then accepts EGAM if the test $(BREMS(E0, EGAM) \cdot GT \cdot RANDOM)$ is satisfied. Otherwise it chooses a new R1 and new EGAM. See text, Section III.

B. BREMS (E0, EGAM)

BREMS (called by subroutine PHOTON) provides the function $kN(k)$ that describes the photon spectrum. Here, $k = EGAM$ is the photon energy, and E0 is the high-energy limit of the spectrum. The user will desire to substitute a version of BREMS that describes the photon spectrum used in his application. For rough work, a "rectangular" spectrum $BREMS = 1.0$ may be used. Because PHOTON accepts or rejects a particular choice of EGAM by comparison with a random number in the range 0.0 - 1.0, the value of BREMS must in no case be greater than 1.0. In general, however, values of BREMS close to, but not larger than 1.0 are desirable for efficiency.

The current version of BREMS contains a table TH of values $kN(k)$ for an E0 = 18,000-MeV bremsstrahlung spectrum, and a table ET containing a transmission curve for one radiation length of LiH beam hardener. The spectrum was calculated by means of a program written by R. A. Early;⁸ the transmission curve was taken from a paper by D. C. Gates et al.⁵

C. PAIRWL (IGEN, EGAM, PPOS, THPOS, PHIP, PNEG, THNEG, PHIN, PSIGMA)

This routine calculates the total cross section for pair production from hydrogen, using Wheeler-Lamb screening (see paragraph E on TRIPL) and generates a random pair event for Monte-Carlo use, at a chosen photon energy (EGAM). See Ref. 4 .

Parameter List

1. IGEN = 0: 'PAIRWL' computes only the cross section (PSIGMA) for a given EGAM.
IGEN = 1: 'PAIRWL' computes PSIGMA as for IGEN=0 and then chooses a random event with the parameters described below.
2. EGAM (MEV) is the photon energy supplied by the calling program (LAB).
3. PPOS (MEV/C) is the momentum (LAB) of the positron (e^+), given by the partition function of Wheeler-Lamb.
4. THPOS (RAD) is the polar angle (LAB) of the positron (e^+), presently set to 0.0.
5. PHIP (RAD) is the azimuthal angle (LAB) of the positron (e^+), presently undefined.
6. PNEG (MEV/C) is the momentum (LAB) of the electron (e^-), uniquely determined by EGAM and PPOS.
7. THNEG (RAD) is the polar angle (LAB) of the electron (e^-), presently set to 0.0.
8. PHIN (RAD) is the azimuthal angle (LAB) of the electron (e^-), presently undefined.
9. PSIGMA (cm^2) is the total cross section.

The Wheeler-Lamb equation is defined by the function program PRFUNC, which in turn calls the subroutine WELAMB to obtain the screening corrections ψ_1 and ψ_2 defined in Ref. 4. In calculating PSIGMA, the function is integrated by using the trapezoidal rule (TZRULE) over forty steps.

The Monte-Carlo pair event is chosen by using to advantage the fact that the partition function $(EPOS - MC^2)/(EGAM - 2*MC^2)$ is rather flat. EPOS is the positron (LAB) energy and MC^2 represents the electron rest energy. A trial value of EPOS is chosen uniformly between MC^2 and $(EGAM - MC^2)$. Then the value is accepted if the test is satisfied:

$$(DSDEP . GT . RANDOM * DSMAX),$$

where DSDEP is $d\sigma/dE^+$ for that value of EGAM and EPOS, and DSMAX is merely a constant with regard to EPOS:

$$DSMAX = 2.0 * PSIGMA/EGAM$$

The factor 2.0 insures that DSMAX is everywhere greater than DSDEP. See text, Section III.

D. WELAMB (G, X1, X2, X3, X4, X5, IFLAG)

This routine is a modification of a subroutine by the same name which is used in the program by R. A. Early (Ref. 8). It calculates the screening functions described in the paper of Wheeler and Lamb (Ref. 4).

Parameter List

- G is the "epsilon" defined in Ref. 4.
- X₁ is ψ_1 for the hydrogen atom.
- X₃ is ψ_2 for the Fermi-Thomas atom.
- X₅ is ψ_2 for hydrogen introduced by authors.

IFLAG = 0 if G < 120.0; this argument is not examined in PAIRWL at present.

E. TRIPL (IGEN, EGAM, PPOS, THPOS, PHIP, PNEG, THNEG, PHIN, PREC, THREC, PHIR, TSIGMA)

This routine calculates the total cross section for triplet production from hydrogen, using Wheeler-Lamb screening (Ref. 4). Note that the screening corrections for hydrogen have been recently recalculated with greater accuracy by T. M. Knassel and were found to be correct to within the accuracy of the Wheeler-Lamb graph (see Ref. 9).

The routine also generates a random triplet event for Monte-Carlo use, at a chosen photon energy (EGAM). Note that at present, only the two higher-energy members are provided; the "recoil" has not yet been introduced.

Parameter List

- 1. IGEN = 0: 'TRIPL' computes only the cross section (TSIGMA) for a chosen EGAM.
- IGEN = 1: 'TRIPL' computes TSIGMA as for IGEN=0 and then chooses a random event with the parameters described below.

- | | | |
|------------------------------|---|---|
| 2. EGAM (MEV) | | is the photon energy supplied by the calling program (LAB). |
| 3. PPOS (MEV/C) | | is the momentum (LAB) of the positron (e^+), distributed according to the partition function of Wheeler and Lamb. |
| 4. THPOS (RAD) | | is the polar angle (LAB) of the positron (e^+), presently set to 0.0. |
| 5. PHIP (RAD) | | is the azimuthal angle (LAB) of the positron (e^+), presently undefined. |
| 6. PNEG | } | are similar quantities as in 3-5, but for the high-energy electron. They are uniquely determined by EGAM and PPOS. |
| 7. THNEG | | |
| 8. PHIN (e^-) | | |
| 9. PREC | } | are similar quantities for the "recoil electron"; presently undefined. |
| 10. THREC | | |
| 11. PHIR | | |
| 12. TSIGMA (cm^2) | | is the total cross section. |

The Wheeler-Lamb equation is defined by the function program TPFUNC, which in turn calls the subroutine SCREEN to obtain interpolated screening corrections ψ_1 and ψ_2 . In calculating TSIGMA the function is integrated by using the trapezoidal rule (TZRULE) over forty steps.

The generation of a Monte-Carlo event for photon-energy EGAM is done as in the subroutine PAIRWL.

F. SCREEN (EPSIL, PSI1, PSI2)

This routine calculates the screening functions described in the paper by Wheeler and Lamb (Ref. 4). EPSIL is the "epsilon" in Wheeler and Lamb; PSI1 and PSI2 are the ψ_1 and ψ_2 for hydrogen.

G. COMPT (IGEN, EGAM, TELEC, THELEC, PHIEL, ESCAT, CSIGMA)

This routine calculates the total cross section for Compton scattering and generates a random Compton event for Monte-Carlo use, at a given photon energy (EGAM).

Parameter List

- | | |
|--------------|--|
| 1. IGEN = 0: | 'COMPT' computes only the total cross section (CSIGMA) for a photon energy (EGAM) supplied by calling program. |
|--------------|--|

- IGEN = 1: 'COMPT' computes CSIGMA as for IGEN = 0, and then chooses a random event with the parameters described below.
2. EGAM (MEV) is the photon energy supplied by the calling program (LAB).
 3. TELEC (MEV) is the kinetic energy of recoil for e^- (LAB), chosen randomly according to the distribution given by Eq. (2.8), p. 683 of Ref. 3.
 4. THELEC (RAD) is the polar angle of recoil for e^- (LAB) uniquely determined by EGAM and TELEC.
 5. PHIEL (RAD) is the azimuthal angle of recoil for e^- (LAB) where PHIEL is uniformly random in the range 0. to 2π .
 6. ESCAT (MEV) is the energy of the scattered photon (LAB).
 7. THSCAT (RAD) is the polar angle of the scattered photon (LAB). Both arguments (6) and (7) are uniquely determined by the parameters (2) through (5).
 8. CSIGMA (cm^2) is the total cross section, computed by using the Klein-Nishima formula (see Eq. (2.11), p. 684, Ref. 3).

The Monte-Carlo section of the program is sectioned into low energy (EGAM/ELMASS < 10.) and high energy (EGAM/ELMASS > 10.) parts. EGAM is photon energy and ELMASS = 0.5110 MeV. The lower-energy part uses to advantage the fact that the kinetic energy distribution of the recoil electron is roughly uniform. At higher energies we use the fact that the Klein-Nishima formula is dominated by the term (ESCAT/EGAM). See text, Section III.

(NOTE: The term 'DIV2' in the program must be computed in double precision, otherwise loss of significance will result in division by zero.)

H. Subroutine HG PLOT (DATA, WEIGHT, DMIN, DMAX, N)

This routine accumulates a weighted frequency count of the data in the range (DMAX - DMIN) and plots the resulting histogram of 100 intervals. The weight is transmitted by WEIGHT.

1. DATA is the point in the range to be counted.
2. WEIGHT is the weighting factor to be applied to the data point.

3. DMIN is the minimum value of the range.
4. DMAX is the maximum value of the range.
5. N is a code (> 0) specifying a particular graph for which a frequency count is accumulated. If ($N < 0$) the graph (N) is plotted.

The subroutine is always called at two locations of the calling program; one for accumulating the data points over 100 bins and the other for printing the histogram. For positive 'N' the routine takes the current value of DATA and adds its WEIGHT into the appropriate bin. Those points falling above and below the range given by DMIN and DMAX are counted in two other separate arrays.

For negative 'N' the routine begins by scaling the frequency counts down by two until the largest of the bins fits a table with a maximum of 50 characters. In addition to generating a plot, the routine prints out a table of the actual bin counts and a table of the average WEIGHT for each bin.

This program also calls the subroutines LINE, NUMB and COMPN2 to generate scaled and labelled axes. The maximum number of plots is currently set to 25. At the present time page formatting such as page skipping and the writing of titles must be controlled externally before the plot is called.

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