SLAC-R-634

# Strings Without Supersymmetry

Allan W. Adams

Stanford Linear Accelerator Center
Stanford University
Stanford, CA 94309

SLAC-Report-634

Prepared for the Department of Energy
under contract number DE-AC03-76SF00515

Printed in the United States of America. Available from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Road, Springfield, VA 22161.

PhD. Thesis, Stanford University, Stanford, CA 94305

# STRINGS WITHOUT SUPERSYMMETRY

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF PHYSICS
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Allan W. Adams
February 2003

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosopy.

Eva M. Silverstein, Principal Adviser

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosopy.

Shamit Kachru

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosopy.

Savas Dimopolous

Approved for the University Committee on Graduate Studies:

# Acknowledgements

The ideas presented in this text were developed in discussion and collaboration with many colleagues and friends. My most profound debt is to Eva Silverstein, my advisor and closest collaborator; much of the work behind this dissertation began in the countless hours we spent at chalkboards at Stanford and Santa Barbara. Deep thanks also to Joseph Polchinski, whose collaboration in our study of localized tachyons was invaluable. I am also grateful to Michal Fabinger, John McGreevy, Jarah Evslin, Uday Varadarajan, Eric Sharpe and Sang-Jin Sin for related collaborations.

In addition to these collaborators, my studies benefited enormously from discussions with many, many colleagues, including (though certainly not limited to!) O. Aharony, N. Arkani-Hamed, T. Banks, M. Berkooz, R. Bousso A. Chari, A. Cohen, O. DeWolfe, S. Dimopoulos, M. Dine, O. Ganor, S. Giddings, J. Gomis, D. Gross, S. Gukov, A. Hashimoto, S. Hellerman, P. Hořava, G. Horowitz, S. Kachru, N. Kaloper, D. B. Kaplan, D. E. Kaplan, M. Kleban, I. Klebanov, P. Kraus, D. Kutasov, A. Lawrence, A. Maloney, E. Martinec, R. Meyers, S. Minwalla, A. Nelson, H. Ooguri, T. Pantev, M. Peskin, M. Roček, S. Sethi, S. Shenker, M. Strassler, A. Strominger, L. Susskind, W. Taylor, S. Thomas, J. Troost, C. Vafa, N. Weiner, K. Zarembo and B. Zwiebach.

I would also like to thank many friends and colleagues at UC Berkeley, where my graduate work began, including Ed Boyda, Dan Brace, Jarah Evslin, Lawrence Hall, Marty Halpern, Hitoshi Murayama, Hiroshi Ooguri, Aaron Pierce, Surya Ganguli, James Gill, Ben Metcalf, Harlan Robbins, Jonathan Tannenhauser, Radu Tatar and Uday Varadarajan.

I have also benefitted greatly from two and a half years of discussions and adventures with many people in the Theory Group at SLAC and the ITP at Stanford, including Puneet Batra, John Brodie, Keshav Dasgupta, Savas Dimopoulos, Mike Dine, Lance Dixon, Michal Fabinger, Simeon Hellerman, Veronika Hubeny, Shamit Kachru, Renatta Kallosh, Nemanja Kaloper, David (E.) Kaplan, Amir-Kian Kashani-Poor, Matt Kleban, Albion Lawrence, Andre Linde, Xiao Liu, Liam McAllister, John McGreevy, Michael Peskin, Aaron Pierce, Mark van Raamsdonk, Mike Schulz, Shahin Sheikh-Jabbari, Steve Shenker, Eva Silverstein, Scott Thomas and Yonatan Zunger.

I am also grateful to: the Kavli Institute for Theoretical Physics and the participants and organizers of the M-Theory program in the Spring of 2001; the Aspen Center for Physics and the participants and organizers of the "Advances in Field Theory" Workshop in the summer of 2002; the participants and organizers of PASI 2002; and the theory groups at Harvard, Chicago and Duke for stimulating conversations, feedback, and hospitality.

Many thanks also to Lilian DePorcel, Sharon Jensen and Karin Slinger, without whose tireless labors the ITP and SLAC Theory groups would have been reduced to utter dysfunction.

I would also like to acknowledge the vital importance of countless internet cafes in Tunisia, Egypt, Austria, England, France, Italy, Switzerland, Chile which allowed my work to continue while affording me some necessary perspective.

This research (kind reader, my rent) has been supported variously by an NSF Graduate Research Fellowship, NSF grants PHY-00-97915, PHY-99-07949, PHY-97-22022 and PHY-95-14797 DOE contracts DE-AC03-76SF00098 and DE-AC03-76SF00515, and the DOE OJI and Sloan Foundation Fellowships of Eva Silverstein. My research has also been made possible by the support and hospitality of the Kavli Institute for Theoretical Physics at UCSB during the winter and spring of 2001 and the Aspen Center for Physics during the summer of 2002.

On a more personal note, I am enormously grateful to Shamit Kachru, Stephen Shenker, Lenny Susskind and especially Eva Silverstein, whose inexplicable faith in supporting my transfer to Stanford was literally life-altering.

My greatest, most heartfelt thanks go to my advisor, mentor and friend, Eva Silverstein. Eva's faith and patience We have passed countless hours working and playing together at the chalkboard, full of laughter and excitement, equations and puns; I cannot imagine a more wonderful advisor.

Finally, I would like to thank my friends and family for their love and support, continuing my education in topics not covered in any university.

Stanford University Allan W. Adams
February 2003

Contents

# 1. Introduction

String theory was originally formulated in terms of a perturbative expansion around classical solutions to the effective spacetime equations of motion, the Einstein equations of general relativity. Quantizing strings in these backgrounds resulted in remarkably rich spectra, including a graviton, which led to much excitement. Unfortunately, the spectra also contained tachyons, whose equations of motion have exponentially growing solutions which make perturbation theory inconsistent. Since these string theories were defined pertubatively, this looked like a disaster.

One might say the same of perturbation theory for a rigid pendulum about its unstable equilibrium point, *i.e.*pointing straight up. In the perturbative approximation of very small displacement, $\theta << 1$, the potential energy is simply $V(\theta) = -\theta^2$, which is a tachyonic potential. When we perturb the system away from equilibrium by, say, blowing on it, it immediately falls down the potential with an exponentially growing velocity - any small fluctuation grows rapidly. Perturbation theory is thus inconsistent. However, this spectacular failure of perturbation theory does not mean that the physical system is sick, but that we have chosen a poor point about which to conduct a small-fluctuation analysis; what we need is knowledge of the physics beyond perturbation theory. Of course, for the pendulum this is easy: the full potential is $V(\theta) = cos(\theta)$, and when perturbed from vertical it simply swings down to the stable equilibrium point at $\theta = \pi$, about which perturbation theory *is* valid. In this "vacuum", the perturbative hamiltonian is hermetian and perturbations remain bounded under evolution.

Unfortunately, in canonical world-sheet string theory, we do not have such non-perturbative information; the stringy analog of the full potential remains utterly mysterious. When the tachyon turns on, where does it go? Perturbation theory, our only guide, says nothing. The perturbative theory appears dead in the water.

Supersymmetry provides a beautiful escape from this unpleasant state of affairs by ensuring a tachyon-free spectrum (a hermetian hamiltonian), resulting in a self-consistent and well-posed[1] perturbative expansion. Supersymmetry thus does not solve the problem of tachyon condensation; it eliminates the problem altogether. Of course, since supersymmetry is not manifest on any observable scale, one might worry that this is not a terribly well-motivated addition to the theory. But then the same could be said of the full gauge symmetry of the electroweak theory; could not supersymmetry be spontaneously broken well below the string scale? In any case, such objections are simply irrelevant: the superstring is the only well-posed perturbative string in town.

Nearly thirty years of work on supersymmetric string theory has led to spectacular progress in understanding non-perturbative aspects of string theory, resulting in a host of powerful new techniques for studying regimes in which the perturbative worldsheet description, even with supersymmetry, breaks down. Perhaps most important are D-branes, stringy solitons whose mass (in string units) scales inversely with the string coupling, allowing them to probe length scales (and time scales) far shorter than the string scale[2] the natural short-distance cutoff in perturbative string theory. Careful study of these D-branes led to the AdS/CFT correspondence, which reveals that string theory in anti-de Sitter spacetimes is exactly dual to $\mathcal{N} = 4$ SYM, providing not just a non-perturbative probe but a full non-perturbative definition of string theory, at least in this class of spacetimes. Another set of tools exploit not spacetime supersymmetry but worldsheet supersymmetry, including gauged linear sigma models, topological sigma models, and mirror symmetry. Of no less importance has been the introduciton of many tools of modern algebraic geometry, especially toric geometry, which are intimately related to both worldsheet supersymmetry and spacetime geometry.

---

[1] *Mostly* well posed - a number of problems remain, such as the exponential growth of hard-scattering amplitudes with genus, making the string expansion asymptotic at best.

[2] In particle theories without gravity, one can in principle probe arbitrarily small length scales by colliding particles at ever higher energies - which is how particle accelerators work. In string theory, however, this is not true, since as we pump energy into a string, it actually gets longer and floppier, and ends up probing a *larger* length scale than the static string. The minimum length scale probed by perturbative strings is thus limited to the fundamental string scale.

It is armed with these awesomely powerful tools that we return to the basic problems of non-supersymmetric string theory, in particular the fate of closed string tachyons and, in non-tachyonic models, tadpole-driven instabilities. In the remainder of this dissertation, we will describe three approaches to these problems, employing modern non-perturbative techniques as well as old-fashioned worldsheet technology applied using intuition from a modern, non-perturbative perspective.

In Chapter 2 we will study tachyons in non-supersymmetric orbifolds of an AdS/CFT duality. As the CFT provides an exact non-perturbative definition of the dual string theory, we will be able to faithfully phrase the question of the fate of the closed string tachyon in terms of completely well-posed, concrete computations in the dual orbifold CFT. We will find that the CFT description often supports much of our intuition, with the string theory running apparently non-critical as advocated for decades by Polyakov, but that the details of this process are surprisingly rich and complicated, while in some cases our intuition will be totally violated. These ideas were developed in collaboration with Eva Silverstein.

In Chapter 3 we will study closed string tachyons in locally flat spacetimes where supersymmetry is broken only at isolated orbifold singularities. While these systems do not have complete non-perturbative definitions, D-probe, GLSM and gravitational methods will provide enough non-perturbative information to allow us to follow the condensation of these localized tachyons in a completely controlled and reliable way. Remarkably, tachyon condensation will be shown to dynamically resolve the supersymmetry violating orbifold singularities, driving the spacetime to a stable and generically supersymmetric endstate. These ideas were developed in collaboration with Joe Polchinski and Eva Silverstein.

It should be emphasized that these well-understood cases form a set of measure zero in the full space of non-supersymmetric string vacua, generic examples of which contain tachyons in the bulk of a non-AdS spacetime; thus many questions remain largely untouched. To get a flavour for the challenge involved, recall that the c-theorem (which does not apply to localized tachyon condensation in non-compact orbifolds) suggests that bulk tachyon condensation drives a theory sub-critical. This can be seen explicitly in a number of simple cases. But is this always the result? What happens when we condense, say, the Type 0 tachyon? Does the theory get driven non-critical, or through strong string coupling, where the c-theorem does not apply?

In Chapter 4 we address non-supersymmetric vacuua with entirely non-tachyonic spectra. In all known cases, such vacuua have non-vanishing (though often finite) one-point functions for their massless moduli at one loop in the string expansion, which also destablized the perturbative vacuum. This is one aspect of the cosmological constant problem.

The cosmological constant problem, one of the basic problems of quantum gravity, involves the physics of both the strict UV and the deep IR, and thus seems unlikely to be solved by playing with the details of UV physics alone. Moreover, since we have probed the equivalence principle only up to length scales well below the Hubble scale, we are truly ignorant of Hubble scale physics. Along these lines, and motivated by our experiences studying the dynamics of non-supersymmetric orbifolds of AdS/CFT dualities described in Chapter 2, Chapter 4 describes some early concrete explorations of quantum field and string theories with modified IR physics. In particular, it will describe a proposal for the IR modification of perturbative string theory which was hoped to result in a perturbatively finite and unitary flat-space S-matrix without massless moduli. The simple version discussed in this chapter, unfortunately, does not correctly deal with worldsheet anomalies, and thus is not a well-posed definition in and of itself; nontheless, it provides a first step towards such a theory, towards which we continue to work. While this is a somewhat speculative approach, even negative results could teach us a great deal about the physics of non-supersymmetric string vacua. These ideas were developed in collaboration with John McGreevy and Eva Silverstein.

These three approaches, together with a variety of modifications which have appeared in the literature, provide an important first step in the application of tools and lessons from supersymmetric theories to the fundamental problems in non-supersymmetric string theories. A very small first step, admittedly: much remains to be understood, most notably, the fate of bulk tachyons, regarding which we have had very little to say. Nontheless, the limited successes described in these chapters should give us hope that these more thorny and dificult questions may be fruitfully addressed with similar tools. With that in mind, let's begin the story.

# 2. Closed String Tachyons, AdS/CFT and Large-N QCD

## 2.1 Introduction and Summary

Orbifold examples [1][2][3] provide one of the simplest testing grounds and applications of AdS/CFT duality [4][5]. With less than maximal SUSY, the physics at low energies is less constrained, and new elements of the AdS/CFT dictionary emerge. One such element is the relation between the gravity-side cosmological term (dilaton potential) which typically gets generated in the absence of SUSY, and the finite-N beta functions and dimension spectrum of the gauge theory [1][6][7].

Another element that arises upon breaking supersymmetry is the possibility of a stringy tachyon in the twisted sector on the gravity side. This typically (in fact in all cases known to the authors) happens at large AdS radius when the orbifold is symmetric and fixes some or all of the points on the $S^q$ component of the gravity background.[3] Freely-acting orbifolds on the $S^q$ have no twisted-sector tachyon at large radius, since the twisted-sector states must wind around the sphere and are therefore very heavy. In this chapter, we find an interesting pattern in the corresponding instability structure of the small radius limit of these theories by investigating the dynamics of twisted operators in the appropriate weakly-coupled dual quiver gauge theories [8].

Building on work of Tseytlin and Zarembo [9], we study radiative corrections in $IIB/(-1)^F$ (Type 0) on $AdS_5 \times S^5$ and more general non-supersymmetric non-freely-acting orbifolds by $\Gamma = Z_n$, identifying a Coleman-Weinberg effective potential [10] which leads to growth of the VEVs of certain twisted operators quadratic in

---

This chapter is reprinted, with changes, from Allan Adams and Eva Silverstein, "Closed String Tachyons, AdS/CFT and Large-$N$ QCD" Phys. Rev. D **64**, 086001 (2001), by permission of the publisher. 

[3] A tachyon indeed appears in the twisted sector of the non-SUSY model studied in detail in [1]; the statement otherwise was in error.

the gauge theory scalars. This instability along a (partial) Coulomb branch of the quiver gauge theory describes D-branes splitting into fractional branes at the fixed point locus of the orbifold. This one-loop contribution to the effective potential involves double-trace operators which affect correlators involving twisted operators at leading order in $1/N^2$ (corresponding to genus zero on the gravity side). It is the leading non-conformal effect at small 't Hooft coupling and introduces non-conformal correlators for twisted operators into the theory at large $N$.

This raises an interesting puzzle some of whose potential resolutions we discuss in §3.2. It has to do with the issue of whether and how the conformal symmetry of the gauge theory (or equivalently the $SO(4,2)$ isometry group of the $AdS_5 \times S^5/Z_n$) should be preserved in correlation functions involving twisted operators in the QFT dual to the standard gravity-side orbifold construction of the $AdS_5 \times S^5$ sigma model. As we will discuss in §2, there is a space of possible orbifold quantum field theories arising from the space of renormalization group trajectories consistent with the quantum symmetry and inheritance of untwisted operator correlation functions from the $\mathcal{N} = 4$ theory. This suggests a corresponding space of dual gravity-side orbifold string backgrounds, generic elements of which may generalize the standard construction.

The one-loop calculation is reliable in this context for a large but finite range of nonzero VEVs for the scalar fields, including values near a local minimum of the effective potential. We find that a renormalization-group improved perturbation theory analysis using one-loop beta functions does not lead to any additional control for the calculation of the potential near the origin of the Coulomb branch. From this we conclude that the instability indicated by this potential is present but, depending on the form of the potential near the origin of the Coulomb branch, may be a global effect accessible only by tunneling.

The resulting instability, present in the large-N limit, drives the erstwhile tachyon (or at least a twisted state with the same quantum numbers as a large-radius tachyon) to condense. It is interesting to note that whatever the behavior near the origin, the instability is consistent with the reality of the operator dimensions observed in [11], though not with an uncorrected extrapolation of the Breitenlohner-Freedman bound to the regime of large curvatures and large RR field strengths.

For a freely-acting orbifold, the quiver gauge theory has no Coulomb branch, as the D-branes are at the same codimension as the orbifold fixed point locus in spacetime and have no directions in which to split up into separated fractional branes. One correspondingly finds no instability in the effective potential for twisted operators; we therefore conjecture that the tachyon-freedom of this type of model persists to small radius. This constitutes an interesting prediction for the RR string theory in this limit, and a satisfying class of examples to contrast with the tachyonic models.

We also study a case with non-free orbifold action and discrete torsion which projects out the lowest-lying tachyon, leaving tachyonic modes with angular momentum along the sphere. At weak coupling we find no one-loop instability. However there is a Coulomb branch (again describing fractional D-branes) in this case, and we find no symmetries preventing instabilities from getting generated at higher order.

We then consider in more detail the effects of condensing the twisted modes in the unstable (non-freely-acting) cases. On the gravity side at large radius, condensing a tachyon is expected to drive the theory to a subcritical dimension (with compensating gradients for the dilaton and other gravity fields), since the zero-momentum tachyon is a relevant operator on the string worldsheet [12][13][14]. One way in which this can happen is to lose and/or deform the dimensions corresponding to the $S^5/\Gamma$ and the radial direction in $AdS_5$. This suggests losing the scalars and perhaps non-chiral fermions on the gauge theory side, since these matter fields have quantum numbers associated to the sphere. Similarly, non-perturbative instabilities have been argued to drive the theory to an endpoint with a loss of degrees of freedom (decays to "nothing") [15].

We find on the field theory side that the Coleman-Weinberg potential indeed drives the theory toward one with fewer degrees of freedom at least in the IR: in some cases, pure glue $SU(N)$ QCD plus some decoupled matter and U(1) factors. This can be seen algebraically or from a simple geometric picture of the low-energy/near horizon limit of symmetrically distributed fractional D-branes. We conjecture that this pure glue QCD theory is described by the endpoint of tachyon condensation in the dual gravity theory, at least at long distance on the QFT side. As just discussed, this is likely to be a $D < 10$ theory on the gravity side (which may or may not be a perturbative string theory, depending on the effective string coupling that emerges in the subcritical theory when the tachyon has become large and mixed with the dilaton and other string fields). This result is suggestive of Polyakov's proposal for QCD as a noncritical string [16]; our analysis suggests that if realized it could be connected to ordinary AdS/CFT by tachyon condensation. [4] More complicated models involving both open and closed strings on the gravity side may leave surviving quark flavors as well, but we leave this goal of getting full QCD for future work.

Of course there are other avenues toward the String/M dual of QCD, arising from the low-energy limit of relevant perturbations of the $\mathcal{N} = 4$ theory as in for example [18]. Our connection to non-SUSY AdS/CFT examples via tachyon condensation appears complementary to these. It involves a dynamical mechanism for eliminating the extra matter on the gauge theory side, but the starting point is a non-supersymmetric system less well-understood than the $\mathcal{N} = 4$ theory. As with many applications of AdS/CFT, it can be taken as a lesson about the gravity side: the gauge theory remaining after condensation of the twisted operators gives a dual answer to the question of what happens to the closed string theory after tachyon condensation.

There have been interesting discussions on closed string tachyons in D-brane and/or AdS/CFT systems in [3][11][19][20][21]. The role of tachyons, tachyon condensation, and/or tachyon-freedom in other conjectured non-supersymmetric closed string dualities has been studied in other contexts previously for example in [22][23][6][24][25][14][26]. Calculations making use of the relevance of open-string tachyon vertex operators appeared in for example [27]. It would be very interesting if these techniques could be transported back to closed-string theory to investigate further the hints of QCD emerging in the systems we study here. Indeed, there is some resemblance of the Coleman-Weinberg potentials we discuss here and formulas for an (open string) tachyon potential in [27].

It would be interesting to study potential relations of our results on instabilities on the gauge theory side to the kinds of gravitational instabilities studied in [15][28][29] and in particular in the AdS/CFT context in [30]. Aside from a few comments, we leave this for future work, and focus here on the field theory side at weak coupling.

---

[4] This possibility was suggested earlier in general terms by Minahan at the end of [17].

The chapter is organized as follows. In §2, we calculate the one-loop Coleman-Weinberg potential for a large class of $Z_n$ orbifold models with fixed points, exhibiting a nontrivial regime of validity of the one-loop analysis out on the classical Coulomb branch. In §3, we find two simple minima of the potential within this regime of validity at which the low-energy gauge theory contains pure glue QCD sectors. We study the N-dependence of our results from §2 and establish that at large N the dynamically generated potential affects correlators involving twisted operators, and does not affect untwisted correlators. We discuss a puzzle this raises and several possibilities which may lead to its resolution, and review the evidence we have gathered for the persistence of the tachyon to small radius in these examples. In §4 we turn to the case of freely-acting orbifolds (where there is no tachyon at large radius) and show that the branch along which such an instability would arise is absent in this case. In §5 we analyze an orbifold with discrete torsion which has no Coleman-Weinberg instability at one loop but does have a Coulomb branch along which one could emerge at higher orders. Finally in §6 we discuss future directions.

## 2.2 $Z_n$ Orbifolds with Fixed Points and Effective Potential

Given the duality between $\mathcal{N} = 4$ SYM and IIB string theory on $AdS_5 \times S^5$, we can obtain new dual pairs by orbifolding both sides of this "parent" duality by a discrete group $\Gamma \subset SO(6)$ [1][2]. The gravity side is Type IIB string theory on $AdS_5 \times S^5/\Gamma$, and the QFT side is a quiver gauge theory [8] obtained by taking the low-energy limit of the worldvolume theory on D3-branes at the corresponding $C^3/\Gamma$ orbifold singularity. [5] On the gravity side, correlations of untwisted operators are inherited at genus zero. The corresponding planar diagrams, in particular those

[5] We are working with orbifolds of the maximally supersymmetric version of the duality. The duality being a statement of equivalence between two descriptions of one and the same system, the orbifold (including its instabilities) tautologically exists on one side if it exists on the other. Note that this equivalence is between the quantum theories, which introduces subtleties having to do with the manifestation on the gravity side of the choice of renormalization condition in the gauge theory side as we will discuss shortly.

corresponding to the beta functions [1], of the quiver gauge theories are inherited from those of the parent $\mathcal{N} = 4$ theory [31].[6]

Twisted states in the orbifold string theory correpond to "twisted operators" of the orbifold gauge theory, which are gauge-invariant operators that do not descend from gauge-invariant operators of the parent $\mathcal{N} = 4$ theory. On the gravity side, non-freely-acting geometrical orbifolds have tachyons in the twisted sector at least at large radius, while freely-acting orbifolds do not have tachyons, since twisted-sector states are very massive; we will return to them in §4. The effective potential for twisted operators is not constrained by the $\mathcal{N} = 4$ theory, so we should expect an effective potential for twisted operators at order $N^2$, whereas the effective potential for untwisted operators should appear only at order $N^0$; we will see this explicitly below.

We will find it instructive to study several different cases. In this section we will focus on non-freely-acting orbifolds, which fix some locus on the $S^5$. The most extreme case of this is the "Type 0" theory [3], which is an orbifold by $(-1)^F$ which acts only on the spinors and thus fixes the entire spacetime [32]. The corresponding quiver theory has gauge group $SU(N)^2$ with six real adjoint scalars $X_1^i = (\mathbf{N^2-1,1})$, $X_2^i = (\mathbf{1,N^2-1})$, $i = 1,\ldots,6$, four Weyl fermions $\chi^\alpha$ in the bifundamental representation $(\mathbf{N},\bar{\mathbf{N}})$, and four Weyl fermions $\psi^\alpha$ in the $(\bar{\mathbf{N}},\mathbf{N})$.

These representations arise from projections of a parent $SU(2N)$ $\mathcal{N} = 4$ theory. In a convenient basis, the gauge fields and scalars sit in diagonal blocks of this $SU(2N)$ theory and the fermions sit in the off-diagonal blocks. The tree-level interactions of the orbifold theory are those of the $SU(2N)$ $\mathcal{N} = 4$ theory which involve fields which survive the orbifold projection. In addition to the minimal gauge couplings, one has quartic scalar interactions of the $X_1^i$ which are identical to those of an $SU(N)$ $\mathcal{N} = 4$ theory and likewise for $X_2^i$. The bifundamental fermions mix the two $SU(N)$ gauge groups via tree-level Yukawa interactions of the form[7]

$$g_{YM}\left[tr(\bar{\psi}X_1\psi) + tr(\bar{\chi}\chi X_1) + tr(\bar{\chi}X_2\chi) + tr(\bar{\psi}\psi X_2)\right] \tag{2.2.1}$$

[6] It is worth emphasizing that we here take the regular representation for the action $\gamma_{ab}$ of the orbifold group on the Chan-Paton indices, one where $tr\gamma = 0$. With any other choice of action, the D-branes are a source for the twisted-sector tachyon in the asymptotically flat region away from the core of the D-branes, which does not decouple from the near-horizon low energy D-brane theory [20].

[7] In this section we henceforth assume canonical normalization for the kinetic terms with no factors of $1/g_{YM}^2$ in front of the action.

The orbifold on the gravity side has a "quantum symmetry" $\Gamma$, [33], under which twisted states transform non-trivially. In the Type 0 case, this is a $Z_2$ symmetry which is manifested on the gauge theory side by a symmetry exchanging the two $SU(N)$ factors in the gauge group (and correspondingly exchanging $X_1^i \leftrightarrow X_2^i$ and $\psi \leftrightarrow \chi$). The lowest-dimension twisted operators in our theory are thus $trX_1^2 - trX_2^2$ and $trX_1^iX_1^j - trX_2^iX_2^j$, which have dimension two at leading order. In the full D-brane system, these operators couple to two derivatives of the tachyon field in the directions transverse to the D-branes [34], while the tachyon itself couples to the dimension four operator $tr(F_1^2 + DX_1DX_1 - F_2^2 - DX_2DX_2)$ [34][11].

The interactions of untwisted states are inherited at genus zero, but those of twisted states are constrained a priori only by the quantum symmetry. Our main interest will be contributions to the scalar potential generated by quantum corrections to the gauge theory at leading order in the $1/N^2$ expansion. We will discuss the calculation of the effective potential in non-freely-acting $Z_n$ orbifolds in the remainder of this section, and then interpret the results in terms of tachyons and explore the instability structure on the gauge theory side in §3.

In [9], Tseytlin and Zarembo calculated the bosonic potential energy lifting the classical moduli space of the Type 0 theory at one-loop order in the gauge theory. Because of the quartic scalar interactions, the classical moduli space of this theory is parameterized by the eigenvalues of commuting matrices $\langle X_1^i\rangle \equiv diag(x_1^{i,1}, \ldots, x_1^{i,N})$ and $\langle X_2^i\rangle \equiv diag(x_2^{i,1}, \ldots, x_2^{i,N})$ with $\Sigma_a x_1^{i,a} = 0 = \Sigma_a x_2^{i,a}$. Going to a generic point on this moduli space and integrating out massive particles, one obtains a simple expression for the vacuum energy as a function of $x_1^{i,a}$ and $x_2^{i,a}$ [9]:

$$V_{eff} \sim \frac{g_{YM}^4}{8\pi^2}\Sigma_{a,b=1}^N \Bigg[|x_1^a - x_1^b|^4 log\frac{|x_1^a - x_1^b|^2}{\tilde{M}^2} + |x_2^a - x_2^b|^4 log\frac{|x_2^a - x_2^b|^2}{\tilde{M}^2} - 2|x_1^a - x_2^b|^4 log\frac{|x_1^a - x_2^b|^2}{\tilde{M}^2}\Bigg] \tag{2.2.2}$$

where $|x|^2 \equiv x^ix^i \equiv \vec{x}^2$ and $\tilde{M}$ is related to a subtraction point to be discussed shortly. The first two terms in (2.2.2) arise from integrating out the off-diagonal entries in $(\vec{X}_1)_{ab}, (A_1^\mu)_{ab}$ and $(\vec{X}_2)_{ab}, (A_2^\mu)_{ab}$, which have masses $g_{YM}|x_1^a - x_1^b|$ and $g_{YM}|x_2^a - x_2^b|$ respectively. The last arises from integrating out the fermions $(\psi)_{ab}$ and $(\chi)_{ab}$ in the bifundamental $(\mathbf{N}, \bar{\mathbf{N}}) \oplus (\bar{\mathbf{N}}, \mathbf{N})$, whose masses are $g_{YM}|x_1^a - x_2^b|$.

This expression can be understood (and later generalized) as follows. Integrating out a particle of mass $m^2$ leads to a contribution

$$(-1)^F \int d^4p \;\; log(p^2+m^2) \sim (-1)^F \int_0^\infty \frac{dt}{t^3} e^{-tm^2} \tag{2.2.3}$$

where $F$ is the spacetime fermion number and where we are ignoring coefficients of order one. This expression has quadratic and logarithmic divergences (as well as a quartic divergence in the vacuum energy to which the field theory is insensitive). We therefore require counterterms; following the analysis of [10] one obtains an expression of the form

$$\begin{aligned} V_{eff} = \Sigma_{a,b=1}^N \Big[ & |x_1^a - x_1^b|^4 (\lambda_{11}^{ab} + (Ag_{YM}^4 + B(\lambda_{11}^{ab})^2) log \frac{|x_1^a - x_1^b|^2}{M^2 e^{25/6}}) \\ & + |x_2^a - x_2^b|^4 (\lambda_{22}^{ab} + (Ag_{YM}^4 + B(\lambda_{22}^{ab})^2) log \frac{|x_2^a - x_2^b|^2}{M^2 e^{25/6}}) \\ & - 2|x_1^a - x_2^b|^4 (\lambda_{12}^{ab} + (Ag_{YM}^4 + B(\lambda_{12}^{ab})^2) log \frac{|x_1^a - x_2^b|^2}{M^2 e^{25/6}}) \Big] \end{aligned} \tag{2.2.4}$$

where $A$ and $B$ are constants of order 1 and $g_{YM}$ and $\lambda_{ij}^{ab}$ are renormalized couplings. This potential includes the 1-loop contributions plus counterterms chosen to satisfy the renormalization conditions

$$\frac{d^4 V_{eff}}{d{x_{ij}^{ab}}^4}|_{x_{ij}^{ab}=M} = \lambda_{ij}^{ab} \tag{2.2.5}$$

where $x_{ij}^{ab} = x_i^a - x_j^b$. The coupling constants determined at one value of the subtraction point $M$ are related to those at a different point by the renormalization group. A choice of renormalization group trajectory is a choice of field theory and presumably corresponds to a choice of what the precise configuration of dual gravity-side string fields is. At small radius, we do not have an independent handle on the gravity side, so we will simply consider the whole set of possible trajectories consistent with the symmetries and inheritance properties of the orbifold. This issue will be discussed further in §3.2. As discussed in [9], there are planar contributions proportional to $N$ in the individual terms in the effective potential that must cancel in the orbifold theory by inheritance. This plus the quantum symmetry leads to the simplification that in the quantum field theory dual to the orbifold background, we should have $\lambda_{ij}^{ab} \equiv \lambda$.

As discussed in [9] (and as will be generalized and studied further in §3) there is an unstable direction in the potential in which $x_1^1 = \rho = -x_1^2$, with all other $x_i^a = 0$. Plugging this into (2.2.4), one finds

$$V_{eff} \sim \rho^4 \left[ \lambda + (A g_{YM}^4 + B\lambda^2) log \frac{2^{8/3}\rho^2}{M^2 e^{25/6}} \right] \tag{2.2.6}$$

Let us now renormalize at a subtraction point of order the VEV of $\rho$, e.g. $M = 2^{4/3}\langle\rho\rangle$, where

$$\frac{dV_{eff}}{d\rho}|_{\rho=\langle\rho\rangle} = 0. \tag{2.2.7}$$

Imposing this condition, one obtains in the theory expanded about the minimum of the potential a relation between $\lambda$ and $g_{YM}$ as in [10] of the form

$$\lambda = C g_{YM}^4 \tag{2.2.8}$$

where $C$ is a constant of order 1. So the renormalized quartic scalar coupling along the Coulomb branch is of order $g_{YM}^4$, as befits a contribution at one-loop order in perturbation theory. Plugging this back into (2.2.4) and defining

$$\tilde{M}^2 \equiv e^{\frac{25}{6}-\frac{C}{A}} M^2, \tag{2.2.9}$$

we recover the result (2.2.2).

The one-loop result is reliable where the logarithms in (2.2.4) are not big enough to compensate the small couplings $\lambda, g_{YM}$ and make different orders in perturbation theory commensurate. As in the original analysis of the massless Abelian Higgs model in [10], our result is reliable near minima of the potential but not at the origin $x_{ij}^{ab} \to 0$ or in the asymptotic region $x_{ij}^{ab} \to \infty$, since there the logarithms are large. In some theories, expressing the effective potential in terms of the running coupling (the solution of the Callan-Symanzik equations) results in a weakly coupled description for a larger range of $x$. As demonstrated below, this is not the case at one-loop order at large N in our theories, which have a somewhat remarkable RG structure due to the vanishing of untwisted beta functions at large N.[8]

The 1-loop $\beta$ functions are easily computed. For the Type 0 theory and the other quiver theories we are about to analyze, Large-N inheretance ensures that the gauge and yukawa couplings have vanishing $\beta$ functions at leading order in $\frac{1}{N}$[31].

[8] We thank D. Gross for interesting discussions on this.

The $\beta$ function for the quartic scalar coupling $(trX_1^2 - trX_2^2)^2$ can be calculated directly from our calculation of the renormalized coupling:

$$\beta_\lambda \sim \lambda^2 + g_{YM}^4 \tag{2.2.10}$$

the resulting RG equations are solved by

$$\lambda = g_{YM}^2 \; Tan(g_{YM}^2 ln\frac{\rho^2}{M^2} + g_o^2) \tag{2.2.11}$$

As $\rho$ gets either very large or small compared to $M^2 \; e^{\frac{\pi}{g_{YM}^2}}$, this solution becomes strongly coupled and untrustworthy. So in these theories at one-loop order at large N, RG improvement does not help. While we can trust our one-loop effective potential near its local minimum, we cannot trust the dynamics near the origin, or at large values of the scalar VEV. Since the gauge coupling is protected from developing a $\beta$ function at large $N$ by inheritance, the main effect of higher loops will be to add higher monomials in $\lambda$, whose effects will depend strongly on their signs. We will leave this much more involved two-loop calculation to future work, and in this chapter content ourselves with having identified at least a global instability. This leaves open the possibility that the region near $x \sim M$ could only be accessible via tunneling from the region of the origin. We will comment further on this in §3.

More generically one can consider locally-free orbifold actions. One example we will study in detail is a $C/Z_3$ orbifold, under which a single complex plane with coordinate $z^1$ is rotated by $\alpha \equiv e^{2\pi/3}$: $z^1 \to \alpha^2 z^1$, $z^{2,3} \to z^{2,3}$. This acts by a phase $\alpha^{\pm 1} \equiv e^{\pm 2\pi/3}$ on all the spacetime spinors, and so projects out all the massless gravitinos. The quiver theory in this case has gauge group $SU(N)^3$. The matter content consists of four real scalars in the adjoint:

$$\begin{array}{ll} X_1^i & (\mathbf{N^2-1,1,1}) \\ X_2^i & (\mathbf{1,N^2-1,1}) \\ X_3^i & (\mathbf{1,1,N^2-1}), \end{array} \tag{2.2.12}$$

for $i = 1, \ldots, 4$; one complex scalar in the bifundamental representations:

$$\begin{array}{ll} U & (\mathbf{N,\bar{N},1}) \\ V & (\mathbf{1,N,\bar{N}}) \\ W & (\mathbf{\bar{N},1,N}), \end{array} \tag{2.2.13}$$

and four Weyl fermions in the bifundamental representations:

$$\begin{aligned} &\chi_U^\alpha \quad (\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1}) \\ &\chi_V^\alpha \quad (\mathbf{1}, \mathbf{N}, \bar{\mathbf{N}}) \\ &\chi_W^\alpha \quad (\bar{\mathbf{N}}, \mathbf{1}, \mathbf{N}), \end{aligned} \tag{2.2.14}$$

for $\alpha = 1, \ldots, 4$. The interactions in this case are inherited from an $SU(3N)$ $\mathcal{N} = 4$ theory. In a convenient basis, the gauge bosons and adjoint scalars sit in diagonal $N \times N$ blocks of the adjoint matrices of the parent theory, and the bifundamental scalars (2.2.13) and fermions (2.2.14) sit in off-diagonal blocks.

The Higgs branch of this gauge theory, along which $U = V = W$ (as enforced by the quartic scalar interactions inherited from the $\mathcal{N} = 4$ D-terms) describes motion of the D3-branes away from the orbifold fixed locus. The theory also has a Coulomb branch, where $U = V = W = 0$ and components of the $X_k^i$ for different $k$ get independent VEVs. This describes motion of "fractional" D-branes away from each other along the orbifold fixed locus $z^1 = 0$.

In this case, one finds an effective potential analogous to that of [9] (2.2.2):

$$\begin{aligned} V_{eff} \sim \frac{g_{YM}^4}{8\pi^2}\frac{3}{4}\Sigma_{a,b=1}^N \Big[ &|x_1^a - x_1^b|^4 log\frac{|x_1^a - x_1^b|^2}{\tilde{M}^2} + |x_2^a - x_2^b|^4 log\frac{|x_2^a - x_2^b|^2}{\tilde{M}^2} \\ &+ |x_3^a - x_3^b|^4 log\frac{|x_3^a - x_3^b|^2}{\tilde{M}^2} - |x_1^a - x_2^b|^4 log\frac{|x_1^a - x_2^b|^2}{\tilde{M}^2} \\ &- |x_2^a - x_3^b|^4 log\frac{|x_2^a - x_3^b|^2}{\tilde{M}^2} - |x_3^a - x_1^b|^4 log\frac{|x_3^a - x_1^b|^2}{\tilde{M}^2} \Big] \end{aligned} \tag{2.2.15}$$

Here (similarly to the discussion following (2.2.2) ) the first three terms come from integrating out the four real scalars and the gauge fields which transform in the adjoint representation, which in this theory involves 3/4 of the bosons in the theory, hence the factor of 3/4 relative to the Type 0 result. The last three terms come from integrating out the bifundamental matter, which in this theory consists of one complex scalar (1/4 of the total bosons) and all of the fermions, leading to the factor of -3/4 appearing in front of these terms in (2.2.15).

It is now clear how to generalize this result to arbitrary $Z_n$ orbifolds. Consider for example a non-freely acting $Z_n$ orbifold with rotation angles $2\pi(\frac{r_1}{n}, \frac{r_2}{n}, 0)$ in the three complex planes paramaterized by $z^1, z^2, z^3$ transverse to the D3-branes (with $r_1 \pm r_2$ even so that the orbifold acts as a $Z_n$ on all spinors). The quiver gauge theory has a gauge group $SU(N)^n \equiv \Pi_{k=1}^n SU(N)_k$ with one complex scalar corresponding to $Z^3$ transforming in the adjoint $\Sigma_k(\mathbf{N^2-1})_k$. The complex scalar corresponding to $Z^1$ transforms in the bifundamental representation $\Sigma_{k=1}^n(\mathbf{N_k}, \bar{\mathbf{N}}_{\mathbf{k+r_1}})$, and that corresponding to $Z^2$ transforms in the bifundamental representation $\Sigma_{k=1}^n(\mathbf{N_k}, \bar{\mathbf{N}}_{\mathbf{k+r_2}})$. Half of the fermions transform in the bifundamental representation $\Sigma_{k=1}^n(\mathbf{N_k}, \bar{\mathbf{N}}_{\mathbf{k+\frac{r_1+r_2}{2}}})$, and the other half transform in the bifundamental representation $\Sigma_{k=1}^n(\mathbf{N_k}, \bar{\mathbf{N}}_{\mathbf{k+\frac{r_1-r_2}{2}}})$. From this one obtains the effective potential

$$V_{eff} = \frac{g_{YM}^4}{16\pi^2}\Sigma_{a,b=1}^N\Sigma_{k=1}^n\Bigg[|x_k^a - x_k^b|^4 log\frac{|x_k^a - x_k^b|^2}{\tilde{M}^2} + \frac{1}{2}|x_k^a - x_{k+r_1}^b|^4 * \\ log\frac{|x_k^a - x_{k+r_1}^b|^2}{\tilde{M}^2} + \frac{1}{2}|x_k^a - x_{k+r_2}^b|^4 log\frac{|x_k^a - x_{k+r_2}^b|^2}{\tilde{M}^2} \\ - |x_k^a - x_{k+\frac{r_1+r_2}{2}}^b|^4 log\frac{|x_k^a - x_{k+\frac{r_1+r_2}{2}}^b|^2}{\tilde{M}^2} - |x_k^a - x_{k+\frac{r_1-r_2}{2}}^b|^4 log\frac{|x_k^a - x_{k+\frac{r_1-r_2}{2}}^b|^2}{\tilde{M}^2}\Bigg] \tag{2.2.16}$$

Finally we note that for orbifolds which act freely on the $S^5$, with nontrivial rotation angles $2\pi(r_1/n, r_2/n, r_3/n)$ on the coordinates $z^1, z^2, z^3$, there are no adjoint scalars and no Coulomb branch. This follows geometrically from the fact that the D-branes span the same dimensions as the orbifold plane and cannot move apart into separate fractional branes at the fixed point. We will return to this in §3.

### 2.3 Tachyon Condensation and QCD

We have seen in the above section that the quiver theories corresponding to orbifolds with fixed points on the $S^5$ develop a Coleman-Weinberg potential on the classical moduli space at one loop, and we will see in this section that there are interesting unstable directions in which twisted operators get VEVs.

#### *2.3.1 Counting Powers of N*

Let us first clarify and interpret in terms of the gravity side the $N$-dependence of the results (2.2.2)(2.2.15)(2.2.16).[9] Let us first determine the N-dependence of the

[9] The results on $N$-dependence here, some aspects of which appear in [9], were developed in discussions with O. Aharony.

1-loop potential term in the field theory. In all of the orbifold theory potentials we have derived, as noted for the Type 0 case in [9], the coefficient of the logarithmically divergent 4-point interaction among the scalars contains no powers of $N$ beyond that in the factor of $g_{YM}^4$ after terms of the form $g_{YM}^4 N\Sigma_a|x_a|^4$ cancel out of $V_{eff}$, leaving terms proportional to $g_{YM}^4(TrX_1^iX_1^j - TrX_2^iX_2^j)^2$ and $g_{YM}^4(TrX_1^2 - TrX_2^2)^2$ at the level of four-point graphs. Let us rescale the $X$'s so that a factor of $1/g_{YM}^2 = N/\lambda_{'tHooft}$ appears multiplying the whole tree-level action. Then $g_{YM}^2$ counts loops, and the one-loop potential scales like $g_{YM}^0 = 1$, down by a factor of $g_{YM}^2 \sim 1/N$ from tree level.

With this normalization of the fields, correlation functions of the single-trace operator $NtrX^2$ scale like $N^2$ plus terms subleading in $1/N^2$. These correspond to connected genus-zero amplitudes involving single-particle states on the gravity side [4][5].

Normalizing operators of the form $(trX^2)^2$ with a power of $N^2$:

$$\mathcal{O}_{double-trace} = N^2(trX^2)^2, \tag{2.3.1}$$

we obtain $l$-point correlation functions of the $\mathcal{O}_{double-trace}$ which scale like $N^{2l}$ in the free theory, corresponding to $l$ disconnected genus zero diagrams describing $l$ strings propagating across the AdS. This is in line with the interpretation of multitrace operators as multiparticle states on the gravity side in the unperturbed theory.

As discussed below (2.2.1), the twisted operators in our theory are of the form $trX_k^2 - trX_{k'}^2$. Because they transform non-trivially under the quantum symmetry of the orbifold, terms in the Lagrangian linear in these operators are not generated dynamically, but terms quadratic in these operators which are invariant under the quantum symmetry are (they are implicit in the potentials (2.2.2)(2.2.15)(2.2.16) calculated out along the Coulomb branches in the last section). These are double-trace operators, which are thought to correspond to multiparticle excitations of the dual gravity theory [4][5]. As just discussed, as they appear at one-loop these contributions scale like

$$\delta S \sim \int (trX^2)^2. \tag{2.3.2}$$

Now consider adding a contribution of the order $(trX^2)^2$ to the action (as occurs dynamically in our theory (2.3.2)). Bringing down a power of (2.3.2) into correlation functions, one finds the leading-N effect from factorized terms of the form

$$\begin{aligned}\langle : (TrX_k^2 - TrX_{k'}^2)^2 : \mathcal{O}_1, \ldots, \mathcal{O}_l \rangle \sim \langle (TrX_k^2 - TrX_{k'}^2)\mathcal{O}_1, \ldots, \mathcal{O}_{l'} \rangle \\ * \langle (TrX_k^2 - TrX_{k'}^2)\mathcal{O}_{l'+1}, \ldots, \mathcal{O}_l \rangle\end{aligned} \tag{2.3.3}$$

where we have replaced $(TrX^2)^2$ in (2.3.2) with the more precise form $(TrX_k^2 - TrX_{k'}^2)^2$ we have for deformations of our theories. These go like $N^2$. However if all the $\mathcal{O}_1, \ldots, \mathcal{O}_k$ are *untwisted* operators, then each factor in the factorized leading-N contribution vanishes, and one is left with an effect that is down by $1/N^2$ from genus-zero effects. This is in accord with large-N inheritance on the gravity side [1] and the field theory side [31], which ensures that at large N the correlators of untwisted operators are the same as in the $\mathcal{N} = 4$ theory. The twisted operators do not exist in the parent theory, and are not constrained by inheritance.

It is interesting that a mass scale $\tilde{M}$ appears in correlators of twisted operators at genus zero. In particular, the couplings of the double trace operators $\Sigma_k(TrX_k^2 - TrX_{k'}^2)^2$ have nontrivial beta functions at one-loop (as can be seen from the four-point function contribution to the effective potentials calculated in §2). So even before we go out on the Coulomb branch, the theory is nonconformal at leading order in N in a nontrivial regime of $\lambda_{'tHooft}$. This is invisible to the untwisted operators alone at this order in N, in accord with [31]. Even so, this is puzzling because of the general arguments advanced in [1] for the large-N conformality of these theories. In the next subsection, we will discuss this puzzle and several possible resolutions which it will be interesting to pursue once we have pushed the relevant technology to the necessary level.

### *2.3.2 Orbifolding and Symmetries: A Puzzle*

As discussed in [1], there is a fairly general reason to believe that orbifold field theories should have conformal invariance at large N, including the physics of twisted operators.[10] The worldsheet sigma model describing strings propagating on the parent space $AdS_5 \times S^5$ has a symmetry corresponding to the $SO(4,2)$ isometries of the $AdS_5$, which commutes with the $SO(6)$ of the $S^5$ and in particular commutes with an action of $\Gamma \subset SO(6)$ on $S^5$. This symmetry commutes

[10] We thank T. Banks and S. Kachru for discussions of this.

with the Hamiltonian of the worldsheet theory, and therefore all of its correlation functions respect it. This parent sigma model has many operators, some subset of which $\{V_{parent}\}$ constitute mutually local dimension (1,1) vertex operators describing physical string states. When we orbifold, for example by the $Z_n$ actions we are considering in this chapter, we include only those vertex operators $\{V_{untwisted}\} \equiv (\{V_{invariant}\} \subset \{V_{parent}\})$ which are invariant under the orbifold group action. Having done this one can (and should at the one-loop level) add "twisted" operators which are further operators from the set of operators in the parent sigma model which are mutually local with respect to the reduced set of operators $\{V_{untwisted}\}$. So finally $\{V_{orbifold}\} = \{V_{untwisted} + V_{twisted}\}$ gives the full set of vertex operators for the orbifold theory. The Hamiltonian of the full worldsheet sigma model is the same in all of these theories, and commutes with the $SO(4,2)$. So the orbifold theory should have this symmetry and the QFT dual to it by AdS/CFT should be conformally invariant for all $\lambda_{'tHooft}$ at leading order in the $1/N^2$ expansion. This argument appears rather general (though unforeseen subtleties involving RR fields may render it inapplicable to our case).

On the other hand, at weak coupling in the quiver gauge theory one finds (as we have discussed) nontrivial beta functions for the double-trace quartic scalar interactions of the form $\Sigma_{k,k'}\lambda(TrX_k^2 - TrX_{k'}^2)^2$. Although it is made out of twisted operators, this contribution to the Lagrangian does not itself transform under the quantum symmetry and if we do not condense the twisted operators we should not have left the orbifold point.

We do not yet know the resolution of this puzzle, but can see several interesting possibilities (which are not all mutually exclusive):

(1) The above argument about the symmetries is correct and applies to the RR sigma models of interest here. This would suggest that there is a line of fixed points corresponding to the radius of $AdS_5 \times S^5/Z_n$. Since starting at weak coupling on the field theory side there is not such a fixed line, this line of fixed points would have to be fundamentally strongly coupled.

Then the theories we consider here, with running $\lambda_{ab}^{ij}$, are deformations away from the line of CFTs dual to the standard orbifold of $AdS_5 \times S^5$. But these theories share many properties with the standard orbifold, in particular the quantum symmetry and the inheritance of untwisted operator correlation functions at large

N. Therefore even if (1) is true we feel it is important to understand the gravity-side description of our (perhaps nonstandard) orbifold models. This leads to possibility

(2) The double-trace operators in the effective Lagrangian of our models correspond to a novel type of worldsheet string theory on the gravity side, such as the one under investigation independently in a supersymmetric context with marginal double-trace perturbations [35].[11] This novel string theory, if it exists and applies to our models here, may not have all the properties required for the above symmetry argument. As discussed above, a relative of this possibility is the possibility that RR sigma models do not satisfy the assumptions in the symmetry argument presented above.

Finally, there is always the possibility

(3) Phase transitions and/or other unconstrained non-supersymmetric dynamics ruin the application of the duality to this non-supersymmetric context. Because we began with a parent system with two dual descriptions, the procedure applied to one of them producing the orbifold theory ought to have a translation into the dual variables if it exists nonperturbatively. This translation to the dual may not be a standard orbifold construction, however, which may relate to point (2). Indeed a phase transition in this type of system is suggested by the large-radius duality map, which maps the tachyon to a complex-dimension operator in the field theory [11]. It would be very interesting to understand better what this means for the duality, but in this chapter we will continue to focus on the small-radius (weak 't Hooft coupling) regime.

Because of the RR fields and strong coupling issues, establishing the precise resolution of this puzzle appears out of reach of current technology, and we will leave it for future work. We think it is likely that there is a resolution (perhaps along the lines of (1) and/or (2)) which preserves the duality and teaches us something new about the gravity side, and we will proceed with our analysis on the assumption that the duality holds. In particular, our analysis has generated further concrete evidence in favor of the duality (in addition to generating the puzzle discussed in this subsection). However, possibility (3) should be kept in mind.

[11] We thank O. Aharony and M. Berkooz for sharing with us their ideas on this.

### 2.3.3 Tachyons and AdS/CFT Duality

It has been suggested [3][11] that the Type 0 tachyon is lifted at small $AdS$ radius to satisfy the Breitenlohner-Freedman bound [36]. Heuristically this might be expected from the fact that the $AdS$ curvature reaches string scale for small enough 't Hooft coupling, so that a string-scale tachyon need not violate the bound [3]. This sort of behavior has been seen in the $AdS_3$ context in [37]. Further, the twisted operator $trF_1^2 + DX_1DX_1 - F_2^2 - DX_2DX_2$, to which the tachyon couples directly at large radius, is actually slightly irrelevant at weak coupling [11], which according to an uncorrected extrapolation of the large radius duality map would translate to a non-tachyonic mass in the bulk gravity theory.

However, we have seen that the weakly coupled dual field theory has instabilities in the potential at leading order in $1/N^2$ which cause certain twisted operators (which have the same discrete quantum numbers as the large-radius tachyons) to condense, either directly or via tunneling depending on the small-$X$ behavior of the potential. That the instability appears at genus zero shows that it persists even in the strict large-N limit. This demonstrates an instability of the string theory which causes modes from the orbifold twisted sectors to condense even at small radius.

As we have discussed, because of the running couplings in the theory, our one-loop analysis is not sufficient to determine whether the instability is perturbative or requires a non-perturbative tunneling process to access. If it is non-perturbative, the situation is reminiscent of those described in [15], where a tachyonic instability in one limit of moduli space appears to turn into a non-perturbative instability mediated by a gravitational instanton in another limit.

### 2.3.4 Patterns of Symmetry Breaking

In the remainder of this section we will provide a preliminary discussion of the physics that results when the twisted operator VEVs turn on. We will begin with some heuristic intuition from the gravity side, and then analyze concretely some aspects of the Higgs structure of the model given the scalar potentials calculated in the previous section.

On the gravity side, we expect perturbative tachyon condensation to produce a subcritical dimension target spacetime [12][13]. This is because the zero-momentum tachyon vertex operator is a relevant operator on the string worldsheet. The worldsheet beta function equations are then satisfied by a nontrivial field configuration for the dilaton, metric, and other string fields; in particular dilaton gradients contribute effective central charge to compensate for that lost by going to a subcritical dimension (as occurs for example in the case of a linear dilaton with flat string-frame metric) [12]. In the context of the AdS/CFT correspondence, the dimensions of the $S^5/Z_n$ and the radial direction of AdS arose from the directions transverse to the D3-branes, which are parameterized by worldvolume scalars. It is natural to expect therefore that losing and/or deforming the $S^5$ and radial dimensions would correspond to losing the scalars in the dual quiver gauge theory, and perhaps also the fermions which also transform under $SO(6)$ rotations.

In situations with non-perturbative instabilities on the gravity side [15] one also has a sense in which degrees of freedom are lost, as one "tunnels to nothing".[12] In our situation, as we will discuss shortly, one does not always expect to decay to nothing, but one can decay to something which is in some sense less than what one had to begin with: from the full quiver gauge theory to a long-distance sector with pure glue QCD.

Let us discuss some patterns of symmetry breaking that emerge from our potentials (2.2.2)(2.2.15)(2.2.16). There are instabilities in the effective potential correponding to VEVs for twisted operators in the gauge theory, manifested in the D-brane language convenient for the calculations in §2 by relative motion of fractional branes along the orbifold plane described by VEVs for diagonal entries of the adjoint scalar matrices. Let us investigate the effect of turning on these VEVs.

Let us analyze the Type 0 case (2.2.2) for simplicity; similar patterns will emerge in the higher $Z_n$ cases and can be analyzed in a similar way. Consider the direction in field space in which $X_1^i$ gets a VEV

$$\langle X_1^i \rangle = diag(\rho_1^i, \rho_2^i, \ldots, \rho_{N-1}^i, -\rho_1^i - \rho_2^i - \ldots - \rho_{N-1}^i), \tag{2.3.4}$$

which satisfies the $SU(N)$ condition

$$\Sigma_{a=1}^N \rho_a = 0, \tag{2.3.5}$$

and in which $\langle X_2^i \rangle = 0$.

[12] From other points of view one appears to tunnel to flat space via a Schwinger effect [28][29], a situation whose interpretation and whose relation to our results here would be very interesting to clarify.

Plugging this into the effective potential (2.2.2), we obtain

$$V(\vec{\rho_a}) \sim g_{YM}^4\left(\Sigma_{a,b}|\vec{\rho}_a - \vec{\rho}_b|^4 log\frac{|\vec{\rho}_a - \vec{\rho}_b|^2}{\tilde{M}^2} - 2N\Sigma_a|\vec{\rho}_a|^4 log\frac{|\vec{\rho}_a|^2}{\tilde{M}^2}\right) \tag{2.3.6}$$

where we have replaced the transverse $R^6$ index $i$ by vector notation.

Let us consider the minimization of this potential with respect to the $\rho_a^i$. The first term in (2.3.6) describes the force between "electric" branes (those with $SU(N)_1$ on their worldvolume). The second term describes the force between the electric branes and the "magnetic" branes (those with $SU(N)_2$ on their worldvolume) which are all sitting at the origin, $\vec{X}_2 = 0$. At sufficiently small distances between the branes, the former is repulsive and the latter is attractive.

There is a relatively simple configuration where the $\vec{\rho}_a$ are arranged symmetrically (equally spaced) on an $S^5$ of radius $\rho$. This satisfies the $SU(N)$ constraint (2.3.5). It is an extremum of the effective action in the angular directions. By playing the mutual repulsion of the electric branes against their attraction to the magnetic branes at the center, we will find a minimum for the radial mode $\rho$, generalizing the one along the direction $\langle X_1^i\rangle = diag(\rho, 0, \ldots, 0, -\rho)$ discussed in [9].

Approximating the sum over branes indexed by $a$ by an integral over the angles of the $S^5$, we obtain an approximate form of the potential which will be sufficient to indicate the presence of the anticipated minimum:

$$\int d\Omega_1 d\Omega_2|\vec{\rho}(\Omega_1) - \vec{\rho}(\Omega_2)|^4 \ln\frac{|\vec{\rho}(\Omega_1) - \vec{\rho}(\Omega_2)|^2}{\tilde{M}^2} - 2N\int d\Omega_1|\vec{\rho}(\Omega_1)|^4 \ln\frac{|\vec{\rho}(\Omega_1)|^2}{\tilde{M}^2} \tag{2.3.7}$$

Using this, separating the radius $\rho$ of the sphere of fractional D-branes from the angular variables, we obtain

$$V(\rho) \sim N^2 g_{YM}^4 \rho^4 log\left(\frac{\rho^2 e^{379/240}}{\tilde{M}^2}\right) \tag{2.3.8}$$

The potential (2.3.8) has a minimum at $\rho$ of order $\tilde{M}$,

$$\rho_{min}^2 = \tilde{M}^2 e^{-1/2} e^{-379/240} \tag{2.3.9}$$

as in our earlier discussion of the Coleman-Weinberg potential. We have therefore balanced the attractive and repulsive forces as anticipated. In particular, one finds the force between the electric branes and the magnetic branes is attractive in this regime. Because the forces grow with distance, this suggests that the magnetic branes at the origin are stable against small fluctuations, which means the $X_2^i$ scalars are massive. Indeed this follows from an analysis of small fluctuations in the $\vec{x}_{2b}$ directions, as follows.

Expanding (2.2.2) around the background symmetric distribution of $\vec{\rho}_a$, we find the following mass terms for the $x_{2,b}^i$:

$$-2g_{YM}^4\Sigma_{a,b}\left(2|\vec{x}_{2,b}|^2|\vec{\rho}_a|^2 log\frac{|\vec{\rho}_a|^2e^{1/2}}{\tilde{M}^2}+4x_{2,b}^l x_{2,b}^m \rho_a^l \rho_a^m log\frac{|\vec{\rho}_a|^2e^{3/2}}{\tilde{M}^2}\right) \qquad (2.3.10)$$

Summing over the spherically distributed $\vec{\rho}_a$, the off-diagonal terms in the mass matrix sum to zero and we see that the diagonal terms are nonzero and positive at the minimum (2.3.9) (since $log\frac{\rho_{min}^2e^{1/2}}{\tilde{M}^2}=-379/240<0$ and $log\frac{\rho_{min}^2e^{3/2}}{\tilde{M}^2}=1-379/240<0$).[13]

We have now accumulated enough information to determine the effect on the gauge theory of turning on this VEV. It breaks one $SU(N)$ to $U(1)^{N-1}$ and leaves the other $SU(N)$ intact. From (2.2.1) one finds that all the fermions get masses once our VEVs (2.3.4) are turned on. As we have just seen, the scalars $X_2^i$ get positive mass squared.

The angular fluctuations of the $X_1^i$ on the other hand are dominated by repulsive interactions between the electric branes, so these fluctuations appear to be unstable. Once this configuration of twisted VEVs in the gauge theory is turned on, the long-distance physics of the gauge theory consists of QCD plus decoupled U(1) factors and unstable scalars.

There is another configuration in which QCD sectors emerge at low energies without instabilities in the other sectors; this is a likely endpoint of the spherical configuration we began with. It is another natural generalization to large $N$ of the instability discussed in [9]. Consider again $\langle\vec{X}_2=0\rangle$. Take $\langle X_1\rangle = diag(\vec{r},\vec{r},\ldots,\vec{r},-\vec{r},\ldots,-\vec{r})$ where the first $N/2$ diagonal entries are $\vec{r}$ and the last $N/2$ entries are $-\vec{r}$. Geometrically, this describes $N/2$ electric branes at $-\vec{r}$ and $N/2$ electric branes at $+\vec{r}$, with $N$ magnetic branes at the origin.

[13] Note that there is no coupling-constant dependence in the computation of the signs of forces and scalar $m^2$'s which depend only on the values of order one numbers arising from the geometry of the configuration at the minimum (2.3.9).

In this case, there is a minimum at

$$\frac{r_{min}^2 e^{1/2}}{\tilde{M}^2} = 2^{-8/3} \tag{2.3.11}$$

about which the fluctuations of both $\vec{X}_1$ and $\vec{X}_2$ work out to be massive, which again is consistent with naive expectation from the signs of the forces in the vicinity of the minimum (2.3.11). The Yukawa couplings (2.2.1) yield masses for all the bifundamental fermions in this configuration.

This more stable configuration leaves, on the gauge theory side at distances long compared to the masses of the scalars and fermions, a pure glue QCD sector with gauge group $SU(N)$, decoupled from two others with gauge groups $SU(N/2)$ and a relative $U(1)$ factor.

Because of the limited range of validity of the 1-loop calculation of $V_{eff}$, we do not know if the potential is bounded or unbounded from below at large $\langle X \rangle$. If it turns out to be unbounded, it is tempting to suggest that for infinitely large VEVs for the twisted operators in the gauge theory, the theory may reduce to separate pure glue QCD plus decoupled $U(1)$ sectors. However because of the long-range forces on this fractional D-brane branch of the gauge theory, it is not clear what the masses of the $X_2$'s will be as a function of the VEVs of the $X_1$'s, and it is logically possible that the $X_2$ scalars would come back down to zero mass and/or become unstable as $X_1$ increases beyond the regime of validity of our current calculation.

In our $Z_n$ orbifolds, before going out on the Coulomb branch the gauge group is $SU(N)^n \equiv \Pi_{k=1}^n SU(N)_k$. Turning on VEVs for the diagonal elements of the $X$'s similarly leads to the near-horizon limit of various fractional D-brane configurations whose low-energy theories involve gauge symmetry breaking and some massive and/or decoupled scalar and fermion matter. It would be interesting to classify all the possible behaviors in arbitrary $Z_n$ orbifold models based on the potentials derived in the last section, but we will leave that for future work.

It is not clear from this analysis whether this M-theory dual to the remaining gauge theory will be a perturbative string theory.[14] Indeed in ordinary large-radius string backgrounds the tachyon mixes with the dilaton, and its condensation leads to strong dilaton gradients; the analogous phenomenon should be expected in our

[14] The results [38] perhaps suggest otherwise.

small-radius AdS/CFT system. (Correspondingly on the gauge theory side, the VEVs for twisted operators that we have turned on can induce large renormalizations of all the couplings in the gauge theory.) Even before turning on VEVs for twisted operators along the unstable directions, a novel kind of string theory may be required on the gravity side of a dual pair in which the field theory is deformed by a double-trace operator of this kind, as discussed in §3.2 [35].

In any case, we have arrived at an interesting answer via AdS/CFT duality to the question of what can happen when one condenses a tachyon in closed string theory: in this system, it rolls to the gravity dual of a gauge theory with less symmetry and reduced matter content, but sometimes retaining pure glue QCD factors in the infrared.

As discussed above, condensing the tachyon is expected to lead to a subcritical matter sector on the string worldsheet, and we have just learned that the corresponding process on the gauge theory side can lift or decouple the extra matter and gauge fields beyond pure glue QCD. Noncritical string theory was conjectured to be dual to QCD in [16]. Our results provide some further evidence in this direction.

### 2.4 Freely Acting Orbifolds and Tachyon-freedom

Consider an orbifold group $\Gamma \subset SO(6)$ which fixes an isolated point in $R^6$. In the presence of $N$ D3-branes centered at the fixed point, the spacetime geometry blows up into a near-horizon region which is completely smooth, with the orbifold acting freely on the $S^5$.

Because the orbifold fixes an isolated point in $R^6$, the codimension of the singularity is the same as the codimension of the D3-branes, so the spacetime has no directions along which fractional branes could separate. Correspondingly, the scalars in the resulting quiver gauge theory are all in bifundamental representations, in contrast to the above non-freely-acting cases where the scalars describing motion along the orbifold fixed locus remained in the adjoint. Thus, for freely acting orbifolds, there is no Coulomb branch.

Since in these cases the classical moduli space does not include a branch where twisted states can develop a VEV, the theory will remain stable to all orders in $\lambda_{'tHooft}$. We therefore suspect that there are no twisted instabilities for any radius (any 't Hooft coupling $\lambda_{'tHooft}$) in this system, though there is a logical possibility that one develops at a $\lambda_{'tHooft}$ of order one. (If so, it would have to disappear again for large $\lambda_{'tHooft}$ as discussed above. This result is a new prediction for the gravity description at small radius.)[15]

### 2.5 $Z_n \times Z_n$ Orbifolds with Discrete Torsion

In more general orbifolds than those we have considered so far, such as $Z_n \times Z_n$ orbifolds, one can project out the lowest-lying twisted-sector tachyons with a nontrivial choice of discrete torsion. However, at large radius, tachyonic modes dressed with angular momentum along the $S^5/\Gamma$ will survive this projection. It is interesting to consider whether this instability persists at small radius (weak 't Hooft coupling) in these theories.

In this section we will study in particular a $(C/Z_3) \times (C/Z_3)$ orbifold with nontrivial discrete torsion. Let the first $Z_3$, generated by $g_1$, act on the $z^1$ direction, and the second generated by $g_2$ on the $z^2$ direction, with the third complex plane parameterized by $z^3$ left invariant. The $g_1$ twisted sector vacuum energy is -1/3, as is that of the $g_2$ twisted sector, and in the absence of nontrivial discrete torsion these states correspond to physical tachyons in spacetime. A nontrivial choice of discrete torsion projects out each of these vacua ($g_1$ projecting out the vacuum in the $g_2$ twisted sector and vice versa). However, there are momentum states invariant under the both $g_1$ and $g_2$ which still have tachyonic masses $m^2 < 0$ in spacetime.

Naive intuition might suggest that these momentum-mode tachyons may get lifted as we go toward small radius since the momentum contribution to the $m^2$ of the state grows as the radius shrinks. Again naive intuition is liable to fail in these highly curved Ramond backgrounds, and as in the previous examples, the QFT instability analysis is the appropriate method for answering this question at small radius given the limitations of current technology on the gravity side.

---

[15] Very naive gravity-side intuition might have suggested that a tachyon would arise at small radius, since the positive mass squared of twisted states at large radius is driven by their winding energy around the $S^5/\Gamma$, and when the $S^5/\Gamma$ becomes small the winding energy would appear to be negligible. However, the substringy dynamics of curved Ramond backgrounds is hardly a place where naive intuition applies. Our QFT analysis of the lack of instability in this system at small radius is a prediction for the worldsheet RR sigma model.

The worldvolume theory of D-branes on orbifolds with discrete torsion was worked out in [39] (and studied in the context of AdS/CFT in [40]). The result for our case in particular is as follows. The theory is an $SU(3N)$ gauge theory with three complex scalars $Z^{1,2,3}$ and four Weyl spinors $\chi^{1,\ldots,4}$ in the adjoint, i.e. a theory with the field content of $\mathcal{N}=4$ $SU(3N)$ SYM. The interactions, however, differ from those of the $\mathcal{N}=4$ theory. The quartic scalar couplings involving $Z^1$ and $Z^2$ are deformed from the usual commutators to take the form:

$$\begin{aligned}\mathcal{L}_{scalar} = tr&\left[(Z^1Z^2-\alpha Z^2Z^1)(Z_{\bar{2}}Z_{\bar{1}}-\alpha^{-1}Z_{\bar{1}}Z_{\bar{2}})\right]\\ tr&\left([Z^1,Z^3][Z^1,Z^3]^\dagger\right)+tr\left([Z^2,Z^3][Z^2,Z^3]^\dagger\right)\end{aligned} \tag{2.5.1}$$

where $\alpha=e^{2\pi i/3}$. Similarly the Yukawa couplings are deformed to the form

$$\begin{aligned}&\alpha^{-1}\Gamma_1^{\alpha\beta}tr\left[\chi^\alpha(Z^1\chi^\beta-\alpha^{-1}\chi^\beta Z^1)\right]+\alpha\Gamma_2^{\alpha\beta}tr\left[\chi^\alpha(Z^2\chi^\beta-\alpha\chi^\beta Z^2)\right]\\ &+\Gamma_3^{\alpha\beta}tr\left(\chi^\alpha[Z^3,\chi^\beta]\right)\end{aligned} \tag{2.5.2}$$

The Coulomb branch describing fractional branes is parameterized by diagonal $Z^3$ matrices. We can now immediately observe a difference between this case and the cases discussed in §2. Namely, the one-loop Coleman-Weinberg potential will be absent here, since all the tree-level vertices involving $Z^3$ are exactly the same as in an $SU(3N)$ $\mathcal{N}=4$ theory. On the other hand, higher-loop contributions to the effective potential of the gauge theory mix $Z^3$ with all the other fields, and we expect such contributions will get generated. It would be interesting to explore their signs to see if an instability exists in this case at higher orders in $\lambda_{'tHooft}$.

## 2.6 Discussion and Future Directions

In this chapter we have identified global instabilities in certain weakly-coupled non-supersymmetric gauge theories whose AdS/CFT duals contain twisted-sector tachyons at large radius. These instabilites, which induce VEVs for twisted field theory operators, appear at one-loop in the gauge theory and genus zero in the string theory, though their effect on untwisted operators is suppressed by factors of $(1/N^2)$, as expected from large-N inheritance.

At higher orders in $1/N^2$ there will be a rich set of dynamically generated contributions to the effective action which are not constrained by large-N inheritance from the parent $\mathcal{N} = 4$ theory. This can (and presumably does) include quadratically divergent scalar masses (as well as quartically divergent vacuum energy which does not affect the QFT dynamics). It would be interesting to calculate these effects and understand their description on the gravity side of the correspondence. These finite-N effects can have a dramatic effect on the matter content and dynamics, and it is necessary to calculate these in order to understand the finite-N system. Some interesting perturbative calculations in these theories were done for example in [41]. While we feel such an analysis is further motivated by our work here, it is somewhat subtle to carry out since the QFT couplings appropriate to the gravity dual may themselves be shifted from the orbifold values by contributions of order $1/N^2$.[16]

As discussed in §2, it would be very interesting to ascertain the behavior of the effective potential near the origin of the Coulomb branch. It would also be interesting to see whether a higher-loop analysis leads to persistant instability at large $\langle X \rangle$, and to study the meaning of (and possible constrints on) the quiver theory renormalization conditions from the gravitational dual. These last issues mirror the difficulty on the gravity side of determining the form of the tachyon potential when the tachyon VEV is large and mixes strongly with other string fields.

One important generalization to consider is a case where quark flavors survive the tachyon condensation process, so that we get more than just the pure glue QCD theory. Recall that in our tachyon condensation process in §3 the fermions decoupled and/or became massive as the twisted operator's VEV turned on. A case where flavors survive may well involve a second set of D-branes in addition to the D3-branes contributing the $SU(N)$ gauge group, so that the gravity side has open strings as well as closed strings. It would be interesting to perform a general analysis of symmetry breaking patterns for the $Z_n$ orbifolds considered here and more general ones; the configurations we discussed in §3.4 are particularly simple and there may be a rich set of interesting possibilities implicit in the potentials (2.2.2)(2.2.15)(2.2.16).

It would be interesting to explore the tachyon potential in closed string field theory in this system, and compare the Coleman-Weinberg potential to the closed-string analogues of formulas in [27] for the tachyon potential in open string field

[16] We thank M. Strassler for reminding us of this difficulty.

theory. This is out of range of current technology, and our QFT calculations are simply predictions for the behavior of the appropriate gravity side sigma model.

It would also be interesting to study the tachyon potential on the gravity side at large radius, to see what happens to the $S^5/\Gamma$ and to the RR fields upon tachyon condensation in that regime. We may be able to get a handle on this by studying the geometry of fractional D-branes splitting apart, via nonsuper-gravity at large radius. It would also be very interesting to explore potential relations to the work of [30] and [29]. With respect to the latter, one would need to repeat our analysis of D3-branes in the Type 0B theory for the (nonconformal) even-dimensional D-branes of the Type 0A theory, where the conjectured dualities and instabilities of [29] might apply most directly.

In general, it is important to improve our understanding of the gravity side of the duality (and the duality map) in order to resolve the puzzle of the violation of conformal invariance on the field theory side discussed at length in §3.2.

Finally, it would be interesting to study further examples of tachyonic and non-tachyonic non-supersymmetric AdS/CFT duals, to see how general the pattern found here of large radius instabilities persisting to small radius proves. We covered a large class of examples in our analysis here, but there are many more cases that could be considered.

# 3. Don't Panic! Closed String Tachyons in ALE Spacetimes

## 3.1 Motivation and Outline

An understanding of the vacuum structure of String/M theory after supersymmetry breaking is crucial for phenomenology and cosmology. It is also relevant to the question of unification; it is important to understand the extent and nature of connections between different vacua in the theory. A basic issue is the fate of theories that have tachyons in their tree-level spectra. This has long been a source of puzzlement, but for open strings there has been a great deal of progress.[17] Open string tachyons generally have an interpretation in terms of D-brane annihilation, binding, or decay, and a quantitative description of these processes has been achieved by an assortment of methods from conformal field theory, string field theory, and noncommutative geometry. This has also led to a deeper understanding of the role of K theory, and the reanimation of open string field theory.

For closed string tachyons the understanding is much more rudimentary. These should be connected with the decay of spacetime itself, rather than of branes in embedded in a fixed spacetime. In this chapter we study a class of tachyonic closed string theories in which the decay can be followed with reasonable confidence. The key feature of these theories is that the bulk of spacetime is stable, and the tachyons live only on a submanifold. Thus they are similar to the tachyonic open string theories, and we will note certain close parallels, though in the absence of closed string field theory we will not be able to achieve as complete a quantitative control.

---



[17] A small sampling of references on this subject is [42][43][44][45][46][47][48][49][50][51].

The theories we study are noncompact, nonsupersymmetric orbifolds [52][53] of ten-dimensional superstring theories [54][55][56][57][58]. The simplest case is to identify two dimensions under a rotation by $2\pi/n$, forming a cone with deficit angle $2\pi - 2\pi/n$ ($n$ must be odd, for reasons to be explained in §2). This is the simplest example of an Asymptotically Locally Euclidean (ALE) space, which is defined generally as any space whose geometry at long distance is of the form $\mathbb{R}^k/\Gamma$, with $\Gamma$ some subgroup of the rotation group. The tip of the cone, which is singular, is a seven-dimensional submanifold. The rotation leaves no spinor invariant and so supersymmetry is completely broken, and there are tachyons in the twisted sector of the orbifold theory. Where do these tachyons take us?

There are several plausible guesses, based on experience in other systems:
(I) A hole might appear at the tip, and then expand to consume spacetime. Such a reduction in degrees of freedom is naively suggested by the relevance of the tachyon vertex operator at zero momentum [59], and by the presence of a nonperturbative Kaluza-Klein instability in certain backgrounds [60], and has been argued to be the fate of other tachyonic closed string theories [61][62][63][64][65].
(II) The tip might begin to elongate, asymptotically approaching the infinite throat geometry that is often found in singular conformal field theories [66].
(III) The tip might smooth out, by analogy to the effect of the marginal twisted sector perturbations in supersymmetric orbifolds. This smoothing might stop at the string scale, or continue indefinitely.

We will argue that it is the last of these that occurs, as was also suggested recently in [58]. At late times, when a general relativistic analysis is valid, an expanding dilaton pulse travels outward with the speed of light. This is depicted in figure 1. The region interior to the pulse is flat, with vanishing deficit angle. The energy contained within the pulse produces the jump to the asymptotic deficit angle of $2\pi - 2\pi/n$ [67]. More generally, by following special directions in the space of tachyons, the decay can take place in a series of steps, where for example $\mathbb{C}/\mathbb{Z}_{2l+1}$ decays via a dilaton pulse to $\mathbb{C}/\mathbb{Z}_{2l-1}$, or to any $\mathbb{C}/\mathbb{Z}_{2l'+1}$ orbifold with $l' < l$.

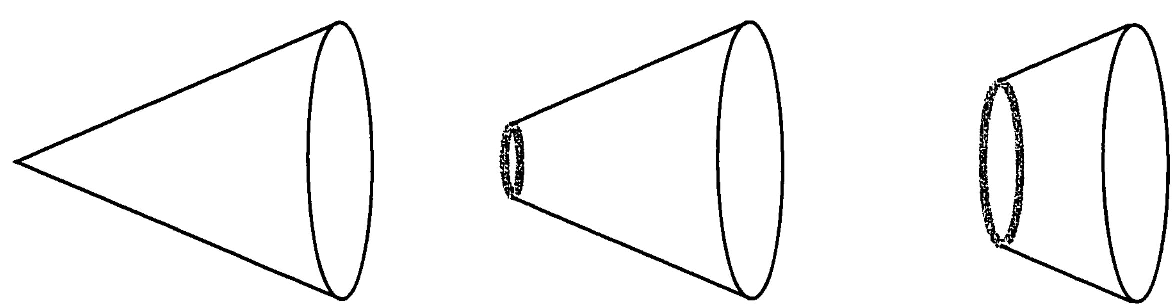

Figure 1: Decay of the conic singularity. The end of the cone is replaced by a flat base. The outward-moving dilaton pulse is shown in gray.

We will analyze this process in two complementary regimes. When the tachyon expectation value is small and so the smoothed region small compared to the string scale, we use D-brane probes [68], whose world-volume theory is a quiver gauge theory [69]. This is the *substring regime.* D-branes on supersymmetric orbifolds have been studied extensively; we extend these techniques to study non-supersymmetric orbifold compactifications with closed string tachyons. The probes see a smoothed geometry when the tachyon is nonzero. When the smoothing region exceeds the string length scale, we can instead use a general relativistic description, and we argue that the solution has the form in figure 1. This is the *gravity regime.* Because of $\alpha'$ corrections, we do not have a controlled approximation that connects the two regimes, but we argue that together they give a simple and consistent picture of a transition from a conic singularity to flat space via tachyon condensation. These same complementary descriptions have been applied to type I instantons and to supersymmetric ALE spaces [69].

If one or more of these singularities is part of a compact manifold, then the initial stages of tachyon condensation will be the same near each orbifold fixed point, producing a smooth compact geometry. Unlike the noncompact system, which evolves forever, we will argue that the compact space collapses toward a Big Crunch in finite time.

If we put $N$ D3-branes on the fixed plane, and consider the near horizon limit, then the system is expected to be dual to a nonsupersymmetric gauge theory [70]. The fixed plane is partly transverse to the D3-brane, and in the large-radius limit of the $AdS_5 \times S^5/\mathbb{Z}_n$ background we find a dramatic instability that grows toward the boundary of AdS. We argue that at large 't Hooft coupling these nonsupersymmetric field theories are unphysical. This contrasts with the more benign infrared Coleman-Weinberg instabilities evident on the field theory side at weak 't Hooft coupling (small radius), which arise in theories whose gravity duals have tachyons at large radius [57][71].

The analysis can also be applied to orbifolds the form $\mathbb{C}^q/\Gamma$, where $\Gamma$ is a discrete subgroup of the rotation group that fixes a point in $\mathbb{C}^q$. If $\Gamma \not\subset SU(q)$ then the background does not preserve supersymmetry, and there are twisted-sector tachyons localized at the fixed point. For $q = 1$ there is no $\Gamma$ that preserves supersymmetry, but for $q \geq 2$ there will be.

We study several $\mathbb{C}^2/\mathbb{Z}_n$ examples, where analysis of the quiver theories on D-brane probes leads to predictions for transitions between different orbifolds. There is a new effect that can occur in this case: we exhibit infinite sequences of examples with transitions from *nonsupersymmetric* ALE spaces to *supersymmetric* ones. We again expect a gravitational background for large tachyon VEV that involves an expanding shell of dilaton gradient, combined with metric curvature. In this case the total energy of the transition region must vanish, since both the initial and final ALE spaces have vanishing energy as measured by the falloff of the gravitational fields at infinity. (We will explain why this is not inconsistent with positive energy theorems.)

For large $n$ in $\Gamma = \mathbb{Z}_n$, an angular direction is small for a significant range of radii and it is useful to go to a $T$-dual picture involving NS5-branes [72]. These transitions therefore provide a closed string analogue of the open string brane descent relations [73], in that we can realize for example any $A_k$ space (and therefore the equivalent dual system of $k+1$ NS5-branes) via tachyon condensation (and/or marginal deformation) from a non-supersymmetric ALE space. Also, by adding an R-R flux, which has little effect on the geometry, we obtain a system which has a conjectured dual description in terms of fluxbranes [74][75][64][65][76][77].

This realization of supersymmetric ALE spaces (and therefore NS branes) by closed-string tachyon condensation is very reminiscent of similar constructions in open string theory [51]. In this regard, we should emphasize that certain puzzles that arise in the open string case arise here as well [58].[18] In particular, in open string tachyon condensation, one finds gauge fields without sufficient perturbative charged matter to Higgs them [78][79], but open string field theory calculations at disk order suggest that they are nonetheless lifted classically [51][80][81]. In the tachyonic closed string systems we study here, there are twisted RR gauge potentials without perturbative charged matter; our evidence suggests that the defect is nonetheless smoothed and the RR potential lifted. In both the open and closed string cases, it would be very interesting to understand the classical stringy

[18] We thank M. Berkooz, P. Kraus, E. Martinec, and other participants of the Amsterdam Summer workshop for discussions on this.

effect, evidently going beyond ordinary effective quantum field theory, which allows gauge fields to be so lifted. In the closed string case, it would be interesting to understand an analogue of the quantum confinement effect identified in the open string case in [82]; here as there one has D-branes charged under the gauge group of interest (in our case these are the fractional D-branes, whose condensation might lead to confinement of twisted strings into untwisted strings).

The organization of this chapter is as follows. In §2 we discuss the $\mathbb{C}/\mathbb{Z}_n$ orbifold, including the twisted sector spectrum and the quantum symmetry group of the orbifold theory. We also discuss the difference between orbifolds and ALE spaces that do not have orbifold descriptions. We consider D-brane probes in the orbifold theory, deriving the quiver representation. In §3 we analyze the quiver theory/linear sigma model for the $\mathbb{C}/\mathbb{Z}_n$ orbifolds and their twisted deformations, discussing both generic decays and decays that leave lower-order orbifolds. In §4 we develop the general relativistic description of these same solutions. We discuss renormalization group evolution and time evolution. These are similar, in that both lead to a smoothed region that grows without bound, but there are differences in the details. We discuss the consistency between the renormalization group analysis and the $c$-theorem. We then discuss the fate of compact spaces with nonsupersymmetric orbifold points, and the consequences of our results for AdS/CFT duality. In §5, we analyze transitions in the $\mathbb{C}^2/\Gamma$ case by means of the quiver theories, and exhibit decays from non-SUSY to SUSY ALE spaces. In the general relativistic regime we explain how our results are consistent with positive energy theorems. Finally, in §6 we discuss dual systems, including fluxbranes, and in §7 mention some directions for further research.

## 3.2 The $\mathbb{C}/\mathbb{Z}_n$ Orbifold

### *3.2.1 Closed String Spectrum*

Let us start by reviewing some of the basic properties of the $\mathbb{C}/\mathbb{Z}_n$ orbifold conformal field theory [54][55][56]. These orbifolds are defined by identifying the 8-9 plane under a rotation $R$ through $2\pi/n$. This allows two possible actions on the spinors,

$$R = \exp(2\pi i J_{89}/n) \quad \text{or} \quad \exp(2\pi i J_{89}) \exp(2\pi i J_{89}/n) \ , \tag{3.2.1}$$

where $J_{89}$ is the rotation generator. For either choice, $R^n$ acts trivially on spacetime and so is either 1 or $\exp(2\pi i J_{89}) = (-1)^{\mathbf{F}}$. If $R^n = (-1)^{\mathbf{F}}$, the orbifold group (which is actually $\mathbb{Z}_{2n}$ in this case) includes this operator and so projects out spacetime fermions and introduces tachyons in the bulk. Because we want to have all tachyons localized at the fixed point we must have $R^n = 1$. For the two choices (3.2.1) one finds

$$R^n = (-1)^{\mathbf{F}} \quad \text{or} \quad (-1)^{(n+1)\mathbf{F}} \; . \tag{3.2.2}$$

Thus, only the second choice of $R$ is acceptable, and only for $n$ odd:

$$R = \exp\left(2\pi i \frac{n+1}{n} J_{89}\right) \; , \quad n = 2l + 1 \; . \tag{3.2.3}$$

In the sector twisted by $R^k$ $(1 \leq k \leq n-1)$, in the light-cone Green-Schwarz description there are six real untwisted scalars, one complex scalar twisted by $k/n$, and four complex fermions twisted by $k/2 + k/2n$. The standard calculation of the zero-point energy gives

$$\frac{\alpha'}{4} m^2 = \begin{cases} -k/2n \; , \; k \text{ even} \; , \\ (k-n)/2n \; , \; k \text{ odd} \; . \end{cases} \tag{3.2.4}$$

Thus the lowest state is tachyonic in every twisted sector. There are also excited state tachyons in many sectors. For example, when $k = 1$ the lowest twisted scalar excitation takes $(1-n)/2n$ to $(3-n)/2n$ and so this state is tachyonic for $n > 3$. Our analysis will be rather coarse, and so we will generally not distinguish the ground state in each sector from excited states with the same symmetries.

We wish to ask, where do these tachyonic perturbations take the system? Since they are in the twisted sectors, their initial effect is in the neighborhood of the fixed point. There are two contexts to consider. First, we could add the tachyonic vertex operators to the Hamiltonian. Tachyonic states correspond to relevant vertex operators, so they change the IR behavior of the world-sheet theory. We are then interested in determining the renormalization group (RG) flow. Second, we could consider a time-dependent string solution that begins as a small but exponentially growing tachyonic perturbation of the orbifold. We are then interested in the subsequent time evolution.

Physically these are distinct questions. The first is an off-shell question from the point of view of string theory, but well posed as question in two-dimensional quantum field theory. The second is an on-shell question in string theory. In fact we will find, as has been seen in other contexts, that the scale and time evolutions are similar. In both cases the question can be posed in the classical string limit, with no string loop effects. If the world-sheet theory were to become singular, for example if the dilaton were to become large, then this framework would break down, but we will find that at least generically this does not happen.

The orbifold preserves an $SO(7,1) \times U(1)$ subgroup of the parent $SO(9,1)$. In addition there is a new "quantum" symmetry that appears in the orbifolded theory [83]: the twist is conserved, mod $n$.[19] The lowest tachyons in general break the quantum symmetry completely but leave the $SO(7,1) \times U(1)$ unbroken (in the RG case) or break it to $SO(7) \times U(1)$ (in the time-dependent case). In some cases we will consider perturbations that leave part of the quantum symmetry unbroken, while in others we will find that a new quantum symmetry, unrelated to the original one, emerges asymptotically.

Actually, the evolution is more restricted than would follow from spacetime symmetry alone. The $X^M$ and $\psi^M$ (of the RNS description) upon which the $SO(7,1)$ or $SO(7)$ acts do not appear in the perturbation. These fields therefore remain free, whereas the symmetry would allow a warp factor depending on the other coordinates.

We will consider processes where a $\mathbb{Z}_{2l+1}$ singularity emits a radiation pulse with just the appropriate energy to leave behind a $\mathbb{Z}_{2l-1}$ singularity. We could also imagine the time-reversed process, sending in a pulse with the appropriate energy.

This raises an interesting issue. We have found that there are no $\mathbb{Z}_{2l}$ orbifolds with supersymmetry broken only locally at the tip of the cone, but what if we consider a solution of type II string theory which describes a pulse sent inward with just the right energy to create such a singularity? When the pulse reaches the origin, the geometry is a cone with deficit angle $2\pi - 2\pi/2l$. The difference between this case and the time reversal of our orbifold decay process is that here there is no simple description of the singularity. Away from the singularity, the lines $\theta = 0$ and $\theta = 2\pi/2l$ are identified under a rotation $\exp(2\pi i J_{89}/2l)$ or $\exp(2\pi i J_{89})\exp(2\pi i J_{89}/2l)$. This

[19] This is not the same spacetime $\mathbb{Z}_n$ group used to construct the orbifold. All states are invariant by definition under that symmetry, while the quantum $\mathbb{Z}_n$ is carried by the twisted sector states.

is a sensible configuration, and the dynamical process allows one to reach it. However, on the $2l$-fold covering space, the lines $\theta = 0$ and $\theta = 2\pi$ are identified under the action of $(-1)^{\mathbf{F}}$, and so there is a branch cut in the spinor fields. This is the essential difference from the orbifold: in the orbifold the untwisted fields are single-valued on the covering space. We could similarly consider a wedge of any opening angle $\theta_0$, where the plane is generically not a covering space. Again, dynamically we could construct a state that has this behavior away from the singularity, but that within a string distance of the singularity has some complicated description, not based on a free CFT, if the singularity is resolved at all. Indeed, we will find many example of orbifolds decaying to such spaces; we will use the terms 'quasi-orbifold' or 'quasi-ALE (QALE)' to refer to these more general spaces that are locally Euclidean but are not obtained as orbifolds of a single-valued theory on Euclidean space.

### *3.2.2 Open String Spectrum*

We now consider a D$p$-brane probe of the geometry. Here as in many other contexts, D-brane probes [69] and closely related linear sigma model techniques [84] are useful for obtaining a broader view of the space of closed string backgrounds than is available from perturbation theory about a specific world-sheet CFT. In studying a D-brane probe, the low energy quantum field theory on its world-volume is only valid in the substring regime, where the VEVs of world-volume scalars (scaled to have dimensions of length) are sufficiently small compared to the string length $\sqrt{\alpha'}$. We will also study the D-brane probes in the *classical* limit, and in doing so will self-consistently find results consistent with the string coupling remaining bounded throughout the tachyon decay process. It would be an interesting, but distinct, question to relax the $g_s \to 0$ limit we consider here and analyze the quantum dynamics on D-brane probes in these backgrounds, a question that could be considered both before and after the tachyons condense.

The classical world-volume theories of D-branes probing orbifold singularities were worked out in a beautiful paper by Douglas and Moore [69]. The orbifold group $\Gamma$ has both a geometric action $\hat{R}$ and an action $\gamma_R$ on the Chan-Paton indices,

$$|\psi, i, j\rangle \;\to\; \gamma_{R\,ii'}|\hat{R}\psi, i', j'\rangle\gamma^{-1}_{R\,j'j}\;. \tag{3.2.5}$$

For branes that are free to move away from the orbifold singularity, there must be a distinct image for each element of $\Gamma$ and so the Chan-Paton indices transform in the regular representation. These branes have integer tensions and charges. Irregular representations correspond to fractional branes bound to the fixed locus. We will be interested in the regular case; as we have noted in the introduction, fractional branes are confined once the singularity is resolved, but the full mechanism is not understood.

We will consider a D$p$-brane probe that is extended in the directions $\mu = 0, 1, \ldots, p$ and localized in the directions $m = p+1, \ldots, 7$ and in the orbifolded 8-9 plane. The treatment will be uniform for the IIA or IIB theories, and for all $p$ in the respective theories. We take a single copy of the regular representation, but the discussion readily extends to $N$ copies. For $\Gamma = \mathbb{Z}_n$, $R$ cyclically permutes the D-brane images and so $\gamma_{R\,jk} = \delta_{j+1,k}$. The indices $j, k$ are understood to be defined mod $n$, so in particular $\gamma_{R\,n1} = 1$. It is more convenient to work in a basis in which the spacetime action is not so evident but the spectrum and its quiver representation are simple:

$$\gamma_{R\,jk} = e^{2\pi i j/n} \delta_{jk} \ . \tag{3.2.6}$$

The low energy theory is itself an "orbifold" of the $\mathcal{N} = 4$ world-volume theory of a D-brane in flat space, obtained by projecting out gauge theory fields that are not invariant under the action (3.2.5). The massless open string fields are the vector potential $A_{\mu jk}$, the collective coordinates $X^m_{jk}$ and $Z_{jk} = (X^8 + iX^9)_{jk}$, and the spinor $\xi_{jk}$ in the $\mathbf{8}$ of $SO(7,1)$ and with $J_{89} = +\frac{1}{2}$. The real and imaginary parts of $\xi$ form the $\mathbf{16}$ of $SO(9,1)$; we will suppress the $SO(7,1)$ spinor index. The orbifold projection (3.2.5) on the operation (3.2.3), (3.2.6) retains fields with $j - k + (n+1)J_{89} = 0$. The surviving fields are then

$$A_{\mu jj} \ , \quad X^m_{jj} \ , \quad Z_{j,j+1} \ , \quad \xi_{j,j-l} \ , \tag{3.2.7}$$

where $j$ runs from 1 to $n = 2l+1$ and indices are defined mod $n$. The conjugates are $\bar{Z}_{j+1,j}$ and $\bar{\xi}_{j,j+l}$.

Thus the gauge group is $U(1)^n$, with the collective coordinate $Z_{j,j+1}$ having charge $+1$ under $U(1)_j$ and charge $-1$ under $U(1)_{j+1}$, while $\xi_{j,j-l}$ has charge $+1$ under $U(1)_j$ and charge $-1$ under $U(1)_{j-l}$. The spectrum can be succinctly expressed through "quiver" diagrams [69]. For each factor in the product gauge group, the diagram has a node; for $\Gamma = \mathbb{Z}_n$ these are in one-to-one correspondence with the range of the Chan-Paton indices. A field with charge +1 under $U(1)_j$ and charge $-1$ under $U(1)_k$ is denoted by an arrow from node $k$ to node $j$. For more general representations of $\Gamma$ the gauge group is a product $\prod_j U(N_j)$ and the arrows represent bifundamentals. Arrows beginning and ending on the same node are adjoints, which of course are neutral in the case of $U(1)^n$. For the example $\Gamma = \mathbb{Z}_5$, figure 2 shows the separate quiver diagrams for the various fields.

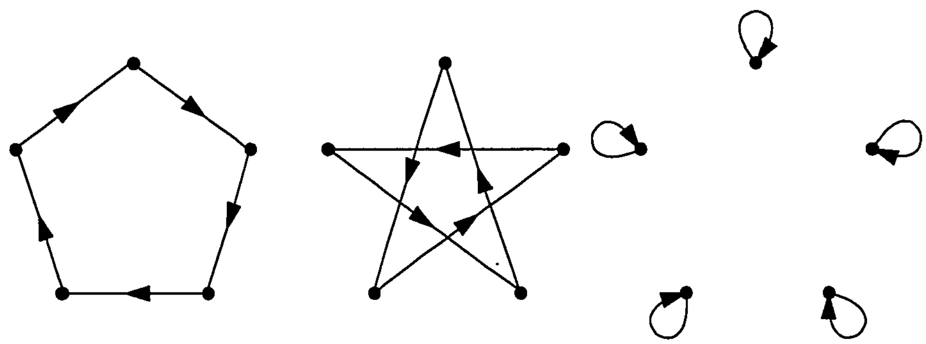

**Fig. 1:** Figure 2: Quiver diagrams for the $\mathbb{C}/\mathbb{Z}_5$ orbifold: for $Z$, for $\xi$, and for $A_\mu$ and $X^m$.

Note that the quiver theory spectrum is invariant under cyclic permutation of the gauge groups; call this symmetry $\Gamma_Q$. All gauge invariant operators inherited from the parent theory, such as $\sum_j F^2_{\mu\nu jj}$, are scalars under $\Gamma_Q$. Gauge invariant operators not descending from gauge invariant operators in the parent theory, such as $F^2_{\mu\nu 11}$, are not $\Gamma_Q$ scalars. Since the lagrangian itself descends from a gauge invariant operator in the parent theory, it is a scalar, and so $\Gamma_Q$ is truly a symmetry of the system. This symmetry is just the realization in the quiver theory of the orbifold quantum symmetry. In particular, bulk twisted modes couple to gauge-twisted operators (*i.e.* not $\Gamma_Q$ scalars) on the brane only in $\Gamma_Q$ invariant combinations — a very useful fact in fleshing out the AdS/CFT dictionary, for example. We will see that quiver diagrams are a very effective tool for following the behavior of the probe theory as the singularity decays.

The potential for the scalars is classically, at the orbifold point,

$$V = \frac{1}{2}\sum_{j,m}(X^m_{jj} - X^m_{j+1,j+1})^2|Z_{j,j+1}|^2 + \frac{1}{2}\sum_j\Big(|Z_{j,j+1}|^2 - |Z_{j-1,j}|^2\Big)^2 , \qquad (3.2.8)$$

where the overall normalization will not be important. We are interested in the Higgs branch, where $X^m_{jj}$ is independent of $j$ and the $Z_{j,j+1}$ are nonzero. This corresponds to a D-brane probe of the orbifold geometry. On the Coulomb branch the $X^m_{jj}$ depend on $j$ and the $Z_{j,j+1}$ vanish. This branch corresponds to the probe separating into fractional D-branes trapped at the singularity, and it disappears in the deformed geometry. On the Higgs branch there is a Yukawa interaction

$$L_{\mathrm{Y}} = \sum_j \xi_{j,j-l}\xi_{j-l,j+1}\bar{Z}_{j+1,j} \ . \tag{3.2.9}$$

Note that each interaction forms a closed loop on the quiver diagram.

We now consider the geometry of the Higgs branch. The vanishing of the potential (3.2.8) implies that the magnitude $|Z_{j,j+1}|$ is independent of $j$. Of the $n$ $U(1)$ symmetries, the diagonal decouples. The remaining $n-1$ gauge symmetries can be used to set the phases of the $Z_{j,j+1}$ equal as well, so that the common value $Z_{j,j+1} = Z$ parameterizes the branch. The branch is thus two-dimensional, as it should be for the interpretation of a probe. The gauge choice leaves unfixed a $\mathbb{Z}_n$ gauge symmetry, whose generator is

$$\exp\left(-\frac{2\pi i}{n}\sum_j jQ_j\right) \ . \tag{3.2.10}$$

This identifies $Z \to e^{2\pi i/n}Z$, so the probe moduli space is indeed the $\mathbb{C}/\mathbb{Z}_n$ spacetime. For each of the fields (3.2.7) there is one massless mode, where the field is independent of $j$. This is the correct spectrum for a D-brane probe.

The moduli space metric, as measured by the probe kinetic term, is obtained by integrating out the higgsed gauge fields. In a general gauge, the potential requires that on the moduli space $Z_{j,j+1} = re^{i\theta_j}$. The kinetic terms are then

$$\begin{aligned} L_{\mathrm{k}} &= \sum_{j=1}^{n}\Big|(\pi_\mu + iA_{\mu\,jj} - iA_{\mu\,j+1,j+1})Z_{j,j+1}\Big|^2 \\ &= \sum_{j=1}^{n}\Big[(\pi r)^2 + r^2(\pi_\mu\theta_j - B_{\mu j})^2\Big] \ . \end{aligned} \tag{3.2.11}$$

We have defined the relative gauge potentials, $B_{\mu j} = A_{\mu\,jj} - A_{\mu\,j+1,j+1}$, which tautologically satisfy the constraint $\sum B_{\mu j} = 0$. The total $U(1)$ is unbroken and decouples. Integrating out the broken gauge fields subject to the constraint gives

$$B_{\mu j} = \pi_\mu(\theta_j - \tilde{\theta}) \ , \quad \tilde{\theta} = \frac{1}{n}\sum_{k=1}^{n}\theta_k \ . \tag{3.2.12}$$

Inserting this into the kinetic term gives the manifestly gauge invariant result

$$L_{\rm k} = n\left[(\pi r)^2 + r^2(\pi\tilde{\theta})^2\right] . \tag{3.2.13}$$

As deduced above, the periodicity of $\tilde{\theta}$ is $2\pi/n$. Rescaling to $\theta = n\tilde{\theta}$, with canonical period $2\pi$, the kinetic term becomes

$$L_{\rm k} = n\,(\pi r)^2 + \frac{r^2}{n}(\pi\theta)^2 , \tag{3.2.14}$$

corresponding to the metric of a flat $\mathbb{Z}_n$ cone,

$$ds^2 = n\,dr^2 + \frac{r^2}{n}d\theta^2 , \tag{3.2.15}$$

as expected. For future reference note that we can define $\theta$ as

$$\theta = \arg(Z_{n1}Z_{12}\ldots Z_{n-1,n}) ; \tag{3.2.16}$$

the RHS is gauge invariant, so the period of $\theta$ is manifestly $2\pi$.

The gauge bosons that have been integrated out have masses of order $r/\alpha'$, while excited string states with masses of order $\alpha'^{-1/2}$ have been ignored. The result is therefore valid in the substringy regime [68], $r \ll \alpha'^{1/2}$. We have also ignored quantum corrections in the world-volume theory. This is valid because the world-volume fluctuations are open string fields, and we have taken $g_{\rm s} \to 0$ at the beginning — we have posed the problem in classical string theory.

There is a closely related context in which world-volume quantum corrections would be important. The world-volume theory of the D1-brane provides a linear sigma model construction analogous to those in [84] of the F-string orbifold CFT [85]. In this one must let the quantum world-volume theory flow to the IR fixed point. In the present case we know independently, from the orbifold construction, that the fixed point action is the free action (3.2.13).

## 3.3 Decay of $\mathbb{C}/\mathbb{Z}_n$ in the Substring Regime

### *3.3.1 Generic Tachyon VEVs: Breaking the Quantum Symmetry*

In the initial stage of the instability, the tachyon VEV is small and so the geometry is modified only in the substringy region near the tip of the cone. D-brane probes are therefore the effective tool for investigating the geometry. The closed string background determines the low energy quantum field theory on the probe. This can be obtained directly from a calculation of the disk amplitude with a tachyon vertex operator plus open string vertex operators, as in the appendix of ref. [69]. For our purposes, however, it will suffice to identify the world-volume theory by matching with the quantum numbers of the closed string tachyons.

¿From the discussion in §2.1, the tachyons generically break the quantum symmetry completely, so this will be broken in the world-volume theory. We are in the substringy regime, so we are interested in the leading effects in powers of $Z$. In the potential, this would be a mass term

$$\Delta V = \sum_{j=1}^{n} m_j^2 |Z_{j,j+1}|^2 \ . \tag{3.3.1}$$

A term of definite quantum charge $k$ would have a coefficient proportional to $e^{2\pi ijk/n}$. Since there are tachyons with all charges except for the untwisted $k = 0$, one obtains arbitrary masses subject to the constraint $\sum_j m_j^2 = 0$. It is then useful to reexpress the mass term as

$$\Delta V = -\sum_{j=1}^{n} \lambda_j \left( |Z_{j,j+1}|^2 - |Z_{j-1,j}|^2 \right) \ , \quad \sum_{j=1}^{n} \lambda_j = 0 \ . \tag{3.3.2}$$

The notation is suggested by the supersymmetric case, where $\lambda_j$ would be the Fayet-Iliopoulos (FI) coefficient for $U(1)_j$.

On the moduli space we now have

$$|Z_{j,j+1}|^2 - |Z_{j-1,j}|^2 = \lambda_j \ . \tag{3.3.3}$$

For generic $\lambda_j$ the $Z_{j,j+1}$ are therefore distinct, and one of these has magnitude less than the rest, say $Z_{12}$. When this vanishes the remaining $n-1$ $Z_{j,j+1}$ are still nonzero. It follows that $U(1)^n$ is broken to $U(1)$ everywhere on the moduli space, and so there is no orbifold point. The moduli space is smoothed; topologically it is $\mathbb{R}^2$. The gauge-invariant combination $Z_{n1}Z_{12}\ldots Z_{n-1,n}$, which now vanishes linearly when $Z_{12} \to 0$, is a good coordinate.

We can confirm these conclusions by finding the probe metric. Define $\rho_j$ iteratively,

$$\rho_j^2 = \rho_{j-1}^2 + \lambda_j \ , \quad \rho_1 = 0 \ . \tag{3.3.4}$$

Then with $Z_{j,j+1} = r_j e^{i\theta_j}$, eq. (3.3.3) implies

$$r_j^2 = r^2 + \rho_j^2 \ , \quad r \equiv r_1 \ . \tag{3.3.5}$$

The kinetic term is now

$$L_{\rm k} = \sum_{j=1}^{n} \Big[ (\pi r_j)^2 + r_j^2 (\pi_\mu \theta_j - B_{\mu j})^2 \Big] \ . \tag{3.3.6}$$

Enforcing the constraint $\sum_j B_{\mu j} = 0$ with a Lagrange multiplier $\lambda_\mu$, the equation of motion for $B_{\mu j}$ is $r_j^2(\pi_\mu \theta_j - B_{\mu j}) = \lambda_\mu$. Inserting this into the constraint determines the multiplier,

$$\lambda_\mu \sum_{j=1}^{n} \frac{1}{r_j^2} = \pi_\mu \theta \ , \tag{3.3.7}$$

where $\theta = \sum_j \theta_j$ is defined as in eq. (3.2.16) and so has period $2\pi$. The action then takes the simple form

$$L_{\rm k} = n(r)(\pi r)^2 + \frac{r^2}{n(r)} (\pi \theta)^2 \ , \tag{3.3.8}$$

where

$$n(r) = \sum_{j=1}^{n} \frac{r^2}{r^2 + \rho_j^2} \ . \tag{3.3.9}$$

The corresponding metric is

$$n(r) dr^2 + \frac{r^2}{n(r)} d\theta^2 \ ; \tag{3.3.10}$$

for constant $n(r)$ this is the metric (3.2.15)of a cone cone of deficit angle $2\pi/n$. The function $n(r)$ interpolates smoothly from $n(0) = 1$ (the term $j = 1$) to $n(\infty) = n$. Thus the metric (3.3.10) is nonsingular at the origin and connects smoothly onto the original $\mathbb{C}/\mathbb{Z}_n$ geometry asymptotically, as in figure 3.

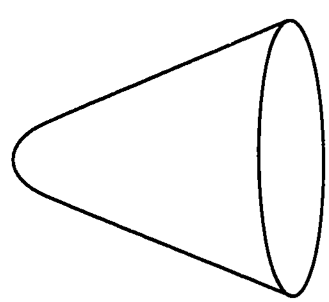

Figure 3: $\mathbb{C}/\mathbb{Z}_n$ singularity with a twisted tachyon VEV, as seen by a D-brane probe.

This smoothed geometry differs somewhat from what will eventually emerge in the gravity regime, depicted in figure 1. The base of the cone is rounded rather than flat. Also, the dilaton is constant: a nontrivial dilaton would lift the moduli space, through the dependence of the DBI term. We will see that in the gravity regime a dilaton must be present, so evidently this is a higher-order effect.

The exact physical meaning of the D-brane probe calculation here is a bit indirect. D-brane probes can observe substringy geometry only on times long compared to the string scale [68][86], while the decay process that we are probing takes place on the string time scale. There are at least two contexts where the calculation above has a precise meaning. First, we could consider a tachyon background which is constant in time and oscillatory in space, where the wavelength is then long compared to the substringy geometry. Second, at large $n$ some tachyon masses-squared are of order $1/n$. Even when neither of these contexts is relevant, we expect that the qualitative conclusion about the geometry is correct, and this is all that we will need.

Again, our analysis of the gauge theory is entirely classical. The non-supersymmetric gauge theories do not look unstable in this approximation at the orbifold point. The tachyon instability is a closed string tree effect and so a one-loop open string effect. In the context of AdS/CFT duality, we would expect to see this instability in the gauge theory; we will return to this point in §4.

Finally, note that the resolved geometry is topologically trivial. Thus, unlike the supersymmetric ALE singularity, there is no interpretation in terms of collapsing cycles at real codimension two. However, in §3.3, and in §5 where we consider the case of real codimension four orbifold singularities, we will see many parallels with the supersymmetric case.

### *3.3.2 World-sheet Linear Sigma Model*

As we noted above, the D1-brane gauge theory provides the starting point for a LSM representation of the F-string world-sheet theory. Let us digress slightly to explain the picture of the tachyon decay process which emerges from this point of view.

The LSM description involves considering a simple gauge theory in the UV which flows to the world-sheet CFT of interest in the IR [84]. In the context of quiver theories on D1-branes at orbifold points, the classical moduli space is the orbifold space (as we reviewed in §2.2), which is the target space for the F-string world-sheet CFT. Based on this and the discrete symmetries of the theory arising for appropriate choice of theta angles, it was argued in [85] that the D1-brane quiver theory provides a linear sigma model formulation of the orbifold CFT, with the caveat that without supersymmetry one must fine-tune away the quantum potential on the moduli space in order to reach the orbifold CFT in the IR (which then enjoys an accidental supersymmetry).

We are interested in the effect on the world-sheet CFT when the tachyon VEVs are turned on in spacetime, which means in terms of a renormalization group analysis that a relevant operator is added to the world-sheet CFT action (taking the tachyons at zero spacetime momentum). We would like to describe this deformation from the UV LSM quiver theory. As we have discussed, the tachyons transform under the quantum symmetry in the IR CFT, and this symmetry exists already in the UV quiver theory. Therefore we can identify twisted operators in the UV theory which will generically mix with the twisted-sector tachyon vertex operators in the IR. The twisted couplings of interest include the $\lambda_j$ in (3.3.2) above. These are the most relevant twisted deformations in the UV, and we will focus on their effects.

The RG flow diagram of this theory appears as in the following figure.

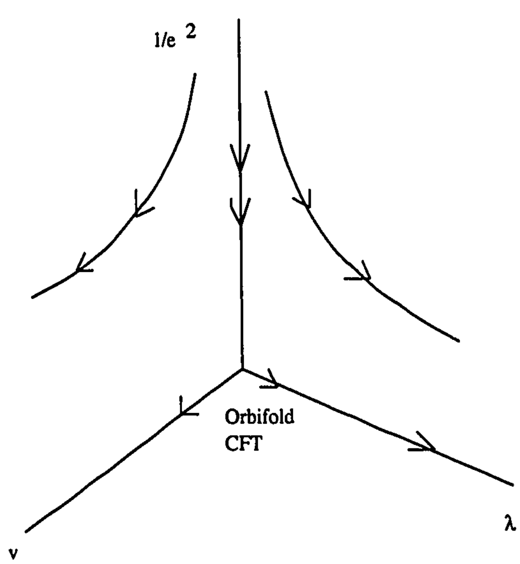


Figure 3.5: Flow diagram for the linear sigma model.

We consider flow toward the IR, keeping track of the indicated couplings $e$ (the gauge coupling) and $\lambda$, and on a third axis the relevant couplings $v$ in the scalar potential of the theory which drive the flow away from the desired IR world-sheet theory; these last we tune away as discussed in [85]. The flow proceeds toward stronger gauge coupling $e$. As we turn on $\lambda$, the vacuum manifold of the LSM smoothes out, as we discussed above. For large $\lambda$, integrating out the massive degrees of freedom in the LSM we obtain a nonlinear sigma model whose RG flow proceeds toward infinite flat space, as we will see in §4. For small $\lambda$, as we flow toward the IR we expect generically for $\lambda$ to mix with the tachyon vertex operators, which are relevant operators so that the flow proceeds away from the orbifold CFT fixed point.

Putting this together, the simplest joining of the two regimes leads again to a picture where the tachyon VEV induces flow from the orbifold CFT to smooth flat space.

### *3.3.3 Special Tachyon VEVs: Annealing the Quiver*

We have considered a generic tachyon VEV, which in the quiver theory breaks all $U(1)$s and resolves the singularity completely. It is interesting to consider instead partial resolutions of the singularity. Depending on the choice of twisted deformation we turn on, we will find that such resolutions can lead to quasi-orbifolds, which have no free world-sheet CFT description, or to real orbifolds, which do. We will start with an example of the former case and then proceed to the transitions between real orbifolds that are our main interest.

Consider for example the case that $\lambda_1 = -\lambda_2 > 0$, for which eq. (3.3.3) implies that one bifundamental is greater than the rest,

$$|Z_{12}|^2 = |Z_{j,j+1}|^2 + \lambda_1 \ , \quad j \neq 1 \ . \tag{3.3.11}$$

The maximum unbroken gauge symmetry is now $U(1)^{n-1}$, where all $Z_{j,j+1}$ other than $Z_{12}$ vanish, so we expect that the symmetry is partially resolved to $\mathbb{Z}_{n-1}$.

The theory near the fixed point can be elegantly described in terms of *annealed* quiver diagrams. As an explicit example, let us analyze the $\mathbb{C}/\mathbb{Z}_5$ orbifold, whose quiver diagrams were given in figure 2. Figure 4 shows the first step in the annealing. In the neighborhood of the fixed point, the bifundamental $Z_{12}$ has a relatively large

VEV and breaks $U(1)_1 \times U(1)_2$ to the diagonal $U(1)$. Thus, in the second line of figure 4 we have collapsed nodes 1 and 2.

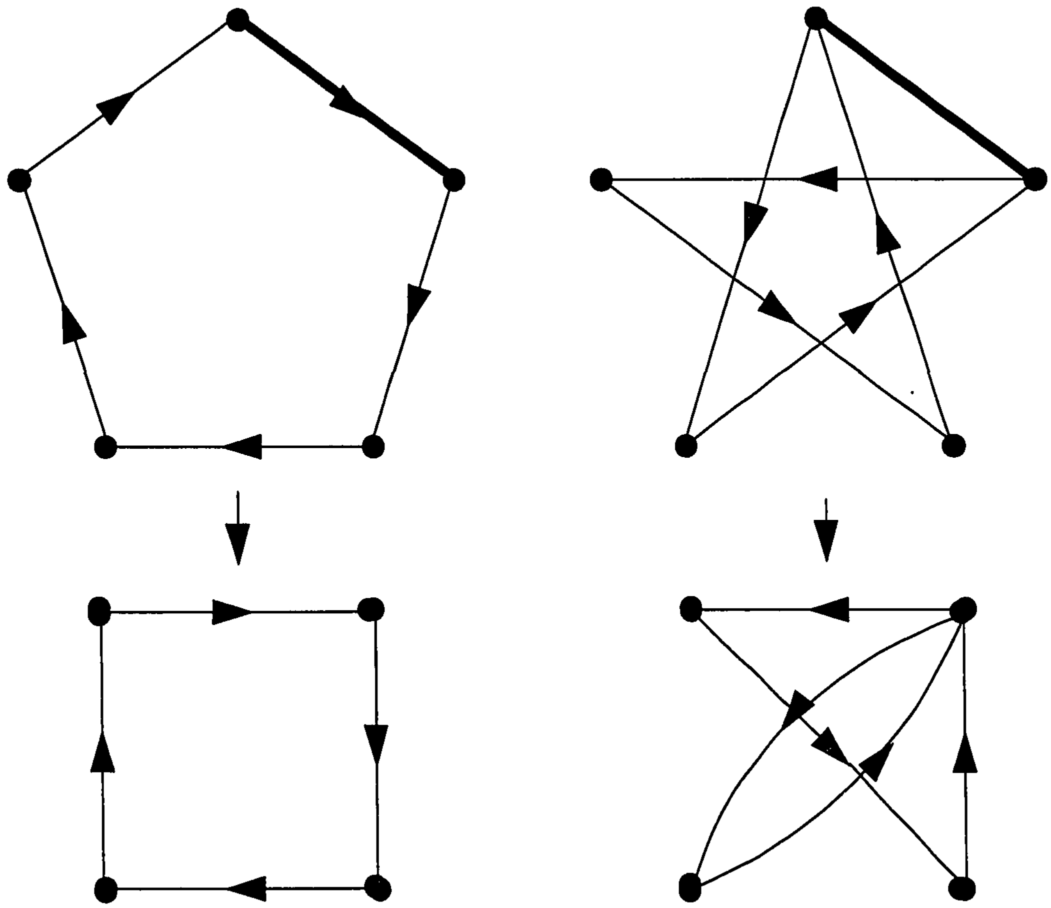

Figure 4: Partially annealed $\mathbb{C}/\mathbb{Z}_5$ scalar and fermion quivers. The scalar $Z_{12}$ is indicated in bold. In the low energy theory the nodes 1 and 2 are identified.

The scalar $Z_{12}$ decouples from the low energy theory, its magnitude fixed by the potential and its phase absorbed by higgsing; thus it is omitted from the annealed diagram. The adjoint scalar $X^m_{11} - X^m_{22}$ accompanying the broken $U(1)$ is also lifted by the potential. Finally, the mass term $\xi_{14}\xi_{42}\overline{Z}_{21}$ removes two fermions, so the final quiver diagram is shown in figure 5.

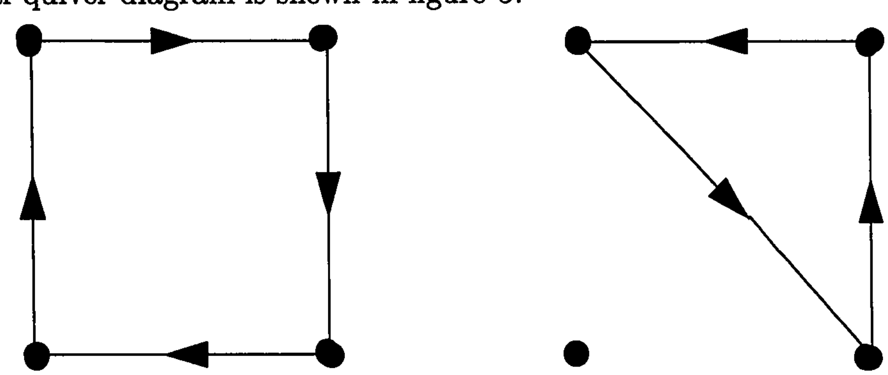

Figure 5: Final annealed $\mathbb{C}/\mathbb{Z}_5$ scalar and fermion quivers.

The scalar spectrum is the same as for a $\mathbb{C}/\mathbb{Z}_4$ orbifold in bosonic string theory, and the metric (3.3.9) seen by a D-brane probe has a $\mathbb{Z}_4$ singularity. The geometry is as in figure 6, with a $\mathbb{Z}_4$ singularity in a space whose asymptotic geometry is $\mathbb{C}/\mathbb{Z}_5$.

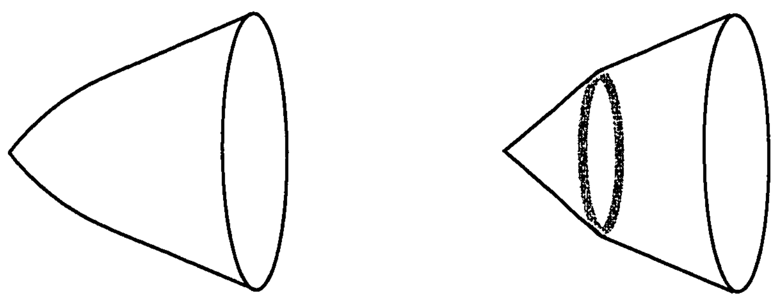

Figure 6: Asymptotic $\mathbb{C}/\mathbb{Z}_n$ geometry with a $\mathbb{C}/\mathbb{Z}_{n'}$ singularity, with $n' < n$, as seen in the substringy and gravity regimes.

The fermion spectrum is not of quiver form. This is not surprising, as we know that there is no orbifold construction of the supersymmetric type II string on the $\mathbb{C}/\mathbb{Z}_4$ singularity. Rather, this must be a quasi-orbifold, not based on a free CFT, as discussed in §2. However, by turning on additional Fayet-Iliopoulos terms, and so a second scalar VEV, we can flow to the $\mathbb{Z}_3$ quiver as shown in figure 7; it is easy to check that the Yukawa terms lift no additional fermions.

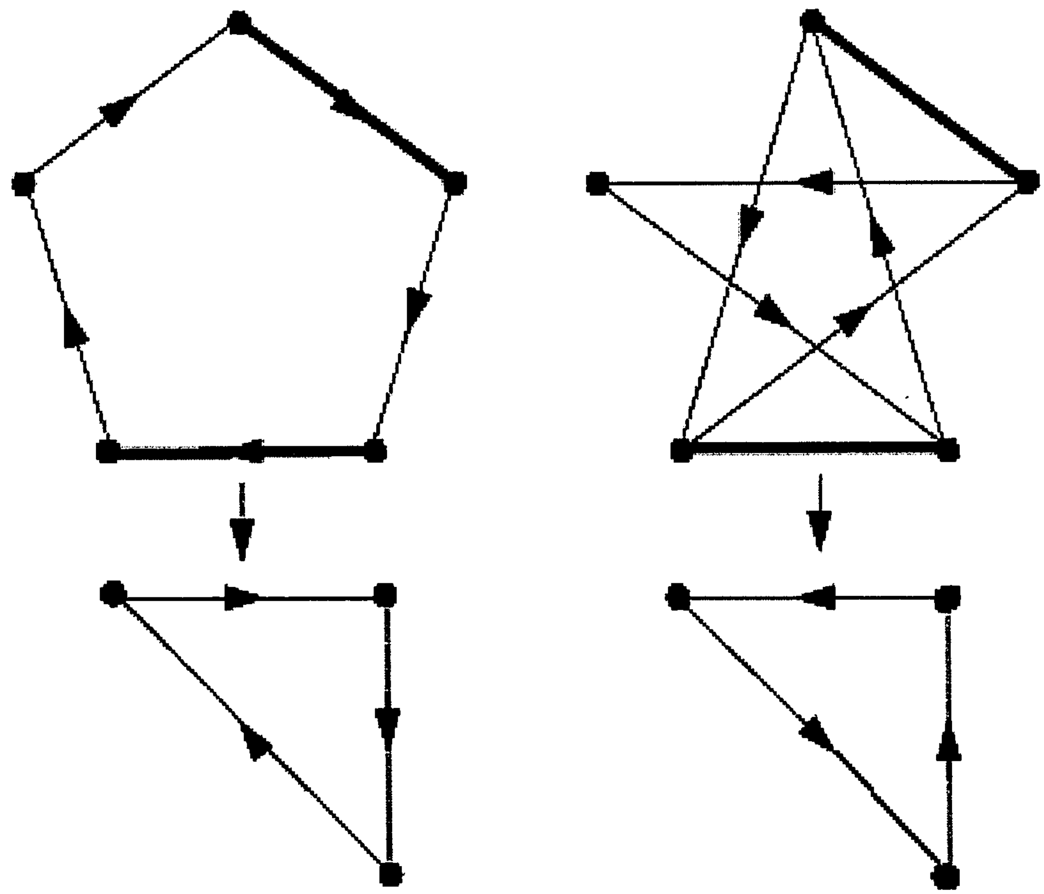

Figure 7: The massless sector of $\mathbb{C}/\mathbb{Z}_5$ with two scalars turned on gives the $\mathbb{C}/\mathbb{Z}_3$ quiver!

More generally, the $\mathbb{C}/\mathbb{Z}_{2l+1}$ singularity can decay to the $\mathbb{C}/\mathbb{Z}_{2l-1}$ singularity, if the FI terms are such that $Z_{12}$ and $Z_{l+1,l+2}$ decouple. (It can also flow to a variety of quasi-orbifold singularities.) For the true orbifold case, the quiver diagram has an obvious $\mathbb{Z}_{2l-1}$ symmetry. This is not a subgroup of $\mathbb{Z}_{2l+1}$, but emerges as an accidental symmetry (in the technical sense) of the low-energy theory. This process can be repeated until we reach the trivial $\mathbb{C}/\mathbb{Z}_1$ orbifold, without tachyons.

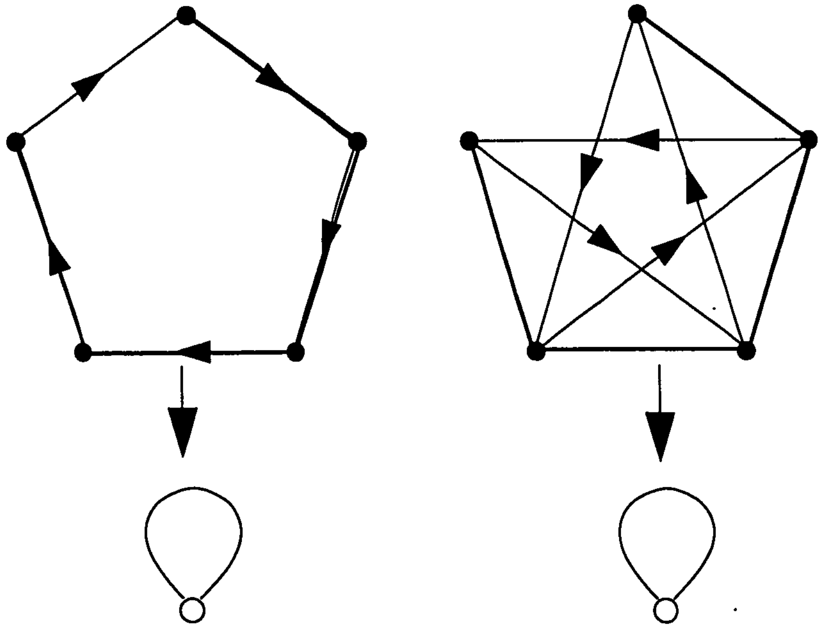

Figure 8: Decay to the $\mathbb{C}/\mathbb{Z}_1$ space.

The spectrum is simply a free chiral supermultiplet, with SUSY reappearing as an accidental symmetry under successive quiver annealings. The order of liftings does not particularly matter. As long as all but one scalar receive generic VEVs the result is inevitably the quiver for SUSY flat space, regardless of the geometries or effective quivers at intermediate scales. Since the bifundamentals couple to relative gauge potentials, this maximally higgses the system; we cannot lift all the scalars without changing the number of degrees of freedom in our theory. In complex codimension two, the story will be much richer, as there is an infinite family of SUSY quivers to which a generic tachyonic quiver can decay.

Note that the decays to lower-order singularities require specific FI terms of no particular quantum symmetry, so they arise from a linear combination of different tachyons. Since the tachyons have different lifetimes, the singular point will not be a static configuration. Presumably it is possible to fine-tune the initial conditions so that the geometry develops the lower order singularity as it enters the gravity regime, where it will remain on top of its tachyon potential.

### 3.4 Decay of $\mathbb{C}/\mathbb{Z}_n$ in the Gravity Regime

In the preceding section we found that the initial effect of the tachyonic singularity is to smooth the geometry. As with any tachyon, an essential question is the nature of the final state: does the tachyon potential have a minimum, or does the instability continue without end? The D-brane probe analysis in the previous section breaks down when the size of the smoothed region reaches the string scale. We do not have tools to probe this regime, so will study the question by going beyond it to the regime of small curvature. If we were to find that the RG flow in that regime carries us back to higher curvature, this would indicate the presence of a minimum with curvature of order the string scale. In fact, we will find that the flow goes toward ever-smaller curvature.[20] Thus the geometry evolves forever, generating an arbitrarily large region of arbitrarily small curvature, which contains a lower-order singularity if the initial state has been appropriately fine-tuned.

### *3.4.1 RG Flow*

We now study the RG flow of the world-sheet NLSM corresponding to a background of the massless closed string fields. Owing to discrete symmetries, we need only consider the metric and dilaton. Note that there is no explicit tachyon field in this regime. The instability, whose initial stage is represented by a tachyon in the orbifold description, would now be a property of the solutions to the low energy field equations. The RG equations are

$$\begin{aligned} \dot{G}_{MN} &= -\beta[G_{MN}] + \nabla_M \xi_N + \nabla_N \xi_M \ , \\ \dot{\Phi} &= -\beta[\Phi] + \xi^M \nabla_M \Phi \ , \end{aligned} \tag{3.4.1}$$

where $G_{MN}$ is the string metric, a dot denotes the logarithmic derivative with respect to world-sheet length scale $\ell\partial_\ell$, and

$$\begin{aligned} \beta[G_{MN}] &= \alpha' R_{MN} + 2\alpha' \nabla_M \nabla_N \Phi \ , \\ \beta[\Phi] &= \alpha' (\nabla\Phi)^2 - \frac{\alpha'}{2} \nabla^2 \Phi \ . \end{aligned} \tag{3.4.2}$$

The vector field $\xi_M$ is arbitrary and represents the freedom to make a spacetime coordinate change with the change of world-sheet scale. A convenient choice is $\xi_M = \alpha' \nabla_M \Phi$, so that

$$\dot{G}_{MN} = -\alpha' R_{MN} \ , \quad \dot{\Phi} = \frac{\alpha'}{2} \nabla^2 \Phi \ . \tag{3.4.3}$$

[20] We cannot exclude the possibility of a fixed point with curvature of order the string scale, but the fact that both the substring and gravity geometries evolve toward smaller curvature strongly suggest that this flow continues smoothly through the stringy regime.

In these coordinates the flow of the metric does not depend on the dilaton; this is possible because the dilaton does not appear in the flat world-sheet action.

The perturbation leaves a $(7+1)$-dimensional free field theory, so the problem is essentially two dimensional. It is convenient to work in conformal gauge, because the flow (3.4.3) preserves that gauge. Thus,

$$ds^2 = e^{2\omega}(d\rho^2 + \rho^2 d\theta^2) \ , \tag{3.4.4}$$

where for generality we consider an arbitrary periodicity $0 \leq \theta \leq 2\pi/\nu$. In this gauge, the metric (3.2.15) for a cone of opening angle $2\pi/n$ corresponds to

$$\omega = \left(\frac{\nu}{n} - 1\right) \ln \rho + \text{constant} \ . \tag{3.4.5}$$

In conformal gauge the RG is

$$\dot{\omega} = \frac{\alpha'}{2} e^{-2\omega} \hat{\nabla}^2 \omega \ , \tag{3.4.6}$$

where $\hat{\nabla}^2$ is the Laplacian for the flat metric $d\rho^2 + \rho^2 d\theta^2$, which is $\partial_\rho^2 + \rho^{-1}\partial_\rho$ for a cylindrically symmetric solution.

Let us analyze this first for the transition $n \to n-2$ at large $n$, where the change in the metric is small. The geometry is as depicted in figure 6, with a $\mathbb{C}/\mathbb{Z}_{n-2}$ cone at the origin, going smoothly to a $\mathbb{C}/\mathbb{Z}_n$ cone at large radius. In coordinates with $\nu = n-2$, the boundary conditions are

$$\omega(\rho \to 0) = \text{finite} \ ; \quad \omega(\rho \to \infty) \to -\frac{2}{n} \ln \rho \ . \tag{3.4.7}$$

We can then linearize, $\dot{\omega} = \alpha' \hat{\nabla}^2 \omega / 2$. A simple solution, obtained by the Fourier transform on the covering space, is

$$\omega(\rho, \ell) = -\frac{1}{n}\left(\ln\ln(\ell/\ell_0) + \int_0^{u_0} \frac{du}{u}(1 - e^{-u})\right) \to -\frac{1}{n}\int_0^{u_0} \frac{du}{u}(1 - e^{-u}) \ , \tag{3.4.8}$$

where $u_0 = \rho^2/2\alpha' \ln(\ell/\ell_0)$; in the second form the $\ln\ln$ term has been conveniently absorbed in an $\ell$-dependent rescaling of $\rho$. The solution depends only on $u_0$, so the radius $\rho_t$ of the transition region grows with increasing world-sheet length scale, $\rho_t \sim \ln(\ell/\ell_0)^{1/2}$. Pointwise in the IR the system approaches a $\mathbb{C}/\mathbb{Z}_{n-2}$ cone everywhere. Asymptotically, all solutions to the diffusion equation with the given boundary conditions will have the same form. The dilaton satisfies a diffusion equation as well and any initial dilaton gradient will similarly diffuse outward.

For the full nonlinear evolution (3.4.6) we do not have a simple analytic result, but given the diffusive nature of the equation we expect that in general the smoothed area depicted in figure 3 grows without bound. Hence our conclusion that the flow found in the substring region, toward smaller curvature, continues indefinitely in the gravity region.

There are two reasons that one might doubt this result. The first is the Zamolodchikov $c$-theorem, showing irreversibility of the flow of the central charge [87]. Here we start with an orbifold CFT of canonical central charge (15 in all for the type II string). In the IR, we claim that the theory flows pointwise to flat spacetime, again with canonical central charge. The reason that this is consistent is that the noncompactness of the target space invalidates the $c$-theorem [88]. There are other cases of CFT theorems that are invalid in noncompact target spaces. The classic example is the holomorphicity of conserved currents, which does not hold for the world-sheet currents associated with rotational invariance in noncompact directions [89]. For the $c$-theorem, the basic objects are the vacuum expectation values of operator products. The string world-sheet vacuum fills out the entire target manifold, a familiar IR effect, so the region of curvature makes a contribution of measure zero.

A second reason that one might have expected the opposite result is the example of compact spaces of positive curvature, which flow to greater curvature. We claim that the difference of boundary conditions in the compact and noncompact cases accounts for the differing behaviors. In fact, there is a simple monotonicity result that makes this clear. From the differential equation (3.4.3) it follows that

$$\ell\partial_\ell \int d^2x\sqrt{G} = -\frac{\alpha'}{2}\int d^2x\sqrt{G}R\ . \tag{3.4.9}$$

For a manifold of spherical topology, the RHS is $-4\pi\alpha'$ and so the volume is monotonically decreasing. The curvature must at some point become stringy, and the low energy theory break down. For the noncompact manifold the integral is not defined, but one can consider the integral interior to a circle of some given radius (over a region such as depicted in figure 3). The smoothing of the singularity does have the effect of reducing this volume, whereas flow back toward a singular cone would increase the volume in contradiction to the flow (3.4.9).

Finally, we might also be interested in the case that the original singularity is part of a compact space. Most simply, consider $T^2/\mathbb{Z}_3$, which is a flat space of spherical topology, with three $\mathbb{C}/\mathbb{Z}_3$ singularities each of deficit angle $4\pi/3$. From the $c$-theorem, or from eq. (3.4.9), one concludes that the space eventually flows to large curvature. The three singularities begin to smooth, until the smoothed regions merge to form a rough sphere, which then evolves toward smaller radius.

### *3.4.2 Dynamical Evolution*

We now consider on-shell evolution,

$$\beta[G_{MN}] = \beta[\Phi] = 0 \ , \tag{3.4.10}$$

with the same $\beta$-functions (3.4.2). This is now a three-dimensional problem, since the solution depends on time. It is convenient to work in the Einstein frame, where this system is just a massless scalar canonically coupled to the metric. The initial metric is again assumed to interpolate from $\mathbb{C}/\mathbb{Z}_n$ at infinity to $\mathbb{C}/\mathbb{Z}_{n'}$ at the origin, with $n' < n$. This is true in both the Einstein and the string frames, because we assume that the dilaton is nonsingular at the origin, while it goes to a constant at infinity (where the evolution has not yet reached).

We do not have analytic solutions for this problem, but it is easy to deduce the general form of the solutions. The constraint equations require that the change in deficit angle be accompanied by energy density of matter. Since we can solve the equations with the NS three-form field strength set to zero, so that the only matter involved is the massless dilaton, this energy must be dilaton gradient and kinetic energy. This dilaton field will radiate outward at the speed of light, as in figure 1. The time scale of the initial decay, before the gravity regime, is the string scale, so this sets the initial width of the dilaton pulse and the kink in the geometry, which then gradually broadens due to dispersion. For $n' = n - 2$ at large $n$, an analytic treatment is again simple. The dilaton satisfies a massless wave equation in flat spacetime, and the backreaction on the metric is a perturbative effect.

As a check, let us look for static solutions in the gravity regime, which would have corresponded to minima of the tachyon potential that are visible in this regime. We will take the most general form with $SO(7,1) \times SO(2)$ spacetime symmetry:

$$ds^2 = e^{2\sigma(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2f(r)}(dr^2 + r^2 d\theta^2) \ . \tag{3.4.11}$$

This is slightly more general than elsewhere, in that we allow the $(7,1)$ directions to be warped; note also a slight change of notation, $\mu,\nu = 0,\ldots,7$. The dilaton field equation is

$$\frac{\Phi''}{\Phi'} - 2\Phi' = -\frac{1}{r} - 8\sigma' \quad\Rightarrow\quad (e^{-2\Phi})' = \frac{c_1}{r}e^{-8\sigma} \tag{3.4.12}$$

with integration parameter $c_1$. The $\mu\nu$ curvature equation reads

$$\frac{(r\sigma')'}{r\sigma'} + 8\sigma' = 2\Phi' \quad\Rightarrow\quad e^{2\Phi} = c_2 r\sigma' e^{8\sigma} \tag{3.4.13}$$

with integration parameter $c_2$. Putting these together gives $\Phi' = -c_1c_2\sigma'/2$, and so

$$e^{\Phi} \propto (\ln r/r_0)^{-c_1c_2/2(8+c_1c_2)}\ , \quad e^{\sigma} \propto (\ln r/r_0)^{1/(8+c_1c_2)}\ . \tag{3.4.14}$$

These are doubly unacceptable: they do not go over to the unperturbed behavior at large $r$, and they have a singularity at finite $r = r_0$. Only the flat cone, the exceptional solution with $\Phi$ and $\sigma$ constant, survives.

It is interesting again to consider the $T^2/\mathbb{Z}_3$ orbifold, with nonsupersymmetric singularities in a compact space. We cannot follow the behavior analytically, but might expect that after the dilaton pulses have begun to cross the compact space, the time-averaged behavior will be that of a positively curved radiation dominated spacetime. Thus, it will reach a Big Crunch in finite time, beyond which we cannot follow the evolution. One supposition would be that the compact dimensions effectively disappear, leaving an eight-dimensional noncritical string theory [59]. However, the simplest background in that theory — the linear dilaton — has the wrong symmetries to be the endstate of our evolution, as the dilaton gradient is spacelike.

### *3.4.3 Application to AdS/CFT*

In ref. [70] it was argued that orbifolding should commute with AdS/CFT duality, so that the dual of the orbifolded gauge theory is IIB string theory on the orbifolded spacetime. This expectation is based on the fact that a duality like the AdS/CFT correspondence concerns a single system with two dual descriptions; if orbifolding makes sense on one side of the duality then the procedure can be mapped to the equivalent dual description of the system given a complete duality dictionary translating between them. In the absence of supersymmetry, if the orbifolding procedure produces a consistent physical system, this requires any instabilities that arise to match up between the two equivalent descriptions. The leading instability that arises in a string background is that of interest here, namely the tachyons. In freely-acting orbifolds on the sphere component of the $AdS_p \times S^q$ geometry, the spectrum is classically tachyon-free. Non-freely acting orbifolds on the $S^q$ do have tachyons, and this has been considered at small 't Hooft parameter [57], where the gauge theory is perturbative. Now let us consider the situation at large 't Hooft parameter, where the AdS description is good.

The AdS description starts with $N$ coincident D3-branes extended in the 0123 directions. The orbifold produces a (7+1)-dimensional fixed plane. This plane contains the D3-branes and extends in four transverse directions. The AdS curvature is small on the string scale and so locally on the fixed plane the initial instability is the same as in flat spacetime. In particular, the decay will release a given energy per unit volume of the fixed plane, as measured in a local inertial frame. The invariant volume element is $(r/R_{\rm AdS})^3 d^3x\, (r/R_{\rm AdS})^{-4} r^3 dr$, where $x$ coordinatizes the field theory dimensions and $r$ is a coordinate along the radial direction of $AdS_5 \times S^5$, with metric $(r/R_{\rm AdS})^2 dx^2 + (r/R_{\rm AdS})^{-2}(dr^2 + r^2 d\Omega^2)$. The translation to the global conserved energy brings in an additional factor of $r/R_{\rm AdS}$, so the total energy released per unit gauge theory volume is simply

$$\int_0^\infty dr\, r^3 \sim \Lambda^4 \ . \tag{3.4.15}$$

That is, it diverges quarticly in the gauge theory.

We can make a simple model of how this divergence might arise in the gauge theory. Consider a state a $U(1)$ gauge theory where we add a + and a − charge in a volume of linear size $\ell$. The kinetic energy is of order $2\ell^{-1}$, but this is reduced somewhat by the gauge theory potential, for a net $\{2 - O(g^2)\}\ell^{-1}$. Extrapolation would suggest a possible instability at large $g^2$ (to be precise, this theory will have a Landau pole in the UV, so we must imagine a cutoff). In a globally supersymmetric theory, positivity of the energy is guaranteed and so this instability is absent; thus, supersymmetric field theories can make sense at large coupling. However, for nonsupersymmetric theories there is no guarantee that they make sense at strong coupling. Indeed the result (3.4.15) suggests an instability of just this sort: in a conformal theory we can produce pairs on any scale $\ell$, and the integral over all scales produces a quarticly divergent result. Note that this is much more severe than the instabilities normally encountered in field theories (such as symmetry breaking), which are IR effects and release a finite energy per unit volume. It is difficult to see how this instability could have any sensible final state.

Indeed, the AdS picture is similarly pathological. We can quantitatively study the large-$n$ case $\mathbb{Z}_n \to \mathbb{Z}_{n-2}$, because the dilaton essentially satisfies a free wave equation on the $AdS_5 \times S^5$ covering space,

$$\frac{R_{\text{AdS}}^2}{r^2}\partial_t^2\Phi = \frac{r^2}{R_{\text{AdS}}^2}\partial_\perp^2\Phi\ . \tag{3.4.16}$$

The orbifold breaks the $SO(6)$ symmetry of $S^5$ so the dilaton is a superposition of different angular states. For angular momentum $L$,

$$\partial_\perp^2 = \partial_r^2 + \frac{5}{r}\partial_r - \frac{L(L+4)}{r^2}\ . \tag{3.4.17}$$

Imagine that the decay starts everywhere at once at $t = 0$. This condition is conformally invariant so the dilaton is a function only of $rt$. The wave equation (3.4.16) then becomes an ordinary differential equation for $\Phi_L(rt)$, and $rt = R_{\text{AdS}}^2$ is a singular point. From the dominant terms near the singular point one finds that

$$\Phi_L \sim (R_{\text{AdS}}^2 - rt)^{-3/2} \tag{3.4.18}$$

for every partial wave $L$. Thus, the energy density diverges at finite time for any $r$; this occurs when a geodesic from $(r,t) = (\infty, 0)$ reaches a given radius, carrying the information about the divergent energy release at large radius.

Again, this instability is a property of large 't Hooft parameter, and is not inconsistent with the much milder instability found at small 't Hooft parameter in ref. [57]. Note that we have assumed that the 't Hooft parameter does not run, as holds at large $N$. If the full $\beta$ function were in fact asymptotically free, then the theory would be stable in the UV, and the instability that we are discussing would set in only below some scale. In this event it is possible that there would be a stable final state.

## 3.5 $\mathbb{C}^2/\mathbb{Z}_n$ Orbifolds and Non-SUSY to SUSY Flows

### 3.5.1 Orbifolds and Quivers

One of the interesting results of the study of open string tachyons has been the possibility of realizing stable branes, in particular SUSY branes, by open string tachyon condensation [51][73]. In this section, we study closed string tachyon condensation on $\mathbb{C}^2/\mathbb{Z}_n$ orbifolds by generalizing the D-brane probe approach of §3 to this case. We will exhibit various transitions from non-supersymmetric, tachyonic $\mathbb{C}^2/\mathbb{Z}_n$ orbifolds to supersymmetric ALE spaces, and provide an infinite sequence of such flows which allows us to realize any SUSY ALE space via closed-string tachyon condensation (or more generally a combination of marginal deformation and tachyon condensation).

The discussion will parallel the $\mathbb{C}/\mathbb{Z}_n$ case. All the orbifolds that we consider will be based on a twist of the form

$$R = \exp\left\{\frac{2\pi i}{n}(J_{67} + kJ_{89})\right\} , \tag{3.5.1}$$

depending on two integers $n$ and $k$ (mod $2n$). We will denote the group generated by $R$ as $\mathbb{Z}_{n(k)}$. On spinors with $J_{67}$ and $J_{89}$ charge $s_{67} = s_{89} = \pm\frac{1}{2}$, $R$ acts as $e^{\pm 2\pi i(k+1)/2n}$. On spinors with $-s_{67} = s_{89} = \pm\frac{1}{2}$ it acts as $e^{\pm 2\pi i(k-1)/2n}$. The condition that $R^n = 1$ on spinors forces $k$ to be odd. If $k$ is $\pm 1$, then $R$ leaves half of the $D = 10$ spinors invariant and so produces the familiar supersymmetric $A_{n-1}$ orbifold (for reviews see [90][91]). For other values of $k$, at least some of the twisted sector ground states are tachyonic. If $2n$ is divisible by $k+1$ or by $k-1$, then $R^{2n/(k+1)}$ or $R^{2n/(k-1)}$ leaves some spinors invariant. The associated twisted sector ground state is massless, and indeed is the same as the corresponding twisted sector state in the supersymmetric orbifold (but note that the respective cases $k+1=2$ and $k-1=2$ are trivial).

Let us now consider a D-brane probe in this background. Define

$$Z^1 = X^6 + iX^7 , \quad Z^2 = X^8 + iX^9 . \tag{3.5.2}$$

For the world-volume spinor in the **16** of $SO(9,1)$, its component with $(s_{67}, s_{89}) = (-\frac{1}{2}, -\frac{1}{2})$ will be denoted $\chi$ and it component with $(s_{67}, s_{89}) = (-\frac{1}{2}, +\frac{1}{2})$ will be denoted $\eta$ (the remaining two components are the conjugates). The $SO(5,1)$ spinor indices, respectively **4** and **4'**, are suppressed. Using the techniques discussed in [69] and §2, one finds the world-volume theory to be a $U(1)^n$ quiver theory with matter content

$$A_{\mu jj}\ ,\quad X^m_{jj}\ ,\quad Z^1_{j,j+1}\ ,\quad Z^2_{j,j+k}\ ,\quad \chi_{j,j-q-1}\ ,\quad \eta_{j,j+q}\quad (k\equiv 2q+1)\ . \tag{3.5.3}$$

The classical scalar potential is

$$V=\mathrm{Tr}\left\{\frac{1}{2}[Z^1,\bar Z^1]^2+\frac{1}{2}[Z^2,\bar Z^2]^2+\left|[Z^1,Z^2]\right|^2+\left|[Z^1,\bar Z^2]\right|^2\right\}\ . \tag{3.5.4}$$

Using the Jacobi identity this can also be rewritten

$$\begin{aligned} V&=\mathrm{Tr}\left\{\frac{1}{2}\left([Z^1,\bar Z^1]-[Z^2,\bar Z^2]\right)^2+2\left|[Z^1,\bar Z^2]\right|^2\right\}\\ &=\mathrm{Tr}\left\{\frac{1}{2}\left([Z^1,\bar Z^1]+[Z^2,\bar Z^2]\right)^2+2\left|[Z^1,Z^2]\right|^2\right\}\ . \end{aligned} \tag{3.5.5}$$

The Yukawa terms are

$$L_{\mathrm{Y}}=\mathrm{Tr}\left\{[Z^1,\chi]\,\eta+[Z^2,\chi]\,\bar\eta+\mathrm{h.c.}\right\}\ . \tag{3.5.6}$$

### *3.5.2 The Example $\mathbb{Z}_{2l(2l-1)}$: Non-SUSY $\mathbb{Z}_{2l}$ to SUSY $\mathbb{Z}_2$*

Now we analyze the case $k=n-1$, where $n=2l$ must be even because $k$ is odd. Here

$$R=\exp\{2\pi i J_{89}\}\exp\{2\pi i(J_{67}-J_{89})/2l\} \tag{3.5.7}$$

is the same as in the supersymmetric case except for a factor of $\exp\{2\pi i J_{89}\}=(-1)^{\mathbf{F}}$, which breaks supersymmetry. Note however that the special case $l=1$, the $\mathbb{C}^2/\mathbb{Z}_{2(1)}$ orbifold, is supersymmetric: $R=\exp\{2\pi i(J_{67}+J_{89})/2\}$ leaves invariant spinors such that $s_{67}=-s_{89}$.

Before exciting tachyons, the geometry is the same as for the supersymmetric orbifold. In particular, on the probe moduli space the condition that $V$ vanish gives

$$Z^1_{j,j+1}=Z^1\ ,\quad Z^2_{j+1,j}=Z^2\quad (\text{independent of } j) \tag{3.5.8}$$

up to gauge transformation. Thus the probe has two complex moduli, as it should. The origin, where the $U(1)^{2l}$ gauge symmetry is restored, is a $\mathbb{Z}_{2l}$ singularity as in §2.2.

Before discussing the generic decay, it is interesting to consider first deformations that preserve a $\mathbb{Z}_2 \subset \mathbb{Z}_{2l}$ quantum symmetry. This $\mathbb{Z}_2$ acts on the $l^{\text{th}}$ twisted sector as $(-1)^l$, so only states twisted by powers of $R^2$ can have VEVs. Since $R^2 = \exp\{2\pi i(J_{67} - J_{89})/l\}$, these sectors are exactly the same as for the *supersymmetric* $\mathbb{C}^2/\mathbb{Z}_{l(-1)}$ orbifold. In particular, there are no tachyons, so we are actually considering marginal deformations. The perturbation of the gauge theory is then a supersymmetric $D$-term

$$\Delta V = -\sum_{j=1}^{2l} \lambda_j D_j \ , \quad D_j = |Z^1_{j,j+1}|^2 - |Z^2_{j+1,j}|^2 - |Z^1_{j-1,j}|^2 + |Z^2_{j,j-1}|^2 \ , \quad (3.5.9)$$

where the sign of each term is determined by the $U(1)_j$ charge (note that for a $k = -1$ orbifold $Z^1$ and $Z^2$ are in chiral superfields, while for $k = +1$ it would be $Z^1$ and $\overline{Z}^2$). An overall additive constant is ignored. As in §2.2, $\sum_{j=1}^{2l} \lambda_j = 0$, while the $\mathbb{Z}_2$ quantum symmetry requires that $\lambda_j = \lambda_{j+l}$. Now consider the deformed moduli space; focus on the second of forms (3.5.5) and note that the first term there is just $\sum_{j=1}^{2l} D_j^2$. The vanishing of the second term requires that

$$Z^1_{j,j+1} Z^2_{j+1,j} \equiv \alpha \quad (3.5.10)$$

be independent of $j$. Minimizing the $D$-terms then sets $D_j = \lambda_j$, which determines all of the magnitudes in terms of $|Z^1_{12}|$ and $\alpha$. Finally, the phases can be gauged away except for $\sum_{j=1}^{2l} \arg Z^1_{j,j+1}$, giving four real moduli in all.

There are still singularities. Consider the subspace $\alpha = 0$. The condition $D_j = \lambda_j$ determines

$$|Z^1_{j,j+1}|^2 - |Z^2_{j+1,j}|^2 = \rho_j + x \ , \quad (3.5.11)$$

where $\rho_j = \rho_{j-1} + \lambda_j$ and $x$ is undetermined. When $x = -\rho_{j_0}$ for some $j_0$, both $Z^1_{j_0,j_0+1}$ and $Z^2_{j_0+1,j_0}$ vanish. Further, the $\mathbb{Z}_2$ quantum symmetry implies that $Z^1_{j_0+l,j_0+l+1}$ and $Z^2_{j_0+l+1,j_0+l}$ vanish as well. There are then two unbroken $U(1)$'s, namely $\sum_{j=j_0+1}^{j_0+l} Q_j$ and $\sum_{j=j_0+l+1}^{j_0} Q_j$, where $Q_j$ is the $U(1)_j$ charge and $j$ is defined mod $2l$. Thus, these $l$ points of restored gauge symmetry, which are generically distinct, are $\mathbb{Z}_2$ singularities on the moduli space.

Thus far the discussion is the same as for the resolution of a *supersymmetric* $\mathbb{C}^2/\mathbb{Z}_{2l(-1)}$ singularity while preserving a $\mathbb{Z}_2$ quantum symmetry: the result there would be a $\mathbb{Z}_{2(-1)}$ orbifold of a smooth $\mathbb{Z}_l$ ALE space. The difference for us is that the final orbifold operation here contains an extra factor of $(-1)^{\mathbf{F}}$, so it must be $\mathbb{Z}_{2(1)}$. Naively one might expect this orbifold point to be nonsupersymmetric, but the discussion at the beginning of this subsection shows that it is supersymmetric with the opposite supersymmetry from that respected by $R^2$. One can think of the final picture as follows: we resolve the $\mathbb{C}^2/\mathbb{Z}_{l(-1)}$ orbifold generated by $R^2$ into a smooth ALE space preserving half of the supersymmetry, and then make a $\mathbb{Z}_{2(1)}$ orbifold which locally would preserve the other half. In other words, we have a space of $SU(2)_1 \subset SO(4) = SU(2)_1 \times SU(2)_2$ holonomy, with $l$ orbifold singularities whose holonomy is in $SU(2)_2$. The space as a whole has no supersymmetry, but half of the supersymmetry survives in the smooth region and the other half locally at the orbifold points. In the limit that the marginal deformation is taken to infinity, we simply have a supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2(1)}$ space, without tachyons.

We can verify this by examining the quivers. At the orbifold point, the potential for the vanishing fields $Z^1_{j_0,j_0+1}$, $Z^2_{j_0+1,j_0}$, $Z^1_{j_0+l,j_0+l+1}$ and $Z^2_{j_0+l+1,j_0+l}$ is quartic so they are massless, while all other scalars are massed up. One unbroken $U(1)$ acts on indices $j = j_0,\ j_0+l+1$, and the other on indices $j = j_0+1,\ j_0+l$. Expanding the Yukawa coupling (3.5.6) in components, one finds that $\eta_{j_0+1,j_0+l}$ and $\eta_{j_0+l+1,j_0}$ do not appear in terms with scalar expectation values, so these remain massless (note that these are neutral under the unbroken $U(1)$'s). There must therefore also be two massless linear combinations of $\chi$'s; these come in the bifundamental representation of the unbroken $U(1)^2$. The correlation between $U(1)$ charges and $SO(5,1)$ quantum numbers is the same as for the $\mathbb{C}/\mathbb{Z}_{2(1)}$ orbifold, namely the spectrum (3.5.3) at $k = 1$ with $\eta$ in the adjoint and $\chi$ in the bifundamental representation of the gauge group.

We now turn to the generic twisted state background. The full D-brane probe analysis is less useful here, for two reasons. The first is that without any connection to supersymmetry, the quantum symmetry alone does not fix the form of the quadratic mass terms (specifically, the ratio of $Z^1$ and $Z^2$ masses); it requires the calculation of a disk amplitude, as in the appendix to ref. [69]. More critically, for general mass terms allowed by the quantum symmetry, there is no probe moduli space. This is not a problem — from the spacetime point of view it is the same effect that a dilaton background would have — but it makes it difficult to give a geometric interpretation in the substring regime.

Fortunately, we can largely deduce the fate of the instability by expanding around the deformation already considered. Let us first deform $\mathbb{C}^2/\mathbb{Z}_{2l(2l-1)}$ along directions that preserve the $\mathbb{Z}_2$ quantum symmetry as above, so as to have an orbifold of $SU(2)_2$ holonomy in a space of $SU(2)_1$ holonomy. The orbifold locally is supersymmetric and so has marginal deformations in the twisted sector. These correspond to blowing the orbifold points up into smooth $\mathbb{Z}_2$ ALE spaces of $SU(2)_2$ holonomy. Thus we have small patches of $SU(2)_2$ holonomy in a larger region of $SU(2)_1$ holonomy. This is only an approximate solution to the equations of motion, and will in time evolve to a space of generic holonomy and expand indefinitely as in the $\mathbb{C}/\mathbb{Z}_n$ case.

Note that the second blowing-up will not be exactly marginal, as the coupling to the $SU(2)$ curvature will break supersymmetry and presumably drive the marginal direction to be tachyonic. If the extent of initial blowing-up is reduced, so as to condense the two steps towards one, the $\mathbb{Z}_{2(1)}$ twisted state will become more tachyonic, so we seem to connect smoothly onto the original string-scale tachyon.

There is a seeming paradox here, whose resolution provides an elegant check on our picture. The initial $\mathbb{C}^2/\mathbb{Z}_{2l(2l-1)}$ orbifold is an exact CFT, and so its tree-level energy (as measured by the $1/r^2$ falloff of the metric) is zero. There is a tree-level tachyon, and so the final state should have negative energy when the kinetic energy of the outgoing pulse is subtracted.[21] Does this not violate a positive energy theorem? In fact, there is no such theorem: negative energy configurations of asymptotic ALE geometry exist [92].[22] There is a negative energy theorem for any geometry that admits spinor fields going to a constant at infinity [94][95]. The geometries of ref. [92] admit spinors, so it must be that any smooth spinor field is *antiperiodic* under the asymptotic ALE identification. This is precisely the geometry of the $\mathbb{Z}_{2l(2l-1)}$ orbifold.

In the above example and the others we will consider in this section, we have studied in detail the substring regime using D-brane probes, and in the case of

[21] This paradox did not arise for $\mathbb{C}/\mathbb{Z}_n$, because in two dimensions a conic deficit angle *is* an ADM energy.

[22] We would like to thank G. Horowitz for informing us about these spaces and explaining their significance, as well as sharing insights from his investigations into GR solutions for the $\mathbb{C}^2/\mathbb{Z}_n$ cases [93].

marginal deformations, we have also studied the regime far away from the original orbifold point using inheritance from a related SUSY orbifold. Once tachyons turn on and the system evolves into the gravity regime, we have not analyzed the subsequent GR solutions as explicitly as in the $\mathbb{C}/\mathbb{Z}_n$ case. However, the following indicates that the behavior is as before. Consider a configuration of negative energy. If the size of the configuration is scaled up by a factor $\lambda$, the energy scales as $\lambda^2$ ($\lambda^4$ from the volume and $\lambda^{-2}$ from the derivatives). This implies that the potential is unbounded below in this direction.

### *3.5.3 The Example $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$: Non-SUSY $\mathbb{Z}_{2l}$ to SUSY $\mathbb{Z}_l$*

These results have a resemblance to phenomena that have been observed in open string systems. The existence of a tachyon, which disappears as one goes along a marginal direction, is the same as in a D-brane/anti-D-brane system, where the string-scale tachyon at small separation goes over to a long-range attraction as the branes are separated. The decay of a nonsupersymmetric configuration to a supersymmetric configuration plus outgoing radiation is also familiar.

There are many other similar flow patterns that can be deduced by studying the quiver theories as we have done for the above case. One interesting sequence is for $n = 2l$ and $k = 3$,

$$R = \exp\left\{ \frac{2\pi i}{2l} (J_{67} + 3J_{89}) \right\} , \tag{3.5.12}$$

whose quiver diagrams are shown in figure 9.

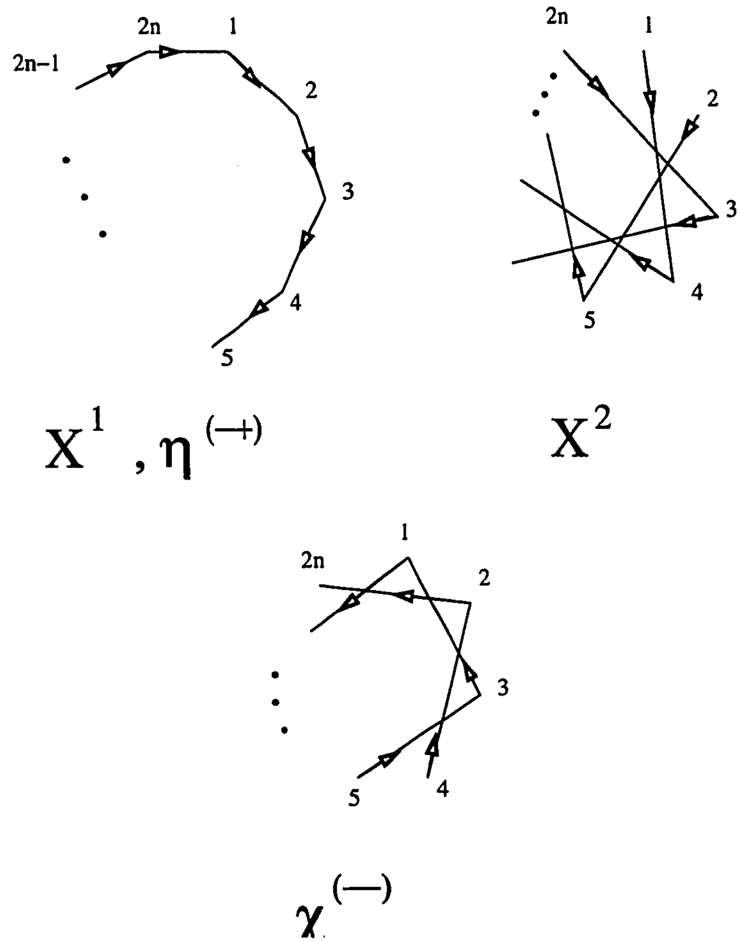


Figure 9: $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$ quivers

In this case $R^l = \exp\{i\pi(J_{67} - J_{89})\}$ is the same as for the supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2(-1)}$ orbifold. In parallel with the previous example, we first excite only marginal states from the sector twisted by $R^l$. This preserves as $\mathbb{Z}_l$ subgroup of the original $\mathbb{Z}_{2l}$ quantum symmetry.

The Fayet-Iliopoulos terms then satisfy

$$\lambda_j = (-1)^{j+1}\lambda \tag{3.5.13}$$

where we take $\lambda > 0$. The quantum symmetry requires that $Z^1_{2p-1,2p}$, $Z^1_{2p,2p+1}$, $Z^2_{2p-1,2p+2}$, and $Z^2_{2p,2p+3}$ be independent of $p$, and the $D$-terms are minimized when

$$|Z^1_{2p-1,2p}|^2 + |Z^2_{2p-1,2p+2}|^2 = |Z^1_{2p,2p+1}|^2 + |Z^2_{2p,2p+3}|^2 + \lambda \ , \tag{3.5.14}$$

while $Z^1_{2p-1,2p}Z^2_{2p,2p+3} = Z^1_{2p,2p+1}Z^2_{2p-1,2p+2}$. When $Z^1_{2p-1,2p} = \lambda^{1/2}$ with all other VEVs vanishing, a $U(1)^l$ is restored — namely $Q_{2p-1} + Q_{2p}$ for all $p$ — so this is a $\mathbb{Z}_l$ singularity. Before taking into account interactions that give mass to some fields, the quiver diagrams thus collapse to those depicted in Figure 10.

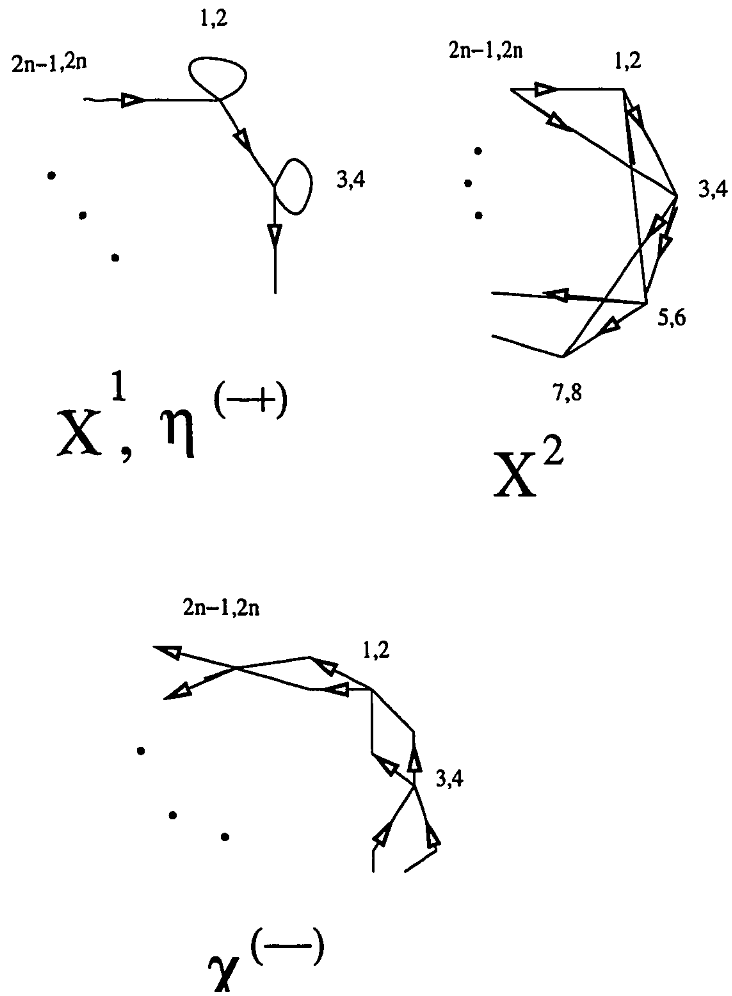


Figure 10: $\mathbb{C}^2/\mathbb{Z}_l$ quivers from collapse of $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$, including massive fields

We next must determine which of the fields in figure 10 mass up in the transition. On the $Z^1$ diagram, the adjoint representations are removed: the potential fixes the magnitudes and the Higgs mechanism removes the phases, leaving the result in Figure 11. On the $Z^2$ diagram, the $|[Z^1, Z^2]|^2$ term gives masses to $Z^2_{2p,2p+3}$, so that the components depicted in figure 11 remain massless. Of the fermions, only half of the components appear in the mass matrix, namely $\eta_{2p,2p}$ and $\chi_{2p+1,2p-1} - \chi_{2p,2p-2}$, leaving the fermions depicted in figure 11. In particular, the $\eta$ are in the adjoint representation and the $\chi$ are in the $(q, q+1)$ bifundamental.

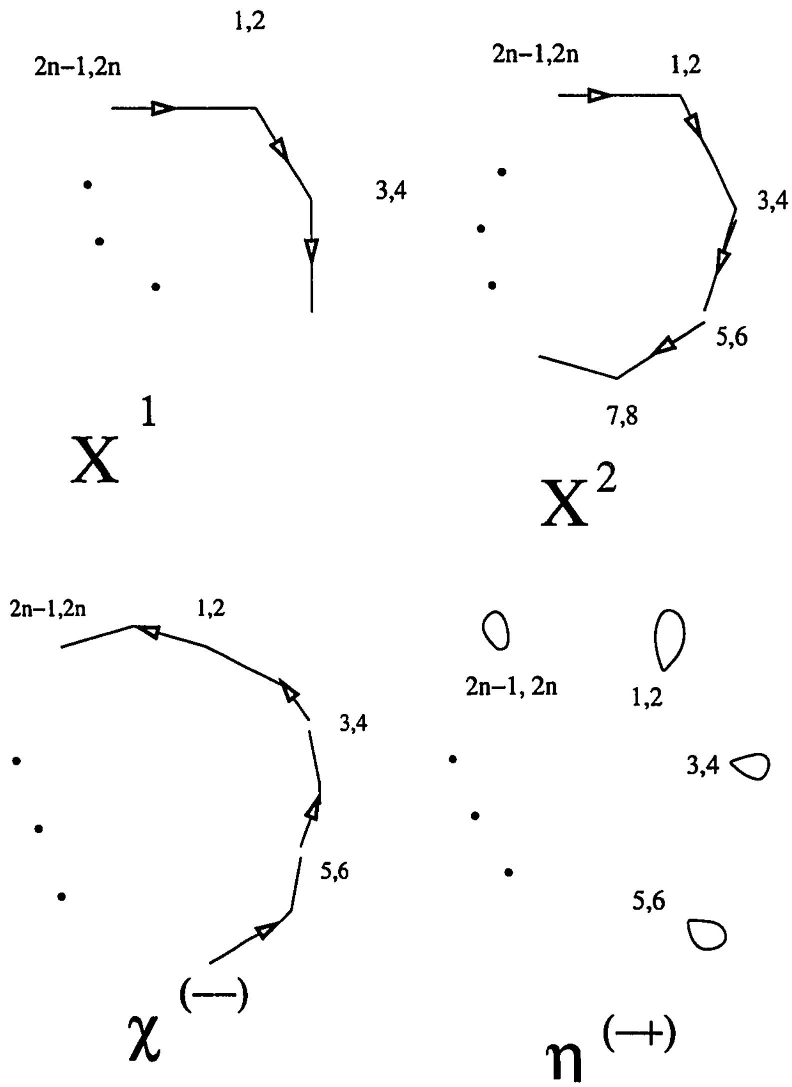


Figure 11: $\mathbb{C}^2/\mathbb{Z}_{2l(3)}$ quivers after twisted-state condensation

Altogether, we are left in Figure 11 with the quiver theory corresponding to D-branes at a $\mathbb{C}^2/\mathbb{Z}_{l(1)}$ orbifold point, which is supersymmetric but with the opposite supersymmetry from the $R^l$ orbifold. Thus the interpretation is parallel to the previous example: the marginal direction blows up the orbifold into a manifolds of smooth $SU(2)_1$ holonomy, which is orbifolded by $\mathbb{Z}_{l(1)} \subset SU(2)_2$.

As a check, consider the low energy theory near the fixed point. We have $R = \exp(2\pi i J/2l)$, where $J = J_{67} + 3J_{89}$. This operator in the original theory becomes

$$J_{67} + 3J_{89} + \frac{1}{2}\sum_{p=1}^{l}(Q_{2p} - Q_{2p-1}) \ , \qquad (3.5.15)$$

in the low energy theory, because this is the linear combination including the broken generators that leaves the background invariant. This acts on the massless fields as

$$Z^1_{2p,2p+1} \to 2Z^1_{2p,2p+1} \ , \quad Z^2_{2p-1,2p+2} \to 2Z^2_{2p-1,2p+2} \tag{3.5.16}$$

and so it acts as $\tilde{J} = 2(J_{67} + J_{89})$ in the low energy theory. The orbifold operation $\exp(2\pi i\tilde{J}/2l)$ is then $\mathbb{Z}_{l(1)}$ in the low energy theory.

There is another orbifold point, where $Z^2_{2p-1,2p+2} = \lambda^{1/2}$ with all other VEVs vanishing. The analysis of the previous paragraph shows that this is a $\mathbb{Z}_{l(-3)}$, which is nonsupersymmetric for $l > 2$.

In summary, we can obtain all supersymmetric ALE orbifolds by descent from nonsupersymmetric ones. The $\mathbb{C}^2/\mathbb{Z}_{4(3)} \to \mathbb{C}^2/\mathbb{Z}_{2(1)}$ flow is common to both this sequence and the one discussed in §5.2.

### *3.5.4 The Example $\mathbb{C}^2/\mathbb{Z}_{5(3)} \to \mathbb{C}/\mathbb{Z}_{2(1)}$: Tachyon Condensation*

Since both of the above examples involved marginal as well as tachyonic deformations, it is interesting to ask whether there are in fact examples where such transitions between non-supersymmetric and supersymmetric ALE spaces proceed exclusively by tachyon condensation, without any marginal component. The following simple example exhibits this possibility (which we expect to be generic). We will make the assumption that the twisted deformations we turn on in the quiver worldvolume QFT can be accessed by adjusting modes in the tower of twisted states in the closed string sector. It would be interesting to check this generic assumption more explicitly as in [69].

Start with the orbifold $\mathbb{C}^2/\mathbb{Z}_{5(3)}$. We can choose three independent $\lambda_j$ such that the D-terms induce VEVs for $Z^1_{45}$, $Z^1_{51}$, and $Z^1_{23}$. This preserves a $U(1)^2$ subgroup of the $U(1)^5$ gauge symmetry, generated by combinations of charges $Q_4 + Q_5 + Q_1$ and $Q_2 + Q_3$. Plugging these VEVs into the component expansion of the interaction terms (3.5.5)(3.5.6) as before, we find that the spectrum reduces to that of the $\mathbb{C}/\mathbb{Z}_{2(1)}$ quiver theory, with gauge group $U(1)^2$, $\eta$ in the adjoint and $\chi$, $Z^1$, and $Z^2$ transforming as bifundamentals. This theory does not have effectively supersymmetric subsectors, in contrast to those in §5.2 and §5.3. So given our assumption about the availability of these deformations in the closed string spectrum (including those that put the Lagrangian in supersymmetric form), this provides an example of a truly tachyonic transition from a non-supersymmetric ALE space to a supersymmetric one.

## 3.6 Dualities, Fluxbranes, and the Type 0 Tachyon

The results in the preceding sections describe transitions between different ALE spaces (including flat space) by processes in which the string coupling remains bounded. While this is sufficient for our purposes, it is also instructive to consider the predictions that these results imply for processes in dual descriptions of the system. In particular we will consider $T$-dual descriptions, in the angular direction, of the orbifolds that we have considered, as well as the addition of R-R Wilson lines. The duals thus involve NS5-branes, fluxbranes, and the type 0 tachyon.

### *3.6.1* $\mathbb{C}/\mathbb{Z}_n$ *at Large* $n$

An angular direction along which we rotate in performing a $\mathbb{Z}_n$ orbifold projection ends up $n$ times smaller than in the parent theory, so for $n$ large it is of interest to $T$-dualize along this direction in the region near the origin. Near but not at the origin, there is a subspace that looks approximately like a cylinder, with twisted strings playing the role of winding modes around the $S^1$ direction of the cylinder:

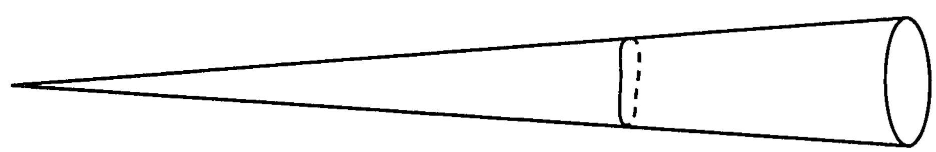

Figure 12: The $\mathbb{C}/\mathbb{Z}_n$ cone at large $n$, with a twisted closed string.

In other words, there are many low-lying twisted states, which become Kaluza-Klein states in the $T$-dual description. The formal $T$-dual of the cone metric (3.2.15) is

$$ds^2 = ndr^2 + \frac{n\alpha'^2}{r^2}d\tilde{\theta}^2 \ . \tag{3.6.1}$$

Also, the dilaton is now position-dependent,

$$e^{\Phi} = g_s\frac{\sqrt{n\alpha'}}{r} \ . \tag{3.6.2}$$

In the large-$n$ limit the orbifold operation (3.2.3) is a small rotation times $(-1)^{\mathbf{F}}$, so on the circle that we are $T$-dualing fields are twisted by $(-1)^{\mathbf{F}}$. Such a twist has three effects on the $T$-dual description. First, the bulk theory is twisted by $(-1)^{\mathbf{F}}$, so it is the type 0 theory (type 0B if we began with IIA, and type 0A if we began with IIB). Second, in going around the $T$-dual circle there is a twist by $(-1)^{\mathbf{Q}}$, where $\mathbf{Q}$ is the quantum symmetry dual to the twist $(-1)^{\mathbf{F}}$. That is, type 0 fields that descend from the type II theory are periodic, while type 0 fields that do not descend (including the type 0 tachyon) are antiperiodic. Third, the periodicity of the dual coordinate is halved, $0 \leq \tilde{\theta} \leq \pi$.

The $T$-dual description is valid out to $r \sim n\sqrt{\alpha'}$, beyond which the $T$-dual circle is small and the original circle is large. It also breaks down for $r < \sqrt{\alpha'}$, where the curvature becomes large. Thus, the apparent divergence of the dilaton (3.6.2) is irrelevant, as we could have expected since the orbifold description is manifestly weakly coupled. We do not have any good description in this region; it is some sort of effective 'wall' in spacetime, whose properties can be deduced from the exact orbifold description. One property of the wall that is not evident in the metric (3.6.1) is the breaking of translation invariance in the $\tilde{\theta}$-direction. The twisted modes of the orbifold transform under the finite $\mathbb{Z}_n$ quantum symmetry rather than the infinite group $\mathbb{Z}$ characterizing true winding modes on a cylinder. In the $T$-dual description, this means that the continuous translation symmetry along the dual angular circle is broken to a discrete $\mathbb{Z}_n$ symmetry [96]. This suggests that the wall is actually a line of $n$ branes (defined broadly as defects which break translation invariance) spaced equally along the $T$-dual circle. In the case of $\mathbb{C}^2/\mathbb{Z}_n$, this picture is well understood, as we will review shortly, but for $\mathbb{C}/\mathbb{Z}_n$ we do not know of any suitable candidate branes.

The twisted state tachyon of the original theory is just the bulk type 0 tachyon in the $T$-dual description. The multiplicity of excited tachyons associated with the eight-dimensional fixed plane on the orbifold side maps on the $T$-dual side to the multiplicity of modes of the ten-dimensional type 0 tachyon. Because of the $(-1)^{\mathbf{Q}}$ twist the decay is most rapid at small $r$. The type 0 tachyon in ten dimensions has $\frac{\alpha'}{4}m^2 = -\frac{1}{2}$, and the antiperiodic boundary condition should shift this upward by an amount of order the inverse radius of the dual circle. Indeed, the most tachyonic mode has

$$\frac{\alpha'}{4}m^2 = -\frac{1}{2} + \frac{1}{2n} \tag{3.6.3}$$

For the partial resolution $n \to n-2$, it seems that the wall relaxes into a lower energy state while a metric and dilaton perturbation (given by the $T$-dual of the picture in §4) propagates outward. For the full decay $n \to 1$, the tip of the cone and the associated low-lying states disappear at the speed of light. In the $T$-dual picture, it seems that the wall, where our control breaks down, is propagating to larger $r$ at the speed of light. At larger $r$, the angular direction gets smaller in this T-dual picture. It would be interesting to try to extract from this a prediction for the type 0 tachyon, but this is not immediate in our system here because the initial wall is present to act as a seed for the decay.

### *3.6.2* $\mathbb{C}^2/\mathbb{Z}_n$ *and NS5-Branes*

For the orbifold $\mathbb{C}^2/\mathbb{Z}_n$ at large $n$ and fixed $k$, the angular direction generated by $J_{67} + kJ_{89}$ is again small and a $T$-dual picture is valid. This is best understood in the supersymmetric cases $k = \pm 1$: the $T$-dual description has $n$ evenly spaced NS5-branes [72]. The sequences of transitions between non-supersymmetric and supersymmetric four-dimensional ALE spaces detailed in §5 (and presumably many others like them) allow us to produce any supersymmetric ALE space by closed-string tachyon condensation or marginal deformation. Using the $T$-duality, we can restate this in terms of NS5-branes. Namely, any number of NS5-branes can be obtained by condensation of modes in a non-supersymmetric closed string background.

It is also interesting to look for a brane description of the tachyonic starting point. In particular, in the case $\mathbb{C}^2/\mathbb{Z}_{2l(2l-1)} \to \mathbb{C}^2/\mathbb{Z}_{2(1)}$ one might have expected that since the bosonic action is the same as in a supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2l}$ orbifold, the $T$-duality transformation would produce a similar configuration of $2l$ NS5-branes. However, the factor $(-1)^{\mathbf{F}}$ in the twist (3.5.7) modifies the $T$-duality as described in §6.1. The $T$-dual circle is only half as large, so there are only $l$ NS5-branes, while the bulk theory is type 0 theory with a $(-1)^{\mathbf{Q}}$ twist around the $T$-dual circle. The marginal deformations that we discussed descend from those of the supersymmetric $\mathbb{Z}_l$ theory, and so correspond to the positions of the $l$ NS5-branes. The tachyons, in the sectors of odd $\mathbb{Z}_2$ quantum symmetry, are modes of the type 0 tachyon.

It would be interesting to pursue this type of dual description of the non-supersymmetric ALE orbifolds further. It is straightforward to apply the general $T$-duality transformation [97], but this results in a smeared 5-brane solution and we do not know the localized form in general.

### 3.6.3 Adding RR Flux

A simple generalization is to add an RR Wilson line to the $\mathbb{C}/\mathbb{Z}_n$ orbifold,[23] $C_\theta = 1$ in coordinates where the identification is $\theta \sim \theta + 2\pi/n$. The net phase is then $2\pi/n$. In M theory this corresponds to an orbifold by a $2\pi/n$ rotation accompanied by a shift by $1/n$ around the M theory circle. In a dual description where a linear combination of the eleventh direction and the angular direction of the orbifold is taken to be the M direction, this is a fluxbrane [74][75][64][65][76][77]. Because of the factor of $(-1)^{\mathbf{F}}$ in the orbifold, it is a fluxbrane in the type 0A theory [98] of strength $BR^2 = 1/n$, or a fluxbrane in the type IIA theory of strength $BR^2 = 1 + 1/n$.

This duality is a strong-weak coupling duality, so that both sides are not simultaneously weakly coupled. However, on the fluxbrane side the coupling varies with radial distance from the origin, becoming weaker toward the origin. If we fix the string coupling to be $g_s < 1$ on the orbifold side, on the fluxbrane side one has a region $r < l_s g_s^{1/3} n = n l_{P,11}$ near the origin which has string coupling $g_s^{(f)} < 1$. For large $n$ and $g_s > 0$, this region can cover many string lengths. We will study the predictions of our results combined with the conjectured orbifold/fluxbrane duality for decay of the Type 0A tachyon in this region. In our analysis in the bulk of this chapter, we worked in the classical string limit. As we have just learned, in order to dualize to a fluxbrane side with a significant region of weak coupling near the origin, we must relax this limit somewhat, and consider a nonvanishing orbifold string coupling, though we can keep it weak. For the remainder of this section, we will assume that the decay process we studied proceeds similarly at weak but nonzero coupling.

RR field strengths couple to extra powers of $g_s$ in the action and so for weak string coupling they have only a small effect on the tachyon decay process we have studied. The decay will proceed as we have described, with the RR flux ultimately dispersing when we reach the flat space endpoint. For the partial decay $n \to n-2$, the outgoing pulse must contain a negative RR flux $2\pi(\frac{1}{n} - \frac{1}{n-2})$. In the dual fluxbrane, the flux near the origin *increases* in the transition, from $\frac{1}{n}$ to $\frac{1}{n-2}$. According to the conjectured duality dictionary in [64][65], this addition of flux takes the $0A$ theory closer to the flat space IIA theory.

---

23 We thank A. Strominger for discussions on this issue.

Therefore, by assuming the dualities described in [64][65][76], and combining them with our results on classical tachyon decay in orbifolds, we predict that the type 0A tachyon in ten dimensions decays toward the flat ten-dimensional IIA vacuum. This agrees with the conjecture for the fate of the Type 0A tachyon made in [65] based on extrapolating to a regime where *non-perturbative* decays from 0A to IIA occur [60][99]. Our route to this conclusion is somewhat more direct, as we use our classical results on tachyon decays in orbifolds rather than non-perturbative instanton effects. However, these statements are still predicated on the conjectural non-supersymmetric strong-weak coupling duality [98][64] assumed in [65]. Therefore we regard this as a mild consistency check of the proposal that the type 0A tachyon drives the theory to the type IIA vacuum.

## 3.7 Conclusions

In this chapter we have exhibited strong evidence that tachyonic non-supersymmetric ALE spaces decay to supersymmetric ALE spaces (including flat space). There are several interesting lessons and directions for future work that emerge from our analysis.

On the theoretical side, as we have emphasized at various points, our results are rather similar to ones that emerge in the study of open string tachyon condensation and its relation to unstable brane annihilation. It would be very interesting to understand how far the analogy between twisted strings and open strings goes. Is there a notion of confinement of twisted strings into ordinary untwisted closed strings? Is there a simplification of closed string field theory if one focuses on twisted states and regards untwisted strings as derivative degrees of freedom obtained in internal legs of the diagrams? What does the similarity between closed string and open string processes say about the extent of applicability of K-theoretic techniques as a function of $g_s$?

We should reemphasize, as discussed in the introduction, that there is a similar puzzling issue in the two cases. Namely as in the open string case, our results point to the need for a strictly classical stringy mechanism, different from the Higgs mechanism, for lifting gauge bosons living on decaying defects. It is perhaps a clue that the disappearance of these gauge bosons and the other phenomena we have observed occurs in the closed string as well as open string context: whatever the physics is that gives rise to these processes, it is not tied uniquely to the open string perturbation expansion since it arises for twisted closed strings as well.

Another related lesson is the existence of a large class of non-supersymmetric configurations which, while unstable, do not "decay to nothing", as a class of non-SUSY models without massless fermions are known to do [60][99][62]. Instead, they decay via a relatively well-controlled weakly coupled process to stable supersymmetric configurations. It would be very interesting to understand the fate of *compact* non-supersymmetric orbifolds of the superstring with massless fermions, particularly since as we discussed the time-dependent physics in the compact case is very similar to that of a cosmology heading toward a big crunch singularity.

In terms of model-building, these results, while mostly negative for supersymmetry breaking, at least may help direct attention to more stable possibilities than geometrical orbifolds for breaking SUSY. The fact that the noncompact models decay to SUSY spaces provides a new indication of the intrinsic role of SUSY within the theory. Again, the question of the fate of compact examples which are most relevant for phenomenological model-building is still open.

Finally, it would be interesting to extend these results to other cases, such as intersecting ALE spaces probed by different combinations of D-branes, and the type I theory. In particular, dualities suggest that the case of singular ALE spaces intersecting at angles introduces novel phenomena [100], and it will be interesting to see if our techniques in this chapter can provide insight into this case (or a deformation of it).

Noncompact tachyonic orbifolds of the heterotic string may have a similar fate to those we discussed here, but in that case there are no D-brane probes available to study the substring regime. The heterotic case will require an understanding of the dynamics of the vector bundle formed by the gauge bosons as well as the configuration of dilaton and metric. Under RG flow the cases with a standard embedding of the orbifold action into the gauge group will behave as our models here; it would be interesting to study also the time-dependent on-shell spacetime solutions in the heterotic string.

# 4. Decapitating Tadpoles

## 4.1 Motivation and Summary

Consider a flat space field theory or string theory with one or more classically massless scalars. After supersymmetry breaking, these scalars (and the trace of the graviton) typically develop tadpoles at generic points on the classical moduli space. As a result, perturbation theory around generic points on the classical moduli space does not produce a sensible S-matrix. This is because the zero-momentum tadpole can attach itself to any diagram by the massless propagator,

$$\frac{1}{k^2}|_{k=0} = \infty, \tag{4.1.1}$$

rendering all amplitudes quantum mechanically divergent.

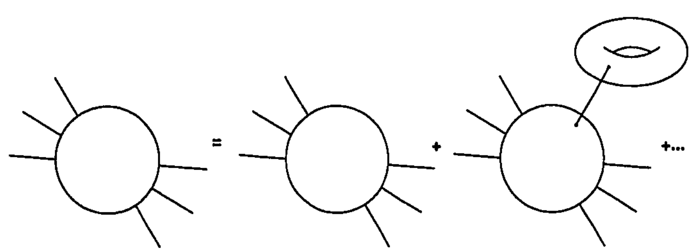


**Fig. 2:** In the presence of tadpoles, the flat space S matrix does not exist due to divergences.

---

This chapter is reprinted, from Allan Adams, John McGreevy and Eva Silverstein, "Decapitated Tadpoles", hep-th/0209226

This IR divergence is usually interpreted as a signal that one must shift the massless field to an extremum of the radiatively generated effective potential. In string theory, this is accomplished by adding the corresponding vertex operator to the worldsheet action [101][102](and *e.g.* [103][104][105]). The equations of motion satisfied by the shifted field can be deduced cleanly from the condition that BRST trivial modes decouple in the string S-matrix [106][107].[24]

We would like to suggest that there is another way to construct a perturbatively consistent (*i.e.* unitary) theory beginning with this classical background. Instead of shifting the massless fields, we will consider changing their propagators. For example, for scalars, we will consider (an IR and UV regulated version of)

$$\frac{-i}{k^2+i\epsilon} \rightarrow \frac{-i(1+F(k))}{k^2+i\epsilon} \tag{4.1.2}$$

where $F(k)$ is chosen to preserve unitarity (and in string theory, worldsheet consistency conditions) while satisfying $F(0)=-1$ in order to cancel the contribution of the zero mode.[25] This effectively changes the equations of motion for the field whose tadpoles we are decapitating, so that any point on the classical moduli space becomes a solution of the deformed equations of motion.

This change is effected in string theory by the perturbative application of the following non-local string theory (NLST) [108][109] deformation of the worldsheet action (again to be regulated in the IR and UV in a manner to be explained in detail in the body of the chapter)

$$\delta S_{ws} = \int \frac{d^dk}{(2\pi)^d}\frac{F(k)}{k^2+i\epsilon}\int V^{(k)}\int V^{(-k)} \tag{4.1.3}$$

[24] In the case where the scalar being shifted to its extremum is the ubiquitous dilaton, this often leads to either a trivial S-matrix, in the case that the string coupling is driven to zero, or a background which is not well described by perturbation theory, in the case that the dilaton is driven to strong coupling in some region of spacetime. In backgrounds of recent interest that fix the dilaton at a nonzero value via flux stabilization or nongeometrical monodromies, this problem may be avoided (though so far in those cases spacetime techniques have proven more practical than worldsheet analysis).

[25] Note that the tadpole only sources the zero mode ($\int d^dx\lambda_1\phi(x)=\lambda_1\phi_0$), as is clear diagrammatically from the fact that energy momentum conservation forces the tadpole propagator to $k=0$.

where $F(k)$ is chosen to have support only on-shell, on the cone $k^2 = 0$. For simplicity, we will in fact take $F(k)$ to only have support at $k = 0$, though for scalars there may be other options, and will define it as the limit of a smooth function. Here, $\int V$ is an integrated vertex operator; the two factors of the bilocal product can be inserted on the same Riemann surface or on otherwise disconnected surfaces. Diagrammatically, each propagator line is thus replaced by the right hand side of (4.1.2), so here is the basic mechanism for removal (which we will refer to as "decapitation") of tadpoles:

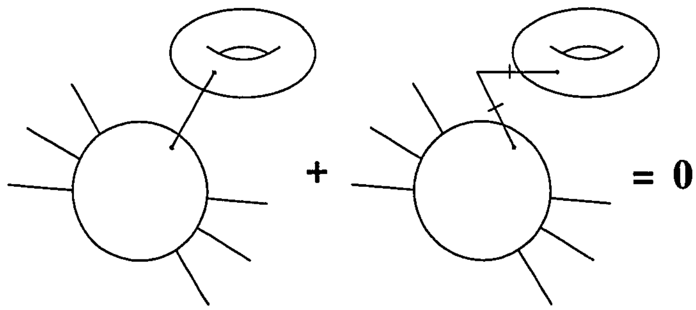


**Fig. 3:** Cancellation of tadpole divergence via deformation of propagator. The wedge denotes the contribution of the $F(k)$ term from (4.1.3) in (4.1.2).

In addition to decapitating dilaton and moduli scalar tadpoles, we will decapitate the tadpole associated with the trace of the zero momentum graviton in a similar way.

At the same time that we decapitate the tadpole, we remove the zero modes of the massless scalars and graviton from the set of external states we consider in the S matrix. More generally we will focus on the physical S matrix with generic incoming momenta or with external states constructed from smooth wavepackets. In string theory, this is accomplished by rescaling the vertex operators describing external states in our S-matrix by

$$V^{(p)} \to \sqrt{1+F(p)}\, V^{(p)}. \tag{4.1.4}$$

We will choose $F(k)$ so that (4.1.3) does not contribute to S-matrix elements except via its cancellation of the massless tadpoles. This naively ensures the perturbative unitarity of the resulting theory, provided that the tadpole-free diagrams in the original theory satisfy the cutting rules; this is manifest in simple field theoretic examples and is thought to hold in superstring perturbation theory.

*However, in string theory, simply removing the divergences is not enough to ensure the perturbative unitarity of the resulting diagramatic expansion, as pointed out in the context of this construction by Joe Polchinski [110]. In amplitudes with BRST trivial vertex operators, the undeformed theory has a finite contribution from the tadpole which is not cancelled by our deformation (4.1.3) as it stands. Therefore the claims of consistency made in the remainder of this chapter on the string theory case based on (4.1.3) alone are wrong. An additional set of NLST deformations which cancel the BRST anomaly as well as the divergences are under investigation to see whether they lead to a fully consistent theory. While the procedure outlined in the remainder of this chapter does not result in a unitary string S-matrix for the string theory case, it is worth noting that this problem does not arise for the field theory case of our procedure.*

As we will explain in detail in the bulk of the chapter, this effectively removes the *spacetime average* of the tadpole for the field in a radiatively stable way, while retaining the quantum-generated self energy for nonzero-momentum modes, including mass renormalization lifting moduli. In simple examples (where the tadpole *is* constant in spacetime) this leads to a nontrivial nonsupersymmetric perturbative S-matrix in flat space. We will study this explicitly for theories for which the tadpole is generated perturbatively.[26] The S matrix so constructed agrees at tree level with the classical S matrix of the undeformed theory, but exists quantum mechanically (at least in perturbation theory). In this S-matrix the fluctuating (nonzero) modes of the moduli are lifted, while the zero mode values (VEVs) of the moduli constitute parameters (couplings) on which the S-matrix amplitudes depend.

In quantum field theory, the perturbative S matrix we construct this way is equivalent (for external states carrying generic momenta or arranged into smooth wavepackets) to that which one would obtain from simply fine tuning away order by order the linear term in the potential expanded about any value for the VEV of the scalar field (or fine tuning away the cosmological constant in the case of gravity). Such a prescription would not be radiatively stable. Our prescription of a nonlocal shift in the propagator is radiatively stable. So, by enlarging the space of possible backgrounds to include nonlocal deformations, one can realize in a radiatively stable

[26] We expect that similar results will hold in situations with dynamical supersymmetry breaking.

manner a system which would otherwise require unnatural fine tuning. In perturbative string theory, one cannot directly fine tune the spacetime effective action in any case, but the decapitation prescription (4.1.3) can be implemented directly and again provides the same effect in a radiatively stable way. It is also worth noting that it seems likely that the full theory in the presence of $F(k)$, including the possibility of expanding around backgrounds other than flat space, is not equivalent to that which one would obtain from fine tuning away the tadpole.

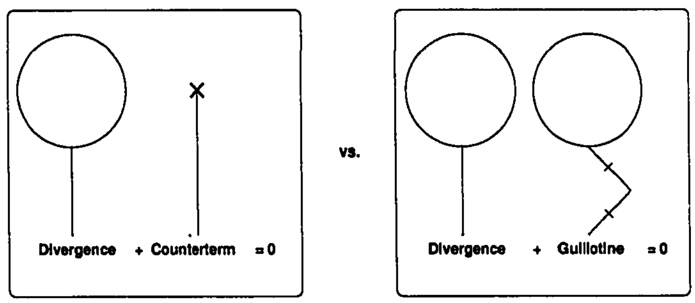


**Fig. 4:** A counterterm for the tadpole requires delicate order-by-order fine-tuning, and depends critically on the UV cutoff. By contrast, decapitation automatically generates contributions cancelling the tadpoles to all orders once the tree level deformation has been specified, and thus does not involve fine tuning.

Even if we focus on the radiatively stable description in terms of the modified tree-level propagator, we cannot regard this prescription as a solution to the cosmological constant problem per se since in the real world the tadpole is not constant in spacetime. Our prescription removing the zero mode does not address the issue of phase transitions (variation in time) and does not cancel the cosmological term in different localized spatial domains (variation in space) [111]. Indeed, one of the appealing features of our construction is that the metric responds normally to localized sources of stress-energy; it is only the tadpoles due to the cosmological term which are removed by the procedure. It will be interesting to explore more systematically the space of consistent IR modifications, and to try to implement in string theory deformations with a better chance of solving the real-world vacuum energy problem.

Finally, it should be mentioned that, although we will argue for perturbative consistency (unitarity) of our S-matrix, we will have nothing to say here about nonperturbative stability and consistency.

Our argument may appear at odds with standard assumptions about the unity and predictivity of string theory, which are supported by some spectacular results of recent years. Ordinary string/M theory has been unified significantly by string dualities, and formulated nonperturbatively in some backgrounds by matrix theory and AdS/CFT. However, these beautiful results, while conceptually unifying the framework, have not yet rendered the theory highly predictive. Indeed, the space of a priori possible string phenomenologies has grown tremendously with the advent of nonperturbative gauge symmetries, D-branes, and dual descriptions of large N gauge sectors; focusing on elegant possibilities such as [112] may be well motivated from phenomenological considerations and simplicity but has not yet been seen as a prediction of the full theory, which can apparently accommodate arbitrarily large gauge groups and matter content. In addition, the different backgrounds of the theory, while mathematically arising from a unified framework, may not be physically connected due to their very different UV and/or IR behavior [113][114]. In the context of AdS/CFT the equivalence of quantum field theory and string theory shows that string theory need not be more predictive than field theory. In the context of string compactification there is growing evidence that many quantities in the low energy theory can be effectively tuned by choosing the background [115][116][117]. The most urgent issue in evaluating a potential new class of backgrounds of string theory is its physical consistency. The question of vacuum selection in the full quantum theory is an issue that must certainly be addressed but may well fall outside the scope of perturbation theory. In any case, if our backgrounds can ultimately be eliminated by some concrete physical consistency requirement going beyond those we address in this chapter, it would serve as further evidence for the unity and predictivity of string theory.

Regardless, our proposal, which will be checked in detail the bulk of this chapter, may seem outlandish on first sight. Let us begin therefore by sharing some of the motivations leading to this idea, before embarking on a systematic analysis of our prescription and its physical features.

### 4.1.1 Motivation from AdS/CFT double-trace couplings

Bilocal deformations of the general form of (4.1.3), namely

$$\delta S_{ws} \sim \sum_{I,J} c_{IJ} \int V^{(I)} \int V^{(J)} \tag{4.1.5}$$

have been derived perturbatively on the string theory side of AdS/CFT dual pairs perturbed by double trace deformations [108][109]. In some AdS/CFT examples [70][118][119][120][121][122], running marginally-relevant double-trace couplings on the field theory side are generated dynamically [57][71][121][122][123] and affect some amplitudes in the theory at large $N$ [108][57].

On the field theory side, the space of couplings includes both single-trace and arbitrary multitrace deformations. These couplings are all on the same footing in field theory (aside from their effect on the structure of the 't Hooft expansion). In specifying a field theory, one chooses a renormalization group trajectory accounting for the behavior of all the couplings. Depending on how one organizes the perturbation expansion, this may involve cancelling divergent amplitudes with counterterms. The coefficients of these counterterms are determined by appropriate renormalization conditions.

Applying the dictionary of [108], this suggests that one should enlarge the space of string backgrounds one considers to include those deformed from ordinary string theory by perturbations of the form (4.1.5). As in field theory, and in the case of local deformations of string theory, appropriate consistency conditions will restrict this space of backgrounds to a physical subspace.

Moreover, in the context of AdS/CFT, UV divergences requiring counterterms on the field theory side map to IR divergences on the string theory side. These IR divergences may therefore entail a corresponding renormalization prescription, including contributions of the form (4.1.5) required to cancel divergences, similarly to the way counterterms for double-trace couplings cancel UV divergences on the field theory side [124][125].

This idea is difficult to apply directly in the context of AdS/CFT with dynamically generated double-trace interactions in perturbation theory, because of the usual difficulty involved in describing the string theory side at large curvature. In this chapter, we will take this as motivation and apply these ideas directly to flat space string theory, studying the deformation of the form (4.1.3) and placing on its coefficient $F(k)$ appropriate "renormalization conditons" to ensure the finiteness and consistency of the resulting S matrix.[27]

### 4.1.2 Outline of the chapter

In section 2 we will present our prescription in detail and show how it cancels tadpole divergences in a radiatively stable manner and lifts the nonzero modes of the moduli. In section 3 we will address the question of other effects of the deformation, and show that the deformation does not contribute for generic external momenta (and therefore smooth wavepackets) to S-matrix elements except via its cancellation of massless tadpoles. This in particular ensures spacetime unitarity and Lorentz invariance of the resulting S-matrix, given plausible assumptions about superstring perturbation theory. In section 4 we will assemble and discuss some basic physical features of the construction, and discuss many future directions.

### 4.1.3 Related work

The notion of modifying gravity in the IR and generalizing renormalization to that context is an old idea which has also been explored recently in [127][128][129][111][130]. The work [127][129] has pursued the possibility of a consistent modification of gravity in the IR arising in a brane configuration in a higher dimensional bulk spacetime in the presence of an Einstein term with large coefficient on the brane worldvolume. The work [111] has provided many insights into the requirements an IR modification of gravity must satisfy in order to be able to address the cosmological constant problem including the effects of phase transitions, while maintaining consistency with known physics, and has proposed concrete examples and mechanisms for satisfying these requirements. It would be interesting if an NLST prescription such as the one we employ here to produce a consistent

[27] Another approach to flat space was adopted in [109], by taking a scaling limit of double-trace deformed AdS/CFT to flat space; there one found divergences from insertion of a bilocal product of 0-momentum vertex operators, not smoothed by an integral over $k$ as we have done in (4.1.3). In [126], NLST deformations naturally arose in describing the squeezed states obtained from particle creation in an asymptotically flat time-dependent background; again this is different because our deformation (4.1.3) involves both positive and negative frequency modes and does not constitute a squeezed state in the original flat space background.

flat-space nonsupersymmetric S-matrix could provide a way to formulate a consistent string-theoretic embedding of the effective field theory examples of [111]. The approach of [128] is complementary to ours in a sense we will remark on in the following. Bilocal worldsheet terms appeared in the work [131] on generating effective field theory from string theory, as well as in the more recent context of the AdS/CFT double trace deformations just reviewed.

## 4.2 The prescription, and cancellation of tadpole divergences

In this section we will lay out in detail the prescription motivated and summarized in the last section.

### *4.2.1 Tadpoles, Divergences, and Regulators*

In string theory, similarly to field theory, the contribution of a massless tadpole to an S-matrix element is by a factor of the zero momentum propagator $G_2(k=0)$ times the one-point function of the massless vertex operator at zero momentum. This multiplies the rest of the diagram given by one insertion of the massless vertex operator at zero momentum along with the insertions of vertex operators describing the external states in the amplitude,

$$\mathcal{A}^h|_{Tadpole} \sim \langle \int V_1^{(k_1)} \dots \int V_n^{(k_n)} \int V^{(0)} \rangle_{\Sigma_{\tilde{h}}} \times G_2(k=0) \times \langle \int V^{(0)} \rangle_{\Sigma_{h-\tilde{h}}}. \quad (4.2.1)$$

This is represented diagramatically as follows:

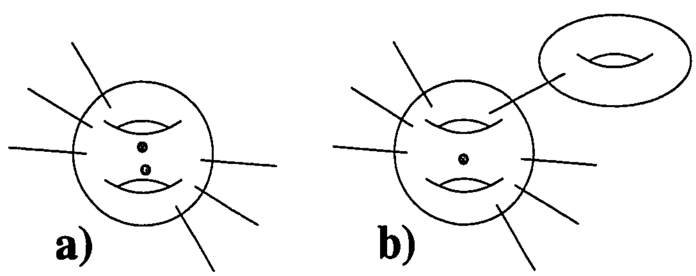


**Fig. 5:** a) a generic $h$-loop amplitude; b) the contribution of the one-loop tadpole to this amplitude is a product of the $(h-1)$-loop amplitude, the one-loop tadpole, and a propagator.

At least in the bosonic string, both the massless tadpole diagram and the remaining contributions to the amplitude can be represented as a collection of field theoretic diagrams constructed from (an infinite number of) hermitian irreducible vertices and propagators [132][133]. In this decomposition, all spacetime IR divergences arise from propagator contributions, not from the effective vertices. In the superstring we expect a similar decomposition to hold, and we will assume this, though to our knowledge this has not been proven. This field theoretic decomposition will be important in the following, particularly for our analysis of unitarity in §3 (as in [133]).

While the tadpole is finite in the absence of tachyons (thanks to the soft UV properties of string loops) the on-shell massless propagator is divergent, and requires regularization. We will discuss two natural ways to do this in the case of scalar fields, one of which generalizes to the graviton. We will work in signature $(+,-,\ldots,-)$, and denote by $d$ the number of dimensions in which the field whose tadpole we are decapitating propagates.

We begin by discussing classically massless scalar fields. In field theory, a simple method of IR regulation, in situations where it is consistent with gauge invariance, is the by-hand introduction of a small mass $\mu$ to be taken to zero at the end of each calculation giving the regulated propagator,

$$\frac{1}{k^2-\mu^2+i\epsilon}. \tag{4.2.2}$$

In string theory, infrared regularization is most directly expressed in terms of a cutoff on the appropriate Schwinger parameter arising in the propagator of the field theory decomposition summarized above (for a discussion of IR regulation in string theory, see e.g. [134][135]). In particular, the closed string propagator is

$$\lim_{T_c\to\infty,T_0\to 0}\sum_{states}\int_{T_0}^{T_c} dT e^{-T(L_0+\tilde{L}_0)} \tag{4.2.3}$$

In flat space, for a state corresponding to a spacetime excitation with mass $m$ and momentum $k$ this gives

$$\begin{aligned} G_2(k;T_c,T_0) &\sim \int_{T_0}^{T_c} dT e^{T(k^2-m^2+i\epsilon)} \\ &= \frac{1}{k^2-m^2+i\epsilon}\left(e^{T_c(k^2-m^2+i\epsilon)}-e^{T_0(k^2-m^2+i\epsilon)}\right) \end{aligned} \tag{4.2.4}$$

Taking $T_c \to \infty$, $T_0 \to 0$ reproduces the usual pole $\frac{1}{k^2-m^2+i\epsilon}$. For finite (but large) $T_c$, as $k^2 \to m^2$ we obtain an IR regulated result

$$G_2(k^2 \to m^2; T_c) \sim T_c \tag{4.2.5}$$

One may define these momentum integrals in appropriate circumstances by Euclidean continuation; in that case, $T_0$ represents a UV cutoff which we may also employ. We can relate the two regulation schemes near the IR limit $k \to 0$ by taking $T_c$ to be a function of $k^2$ and $\mu^2$ given by the solution to

$$\frac{1}{k^2+i\epsilon}\left(e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)}\right) \equiv \frac{1}{k^2-\mu^2+i\epsilon}. \tag{4.2.6}$$

We will mostly consider the hard (*i.e.* $\mu$-independent) $T_c$ regulator, but will use the $\mu$ regulator in sufficiently simple quantum field theory examples.

### 4.2.2 The Deformation

We will consider our deformation both in perturbative quantum field theory and string theory. In the $\mu$ regularization scheme in quantum field theory, we deform the propagator by

$$\frac{iF(k)}{k^2-\mu^2+i\epsilon}. \tag{4.2.7}$$

where $F(k)$ will be specified shortly. One can also employ the Schwinger parameterization and regularization in quantum field theory.

In string theory, in terms of the Schwinger cutoff, we implement the following NLST deformation, adding to the worldsheet action

$$\delta S_{ws} \propto \int d^dk \frac{F(k)}{k^2+i\epsilon}\left(e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)}\right) \int V^{(k)} \int V^{(-k)} \tag{4.2.8}$$

where $\int V$ are integrated vertex operators corresponding to the massless particles whose tadpoles we wish to decapitate.

As in [108][109], we treat this deformation perturbatively. This introduces an infinite array of new diagrams in which the vertex operators in (4.2.8) attach to Riemann surfaces in all possible combinations (including diagrams in which the two members of the bi-local pair of vertex operators sit on different, otherwise disconnected, Riemann surfaces).

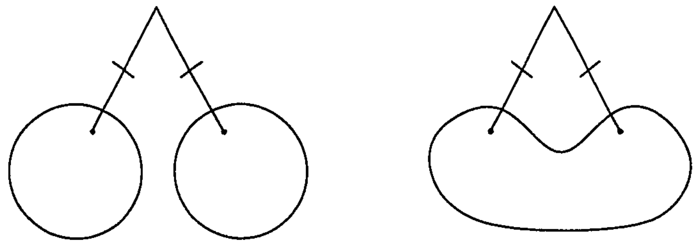

**Fig. 6:** The bi-local deformation can connect two Riemann surfaces or attach to a single Riemann surface.

Because our two vertex operators in the bi-local term carry momentum $k$ and $-k$ respectively, they occur precisely in the same way as the propagator for the corresponding low-energy field, and thus the effect of the deformation is to shift the propagator:

$$G_2(k;T_c) \to \frac{i(1+F(k))}{k^2+i\epsilon}\left(e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)}\right). \tag{4.2.9}$$

In terms of the $\mu$ cutoff, the full momentum-space propagator is parameterized as

$$\frac{i(1+F(k))}{k^2-\mu^2+i\epsilon}. \tag{4.2.10}$$

In identifying our deformation with a shift in the propagator, we have not implemented any extra subtraction prescription (such as normal ordering) to remove divergences when the $V^{(k)}$ approach the $V^{(-k)}$. As we will see in detail in §3, this divergence integrates to zero once we regulate the theory and does not require any such subtraction procedure. (That is, in the field theoretic organization of the string diagrams which we are using [132][133], all such divergences arise in the propagator, which we have regulated.)

$F(k)$ is constrained as follows.

1. In order to preserve conformal invariance of the worldsheet theory, we demand that $F(k)$ vanish when $k$ is off-shell.
2. In order to cancel the divergences coming from tadpoles, we need

$$F(0) = -1 + \mathcal{O}(\frac{1}{T_c}), \tag{4.2.11}$$

and in order to precisely cancel the zero mode propagator, we will require

$$F(0) = -1. \tag{4.2.12}$$

The latter condition ensures that we remove the full zero mode propagator from the tadpole contribution, rather than leaving behind a contribution scaling like an extra massive tadpole as would occur if we kept a nontrivial $\mathcal{O}(\frac{1}{T_c})$ contribution allowed by (4.2.11).

3. We require $F(k)$ to be consistent with unitarity of the resulting perturbative S matrix. The simplest way to ensure this, which we will employ here, is to choose an $F(k)$ such that the deformation of the propagator does not contribute except in precisely cancelling the tadpole contributions, leaving behind tadpole-free diagrams which satisfy the cutting rules.

One choice of $F$ we have found consistent with the criteria 1 – 3 is

$$F(k) = \lim_{\eta\to 0} F_\eta(k) = \lim_{\eta\to 0} \frac{\eta^2}{(k^0 + |\vec{k}| - i\eta)(k^0 - |\vec{k}| - i\eta)}. \tag{4.2.13}$$

In a tadpole diagram, (4.2.7)(4.2.8) appears integrated with the energy-momentum conserving delta function $\delta^d(k)$ for the propagator in the tadpole part of the diagram. This picks out the integrand evaluated at $k = 0$, for which the factor (4.2.13) becomes $\frac{\eta^2}{(-i\eta)^2} = -1$. The entire propagator strictly at $k = 0$ is then (in the Schwinger parameterization)

$$G_2(k=0) = \lim_{T_c\to\infty, T_0\to 0} (1-1)(T_c - T_0) = 0, \tag{4.2.14}$$

or, in the massive QFT regularization scheme,

$$(1-1)\frac{i}{-\mu^2 + i\epsilon} = 0, \tag{4.2.15}$$

as depicted in fig. 3. Again, we refer to this mechanism for decoupling the zero mode as decapitating the tadpole.

$F_\eta(k)$ in (4.2.13) can be written as

$$F_\eta(k) = \pi^2(k^0 + |\vec{k}| + i\eta)(k^0 - |\vec{k}| + i\eta)\delta_\eta(k^0 + |\vec{k}|)\delta_\eta(k^0 - |\vec{k}|) \tag{4.2.16}$$

where $\delta_\eta(x) = \frac{1}{\pi}\frac{\eta}{x^2+\eta^2}$ is a regulated Dirac delta distribution. As such, when $F(k)$ is integrated against a smooth function, it vanishes. As we have seen, when integrated against $\delta^d(k)$ (which is of course not smooth at $k = 0$) it is $-1$, so that the deformation cancels the tadpole divergence. We will see that these properties of $F(k)$ imply that its only contribution to physical S-matrix elements (ones at generic external momenta or set up as scattering amplitudes of smooth wavepackets) is precisely its cancellation of the tadpole divergences.

Although we will work with the specific form (4.2.13) for $F(k)$, any choice satisfying criteria 1–3 is suitable. Any such $F$ which preserves Lorentz symmetry will give an identical perturbative S matrix, so any parameters involved in this choice are not physical, at least perturbatively.

We will perform computations with the following order of limits: we first send $\epsilon \to 0$ and $\eta \to 0$, then remove our IR regulator by taking $T_c \to \infty$ (alternatively, $\mu \to 0$). The $\epsilon \to 0$ and $\eta \to 0$ prescriptions are applied integral by integral, diagram by diagram (*i.e.* these limits are taken before summing over infinite series of diagrams). We refer to this regularization scheme as *the padded room.*

This prescription involves two minor subtleties. Before taking $\eta \to 0$, our deformation (4.2.7)(4.2.8) includes off- shell (non-BRST invariant) vertex operators $V^{(\pm k)}$ with $k^2 \neq 0$. Calculating the effects of our deformation perturbatively, as we are doing, thus involves diagrams with insertions of off-shell vertex operators. In tadpole diagrams, energy-momentum conservation projects the deformation onto $k = 0$ so this issue does not arise. In other diagrams, we need to define our prescription and check that the non gauge-invariant contributions vanish as $\eta \to 0$ (the limit we are taking in which $F(k)$ has support only at $k = 0$). Our prescription for the finite $\eta$ theory before taking the limit $\eta \to 0$ is to work in a specific gauge (fixing the worldsheet metric up to moduli to be integrated over) and calculate correlation functions of the (on-shell and off-shell) vertex operators in the worldsheet CFT on this Riemann surface as in [106]. We will see in §3 that the integration over $k$ in (4.2.7)(4.2.8) involves $F(k)$ convolved with a smooth integrand in the regulated theory, so that the deformation makes a vanishing contribution as $\eta \to 0$. This will depend simply on the local behavior of the $V^{(\pm k)}$ near other vertex operators and degenerations of the surface.

In regulating the theory to produce a finite integral over $k$ for general diagrams, note that a UV regulator is also important in intermediate steps of the calculation. For finite $\eta$, the wedge propagator scales as $\eta^2/k^4$ for large $k$, (in the UV), which is not soft enough to prevent UV divergences in the diagrams we are adding with wedge propagators in loops. These must be regulated. Once we regulate in the UV, all such diagrams are proportional to (positive powers of) $\eta$, and these terms all vanish diagram-by-diagram in the UV regulated theory once we impose our limit

($\eta \to 0$). In the Schwinger parameterization, we can regularize in the UV with our parameter $T_0$ in computations in which the loop integrals are defined by Euclidean continuation.[28] Alternatively we can simply cut off the $k$ integrals at some scale $M_{UV}$. We will see in §3 that all such loop contributions will vanish regardless of the details of the choice of UV regulator. (Note that in the tadpole diagrams, the UV behavior is irrelevant since the momentum $k$ is strictly zero.)

This prescription (4.2.12)(4.2.14)(4.2.15) for cancelling divergences caused by radiative tadpoles is reminiscent of the prescription for renormalization of UV divergences via counterterms in quantum field theory. Although our deformation has a large effect in cancelling the divergences from tadpoles, it can be treated perturbatively via (stringy) Feynman diagrams much like counterterms in quantum field theory. In both cases, the (infinite) corrections appear in one to one correspondence with divergences in the uncorrected theory, cancelling them precisely.

### *4.2.3 Radiative corrections: stability and moduli masses*

It is important to ask whether the specific form of $F(k)$ required by the criteria of the previous subsection is preserved by loop corrections. By construction it is immediate that loop corrections to the "head" of the tadpole do not affect the decapitation, which occurs at the level of the "neck" (*i.e.* at the level of the propagator, regardless of the form of the one-point amplitude to which it attaches).

In fact, loop corrections to the propagator itself also preserve the cancellation of divergences. To see that this is the case, take the 1PI self-energy, $\Sigma$, and use it to correct the propagator including the modification (4.2.15) in the tree-level propagator. One finds (for example in the field theoretic regularization scheme)

$$G_{2,Ren}(k;\mu) = \frac{1+F}{k^2-\mu^2}\left(1+\Sigma\frac{1+F}{k^2-\mu^2}+\cdots\right) = \frac{1+F}{k^2-\mu^2-(1+F)\Sigma}. \quad (4.2.17)$$

The fact that $1+F(k)$ remains in the numerator of the corrected propagator clearly shows that the cancellation persists at zero momentum and the renormalization of

[28] In rotating from Lorentzian to Euclidean loop momentum integrals, an extra pole must be included from (4.2.13); however this pole does not contribute anything in our regulated theory, as will become clear in §3.

the propagator does not change the fact that the tadpoles (now with renormalized propagator for the neck) are decapitated.

Furthermore, this exhibits the following important physical feature of our construction. Nonzero modes in (4.2.17) are not affected by $F(k)$, and are subject to generic mass renormalizations included in the quantum self-energy $\Sigma$. For models in which this renormalization produces positive mass squared for all the scalars (*i.e.* models in which the second derivative of the effective potential is positive in all directions about the starting value), the fluctuating modes of the moduli are lifted! One example of this is the $O(16) \times O(16)$ heterotic string, whose one-loop potential energy in Einstein frame is proportional to $+e^{(5/2)\Phi}$. Another example would be a pair of D-branes with a repulsive force between them.

On the other hand, there are models in which some of the moduli have negative mass squared at one loop, leading to tachyonic instabilities for nonzero modes. The resulting striped phases may be interesting to study, but for now let us discard these cases since these instabilities will drive us away from the simplest case of Poincaré invariant flat space. Examples of this latter class of 1-loop tachyonic backgrounds include Scherk-Schwarz compactifications and D-brane–anti-D-brane systems.

In this analysis it is important to follow the padded room regularization prescription specifying that the limit $\eta \to 0$ be taken diagram by diagram. In particular, for finite $\eta$, the right hand side of (4.2.17) has poles in the complex $k$ plane corresponding to solutions of the linearized field equations with exponential growth along the spacetime coordinates $x^{\mu}$.[29] As we take $\eta \to 0$, these solutions revert to oscillating solutions; summing the resulting diagrams then gives the finite result above. If instead we were to sum these diagrams before taking $\eta \to 0$, thereby studying the RHS of (4.2.17) first at finite $\eta$, we would expect divergences arising from these exponentially growing solutions (similar to divergences caused by tachyons in loop diagrams). Importantly, this order of operations is explicitly disallowed in our regularization prescription; the limit $\eta \to 0$ is part of the definition of each diagram and must be taken before doing the sum in (4.2.17). In fact, as we will see in §3, diagram by diagram our deformation does not contribute in loop propagators; $F(k)$ integrated against the rest of the amplitude vanishes unambiguously, diagram by diagram.

[29] We thank the authors of [111] and N. Kaloper and E. Martinec for emphasizing this issue to us.

A related issue is the question of whether nonperturbatively the decapitated theory has other background solutions, different from flat space, with consistent (in particular, unitary) physics. (For example, in the presence of our deformation, could one still start with a solution in scalar field theory with the scalar field rolling down the potential hill and expand around this solution to produce a consistent theory?) If there exist other solutions which are in fact connected physically to our flat space solution, it would be interesting to study nonperturbative dynamics that may select which background will arise naturally when this framework is considered in a cosmological context. This very interesting question we leave for future work.

### *4.2.4 Decapitating the graviton tadpole*

We so far formulated our deformation for massless scalar fields. The tadpole generated for the (trace of the) graviton is the cosmological constant and is of particular interest.[30]

Since the graviton tadpole (cosmological constant) is one of the main motivations for pursuing this direction, we wish to generalize our prescription to a modification of the graviton propagator which cancels its zero mode. In particular, for the procedure under discussion to be useful in a simple closed string example (like the $O(16) \times O(16)$ heterotic string) we need to decapitate the graviton also so as to avoid generating large curvature.

It may also be interesting in some circumstances to decapitate the scalars but shift the gravity background in the standard way to obtain dS or AdS space. That said, we content ourselves in the following to the most simple case of asymptotically flat space, leaving generalizations to future work.

In expanding about flat space, Lorentz invariance implies that the only tadpole contribution from the gravitational sector comes from the trace of the graviton. The trace can be gauged away for nonzero momentum, but at zero momentum the gauge transformation required to do so would not vanish at infinity. The worldsheet manifestation of this is the presence of an extra BRST-invariant vertex operator at zero momentum transforming as a spacetime scalar, which we will denote by $V_{trG}^{(k=0)}$.

[30] We could restrict our attention to scalars by considering tadpoles for the scalars arising in the open string sector on D-branes with broken supersymmetry (see the next subsection).

(This mode is degenerate with but independent from the zero-momentum mode of the dilaton.) Defining

$$V_{trG}^{(k)} \equiv: V_{trG}^{(k=0)} e^{ikX} : \qquad (4.2.18)$$

we add to the worldsheet action

$$\delta S_{ws}^{G} = \int \frac{d^d k}{(2\pi)^d} \frac{F(k)}{k^2 + i\epsilon} \left( e^{T_c(k^2+i\epsilon)} - e^{T_0(k^2+i\epsilon)} \right) \int V_{trG}^{(k)} \int V_{trG}^{(-k)} \qquad (4.2.19)$$

As in the case of the scalar fields, before taking $\eta \to 0$ this involves off-shell vertex operators $V_{trG}^{(\pm k)}$ with $k \neq 0$ included in (4.2.19). Again, we can compute in a fixed gauge and show that these contributions vanish when $\eta \to 0$.

As in the scalar case, this suffices to cancel all tadpole divergences at any loop order. (Note that in contrast to the scalar case, the self-energy of the graviton of course does not include a mass by gauge invariance.) Since we only modified the zero mode of the graviton, we do not expect problems with gauge (diffeomorphism) invariance to be introduced by our prescription; gauge transformations which die at infinity cannot act on the strict zero mode of the graviton. Acting only on the zero mode also ensures that the graviton responds to ordinary local sources of stress-energy in the usual way, as we will exhibit for the S-matrix in §3.

### *4.2.5 Open string examples*

It is worth emphasizing that we may consider tadpoles for scalars independently of gravitons by considering a non-supersymmetric combination of D-branes in a supersymmetric bulk theory. In such a situation, any closed string tadpoles can be absorbed in radial variation of the fields (if the D-branes are at sufficiently high codimension). In order to produce an S- matrix with positive mass squared for the nonzero modes of the scalars, we can for example choose a pair of branes which repel each other at long distance. (Note that we may not choose an attractive potential $V(r) \sim -\frac{1}{r^n}$ such as arises in a simple D-brane-anti-D-brane system since $V''(r) < 0$ in that case; we can instead choose a repulsive potential $V(r) \sim +\frac{1}{r^n}$ which has $V''(r) > 0$.) In such a system we may decapitate the tadpoles for scalars on one or both of the branes (shifting the nondecapitated fields to the appropriate time-dependent solutions describing motion of the corresponding brane).

### *4.2.6 BRST analysis*

In [106][107], the loop corrected equations of motion for massless fields were derived by requiring that BRST trivial modes decouple from string S-matrix elements. One considers a diagram

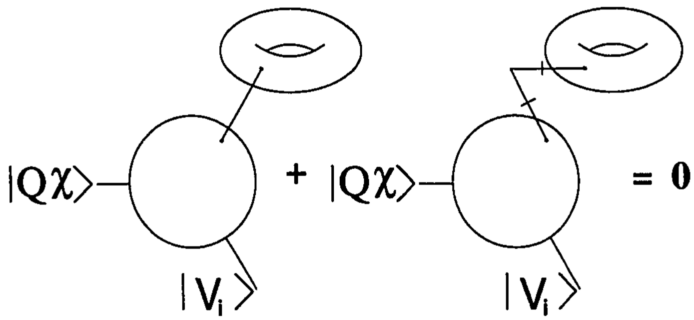


**Fig. 7:** Before decapitation, the tadpole spoils the decoupling of BRST trivial modes. Decapitation adds a diagram precisely cancelling **only the divergent part of** this anomaly: see text.

with one BRST trivial vertex operator $Q_B\chi \equiv \oint_z j_B\chi(z,\bar{z})$ and any number of physical vertex operators $V_i$. One can deform the contour of integration away from $\chi(z,\bar{z})$ so that the BRST operator $Q_B$ acts on the other insertions in the diagram. $Q_B$ kills the remaining (physical) vertex operators. *On a degenerating tadpole neck, it contributes a finite anomalous piece which our deformation as it stands does not cancel. A modification aimed at correcting this is in progress.*

*4.2.7 Effective field theory description*

A useful heuristic way to describe our prescription is to consider the momentum-space effective action for a scalar field $\phi$ whose tadpole we are decapitating. The presence of the discontinuous object $F(k) = \lim_{\eta\to 0} F_\eta(k)$ complicates the analysis of the field theory (the limit $\eta \to 0$ being taken diagram by diagram in the S matrix as we explained in §2.1§2.2). We will ignore all such subtleties in this subsection with the aim of gaining some further intuition for the physics of the deformation. Taking into account the modification we have made to the propagator, this effective action is

$$\int d^dk\left[\phi(k)\left(\frac{k^2-\mu^2}{1+F(k)}-\lambda_2\right)\phi(-k)\right]-\lambda_1\phi(0)-\int d^dk\int d^dk'\lambda_3\phi(k)\phi(k')\phi(-k-k')-\dots \tag{4.2.20}$$

This leads to the equation of motion

$$\phi(-k) = \lambda_1\delta^d(k)\frac{1}{\frac{k^2-\mu^2}{1+F(k)}-\lambda_2} \tag{4.2.21}$$

plus subleading terms involving the higher ($\lambda_{n>2}$) terms in the effective potential. Because of the $F(0) = -1$ contribution, the right hand side here is of the form

$f(y)\delta(y)$ with $f(0)=0$, so this vanishes. That is, $\phi(0)$ is not forced to shift by the tadpole once we include our modification of the kinetic term (corresponding to our original modification of the propagator).

This description involving a nonlocally modified action may be useful but we will mostly stick to the S-matrix formalism (natural in perturbative string theory) we have been developing.

### *4.2.8 In contrast*

Before returning to the S-matrix description, it is worth noting at this point that our prescription is different from two somewhat similar manipulations that might be confused with it.

*Removing the zero mode by boundary conditions*

First, in field theory one might consider removing the zero mode of a massless field by putting the system in a box with appropriate boundary conditions. For example, consider a scalar field with a tadpole (say a linear potential) in a box. Imposing Dirichlet boundary conditions removes the zero mode. However, since this does not change the basic equation of motion, half of the remaining modes still respond to the linear term in the potential. Adiabatically decompactifying the box therefore leads to an unstable theory.

Decapitation works not by selecting particular solutions of the original equation of motion, but by changing the equations of motion. In our case (4.2.20), there is no linear term for nonzero modes, and hence no instability left in the system once we remove the zero mode by our decapitation prescription. Also, our analysis of decapitation involves a regulation prescription compatible with an S matrix description, whereas introducing a box as an IR regulator would not have this feature.

*String IR modifications*

As discussed in [108], the bilocal deformation $\delta S \sim \int V \int V$ can be obtained by deforming the action locally by

$$\delta S = \int d^2 z \lambda V \tag{4.2.22}$$

and integrating over $\lambda$ with a Gaussian weight.

Recently a modification of string theory has been proposed in [128] which involves considering fluctuating couplings $\lambda$ on the worldsheet. In our case (4.2.22), $\lambda(k)$ is a constant on the worldsheet, whereas in [128], $\lambda = \lambda(z,\bar{z})$ is a fast varying function of the worldsheet coordinates, and in particular explicitly does not include a worldsheet zero mode.

### 4.3 Effects of deformation on general diagrams and unitarity

We have so far established that our modification removes the tadpole divergences associated with massless fields. We must now address the question of what other effects the modification has, and in particular determine whether the S-matrix resulting from our deformation is unitary.

Because of the simplicity of the $F(k)$ we chose for our deformation, we will see in fact that it does not contribute to physical S-matrix amplitudes beyond its cancellation of tadpole divergences, and that unitarity is therefore satisfied.

In particular, as we have seen, $F(k)$ vanishes when integrated against any smooth function (its nonvanishing contribution cancelling the tadpole arises from its integration against a delta function $\delta^d(k)$). The question is then whether the $k$-dependence of the integrand in amplitudes obtained by bringing down powers of (4.2.8) and (4.2.19) is sufficiently smooth, modulo (non-smooth) $\delta^d(k)$ factors coming from tadpole contributions. (Note that we are working with a UV cutoff which ensures no divergence from the UV end of the $k$ integration.) Generic diagrams involving smooth wavepackets integrated over external momenta as well as ordinary loop momentum integrals indeed turn out to have this property in the padded room, *i.e.* in our regularization prescription.

Thus we are interested in the $k$-dependence of amplitudes with insertions of $V^{(\pm k)}$, near potential singularities in the integrand. The $V^{(\pm k)}$ can be slightly off shell before we take the limit $\eta \to 0$, and we define their amplitudes by working in a gauge-fixed worldsheet path integral. The possible singularities in the integrand arise as the $V^{(\pm k)}$ approach other vertex operators $V^{(p_i)}$ or degenerating internal lines carrying momenum $p_i$. In both cases, the behavior is determined locally on the Riemann surface and has the structure $\int \frac{d^2z}{|z|^{2+2k\cdot p_i}} \sim \frac{1}{(k+p_i)^2-m_i^2+i\epsilon}$. As in the UV, these potential divergences are cut off in the IR by our regularization prescription.

#### *4.3.1 Non-1PI contributions*

Let us consider first diagrams for which cutting an $F(k)$ contribution to the propagator (which we will refer to as a "wedge propagator" contribution) breaks the diagram in two. For this non-1PI propagator there are two cases. One is what we have already accounted for: the wedge propagator attaches to the head of a tadpole (with no incoming momentum); in this case the wedge contribution cancels the divergence from the tadpole (in fact the whole massless propagator contribution) by construction. The second case is that the the wedge propagator in question connects to a subdiagram with incoming momenta $q_i$, so that generically there is nonzero momentum $k \equiv \sum_{i=1}^{n} q_i$ flowing through the wedge propagator.

At generic incoming momentum, since $k \equiv \sum_{i=1}^{n} q_i \neq 0$, $F(k)$ does not contribute (since $F(k \neq 0) = 0$). Similarly, if we consider a smooth wavepacket in the incoming momenta $q_i$, the relevant part of the amplitude is

$$\int \prod_i d^d q_i f(q_i) F(\sum_i q_i) \frac{i}{(\sum_i q_i)^2 + i\epsilon} \left( e^{[(\sum_i q_i)^2 + i\epsilon]T_0} - e^{[(\sum_i q_i)^2 + i\epsilon]T_c} \right) \quad (4.3.1)$$

We can change basis in the $q_i$ to obtain an integral over $\sum_i q_i$ (the argument of $F$ in this amplitude); it is then clear that the integrand is sufficiently smooth at $\sum_i q_i = 0$ and because of the convolution with $F$ this amplitude vanishes.[31]

*Forces between D-branes*

One type of one-particle reducible diagram of particular interest is that describing the force between D-branes, so let us study this explicitly. Here we have at leading order

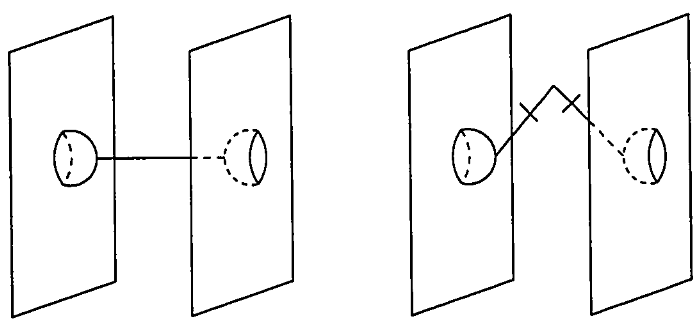

**Fig. 8:** Diagrams contributing to the force between parallel branes.

The correction term is proportional to

$$\int d^n \vec{k}_\perp \frac{F(\vec{k}_\perp)}{\vec{k}_\perp^2 + i\epsilon} \left( e^{-T_c(\vec{k}_\perp^2 + i\epsilon)} - e^{-T_0(\vec{k}_\perp^2 + i\epsilon)} \right) \quad (4.3.2)$$

where $\vec{k}_\perp$ denotes the momenta in the $n$ transverse directions to the D-brane. This contribution vanishes, as can be seen by plugging in the above expression for $F$ in

[31] In fact for normalizable wavepackets $f(q_i)$ is not only smooth at $\sum_i q_i = 0$ but vanishes there.

terms of delta functions (4.2.16). So as expected from the general arguments above, we see explicitly here that the force between gravitational sources such as D-branes is not changed by our decapitation of the tadpoles of the closed strings exchanged.

### *4.3.2 1PI contributions*

Consider a general contribution involving wedges which carry loop momentum (*i.e.* a diagram which is 1PI with respect to cutting at least some of the wedges). We would like to know if this diagram is nonzero (and if it is nonzero, we would like to know if it preserves unitarity of the S-matrix).

Let us focus on one wedge at a time, with momentum $k$. If the Riemann surfaces are smooth and the vertex operators are separated from each other and from the $V^{(k)}$'s, then the integrand will be nonsingular. The potential divergences as $k$ varies come from the degenerations of the Riemann surface approaching the $V^{(\pm k)}$'s and/or the approach of vertex operators to each other. These can always be viewed as IR divergences or poles in the S-matrix. So we can focus on the region of the moduli space of the Riemann surface near IR limits and poles. (Again, note that any UV divergences are cut off.)

Using this, the structure of the potentially singular part of the $k$-dependent integrand in the amplitude is

$$\int d^dkF(k)\frac{i}{k^2+i\epsilon}(e^{T_c(k^2+i\epsilon)}-e^{T_0(k^2+i\epsilon)})$$
$$\prod_i\int d^dp_if(p_i)\frac{1}{(k+p_i)^2-m_i^2+i\epsilon}(e^{T_c((k+p_i)^2-m_i^2+i\epsilon)}-e^{T_0((k+p_i)^2-m_i^2+i\epsilon)}), \tag{4.3.3}$$

times a factor of $T_c$ if the $V^{(\pm k)}$ approach each other (*c.f.* (4.2.5)). Here the $p_i$ are linear combinations of some subset of the momenta (including in general both loop and external momenta). That is, the propagators in (4.3.3) come from pieces of the diagram in which a $V^{(\pm k)}$ line hits a line carrying momentum $p_i$. In the case that $p_i$ is a linear combination of external momenta, then we take the function $f(p_i)$ to be a nontrivial smooth wavepacket.[32] When $p_i$ involves a loop momentum, then

[32] This wavepacket should die off fast enough for large momentum so as not to introduce new UV divergences; we may in any case include a UV cutoff $M_{UV}$ on the external momentum integrals as well as on the internal ones.

$f(p_i)$ encodes any further momentum dependence in the amplitude beyond the pole contribution, and again is a smooth function.

As before, whether this contribution survives is determined by whether the integrand as a function of $k$ can become singular as $k$ varies. This is clearly averted here since the only singularities of the integrand are the poles from the propagators, and for finite $T_c$, the expansion of the exponentials for small $(k+p_i)^2 - m_i^2$ kills the factor of $(k+p_i)^2 - m_i^2$ in the denominator. So we see that the $F$ terms do not contribute in loop (1PI) propagators, just as we found for non-1PI propagators in physical S-matrix amplitudes.

### *4.3.3 New tadpole diagrams which vanish*

It is worth mentioning that the tadpole contributions formally include the following diagrams introduced by our modification:

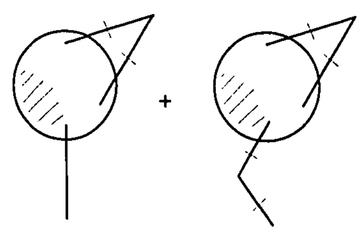

**Fig. 9:** 0+(-0)=0.

However, these diagrams cancel. Not only do they cancel each other exactly via decapitation, but they are separately zero because as we have just derived, the $F$'s do not contribute in loops. This is related to the comment in §2 about the absence of a need for a normal-ordering prescription for the product $V^{(k)}V^{(-k)}$.

### *4.3.4 Explicit evaluation at one loop*

The above general arguments suffice to establish that our deformation proportional to $F$ does not contribute except in decapitating the tadpoles. It is nonetheless instructive to work out explicitly a simple 1-loop example in quantum field theory to illustrate the effect.

Let us consider a one-loop graph involving two virtual massless scalar particles (whose tadpoles we are decapitating) with total momentum $p$ running through it and loop momentum $k$. This is given by (up to an overall real constant)

$$\lim_{\mu\to 0}\lim_{\eta\to 0}\int \frac{d^dk}{(2\pi)^d}\frac{1+F_\eta(k)}{k^2-\mu^2+i\epsilon}\frac{1+F_\eta(k-p)}{(k-p)^2-\mu^2+i\epsilon} \tag{4.3.4}$$

Let us perform the $k^0$ integral by treating it as a contour integral, closing the contour at infinity in the lower half plane. This is possible because the integrand falls off for large $|k^0|$. This picks up the residues of poles at $k^0 = \sqrt{\vec{k}^2+\mu^2-i\epsilon}$ and $k^0 = p^0 + \sqrt{(\vec{k}-\vec{p})^2+\mu^2-i\epsilon}$. (Note that this follows even in the presence of the $F$ terms because we constructed $F_\eta(k)$ to have no poles in the lower half $k_0$ plane.) Letting $E_k \equiv \sqrt{|\vec{k}|^2+\mu^2}$, this gives

$$\begin{aligned} i\int \frac{d^{d-1}\vec{k}}{(2\pi)^{d-1}}\Bigg(&\frac{1+F_\eta(E_k-i\epsilon,\vec{k})}{2\sqrt{\vec{k}^2+\mu^2-i\epsilon}}\frac{1+F_\eta(E_k-i\epsilon-p^0,\vec{k}-\vec{p})}{(E_k-i\epsilon-p^0)^2-(\vec{k}-\vec{p})^2-\mu^2+i\epsilon}+ \\ &+\frac{1+F_\eta(E_{k-p}-i\epsilon,\vec{k}-\vec{p})}{2E_{k-p}-i\epsilon}\frac{1+F(p^0+E_{k-p}-i\epsilon,\vec{k})}{(p^0+E_{k-p}-i\epsilon)^2-\vec{k}^2-\mu^2+i\epsilon}\Bigg) \end{aligned} \tag{4.3.5}$$

For generic external momentum $p$, the denominators in this expression never vanish (for finite $\mu$, which is taken to zero at the very end of the computation) where either of the $F$ factors have support. Hence as argued for general diagrams in the above subsections, here we see explicitly that the deformation does not contribute in loop propagators.

### *4.3.5 Unitarity*

Because the $F$ terms do not contribute to amplitudes except in cancelling massless tadpole contributions, we expect that perturbative unitarity is satisfied. This is manifest in simple quantum field theories such as $\phi^3$ theory expanded about $\phi_0 = 0$: once the diagrams including tadpoles are removed the remaining diagrams satisfy the cutting rules for perturbative unitarity (see figure 9). This result is clear also from the equivalence of the S matrix resulting from decapitation and that obtained by simply fine tuning away the tadpole contribution order by order; the latter also removes the tadpole diagrams leaving behind finite ones satisfying the cutting rules.

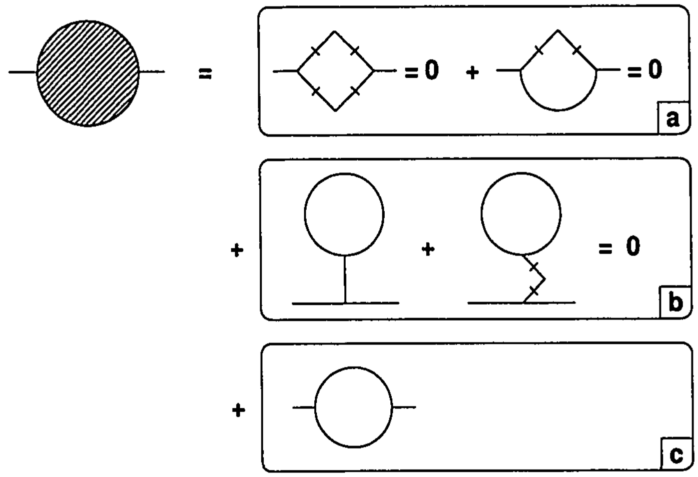


**Fig. 10:** The one loop two-point function in decapitated $\phi^3$ theory. (a) All diagrams with wedges in loops vanish identically. (b) Decapitation ensures that all tadpole diagrams cancel. (c) The remaining diagram respects the cutting rules by construction.

In ordinary bosonic string theory, one can formally argue for perturbative unitarity by decomposing string diagrams into quantum field theory diagrams made from propagators and hermitian vertices (the latter containing no boundaries of moduli space and therefore no poles) (see *e.g.* [132] and the discussion in [133], chapter 9), and then appealing to the field theory argument based on Cutkosky rules. As we have shown in this section, the effect of our deformation is precisely to cancel massless tadpole contributions in this field theory language; the remaining diagrams satisfy the cutting rules as usual if they do in the undeformed theory. So if the superstring perturbation theory works similarly to the bosonic case in this regard, *i.e.* if it is decomposable into diagrams, formed from propagators and vertices, which satisfy the cutting rules for unitarity (which seems plausible though it has not been proved), then we can conclude that our deformation produces a unitary theory.

Since the remaining diagrams describe forces that fall off with distance, we expect cluster decomposition to hold in our theories. (This is again manifest in the perturbative quantum field theory examples where the result is equivalent to that one would obtain tuning away the tadpoles.)

Note that since we have shown that tadpole-free diagrams are unaffected by our modification, the analogous modifications of perturbative supersymmetric theories would have no effect on the physical S matrix. (An interesting future direction is to apply our construction, perhaps field theoretically, to models of low-energy supersymmetry with dynamical (nonperturbative) supersymmetry breaking.)

## 4.4 Discussion

Having argued for the unitarity of our S-matrix, let us now recap and assemble the salient physical features of our system. Our prescription leads to a class of unitary non-supersymmetric perturbative S-matrices in flat space, parameterized by the VEVs of the classical moduli, whose fluctuating modes are generically lifted. We accomplished this by rendering non-dynamical the zero modes of fields (moduli and the graviton) which would otherwise be destabilized by tadpoles, via a modification of the propagator for these fields in the deep infrared. On the worldsheet this modification arises as a perturbative NLST deformation. The tree-level S-matrix is the same as in the unmodified theory; in particular the response of gravity to localized sources of stress-energy is as in ordinary general relativity and has not been removed by our mechanism.

The tadpoles in our examples are uniform over spacetime, and have been effectively removed. It is worth emphasizing that this is not true of the cosmological term in the real world, which is subject to phase transitions (variation in time) as well as possible variation among different spatial domains. Further, we have not so far identified a dynamical mechanism for selecting our theory. In this regard, it will be very interesting to study more systematically the space of consistent IR deformations along these lines.

One result of our analysis which is in some sense disappointing is the presence of parameters descending from the VEVs of the moduli fields. Again, these arise because we can implement our decapitation construction expanding about any point in the classical moduli space having positive 1-loop quadratic terms in the potential for all the moduli. The point in the moduli space from which we start controls the couplings in the S matrix, while the decapitation construction removes the tadpoles which would otherwise generically drive the moduli away from the starting point. Our construction (for any choice of $F(k)$ satisfying our criteria in §2) does not entail any parameters coming from $F(k)$, though it is possible that more general choices of $F(k)$ that do affect non-tadpole diagrams could also lead to consistent perturbative S matrices in flat space or otherwise.

Continuous parameters are of course also seen in flat space SUSY models with moduli spaces and in SUSY and non-SUSY versions of (deformations of) the AdS/CFT correspondence (where the values of the field theory couplings in the UV form a continuum of parameters). The novelty here is the persistence of such a continuum after supersymmetry breaking, in a background preserving maximal (Poincaré) symmetry. (This also has something of an analogue in known backgrounds–in flux compactifications even after supersymmetry breaking, one has a finely spaced set of discrete parameters which can allow one to effectively tune contributions to the low-energy effective action, including the cosmological term [115][116][117][136][137].)

This work leaves open the possibility that our perturbative string theories may not complete to nonperturbatively consistent theories. It was only relatively recently that ordinary perturbative string theories have been (in many cases) understood to fit into a nonperturbative framework via string/M theory duality, matrix theory, and AdS/CFT. We do not have any concrete results on this question; perhaps something could be learned by considering nonperturbative features of decapitation in spontaneously broken gauge theories.[33] Also, it is possible that the assumption we make about the undeformed superstring diagrams satisfying perturbative unitarity relations as in quantum field theory along the lines of the bosonic case [132][133] is wrong because of subtleties associated with superstring perturbation theory. This loophole we find less plausible but in the absence of a proof it certainly remains a possibility.

Although (as in the previously known cases listed above) the parameters add to the lack of predictivity in perturbative string theory, there is a very appealing robust prediction in this class of models. Namely, our construction provides a mechanism for solving the moduli problem, in that for generic values of the parameters in our S-matrix, the fluctuating modes of the moduli are lifted.

While in this chapter we considered perturbative diagrams producing tadpoles, our construction may also apply to situations in which SUSY is broken dynamically at low energies. As an IR effect, we can describe our modification in field theory terms, and low-energy field theoretic SUSY breaking models may be amenable

[33] Work on a related question of whether or not similar modifications might be consistent in the Higgs sector of the Standard Model is in progress [138].

also to such a deformation. (Also, in some circumstances classical SUSY breaking superpotentials may be dual to dynamical ones.)

Similarly we may ask about non-flat backgrounds. It will be interesting to consider whether we can decapitate scalar tadpoles but not the graviton tadpole, leading to a de Sitter or anti de Sitter solution. It is also important to understand much better the space of consistent string backgrounds, in particular to understand how much fine tuning of initial conditions is required to land on the flat space backgrounds we have exhibited in this chapter.

Along similar lines, one may consider IR deformations of this sort which involve different forms for $F(k)$. In particular one can imagine introducing a length scale $L$ above which the decapitation acts nontrivially, rather than simply acting at zero momentum. As in [111], this may bring the approach closer to applying to the real world cosmological term.

An important theme of this subject is the application of renormalization ideas to infrared divergences. Our prescription here is analogous to renormalization via counterterms in that the finite result we obtain arises from cancellation of quantities that diverge as the cutoff is removed. It would be very interesting to pursue the possibility of IR renormalization using instead an analogue of Wilsonian renormalization involving coarse-graining in momentum space.

# References


[1] S. Kachru and E. Silverstein, "4d conformal theories and strings on orbifolds," Phys. Rev. Lett. **80**, 4855 (1998) hep-th/9802183.

[2] A. E. Lawrence, N. Nekrasov and C. Vafa, "On conformal field theories in four dimensions," Nucl. Phys. B **533**, 199 (1998) hep-th/9803015; M. Berkooz, "A supergravity dual of a (1,0) field theory in six dimensions," Phys. Lett. B **437**, 315 (1998) hep-th/9802195.

[3] I. R. Klebanov and A. A. Tseytlin, "A non-supersymmetric large N CFT from type 0 string theory," JHEP**9903**, 015 (1999) hep-th/9901101; I. R. Klebanov and A. A. Tseytlin, "D-branes and dual gauge theories in type 0 strings," Nucl. Phys. B **546**, 155 (1999) hep-th/9811035; I. R. Klebanov and A. A. Tseytlin, "Asymptotic freedom and infrared behavior in the type 0 string approach to gauge theory," Nucl. Phys. B **547**, 143 (1999) hep-th/9812089; I. R. Klebanov, N. A. Nekrasov and S. L. Shatashvili, "An orbifold of type 0B strings and non-supersymmetric gauge theories," Nucl. Phys. B **591**, 26 (2000) hep-th/9909109.

[4] J. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," ATMP **2** (1998) 231, hep-th/9711200.

[5] S. Gubser, I. Klebanov and A. Polyakov, "Gauge Theory Correlators from Noncritical String Theory," Phys. Lett. **B428** (1998) 105, hep-th/9802109; E. Witten, "Anti-de Sitter Space and Holography," ATMP **2** (1998) 253, hep-th/9802150.

[6] S. Kachru, J. Kumar and E. Silverstein, "Vacuum energy cancellation in a non-supersymmetric string," Phys. Rev. D **59**, 106004 (1999) hep-th/9807076.

[7] C. Angelantonj and A. Armoni, "RG flow, Wilson Loops, and the Dilaton Tadpole", Phys. Lett. B **482** 329 (2000); "Non-Tachyonic type 0B Orientifolds, Non-supersymmetric Gauge Theories, and Cosmological RG Flow", Nucl. Phys. B **578** 239 (2000).

[8] M. R. Douglas and G. Moore, "D-branes, Quivers, and ALE Instantons," hep-th/9603167.

[9] A. A. Tseytlin and K. Zarembo, "Effective potential in non-supersymmetric SU(N) x SU(N) gauge theory and interactions of type 0 D3-branes," Phys. Lett. B **457**, 77 (1999) hep-th/9902095.

[10] S. Coleman and E. Weinberg, "Radiative Corrections As The Origin Of Spontaneous Symmetry Breaking," Phys. Rev. D **7** (1973) 1888.

[11] I. R. Klebanov, "Tachyon stabilization in the AdS/CFT correspondence," Phys. Lett. B **466**, 166 (1999) hep-th/9906220.

[12] S. P. de Alwis, J. Polchinski and R. Schimmrigk, "Heterotic Strings With Tree Level Cosmological Constant," Phys. Lett. B **218**, 449 (1989).

[13] I. Antoniadis and C. Kounnas, "Superstring Phase Transition at High Temperature", Phys. Lett. **B261** (1991) 369.

[14] S. Kachru, J. Kumar and E. Silverstein, "Orientifolds, RG flows, and closed string tachyons," Class. Quant. Grav. **17**, 1139 (2000) hep-th/9907038.

[15] E. Witten, "Instability Of The Kaluza-Klein Vacuum," Nucl. Phys. B **195**, 481 (1982); M. Fabinger and P. Horava, "Casimir effect between world-branes in heterotic M-theory," Nucl. Phys. B **580**, 243 (2000) hep-th/0002073;

[16] A. M. Polyakov, "The wall of the cave," Int. J. Mod. Phys. A **14**, 645 (1999) hep-th/9809057; A. M. Polyakov, "String theory and quark confinement," Nucl. Phys. Proc. Suppl. **68**, 1 (1998) hep-th/9711002.

[17] J. A. Minahan, "Asymptotic freedom and confinement from type 0 string theory," JHEP**9904**, 007 (1999) hep-th/9902074.

[18] J. Polchinski and M. J. Strassler, "The string dual of a confining four-dimensional gauge theory," hep-th/0003136.

[19] E. Silverstein and Y. S. Song, "On the critical behavior of D1-brane theories," JHEP**0003**, 029 (2000) hep-th/9912244.

[20] V. Niarchos, "Density of states and tachyons in open and closed string theory," hep-th/0010154.

[21] F. Bigazzi, "RG flows toward IR isolated fixed points: Some type 0 samples", hep-th/0101232.

[22] J. D. Blum and K. R. Dienes, "Strong/weak coupling duality relations for non-supersymmetric string theories," Nucl. Phys. B **516**, 83 (1998) hep-th/9707160.

[23] O. Bergman and M. R. Gaberdiel, "A non-supersymmetric open-string theory and S-duality," Nucl. Phys. B **499**, 183 (1997) hep-th/9701137.

[24] J. A. Harvey, "String duality and non-supersymmetric strings," Phys. Rev. D **59**, 026002 (1999) hep-th/9807213.

[25] S. Kachru and E. Silverstein, "Self-dual nonsupersymmetric type II string compactifications," JHEP**9811**, 001 (1998) hep-th/9808056.

[26] G. T. Horowitz and L. Susskind, "Bosonic M theory," hep-th/0012037.

[27] E. Witten, "Some computations in background independent off-shell string theory," Phys. Rev. D **47**, 3405 (1993) hep-th/9210065; J. A. Harvey, D. Kutasov and E. J. Martinec, "On the relevance of tachyons," hep-th/0003101; D. Kutasov, M. Marino and G. Moore, "Some exact results on tachyon condensation in string field theory," JHEP**0010**, 045 (2000) hep-th/0009148; A. A. Gerasimov and S. L. Shatashvili, "On exact tachyon potential in open string field theory," JHEP**0010**, 034 (2000) hep-th/0009103; S. L. Shatashvili, "On the problems with background independence in string theory," hep-th/9311177; S. L. Shatashvili, "Comment on the background independent open string theory," Phys. Lett. B **311**, 83 (1993) hep-th/9303143.

[28] F. Dowker, J. P. Gauntlett, G. W. Gibbons and G. T. Horowitz, "The Decay of magnetic fields in Kaluza-Klein theory," Phys. Rev. D **52**, 6929 (1995) hep-th/9507143.

[29] M. S. Costa and M. Gutperle, "The Kaluza-Klein Melvin solution in M-theory," hep-th/0012072.

[30] J. Maldacena, J. Michelson and A. Strominger, "Anti-de Sitter fragmentation," JHEP**9902**, 011 (1999) hep-th/9812073.

[31] M. Bershadsky, Z. Kakushadze and C. Vafa, "String expansion as large N expansion of gauge theories," Nucl. Phys. B **523**, 59 (1998) hep-th/9803076.

[32] N. Nekrasov and S. Shatashvili, "On Non-Supersymmetric CFT in Four Dimensions", Phys. Rept. **320**, 127 (1999); E. Silverstein, unpublished.

[33] C. Vafa, "Quantum Symmetries Of String Vacua," Mod. Phys. Lett. A **4** (1989) 1615.

[34] M. R. Garousi, "String scattering from D-branes in type 0 theories," Nucl. Phys. B **550**, 225 (1999) hep-th/9901085.

[35] O. Aharony, M. Berkooz, and E. Silverstein, work in progress.

[36] P. Breitenlohner and D. Z. Freedman, "Positive Energy In Anti-De Sitter Backgrounds And Gauged Extended Supergravity," Phys. Lett. B **115**, 197 (1982).

[37] J. Maldacena and H. Ooguri, "Strings in AdS(3) and SL(2,R) WZW model. I," hep-th/0001053.

[38] D. Kutasov and N. Seiberg, "Number of degrees of freedom, density of states and tachyons in string theory and CFT," Nucl. Phys. B **358**, 600 (1991).

[39] M. R. Douglas and B. Fiol, "D-branes and discrete torsion. II," hep-th/9903031. M. R. Douglas, "D-branes and discrete torsion," hep-th/9807235.

[40] D. Berenstein and R. G. Leigh, "Discrete torsion, AdS/CFT and duality," JHEP**0001**, 038 (2000) hep-th/0001055.

[41] C. Csaki, W. Skiba, and J. Terning, "Beta Functions of Orbifold Theories and the Hierarchy Problem", Phys. Rev. D **61**, 025019 (2000); P. H. Frampton

and W. F. Shively, "Conformal N=0 d=4 Gauge Theories from AdS/CFT Superstring Duality?", Phys. Lett. B **454** 49 (1999).

[42] K. Bardakci, "Dual Models and Spontaneous Symmetry Breaking", Nucl.Phys. B68(1974)331; K.Bardakci and M.B.Halpern, "Explicit Spontaneous Breakdown in a Dual Model", Phys.Rev.D10 (1974)4230; K.Bardakci and M.B.Halpern, "Explicit Spontaneous Breakdown in a Dual Model II: N Point Functions", Nucl. Phys. B96(1975)285; K.Bardakci, "Spontaneous Symmetry Breakdown in the Standard Dual String Model", Nucl.Phys.B133(1978)297.

[43] V. A. Kostelecky and S. Samuel, "On A Nonperturbative Vacuum For The Open Bosonic String," Nucl. Phys. B **336**, 263 (1990);

[44] T. Banks and L. Susskind, "Brane - Antibrane Forces," hep-th/9511194.

[45] A. Sen, "Non-BPS states and branes in string theory," hep-th/9904207.

[46] J. A. Harvey, D. Kutasov and E. J. Martinec, cp "On the relevance of tachyons," hep-th/0003101.

[47] A. A. Gerasimov and S. L. Shatashvili, "On exact tachyon potential in open string field theory," JHEP **0010**, 034 (2000) [hep-th/0009103].

[48] D. Kutasov, M. Marino and G. Moore, "Some exact results on tachyon condensation in string field theory," JHEP **0010**, 045 (2000) [hep-th/0009148].

[49] D. J. Gross and W. Taylor, "Split string field theory. I," JHEP **0108**, 009 (2001) [hep-th/0105059].

[50] L. Rastelli, A. Sen and B. Zwiebach, "Vacuum string field theory," hep-th/0106010.

[51] Owing to a lack of written reviews, for reviews and further references we suggest
http://online.itp.ucsb.edu/online/mp01/sen1/ ,
http://www.slac.stanford.edu/spires/find/hep/www?rawcmd=ti+tachyon%23
.

[52] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, "Strings On Orbifolds," Nucl. Phys. B **261**, 678 (1985).

[53] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, "Strings On Orbifolds. 2," Nucl. Phys. B **274**, 285 (1986).

[54] A. Dabholkar, "Strings on a cone and black hole entropy," Nucl. Phys. B **439**, 650 (1995) [hep-th/9408098].

[55] A. Dabholkar, "Quantum corrections to black hole entropy in string theory," Phys. Lett. B **347**, 222 (1995) [hep-th/9409158].

[56] D. A. Lowe and A. Strominger, "Strings near a Rindler or black hole horizon," Phys. Rev. D **51**, 1793 (1995) [hep-th/9410215].

[57] A. Adams and E. Silverstein, "Closed string tachyons, AdS/CFT, and large N QCD," Phys. Rev. D **64**, 086001 (2001) [arXiv:hep-th/0103220].

[58] E. J. Martinec and W. McElgin, "String theory on AdS orbifolds," hep-th/0106171.

[59] S. P. de Alwis, J. Polchinski and R. Schimmrigk, "Heterotic Strings With Tree Level Cosmological Constant," Phys. Lett. B **218**, 449 (1989).

[60] E. Witten, "Instability Of The Kaluza-Klein Vacuum," Nucl. Phys. B **195**, 481 (1982).

[61] S. Kachru, J. Kumar and E. Silverstein, "Orientifolds, RG flows, and closed string tachyons," Class. Quant. Grav. **17**, 1139 (2000) [hep-th/9907038].

[62] M. Fabinger and P. Horava, "Casimir effect between world-branes in heterotic M-theory," Nucl. Phys. B **580**, 243 (2000) [hep-th/0002073].

[63] G. T. Horowitz and L. Susskind, "Bosonic M theory," hep-th/0012037.

[64] M. S. Costa and M. Gutperle, "The Kaluza-Klein Melvin solution in M-theory," JHEP **0103**, 027 (2001) [hep-th/0012072].

[65] M. Gutperle and A. Strominger, "Fluxbranes in string theory," JHEP **0106**, 035 (2001) [hep-th/0104136].

[66] E. Witten, "Some comments on string dynamics," hep-th/9507121.

[67] S. Deser, R. Jackiw and G. 't Hooft, "Three-Dimensional Einstein Gravity: Dynamics Of Flat Space," Annals Phys. **152**, 220 (1984).

[68] M. R. Douglas, D. Kabat, P. Pouliot and S. H. Shenker, "D-branes and short distances in string theory," Nucl. Phys. B **485**, 85 (1997) [hep-th/9608024].

[69] M. R. Douglas and G. Moore, "D-branes, Quivers, and ALE Instantons," hep-th/9603167.

[70] S. Kachru and E. Silverstein, "4d conformal theories and strings on orbifolds," Phys. Rev. Lett. **80**, 4855 (1998) [arXiv:hep-th/9802183].

[71] A. A. Tseytlin and K. Zarembo, "Effective potential in non-supersymmetric SU(N) x SU(N) gauge theory and interactions of type 0 D3-branes," Phys. Lett. B **457**, 77 (1999) [arXiv:hep-th/9902095].

[72] H. Ooguri and C. Vafa, "Two-Dimensional Black Hole and Singularities of CY Manifolds," Nucl. Phys. B **463**, 55 (1996) [hep-th/9511164].

[73] N. Berkovits, A. Sen and B. Zwiebach, "Tachyon condensation in superstring field theory," Nucl. Phys. B **587**, 147 (2000) [hep-th/0002211].

[74] F. Dowker, J. P. Gauntlett, D. A. Kastor and J. Traschen, Phys. Rev. D **49**, 2909 (1994) [hep-th/9309075].

[75] F. Dowker, J. P. Gauntlett, S. B. Giddings and G. T. Horowitz, Phys. Rev. D **50**, 2662 (1994) [hep-th/9312172].

[76] M. S. Costa, C. A. Herdeiro and L. Cornalba, "Flux-branes and the dielectric effect in string theory," hep-th/0105023.

[77] P. M. Saffin, "Gravitating fluxbranes," Phys. Rev. D **64**, 024014 (2001) [gr-qc/0104014].

[78] M. Srednicki, "IIB or not IIB," JHEP **9808**, 005 (1998) [hep-th/9807138].

[79] E. Witten, "D-branes and K-theory," JHEP **9812**, 019 (1998) [hep-th/9810188].

[80] I. Ellwood and W. Taylor, "Open string field theory without open strings," Phys. Lett. B **512**, 181 (2001) [hep-th/0103085].

[81] I. Ellwood, B. Feng, Y. He and N. Moeller, "The identity string field and the tachyon vacuum," JHEP **0107**, 016 (2001) [hep-th/0105024].

[82] P. Yi, "Membranes from five-branes and fundamental strings from Dp branes," Nucl. Phys. B **550**, 214 (1999) [hep-th/9901159].

[83] C. Vafa, "Quantum Symmetries Of String Vacua," Mod. Phys. Lett. A **4**, 1615 (1989).

[84] E. Witten, "Phases of $N = 2$ theories in two dimensions," Nucl. Phys. B **403**, 159 (1993) [hep-th/9301042].

[85] E. Silverstein and Y. S. Song, "On the critical behavior of D1-brane theories," JHEP **0003**, 029 (2000) [hep-th/9912244].

[86] M. Li and T. Yoneya, "D-particle dynamics and the space-time uncertainty relation," Phys. Rev. Lett. **78**, 1219 (1997) [hep-th/9611072].

[87] A. B. Zamolodchikov, "Irreversibility Of The Flux Of The Renormalization Group In A 2-D Field Theory," JETP Lett. **43**, 730 (1986) [Pisma Zh. Eksp. Teor. Fiz. **43**, 565 (1986)].

[88] J. Polchinski, "Scale And Conformal Invariance In Quantum Field Theory," Nucl. Phys. B **303**, 226 (1988).

[89] T. Banks and L. J. Dixon, "Constraints On String Vacua With Space-Time Supersymmetry," Nucl. Phys. B **307**, 93 (1988).

[90] D. Anselmi, M. Billo, P. Fre, L. Girardello and A. Zaffaroni, "ALE manifolds and conformal field theories," Int. J. Mod. Phys. A **9**, 3007 (1994) [hep-th/9304135].

[91] C. V. Johnson, "D-brane primer," hep-th/0007170.

[92] C. LeBrun, "Counterexamples to the Generalized Positive Action Conjecture", Comm. Math. Phys. **118**, 591 (1988).

[93] G. Horowitz, work in progress

[94] E. Witten, "A Simple Proof Of The Positive Energy Theorem," Commun. Math. Phys. **80**, 381 (1981).

[95] G. W. Gibbons and C. N. Pope, "Positive Action Theorems For Ale And Alf Spaces," ICTP-81-82-20.

[96] R. Gregory, J. A. Harvey and G. Moore, "Unwinding strings and T-duality of Kaluza-Klein and H-monopoles," Adv. Theor. Math. Phys. **1**, 283 (1997) [hep-th/9708086].

[97] T. H. Buscher, "A Symmetry Of The String Background Field Equations," Phys. Lett. B **194**, 59 (1987); "Path Integral Derivation Of Quantum Duality In Nonlinear Sigma Models," Phys. Lett. B **201**, 466 (1988).

[98] O. Bergman and M. R. Gaberdiel, "Dualities of type 0 strings," JHEP **9907**, 022 (1999) [hep-th/9906055].

[99] F. Dowker, J. P. Gauntlett, G. W. Gibbons and G. T. Horowitz, "The Decay of magnetic fields in Kaluza-Klein theory," Phys. Rev. D **52**, 6929 (1995) [hep-th/9507143].

[100] A. Uranga, work in progress; K. Dasgupta, private communication.

[101] W. Fischler and L. Susskind, "Dilaton Tadpoles, String Condensates And Scale Invariance," Phys. Lett. B **171**, 383 (1986).

[102] W. Fischler and L. Susskind, "Dilaton Tadpoles, String Condensates And Scale Invariance. 2," Phys. Lett. B **173**, 262 (1986).

[103] S. R. Das and S. J. Rey, "Dilaton Condensates And Loop Effects In Open And Closed Bosonic Strings," Phys. Lett. B **186**, 328 (1987); S. J. Rey, "Off-Shell Prescription For Dilaton Tadpoles," Phys. Lett. B **203**, 393 (1988); S. J. Rey, "Unified View Of BRST Anomaly And Its Cancellation In String Amplitudes," Nucl. Phys. B **316**, 197 (1989).

[104] C. G. Callan, C. Lovelace, C. R. Nappi and S. A. Yost, "Loop Corrections To Superstring Equations Of Motion," Nucl. Phys. B **308**, 221 (1988).

[105] H. Ooguri and N. Sakai, "String Multiloop Corrections To Equations Of Motion," Nucl. Phys. B **312**, 435 (1989).

[106] J. Polchinski, "Factorization Of Bosonic String Amplitudes," Nucl. Phys. B **307**, 61 (1988).

[107] H. La and P. Nelson, "Effective Field Equations For Fermionic Strings," Nucl. Phys. B **332**, 83 (1990).

[108] O. Aharony, M. Berkooz and E. Silverstein, "Multiple-trace operators and non-local string theories," JHEP **0108**, 006 (2001) [arXiv:hep-th/0105309].

[109] O. Aharony, M. Berkooz and E. Silverstein, "Non-local string theories on $AdS_3 \times S^3$ and stable non-supersymmetric backgrounds," Phys. Rev. D **65**, 106007 (2002) [arXiv:hep-th/0112178].

[110] Guardian of the Purity of String Theory, Private Communication

[111] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, G. Gabadadze, to appear.

[112] E. Witten, "Strong Coupling Expansion Of Calabi-Yau Compactification," Nucl. Phys. B **471**, 135 (1996) [arXiv:hep-th/9602070].

[113] O. Aharony and T. Banks, "Note on the quantum mechanics of M theory," JHEP **9903**, 016 (1999) [arXiv:hep-th/9812237].

[114] T. Banks, "On isolated vacua and background independence," arXiv:hep-th/0011255.

[115] R. Bousso and J. Polchinski, "Quantization of four-form fluxes and dynamical neutralization of the cosmological constant," JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].

[116] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," arXiv:hep-th/0105097.

[117] A. Maloney, E. Silverstein and A. Strominger, "De Sitter space in noncritical string theory," arXiv:hep-th/0205316.

[118] A. E. Lawrence, N. Nekrasov and C. Vafa, "On conformal field theories in four dimensions," Nucl. Phys. B **533**, 199 (1998) [arXiv:hep-th/9803015].

[119] M. Bershadsky, Z. Kakushadze and C. Vafa, "String expansion as large N expansion of gauge theories," Nucl. Phys. B **523**, 59 (1998) [arXiv:hep-th/9803076].

[120] I. R. Klebanov and A. A. Tseytlin, "A non-supersymmetric large N CFT from type 0 string theory," JHEP **9903**, 015 (1999) [arXiv:hep-th/9901101].

[121] E. Witten, "Multi-trace operators, boundary conditions, and AdS/CFT correspondence," arXiv:hep-th/0112258.

[122] M. Berkooz, A. Sever and A. Shomer, "Double-trace deformations, boundary conditions and spacetime singularities," JHEP **0205**, 034 (2002) [arXiv:hep-th/0112264].

[123] M. Strassler, to appear

[124] V. Balasubramanian and P. Kraus, "A stress tensor for anti-de Sitter gravity," Commun. Math. Phys. **208**, 413 (1999) [arXiv:hep-th/9902121].

[125] S. Thomas, in progress.

[126] O. Aharony, M. Fabinger, G. T. Horowitz and E. Silverstein, "Clean time-dependent string backgrounds from bubble baths," JHEP **0207**, 007 (2002) [arXiv:hep-th/0204158].

[127] G. Dvali, G. Gabadadze and M. Shifman, "Diluting cosmological constant in infinite volume extra dimensions," arXiv:hep-th/0202174.

[128] D. Friedan, "A tentative theory of large distance physics," arXiv:hep-th/0204131.

[129] G. Dvali, G. Gabadadze and M. Shifman, "Diluting cosmological constant via large distance modification of gravity," arXiv:hep-th/0208096.

[130] J. W. Moffat, arXiv:hep-th/0207198.

[131] A. A. Tseytlin, "On 'Macroscopic String' Approximation In String Theory," Phys. Lett. B **251**, 530 (1990).

[132] B. Zwiebach, "Closed string field theory: Quantum action and the B-V master equation," Nucl. Phys. B **390**, 33 (1993) [arXiv:hep-th/9206084].

[133] J. Polchinski, "String Theory. Vol. 1: An Introduction To The Bosonic String," *Cambridge, UK: Univ. Pr. (1998) 402 p.*

[134] E. Kiritsis and C. Kounnas, Nucl. Phys. Proc. Suppl. **41**, 331 (1995) [arXiv:hep-th/9410212].

[135] E. Kiritsis and C. Kounnas, Nucl. Phys. B **442**, 472 (1995) [arXiv:hep-th/9501020].

[136] S. Kachru, X. Liu, M. B. Schulz and S. P. Trivedi, "Supersymmetry changing bubbles in string theory," arXiv:hep-th/0205108.

[137] A. R. Frey and J. Polchinski, "N = 3 warped compactifications," Phys. Rev. D **65**, 126009 (2002) [arXiv:hep-th/0201029].

[138] A. Adams, O. Aharony, J. McGreevy, E. Silverstein, ..., work in progress