# A MEASUREMENT OF THE RESONANCE PARAMETERS OF THE NEUTRAL INTERMEDIATE VECTOR BOSON* 

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* Ph.D. thesis


## Abstract

This thesis presents a measurement of the $Z^{0}$ Boson resonance parameters. The measurement was performed at the Stanford Linear Collider using the Mark II detector. Based on a sample of 480 Hadronic and Leptonic decays, the mass is found to be $91.14 \pm 0.12 \mathrm{GeV} / \mathrm{c}^{2}$, the total width is $2.42_{-0.35}^{+0.45} \mathrm{GeV}$, and the peak cross section for all Hadronic events, and for Muon and Tau events with $\cos \theta_{\text {Thrust }}<0.65$ is $45 \pm 4 n b$.

By constraining the visible width to the Standard Model value for 5 quarks and 3 charged leptons, and allowing the invisible width to be a parameter, the width to invisible decay modes is found to be $0.46 \pm 0.10 \mathrm{GeV}$. Assuming this width comes from massless neutrinos, this measurement corresponds to $2.8 \pm 0.6$ neutrino species. This measurement sets an upper limit of 3.9 neutrino generations at the $95 \%$ confidence level, ruling out a fourth generation of Standard Model neutrinos at this level.

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## Chapter 1

## Introduction

The Standard Model of electroweak interactions proposed by Glashow, Weinberg, and Salam [1] has been remarkably successful in describing a wide range of experiments designed to measure electroweak interactions [2]. One of the predictions of the Standard Model is the existence of the Intermediate Vector Bosons which mediate the weak force (The $W$ and $Z^{0}$ bosons). The discovery in 1983 of the $W$ and $Z^{0}$ bosons by UA1 $[3,5]$ and UA2 $[4,6]$ at CERN confirmed the Standard Model prediction for the structure of the electroweak gauge bosons. The mass of the $Z^{0}$ boson is an input parameter to the Standard Model, and hence the relatively large errors on the $Z^{0}$ mass as determined at the CERN $p \bar{p}$ collider ( $M_{Z}=93.1 \pm 1.0 \pm 3.1$ at UA1 [7] and $M_{Z}=91.5 \pm 1.2 \pm 1.7$ at UA2 [8] where the first error is statistical and the second error is due to uncertainty in the energy scale of the detectors) limited the sensitivity of tests looking for deviations in the predictions of the Standard Model.

Direct production of the $Z^{0}$ Boson in electron positron collisions provides both an accurate measurement of a fundamental input parameter (the mass of the $Z^{0}$ ), and a sensitive test of Standard Model predictions for the couplings of matter to the gauge
bosons. In addition, the decays of the $Z^{0}$ provide a probe of the particle content of the Standard Model since the $Z^{0}$ lineshape depends on the number of particles which the $Z^{0}$ can decay into. Specifically, accurately determining the lineshape gives a measurement of how many species of light neutrinos the $Z^{0}$ can decay into. The Stanford Linear Collider (SLC) has provided 480 events for studying the properties of $Z^{0}$ decays in a run from April 1989 to October 1989. These data will constitute the basis for the measurements presented here.

This thesis describes a precise measurement of the $Z^{0}$ resonance parameters performed using the Mark II detector at the SLC. In chapter 2, a theoretical framework for the expected $Z^{0}$ line shape is presented. In order to unfold the parameters of the $Z^{0}$ resonance, radiative corrections must be applied. Details of how these corrections are performed are also explained.

The SLC and the Mark II detector are very complicated devices. Detailed descriptions of the components of the Mark II used in this measurement are provided in chapter 3. The principle component of the SLC which is used in this analysis is a pair of precision spectrometers for determining the center-of-mass energy $\left(E_{c m}\right)$ of the $e^{+} e^{-}$collisions. The spectrometers are described in detail in this chapter.

Following the description of the experimental apparatus, a discussion of the event selection and background for $Z^{0}$ candidate events is provided in chapter 4. In order to normalize the measurement of the $Z^{0}$ cross section, one compares the cross section for $Z^{0}$ events to the cross section for QED events where the expected cross section is accurately known. A description of the selection and background for these normalizing events is also presented.

After discussing the selection of the data, the actual data, and fits describing the data for a variety of assumptions are given in chapter 5. Descriptions of the fitting
procedure and systematic errors are also provided in this chapter.

## Chapter 2

## Theory

The three parameters of a general resonance curve are its position (related to the mass of the particle), peak height (which depends on the cross section for producing the particle) and width. For the $Z^{0}$, extracting these parameters from the data requires little knowledge of the Standard Model. The Standard Model does, however, play a large role in interpreting the observed resonance parameters. In fact, given the position of the resonance, and a knowledge of the particle content of nature, the Standard Model predicts exactly the height and width of the resonance. The width of the resonance is proportional to the number of species into which the resonance decays such that the more species which are available for the $Z^{0}$ to decay into the wider the resonance becomes. As the width increases, the peak cross section is reduced, hence the peak cross section provides a second quantity which is proportional to the number of species into which the $Z^{0}$ decays. Assuming the validity of the Standard Model one can thus work backwards and predict the number of fundamental particles which interact with the $Z^{0}$ based on the observed resonance parameters. Specifically, one can indirectly measure the number of light neutrinos in the universe by measuring
the peak cross section as well as the width of the resonance.
This chapter reviews the pieces of the Standard Model which are needed for calculation of the production and decay of $Z^{0}$ Bosons in $e^{+} e^{-}$collisions.

The shape of the resonance curve can be substantially altered by the fact that any of the particles participating in the production of $Z^{0}$ Bosons, or produced in the decay of the $Z^{0}$, can radiate either photons or gluons. Radiative Corrections must be applied to the data in order to extract the parameters of the resonance. An analytic form for these corrections is described in this chapter.

The last ingredient of the measurement is setting the overall normalization for the cross section measurement. This is done using elastic $e^{+} e^{-}$scattering (Bhabha scattering). A discussion of the calculation of the Bhabha cross section is presented at the end of this chapter.

### 2.1 Elements of the Standard Model

The Standard Model provides a description of electroweak interactions in terms of a $S U(2) \otimes U(1)$ gauge field theory. The bosonic gauge fields mediate interactions between the fermionic particles of the theory. The couplings between the bosons and the fermions are described in terms of a small number of parameters. Knowledge of these couplings allows one to calculate expected values for observable cross sections.

### 2.1.1 Constituents of the Standard Model

The particle spectrum of the Standard Model is motivated by the observed spectrum of fundamental fermions. For leptons, there is an $S U(2)$ doublet for the left handed
leptons $\binom{\nu_{l}}{l}_{L}$ and a right handed singlet $(l)_{R}$ for the charged lepton. The quarks are represented by a left handed doublet $\binom{q}{q^{\prime}}_{L}$ and two right handed singlets $(q)_{R}$ and $\left(q^{\prime}\right)_{R}$.

This structure is called a family. Three families have been observed so far, they are for the leptons

$$
\left.\begin{array}{c}
\binom{\nu_{e}}{e}_{L} \\
\binom{\nu_{\mu}}{\mu}_{L}  \tag{2.1}\\
(e)_{R}
\end{array} \underset{(\mu)_{R}}{\binom{\nu_{\tau}}{\tau}_{L}} \begin{array}{c}
Q \\
1 / 2
\end{array} \begin{array}{c}
0 \\
-1 / 2
\end{array}\right)-1
$$

and for the quarks

$$
\begin{array}{ccccc}
\binom{u}{d}_{L} & \binom{c}{s}_{L} & \binom{t}{b}_{L} & \begin{array}{c}
1 / 2 \\
-1 / 2
\end{array} & -1 / 3  \tag{2.2}\\
(u)_{R} & (c)_{R} & (t)_{R} & 0 & 2 / 3 \\
(d)_{R} & (s)_{R} & (b)_{R} & 0 & -1 / 3
\end{array}
$$

The column for $T_{3}$ indicates the value of the third component of weak isospin for the members of the doublets. The column for Q gives the charge electric charge, where $Q=T_{3}+Y$. All of these particles have been observed with the exception of the $t$ (top) quark and $\nu_{\tau}$.

The bosons of the theory are the massless photon, the carrier of the electromagnetic field, and the massive vector bosons ( $W^{ \pm}$and $Z^{0}$ ) which mediate the weak
interactions. These bosons are the mass eigenstates of the $S U(2)$ and $U(1)$ fields of the electroweak theory. One writes the mass eigenstates in terms of the $S U(2)$ fields $\left(b_{3}^{\mu}\right)$ and the $U(1)$ fields $\left(Y^{\mu}\right)$ in the following manner: for the neutral fields;

$$
\binom{b_{3}^{\mu}}{Y^{\mu}}=\left(\begin{array}{rr}
\cos \theta_{W} & \sin \theta_{W} \\
-\sin \theta_{W} & \cos \theta_{W}
\end{array}\right)\binom{Z^{\mu}}{A^{\mu}}
$$

while the charged fields are written as;

$$
W_{ \pm}^{\mu}=\frac{1}{\sqrt{2}}\left(b_{1}^{\mu} \mp i b_{2}^{\mu}\right)
$$

where $\theta_{W}$ is called the Weinberg angle.
In addition to these particles, the Standard Model also accommodates a scalar particle, the Higgs. The Higgs provides a mechanism for giving masses to the gauge bosons which are otherwise predicted to be massless and also gives rise to masses for the fermions. The Standard Model provides no prediction for the mass of the Higgs and it has yet to be observed in a wide range of experiments.

### 2.1.2 Couplings of fermions to gauge fields

Given the particle content of the Standard Model, one needs to know the strength of the couplings of the various particles in order to calculate cross sections. The two couplings which are needed to in order to calculate the the cross section for $Z^{0}$ production in $e^{+} e^{-}$annihilation are shown in Figure 2.1. These diagrams represent the coupling of fermion antifermion pairs to the $Z^{0}$ and the photon.

Following roughly the notation of Quigg [9], diagram (b) in Figure 2.1 has a coupling of

$$
\begin{equation*}
-i Q_{f} \bar{f} \gamma f \tag{2.4}
\end{equation*}
$$



Figure 2.1: Fermion Antifermion coupling to neutral Bosons
For the $Z^{0}$ coupling in diagram (a) the following coupling is used

$$
\begin{equation*}
\frac{-i}{\sqrt{2}}\left(\frac{G_{F} M_{Z}^{2}}{\sqrt{2}}\right)^{1 / 2} \bar{f} \gamma\left[R_{f}\left(1+\gamma_{5}\right)+L_{f}\left(1-\gamma_{5}\right)\right] f . \tag{2.5}
\end{equation*}
$$

Here $R_{f}$ is the coupling of the fermion to the right handed components (which are projected out by the $1+\gamma_{5}$ ) and $L_{f}$ is the coupling to the left handed components. $G_{F}$ is the fermi constant, and $M_{Z}$ is the mass of the $Z^{0}$.

The couplings $R_{f}$ and $L_{f}$ for any fermion are given by

$$
\begin{align*}
R_{f} & =2\left[\left(T_{3}\right)_{f_{R}}-Q_{f} \sin ^{2} \theta_{W}\right] \\
L_{f} & =2\left[\left(T_{3}\right)_{f_{L}}-Q_{f} \sin ^{2} \theta_{W}\right] . \tag{2.6}
\end{align*}
$$

Here $Q_{f}$ is the charge of the fermion, $\left(T_{3}\right)_{f_{R}}$ and $\left(T_{3}\right)_{f_{L}}$ the right and left handed third components of weak isospin as given in (2.1). As an example, the left and right couplings for the electron are

$$
\begin{align*}
R_{e} & =2\left[\left(T_{3}\right)_{e_{R}}-Q_{f} \sin ^{2} \theta_{W}\right]=2 \sin ^{2} \theta_{W} \\
L_{e} & =2\left[\left(T_{3}\right)_{e_{L}}-Q_{f} \sin ^{2} \theta_{W}\right]=-1+2 \sin ^{2} \theta_{W} . \tag{2.7}
\end{align*}
$$

It is sometimes useful to write equation 2.5 in terms of the vector and axial components

$$
\begin{equation*}
\frac{-i}{\sqrt{2}}\left(\frac{G_{F} M_{Z}^{2}}{\sqrt{2}}\right)^{1 / 2} \bar{f} \gamma\left[V_{f}-\gamma_{5} A_{f}\right] f \tag{2.8}
\end{equation*}
$$

Now substituting (2.6) into (2.5) and remembering that $\left(T_{3}\right)_{f_{R}}$ is always 0 , the following is obtained

$$
\begin{equation*}
\frac{-i}{\sqrt{2}}\left(\frac{G_{F} M_{Z}^{2}}{\sqrt{2}}\right)^{1 / 2} \bar{f} \gamma\left[\left(-4 Q_{f} \sin ^{2} \theta_{W}+2\left(T_{3}\right)_{f_{L}}\right) \cdot 1-2\left(T_{3}\right)_{f_{L}} \gamma_{5}\right] f \tag{2.9}
\end{equation*}
$$

Comparing this with (2.8) yields

$$
\begin{align*}
V_{f} & =2\left(T_{3}\right)_{f_{L}}-4 Q_{f} \sin ^{2} \theta_{W} \\
A_{f} & =2\left(T_{3}\right)_{f_{L}} \tag{2.10}
\end{align*}
$$

In terms of $L_{f}$ and $R_{f}$

$$
\begin{align*}
A_{f} & =L_{f}-R_{f} \\
V_{f} & =L_{f}+R_{f} \tag{2.11}
\end{align*}
$$

$2.2 e^{+} e^{-} \rightarrow Z^{0} \rightarrow f \bar{f}$

With a knowledge of the couplings of the Standard Model, it is now possible to calculate the cross section for production of fermion-antifermion pairs at the $Z^{0}$ resonance. Any fermion which has a mass less than half the $Z^{0}$ mass $(e, \mu, \tau$, and their neutrinos, as well ast the $u d c s b$ quarks) will be produced at the $Z^{0}$ resonance. The diagrams which contribute to this process are shown in Figure 2.2. These diagrams represent $e^{+} e^{-}$annihilation into either a photon or a $Z^{0}$. When the invariant amplitude for these two diagrams is calculated, there are three contributions; a pure QED contribution (from $e^{+} e^{-} \rightarrow \gamma \rightarrow f \bar{f}$ ), a pure weak contribution (from $e^{+} e^{-} \rightarrow Z^{0} \rightarrow f \bar{f}$ ), and

(a)

(b)

Figure 2.2: Diagrams contributing to $e^{+} e^{-} \rightarrow f \bar{f}$
an interference term. Near the $Z^{0}$ resonance the pure weak contribution is completely dominant.

For any fermion-antifermion final state, the three contributions of the differential cross section are: the familiar QED piece

$$
\begin{equation*}
\frac{\pi \alpha^{2} Q_{f}^{2} C_{f}}{2 s}\left(1+\cos ^{2} \theta\right) \tag{2.12}
\end{equation*}
$$

where $C_{f}$ is a color factor which is 1 for leptons and 3 for quarks, $\theta$ is the angle of production of the final state with respect to the incoming $e^{+} e^{-}$, and $s$ is the familiar Mandelstam variable ( $s=E_{c m}^{2}$ ), the interference term,

$$
\begin{align*}
& -\frac{C_{f} \alpha Q_{f} G_{f} M_{Z}^{2}\left(s-M_{Z}^{2}\right)}{8 \sqrt{2}\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]} \times \\
& \quad\left[\left(R_{e}+L_{e}\right)\left(R_{f}+L_{f}\right)\left(1+\cos ^{2} \theta\right)+2\left(R_{e}-L_{e}\right)\left(R_{f}-L_{f}\right) \cos \theta\right] \tag{2.13}
\end{align*}
$$

|  | $A_{f}$ | $V_{f}$ | $A_{f}^{2}+V_{f}^{2}$ |
| :--- | :---: | :---: | :---: |
| $e, \mu, \tau$ | -1 | -0.073 | 1.005 |
| $\nu_{e}, \nu_{\mu}, \nu_{\tau}$ | 1 | 1 | 2 |
| $u, c, t$ | 1 | 0.382 | 1.146 |
| $d, s, b$ | -1 | 0.691 | 1.477 |

Table 2.1: This table lists the axial and vector couplings calculated using equation 2.10 and a value of $\sin ^{2} \theta_{W}=0.232$.
and the purely weak term

$$
\begin{align*}
+ & \frac{C_{f} G_{f}^{2} M_{Z}^{4} s}{64 \pi\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]} \times \\
& {\left[\left(R_{e}^{2}+L_{e}^{2}\right)\left(R_{f}^{2}+L_{f}^{2}\right)\left(1+\cos ^{2} \theta\right)+2\left(R_{e}^{2}-L_{e}^{2}\right)\left(R_{f}^{2}-L_{f}^{2}\right) \cos \theta\right] } \tag{2.14}
\end{align*}
$$

### 2.2.1 Total cross section

With the above contributions to the differential cross section it is now possible to calculate a total cross section for production of fermion pairs near the $Z^{0}$ resonance. Integrating the differential cross section over $\cos \theta$ gives the following total cross sections. First, for the QED piece

$$
\begin{equation*}
\sigma_{Q E D}=\frac{4 \pi \alpha^{2} C_{f} Q_{f}^{2}}{3 s} \tag{2.15}
\end{equation*}
$$

Inserting numerical values gives

$$
=\frac{86.8 C_{f} Q_{f}^{2}}{s\left(G e V^{2}\right)} n b
$$

This cross section is shown in Figure 2.3 for the $\mu$ (which is the same as for (the electron and tau), and for $u$ and $d$ type quarks. This term is small compared with

## Pure QED Cross Section



Figure 2.3: QED cross section for fermions
the resonance term which is greater than $1 n b$ for all fermions in the region of the resonance.

The interference term is

$$
\begin{equation*}
\sigma_{\text {inter ference }}=-\frac{C_{f} \alpha Q_{f} G_{f} M_{Z}^{2}\left(s-M_{Z}^{2}\right)}{3 \sqrt{2}\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]}\left[\left(R_{e}+L_{e}\right)\left(R_{f}+L_{f}\right)\right] . \tag{2.16}
\end{equation*}
$$

Notice immediately that this term vanishes at $\sqrt{s}=M_{Z}$ and remains small in the region of the resonance. This term is shown in Figure 2.4

The main contribution to the total cross section for $s \approx M_{Z}$ comes from the pure weak interaction.

$$
\begin{equation*}
\sigma_{W e a k}=\frac{C_{f} G_{f}^{2} M_{Z}^{4} s}{24 \pi\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]}\left[\left(R_{e}^{2}+L_{e}^{2}\right)\left(R_{f}^{2}+L_{f}^{2}\right)\right] . \tag{2.17}
\end{equation*}
$$

Contributions from Gamma-Z Interference


Figure 2.4: Cross section for Gamma-Z interference.

Using the following relationship

$$
L_{f}^{2}+R_{f}^{2}=\frac{A_{f}^{2}+V_{f}^{2}}{2}
$$

the cross section can be rewritten as

$$
\begin{equation*}
\sigma_{\text {Weak }}=\frac{C_{f} G_{f}^{2} M_{Z}^{4} s}{96 \pi\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]}\left[\left(A_{e}^{2}+V_{e}^{2}\right)\left(A_{f}^{2}+V_{f}^{2}\right)\right] . \tag{2.18}
\end{equation*}
$$

This term is clearly resonant in the region of $\sqrt{s}=M_{Z}$

### 2.2.2 Partial widths to fermion pairs

The width of the $Z^{0}$ resonance depends on the number of species into which it decays.
Each species which couples to the $Z^{0}$, and is kinematically accessible increases the

(a)
(b)

Figure 2.5: These diagrams illustrate the types of final state radiation which can occur in fermion pair production. Diagram (a) shows final state photon emission, while diagram (b) shows final state gluon emission.
width of the resonance. As derived in many sources [9], the contribution to the width of the $Z^{0}$ by a pair of fermions is given by

$$
\begin{equation*}
\Gamma_{f f}=\frac{C_{f} G_{f} M_{Z}^{3}}{24 \sqrt{2} \pi}\left(A_{f}^{2}+V_{f}^{2}\right) \tag{2.19}
\end{equation*}
$$

where $C_{f}$ is the color factor which is 3 for quarks and 1 for leptons.
Table 2.1 gives the values for the axial and vector couplings assuming a value for $\sin ^{2} \theta_{W}$ of 0.232 . With these values, and assuming a $Z^{0}$ mass of $91.14 \mathrm{GeV} / \mathrm{c}^{2}$, the widths for the different fermionic final states are calculated to be

$$
\begin{align*}
\Gamma_{e e} & =83 \mathrm{MeV}  \tag{2.20}\\
\Gamma_{\nu \nu} & =166 \mathrm{MeV}
\end{align*}
$$

$$
\begin{aligned}
\Gamma_{u u} & =285 \mathrm{MeV} \\
\Gamma_{d d} & =367 \mathrm{MeV}
\end{aligned}
$$

These widths are modified because of the effects of final state radiation. The diagrams which contribute are shown in Figure 2.5. Figure 2.5(a) shows photons radiated in the final state. Diagrams of this type can be summed to all orders, and change the width as follows [10]

$$
\begin{equation*}
\Gamma_{f f-\text { corrected }}=\Gamma_{f f}\left(1+\frac{3 \alpha Q_{f}^{2}}{4 \pi}\right) . \tag{2.21}
\end{equation*}
$$

This term in parenthesis is approximately 1.0005 and is ignored in this analysis. Logarithmic terms which arise from summing the final state radiation to all orders are cancelled by terms from final state vertex corrections which appear with the opposite sign [10].

In addition to this term, $q \bar{q}$ final states are subject to final state gluon radiation [Figure 2.5(b)]. This broadens the width by the following term [11]

$$
\begin{equation*}
\Gamma_{q q-\text { corrected }}=\Gamma_{q q}\left(1+\frac{\alpha_{s}}{\pi}+1.41\left(\frac{\alpha_{s}}{\pi}\right)^{2}+64.84\left(\frac{\alpha_{s}}{\pi}\right)^{3}\right) \tag{2.22}
\end{equation*}
$$

Here the term in the parenthesis is approximately $1.05 \pm 0.01$ where the uncertainty comes from the uncertainty in $\alpha_{s}$ (the strong coupling constant).

The overall effect of the radiative corrections on the widths do not alter the leptonic final states significantly, but the hadronic final states change as follows

$$
\begin{align*}
\Gamma_{u u} & =298 \mathrm{MeV}  \tag{2.23}\\
\Gamma_{d d} & =385 \mathrm{MeV}
\end{align*}
$$

The total width is just the sum of all the partial widths. Recent results suggest that the top is not kinematically accessible at the $Z^{0}$ resonance [12]. Assuming the
top is not produced, the total width is just

$$
\begin{aligned}
\Gamma_{t o t} & =\Gamma_{e e}+\Gamma_{\mu \mu}+\Gamma_{\tau \tau}+\Gamma_{\nu e}+\Gamma_{\nu_{\mu}}+\Gamma_{\nu_{\tau}}+\Gamma_{u u}+\Gamma_{d d}+\Gamma_{c c}+\Gamma_{s s}+\Gamma_{b b}(2.24) \\
& =2.5 \mathrm{GeV}
\end{aligned}
$$

Changing the final state correction factor for gluon radiation from 1.05 to 1.04 changes the total width by 12 MeV , hence the uncertainty in $\alpha_{s}$ does not limit the sensitivity of the Mark II measurement of the $Z^{0}$ width when viewed in the light of the relative energy measurement resolution which is 27 MeV .

### 2.3 Details of the $Z^{0}$ line shape

In order to determine the $Z^{0}$ resonance parameters, the cross section is measured at several different energies in the region of the resonance. Unfolding the parameters demands knowing exactly what the expected cross section at these scan points is for any set of resonance parameters. Equation 2.18 provides a useful starting point for this calculation. a more precise form of the cross section requires including the radiative corrections to the cross section. These corrections signifigantly alter the line shape, and hence must be included for an accurate determination fo the bare parameters.

### 2.3.1 Breit-Wigner form

Starting with Equation 2.18 and using Equation 2.19 the total cross section into fermion pairs can be written as

$$
\begin{equation*}
\sigma_{f f}=\frac{12 \pi}{M_{Z}^{2}} \frac{s \Gamma_{e e} \Gamma_{f f}}{\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]} \tag{2.25}
\end{equation*}
$$

The second term in the denominator is modified to reflect the energy dependence of the total width. Following Cahn [19] and replacing $M_{Z}^{2}$ by $s$ and setting

$$
\Gamma(s)=\frac{\sqrt{s} \Gamma_{Z}}{M_{Z}}
$$

yields

$$
\begin{equation*}
\sigma_{f f}=\frac{12 \pi}{M_{Z}^{2}} \frac{s \Gamma_{e e} \Gamma_{f f}}{\left[\left(s-M_{Z}^{2}\right)^{2}+s^{2} \Gamma_{Z}^{2} / M_{Z}^{2}\right]}, \tag{2.26}
\end{equation*}
$$

which is in the form of a relativistic Breit-Wigner resonance as expected.
The total visible cross section can now be written as

$$
\begin{equation*}
\sigma_{v i s}=\sigma_{p e a k} \frac{s \Gamma_{Z}^{2}}{\left[\left(s-M_{Z}^{2}\right)^{2}+s^{2} \Gamma_{Z}^{2} / M_{Z}^{2}\right]} . \tag{2.27}
\end{equation*}
$$

The cross section at the peak is determined by setting $\sqrt{s}=M_{Z}$ and is written as

$$
\begin{equation*}
\sigma_{p e a k}=\frac{12 \pi}{M_{Z}^{2}} \frac{\Gamma_{e e} \Gamma_{v i s}}{\Gamma_{Z}^{2}}, \tag{2.28}
\end{equation*}
$$

where here $\Gamma_{v i s}$ is the total width to species which are visible in the detector, and $\Gamma_{Z}$ is the total width into all species which are kinematically accessible. Given the results of the preceding sections, it is now possible to calculate the total width, and peak cross section using the predictions of the Standard Model and a value for the mass of the $Z^{0}$.

### 2.3.2 Radiative Corrections

The most drastic alteration of the $Z^{0}$ line shape from the Breit-Wigner shape comes from QED radiation in the initial state. Much work has been done to understand the effects of this radiation on the line shape [10,13]. Figure 2.6 shows the diagrams which are most responsible for changing the line shape.

The reason these diagrams change the line shape is easy to understand. When an electron radiates before interaction, the effective center-of-mass energy of the system


Figure 2.6: This figure shows the first order diagrams which are used to calculate the initial state radiation.
is lowered. Above the $Z^{0}$ peak, this has the effect of bringing the energy of the system closer to the pole, and hence raising the cross section. Below the peak, the opposite happens. Initial state radiation lowers the energy of the system, and hence moves the system farther from the resonance, which lowers the cross section. Figure 2.7 illustrates the effects of these radiative corrections. The upper curve is the uncorrected Breit-Wigner, while the lower curve has the radiative corrections applied. In this figure, the corrected cross section is clearly higher above the peak (the so called radiative tail), and reduced below the peak. This shifting of the cross section also has the effect of moving the observed peak position. Since the cross section is raised above the pole, and lowered below, the measured peak position appears to be higher in energy than the natural resonance peak by $220 \mathrm{MeV} / \mathrm{c}^{2}$. The peak cross section is also clearly lowered, and the width is broadened.

This estimate is based on a first order correction. Just as for final state radiation,


Figure 2.7: This figure shows the effect of radiative corrections on the $Z^{0}$ line shape. The upper curve is a relativistic Breit-Wigner without any radiative corrections. The lower curve is the cross section for the same resonance parameters, but with radiative corrections applied. The units on the $y$ axis are arbitrary.


Figure 2.8: This figure shows the effect of different orders of radiative corrections on the $Z^{0}$ line shape. These curves were generated for a $Z^{0}$ mass of 93 GeV for first and second order corrections with and without exponentiation
the first order correction also requires including a term from the vertex correction diagram shown in Figure 2.9 in order to cancel logarithmic divergences in the diagrams of Figure 2.6 [14,15].

The radiative corrections due to the initial state radiation diagrams can be performed to all orders using a process called exponentiation[16,17]. The vertex correction diagram, however, has only been calculated to second order. A Monte Carlo calculation which includes second order vertex corrections and exponentiation of the initial state radiation diagrams is available [18]. Figure 2.8 (which is taken from ref [10]) shows how well the different orders of corrections agree. From this figure one concludes that the exponentiated first order corrections agree at the 2 MeV level


Figure 2.9: This figure shows the first order diagram for the vertex correction. Divergences in this diagram cancel divergences in the initial state radiation diagrams.
with the exponentiated second order corrections, and thus should be adequate for this measurement. It is also clear that a first order calculation without exponentiation is unacceptable.

Other diagrams that could contribute at this level are vacuum polarization diagrams, box diagrams, and interference diagrams between initial and final state radiation. The vacuum polarization diagrams don't alter the line shape, but they do affect the coupling of particles to the $Z^{0}$ and are discussed later. The box and interference diagrams are small, and are not included here.

### 2.3.3 Analytic form for the line shape

In order to extract the $Z^{0}$ resonance parameters from the data, a fit is performed minimizing the difference between the observed cross section and a predicted cross section. The predicted cross section is a function of the resonance parameters the mass, width, and peak cross section. Since these parameters are varied frequently during a fit, an easily calculable analytic form for the cross section is preferable to using Monte Carlo calculations. Several analytic forms are available [19,20,14]. In this analysis, the form of Cahn [19] is used. This expression is based on a calculation of Kuraev and Fadin [14] with exponentiation added; details of the derivation are
provided in Cahn's paper. For reference, the form used in this analysis is provided below.

The input parameters to this function are the center-of-mass energy (s), the mass (M), the total width $(\Gamma)$, and the peak cross section $\left(\sigma_{\text {peak }}\right)$. The quantity

$$
t=\frac{2 \alpha}{\pi}\left(\ln \frac{s}{m_{e}^{2}}-1\right)
$$

which is of order 0.1 on the resonance, determines the strength of the radiative corrections.

The cross section may be written as:

$$
\begin{align*}
\sigma= & \sigma_{\text {peak }} \frac{(1+3 t / 4) \gamma^{2}}{1+\gamma^{2}} \times \\
& {\left[(1+y) a^{t-2} \Phi(\cos \beta, t)-a^{t-1} \frac{t}{1-t} \Phi(\cos \beta, 1+t)\right] } \tag{2.29}
\end{align*}
$$

where the auxiliary function $\Phi$ is defined as

$$
\Phi(\cos \beta, \nu)=\pi \nu \frac{\sin [(1-\nu) \beta]}{\sin \pi \nu \sin \beta}
$$

and also defining the following

$$
\begin{gathered}
y=\left(\frac{\sqrt{s}}{M_{z}}\right)^{2}-1 \\
\gamma=\Gamma / M_{z} \\
a=\sqrt{\frac{\left.y^{2}+\gamma^{2}(1+y)^{2}\right]}{1+\gamma^{2}}},
\end{gathered}
$$

and

$$
\cos \beta=\frac{-\left[y+\gamma^{2}(1+y)\right]}{a\left(1+\gamma^{2}\right)}
$$

(this is equation (4.4) in Cahn). In addition to this piece, there is a correction due to the radiation of hard photons [21] which is given by

$$
\begin{equation*}
\sigma_{\text {hard }}=-\frac{t \Gamma}{\sqrt{s}}\left[\tan ^{-1}\left(\frac{2 M}{\Gamma}\right)-\tan ^{-1}\left(\frac{2(m-\sqrt{s})}{\gamma}\right)\right] \sigma_{\text {peak }} \tag{2.30}
\end{equation*}
$$

Combining this with (2.29) gives the complete approximation to the line shape. The results of this analytic form are shown in Figure 2.7.

### 2.3.4 Comparison to Monte Carlo calculations

The analytic form presented above has been compared to the output of several Monte Carlo generators [10] and analytic calculations. The comparison was done by generating data with a second order exponentiated Monte Carlo [18] using a set of resonance parameters. The radiative corrections $(\delta(s)$ in the equation below) were then calculated for each of the different calculations that were to be tested. The form used for applying the corrections is

$$
\begin{equation*}
\sigma_{v i s}=\sigma_{\text {peak }} \frac{s \Gamma_{Z}^{2}}{\left[\left(s-M_{Z}^{2}\right)^{2}+s^{2} \Gamma_{Z}^{2} / M_{Z}^{2}\right]}[1+\delta(s)] \tag{2.31}
\end{equation*}
$$

The corrected cross sections are then fit to a Breit-Wigner form, and a measured set of resonance parameters is determined. The difference between these measured parameters, and the input parameters provides a test of how well the calculations agree with the second order exponentiated Monte Carlo. The difference between the parameters found using Cahn's analytic form and the input parameters was less than $1 \mathrm{MeV} / \mathrm{c}^{2}$ for the Mass, less than 10 MeV for the total width, and less than $0.01 \%$ difference in measured peak cross section. These results suggest that the analytic form used in this analysis will certainly be sufficient at the 10 MeV level for extracting the parameters from the data well below the expected systematic errors on the measurement.

### 2.4 Parameters of the Standard Model

In the preceeding sections, formulas were presented for calculating cross sections in the Standard Model. Implicit in these equations were several parameters of the Standard Model which cannot be calculated, but are simply constants of nature. Parameters which had to be specified before were the $Z^{0}$ mass, $G_{F}, \alpha, \sin ^{2} \theta_{W}$, and $m_{e}$.

The Standard Model does not predict the masses of the fermions, and they can all be considered as parameters. The bare parameters of the Standard Model are just the couplings of the of the $S U(2)$ and $U(1)$ gauge fields to the fermions, and the vacuum expectation value of the Higgs field. These are not directly measurable, but are related by the Standard Model to observables which are measurable. Many different sets of observable parameters can be used to extract the couplings of the electroweak theory. In this thesis, the scheme of Lynn,Peskin, and Stewart [22] is used.

In this scheme, the parameters of the standard model are $\alpha, G_{F}$, and $M_{Z}$. These are the parameters which can be measured with a minimal dependence on radiative corrections (which depend on unknown quantities), and hence are accurately measurable. $G_{F}$ and $\alpha$ have already been measured with great precision, and a precise measurement of the $Z^{0}$ mass is presented here.

This leaves open the question of $\sin ^{2} \theta_{W}$. With this parameterization, $\sin ^{2} \theta_{W}$ is not a parameter of the Standard Model, but is determined by the other parameters. In terms of $\alpha, G_{F}$, and $M_{Z} \sin ^{2} \theta_{W}$ can be written as

$$
\begin{equation*}
\sin ^{2} \theta_{W} \approx \frac{1}{2}\left[1-\sqrt{1-\frac{4 \pi \alpha}{\sqrt{2} M_{Z}^{2} G_{F}(1-\Delta r)}}\right] \tag{2.32}
\end{equation*}
$$

In this equation $\Delta r$ is a term which is calculated to correct for the loop corrections


Figure 2.10: This figure shows the vacuum polarization diagram for the $Z^{0}$. Any particle which couples to the $Z^{0}$ can couple in this loop including particles heavier than half the $Z^{0}$ mass.
which affect the $Z^{0}$ couplings. The diagrams which contribute to these corrections are shown in Figure 2.10. Particles in the loop in this diagram, since they are virtual, can be heavier than those kinematically available in $Z^{0}$ decay, hence this correction is sensitive to physics at a scale heavier than the $Z^{0}$. Specifically, there is a strong dependence on the top quark mass in $\Delta r$ if the top mass is greater than the $W$ mass. The correction factor $\Delta r$ has been calculated for many values of the top and Neutral Higgs masses. To be consistent, a top mass of 90 GeV and a Higgs mass of 100 GeV are used throughout this thesis. With these values, $\Delta r$ is calculated to be 0.0602 [23].

### 2.5 Bhabha scattering

In order to determine the normalization of the measured $Z^{0}$ cross section, small angle Bhabha scattering is measured using two specialized devices in the Mark II. A precise calculation for the expected cross section into these devices is needed in order to determine the integrated luminosity seen by the Mark II detector. Small angle Bhabhas are used because the cross section is large, calculable with reasonable accuracy, and not subject to backgrounds.

(a)

(b)

Figure 2.11: These two diagrams are the lowest order QED diagrams for Bhabha scattering. Diagram (a) is the t-channel diagram which is dominant in the small angle region. Diagram(b) shows the familiar s-channel piece, which is smaller than the equivalent weak process in the region of the $Z^{0}$ resonance.

The lowest order diagrams contributing to Bhabha scattering are shown in Figure 2.11. A calculation of the cross section for these diagrams yields [24].

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{\alpha^{2}}{2 s}\left[\frac{1+\cos ^{4}(\theta / 2)}{\sin ^{4}(\theta / 2)}+\frac{1+\cos ^{2} \theta}{2}+\frac{2 \cos ^{4}(\theta / 2)}{\sin ^{2}(\theta / 2)}\right] \tag{2.33}
\end{equation*}
$$

The first term in this cross section is from the t -channel diagram of Figure 2.11 (a). The second term is the familiar s-channel diagram, and the third term is the interference between these two diagrams. At small angles, the cross section is completely dominated by the t-channel term. The total cross section into small angles is
$\propto 1 / \theta^{3}$, and scales with $1 / s$.
Bhabha scattering is also subject to radiative corrections. To first order there are eight diagrams which contribute. These are just the two diagrams of Figure 2.11 with each of the eight legs radiating a photon. These corrections have been calculated. $[26,27]$

The second order radiative corrections are much more difficult to calculate. There are more than 150 diagrams which need to be calculated.[25] The 40 diagrams of double Bremsstrahlung have been calculated [28], but there is as yet no complete treatment of all second order corrections. Since this measurement is made near the $Z^{0}$ resonance, the interference between the $Z^{0}$ and the t-channel diagram is also calculated [32] and added into the cross section.

In order to calculate Bhabha cross sections seen by the small angle detectors, one must fold in detector acceptance effects. For this reason, the Monte Carlo method is ideal. Several Monte Carlo programs are used. They vary in how they attempt to implement the higher order radiative corrections. HOWLEEG[31] contains the first order corrections and $Z^{0}$-photon interference. There are also versions of this program which include exponentiation to estimate the higher order corrections. BHLUMI[30] uses the Yenni, Frautschi, and Suura (YFS) exponentiation scheme [16] for estimating the contributions of final state Bremsstrahlung to all orders. HOWLBHK[29] is a first order Monte Carlo, but includes all electroweak effects to first order. Comparisons between these programs, are presented in chapter 4.

## Chapter 3

## Experimental Apparatus

The measurment in this thesis was performed using the Mark II detector which has been taking data at the Stanford Linear Collider since 1987. The Mark II detector is an upgrade of the detector which had successfull data taking runs at the SPEAR and PEP storage rings. This chapter describes the Mark II detector, concentrating on the elements of the detector which are used in this analysis.

Measuring the $Z^{0}$ line shape requires measuring a series of cross sections at different energies in the region of the $Z^{0}$ resonance. In order to make an accurate measurement of the absolute position of the resonance peak, an accurate measurement of the absolute electron and positron beam energies are required. An accurate measurement of the peak cross section requires a knowledge of the absolute normalization of the cross section. The accuracy of the width measurement depends on the relative accuracy of the collision energy and cross section measurements.

Spectrometers have been constructed to determine the absolute energy of the SLC beams, and are described in this chapter. The measurment of the cross section requires accurately understanding the efficiencies of the detector components used in the
measuring both the $Z^{0}$ production cross section as well as the Bhabha normalization cross section.

### 3.1 The Stanford Linear Collider

The Stanford Linear Collider (SLC) is the first operating linear collider [33]. The outline of the collider is shown in Figure 3.1. The accelerator operates by first storing electrons and positrons in two damping rings in order to reduce the emittance of the bunches of particles. The bunches of electrons and positrons are then simultaneously accelerated down the two mile linear accelerator(LINAC) to energies up to 50 GeV . At the end of the LINAC, the beams are separated, transported through two arcs, and finally made to collide. The Mark II detector surrounds the interaction point where the two beams collide. After collision, the beams are extracted to two separate dumps. The energies of the beams are measured in the extraction lines as the beams are heading to the dumps.

The data for this thesis were collected during a run from April 1989 to October 1989. The first $Z^{0}$ was observed on April 11,1989.

### 3.2 The SLC Extraction Line Spectrometer

The Extraction Line Spectrometers were constructed to provide a precise measurement of the SLC beam energy [35]. A conceptual diagram of the electron energy measurement beam line is shown in Figure 3.2. A similar spectrometer measures the positron energy. In each extraction line, the beam is passed through a series of three dipole magnets. On passing through the first dipole, the beam makes a swath of synchrotron light. The second dipole is a precisely measured magnet which is the


Figure 3.1: Basic SLC Layout
spectrometer for the device. This magnet makes a bend perpendicular to the first bend, and displaces the beam by an angle of 18.286 mrad. The third dipole bend is parallel to the first bend, and another swath of synchrotron light parallel to the first swath is created.

The swaths from these bends are imaged on phosphorescent screens located a distance ( L ) downstream from the magnets. The distance (D) between the parallel stripes from the horizontal bend magnets is measured, and used to determine the energy of the beam. This is done using the following formula for the bend angle of the spectrometer

$$
\begin{equation*}
\theta=\frac{\left(\int B \cdot d l\right)(0.29978)}{P} \tag{3.1}
\end{equation*}
$$

where $\int B \cdot d l$ is in units of $k g m$ and $P$ is in $(\mathrm{GeV} / \mathrm{c})$. In terms in D and L the beam energy is:

$$
\begin{equation*}
P=\frac{\left(\int B \cdot d l\right)(0.29978) L}{D} . \tag{3.2}
\end{equation*}
$$

In this equation, the quantities which must be measured are; $L$ the distance from the spectrometer to the phosphor screen (about 15 m ), $D$ the displacement between the synchrotron light stripes (about 27 cm ), and $\int B \cdot d l$ the field integral through the spectrometer $(\approx 30 \mathrm{kgm})$. The absolute accuracy with which these measurements are made is what limits the energy measurement.

### 3.2.1 Field mapping of the spectrometer

The absolute field integral of the spectrometer magnets was accurately measured before the magnets were placed in the extraction lines [34]. In addition, devices were developed for continuously monitoring the magnetic field of the spectrometers while they are in operation.


Figure 3.2: Conceptual design of the Extraction Line Spectrometer
The absolute field mapping of the spectrometers was done using two different techniques. The repeatability of each technique, and the comparison between them determines the confidence in the measurement.

The first method is the moving wire technique. A block diagram of the apparatus for this measurement is shown in figure 3.3. For this measurement, a loop of wire is placed through the spectrometer magnet. The ends of the wire, which are secured at both ends of the spectrometer, are then moved. This motion changes the magnetic flux through the coil of wire, and hence induces a voltage on the wire. This voltage is given by

$$
\begin{equation*}
V=-\frac{d \Phi}{d t} \tag{3.3}
\end{equation*}
$$

where $d \Phi$ is the change in magnetic flux induced by the motion of the wire. The quantity which must be measured is $\int B \cdot d l$ along the wire which runs the length of the spectrometer. To this end, the ends of the wire are moved perpendicular to the direction the wire is stretched. In this case, $d \Phi$ is given by

$$
d \Phi=\Delta x \int B \cdot d l .
$$

Here $\Delta x$ is the distance the wire is moved (typically 1 cm ). Combining this with (3.3) gives

$$
\begin{equation*}
\int V d t=-\Delta x \int B \cdot d l \tag{3.4}
\end{equation*}
$$

$\int V d t$ is measured using a sampling Digital Voltmeter(DVM). A typical readout of the wire voltage is shown in Figure 3.4. $\Delta x$, the movement of the ends of the wire, is accurately controlled by having the two ends secured on precision translation tables. The tables are controlled by stepping motors, and the position of the wire is determined to better than 1 micron.

The absolute accuracy of this measurement is estimated to be 40 parts per million ( ppm ), while the short term repeatability in measuring $\left(\frac{\Delta \int B \cdot d l}{\int B \cdot d l}\right)$ was measured to $\mathrm{be} \pm 28 \mathrm{ppm}$.

The second technique of field mapping is a direct measurement of $\int B \cdot d l$. This technique involved moving a magnetic field monitor through the spectrometer and measuring $B$ and $d l$. A block diagram of the apparatus for this moving probe measurement is shown in Figure 3.5.

The magnetic field is measured by two devices. A Nuclear Magnetic Resonance (NMR) probe measures the field to 10 ppm , but is only usable in the uniform field region inside the spectrometer. The rapidly varying field at the end of the spectrometer is measured using a Hall probe which is placed next to the NMR probe. The Hall probe is calibrated using the NMR readings in the uniform field region of the


Figure 3.3: This is a block diagram of the moving wire technique. The spectrometer magnet is labeled B32 in this diagram. The wire is run through the spectrometer, and is attached on either end of the spectrometer to precise translation stages. When the stages are moved, a voltage is induced on the wire, and measured by a high precision sampling DVM.


Figure 3.4: This is a typical readout of the voltage versus time for the moving wire measurement. The ramp up and ramp down of the signal come when the stepping motors accelerate at the beginning and end of the measurement.
spectrometer where both devices are read out. The accuracy of the Hall probe is at worst only 800 ppm , but it is only used to measure approximately $6 \%$ of $\int B \cdot d l$.

The two probes are secured in a special holder which is placed on a rail which runs the length of the spectrometer. The probe holder is moved using a stepper motor in steps whose length varies from $100 \mu \mathrm{~m}$ to 1 cm depending on how rapidly the field is varying. The position of the probes is measured with a laser interferometer system which has an accuracy of better than 1 ppm .

A typical set of maps is shown in Figure 3.6. The absolute accuracy of this method is estimated to be 53 ppm , while the short term repeatability is found to be $\left(\frac{\Delta \int B \cdot d l}{\int B \cdot d l}\right)= \pm 15 \mathrm{ppm}$.

Many measurements at varying field strengths were made using both techniques of field mapping on each of the spectrometers. The mean difference between the two techniques for all these measurements was found to be 72 ppm with point to point variations of 53 ppm .


Figure 3.5: This is a diagram of the moving probe apparatus. A probe holder consisting of both Hall and NMR field probes, and a laser target is placed on a rail which runs through the spectrometer. A stepping motor moves the probe holder along the rail. The position of the probes is measured using a laser interferometer system.


Figure 3.6: This figure shows typical maps of the field made with the moving probe measurement. The x axis is the distance from where the probe motion was started. The $y$ axis is the field measured at that position. (a) is a typical map for one of the spectrometers. (b) and (c) show suppressed zero views for measurements of both spectrometers.

While the absolute field mapping was done, a set of in situ field monitoring devices were simultaneously calibrated. These devices are flipcoils, NMR probes, and transductors. The most accurate of these, the flipcoil, consists of a quartz rod 2.8 m long and 15 mm in diameter with a ten-wire coil pack wrapped around the rod lengthwise. Figure 3.7 shows the layout of the flipcoil. The rod is rotated at 3 RPM, this changes the magnetic flux through the coil, and hence induces a voltage in the coil. This voltage is measured using a sampling digital voltmeter. A typical waveform from the flipcoil is shown in figure 3.8. Once again, the field integral is proportional to the $\int V d t$ detected in the coil. The constant of proportionality is determined during the field mapping. The accuracy of the flipcoil readout is estimated to be 42 ppm .

The flipcoil is placed in the magnet alongside the beampipe. Figure 3.9 shows the layout of the devices in the spectrometer. The beam passes throught the magnet approximately 62 mm away from where the flipcoil measures the field. In order to compare the field at these two locations, the uniformity of the field across the gap was measured using the moving wire method. Non-uniformity in the field contributes 54 ppm uncertainty in the determination of the field at the beam position versus the flipcoil position where it is measured.

The final result for the accuracy of the field measurement is a combination of the absolute field mapping accuracy ( 72 ppm ), the flipcoil measurement accuracy ( 42 ppm ), and the uniformity uncertainty ( 54 ppm ) giving a total accuracy of 100 ppm . This corresponds to a 5 MeV measurement error for a 50 GeV beam.

### 3.2.2 Phosphorescent Screen Monitor

The measurement of the distance between the synchrotron light stripes (D), is accomplished using a device called the Phosphorescent Screen Monitor (PSM) [36].


Figure 3.7: Layout of the Flipcoil


Figure 3.8: This is a typical reading of the voltage versus time from the flipcoil. The area under this curve is proportional to the field integral through the spectrometer.


Figure 3.9: This diagram shows how the beampipe and flipcoil are situated in the spectrometer. The beam and flipcoil are placed symmetrically around the center of the magnet separated by 62 mm .


Figure 3.10: The synchrotron light stripes are intercepted by two phosphor screens. The light from these screens is imaged by two video cameras. In front of each screen is an array of wires used to calibrate the readout of the cameras.

The layout of this device is shown in Figure 3.10. The PSM consists of two phosphor screens which intercept the synchrotron light, and two cameras which image the phosphor screens.

The two phosphor screens are attached to a bar of INVAR (an iron-nickel alloy with low thermal expansion coefficient). In front of each screen is an array of fiducial wires. The wires, which are $100 \mu \mathrm{~m}$ Iconel, are strung with a spacing of $500 \mu \mathrm{~m}$, and are placed several hundred microns above the surface of the phosphor. Wires are selectively removed to create a bar code pattern. The position of each wire within the arrays is measured to an accuracy of $5 \mu \mathrm{~m}$. There is an overall accuracy of $8 \mu \mathrm{~m}$


Figure 3.11: This figure shows a conceptual layout of a typical signal seen on the video camera. The Camera sees both the synchrotron light stripe, and the fiducial wires. The shaded region is specified, and determines where on the screen the signal is digitized.
on the absolute distance between the two wire frames.
The cameras are focused on the plane of fiducial wires. A selected portion of the video signal is digitized by a DSP 2030/4101 signal averager. 128 samples (pixels) are taken on each video scan line, and then summed (in the direction perpendicular to the fiducial wires). Figure 3.11 shows schematically a typical view of the camera. Remotely controlled lights can be turned on to illuminate the fiducial wires from the front. With the light on, the wires are visible, and are used to calibrate the pixel position for each fiducial wire. Figure 3.12 shows a typical calibration. The estimated error on calibrating pixel positions is $10 \mu \mathrm{~m}$ for each screen, or $14 \mu \mathrm{~m}$ for the distance between the two screens.

During operation, the light is turned off, and the wires are no longer visible. A typical peak from synchrotron light is shown in Figure 3.13. The shape is fit to a gaussian in order to determine the centroid of the stripe. The error in finding the difference between the centroids of the two stripes is estimated to be $16 \mu \mathrm{~m}$.


Figure 3.12: This is a typical calibration readout of the PSM. The dips are caused by the fiducial wires not reflecting light as brightly as the phosphor behind them. The large high spots come from the gaps between the wires.

PSM RAW SIGNAL


Figure 3.13: This is a typical readout of synchrotron light from the PSM. A small amount of shadowing caused by the fiducial wires can be seen.

| Source of Error | Contribution |
| :---: | ---: |
| Fiducial Wire Position | $8 \mu \mathrm{~m}$ |
| Pixel Location Calibration | $14 \mu \mathrm{~m}$ |
| Uniformity of Response | $14 \mu \mathrm{~m}$ |
| Parallax Error | $9 \mu \mathrm{~m}$ |
| Centroid Finding | $16 \mu \mathrm{~m}$ |
| Total | $28 \mu \mathrm{~m}$ |

Table 3.1: Summary of systematic errors in the PSM
Another possible source of error arises from misjudging the position of the stripe due to parallax error since the fiducial wires are several hundred microns above the phosphor. This error is estimated to be less than $9 \mu m$. Finally, nonuniformities in the response of the phosphor are estimated to contribute less than $14 \mu \mathrm{~m}$ error. These nonuniformities can arise from the phosphor not being uniformly deposited on the screen, or from the shadowing of the synchrotron light stripe by the fiducial wires. The estimates of the error in uniformity were made by viewing the same synchrotron light stripe at different locations on the phosphor screen (see Figure 3.15).

Table 3.1 summarizes the systematic errors in the measurement of the distance between the two synchrotron light stripes. The net systematic error is estimated to be less than $28 \mu \mathrm{~m}$. The total distance between the stripes is approximately 26 cm This corresponds to an error on the energy measurement of approximately 5 MeV . Another possible source of systematic error arises from drifts in electronics between calibrations. This effect was checked by comparing the the positions of wires found by calibrations over a long period of time. The drift in wire position was generally equivalent to less than 5 MeV over several days (the calibration is typically done three times a day), but a conservative total systematic error of 10 MeV is assigned for the measurement of the distance between the synchrotron light stripes.


Figure 3.14: Data Acquisition for the Energy Spectrometer

### 3.2.3 Data acquisition

The data from the Spectrometer is reported to both the Mark II experiment, and the SLC control system. The data for this measurement is acquired from both GPIB and CAMAC instruments. In order to handle the different instruments, the tasks for data acquisition are divided into Magnetic Measurement, and readout of the PSM video screens. Figure 3.14 shows a block diagram of the Spectrometer data acquisition.

The Magnetic Measurement is done by a Macintosh II computer which is equipped
with a National Instruments GPIB Interface. The NMR readout and the DVM which reads the flipcoil both communicate with the Magnetic Measurements Macintosh (MagMAC) via GPIB. The MagMAC integrates the flipcoil waveform, applies temperature corrections, and computes $\int B \cdot d l$. This Macintosh then reports the results for $\int B \cdot d l$ to the energy measurement devices.

The energy readout for the Mark II is part of the CAMAC system for the entire Mark II. The video digitizers are CAMAC devices which are read out by the Mark II VAX. The magnetic measurements information is written by the MagMAC into a FIFO (a First In First Out buffer) in the Mark II CAMAC crate using a National Instruments 32 bit Digital IO board. The energy is calculated in the Mark II data acquisition VAX, and all data is available for offline calculation of the energy. This system is read out on every Mark II trigger (the Mark II trigger rate is $\leq 4 H z$ ).

The energy calculation for the SLC control system is done using a separate Macintosh computer. This Macintosh controls its own CAMAC crate with a GPIB crate controller. In this crate, there is another set of video digitizers which receive the same video signal as the Mark II video digitizers. The energy Macintosh receives its magnetic measurements from the MagMAC via the appletalk network. The energy MAC calculates the beam energy, and reports its updated values to the SLC control system by writing into a FIFO in a SLC CAMAC crate. The energy is calculated at a rate of about 2 hz .

### 3.2.4 Systematic errors in the energy measurement

The last quantity needed for the energy measurement is the distance between the spectrometer and the phosphor screens. The magnetic center of each spectrometer was measured during field mapping, and determined to be within 0.7 mm of the

| Source of Error | Contribution |
| :---: | ---: |
| Magnetic Measurement | 5 MeV |
| Separation Between Stripes | 10 MeV |
| Survey Error | 5 MeV |
| Rotation Errors | 16 MeV |
| Total | 20 MeV |

Table 3.2: Summary of systematic errors in the ELS
geometric centers of the magnets. Tooling balls on the spectrometer were surveyed relative to the magnetic center of each spectrometer. Tooling balls on the PSM were surveyed relative to the arrays of fiducial wires. The distance between these two sets of tooling balls was then surveyed. The distance between the tooling balls was determined with an estimated error of less than 1.5 mm for a total distance of $\approx 15 \mathrm{~m}$. This gives a contribution to the error on the energy measurement of 5 MeV .

Another possible source of error arises if the horizontal bends are not parallel. If the stripes for the horizontal bends are not parallel, the measured separation between the stripes depends on where on the screen the stripe is viewed. This effect was measured by simultaneously measuring each stripe at two different locations on the phosphor screen. The layout of this test is shown in Figure 3.15. Using this technique, a measured misalignment of 2 mrad was found. This corresponds to a systematic error of 16 MeV . The rotation error is corrected for, but is treated as a systematic error.

The total systematic error on each beam is summarized in Table 3.2. The estimated accuracy in measuring a 50 GeV beam is 20 MeV .

In estimating the error on determining the center-of-mass energy, it is necessary to allow for a possible mismeasurement of the energy which arises when beams of finite dispersion are misaligned at the interaction point. In this case, there is a


Figure 3.15: The misalignment between the horizontal bends was tested by measuring the stripes at two different locations on the phosphor screens. Misalignments give different measurements for the distance between the stripes.
momentum dependence on which particles participate in the collision. Since not all particles participate in collisions when the beams are not overlapping, the measured distribution of energies at the dump does not correspond to the energy distribution of particles which do collide. This error is estimated by measuring the dispersion and crossing error of the beams. During a typical series of runs, an average crossing error is $0.2 \pm 0.05 \mu \mathrm{~m}$. The average dispersion at the IP is less than 1 mm , and the average beam size is $3.5 \mu \mathrm{~m}$. If the beam is systematically offset by this amount, and has the maximum dispersion also systematically shifted from zero, then the largest effect would be a 12 MeV mismeasurment of the energy. Conservatively, an upper limit of 20 MeV on the center-of-mass energy is assigned to this error.

Combining this with the determination of the energy of each beam gives an overall systematic error of 35 MeV on the determination of the absolute center-of-mass energy of the colliding $e^{+} e^{-}$beams. The short term repeatability of this measurement is quite good. Figure 3.16 shows the readout from the spectrometer for 100 consecutive Mark II triggers. This data was taken when the energies of the beams was quite stable. From this data, a short term repeatability of approximately 5 MeV is observed.

Errors which set absolute scales do not affect the relative energy measurement, and for this reason, the survey error, and rotation errors do not contribute to the relative energy error. This gives a 12 MeV error on each beam, and a relative error on the center-of-mass energy of 27 MeV .

The spectrometer is also capable of determining the energy spread of the beams. This is done by measuring the change in the width of the horizontal stripe before and after the beam has passed through the spectrometer. Having measured the energy of the beam, and knowing the dispersion introduced by the spectrometer, the following relation describes the change in the stripe width after going through the spectrometer


Figure 3.16: Readout from the Energy Spectrometer
[37]

$$
\begin{equation*}
\sigma_{a f t e r}=\sqrt{\left(\sigma_{\text {before }}\right)^{2}+\left(\eta_{y} \frac{d P}{P}\right)^{2}} \tag{3.5}
\end{equation*}
$$

Here $\eta_{y}$ is the dispersion from the spectrometer (just the distance between the stripes), $\sigma_{\text {before }}$ is the width of the stripe before the spectrometer bend, and $P$ is the momentum of the beam. The narrowest beam width, that for a beam with no dispersion ( $\sigma_{0}$ ), is determined experimentally to be 0.31 mm . This width comes from the angular spread of the synchrotron radiation (with a very small contribution from the actual size of the beam spot). The spectrometer optics are designed to allow the beam to be focused without dispersion on one of the screens. This is not always done perfectly in practice, so to determine an upper limit on the energy spread, it is calculated in the following manner. First calculating the width due to dispersion in each beam

$$
\begin{aligned}
\sigma_{b}^{\prime} & =\sqrt{\sigma_{b}^{2}-\sigma_{0}^{2}} \\
\sigma_{a}^{\prime} & =\sqrt{\sigma_{a}^{2}-\sigma_{0}^{2}}
\end{aligned}
$$

where $\sigma_{a}$ and $\sigma_{b}$ are the measured stripe widths before and after the spectrometer
bend. The energy spread is then just

$$
\begin{equation*}
\delta_{p}=\left(\sigma_{b}^{\prime}+\sigma_{a}^{\prime}\right) P / \eta \tag{3.6}
\end{equation*}
$$

Typical energy spreads in the SLC are 100 to 200 MeV in each beam. This technique of estimating the energy spread is estimated to be accurate to $10 \%$.

### 3.3 The Mark II detector

The Mark II detector used in this analysis is shown in figure 3.17. A complete description of the detector can be found in ref [38]. This section describes in detail the components of the detector which are used in this analysis.

### 3.3.1 Overall description of the Mark II

Figure 3.18 shows a cutaway view of the detector. The components of the detector from the inside out are

Central Drift Chamber The Drift chamber (DC) is used in combination with the magnet for analyzing charged particles. It is described in section 3.3.2.

Time-of-Flight This system consists of 48 scintillator slabs which lie between the DC and the magnetic coil. This system is used for charged particle identification, and for tagging cosmic rays.

Solenoid Coil The coil is an aluminum cylindrical coil which produces a solenoidal magnetic field of 4.5 Kg in the center of the detector. The field inside the detector has been mapped to an accuracy of $0.1 \%$, and within the tracking volume is uniform to within $3 \%$. Hall probes monitor the magnetic field during data taking. The coil is 1.3 radiation lengths thick.


Figure 3.17: The Mark II Detector
Barrel Calorimeter A lead liquid Argon based calorimeter used for electromagnetic calorimetry. It is described in section 3.3.3

End Cap Calorimter A lead proportional tube calorimeter used for electromagnetic calorimetry at small angles. It is also described in section 3.3.3.

Muon System This system consists of four layers of hadron adsorber with proportional tubes between the layers. The thickness of adsorber is $\geq 7$ nuclear interaction lengths, and covers $45 \%$ of the solid angle around the detector.


Figure 3.18: A side view of the Mark II detector. The systems used in this analysis are highlighted.

Luminosity Monitors These are two devices used for the detection of small angle Bhabha scattering. The Small Angle Monitor (SAM) is a precise device with both tracking and calorimetry. The Mini Small Angle Monitor (MiniSAM) is a less precise device, but has a much higher counting rate. These detectors are described in section 3.3.4

### 3.3.2 Central Drift Chamber

The Central Drift Chamber (DC) consists of twelve concentric cylindrical layers of six wire cells. The wires are $2.3 m$ long. The 12 layers alternate between layers parallel to the beam axis, and layers inclined $\pm 3.8^{\circ}$ to the beam axis providing stereo information for tracking. The inner radius of the chamber is at 19.2 cm ; the inner layer consists of 26 cells. Each succeeding layer has 10 additional cells giving 136 cells in the outermost layer at 151.9 cm .

The layout of wires in a cell is shown in figure 3.19. Charged particles passing through the cell ionize the gas ( $89 \% \mathrm{Ar}, 10 \% \mathrm{C0}_{2}, 1 \% \mathrm{CH}_{4}$ ). A uniform drift field of $900 \mathrm{~V} / \mathrm{cm}$ is set up by the voltages on the field and potential wires $(-4.5 \mathrm{kv}$ on the field wires, and -1.5 kv on the potential wires). Charge drifts at $\approx 52 \mu \mathrm{~m} / \mathrm{ns}$ to the sense wires which are at ground potential.

The sense wires, which are $30 \mu \mathrm{~m}$ gold plated tungsten, are staggered $\pm 380 \mu \mathrm{~m}$ from the cell axis. This allows the tracking to distinguish between tracks which pass on the right side of the sense wire plane from tracks which pass on the left. The signals on the sense wires are amplified and sent to two different systems to determine the timing and pulse shape of the signals.

The timing information is provided by LeCroy 1879 FASTBUS TDCs. The signals are also digitized using 100 MHZ 6 -bit Flash ADCs (FADC). The TDCs are primarily


Figure 3.19: Layout of Wires in a Drift Chamber Cell
responsible for the timing used in track reconstruction, while the FADCs are used to provide pulse shapes used for energy loss $(D e / D x)$ analysis. The FADCs are also used to provide timing information for two track separation when tracks are too close together for the TDCs to resolve the separation.

Tracks are reconstructed by looking for segments which are formed when hits line up in the six wire cells. Segments are then joined together to form tracks. The track finding efficiency was measured using large angle Bhabha electrons which were detected when the Mark II was installed at the PEP storage ring $(\sqrt{s}=29 \mathrm{GeV})$. It is estimated to be $>99 \%$ for tracks which go through all twelve layers of the chamber (see Figure 3.20). Monte Carlo studies of high multiplicity events also show high efficiency in the central region of the detector (typically greater than $96 \%$ ).

The momentum resolution for single tracks was measured using Bhabha scattering


Figure 3.20: A plot of the efficiency of track finding in the Central Drift Chamber vs angle. The solid dots are a measurement using Bhabha events, the open squares are a Monte Carlo simulation of Hadronic $Z^{0}$ decays
at PEP, and found to be

$$
\frac{\sigma(p)}{p^{2}}=0.31 \% G e V^{-1}
$$

when the track is constrained to come from the origin, and

$$
\frac{\sigma(p)}{p^{2}}=0.46 \% G e V^{-1}
$$

when this constraint is dropped.

### 3.3.3 Calorimetry

The electromagnetic calorimetry for the Mark II is performed by two devices, the Liquid Argon Barrel Calorimeter, and The Endcap Calorimeter.

The Barrel Calorimeter is composed of eight independent liquid Argon cryostats. The barrels each cover the complete polar angle region for $\cos \theta<0.682$. The azimuthal angle is completely covered except for $3^{\circ}$ gaps between the barrels. Behind
these gaps are a one inch layer of lead followed by a layer of scintillator which acts as a tag for electromagnetic energy which goes through the gaps.

Inside each module are stacks composed of 2 mm strips of lead with 3 mm gaps between the layers of lead filled with liquid Argon. The gaps are created by ceramic spacers which result in a dead space of $5 \%$ in the liquid Argon. The strips are aligned in one of three directions; parallel to the beam axis, perpendicular to the beam axis, or at $45^{\circ}$ to the beam axis. This design allows the three dimensional position of hits in the Barrel to be reconstructed. The total barrel is 16 radiation lengths of material.

When an electron or photon enter the barrel, and electromagnetic shower occurs in the barrel. Charged particles from the shower pass through the liquid Argon and ionize it. Charge then drifts in a $12 \mathrm{Kv} / \mathrm{cm}$ field to the lead strips. The charge induced on the readout strips is amplified and shaped on the detector. In order to minimize the number of electronics channels, the readout strips are ganged together as shown in Figure 3.21. This reduces the total number of channels to 326 for each module.

On the inner edge of the barrel, there is an 8 mm gap of liquid argon between two 1.6 mm layers of aluminum strips. This layer allows detection of preshowering in the coil, and corrections for this effect are applied in software.

The performance of the barrel was measured using large angle Bhabha scattering events at the PEP storage ring. The distribution is shown in Figure 3.22 along with the distribution from a Monte Carlo simulation of the barrel. From this data, the energy resolution of the barrel is measured to be

$$
\frac{\sigma}{\sqrt{E}}=13.3 \% \oplus 3.3 \%
$$

The Endcap Calorimeters (ECC) are located at $\pm 1.37 m$ from the interaction point, and surround the beam pipe. The ECCs are composed of 72 alternating layers


Figure 3.21: Ganging of layers of Liquid Argon readout

Bhabha Energy Distribution for the Liquid Argon Calorimeter


Figure 3.22: A plot of the energy resolution for 14.5 GeV electrons in the liquid Argon. Overlaid on the data is a Monte Carlo simulation of the detector response.
of lead and proportional tubes which are a total of 18 radiation lengths thick. The calorimeter covers the angular region for $0.71<\cos \theta<0.97$.

Each layer of lead in the ECC is 0.28 cm thick. The layers of proportional tubes consist of 191 aluminum tubes with a $50 \mu \mathrm{~m}$ Stablohm 800 wire strung through the center. The tubes are filled with the same gas as the Central Drift Chamber. The first layer of tubes is oriented vertically, the second horizontally, the third at $+45^{\circ}$, and the fourth at $-45^{\circ}$. This pattern is repeated for the first 20 layers, while the final 16 are alternating horizontal-vertical. The readout from the wires is also ganged in a projective geometry and reduces the total number of channels to 1276 per endcap.

A study of Bhabha electrons at PEP gives an energy resolution of

$$
\frac{\sigma}{\sqrt{E}}=22 \%
$$

where E is in GeV .
The polar angle covered by the electromagnetic calorimetry is shown in Figure 3.23. The coverage is quite uniform, and represents $86 \%$ of the solid angle around the interaction point. The overlap region between the ECC and barrel calorimeters is also visible in this graph. Tracks going in this region are seen by the detectors, but not fully contained by either detector.

### 3.3.4 Luminosity Monitors

The luminosity monitors for the Mark II are used to detect small angle Bhabha scattering events. The two separate devices used for this purpose are the Small Angle Monitor (SAM) and the Mini Small Angle Monitor (MiniSAM). The layout of these two devices is shown in Figure 3.24. The SAM covers the angular range from 50 mrad to 165 mrad. The cross section in the SAM is about $4 / 3$ the $Z^{0}$ peak cross section.


Figure 3.23: Angular coverage of the electromagnetic calorimetery. The solid line is the number of radiation lengths a particle at the given angle goes through. The dashed line is the number of sampling layers the particle goes through.

The MiniSAMwhich covers the angular range from 15.2 to 25 mrad has a counting rate $\approx 7$ times the peak $Z^{0}$ rate.

The two SAM modules, situated at $\pm 1.38 \mathrm{~m}$ on either side of the interaction point, consist of nine layers of tracking followed by six layers of calorimetry. The layout of the SAM is shown in Figure 3.25.

The tracking consists of nine layers of drift tubes constructed from aluminum tubes 9.47 mm wide. Inside each tube are $38 \mu \mathrm{~m}$ gold plated tungsten wires while the gas in the tube is the same gas mixture as used in the Central Drift Chamber. The sense wires are operated at +1800 V with the walls of the tubes at ground potential. The first layer of tubes consists of 30 tubes oriented in the horizontal direction, while the second and third layers are oriented at $\pm 30^{\circ}$. This pattern is repeated for the fourth through ninth layers. The SAM layers are assembled from two pieces which are clamped around the beampipe as shown in Figure 3.26

The calorimetry layers are alternating lead and proportional tubes. The layers of


Figure 3.24: Layout of the Mark II Luminosity Monitors


Figure 3.25: Layout of the SAM


Figure 3.26: Assembly of the SAM Around the Beampipe
lead are each 13.2 mm thick giving a total thickness of 14.3 radiation lengths. The proportional tubes between the lead layers are of the same design as the tracking layers, but are operated at 1700 Volts.

The performance of the system was measured in a test beam of positrons with energies up to 15 GeV [39]. The tracking was found to have an angular resolution of 0.2 mrad while the energy resolution was parameterized to be $45 \% / \sqrt{E}$ (where $E$ is in $G e V$ ).

At the SLC, material in and around the beampipe shadows the outer regions of the SAM. A survey of the material in front of the SAM [40] is shown in Figure 3.27. The inner edge at 50 mrad is masked by a tungsten mask which defines the inner edge of the SAM acceptance. Material in the beampipe can cause preshowering of the electrons which creates extra hits in the tracking layers, often rendering tracking


Figure 3.27: This figure shows the number of radiation lengths of material in front of the SAM at each angle. The Tungsten mask at 50 mrad defines the inner edge. At angles greater than 125 mrad , material in the beampipe can be as much as 4 radiation lengths.
impossible. In very rare cases, events can be lost if the shower develops too early and cannot penetrate all layers of the calorimetry.

A typical SAM event is shown in Figure 3.28. The event picture shows both the north and south modules. The tracking layers in this event have quite a few extra hits. Energy is also clearly seen in all layers of the calorimetry. In the data, the tracking is found to be only about $80 \%$ efficient, while the calorimetry is virtually $100 \%$ efficient for identifying Bhabhas. The energy resolution for the data at the SLC is measured to be $15 \%$ [41]. Details of the event selection and systematic errors for the SAM are found in section 4.4.1.

The MiniSAM consists of two modules which surround the beampipe displaced on either side of the interaction point by 2.05 m . Each module is divided into 4 quadrants constructed of 6 alternating layers of scintillator and 0.79 cm thick tungsten, the total thickness of which is 15 radiation lengths. Figure 3.29 shows the layout of the


Figure 3.28: This is a typical event picture for a Bhabha in the SAM. Energy in the calorimeter is shown as peaks. The shower can clearly be seen in all layers of the calorimetry. The hits found in the tracking layers are represented by the + 's in the picture. Spurious hits are clearly visible.
quadrants.
The scintillator is read out with a wavelength shifter bar running the length of the device. The signals from the wave shifter bar are read out by phototubes on the back end of the MiniSAM, and the timing and pulse height of the signals is recorded. The energy resolution is $\approx 35 \% / \sqrt{E}$ (where $E$ is in $G e V$ ).

The angular acceptance of the MiniSAM is defined by two conical masks of 5.08 cm thick tungsten ( 15 radiation lengths). The rate of Bhabha events into the MiniSAM is critically dependent on the alignment of these masks. Due to uncertainties in the


Figure 3.29: The modules in the MiniSAM are laid out in quadrants on either side of the interaction point. Bhabhas leave deposits of energy in back to back quadrants.
alignments of the masks, the absolute cross section into the MiniSAM cannot be accurately calculated. For this reason, the MiniSAM is used as a relative luminosity monitor. Since the MiniSAM is located so close to the beampipe, it is subject to backgrounds which can affect the efficiency of the device. A complete discussion of the corrections for backgrounds and inefficiencies, as well as the systematic errors are contained in section 4.4.3.

### 3.3.5 Trigger

The Mark II data readout can be triggered from any of a number of different sources. Information from the Central Drift Chamber is used as a trigger for charged particles. The neutral trigger is made using signals from the calorimeters. The luminosity monitors can also trigger a readout of the detector for Bhabha scattering events. In addition to these sources, random beam crossings also trigger readout of the detector, providing events for studying background in the detector.

The charged trigger works by first determining which cells in the Drift Chamber


Figure 3.30: Hardware of the Charged particle Trigger
contain track segments. A cell is hit if a programmable number of wires are hit in the cell. Currently 4 out of the 6 wires in a cell must be hit for a cell to be considered as hit.

The pattern of hit cells in each of the twelve layers of the Drift Chamber is sent to a series of modules which look for patterns of hits which come from tracks. These curvature modules (See Figure 3.30) are each programmed to look for the pattern of tracks in a certain range of momenta. The modules are programmed to require that 8 out of the first 10 layers have track segments in them. The minimum momentum currently programmed requires that a track have $p_{t}>150 \mathrm{MeV} / \mathrm{c}$ (where $p_{t}$ is the momentum transverse to the beam direction). The tracks must also be within the angular range of $|\cos \theta|<0.75$ in order to pass through at least eight layers of the drift chamber.

The efficiency of the charged trigger is estimated using a Monte Carlo simulation
to be $>99 \%$ for single tracks which go through 12 layers of the drift chamber. For Hadronic $Z^{0}$ decays, the charged trigger is estimated to be $97 \%$ efficient.

The relatively low crossing rate for the SLC ( 60 hz for the data here) allows reasonably complex trigger decisions to be made. The calorimeter energy trigger is made in a module called the SSP Software Trigger (SST). A Slac Scanner Processor (SSP) in a FASTBUS crate reads the summed energy for the Endcap and Barrel calorimeter modules and looks for hits in modules lining up in towers which project to the interaction point.

The SST is programmed to require a 3.3 GeV shower in the Barrel Calorimeter, or a 2.2 GeV shower in the Endcap in order to produce a trigger. An offline emulation of the SST run on Monte Carlo data shows that the efficiency of this trigger for Hadronic $Z^{0}$ decays is $95 \%$.

The Mark II trigger is a logical OR of the charged and calorimetric triggers, and is estimated to be $99.8 \%$ efficient for Hadronic $Z^{0}$ decays.

The SAM trigger is an analog sum of the signals on the wires in the SAM calorimeter layers. For each module, there is a sum for the front half, the back half, and a separate sum for the total energy deposited in the module. A trigger requires 4 GeV in half a module, or 7 GeV in a whole module for both the north and south modules. This decision is made in a programmable Memory Logic Module (MLM). For some of the data, only one module is required to have a trigger. The trigger sums are read out from the MLM, and compared offline to sums from the individual wires which are also digitized. This trigger is estimated to be essentially $100 \%$ efficient.

The MiniSAM trigger is also formed in the MLM based on the analog sums of the modules in the MiniSAM. The sums are for all four quadrants in the MiniSAM. A trigger requires 20 GeV of deposited energy in both the North and South MiniSAM.

## Chapter 4

## Event Selection

### 4.1 Philosophy of event selection

The measurement of the $Z^{0}$ resonance parameters requires a careful measurement of the $Z^{0}$ cross section at several energies around the resonance. Measuring the cross section at each energy is done in two steps. First, the number of $Z^{0}$ decays ( $n_{Z}$ ) observed at a given scan point is counted, then the integrated luminosity $(\mathcal{L})$ at that scan point is calculated. Using $n_{Z}$ and $\mathcal{L}$, the cross section for $Z^{0}$ events is then simply

$$
\sigma_{Z}=\frac{n_{Z}}{\mathcal{L}}
$$

At this point, it is necessary to consider the effects of the detector on measuring the cross section. When attempting to measure $Z^{0}$ decays, it is obligatory to define what constitutes a $Z^{0}$ event. This is done by determining a set of selection criteria, or event cuts, which select an event as being a candidate $Z^{0}$ decay. These cuts should be designed to find as many real $Z^{0}$ decays as possible, while rejecting background events. These cuts will not be perfect, so the efficiency of the cuts must be determined. Once
the efficiency $(\epsilon)$ is estimated, the cross section is expressed as

$$
\sigma_{Z}=\frac{n_{Z}}{\mathcal{L} \epsilon}
$$

This effectively removes the biases of the detector from the measurement of the cross section.

The integrated luminosity $(\mathcal{L})$ is the last piece of the equation which needs to be filled in. This is done by counting events in the small angle monitors, and using the known cross section to determine the luminosity by

$$
\mathcal{L}=\frac{n_{\mathcal{L}}}{\sigma_{\mathcal{L}}} .
$$

Here $n_{\mathcal{L}}$ is the number of events which are counted in the luminosity monitors, while $\sigma_{\mathcal{L}}$ is the calculated cross section determined using knowledge of the detectors, and the Bhabha scattering process.

The measured cross section is now determined just by counting two types of events and using

$$
\begin{equation*}
\sigma_{Z}=\frac{n_{Z}}{n_{\mathcal{L}}} \frac{\sigma_{\mathcal{L}}}{\epsilon} \tag{4.1}
\end{equation*}
$$

In measuring $\sigma_{Z}$ it is important to define what types of decays are actually detected, and contribute to the measured cross section. In this thesis, the cross section which is measured is the sum of all detected hadronic decays, and all $\mu$ and $\tau$ events which fall within a certain fiducial volume of the detector. This cross section will be defined as $\sigma_{Z}$ for the rest of the thesis, and is given by

$$
\begin{equation*}
\sigma_{Z}=\sigma_{h a d}+\int_{f i d} \sigma_{\mu}+\int_{f i d} \sigma_{\tau} \tag{4.2}
\end{equation*}
$$

where the integrals are performed over the fiducial region which is defined later in this chapter.


Figure 4.1: A typical Hadronc $Z^{0}$ decay in the Mark II
The rest of this chapter describes the event selection criteria, and the efficiencies that result from the event selection for both Hadronic and Leptonic events. A description of the event selection for the luminosity monitors, and the calculation of the cross section into these devices is also presented at the end of this chapter.

### 4.2 Hadronic event selection

$Z^{0}$ decays into pairs of quarks manifest themselves in the detector as jets of Hadrons which form when the two quarks, which each have half the energy of the beam, fragment into Hadrons. The signature of these events is a large number of charged particles (high multiplicity), and a reasonable fraction of the beam energy seen in charged tracks, and in the calorimetry of the detector. A typical Hadronic event


Figure 4.2: A plot of the two dimensional distance of closest approach to the z axis for charged tracks (in $m$ ). The cut at $\pm 1 \mathrm{~cm}$ is shown by the arrows.
is shown in figure 4.1, visible are a large number of charged tracks throughout the volume of the detector.

### 4.2.1 Cuts for selecting hadrons

The selection criteria for Hadronic decays take into account the fact that the produces $Z^{0}$ particles are at rest in the laboratory frame, and thus, the decay products deposit energy in both the forward and backward (relative the the electron beam direction) hemispheres of the detector. Background events (beam-gas interactions and two photon events) tend to deposit energy in either the forward or backward direction, but not both.

For each event, all candidate tracks are first subject to a small set of quality cuts


Figure 4.3: A plot of the three dimensional distance of closest approach to the interaction point for charged tracks (in $m$ ). The cut at $\pm 3 \mathrm{~cm}$ is shown by the arrows.
to try and reduce the contribution from backgrounds. The small ( $\sigma_{x y} \approx 3.5 \mu m, \sigma_{z} \approx$ 1 mm ) size of the SLC beams means that the $Z^{0}$ events are produced in a small vertex region. Figure 4.2 shows a plot of the distance of closest approach (DCA) of charged tracks in the data to the z axis (where the z axis is along the direction of the $e^{+} e^{-}$ beams). A cut is made requiring that the DCA to the $z$ axis (RXY) be less than 1 cm . This is a two dimensional cut forming a cylinder of radius 1 cm around the z axis. Figure 4.3 shows the DCA for charged tracks to the interaction point. A sharp peak is again evident at the origin, but there is also a tail of background events which come from beam-gas interactions occuring at large $z$ (where $z=0$ is the location where the $e^{+} e^{-}$beams collide). A cut is made demanding that the DCA to the interaction point (RXYZ) be less than 3 cm . This three dimensional cut forms a sphere of radius

3 cm around the origin which when used together with the two dimensional cut forms a cylinder of radius 1 cm and length $\pm 3 \mathrm{~cm}$ around the interaction point. Tracks are constrained to come from this cylindrical volume in order to be considered good tracks. In addition to originating near the interaction point, tracks are required to be emitted at $|\cos \theta|<0.92$, and to have $p_{t}>110 \mathrm{MeV} / \mathrm{c}$ in order to insure accurate track reconstruction.

For photons in the electromagnetic calorimeters, a shower of at least 1 GeV must be reconstructed in either the Endcap, or the Liquid Argon Barrel for the track to be considered a good photon. Backgrounds from muons produced in the SLC upstream of the detector can travel through the Liquid Argon Barrel parallel to a readout strip. Energy from these strips is removed from the calculation of shower energies before the 1 GeV cut is made.

The number of good charged tracks in an event is required to be $\geq 3$ for candidate Hadronic events, eliminating background especially from Leptonic decays.

The visible energy in the forward and backward hemispheres of the detector is determined by calculating the sum of the charged particle momenta in the forward and backward hemispheres for good tracks, and adding in the energy of showers in the calorimeters which pass the quality cuts. No attempt is made to associate the energy from showers in the calorimeters with charged tracks. The visible energy in each hemisphere of the detector is required to be $\geq 0.05 E_{c m}$ (Where $E_{c m}$ is the center-of-mass energy). This requirement demanding substantial energy in each half of the detector is useful in eliminating beam-gas interactions and two photon events which tend to deposit energy in only one hemisphere.

A Plot of the energy in the forward hemisphere $\left(E_{Z}^{+}\right)$divided by $E_{c m}$ vs the energy in the backward hemisphere $\left(E_{Z}^{-}\right)$divided by $E_{c m}$ for a portion of the Mark II data

## Data



Figure 4.4: This plot shows the distribution $E_{Z}^{+} / E_{c m}$ versus $E_{Z}^{-} / E_{c m}$ for data in the Mark II. The cuts at $\geq 0.05 E_{\text {cm }}$ are shown as lines on the plot.
sample is shown in Figure 4.4. The cut at $5 \%$ of $E_{\text {cm }}$ is shown as a line along both axis. Events satisfying these cuts are presumed to be hadronic $Z^{0}$ decays.

### 4.2.2 Efficiency for selecting hadrons

The efficiency of the selection criteria is determined using a Monte Carlo simulation of Hadronic $Z^{0}$ decays in the Mark II detector. Figure 4.5 shows $E_{Z}^{+} / E_{c m}$ versus $E_{\bar{Z}}^{-} / E_{c m n}$ for a simulated sample of Hadronic decays (udscb quarks) of the $Z^{0}$. Clearly, a large fraction of the events pass the cuts, giving a high efficiency. From the Monte Carlo, the overall hadronic efficiency ( $\epsilon_{\text {had }}$ ) is determined to be

$$
\epsilon_{\text {had }}=0.953 \pm 0.006
$$



Figure 4.5: This plot shows the distribution $E_{Z}^{+} / E_{c m}$ versus $E_{Z}^{-} / E_{c m}$ for a Monte Carlo simulation of Hadronic $Z^{0}$ decays in the Mark II.

The error of $\pm 0.006$ comes primarily from varying the Monte Carlo generators, and parameters of the simulation. Three different event generators where used; the Lund Shower model [42], the Webber model[43], and the Lund Matrix Element model [44]. The difference in the efficiency calculated by these different generators contributes an error of $\pm 0.004$ to $\epsilon_{\text {had }}$. The energy scale of the calorimeters is not yet known precisely (since there are not yet enough large angle Bhabha scattering events to calibrate the calorimeters). Varying the energy scale in the Monte Carlo simulation contributes an error of $\pm 0.003$ to $\epsilon_{\text {had }}$. Finally, uncertainties in the comparisons between the Monte Carlo efficiency for track reconstruction at large $|\cos \theta|$ and the tracking reconstruction in the data contribute an error of $\pm 0.003$ to $\epsilon_{\text {had }}$.


Figure 4.6: This plot shows the distribution $E_{Z}^{+} / E_{c m}$ versus $E_{\bar{Z}}^{-} / E_{c m}$ for a Monte Carlo simulation of two photon events in the Mark II.

### 4.2.3 Backgrounds

Backgrounds from two photon interactions were studied using a Monte Carlo simulation of two photon events in the Mark II detector [45]. The results of this simulation which represents 25 times the integrated luminosity of the $Z^{0}$ data sample, are shown in Figure 4.6. One event is found to pass the cuts implying that 0.04 events can be expected tp pass the Hadronic selection cuts.

Backgrounds from beam-gas interactions, which occur when the beams interact with residual gas atoms in the beampipe, were studied by looking at events which have an event vertex at $3<|z|<50 \mathrm{~cm}$. Charged tracks have the same selection criteria, except that the DCA to the IP is demanded to be $>3 \mathrm{~cm}$ and $<50 \mathrm{~cm}$. A plot of $E_{Z}^{+} / E_{c m}$ versus $E_{Z}^{-} / E_{c m}$ for these events is shown in Figure 4.7. No events

## Background



Figure 4.7: This plot shows the distribution $E_{Z}^{+} / E_{c m}$ versus $E_{Z}^{-} / E_{c m}$ for beam-gas events in the Mark II.
pass the cuts implying that the number of events in the region $3<|z|<50 \mathrm{~cm}$ is $<2.3$ at the $90 \%$ confidence level. Since the volume for this region is 15 times as large as the volume for accepted Hadronic decays, the number of events in the Hadronic sample is estimated to be $<0.2$ at the $90 \%$ confidence level.

Backgrounds from Leptons are also a possibility in the case of $\tau$ decays. $\tau$ pairs which decay into 4 or 6 particle final states can pass the Hadronic selection criteria, however, in this analysis, many Leptonic decays are also included in the cross section from which the resonance parameters are extracted. Certain low angle $\tau$ decays which can still be considered a background are discussed in the next section.

### 4.3 Lepton selection

In order to decrease the statistical error in determining the $Z^{0}$ cross section, Leptonic decays of the $Z^{0}$ are also included in the data sample. To reduce the uncertainties in the triggering and identification of Leptons, Lepton candidates are required to be contained in the central region of the detector, and pass through the Barrel calorimeter. To insure this, Lepton candidates must have $\left|\cos \theta_{\text {thrust }}\right|<0.65$ where $\theta_{\text {thrust }}$ is the polar angle of the thrust axis with thrust defined as the axis $i$ which maximizes

$$
\text { Thrust }=\max \frac{\Sigma_{j}\left|P_{i j}\right|}{\Sigma\left|P_{j}\right|}
$$

This cut includes 0.556 of the total cross section, where this fiducial factor of 0.556 is just the integral of $\left(1+\cos ^{2} \theta\right)$ from -0.65 to 0.65 . Figure 4.8 shows a typical $\tau$ event in the Mark II where one $\tau$ decays into an electron and two neutrinos, and the other decays into a $\mu$ and two neutrinos. The electron is stopped in the calorimetry, while the mu penetrates the hadron adsorber leaving a track in the muon detecting tubes. Figure 4.9 shows a typical $\mu$ event where both produced muons penctrate the calorimeter, and all layers of the hadron adsorber.

Electrons, which can have substantial non-weak cross sections, are not included in this analysis, and thus must be separated from the $\mu$ and $\tau$ candidates.

### 4.3.1 Cuts for selecting Leptons

Charged tracks in Lepton candidate events are first subject to the same quality cuts as the hadronic events. Again, tracks must come from a cylinder of radius 1 cm and length $\pm 3 \mathrm{~cm}$ around the interaction point, they must have a minimum $p_{t}$ of $110 \mathrm{MeV} / \mathrm{c}$ and must emerge at $|\cos \theta|<0.82$. After the track quality cuts, an energy cut is made again requiring $5 \%$ of $E_{c m}$ energy deposited in each hemisphere


Figure 4.8: A typical $\tau$ decay in the Mark II
of the detector.
At this stage, the number of good charged tracks is counted ( $n_{\text {good }}$ ). If $2 \leq n_{\text {good }} \leq$ 6 , the event is considered for further analysis as a lepton. For these events, the thrust is calculated, and the following cuts are made on the thrust

Thrust $>0.95$

$$
\left|\cos \theta_{\text {thrust }}\right|<0.65
$$

After this cut, events with more than 2 tracks are considered $\tau$ events, while events with only 2 good tracks are examined to separate electrons from $\tau$ and $\mu$ events. Electrons which pass through the calorimeter shower and deposit their full energy in the calorimeter. Muons which pass through the calorimeter leave only minimum ionizing energy in the Barrel. The decay products of $\tau$ decays can be either


Figure 4.9: A typical $\mu$ decay in the Mark II
hadronic or leptonic. Hadrons can leave energy in the calorimeter (via $\pi^{0}$ decay for example) as can electrons, but in either case, these particles must share substantial momentum with other decay products ( $\pi^{ \pm}, \nu$ ), and hence will not deposit the full momentum of the $\tau$ in the calorimeter.

Clearly the Barrel is a powerful tool for separating electrons from $\tau$ and $\mu$ candidates in the central region of the detector. For each event, the total energy in the Barrel Calorimter is summed ( $E_{C a l}$ ), and divided by $E_{c m}$. Electrons leave nearly the full energy in the calorimter, while $\mu$ and $\tau$ events leave considerably less, hence $E_{C a l} / E_{c m}$ is $\approx 1$ for electrons, but much smaller for $\mu$ and $\tau$ events. Figure 4.10 shows a Monte Carlo study [46] of the quantity $E_{C a l} / E_{c m}$ for $e, \mu$, and $\tau$ events. Based on this study a cut on $E_{C a l} / E_{c m}$ of 0.8 clearly separates the electrons from the $\mu$ and $\tau$


Figure 4.10: $E_{C a l} / E_{c m}$ for Monte Carlo Lepton events
events.

### 4.3.2 Efficiencies for Lepton selection

The efficiency of these cuts was determined using Monte Carlo simulations of lepton decays in the Mark II[46]. The efficiencies were found to be

$$
\begin{aligned}
& \epsilon_{\mu}=97 \pm 2 \% \\
& \epsilon_{\tau}=95 \pm 2 \% .
\end{aligned}
$$

The uncertainty comes primarily from uncertainties in modeling the efficiencies in the veto counters located behind the cracks in the Liquid Argon Barrel.

The overall efficiency for Hadrons and Leptons combined ( $\epsilon_{t o t}$ ) is determined by weighting the individual efficiencies by how many events of each type are seen. This
overall efficiency is thus given by

$$
\begin{equation*}
\epsilon_{\text {tot }}=\frac{n_{\text {tot }}}{\frac{n_{\tau}}{\epsilon_{T}}+\frac{n_{\mu}}{\epsilon_{\mu}}+\frac{n_{\text {had }}}{\epsilon_{\text {had }}}} \tag{4.3}
\end{equation*}
$$

where $n_{\tau}, n_{\mu}, n_{\text {had }}$ are the number of $\tau, \mu$, and Hadronic events respectively, and $n_{\text {tot }}$ is the sum of $n_{\tau}, n_{\mu}$ and $n_{\text {had }}$. In order to determine $n_{\tau}$ and $n_{\mu}$, two particle final state leptons are separated into $\mu$ and $\tau$ events by a handscan. since the Leptonic cross section which is included in this analysis is $<7 \%$ as big as the hadronic cross section, the overall efficiency ( $\epsilon_{\text {tot }}$ ) and the uncertainties in $\epsilon_{\text {tot }}$ are dominated by the determination of the Hadronic efficiency. For this reason, and because the efficiencies are very similar, mistakes in assigning the $\mu$ and $\tau$ events does not contribute a sizable error in determining $\epsilon_{\text {tot }}$.

### 4.3.3 Backgrounds

Since $\tau$ events can pass the hadronic event selection criteria, both analysis are compared to find overlapping events. A handscan of the events is performed, and the events are labelled either either Hadronic or $\tau$ decays. In the region $|\cos \theta|<0.65$ the measured cross section is the sum of the Hadronic cross section, and the $\tau$ and $\mu$ events cross section. Events in this region which are identified by both the $\tau$ and Hadronic analysis are summed together and whether the event is called a $\tau$ or a Hadronic event does not change the number of events. In the region $|\cos \theta|<0.65$ the $\tau$ events are a background to the Hadronic sample, and are subtracted. Electrons are estimated to contribute less than 0.9 events as background to the Leptonic data sample. This is primarily from events which pass through the crack in the Barrel Calorimeter which is not instrumented with a tagging scintillator.

### 4.4 Event selection for the luminosity monitors

The other events which must be counted are the events in the luminosity monitors. The data from the luminosity monitors are grouped into two sets; events which are used to determine the absolute luminosity, and those which are used to calculate the relative luminosity. Since the cross section for events in the MiniSAM, and also for events near the inner edge of the SAM cannot be accurately calculated, the cross section for these events is determined empirically. This strategy takes advantage of the fact that the ratio of total events in the luminosity monitors (all SAM and MiniSAM events) to events fully contained in the SAM remains constant across scan points. The cross section for events which are well contained in the SAM (precise events) is calculated with a small systematic error, and the ratio between these precise events, and all luminosity events is determined empiricaly. This ratio has an uncertainty due to the statistical error on the number of events used to determine the ratio, but this statistical error is much smaller than the systematic error estimated for the calculation of the cross section into the MiniSAM.

This section describes the event selection for events in the SAM and MiniSAM, the procedure for determining which events are used for absolute luminosity calculation, and a discussion of the calculation of the absolute luminosity.

### 4.4.1 SAM event selection

Searches for Bhabha scattering events in the SAM proceed along two parallel channels. Events in the SAM are subject to both an automated algorithm for finding electromagnetic showers, and a handscan of candidate events. In order to select candidate events for handscanning, the middle four layers of the SAM calorimetry are


Figure 4.11: Energy distribution for SAM events
searched for deposited energy. If two of these four layers have at least one tube with a minimum of 400 MeV deposited energy above the average energy for the layer, then tat SAM module is tagged as being hit. Looking at only the inner layers reduces background from low energy particles produced by the SLC which can bombard the front and back layers of the SAM, but tend not to penetrate the inner layers. When both the North and South modules are tagged in this manner, the event is handscanned. to determine if the event is a Bhabha scattering event. Monte Carlo simulations indicate that this method of tagging is essentially $100 \%$ efficient. For all data searched so far, both the handscanning, and the shower reconstruction have found all the events which end up in the data sample.

An event is considered a Bhabha scattering event if there is a shower in both the North and South module containing at least $40 \%$ of the energy of the beam. Figure 4.11 shows the energy distribution for events in the SAM. The location of the cut is indicated by the arrow. Events passing these criteria are defined to be mask events.

The critical measurement for determining the cross section, is determining the positions of the scattered electrons and positrons in the SAM. This is especially true at the inner edge of the detector since the cross section drops as $1 / \theta^{3}$. The SAM was originally designed to use tracking with a 0.2 mrad angular resolution to determine the position of particles entering the detector. Backgrounds at the SLC have reduced the efficiency of the tracking, and for this reason determination of the shower position is done using the calorimetry position reconstruction which has a 1 mrad angular resolution. The mask which shadows the SAM at 50 mrad defines the inner edge of the acceptance of the SAM, however, events which fall just inside this mask are not well contained in the calorimetry, and their positions cannot be accurately determined.

This effect is visible in Figure 4.12 which shows the number of measured showers in a given angular range divided by $\theta^{3}$. This distribution should be flat for a perfect detector. Clearly visible in this figure is the inefficiency at the inner edge. Also visible is a falloff in efficiency at the outer edge of the detector which is due to the material which shadows the outer regions of the SAM.

To avoid the systematic errors in calculating the cross section resulting from reconstrucion inefficiencies at the inner edge, events which are used for calculating the absolute cross section are required to be fully contained in the SAM. Beamtest studies show that a shower is fully contained for $\theta>0.65 \mathrm{mrad}$. Events in this region which


Figure 4.12: Events in the $\operatorname{SAM} / \theta^{3}$
are used to determine the absolute cross section are divided into two classes; events where both the electron and positron showers are found at $\theta>0.65 \mathrm{mrad}$ (defined to be precise events), and events where one of the showers is found with 60 mrad $<\theta<65 \mathrm{mrad}$ and the other is found at $\theta>0.65 \mathrm{mrad}$ (defined to be gross events). When events are counted, the gross events are counted with half the weight of precise events.

To summarize, there are three classes of events in the SAM.

Mask events All events with two showers reconstructed in the SAM. These events are used for calculating the relative luminosity.

Precise events Events where both showers are reconstructed with $\theta>0.65 \mathrm{mrad}$. These events are used in the calculation of the absolute cross section.

Gross events These are events where on of the tracks is reconstructed with $\theta>0.65$ mrad , and the other is found with $60 \mathrm{mrad}<\theta<65 \mathrm{mrad}$. These events are
counted with one half the weight of precise events in the calculation of the absolute cross section.

### 4.4.2 SAM cross section

The measured absolute cross section is now just

$$
\mathcal{L}=\frac{n_{\text {precise }}+\frac{n_{\text {gross }}}{2}}{\sigma_{G P}}
$$

where $\sigma_{G P}$ is the calculated cross section for gross and precise events, $n_{\text {precise }}$ is the number of precise events, and $n_{\text {gross }}$ is the number of gross events.

The cross section for gross and precise events is calculated using the Monte Carlo generators described in section 2.5. The average value for the three Monte Carlos is

$$
\sigma_{G P}=25.2 n b(@ 91.1 G e V)
$$

The three calculations agree to $\pm 1.5 \%$. Because the effects of higher order radiative corrections are not fully calculated, a conservative estimate of $2 \%$ is assigned for the systematic error from calculating the cross section.

Detector effects also contribute to the systematic error on the absolute luminosity calculation. These are primarily from the error in the calorimetric position reconstruction for showers at the inner edge of the SAM, and reconstruction inefficiencies in the outer regions of the detector.

The accuracy of the position reconstruction of gross and precise events in the SAM calorimeter was checked by comparing the position determined by the calorimeter to that found with the more accurate resolving power of the tracking for events where the tracking is available. The agreement between the two methods is within the resolution of the devices, except in the region $60 \mathrm{mrad}<\theta<70 \mathrm{mrad}$ where leakage through


Figure 4.13: This graph shows the difference in the position of showers found by the SAM calorimeter versus the tracking. The histogram is a gaussian fit to the data.
the inner edge of the detector can cause the calorimetric estimate of the position to be skewed toward smaller angles. This effect can be seen in figure 4.13 which is a distribution of the difference in the shower position determined by tracking and that determined by calorimetry. The distribution is not centered at zero which shows the skewing due to leakage. The calculated cross section is corrected by $1.6 \pm 1.6 \%$ to compensate for this position skewing.

The material shadowing the SAM at angles $>120 \mathrm{mrad}$ can cause showers to be lost when the electron preshowers, and cannot be reconstructed in the SAM. The efficiency of the SAM for finding showers in the region $\theta>120 \mathrm{mrad}$ was studied by hand scanning of events in this region. Eight events were found where a shower was reconstructed in one SAM module, and even though evidence of energy was seen in the other module, a shower could not be reconstructed. This corresponds to a
correction of $-1.9 \% \pm 1.2 \%$ to $\sigma_{G P}$.
Summarizing, the systematic errors on the absolute cross section measurement are $2 \%$ for uncertainties in calculation of the radiative corrections to the Bhabha scattering cross section, and $2 \%$ systematic error in correction for detector effects. This gives a total systematic error of $2.8 \%$ on the absolute cross section.

### 4.4.3 MiniSAM event selection

The relative luminosity measurement uses not only SAM mask events, but also events in the MiniSAM. The design of the MiniSAM does not allow position reconstruction, but only counting of Bhabha scattering events. The high backgrounds which the SLC can create in the MiniSAM represent a possible source of error in counting events, and the methods of understanding these backgrounds are discussed in this section.

The MiniSAM event selection cuts fall into three classes; energy cuts, timing cuts, and geometrical cuts. The energy cuts look for the shower of a Bhabha scattered electron or positron. First, a set of quality cuts is made demanding that the event is not considered if there is more than 110 GeV of energy deposited in any quadrant in the MiniSAM, or more than 250 GeV of total energy in either the North or South modules. Each module is then searched for electromagnetic showers. A module is considered as containing a shower if one of the four quadrants has a total energy which is at least 25 GeV greater than the average energy of the other 3 quadrants, or if two adjacent quadrants have contain at least 25 GeV more deposited energy than the sum of the other two quadrants in the module. These cuts allow showers to be seen on top of the typical SLC backgrounds.

The timing cuts demand that any quadrant with greater than 11 GeV of deposited energy have a timing consistent with the particle having come from the interaction


Figure 4.14: This figure shows the geometries in the MiniSAM which are considered to be Bhabha events, and those for background events. The hits in the North and South are shown overlaid on a drawing of the four quadrants for each of the geometries. Showers which share energy between quadrants are shown on the border between the quadrants.
point. Backgrounds which hit the back of the modules will have early times, and hence fail this cut.

Bhabha scattering events will have back-to-back geometries, for example, the north top and south bottom modules should be hit together. The four geometries which are consistent with Bhabha scattering are shown in Figure 4.14. Events which have these geometries, and pass the energy and timing cuts are counted as Bhabha scattering events in the MiniSAM.

### 4.4.4 MiniSAM Backgrounds and efficiencies

Also shown in Figure 4.14 are geometries which are not consistent with Bhabha scattering, and must be considered background events. A Monte Carlo simulation


Figure 4.15: A plot of the ratio of MiniSAM events corrected for efficiency to SAM events for varying efficienceis in the MiniSAM. This ratio should be constant independent of efficiency.
of the detector revealed that a data sample equal to the total MiniSAM luminosity should have 3 events with the background geometries, while 17 events of this type are seen in the data.

Making the assumption that the background events are distributed isotropically, these events are subtracted from the data at each scan point. The subtraction varies from $0 \%$ to $3.5 \%$ over the scan points and the total subtraction is $0.4 \%$. At each scan point a systematic error equal to the size of the subtraction is assigned to the MiniSAM data.

The large backgrounds in the MiniSAM can sometimes make the device inefficient for tagging Bhabha scattering events. The efficiency of each run is studied by overlaying Monte Carlo Bhabha scattering events on top of the data from random beam crossing triggers, and then recording how efficiently the reconstrucion algorithm finds
the Bhabha scattering events. Any run where the efficiency of one MiniSAM quadrant is less than $25 \%$, or the average efficiency of all MiniSAM quadrants is less than $50 \%$ is dropped from the analysis entirely ( 8 events in the SAM, and one $Z^{0}$ event are lost because of this cut). For each scan point, the efficiency of the MiniSAM is taken as the average efficiency of all the runs at that energy. The efficiencies for all scan points are greater than $90 \%$. To be sure that the efficiency calculation was consistent, the ratio of MiniSAM events corrected for efficiency to SAM events is plotted in figure 4.15 as a function of MiniSAM efficiency. The ratio should be independent of the efficiency. From this graph, the correction can be seen to be consistent within errors.

## Chapter 5

## Results

This chapter presents the data from the resonance scan, and the fits to the data which extract the resonance parameters. The details of the resonance scan are presented, followed by a discussion of the fitting technique used to extract the resonance parameters. Finally, the results are presented along with a discussion of the systematic errors on the measurements.

### 5.1 Details of the Resonance Scan

The scan to determine the resonance parameters proceeded in several steps. An initial scan of three different energies separated by 1.5 GeV was performed to determine the approximate location of the resonance. The energy was initially set to 92.2 GeV and then lowered to 90.7 GeV and 89.2 GeV . Approximately the same amount of integrated luminosity was collected at each of these scan points. The outcome of this initial scan showed the peak of the resonance to be between 90.7 GeV and 92.2 GeV , and at this point, a second set of 3 scan points separated by 1.5 GeV was made with the energies falling between the energies of the first 3 points. The energies of
these scan points were $90 \mathrm{GeV}, 91.5 \mathrm{GeV}$ and 93 GeV , and approximately the same luminosity was collected at each of these points as was collected at the first three points.

The six initial points allowed the approximate location of the peak to be determined [47], and the energy of the SLC was set at this energy ( 91.5 GeV ) for a run to collect approximately four times the integrated luminosity of each of the first six scan points. Running at the peak for an extended period allows an accurate determination of the peak cross section, which is in turn used to determine the number of species into which the $Z^{0}$ decays. At the beginning of August 1989, the SLC was shut down briefly for maintenance, and the masking surrounding the beampipe inside the Mark II was realigned. following the shutdown, the energy of the machine was raised by 400 MeV , and another extended run was made at this energy just above the determined peak position. Finally, two more scan points were measured approximately 1 GeV above and below the peak position.

At each scan point, the energy is determined by reading the measured energy from the extraction line spectrometers for each SAM mask event. Since these events determine the luminosity measurement, using them to determine the average energy of each scan point gives an unbiased estimate of the energy for events where the beams are in collision. The average energy for these events $(\langle E\rangle)$ is determined at each of the scan points. The electron energy is reasonably stable, as a feedback mechanism in the SLC control system attempts to maintain the electron energy. The positron energy, however, can fluctuate, giving rise to a jitter in the average center-of-mass energy measured. This jitter ( $\sigma_{E}$ ) is measured to be between 40 and 70 MeV at the 10 scan points.

In addition to measuring the energy at each scan point, it is also necessary to
measure the energy spread, as the large energy spread in the SLC beams leads to substantial corrections to the measured cross section. These corrections can be visualized by imagining that the machine energy is set exactly at the position of the peak cross section. The finite energy spread in the beams means that the center-of-mass energy of particles which collide is centered on the peak, but with a finite spread around the peak. Collisions with energies on either side of the peak will have a smaller cross section than at the peak, hence the cross section which is measured will be smaller than the peak cross section. This effect can be corrected for by determining what the average measured cross section is for finite energy spread versus the cross section at the central energy. The sign and magnitude of the correction depend on where the scan point is relative to the resonance peak, and what the measured energy spread is at the scan point. A Monte Carlo study showed that the correction is essentially independent of the distribution assumed for the energies of the electron and positron bunches. Figure 5.1 shows the magnitude of the correction to the cross section for a 300 MeV RMS spread in the center of mass energy with a uniform distribution of energy in each bunch.

At each scan point, the energy spread for all SAM mask events is measured and averaged to determine the average energy spread ( $\langle\delta E\rangle$ ) for the scan point ( $\delta_{e}=$ $\sqrt{\delta_{e^{-}}^{2}+\delta_{e^{+}}^{2}}$ ). This quantity is added in quadrature with the determined jitter $\left(\sigma_{E}\right)$ to determine the overall RMS spread of the center-of-mass energy ( $\delta E_{R M S}$ ). Assuming a uniform distribution in the beam energies of each bunch, the correction at an energy $E$ is given by

$$
\begin{equation*}
\delta E_{c o r}=\frac{\int_{e 1}^{e 2} \sigma(S) d S / \int_{e 1}^{e 2} d S}{\sigma(E)} \tag{5.1}
\end{equation*}
$$

The bounds of the integral are just the limits of a uniform distribution centered at


Figure 5.1: This figure shows the correction to the cross section (in \%) as a function of where on the resonance the energy is set. The upper (grey) curve represents the resonance shape, while the lower curve shows the correction at each position on the resonance for a 300 MeV RMS spread in energy.
energy $E$ with RMS energy spread equal to $\delta E_{R M S}$

$$
\begin{aligned}
& e 1=E-\left(<\delta E_{R M S}>\sqrt{12} / 2\right) \\
& e 2=E+\left(<\delta E_{R M S}>\sqrt{12} / 2\right)
\end{aligned}
$$

$\sigma(S)$ is the $Z^{0}$ cross section, which is also a function of the mass, width, and peak cross section of the $Z^{0}$. This correction is determined during the fits for the resonance parameters, and applied to the cross section as described in the next section.

Table 5.1 summarizes the average energy and energy spread determined for each of the ten scan points.

At scan point 6, the energy readout for the electrons was not available for a small portion of the running since a component of the spectrometer failed. During this time, the electron energy was held stable, and measured using a less precise readout system available from the SLC. Within the resolution of the SLC energy readout, the electron energy for these runs was stable. The energy for these runs was constructed by using the last available readout of the electron energy from the spectrometer with the positron readout which was available from the spectrometer. The estimated

| Scan pt | $\langle E\rangle$ | $\left\langle\delta E_{R M S}\right\rangle$ |
| :---: | :---: | :---: |
| 1 | 92.16 | 0.22 |
| 2 | 90.74 | 0.26 |
| 3 | 89.24 | 0.28 |
| 4 | 91.50 | 0.29 |
| 5 | 89.98 | 0.27 |
| 6 | 92.96 | 0.21 |
| 7 | 91.06 | 0.23 |
| 8 | 91.43 | 0.26 |
| 9 | 92.22 | 0.25 |
| 10 | 90.35 | 0.26 |

Table 5.1: This table lists the measured average energies (in GeV ) and RMS energy spreads (in GeV ) for each of the ten scan points.
systematic error on the energy for this small block of runs is approximately 50 MeV , and it is not treated separately in the fits.

### 5.2 Fitting Technique

### 5.2.1 Method of maximum likelihood

In order to extract the resonance parameters, the method of maximum likelihood is used. The principle of this method is to construct a likelihood function which is a function of the resonance parameters ( $\mathrm{L}\left[\mathrm{M}, \Gamma, \sigma_{0}\right]$ ), and vary the parameters until this likelihood is maximized. The observables in this experiment are the number of luminosity events ( $n_{\mathcal{L}}=n_{\text {mask }}+n_{m s a m}$ ) and the number of $Z^{0}$ decays $\left(n_{Z}\right)$. At each scan point, the probability (P) for observing $n_{Z} Z^{0}$ decays and $n_{\mathcal{L}}$ luminosity events is determined as a function of the resonance parameters and the energy of the scan point. The likelihood function is then just the product over the probabilities at each
scan point

$$
L=\prod_{\mathfrak{i}} P\left(M, \Gamma, \sigma_{0}, E_{i}, n_{\mathcal{L}_{\mathfrak{i}}}, n_{Z \mathfrak{i}}\right)
$$

where i is an index over scan points. The resonance parameters are varied until $-\ln L$ is minimized using the computer program MINUIT [49]. The confidence interval of $n$ standard deviations for any parameter is the point where $-\ln L$ changes by $n^{2} / 2$.

### 5.2.2 Likelihood function

The probabilities at each scan point are first determined by noting that $n_{\mathcal{L}}$ and $n_{\mathcal{Z}}$ are poisson distributed variables with means given by

$$
\begin{aligned}
& \left\langle n_{\mathcal{L}}>=\sigma_{\mathcal{L}} \mathcal{L}\right. \\
& <n_{Z}>=\epsilon_{z} \sigma_{Z} \mathcal{L}
\end{aligned}
$$

where

$$
\sigma_{\mathcal{L}}=\sigma_{m a s k}+\epsilon_{m s a m} \sigma_{m s a m}
$$

is the total luminosity cross section and

$$
\epsilon_{z}=\epsilon_{t o t} \delta E_{c o r}
$$

is the overall efficiency of detecting $Z^{0}$ decays combined with the correction for the finite energy spread of the beams.

The quantity of interest in determining $\sigma_{Z}$ is the ratio $n_{Z} / n_{\mathcal{L}}$. The probability of observing the ratio $n_{Z} / n_{\mathcal{L}}$ given $\sigma_{Z}$ and $\sigma_{\mathcal{L}}$ is given by [48]

$$
\begin{equation*}
P=\frac{\frac{\left(\epsilon_{z} \sigma_{\mathcal{Z}} \mathcal{L}\right)^{n_{Z}} n_{\left(\sigma_{\mathcal{L}} \mathcal{L}\right)^{n}}{ }^{\mathcal{L}}}{n_{Z} n_{\mathcal{L}}!} e^{-\mathcal{L}\left(\epsilon_{z} \sigma_{Z}+\sigma_{\mathcal{L}}\right)}}{\frac{\left(\epsilon_{z} \sigma_{Z} \mathcal{L}+\sigma_{\mathcal{L}} \mathcal{L}^{n_{Z}+n_{\mathcal{L}}}\right.}{\left(n_{Z}+n_{\mathcal{L}}\right)!}} e^{-\mathcal{L}\left(\epsilon_{z} \sigma_{Z}+\sigma_{\mathcal{L}}\right)} \tag{5.2}
\end{equation*}
$$

In calculating the maximum likelihood, $n_{Z}$ and $n_{\mathcal{C}}$ are never varied, hence the factorial terms are just constants added to the $\ln L$ and can be ignored. This allows (5.2) to be simplified as

$$
\frac{\left(\epsilon_{z} \sigma_{Z}\right)^{n_{Z}} \sigma_{\mathcal{L}} n_{\mathcal{L}}}{\left(\epsilon_{z} \sigma_{Z}+\sigma_{\mathcal{L}}\right)^{n_{Z}+n_{\mathcal{L}}}},
$$

and the likelihood function is

$$
\begin{equation*}
L=\prod_{\text {scanpoints }} \frac{\left(\epsilon_{z} \sigma_{Z}\right)^{n_{Z}} \sigma_{\mathcal{L}} n_{\mathcal{L}}}{\left(\epsilon_{z} \sigma_{Z}+\sigma_{\mathcal{L}}\right)^{n_{Z}+n_{\mathcal{L}}}} . \tag{5.3}
\end{equation*}
$$

### 5.2.3 Systematic errors in the fits

Systematic errors are included in the fits by adding parameters to the fit which allow constants which have systematic errors associated with them to vary. For each constant with a value $\bar{x}$ and standard deviation $\sigma_{x}$, a parameter $x$ is added to the fit, and a term is added to the $\ln L$ of the form

$$
\ln L=\ln L+\frac{(x-\bar{x})^{2}}{2 \sigma_{x}^{2}} .
$$

This term can be thought of as a penalty function which allows the constant $x$ to vary around its central value in a manner consistent with its systematic error $\left(\sigma_{x}\right)$.

The systematic errors which are included in this manner are:

- The uncertainty in the absolute energy scale.
- The uncertainties in the efficiencies for Hadrons, Muons, and Taus.
- The uncertainties in the scale factors for SAM Mask, and MiniSAM events (described in section 5.4.1).


### 5.3 Details of the Fits

Three fits are performed on the data which differ in how many parameters are allowed to be free in the fits, and how many are fixed by constraints imposed by the Standard Model.

### 5.3.1 One parameter fit

The first fit treats only the $Z^{0}$ mass as a free parameter. This fit takes advantage of the relationship between the width of the $Z^{0}$, and the peak cross section which is predicted by the Standard Model. The peak cross section is calculated using (2.28)

$$
\begin{equation*}
\sigma_{p e a k}=\frac{12 \pi}{M_{Z}^{2}} \frac{\Gamma_{e c} \Gamma_{f}}{\Gamma_{Z}^{2}} \tag{5.4}
\end{equation*}
$$

Here, $\Gamma_{f}$ is the $Z^{0}$ width into all events which pass the cuts detailed in chapter 4 and is given by

$$
\Gamma_{f}=\Gamma_{h a d}+f\left(\Gamma_{\mu \mu}+\Gamma_{\tau \tau}\right)
$$

where f is the factor of 0.556 which arises from the fiducial cuts on $\mu$ and $\tau$ events, and $\Gamma_{h a d}$ is the $Z^{0}$ width into hadrons (udscb quarks). The partial widths into fermions are calculated using (2.19) and (2.22) which depend only on the $Z^{0}$ mass (and on $\sin ^{2} \theta_{W}$ which is calculated using the $Z^{0}$ mass). $\Gamma_{Z}$ is the total $Z^{0}$ width which is calculated from the partial widths by

$$
\Gamma_{Z}=\Gamma_{h a d}+\Gamma_{e e}+\Gamma_{\mu \mu}+\Gamma_{\tau \tau}+3 \Gamma_{\nu \nu}
$$

By assuming that the $Z^{0}$ decays only into 5 quarks, 3 charged leptons, and 3 neutrinos, the width and peak cross section are completely specified by the Standard Model, and only the mass needs to be varied in the fit.

|  | Mass | $N_{\nu}$ | $\Gamma_{Z}$ | $\sigma_{\text {peak }}$ |
| :--- | :---: | :---: | :---: | :---: |
| One Parm. | Free | 3 | calculated | calculated |
| Two Parm. | Free | Free | calculated | calculated |
| Three Parm. | Free | - | Free | Free |

Table 5.2: Summary of parameters for the fits. Parameters which are marked as calculated are determined by Standard Model constraints.

### 5.3.2 Two parameter fit

This fit is performed in the same manner as the one parameter fit, except that the number of neutrinos is treated as a parameter $\left(N_{\nu}\right)$. In this case $\Gamma_{Z}$ is now

$$
\Gamma_{Z}=\Gamma_{h a d}+\Gamma_{e e}+\Gamma_{\mu \mu}+\Gamma_{\tau \tau}+N_{\nu} \Gamma_{\nu \nu},
$$

and $\Gamma_{f}$ is calculated as in the one parameter fit. Again, the mass must be treated as a free parameter, and the peak cross section is calculated using (5.4). This fit permits the invisible width of the $Z^{0}$ to be a fit parameter. By assuming that all of this width is due to massless neutrinos, the results of the fit can be interpreted as determining the number of neutrino generations.

### 5.3.3 Three parameter fit

The final fit does not make any assumptions about the relationship between the peak cross section and the total width of the $Z^{0}$. In this fit, the mass, total width, and peak cross section are all treated as free independent variables. This is the only fit where the parameters are determined without any Standard Model constraints on the line shape.

The parameters of the three fits are summarized in table 5.2.

### 5.4 Data

### 5.4.1 Luminosity Data

The luminosity calculation requires determining the ratios between the number of events in the Precise region of the SAM where the cross section is calculated, and the total number of events in the SAM and MiniSAM. These scale factors are calculated by taking the ratio between the number of Gross-Precise events $\left(n_{g p}\right)$ in the SAM, and the number of Mask events ( $n_{\text {mask }}$ )

$$
S_{p m}=\frac{n_{g p}}{n_{\operatorname{mask}}}
$$

and the ratio between the number of SAM Mask events and MiniSAM events

$$
S_{m m}=\frac{n_{m a s k}}{n_{m s a m} / \epsilon_{m s a m}}
$$

Once the scale factors are determined, the luminosity cross section for Mask and MiniSAM events is calculated using

$$
\begin{aligned}
\sigma_{m a s k} & =\frac{\sigma_{g p}(E)}{S_{p m}} \\
\sigma_{m s a m} & =\frac{\sigma_{m a s k}}{S_{m m}}
\end{aligned}
$$

and the total luminosity cross section is

$$
\begin{equation*}
\sigma_{\mathcal{L}}=\sigma_{m a s k}+\epsilon_{m s a m} \sigma_{m s a m}+\sigma_{i n t}(E) \tag{5.5}
\end{equation*}
$$

In the above expression $\sigma_{g p}(E)$ is the energy dependent cross section for Gross-Precise events given by

$$
\sigma_{g p}(E)=\sigma_{g p}(91.1) \cdot \frac{(91.1)^{2}}{s\left(G e V^{2}\right)}
$$

while $\sigma_{\text {int }}(E)$ is the correction to the luminosity cross section due to interference between the t-channel photon and the s-channel $Z^{0}$. The interference term for the SAM


Figure 5.2: This figure shows the correction to the SAM cross section due to interference with the $Z^{0}$.
is shown in Figure 5.2, the equivalent term for the MiniSAM is about $40 \%$ as large, and is also added in. Both terms are quite small corrections to the approximately $270 n b$ luminosity cross section.

The scale factors $S_{p m}$ and $S_{m m}$ are limited by the statistics of the number of events used to calculate the ratios. The statistical errors on the scale factors are calculated using Binomial statistics [50] and given by

$$
\begin{equation*}
\delta S_{p m}=\sqrt{\frac{n_{g p} / n_{\text {mask }}\left(1-n_{g p} / n_{m a s k}\right)}{n_{\operatorname{mask}}}}, \tag{5.6}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta S_{m m}=\sqrt{\frac{n_{\text {mask }} / n_{\mathcal{L}}\left(1-n_{\text {mask }} / n_{\mathcal{L}}\right)}{n_{\mathcal{L}}}}, \tag{5.7}
\end{equation*}
$$

where $n_{\mathcal{L}}=n_{\text {mask }}+n_{m s a m}$. $\delta S_{m m}$ is an absolute error on the ratio of Mask to luminosity events which is used to estimate the error on the scale factor for efficiency
corrected MiniSAM events. The relative error is calculated using

$$
\delta S_{m m-\text { rel }}=\frac{\delta S_{m m}}{n_{\text {mask }} / n_{\mathcal{L}}},
$$

and an error of that relative size is assigned to $S_{m m}$.
The calculation of the scale factors was complicated by the fact that the masking for the SAM and MiniSAM was moved between scan points 7 and 8 . The effect on the Mask event cross section due to this move was calculated using a Monte Carlo simulation of the SAM and the masking and was found to be

$$
S_{p m}^{8-10}=1.01 \pm 0.02 S_{p m}^{1-7}
$$

where $S_{p m}^{1-7}$ is the scale factor for the first seven scan points, and $S_{p m}^{8-10}$ is the scale factor for the last 3 scan points. Using this calculation, the two sets of data can still be combined statistically to give two scale factors which each have the smaller statistical error of the combined data set. Defining

$$
\begin{aligned}
P_{1} & =\frac{n_{g p}^{1-7}}{n_{\text {mask }}{ }^{1-7}}, \\
P_{2} & =\frac{n_{g p}{ }^{8-10}}{n_{\text {mask }}{ }^{8-10}},
\end{aligned}
$$

and calculating $\delta_{1}$ and $\delta_{2}$ for the two data sets using (5.6), gives

$$
\begin{equation*}
S_{p m}^{(1)}=\frac{P_{1} / \delta_{1}^{2}+r P_{2} / \delta_{2}^{2}}{1 / \delta_{1}^{2}+r^{2} / \delta_{2}^{2}} \tag{5.8}
\end{equation*}
$$

as the scale factor to be used for the first seven data points. Here $r$ is the factor of 1.01 on the scale factors which results from the movement of the masks. The scale factor for scan points after the mask movements $\left(S_{p m}^{(2)}\right)$ is given by $r \cdot S_{p m}^{(1)}$ and is equal to

$$
\begin{equation*}
S_{p m}^{(2)}=\frac{P_{1} / r \delta_{1}^{2}+P_{2} / \delta_{2}^{2}}{1 / r^{2} \delta_{1}^{2}+1 / \delta_{2}^{2}} . \tag{5.9}
\end{equation*}
$$

| Scan <br> Point | $n_{p}$ | $n_{g}$ | $n_{\text {mask }}$ | $n_{\text {msam }}$ | $\epsilon_{\text {msam }}$ | MSAM <br> Bkgd | Lum. <br> $\left(n b^{-1}\right)$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| 1 | 18 | 1 | 31 | 105 | 0.971 | 2 | $0.53 \pm 0.05$ |
| 2 | 35 | 7 | 54 | 266 | 0.962 | 4 | $1.22 \pm 0.07$ |
| 3 | 14 | 0 | 24 | 166 | 0.990 | 0 | $0.68 \pm 0.05$ |
| 4 | 30 | 4 | 53 | 275 | 0.994 | 1 | $1.24 \pm 0.07$ |
| 5 | 23 | 1 | 36 | 174 | 0.994 | 1 | $0.76 \pm 0.05$ |
| 6 | 20 | 2 | 39 | 214 | 0.976 | 8 | $1.00 \pm 0.07$ |
| 7 | 92 | 5 | 170 | 923 | 0.989 | 1 | $4.11 \pm 0.12$ |
| 8 | 90 | 7 | 164 | 879 | 0.910 | 0 | $4.10 \pm 0.13$ |
| 9 | 78 | 5 | 128 | 680 | 0.983 | 0 | $3.04 \pm 0.11$ |
| 10 | 66 | 5 | 116 | 617 | 0.999 | 0 | $2.60 \pm 0.10$ |
| Totals | 815 | 466 | 37 | 4299 |  | 17 | $19.3 \pm 0.9$ |

Table 5.3: This table summarized the data from the luminosity monitors. $n_{p}$ and $n_{g}$ are the number of Precise and Gross events in the SAM. The column labelled "MSAM bkgd" is the number of MiniSAM background events which are subtracted at the scan point.

The error on these scale factors is given by

$$
\delta S_{p m}=\sqrt{\frac{1}{1 / \delta_{1}^{2}+r^{2} / \delta_{2}^{2}}} \approx \sqrt{\frac{1}{1 / \delta_{1}^{2}+1 / \delta_{2}^{2}}}
$$

The change in the MiniSAM scale factors after the mask moved could not be calculated with sufficient accuracy to allow the two sets of data to be combined. For this reason, the MiniSAM scale factors are calculated separately before and after the mask movement, with each factor having an independent statistical error.

Table 5.3 summarizes the data for the luminosity monitors. The errors on the luminosity at each scan point is only the statistical error on $n_{\mathcal{L}}$. The total luminosity is calculated from the Gross-Precise events and includes the systematic error on the absolute cross section. Table 5.4 summarizes the scale factors which are used to calculate the luminosities in the SAM and MiniSAM.

One final detail is the manner in which the systematic error on the MiniSAM

| Scan <br> Points | $n_{g p}$ | $n_{\text {mask }}$ | $n_{\text {msam }} /$ <br> $\epsilon_{\text {msam }}$ | $S_{p m}$ | $S_{m m}$ | $\sigma_{\text {mask }}$ <br> $\left(n b^{-1}\right)$ | $\sigma_{\text {msam }}$ <br> $\left(n b^{-1}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1-7$ | 242.0 | 407 | 2156.7 | $0.591 \pm 0.017$ | $0.1887 \pm 0.0086$ | 42.6 | 226 |
| $8-10$ | 242.5 | 408 | 2276.0 | $0.597 \pm 0.017$ | $0.1793 \pm 0.0086$ | 42.2 | 235 |

Table 5.4: Summary of scale factors for the luminosity monitors before and after the movement of the masks surrounding the Mark II beampipe.
background subtraction is handled. At each scan point, the statistical error on the number of MiniSAM events is just $\sqrt{n_{m a a m}} / n_{m s a m}$ while the systematic error on the number of background events is just $n_{b k g d} / n_{m s a m}$. This gives a total combined statistical and systematic error of

$$
\begin{equation*}
\frac{n_{m s a m}+n_{b k g d}^{2}}{n_{m s a m}{ }^{2}} \tag{5.10}
\end{equation*}
$$

This combined error is included in the likelihood function by calculating a new value for the number of MiniSAM events $n_{m s a m}{ }^{\prime}$ which has a statistical error equal to (5.10) and is given by

$$
\begin{equation*}
\frac{1}{n_{m s a m}^{\prime}}=\frac{n_{m s a m}+n_{b k g d}^{2}}{n_{m s a m}^{2}} \tag{5.11}
\end{equation*}
$$

This reduced number of events simulates the addition of a systematic error by increasing the statistical error. The cross section for the MiniSAM is multiplied by a factor $\beta=n_{m s a m}{ }^{\prime} / n_{\text {msam }}$ so that .

$$
\sigma_{m s a m}^{\prime}=\beta \sigma_{m s a m}
$$

and the luminosity measured by the MiniSAM

$$
\mathcal{L}_{m s a m}=\frac{n_{m s a m}^{\prime}}{\sigma_{m s a m}^{\prime}}=\frac{n_{m s a m}}{\sigma_{m s a m}}
$$

remains constant.
Treating the errors in this manner, allows a point to point systematic error to be included in the maximum likelihood fit.

| Scan <br> Point | $n_{\text {had }}$ | $n_{\mu}$ | $n_{\tau}$ | ecor | $\sigma_{Z}$ <br> $(n b)$ |
| :--- | ---: | ---: | ---: | :---: | :---: |
| 1 | 11 | 0 | 0 | 1.006 | $21.5_{-6.6}^{+9.2}$ |
| 2 | 33 | 2 | 1 | 0.978 | $31.6_{-5.5}^{+6.8}$ |
| 3 | 3 | 0 | 0 | 1.024 | $4.5_{-2.5}^{+4.5}$ |
| 4 | 33 | 1 | 5 | 0.968 | $34.1_{-5.7}^{+7.0}$ |
| 5 | 8 | 1 | 1 | 1.019 | $13.5_{-4.3}^{+6.0}$ |
| 6 | 13 | 0 | 1 | 1.011 | $14.5_{-4.0}^{+5.4}$ |
| 7 | 114 | 3 | 3 | 0.973 | $31.5_{-3.1}^{+3.4}$ |
| 8 | 108 | 3 | 3 | 0.972 | $30.0_{-2.9}^{+3.3}$ |
| 9 | 67 | 2 | 2 | 1.009 | $24.3_{-3.0}^{+3.4}$ |
| 10 | 60 | 1 | 1 | 1.002 | $25.0_{-3.3}^{+3.8}$ |
| totals | 450 | 13 | 17 |  |  |

Table 5.5: This table presents a summary of the number of Hadronic, $\mu$, and $\tau$ events ( $n_{\text {had }}, n_{\mu}, n_{\tau}$ ), the correction to the cross section for the finite energy spread (ecor), and the cross section for Hadronic events and $\mu$, and $\tau$ events which pass the fiducial cuts.

### 5.4.2 Hadronic and Leptonic Data

At this point, all the elements necessary for calculating the cross section for $Z^{0}$ decays are available. Table 5.5 summarizes the events measured and the determined cross section at each of the scan points. In this table $\sigma_{Z}$ is the cross section for all Hadronic, and for $\mu$ and $\tau$ events with $\left|\cos \theta_{\text {thrust }}\right|<0.65$. The errors on the cross section are calculated using the method in ref [51]. Based on the number of events of each type, the overall efficiency is calculated to be

$$
\epsilon_{\text {tot }}=0.953 \pm 0.006
$$

The data are plotted in figure 5.3.
The cross sections shown in table 5.5 are corrected for efficiency, and also corrected for the finite energy spread of the beams. The size of this energy correction is shown


Figure 5.3: This figure shows the data for $\sigma_{Z}$ and the lineshapes predicted by the three fits. The lower curve is the lineshape from the one parameter fit, while the upper curve represents the lineshape for the two and three parameter fits (the curves are indistinguishable).
in this table under the column labelled "ecor".

### 5.5 Results of the fits

Figure 5.3 shows the results of all three fits superimposed on the data. The second and third fits give nearly indistinguishable line shapes. The results for each of the fit parameters are discussed below.


Figure 5.4: Likelihood function vs Number of Neutrino Generations

### 5.5.1 Mass

The mass is a free parameter in all three of the fits, and all three fits give the same result

$$
M_{Z}=91.14 \pm 0.120 \mathrm{GeV} / \mathrm{c}^{2}
$$

The error on this number is primarily statistical, with the dominant error being the 35 MeV uncertainty in the overall energy scale.

### 5.5.2 Number of Neutrino Generations

The number of neutrino generations is treated as a free parameter only in the two parameter fit. The results of this fit are

$$
N_{\nu} \Gamma_{\nu \nu}=0.45 \pm 0.10 \mathrm{GeV}
$$

which can be interpreted using the Standard Model value for $\Gamma_{\nu \nu}$ as

$$
N_{\nu}=2.8 \pm 0.6 \text { Generations. }
$$

This width can be attributed to any particle which the $Z^{0}$ decays into invisibaly. Assuming that these decays are attributable to neutrinos allows one to set a $95 \%$ Confidence Limit on the number of massless neutrino generations. This is found by examining Figure 5.4, which shoes the value of the likelihood function as a function of the number $N_{\nu}$. The one sided $95 \%$ Confidence Limit is found by moving $1.64 \sigma$ from the position of maximum likelihood, or equivalently to the value where the $-\ln L$ changes by $(1.64)^{2} / 2$ from the maximum value. From the curve in this figure, a $95 \%$ Confidence Limit of 3.9 generations is assigned.

If one assumes that there is a minimum of 3 neutrino generations, and demands that the maximum likelihood be taken as the value of the likelihood for $N_{\nu}=3$ (since the maximum likelihood occurs for $N_{\nu}<3$ ), a $95 \%$ Confidence Limit of 3.9 neutrino generations can still be set. This is possible because the likelihood is extremely flat around the maximum value, and so moving what is taken as the central value by a small amount changes the location where the likelihood drops of to $1.64 \sigma$ by a very small amount.

This measurement rules out a fourth massless neutrino generation at the $95 \%$ Confidence Limit. figure 5.5 shows the data with the Standard Model predictions of the lineshapes for $N_{\nu}=3, N_{\nu}=3$ and $N_{\nu}=4$ superimposed. The data clearly supports 3 generations.

The systematic error on this measurement corresponds to 0.45 neutrino generations. It is a combination of the systematic error in determining the absolute cross section which contributes 0.25 neutrino generations, and the statistical errors in determining the scale factors for the MiniSAM and Mask events which contribute the


Figure 5.5: This figure shows the data with the Standard Model line shapes for 2 (top curve), 3 (middle), and 4 (bottom) neutrino generations overlaid.
remaining error.

### 5.5.3 Total Width and Cross section

The total width and peak cross section are free parameters only in the three parameter fit. In this fit, the Standard Model constraints between the peak cross section, and the total width are removed. The results are

$$
\Gamma_{Z}=2.42_{-0.35}^{+0.45} \mathrm{GeV}
$$

for the width, and

$$
\sigma_{\text {peak }}=45 \pm 4 n b
$$

| Fit | Mass <br> $\left(\mathrm{GeV} / \mathrm{C}^{2}\right)$ | $N_{\nu}$ | $\Gamma_{Z}$ <br> $(\mathrm{GeV})$ | $\sigma_{\text {peak }}$ <br> $(\mathrm{nb})$ |
| :--- | :---: | :---: | :---: | :---: |
| One Parm. | $91.14 \pm 0.12$ | - | - | - |
| Two Parm. | $91.14 \pm 0.12$ | $2.8 \pm 0.6$ | - | - |
| Three Parm. | $91.14 \pm 0.12$ | - | $2.42_{-0.35}^{+0.45}$ | $45 \pm 4$ |

Table 5.6: This table summarizes the results for the parameters from each of the three fits.
for the peak cross section.
The numbers can be compared to the standard model predictions for $\Gamma_{Z}$ and $\sigma_{p e a k}$ by examining the contour plot of the likelihood function as a function of $\Gamma_{Z}$ and $\sigma_{\text {peak }}$ shown in Figure 5.6. The Standard Model predictions for $N_{\nu}=2,3$, and 4 neutrino generations are shown overlaid on this contour plot. The data is clearly consistent with the Standard Model prediction for $N_{\nu}=3$ while both $N_{\nu}=2$ and $N_{\nu}=4$ are excluded at the $68 \%$ Confidence Level.

The statistical errors dominate the measurement of the total width, with the two major contributions to the systematic error being 50 MeV coming from the point to point systematic error on the MiniSAM background subtraction, and 27 MeV from the uncertainty in the relative energy measurements.

The results of the three fits are summarized in table 5.6.

### 5.5.4 Fits to the Hadronic Data

As a check, the three fits were performed on only the Hadronic events. The cross sections for Hadronic events are shown in table 5.7, and plotted with the lineshape from the best fit in Figure 5.7. The results from all three of the fits were consistent with the results from the fits to the combination of Hadronic and Leptonic data,


Figure 5.6: This is a Contour plot of the likelihood function for $\Gamma_{Z}$ versus $\sigma_{p e a k}$. Each contour represents a change in likelihood of $n^{2} / 2$ where n is 1 for the first contour, two for the second, and three for the third contour. The Standard Model predictions for the values of $\Gamma_{Z}$ and $\sigma_{\text {peak }}$ for two, three, and four neutrino generations are shown overlaid on the plot.

| Scan <br> Point | $\sigma_{h a d}$ <br> $(n b)$ |
| :--- | :---: |
| 1 | $21.5_{-6.6}^{+9.2}$ |
| 2 | $29.0_{-5.3}^{+6.5}$ |
| 3 | $4.5_{-2.5}^{+4.5}$ |
| 4 | $28.8_{-5.2}^{+6.4}$ |
| 5 | $10.8_{-3.7}^{+5.5}$ |
| 6 | $13.5_{-3.9}^{+5.2}$ |
| 7 | $30.0_{-3.0}^{+3.3}$ |
| 8 | $28.4_{-2.8}^{+3.2}$ |
| 9 | $22.9_{-2.9}^{+3.3}$ |
| 10 | $24.2_{-3.3}^{+3.8}$ |

Table 5.7: Cross sections for Hadronic Data


Figure 5.7: This figure shows the Hadronic data and the lineshape predicted by the two parameter fit.

| Fit | Mass <br> $\left(\mathrm{GeV} / \mathrm{C}^{2}\right)$ | $N_{\nu}$ | $\Gamma_{Z}$ <br> $(\mathrm{GeV})$ | $\sigma_{\text {peak }}$ <br> $(\mathrm{nb})$ |
| :--- | :---: | :---: | :---: | :---: |
| One Parm. | $91.14 \pm 0.12$ | - | - | - |
| Two Parm. | $91.14 \pm 0.13$ | $2.9 \pm 0.6$ | - | - |
| Three Parm. | $91.14 \pm 0.13$ | - | $2.43_{-0.37}^{+0.47}$ | $42 \pm 4$ |

Table 5.8: This table summarizes the results for the parameters from each of the three fits using only the Hadronic data.
and had slightly larger statistical errors. The results of the three fits are shown summarized in table 5.2.

### 5.6 Determination of the Weinberg Angle

The determination of the $Z^{0}$ mass allows limits to be set on the Weinberg angle $\theta_{W}$. The quantity $\sin ^{2} \theta_{W}$ appears in the determination of three separate ratios in the Standard Model[52]. These are; The ratio of the $W$ mass to the $Z$ mass, the rato of the weak coupling to the electromagnetic coupling, and the ratio between the $S U(2)$ and $U(1)$ components of the weak current (see equation 2.3 ). In each of these cases the radiative corrections applied to the calculation of $\sin ^{2} \theta_{W}(\Delta r$ in equation 2.32) are calculated differently

The calculation of $\Delta r$ depends on two unknown parameters of the Standard Model, the Top mass $\left(M_{t}\right)$, and the Higgs mass $\left(M_{H}\right)$. The correction due to the Top mass is proportional to the ratio ( $\frac{M_{1}^{2}}{M_{W}^{2}}$ ) while the correction due to the Higgs is proportional to $\ln \left(\frac{M_{1}^{2}}{M_{W}^{2}}\right)$, hence the corrections are much more sensitive to the Top mass, than the Higgs mass. Figure 5.8 shows $\sin ^{2} \theta_{W}$ versus $M_{t}\left(\right.$ for $M_{h}=100 \mathrm{GeV}$ ) with the


Figure 5.8: This figure shows the limits on the Marciano-Sirlin $\sin ^{2} \theta_{W}$ versus the Top mass which are determined by the measurment of the $Z^{0}$ Mass.
radiative corrections applied for the Marciano-Sirlin definition of $\sin ^{2} \theta_{W}[53]$

$$
\begin{equation*}
\sin ^{2} \theta_{W}=1-\left(\frac{M_{W}^{2}}{M_{Z}^{2}}\right) \tag{5.12}
\end{equation*}
$$

This calculation is useful for comparing to the ratio $M_{W} / M_{Z}$ measured by the $p \bar{p}$ experiments.

Figure 5.9 shows the calculation of $\sin ^{2} \overline{\theta_{W}}$ [52] (sometimes also called $\sin ^{2 *} \theta_{W}$ [54]) in which $\Delta r$ is calculated to properly determine the couplings of the $Z^{0}$ (or equivalently the ratio of the weak components of the $S U(2)$ and $U(1)$ fields). This is the value of $\sin ^{2} \theta_{W}$ which should be used to calculate the assymetries which depend on the axial and vector couplings of the $Z^{0}$.


Figure 5.9: This figure shows the limits on $\sin ^{2} \overline{\theta_{W}}$ versus the Top mass which are determined by the measurment of the $Z^{0}$ Mass.

The strong dependence of these corrections on the Top mass mean that tests of the couplings of the Standard Model will require an accurate determination of $M_{t}$ as well as the $Z^{0}$ mass. Alternatively precise measurements of the couplings, or of the ratio $M_{W} / M_{Z}$ coupled with the measurement of $M_{Z}$ can lead to a prediction for the top mass via the calculation of the radiative corrections.

## Bibliography

[1] S. L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam and J. C. Ward, Phys. Lett. 13, 168 (1964).
[2] There are many excellent reviews summarizing experimental tests of the Standard Model see for example R. D. Pecci in Proceedings of the Winter School of Physics: Cosmology and Elementary Particles (Puerto Rico, 1988).
[3] G. Arnison et. al.,Phys. Lett. 122B, 103 (1983).
[4] M. Banner et. al.,Phys. Lett. 122B, 476 (1983).
[5] G. Arnison et. al.,Phys. Lett. 126B, 398 (1983).
[6] P. Bagnaia et. al.,Phys. Lett. 129B, 130 (1983).
[7] C. Albajar et. al., CERN-EP/88-168 (1988).
[8] R. Ansari et. al., Phys. Lett. 186B, 440 (1987).
[9] C. Quigg, Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, (Benjamin/Cummings, Menlo Park, 1983).
[10] J. Alexander, G. Bonvicini, P. Drell and R. Frey,Phys. Rev. D 37, 56 (1988).
[11] W. De Boer, Nucl. Instr. Meth. a278, 687 (1989); K. G. Chetyriken et. al., Phys. Lett. B85, 227 (1979); M. Dire, J. Sapirstein, Phys. Rev. Lett. 43, 668 (1979); W. Celmaster, R. J.Gonsalves, Phys. Rev. Lett. 44, 560 (1980).
[12] G. S. Abrams et. al., submitted to Phys. Rev. Lett.
[13] A. Barrose et. al., CERN-EP/87-80 (1987).
[14] E. A. Kuraev and V. S. Fadin, Sov. J. Phys. 41, 466 (1985).
[15] T. Kinoshita, J. Math. Phys. 3, 650 (1962); T. D. Lee and M. Naunberg, Phys. Rev. B133, 1549 (1964); R. Barbieri, J. A. Mignaco and E. Remiddi, Nuovo Cim. A11, 824 (1972); G. J. H. Burgers, Phys. Lett. B164, 167 (1985).
[16] D. R. Yenni, S. C. Frautschi and H. Suura, Annals of Phys. 13, 379 (1961).
[17] M. Greco, G. Pancheri and Y. N. Srivastuva, Nucl. Phys. B101, 234 (1975); J. D. Jackson and D. L. Scharre,Nucl. Instr. Meth. 128, 13 (1975).
[18] F. A. Berends, G. J. H. Burgers and W. L. Van Neerven, Phys. Lett. B185, 395 (1987).
[19] R. N. Cahn, Phys. Rev. D 36, 2666 (1987).
[20] O. Nicrosini and L. Trentadue, UPRF-86-132.
[21] G. Bonneau and F. Martin, Nucl. Phys. B27, 381 (1971).
[22] B. Lynn, M. Peskin and R. Stuart, SLAC-PUB-3725 (1985).
[23] W. Marciano, BNL-42855 (1989).
[24] See for example F. Mandl, Quantum Field Theory, (Wiley and Sons, New York, 1984).
[25] D. Karlen, Ph.D. thesis, Stanford University, SLAC-Report-325, March 1988.
[26] S.M. Swanson,Phys. Rev. 154, 1601 (1967).
[27] A.C. Hearn, P.K. Kuo, and D.R. Yennie,Phys. Rev. 87, 1950 (1969)
[28] F.A. Berends et. al.,Nucl. Phys. B264, 265 (1986).
[29] F.A. Berends,W. Hollik, and R. Kleiss,Nucl. Phys. B304, 712 (1988).
[30] S. Jadach and B.F.L. Ward,UTHEP-88-11-01(1988).
[31] F.A. Berends and R. Kleiss,Nucl. Phys. B228, 537 (1983).
[32] M. Consoli,S. Lo Presti,M. Greco,Phys. Lett. 113B, 415 (1982).
[33] SLC Conceptual Design Report, SLAC-Report-299 (1980).
[34] M. Levi,J. Nash, and S .Watson,Nucl. Instr. Meth. A281, 265 (1989).
[35] J. Kent,et. al.,SLAC-PUB-4922(1989).
[36] M. Levi,et. al.,SLAC-PUB-4921(1989).
[37] Extraction Line Spectrometer For SLC Energy Measurement, SLAC-SLC-PROP-2 (1986).
[38] G. S. Abrams et. al.,Nucl. Instr. Meth. A281, 55 (1989).
[39] M. Petrazda et. al.,Mark II-SLC Note 164(1986).
[40] E. Gero,Mark II-SLC Note 209(1986).
[41] J. Hylen,R. Frey,S. Hong,Mark II-SLC Note - In preperation.
[42] Lund 6.3 Parton Shower Model, T. Sjöstrand, Nucl. Phys. B289, 810 (1987).
[43] WEBBER 4.1 Monte Carlo, G. Marchesini and B. R. Webber, Nucl. Phys. B238, 1 (1984); B. R. Webber, Nucl. Phys. B238, 492 (1984).
[44] Lund 6.3 Matrix Element, T. D. Gottschalk and M. P. Shatz, Phys. Lett. B150, 451 (1985).
[45] F. A. Berends, P. H. Daverveld, and R. Kleiss, Comp. Phys. Comm. C 40, 309 (1986).
[46] KORALZ Monte Carlo, S. Jadach, Z. Was, Comp. Phys. Comm. C 36, 197 (1985).
[47] G. S. Abrams et. al.,Phys. Rev. Lett. 63, 794 (1989); G. S. Abrams et. al., Phys. Rev. Lett. 63, 2173 (1989).
[48] D. Cox and P. Lewis, The Statistical Analysis of Series of Events,(Wiley and Sons, New York, 1966).
[49] F. James and M. Roos,Comp. Phys. Comm. C 10, 343 (1975).
[50] A. G. Frodesen, O. Skjeggestad,H. Tofte, Probability and Statistics in Particle Physics,(Universitetsforlaget, Oslo, 1979).
[51] F. James and M. Roos,Nucl. Phys. B172, 475 (1980).
[52] G. Burgers and F. Jergerlehner, to appear in Proceedings of the Workshop on $Z$ Physics at LEP.
[53] W. Marciano,Phys. Rev. D 20, 274 (1979); A. Sirlin,Phys. Rev. D 22, 971 (1980); W. Marciano and A. Sirlin,Phys. Rev. D 22, 2695 (1980); W. Marciano and A. Sirlin, Phys. Rev. D 29, 945 (1984).
[54] D. Kennedy and B. Lynn,SLAC-PUB-4039 (1988).

