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FLUX TRAPPING AND SHIELDING IN IRREVERSIBLE SUPERCONDUCTORS*

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ABSTRACT

Flux trapping and shielding experiments were carried out on Pb, Nb, Pb-Bi, Nb-Sn, and Nb-Ti samples of various shapes. Movable Hall probes were used to measure fields near or inside the samples as a function of position and of applied field. The trapping of transverse multipole magnetic fields in tubular samples was accomplished by cooling the samples in an applied field and then smoothly reducing the applied field to zero. Transverse quadrupole and sextupole fields with gradients of over 2000 G/cm were trapped with typical fidelity to the original impressed field of a few percent. Transverse dipole fields of up to 17 kG were also trapped with similar fidelity. Shielding experiments were carried out by cooling the samples in zero field and then gradually applying an external field. Flux trapping and shielding abilities were found to be limited by two factors, the pinning strength of the material, and the susceptibility of a sample to flux jumping. The trapping and shielding behavior of flat disk samples in axial fields and thin-walled tubular samples in transverse fields was modeled. The models, which were based on the concept of the critical state, allowed a connection to be made between the pinning strength and critical current level, and the flux trapping and shielding abilities. Adiabatic and dynamic stability theories are discussed and applied to the materials tested. Good qualitative, but limited quantitative agreement was obtained between the predictions of the theoretical stability criteria and the observed flux jumping behavior.

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SYMBOLS

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В	Magnetic Induction	Gauss
Be	B-field in equil. with applied field	
	in the absence of irrev. effects	Gauss
н	Magnetic field	0e
Ha,Ba	Applied field	Oe,Gauss
HC, BC,	Upper critical field	Oe,Gauss
b,h	reduced field B/Bc ₂ , H/Hc ₂	-
HC 1	Lower critical field	0e
Hfi	Field of first flux jump	0e
ΔHfi	Spacing of flux jumps	0e
Hffi	Full flux jump field	0e
НЧ	$(4\pi/10)$ x.Jex(wall thickness)	0e
M	Magnetization	Gauss
Ma	Pavarsible magnetization	62055
Ъ.	Magnetic flux	Gauss-om ²
<u>አ</u>	Flux quantum	2110-7 Gauss-cm ²
10 1	Floatria fiold	volt/cm
с. Т	Current	
1	Current Current	n N
10	Tatal current	n A com2
Jtot	Potal current density	AZ CM-
J	current density (exclusive	5 com 2
-	of the magnetization currents)	A/Cm ⁻
ปฏ	Current density due to magnetization	A/cm ⁻
- JC	Critical state current density	AZCM-
1	lemperature	ĸ
To	Constant related to temperature	
	dependence of Jc	ĸ
Td	-Jc(dJc/dT)"	K
θd	Debye temperature	K 2 H
0	Heat capacity	J/cm ³ -K
β	Coefficient of T ³ portion of C	J/cm ^J -K*
К	Thermal conductivity	W/cm-K
М	Energy density	J/cm ³
Rg	Gas constant	
ao	Inter-vortex spacing	cm
ξ	Coherence length	cm
λ	Penetration depth	cm
ĸ	λ/ξ	-
V	Drift velocity of the flux lines	cm/sec
7)	Viscosity coefficient	
f	Helmholtz free energy/volume	J/cm ³
Fp	Macroscopic pinning force/vol.	dynes/cm ³
fp	Microscopic pinning force	dynes∕cm³
P(B)	Pinning force/length	dynes∕cm
Δ₩	Difference in energy between pinned	
	and unpinned flux lines	J
α,	Constants	
Bo,	in the critical	
γ	state models	
np	Density of pinning centers	cm ⁻³
nl	Number pins/lenth of flux line	cm ⁻¹
V	Volume	cm ³
٧n	Volume fraction of normal phase	-

Vs	Volume fraction of superconducting	phase
Sv	Phase boundary area/vol	cm ⁻¹
Svp	Phase boundary area/vol.	
	perpendicular to given direction	cm ⁻¹
1p	Length of flux line pinned/vol	cm ⁻²
d	Spacing of pinning sheets	cm
c	Length of pinning sheets	CM
a	Width of pinning sheets	cm
q	Disk thickness	cm
Es	Elastic energy stored in flux	
	line lattice	
C 6 6	Shear modulus of flux line lattice	
VSC	Superconducting electron velocity	cm/sec
ns	Number density of superconducting	
	ele <u>c</u> trons	cm ⁻³
n	Number density of charge carriers	cm ⁻³
nd	Demagnetization coefficient	-
Co	Field attenuation factor	-
πe	Efficiency factor	-
t	Time	sec
τm	Magnetic time constant	sec
τth	Thermal time constant	sec
ρ	Electrical resistivity	ohm-cm
ρn	Normal state resistivity	ohm-cm
ρf	Flux flow resistivity	ohm-cm
Dm	Magnetic diffusivity	cm²/sec
Dth	Thermal diffusivity	cm²/sec
**	Denotes exponentiation	-

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CHAPTER I-INTRODUCTION

The importance of the Meissner effect is usually emphasized when the behavior of superconductors in magnetic fields is considered. Ideally, type-I materials expel all magnetic flux and type-II materials go into the mixed state when a sample is cooled in a magnetic field or a field is applied to a cold sample. The field within ideal materials of either type is uniquely determined by the value of the applied field (independently of the magnetic history of the material) and goes to zero as the applied field is turned off. Experimentally it is often difficult to achieve the full Meissner state (B=0 inside the material) in real materials. Most superconductors exhibit some degree of irreversibility, including the presence of trapped flux in the material after an applied field has been reduced to zero. In most cases, the lack of a full Meissner effect and the presence of trapped flux result from the pinning of individual magnetic flux lines, or bundles of flux lines, in local free energy minima created by inhomogeneities in the material. The presence of large numbers of these inhomogeneities, called pinning centers, allows the material to trap in or shield out large scale magnetic fields. The procedure by which flux trapping and shielding is accomplished in practice is shown in Fig. 1-1. In the experiments described here the aim was to: investigate and demonstrate the potentially useful features of the flux trapping and shielding process; define its limitations; and relate the process to other better known properties of superconductors.

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Shielding



Fig. 1-1 Flux trapping and shielding procedures.

The technique of trapping or shielding magnetic fields with large superconducting assemblies has several potentially useful applications. In the field of high energy physics there is a continuing need for large and expensive beam handling and focusing magnets. These magnets are typically used to generate transverse dipole, quadrupole, and sextupole fields in beam tubes with diameters on the order of centimeters. The results of this study indicate that it is feasible to use large hollow superconducting samples to trap such fields and thus realize savings in power consumption. A technique in which one master magnet is used to impress the field on a number of slave superconducting trapping tubes could be used to save capital costs. The trapping properties of irreversible superconductors can also be used to construct improved beam optics for electron microscopes (see for example Kitamura et al. 1966, Weyl et al. 1972, and several articles in the 4th European Conf. of Electron Microscopy, Rome 1968). Since no external power supply is required to maintain the fields, the flux trapping process can be useful in situations in which a high degree of temporal stability is required. The shielding abilities can be used in applications where low field regions, or regions shielded from external fields (Martin et al. 1972) are required. Superconducting shields can also be used to improve the uniformity of solenoidal fields (Bychkov et al. 1975). Our work (Rabinowitz et al. 1973, Garwin et al. 1973, Frankel & Garwin 1977) shows that materials presently available can trap and shield fields of a size and complexity that are useful in the above applications. Additional materials development can be expected to further improve capabilities.

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Previous investigations concerned with flux trapping (Pippard 1955, Calverley & Rose-Innes 1960, Coffey & Gauster 1963, Corson 1964, Cline et al. 1966, Coffey 1967, Schweitzer et al. 1967, Voigt 1968) have generally been limited to cylindrical or tubular specimens in axial fields, although some other geometries have been studied (Hanak 1964, Desorbo & Healy 1964, Benaroya et al. 1966, Castillo & Fast 1968). Pickup coils wound around the samples or a few stationary field probes have typically been used as sensors. Only in a few cases have the details of the field structure near high-field irreversible materials been examined closely, and even in these more detailed studies the specimens were usually cylinders or disks coaxial with applied fields. A variety of geometries and field configurations are investigated in the work comprising this thesis. Movable Hall probes sensitive to field direction are used to obtain detailed information on the magnetic field during the flux trapping or shielding process. In one series of experiments further information is gathered by using a spinning coil arrangement to analyze the harmonic content of the internal field during the trapping process.

The experiments demonstrate the feasibility of trapping transverse multipole fields in hollow superconducting samples. Trapping and shielding of uniform transverse dipole fields in the 10-20 kG range by Nb₃Sn and Nb-Ti samples are demonstrated. Flux trapping and shielding are found to be limited by two factors: the pinning strength of a material, and the material's stability with respect to flux jumps. In the absence of flux jumping, it is possible to predict shielding and trapping behavior on the basis of a material's pinning strength by using

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the concept of the critical state. The usual simple stability theories are found to be qualitatively correct, although they are not successful in quantitatively predicting the wide variety of flux jumping behavior observed in the experiments. In addition, the exponential penetration of transverse fields into hollow diamagnetic cylinders and the effect on shielding ability of butting or overlapping superconducting cylinders are observed.

The thesis has been structured so that the major subsections can be considered as nearly independent units. Chapters 2 and 3 include a rather lengthy review of backround material associated with the problems of flux pinning and flux jumping. A full reading of this material is not necessary in order to follow the discussions of the later chapters, which are concerned more directly with the actual experimental results and their analysis. A brief summary of the content of each chapter is given below.

Flux pinning, which is the feature ultimately responsible for flux trapping, and which determines the upper limits on the size of trapped fields, is discussed in chapter 2. Several basic concepts that are useful for the discussion of flux pinning and flux trapping are introduced first: the various fields used to describe magnetic effects in superconductors are defined, the driving forces on flux lines in the mixed state of type II superconductors are described, and the concept of the critical state is introduced. The methods by which pinning forces are measured are briefly analyzed, and some of the phenomenological expressions for the critical current density derived from them are

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enumerated. The phenomenological expressions are generally useful only over limited field ranges and are often disassociated from the underlying physics. However, they do provide a convenient means for presenting experimental data, and are often used as a basis for engineering calculations of the properties of irreversible superconductors of practical concern. The remaining portion of the chapter deals with the problem of relating the flux pinning force to the microstructure of superconducting materials. A number of different approaches have been taken to this complicated problem. Although it is often difficult to determine which approach or theory is appropriate in a given case, the theories are useful in gaining an understanding of how various metallurgical treatments affect the flux pinning, and consequently the ultimate flux trapping ability of a material. The discussion in this chapter concentrates on theories applicable to two of the materials tested. The experimental and theoretical situation for Pb-Bi alloys is discussed first. Pb-Bi alloys are easily cast into almost any desired shape, and their flux trapping and shielding ability varies with composition from relatively low levels in nearly reversible low Bi-content alloys to potentially useful levels in higher Bi-content alloys. Alloys with compositions near the eutectic (56%Bi) exemplify the case in which a relatively straightforward theory for the pinning has been largely verified. Observations on flux pinning in Nb-Ti alloys are then described. Nb-Ti can also be relatively easily formed into a variety of shapes, and, with proper metallurgical treatment, can trap and shield moderately high fields. Several theories that attempt to account for the pinning in this more complicated material are analyzed.

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Instabilities in superconductors, the features which often limit flux trapping and shielding to levels far below those possible with good flux pinning, are discussed in chapter 3. The methods generally used to investigate flux jumping are briefly described, and the results of the numerous previous observations of the phenomenon are summarized. Observations on flux jumping in the samples measured here were generally in accord with those of previous studies. Observations and theories concerning flux_motion in other, more controlled, situations are briefly reviewed, and related to the case of flux motion during flux jumps. The problems which arise in developing a theory of flux jumping are outlined, and a few of the more widely used theories examined. Estimates of expected flux jump fields for the materials studied here are derived from these theories. The theories are generally in qualitative agreement with experimental observations, but often fail to account fully for the detailed data on flux jumping. Both previous work and the present experiments suggest that the properties and environment of the surface region of a sample are very important in the evolution of flux jumps.

Chapter 4 includes a brief description of the experimental setups and procedures, a discussion of sources of error and uncertainty in the data, and a description of the computer programs used to analyze the data and predict flux trapping and shielding behavior. The experimental setups and procedures were straightforward. Samples of moderate size, with typical dimensions on the order of centimeters, were tested in fields of up to 20kG. Fields were measured and mapped with inexpensive Hall probes mounted in movable holders. Most measurements were made

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with a precision of no better than a few percent because the experiments were intended to provide an understanding of the general behavior of irreversible superconductors rather than highly accurate characterizations of particular samples. The problem of relating the observed field distribution in a sample to its pinning forces and magnetic history was considerably more difficult in the case of the transverse geometries used in these experiments than in the usual case of semi-infinite-cylindrical samples in axial fields. Using simplifying assumptions about the form of the critical state currents and their dependence on the local field, computer programs were written to model flux trapping and shielding by thin disk-shaped samples in axial fields, and by thin-walled tubular samples in transverse fields.

The experimental results are presented in chapter 5. Observations made with five different materials, Pb, Pb-Bi, Nb, Nb₃Sn, and Nb-Ti, are discussed in the five subsections of the chapter. The flux trapping and shielding behavior of the various samples is described and, where possible, analyzed through use of the programs described in chapter 4. The flux jumping behavior of the samples is also described and compared to the theoretical predictions outlined in chapter 3. The early experiments were confined mainly to the low-field materials, Pb and Nb. These initial tests verified the feasibility of trapping relatively low transverse dipole, quadrupole, and sextupole fields in hollow superconducting cylinders. The later experiments with the medium- and high-field superconductors, Pb-Bi, Nb₃Sn, and Nb-Ti, were carried out mainly in transverse dipole fields. They demonstrated the ability of these materials to trap and shield higher fields, indicated the

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importance of flux jumping in limiting trapping and shielding, and provided further information on the mechanisms involved in the flux trapping and shielding process.

Chapter 6 summarizes the basic findings of the work and discusses the feasibility of constructing a large flux trapping or shielding device. The computer programs are used to estimate the dependence of trapped fields on the size and configuration of the trapping device. The problem of flux jumping is also discussed.

Appendix I discusses the penetration of magnetic fields into holes and cavites in diamagnetic bodies. The shielding of relatively low magnetic fields by the superconducting samples tested here is generally in agreement with the theory of shielding by diamagnetic bodies.

A list of the various symbols and their units is given on page xii. Throughout the thesis a practical system of units is used in which most quantities are expressed in their most commonly quoted form. For example, magnetic fields are expressed in units of Gauss and Oersteds, currents in amps, current densities in amps/cm², and energy and power in joules and watts. To convert to this practical system from the cgs system, factors of 1/c are generally replaced by the factor $4\pi/10$. While the system is nonstandard, it does avoid awkward units such as meters, Tesla, J/m³, and A/m² used in the mks system, and units such as ergs, statamperes, and statvolts found in the Gaussian system. Alloy compositions are given in atomic percent, unless specifically noted otherwise. The symbol '**' is used to denote exponentiation.

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The list of references starts on page 228. Since the literature on both flux pinning and flux jumping is extensive, only the most relevant papers on these topics have been listed.

CHAPTER II-FLUX PINNING IN TYPE-II MATERIALS

The problem of understanding irreversible behavior and flux pinning in type-II superconductors has been investigated by using both empirical and theoretical approaches. Early magnetization experiments by Bean (Bean 1962, Bean & Dovle 1962, Bean 1964) uncovered several important features of the newly discovered hard superconductors: hysteresis, large values of remanent magnetization (trapped flux), and the effect of sample size on the magnetization. These observations led to the concept of the critical state and to the development of various models for the form of the unique current density Jc(B) that flows in the critical state. In the critical state a superconductor responds to any changes in the applied field with shielding or trapping currents which flow at a level equal to Jc. The currents in the superconductor flow to whatever depths in the material are required to shield out the field change. These postulates, together with Maxwell's equations and information on Jc(B), have been used to derive expressions for the field distribution and the magnetization (including the remanent magnetization) of samples of simple geometrical form. Although the concept of the critical state was originally derived empirically, it has proved extremely useful in understanding the macroscopic behavior of such samples. One of the results of this work was to demonstrate that the concept of the critical state could be successfully applied to other more complicated geometries.

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The concept of the critical state has been incorporated into the microscopic picture of the magnetic behavior of Type-II superconductors by relating the form of Jc(B) to the form of the defect structure in the material. This defect structure is ultimately responsible for pinning the flux lines in place, and allowing lossless currents to flow in the critical state. The initial steps in the process of relating Jc(B) to the defect structure involve defining the B,H, and M fields in the superconducting material, and calculating the driving force on the flux lines in the critical state. The effective macroscopic pinning force per unit volume, Fp, can then be related to Jc and the local spatially averaged field, B. The final steps involve relating the macroscopic force Fp to the microscopic pinning force between an isolated flux line and a material defect, fp, and deriving forms of fp that are appropriate for the defect structures of real materials. Once a connection between Jc and the defect structure can be made, the field structure as a function of fp and magnetic history can be derived by using the concepts of the critical state. However, although these problems have been discussed extensively, theories which are useful, detailed, and well verified have been obtained only for simple pinning systems.

In this chapter the B, H, and M fields are defined, and the concept of the driving force on flux lines and its relation to the critical state are described. The method of obtaining expressions for the magnetization and trapped flux for simple geometries is then outlined, and some common forms for the empirically determined Jc(B) relations are indicated. Finally, the problem of relating the pinning force and critical currents to the microstructure is discussed for two of the materials used in the

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experiments (Pb-Bi and Nb-Ti). The derivation of the form of the field structure from Jc(B) and the postulates of the critical state, in the more complicated geometrical situations encountered in the experiments, is discussed in chapter 4.

A. Magnetic Fields in Superconductors

As a starting point, B is taken as the fundamental field. Maxwell's equations div(B)=0 (and cur)(H)= $(4\pi/10)J$) are valid inside a superconductor on a microscopic scale. However, it is useful to work with a B-field that is the average of the microscopic field, since the microscopic field in a superconductor in the mixed state varies rapidly over distances of the order of the intervortex spacing ao. If div(B)=0 for the microscopic field, the same will be true for the averaged field. The B-field used here will ordinarily refer to this averaged field. At this point either a quantity H or a quantity M can be defined, since the choice of one will determine the other.

The quantity H can be formally defined by $H=4\pi\partial f/\partial B$, where f=the Helmholtz free energy (see for example Parks 1969, p. 830). In free space f=B²/8 π and H=B. However, this definition of H is not complete until f is determined for a superconductor in the mixed state with pinning. In the Meissner state, where the conventional definitions break down (B is identically zero), H is usually taken as H=-4 π M, and M, the magnetization, is defined as the difference between B and H:

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M=(B-H)/4π. M describes the equilibrium response of the superconductor to magnetic fields, in the absence of pinning or other irreversible effects, and can be related to an equivalent current density Jm=(1/10)curl(M). For a type-II superconductor a relation exists, B=Be(H), that describes the B-field which is in equilibrium with a given H-field. Be and H are related through M by: 4π M(H)=Be(H)-H or Be(H)=H+4 π M(H). The actual forms of Be(H) and M(H), which are determined by minimizing the free energy of the system, are complicated functions of field. Analytic forms that approximate Be(H) and M(H) have been developed, but generally apply over limited field ranges only.

The alternative method of defining M and H consists of first defining M as the measure of the magnetic moment of the material. In most cases it is useful to lump both the reversible response and the irreversible response of the material into the quantity M. M can then be identified with the quantity measured in a conventional magnetization measurement. H is defined as the difference between the B and M fields, $H=B-4\pi M$, and is just equal to the applied field, if the demagnetization factor is small. If a material exhibits a complete Meissner effect, then B inside the material is always zero, and $H=-4\pi M$.

B. Driving Force on Fluxoids and the Critical State

The concepts of the critical state and the driving force on fluxoids are integral parts of most explanations of the irreversible behavior of

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type-II superconductors. The concept of the critical state describes the macroscopic response of an irreversible material to changes in fields or applied currents. The definition of the driving force allows a formal connection to be established between the currents and fields of the critical state, and the pinning forces that arise from the detailed structure of the material.

The problem of calculating the exact form of the force on a fluxoid in an irreversible type-II superconductor in the mixed state is quite complicated. However, several authors (Josephson 1966, Evetts et al. 1968) have concluded that the force on a fluxoid in the mixed state is determined by an integral of a force density of the form JXB, where B is the fundamental microscopic field averaged over distances larger than the vortex spacing, but much smaller than the sample size, J represents the currents associated with variations in the internal H-field, $J=(10/4\pi)curlH$, and H is defined by the first set of definitions given above. In equilibrium, when no other forces are present, JXB=0. In most cases this implies that J=0, curlH=0, and H is uniform inside the material. Since J=(10/4 π)curlH=(10/4 π)curl(B-4 π M), this particular J does not include effects due to the reversible magnetization.

When pinning forces are present, the driving force JXB can be balanced by an average pinning force density Fp due to the presence of inhomogeneities in the material. As a result, J and curlH can be nonzero inside the superconductor and variations in B due to pinning, as well as to the reversible magnetization, are possible. The maximum allowable value of J, Jc, which determines the maximum possible value of

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curlH, is determined by the strength of the pinning forces and the local field. If J is perpendicular to B, then $Jc=(10/4\pi)curlH=10Fp/B$. The critical state (Bean 1962) is just a state of the material in which the superconductor opposes any changes in the field with supercurrents J that flow at this maximum possible level Jc in the required portion of material. For most purposes all irreversible superconductors are assumed to enter the critical state when exposed to variations in the applied field.

C. Measurement of the Macroscopic Pinning Force Fp

From a practical standpoint, the average pinning force can be inferred from several types of measurements. The two most common ones are measurements of the critical current of wire or strip samples, and measurements of the total magnetization of a sample vs applied field. Measurements of the critical current in transverse applied fields provide direct estimates of the pinning force in the material. Magnetization measurements also contain information on the pinning, although additional analysis is required to extract estimates of the average pinning force and critical current from the data (Bean 1962, Kim et al. 1963, Fietz et al. 1964). In certain simple geometries the critical current and pinning force can also be obtained directly from measurements of field profiles (Coffey 1967, Voigt 1968). In complicated geometries it is more difficult, although still possible, to extract estimates of Jc from measurements of the field structure. In

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addition, ac techniques have been developed by which flux profiles and critical currents can be inferred from measurements of the electric field at the surface of a superconducting cylinder immersed in a coaxial ac-modulated magnetic field (Rollins et al. 1974). Torque measurements made with a sample containing trapped flux can also be used to find the pinning force and critical current.

1. Direct Measurements

Critical currents are most often measured by observing the maximum transport current that can be passed through a length of superconducting material with negligible voltage drop. If Ha is much larger than Hc₁ and the sample is fairly thin, then the internal field is close to Ha, curlH is close to the impressed transport current Jc, and the driving force that must be balanced by the pinning force is thus close to JcHa.

Although this method of determining Jc(B) is straightforward in principle, it requires in practice that large currents be passed into a dewar while sensitive measurements are being made of the voltage across a small length of the superconductor. Some of these difficulties are avoided by methods in which critical currents are induced in bulk samples. In one such method, J is determined directly from measurements of field profiles of samples with sufficient symmetry to reduce the relation $J=(10/4\pi)curlB$ to a simple relation between J and the field gradient. For instance, Voigt determined critical currents in Pb-Bi samples by measuring B(r) in the narrow space between two long cylinders placed end to end. With this geometry $Jtot=(10/4\pi)dB/dr$ and a plot of

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the field gradient vs field gives Jtot(B) directly. Jm=10curlMe (Me is the equilibrium magnetization, present even in the absence of pinning forces) is included in Jtot, Jtot(B)=Jc(B)+Jm, but is normally negligible except near the sample surface. Critical currents have also been determined by measuring the field difference across thin-walled superconducting tubes in axial fields (Howard 1976). In this arrangement the field difference across the wall is directly proportional to the critical current density at the average field in the sample wall. For the more complicated sample geometries used here, $J=(10/4\pi)$ curlH does not reduce to a sufficiently simple form to allow Jc to be determined directly. However, as shown in chapter 4, additional analysis can be used to extract estimates of Jc(B) from flux trapping and shielding data.

2. Magnetization Measurements

The magnetization of hard superconductors can be related to the critical current density Jc by applying the concept of the critical state. The measured magnetization, which is a result both of the reversible magnetization, Me, and of the pinning forces, is usually defined (for zero demagnetization factor) by $M=1/4\pi V \int (B-H) dV$ with H=Ha, and B = the averaged microscopic B-field. (With this approach, H is given by H=B-4\pi M, where M includes everything except the applied field.) For a long cylinder in an axial field, curlB= $(4\pi/10)$ Jtot reduces to dB/dr= $(4\pi/10)$ Jtot or dB/Jtot = $(4\pi/10)$ dr. In the simplest treatments, the part of Jtot due to the reversible magnetization is ignored, and Jtot is taken to be Jc(B), which is determined by the pinning forces.

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Once a particular form for Jc(B) is chosen, $\int dB/Jc(B) = 4\pi/10 \int dr$ can be integrated to yield an expression for B(r) in terms of Ba and the parameters in the Jc(B) expression. The magnetization, M, as a function of the applied field and of the parameters in the assumed form of Jc(B), is found by integrating the field difference $1/4\pi V \int B(r) - Ha dV = M$ over the body. The final expression for M can be fitted to the experimental data by varying the values of the parameters in the assumed form of Jc(B). A good fit of the expression for M to the experimental data implies that the actual critical currents in the material are close to the values given by the assumed form of Jc(B). This procedure was used to derive an expression for M(Ha) of a cylindrical sample with $Jc=\alpha/B+Bo$ and with Jc=a/B¹ (commonly used empirical forms for the critical current density, see section D). Computer programs were written to carry out the calculations of M(Ha) and graph out the results. As inputs, the programs were given either an analytic expression for Jc(B) or a totally arbitrary form of Jc(B), specified by a series of value-pairs (Jc,B) between which the computer interpolated the full Jc(B) curve. A comparison between the expressions for M(Ha) and data on the magnetizatin of a Pb-Bi sample is given in section V-C-2.

B

The reversible magnetization of the superconductor can be easily taken into account if Me is assumed to be uniform inside the body. In this case the effect of the reversible magnetization can be described by a surface current that causes a jump in B across the surface, equal to 4π Me(Ha) (Fietz et al. 1964). The effect is usually included in the theory as a boundary condition; the field just inside the material can be taken as Ha+4 π Me(Ha) rather than Ha when evaluating the $\int (B(r)-Ha) dV$.

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This analysis neglects contributions from variations in Me with changes of the local field in the material. For a cylindrical geometry this additional contribution to the total field gradient will be proportional to dM/dB (dM/dr=dM/dBdB/dr). For high x type-II materials, the maximum value of 4π Me(H) (which is equal to Hc₁) is generally well under 1kG. The slope dM/dB is also generally small except near Hc₁. Since most measurements on these materials are made in fields much greater than Hc₁, these effects are usually negligible.

D. Model Forms of Jc(B)

Various forms for Jc(B) have been suggested on the basis of empirical studies of the critical current or magnetization of irreversible superconductors. The more complicated forms of Jc have more adjustable parameters and can therefore be more readily fitted to experimental data. However, the simpler forms are often adequate, and reduce the complexity of expressions for quantities that are derived from Jc(B). Some of the commonly used models are described briefly in the following paragraphs and illustrated in Fig. 2-1, which shows plots of the critical current vs field for model parameters chosen so that $Jc(10kG) = 5 kA/cm^2$.

The Bean model (Bean 1962), which grew out of the earlier Mendelsohn sponge model, was one of the first to explain the irreversible magnetization curves and size effects observed with hard

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superconductors. It also contained the original statement of the concept of the critical state. In the Bean model, the magnitude of the critical shielding or trapping currents was assumed to be a constant independent of field. Since the pinning force is approximated by Fp=JcXB/10, the average pinning force Fp is implied to be proportional to B. The model has produced good results when used in cases in which the field varies only over a limited range. It obviously breaks down near Hc₂ where Jc and Fp must decrease smoothly to zero.

The Kim model (Kim et al. 1962, 1963), in which

 $J_c=\alpha/(B+B_0)=(\alpha/B_0)/(1+B/B_0)$ (or $1/J_c=(1/\alpha)(B_0+B+...)$ with the higher terms dropped), was introduced soon after the Bean model to obtain better fits to a series of tube magnetization measurements made by Kim et al. This model, which retains a fairly simple form for Jc(B), has been applied with success to a variety of materials. The form of Jc(B) is more realistic than the Bean model, although Jc and Fp still do not go to zero near Hc2. At fields much larger than Bo, Jc is proportional to 1/B and Fp=constant. At low fields the model remains well-behaved, with Jc going to α /Bo as B goes to zero. Although α can be identified with the pinning force at high fields, the physical significance of Bo has remained obscure. Several variants of the basic Kim model have been suggested. Urban proposed that an extra factor of (Bc2-B) be added in order to improve the form of Jc(B) at high fields. Voigt introduced an extra factor of $1-b^2$ to improve agreement with data on critical currents in Pb-Bi at higher fields. $(1-b^2)$ is approximately proportional to the density of superconducting electrons.) Models with $Jc=\alpha/B$ can also be viewed as a variant of the Kim model with Bo=0.

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Several models with Jc of the form $Jc=\alpha/B^{**\gamma}$ with $\gamma=1/2, 1, ...$ have been used. For these models $Fp = JcB = \alpha B^{**-\gamma+1}$. Calculations are somewhat easier with this form than with the Kim model. The form also has the advantage that the power of B, which is determined by γ , can remain arbitrary during calculations. However, once γ is fixed, only one parameter is available to provide a good fit to experimental data. As shown in Fig. 2-1, the low field critical current increases rather rapidly with decreasing fields and diverges at B=0. The high field behavior is similar to that of the Kim model.

Fietz et al. (1964) found that $Jc = \alpha exp(-B/Bo) + \gamma$ provided a good fit to data on critical currents in Nb-Zr, and McInturff et al. (1967) have used this expression to fit Jc data on Nb-Ti samples. In the latter case, a fit to the Kim model at lower fields resulted in Jc's that were too small at intermediate field levels. The three parameters of this model have no obvious physical significance, and, unless γ is small, the model becomes unrealistic near Hc₂.

E. Physical Models of Pinning

Numerous approaches can be taken to the problem of developing an expression for the average pinning force of a material. Several different prospective interactions between a flux line and a given type of defect structure must often be considered. Usually, an idealized defect structure must be assumed in order to actually calculate the

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elemental interaction forces and sum them into an effective average volume pinning force. The presence of the lattice of flux lines also adds possible complications and options for different approaches. Even the most detailed of the theories must make simplifying assumptions in order to obtain a tractable expression for the effective pinning force. Since the theories must all be consistent with a common pool of data, there is a tendency for theories which use different approaches or approximations to arrive at remarkably similar forms for the average pinning force. Thus it may be difficult to determine the pinning theory appropriate for a particular material, except in cases where the physical system is particularly straightforward and well understood. In such cases, good quantitative agreement is possible between a microscopic pinning theory and experiment. However, in other, more complicated cases, it may be difficult to achieve even qualitative agreement. In order to illustrate the problems of finding Fp in a relatively straightforward case, and in a more typically complicated case, the situations for Pb-Bi alloys and for Nb-Ti alloys are discussed below. Figure 2-2 includes plots of Jc(B) derived from the various models discussed below. To aid in the comparison of the models, the constants in the theoretical expressions have all been adjusted to give the same current density at b=.5.

1. Lead-Bismuth

Excellent agreement between theory and experiment has been obtained with Pb-Bi alloys. With sufficient Bi content, these alloys form a two-phase structure of normal Bi precipitates in a superconducting Pb-Bi matrix

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(\mathcal{E} -phase). Flux pinning in this material is closely associated with the boundary between the superconducting \mathcal{E} -phase and the normal precipitates (Kramer & Rhodes 1966, Campbell et al. 1968). A model in which the pinning force is calculated from the interaction between flux lines and relatively large normal precipitates has been shown to be consistent with experimental measurements of Jc and M.

Campbell et al. (1968) carried out a careful study of flux pinning in Pb-Bi alloys in which both the critical current and magnetization were measured. The critical currents of a variety of samples could be expressed as a product of a field-dependent and a structure-dependent factor. A good fit to the experimental data was obtained with the field dependent factor equal to Me/B $\frac{1}{2}$. Me was the experimentally determined reversible magnetization of the superconductor, measured in well annealed samples with poor pinning. The structure-dependent factor was found to be directly proportional to Sv, the phase boundary area per unit volume. (Sv ranged from $40-1600 \text{ cm}^{-1}$ in Campbell's samples.) The temperature dependence of the pinning followed the temperature dependence of Me. The experimental data therefore implied the following overall expressions for the critical current density and pinning force : $Jc=33SvMe/B\frac{1}{2}$ (A/cm²) 2-1 Fp=1/10JcXB=3.3SvMeB¹/2. 2-2

Coote, Evetts, and Campbell (1972) took additional magnetization measurements on a Pb-Bi sample over a range of temperatures. The pinning force derived from the magnetization measurements factored into field-dependent and temperature-dependent terms. The field dependence

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of the normalized pinning force was well described by the function $b\frac{1}{2}(1-b)$ (b=reduced field B/Bc₂) for temperatures of 4.45-8.1K (Tc=8.4K). The temperature-dependence of the pinning force at a given field was adequately described by the function $(Hc_2(T))^2$. These results were consistent with the earlier work since the expression: Fp α $b\frac{1}{2}(1-b)(Hc_2)^2Sv/K^3$ 2-3

can be rewritten (using Ginzburg-Landau approximations that are valid for H >> Hc1) ast

 $Fp \propto SvMe(B\Phi_0)\frac{1}{2}/\lambda$ 2-4

which is roughly equivalent to equation 2-2.

As a model for flux pinning in the Pb-Bi samples, Campbell et al. (1968) considered pinning by a series of sheets (phase boundaries) perpendicular to the direction defined by JXB. With sheets spaced apart by a distance d, with length c parallel to B, and with width a, the number of vortices threading each sheet is $.93(\Phi_0B)^{\frac{1}{2}}a$. If P(B) is the pinning force/length exerted on the vortices due to the phase boundary, then:

pinning force/sheet=.93ac(B/40)1P(B),

pinning force per volume $Fp=.93ac/d^{3}(B/\Phi_{0})^{\frac{1}{2}}P(B)$,

and critical current density $J_{c=10Fp/B=9.3SvpP(B)/(B\Phi_0)\frac{1}{2}$,

where $Svp=cd/a^3=phase$ boundary area/unit volume perpendicular to JXB. Campbell argued that a magnetic interaction leads to a pinning force on fluxoids near a boundary P(B)=Me Φ_0/λ . With this expression for P(B) the model predicts:

Fp=.93SvpMe(BΦo)칼/λ,

 $Jc(B)=9.3SvpMe(\Phi_0/B)^{1/2}/\lambda$.

The model form of Jc(B) agrees with the observed form, and comes within a factor of four of the actual magnitude of Jc (which is considered rather good agreement, since theories that predict the proper form for Jc(B) are often in error by much larger factors in the magnitude of Jc). The analysis neglects the effects of flux lattice elasticity. These effects are thought to be unimportant in this case, because the relatively large-flux line deformation produced by the strong line-pinning forces of the large phase boundaries should allow all lines near a boundary to be pinned by the boundary.

Dew-Hughes (1974) has summarized the basic factors which enter most conventional pinning theories, and has presented a series of relations that describe the expected field and temperature dependences for the basic types of pinning. The pinning in Pb-Bi fits into this scheme as magnetic pinning by normal precipitates with an efficiency factor of 1 (a)I flux lines near a pinning center are pinned by it). The range of the pinning interaction is taken as λ and the difference in energy across the phase boundary as Φ_0 Me. These considerations lead directly to the above expression for Jc (equation 2-5).

The situation for Pb-Bi alloys seems to be well understood. The flux lines interact with the relatively large, normal Bi-precipitates via a magnetic interaction. The energy available to pin the flux lines is proportional to the reversible magnetization, Me, and the characteristic length for the interaction is the penetration depth λ . Adequate results are obtained with models that use a simple linear summation of the local

2-5

pinning interactions, and that neglect the effects of the flux line lattice. Both theory and experiment show that critical current densities of at least $5-10\times10^3$ A/cm² are possible with eutectic alloys. These current densities are sufficient to trap or shield fields of the order of 5-10kG in samples with wall thicknesses of the order of centimeters.

2. Niobium-Titanium

The situation in Nb-Ti alloys is considerably more complex. A broad range of complex microstructures can be obtained in the Nb-Ti system by starting with different alloy compositions and using different combinations of cold work and heat treatment (Reuter et al. 1966, Baker & Sutton 1969, Bychkov et al. 1969, Penny et al. 1970, Witcomb & Dew-Hughes 1973*). Moreover, the scale of the microstructure is generally much finer than that of the Pb-Bi alloys. Although the types of microstructures that provide the best pinning, and the metallurgical treatments required to obtain these structures, are fairly well-known from the many empirical studies that have been made on Nb-Ti systems, the theoretical picture is not yet completely clear.

*Only a few selected articles out of the extensive literature on the relationships among pinning, critical currents, and microstructures in Nb-Ti will be referenced here.

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Description of the System

Most commercial Nb-Ti alloys have compositions which fall within the 56-70% Ti range. The highest critical currents in these alloys are generally obtained after combinations of severe cold work and low temperature (c. 400°C) heat treatments, which result in a cellular structure with the highest density of dislocations in the cell walls (Vetrano & Boom 1965, McInturff et al. 1967, Pfeiffer & Hillmann 1968, Gregory 1969, McInturff & Chase 1973). In the higher Ti-content alloys, the heat treatment causes precipitates of α -Ti to form preferentially in the regions with high dislocation densities. Experiments indicate that the pinning is due mainly to the α -Ti precipitates, although dislocation pinning and pinning by the periodic substructure may also play a role. Since the precipitates are associated with the dislocations and substructure, it is not always easy to separate their effects. In the lower Ti-content alloys, the actual presence and possible role of fine a-Ti precipitates are still uncertain. The pinning in these lower Ti-content alloys is usually attributed to modulations in x, due to variations in dislocation density, and to the concomitant changes in electron mean free path within the cell walls. This type of pinning is enhanced by a fine, well-defined cell structure (Neal et al. 1971, Hampshire & Taylor 1972). Calculations of pinning forces based on X variations are in moderately good agreement with experimental data. Theories based on summing interactions between flux lines and simple dislocation structures are difficult to apply to typical Nb-Ti samples, since the dislocation density in the cell walls is high, and the structure complex. The range of application and possible overlap of the dislocation cell wall and precipitate pinning mechanisms are not well known.

A number of investigators (Fietz & Webb 1969, Hampshire & Taylor 1972, Mathur et al. 1974) have fitted data on critical currents and flux pinning in Nb-Ti alloys (as well as other materials) by an expression for the pinning force which is the product of a function of the reduced field and a function of the temperature: Fp=C(b)D(T). The form of C(b)depends on the material and is usually a single-peaked function of b with the peak occurring at a characteristic reduced field hp. Metallurgical treatments affect the form of C(b): treatments which increase the pinning strength of a material generally shift hp toward lower fields. The temperature-dependent factor D(T) generally turns out to be fairly well fitted by the function Hc₂(T)**m, with 1.5≤m≤2.5, depending on the material. Several theories have been developed that result in expressions for Fp which can be separated in this manner. Kramer (1973) has developed a particularly appealing theory by building this scaling behavior directly into its quite general framework. The theory can be applied to different materials by changing the form of the microscopic pinning interaction force fp and by weighting and summing the individual pinning interactions in different Ways. Brand (1972) has adapted a dislocation cell wall theory to account for his observations on the critical currents in cold-worked Nb-Ti. Several other investigators have studied flux pinning in higher Ti-content alloys where the pinning is due to α -Ti precipitates, and in some cases have developed theories applicable to these materials.

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The next portion of this chapter includes brief reviews of these theories. Although the theories turn out to be either difficult to apply to the low to moderate field case ($0 \le B \le 20 \ kG$), or clearly at variance with experimental data on critical currents in Nb-Ti, the discussion provides a useful framework for a somewhat more detailed discussion of pinning in high field superconductors.

Theory of the Scaling Laws for Flux Pinning

Kramer (1973) developed an elegant pinning theory in which different mechanisms are assumed to limit the effective pinning force at low and high fields. At low fields, the fluxoids must individually overcome the pinning force of the material in order to break free and start flowing through the material. At higher fields, where the shear strength of the flux line lattice is weaker, flux motion is initiated when the driving force on the fluxoids exceeds the shear strength of the flux line lattice, and unpinned flux lines shear past pinned ones. At the higher fields, Fp is a decreasing function of field because the shear modulus of the flux line lattice decreases with increasing field. Therefore, if Fp at low fields is an increasing function of b, Fp will peak at an intermediate field level. Treatments which increase the pinning strength of a material will increase Fp at low fields, where a higher driving force will be required to overcome the pinning force, but will leave Fp nearly unchanged at high fields since the shear strength of the flux line lattice will not be affected appreciably. The model thus directly incorporates the experimental observations that treatments which increase the pinning often shift the peak in Fp(b) upward and

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toward lower fields, while leaving the form of Fp(b) nearly unchanged at high fields.

The pinning force at low fields, which is limited by the strength of fp, is computed by finding the energy that is dissipated as a flux line moves through the material. This energy is equated to the product of the average macroscopic pinning force and the drift velocity of the flux line: Fp v = $2np^{-}$ Es v/ao (np is the density of pins, and Es is the elastic energy stored in a flux line as it encounters a pin of strength fp). Further considerations, including the tendency of an initially weakly pinned flux line to encounter a strong pinning center, and the high density of pinning centers found in most hard superconductors, lead to Es α fp⁴/C₆₆ (C₆₆=shear modulus of the flux line lattice), and $Fp=2nplEs(B/\Phi_0)^{\frac{1}{2}}$, where npl is the density of line pins (cm⁻²). The final form of Fp therefore depends rather strongly on the particular form of the local interaction force fp. Kramer chooses a somewhat arbitrary, but reasonable, form: $fp=const n!(Hc_2/x)^2(1-h)$ (where n! is the number of pins per length of flux line). This choice results in: 2-6 $Fp = const h_{2}^{1}/(1-h)^{2}(Hc_{2})^{5/2}$,

where the constant is a complicated function of the structure and density of the pinning centers.

The pinning force at high fields, which is limited by the flux line lattice shear strength, is determined by calculating the energy stored in the flux line lattice at the point at which its shear strength is just exceeded. With some additional assumptions about the structure of the pinning centers and the nature of the flux line lattice, this leads to the expression:

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 $Fp = Cs h_{\frac{1}{2}}^{\frac{1}{2}} (1-h)^2 (Hc_2)^{5/2} / \kappa^2,$

where, according to Kramer, Cs varies from 0.14 for small np to 0.56 for large np. The form of Fp given in equation 2-7 is similar to the one in eq. 2-3, which applies to a model based on completely different assumptions.

In Kramer's theory, the final expressions for the pinning force can be separated into temperature- and field-dependent factors as required. Many of the important factors that govern flux pinning in superconductors are implicitly included, and the theory can be used to make predictions of how the pinning force is affected by changes in the microstructure. The derivation of the expression for the high field pinning force is fairly straightforward, and the final form can be compared directly with experimental data. Unfortunately, the expression developed for the pinning force at low fields is not well suited for direct quantitative comparisons. The constant that determines the magnitude of the pinning force at low fields is a complicated function of the defect structure of the material. The field-dependence of Fp at low fields is very sensitive to the particular form of the interaction responsible for the pinning of individual fluxoids (Fp α fp⁴), as well as to the form of the distribution of the strengths of the pinning centers. The low field result, equation 2-6, is just one plausible form for Fp(h).

Mathur et al. (1974) carried out a series of experiments to test the applicability of Kramer's theory to Nb and Nb-Ti multifilament conductors. Curves of the critical current density Jc as a function of

2-7

field and temperature were derived from magnetization measurements, and Fp(h,T) was generated from the product JcB (assuming B \cong H). The general features of Kramer's theory were confirmed. The Nb data on Fp scaled with temperature as $(Hc_2(T))^{2.56}$ and the field dependence at higher fields scaled roughly as $h_2^1(1-h)^2$. An analysis of the low field form of Fp(h) was not presented. Changes in the form of Fp(h) among different samples were consistent with the theory; the peaks in Fp(h) were shifted upward and towards lower reduced fields in samples with more cold-work. The temperature-dependence of Fp for the Nb-Ti samples was $(Hc_2(T))^{1.84}$. Taken as a group, the normalized Fp data for the Nb-Ti multifilamentary samples coincided at high reduced fields, and fanned out in a manner consistent with the model at lower reduced fields. An analysis of the form of Fp(h) was not presented for the Nb-Ti samples, nor was the composition or heat treatment specified.

Pinning by Dislocation Cell Walls

Brand (1972) made extensive measurements of the critical currents of a series of cold-worked Nb-Ti alloys in order to study the effect of normal state paramagnetism on flux pinning. The alloys were not given any subsequent heat treatments, so the pinning, even in the higher Ti-content samples, should have been due mainly to the dislocation structure. The temperature- and field-dependence of the pinning force, derived from the critical current measurements, separated into two nearly independent factors for most of the samples. There was some breakdown of the scaling in the higher Ti-content alloys. The data were found to be consistent with a dislocation cell wall pinning model. In

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this model, the local pinning force fp was calculated by taking the variation in the fluxoid free energy due to variations in *x*, and dividing it by a scaling length equal to the vortex spacing. The macroscopic average pinning force Fp was then derived from fp by using an elastic theory proposed by Labusch, which predicted Fp proportional to fp² and a factor that is related to the distortion of a flux line in response to a point force, and which is a function of the moduli of elasticity of the flux line lattice. The fp² dependence is different from the fp⁴ dependence in Kramer's model, and the fp¹ dependence inherent in simpler theories, such as Campbell's, for Pb-Bi. The resulting form for Fp:

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Fp=const.b^{3/2}(1-b)(Hc₂)^{5/2}/K³,

implies a critical current density $Jc(B) \propto b^{\frac{3}{2}}(1-b)$, whose form differs from the expression in eq. 2-7 by an extra factor of b. The temperature dependence is given by $(Hc_2(T))^{2.5}$. The expression for Fp(B,T), which peaks at b=.6, has a reasonable form and apparently agrees quite well with the experimental data. However, because of the extra factor of b, the form for Jc(B) has a peak at b=1/3 with Jc decreasing with decreasing field at fields less than b=1/3 (Fig. 2-2). None of the experimental Jc(B) data from which the Fp(B) curves were determined are shown in the thesis, and most other low field measurements of critical currents in Nb-Ti, including measurements made here, show increasing values of Jc with decreasing fields. The theory is therefore questionable at fields below b=.2-.4, although Brand claims that its derivation is valid as long as B >> Hc_1 (Hc_1 in Nb-Ti is less than b=.01 (Baker & Sutton 1969, Karasik et al. 1971)). There is also some question about the proper scaling length to be used in this type of

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pinning theory. Hampshire and Taylor (1972) find good agreement with their data by using a similar model with the penetration depth, λ , rather than the vortex spacing, ao, as the scaling length. With their model they find:

Fp=const. $b(1-b)(Hc_2)^2/\kappa^3$ 2-9 Brand worked only with single-phase cold-worked Nb-Ti and so the conclusions do not necessarily apply to heat-treated Nb-Ti with a two phase structure (Nb-Ti + α -Ti).

Pinning by a-Ti Precipitates

Pfeiffer & Hillmann's (1968) experimental results on heat-treated and cold-worked Nb~67%Ti demonstrated that α -Ti precipitates were a major source of pinning in their material. Enhancements in flux pinning by α -Ti precipitates were also observed in the experiments carried out here with cold-worked and heat-treated Nb-70%Ti strips (section 5-V). In additional work by Hillman and Hauck (1972), cold-worked and heat-treated samples with a density of approximately 10^{10} , 2×10^{-6} cm-diameter precipitates per cm² were subjected to varying degrees of further cold work. With increasing amounts of cold work, Jc increased, reached a maximum value, and then decreased. The amount of cold work required to reach the maximum was an increasing function of the applied field at which Jc was measured, which is contrary to Mathur's findings and the usual interpretation of Kramer's theory. Electron micrographs showed that the α -Ti precipitates were initially concentrated along subbands oriented parallel to the drawing direction. Further cold work stretched out the subbands causing the density of precipitates in the

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drawing direction to decrease, and the density perpendicular to it to increase. The initial increase in Jc was attributed to the formation of a more even distribution of the precipitates. The later decrease in Jc was apparently due to the return of an uneven distribution as the density of precipitates along a direction perpendicular to the drawing direction exceeded the density parallel to it. In addition to the above observations, local maxima in the JcB vs B curve were observed in the further cold-worked samples. The location of the maxima were correlated with points where multiples of the flux line spacing matched the characteristic defect spacing. Shifts in the location of these peaks with further cold work were consistent with this hypothesis. The effect was not observed in a sample with a more random precipitate spacing. Experiments with materials which were cold-worked but not heat-treated demonstrated that pinning by a substructure consisting of well-defined subbands was more effective than pinning by a less well-defined structure, even when the dislocation density was higher in the latter. Our experiments also showed relatively poor pinning in samples which had been severely cold-worked, but had not had any subsequent heat treatment to improve the form of the substructure.

Results of this type suggest that theories which model the interactions between the flux lines and arrays of small normal particles should be appropriate. Campbell and Evett's theory (section E-1) considers pinning by the surface of normal precipitates whose size is generally greater than the penetration depth. In this type of magnetic pinning, the average B-field, which can change over distances of the order of λ , can reach its local equilibrium value, and the interaction energy for

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the pinning is usually taken to be proportional to Me(B). When the normal particles are smaller than λ , the average B-field cannot completely adjust to the local material variations, and pinning is usually attributed to interactions between the cores of the fluxoids and the normal particles. Several expressions for the pinning due to core interactions have been developed, including the ones by Dew-Hughes and A totally different approach presented by Bychkov (see p.42), Shmidt. which does not even consider pinning by material defects, has also been used to develop an expression for the pinning and critical currents in cold-worked and heat-treated Nb-Ti. Although the final form of Jc(B) from this theory fits the data, it is doubtful whether the assumed mechanism is actually limiting the critical currents in these alloys. Kramer's theory can also be applied to the problem of core pinning by choosing an appropriate form for fp and by using appropriate values for the constants in equations 2-6 and 2-7.

Dew-Hughes' general expression for the effective pinning force is: Fp=- η e lp Δ W/X 2-10 where η e, an efficiency factor related to the ability of flux lines to be deformed and pinned by defects, is set equal to 1 when pinning forces are strong compared to the flux lattice elasticity; lp is the length of flux line per unit volume that is directly pinned; Δ W is the difference in energy between a pinned and unpinned flux line; and X is the effective range of the pinning interaction. For pinning by an array of small normal particles, Dew-Hughes suggests using lp=(B/ Φ_0)Vn (Vn=volume fraction of the pinning particles, B/ Φ_0 = flux line length per unit volume), X=a/2 (a=size of the particles), and η e=1 (elasticity effects

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disregarded). For core pinning, he also proposes a form for AW which is equal to the change in the Gibbs free energy of the core: AW $\propto \pi \xi^2 (Hc_2 - H)^2 / \kappa^2$ (H>>Hc₁).

These choices lead to a pinning force and critical current:

 $Fp=(1/4\pi)Vn(Hc_2)^2h(1-h)^2/4.64ax^2$

 $Jc=(10/4\pi)VnHc_2(1-h)^2/4.64ax^2$.

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Fp(h) peaks at h=.33. Substituting values appropriate for Nb-Ti: Hc₂=100 kOe, x=50-100, Vn=.008, and $a=2\times10^{-6}$ cm (the latter two values coming from Hillman's work), yields $Jc=(.7-3\times10^4)h(1-h)^2$ A/cm². These values are a factor of ~10 smaller than the critical currents commonly obtained in cold-worked and heat-treated Nb-Ti at the field levels of interest in this study (0 \leq h \leq .2). A plot of Jc = const (1-h)² is included in Fig. 2-2. The form of this expression is useful at higher fields but increases too slowly with decreasing field to fit typical lower field data on Nb-Ti, including data from measurements made here. Using a more exact form for Δ W at lower fields and using $\eta e \neq 1$ might extend the theory's usefulness somewhat.

Shmidt (1972) has developed a theory of flux pinning based on calculations of the interactions between fluxoids and arrays of long cylindrical cavities oriented parallel to the field direction. Pinning by a cubic array of spherical cavities is considered as a special case. The cavities can be visualized as models of lines of precipitates strung out along the drawing direction of a wire sample, or of small, nearly spherical, precipitates dispersed throughout a sample. Shmidt considers situations that can be classified as cases of core pinning, since the radius of the cavities is assumed to be much smaller than either λ or

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ao. The cavities are assumed to be spaced apart by a distance d, much larger than their radius, but still smaller than λ . At low fields, flux lines that pass through the cavities are assumed to remain pinned in place. At very low fields (ao \approx d) the pinning force is calculated from the maximum force exerted on a fluxoid trying to pass between fluxoids pinned in the cavities. In this case Shmidt obtains:

Jc=0.62Hc/xd, which is independent of the field. Substituting values appropriate for Nb-Ti (Hc=100 Oe, d=10⁻⁵ cm, x=75) yields Jc \approx 8X10⁴ A/cm², slightly less than commonly obtained values. At higher fields, where the vortex density is greater than the cavity density, the pinning force is calculated from the force on a pinned vortex caused by the whole lattice of vortices trying to flow past it. In this case:

Jc= $(3^{1/4}/.4)$ Hc/K N₂ (Φ_0 /B) $\frac{1}{2}$, (N₂ = length of the cavities per unit volume). This expression implies Jcs of somewhat less than 10^6 A/cm² at B=10kG for Nb-Ti. The form is similar to some of the often used empirical forms for Jc(B). As the field increases, the ratio of the number of fluxoids to the number of cavities increases. Eventually, the increasing density of driven vortices, acting on the fixed number of pinned vortices, causes the pinned vortices to break free before the limiting current density calculated above is reached. Shmidt predicts that the field at which this transition takes place is $(a^2/d^2)Hc_2=10-20kG$ (for Hillman's material). At higher fields, Shmidt predicts Jc \propto B⁻¹, a form of Jc which implies Fp(h)=constant, and which is similar to the Kim expression at high fields. Shmidt's analysis, which uses the concept of two different pinning regimes, is in some ways similar to Kramer's. However, it assumes that a pin breaking process acts at high fields, and that a flux lattice shearing process operates

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at low fields. Since Kramer reaches exactly the opposite conclusion, the two theories appear to be incompatible.

Bychkov et al. (1969) developed a critical current expression by calculating the current density at which the kinetic energy of the flowing electrons is equal to the available condensation energy: Hc_2 $fM(H)dH = 1/Vfnm vsc^2 dV$, where nm is the mass density of the carriers. Ha The critical current density Jc is then obtained from Jc=(1/10)n e vsc, where n, the density of carriers, is a field-dependent quantity. The theory is worked out in the limit H >> Hc_1, where the reversible magnetization, the volume fraction of superconducting material, and the proportion of superelectrons can all be approximated by simple linear functions of the field. The resulting form for the limiting current density is:

 $Jc=(Hc/4\pi\lambda)(1-h)^{2},$

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an expression with the same functional dependence on h as Dew-Hughes' (equation 2-11), although derived by a totally different approach. Good agreement with the experimentally determined critical current of a variety of Nb-78at%Ti samples was obtained over fields ranging from Hc₂ down to aproximately 20kG, by treating the constant Hc/ λ as an adjustable parameter. However, this procedure undermines the physical significance of the form of Jc(H), since values of the constant Hc/ λ determined from estimates of Hc and λ for Nb-Ti imply the existence of critical currents much higher than those actually measured. The theory fails to account for the orientation dependence found in the critical currents by many other investigators, and for changes in Jc with metallurgical treatments which do not directly affect Hc or λ . However,

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the theory may prove to be of use in materials such as Nb_3Sn , in which the pinning is sufficient for the limit of Hc/λ to be approached, or in cases where large critical currents flow in force-free configurations.

F. Discussion

The subject of flux pinning in superconductors is very broad, and the present discussion can only touch on some of the more important factors. We can see that empirical models of Jc(B), together with the concepts of the critical state can be used to develop predictions of the field structure inside type-II materials. For simple geometries, this process is straightforward (section C-2), and has been used in the past to develop analytic expressions for the internal fields and magnetization of cylindrical or slab-shaped samples. The problem of applying this procedure to other geometries is discussed in chapter IV.

The concepts of the critical state, along with an expression for the driving force on flux lines in the mixed state, allows contact to be made between the macroscopic picture of shielding or trapping currents flowing at J=Jc, and the underlying physics, although the actual derivation of the critical currents from the microstructure remains a difficult problem in many cases. In particular, the theoretical situation for Nb-Ti is still unsatisfactory. Prospective models are contradictory (eg. Kramer's assumptions appear to be incompatible with Shmidt's), and at times in poor agreement with experimental measurements

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(eg. the models of Dew-Hughes, and Brand, as applied to the regime of h(.2). However, for our purposes all that is required for the design of superconducting trapping or shielding devices is an ability to predict and manipulate the pinning and critical current levels with confidence. In many cases, including the case of Nb-Ti, there is sufficient empirical information available to allow designs to be made without a well-verified pinning theory. In Pb-Bi alloys critical current levels can be varied by-modifying the size and density of the Bi-precipitates, and critical current levels of 10³-10⁴ can be easily obtained. In high Ti-content Nb-Ti alloys the pinning and critical currents can be varied over a very large range by using various combinations of cold-work and heat-treatments. Critical currents of over 10⁵ A/cm² are reliably obtained. With critical currents of this level, wall thicknesses of only tenths of centimeters are required to trap or shield fields of 10kG.

CHAPTER III-INSTABILITIES IN TYPE II MATERIALS

The field distributions that occur in irreversible materials with pinning are metastable, since the energy of the superconductor can be lowered by removing the additional field gradients and currents associated with the pinning forces. As a result, the field structure in hard superconductors occasionally decays rapidly to lower energy configurations. These sudden changes in field are known as flux jumps. In the case of bulk materials, flux jumps occur when the field gradients which develop in response to changes in the applied field collapse suddenly. This allows flux to rapidly enter a sample that has been shielding out a field, or to come out of a sample that has been trapping in a field. Since it turns out that flux trapping and shielding are often limited by such instabilities, it is useful to examine some of the observations and theories of flux jumping. The various quantities used to describe a flux jump and factors that have been observed to affect them are discussed first. Flux motion during a flux jump and flux motion in other less catastrophic situations are then compared. Finally, some of the theoretical approaches to the problem of analyzing and predicting instabilities are discussed, and the resultant theoretical expressions applied to the materials tested here.

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A. Factors Affecting Flux Jumping Behavior

Experiments designed to study flux jumping generally involve the application of linearly increasing or decreasing axial magnetic fields to cylindrical or tubular samples. The response of the superconductor is usually monitored with Hall or magnetoresistance probes which measure the magnetic field inside or near the sample, or with loops of fine wire that sense changes in the flux threading the sample. In some cases, a field detection scheme that uses the Faraday effect in a plate of special glass placed above a sample is employed to study the details of the spatial and temporal evolution of flux jumps. Simple resistance thermometers have been used to measure the sample temperature during testing.

A number of quantities that characterize the flux jumping process in bulk materials can be measured. The most common are: Hfj, the applied field at which a sample, cooled in zero field, first undergoes flux a jump; Δ Hfj, the spacing of any succeeding jumps that occur with further increases in the applied field; τ m, the time constant for field changes that occur during a flux jump; the spatial extent of a flux jump; and Δ T and τ th, the change in sample temperature caused by a flux jump and its characteristic decay time. Other quantities that have been measured include the minimum size of the field or temperature perturbation required to trigger a jump, the velocity of the leading edge of a jump, and the details of the field structure during or just after a flux jump.

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Hfj and ΔHfj are usually determined by periodically testing the stability of the field structure with small field or temperature pulses, or by observing the spacing of the spontaneously occurring flux jumps. *rm* is usually found by observing the changes in field or flux with time on an oscilloscope, or through analysis of high speed films taken with Faraday effect detectors. (Such films provide information on the size and speed of the flux jumps that is not readily available from experiments which use stationary field probes or pickup coils.) Sections 1 through 5 discuss the effects of a variety of factors on the quantities which describe the flux jumping process. These include: the rate of change of the applied field, the temperature of the sample, the thermal environment of the sample, the magnetic environment of the sample, and the physical properties of the sample.

1. Absolute Temperature and Thermal Environment

Both the absolute temperature of the sample and the degree of thermal contact between the sample and the bath have been shown to affect flux jumping activity. Hfj and Δ Hfj generally decrease with decreasing temperature (Neuringer & Shapira 1966, Ogasawara et al. 1970). As the temperature is lowered, more, closer spaced, partial flux jumps tend to occur (Wertheimer & Gilchrist 1967). In some cases, an observed increase in the frequency of flux jumping at lower temperatures may be due to the occurrence of more smaller jumps rather than to decreases in the field difference required to initiate a jump. Correlations between enhancements in stability and closer thermal contact between the superconductor and a high heat capacity cooling bath have been

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demonstrated in a number of experimental studies. Samples are generally more stable when they are immersed in liquid helium than when they are cooled by helium gas. For example, Boyer et al. (1972) measured values of Hfj(T) for a Nb-57%Ti sample (0.7 cm-diam. by 7 cm long) which was immersed in liquid helium at temperatures below 4.2K and suspended in Iow pressure gaseous helium at temperatures above 4.2K. The value of Hfj(4.2K) jumped from 2200 De to 3050 De when the cooling medium was changed from gas-to liquid. Porous samples of Nb₃Sn prepared by powder metallurgy techniques are very stable when they are permeated with liquid helium, and have trapped and shielded fields on the order of 50 kOe without flux jumping (Corson 1964). When liquid He is excluded from the sample interiors, or when the sample is tested in He gas, stability decreases markedly (Corson 1964, Hancox 1965, Smith et al. 1965). In a very interesting experiment, Claude & Williams (1967) observed decreases in flux jumping with increases in the rate of flow of liquid He through a porous NbaSn sample. The importance of the cooling environment is also demonstrated in experiments in which samples are tested at temperatures just above and just below the λ -point. Increases in Hfj are sometimes observed as the temperature is reduced below the λ -point (Lange 1965), even though stability generally decreases with decreasing temperature. In cases like these, the significant changes in Hfj can be attributed either to changes in the ease with which heat can be conducted to the bath, or to changes in the bath's heat capacity, since the absolute temperature of the sample prior to a flux jump, the magnetic environment, and the physical properties of the superconductor are all held essentially constant. These observations suggest that stability might also be affected by surrounding samples with

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supercritical helium. However, although wire samples have been tested with supercritical helium, its effect on the stability of bulk samples has not been determined. In the experiments described here, decreases in stability associated with decreased contact with the helium bath (section V-E-2), as well as increases in flux jumping activity with decreases in temperature (sections V-C-2, V-D-3, V-E-2), were observed.

2. Rate of Change of the Applied Field

At low to moderate values of dHa/dt (the rate of change of the applied field) the field interval between flux jumps increases with decreasing values of dHa/dt (Smith et al. 1965, Claude & Williams 1967, Wertheimer & Gilchrist 1967). At very low dHa/dt, Δ Hfj either reaches a limiting value, or continues to increase until the sample is completely stable (Morton 1965, Wipf & Lubell 1965). With low values of dHa/dt, the temperature rise in the sample due to the power dissipation caused by the flow of flux into or out of the sample is small (usually well below a few tenths of a degree K), because sufficient time is available to conduct the heat to the bath. In such cases, the changes in Hfj with dHa/dt must be due to small local heating or other perturbing effects resulting from the changing applied field, and not to a gross temperature change of the sample.

At higher dHa/dt, power dissipation caused by the rapid flow of flux into or out of the sample, together with the large pulses of heat generated by the flux jumps, can heat the sample appreciably above the bath temperature (Chikaba 1970). Care must therefore be taken to assign

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observed values of Hfj and ΔHfj to the appropriate temperature. At moderate to high values of dH/dt (0.01-10 k0e/sec), flux jumps have been observed to occur at decreasing intervals (Wertheimer & Gilchrist 1967), at fairly regular intervals, independently of dHa/dt (Wipf & Lubell 1965, Harrison et al. 1975), or even with decreasing frequency, as dHa/dt increases (Chikaba et al. 1968). The latter behavior is most likely due to increases in the sample temperature above the bath temperature and to the concomitant increases in sample heat capacity and stability that occur as a result.

A moderately fast field ramp (approximately .01-.1 kOe/sec) was generally used in the present experiments. Decreases in Δ Hfj were observed with increases in the field ramping rate with some samples, while in other cases sample behavior was insensitive to changes in dHa/dt.

3. Sample Geometry

Both the size and shape of a sample can influence flux jumping activity. Stability decreases as the change in flux threading a sample per unit change in applied field, $d\Phi/dHa$, increases, and the shape of a sample affects this ratio. For instance, hollow samples are particularly prone to flux jumping when the flux front reaches the inner wall of the sample (Morton 1965). At this point, there is a large increase in $d\Phi/dHa$ as the central cavity begins filling with flux. This requires an increase in the rate of flux flow into the sample, with a concomitant increase in the rate of power dissipation, proportional to JcE=Jcd Φ/dt . Similarly,

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if the flux front reaches the center of a solid sample without triggering a flux jump, the sample may remain stable since d∳/dHa will increase much more slowly with further increases in the applied field than at fields before full penetration. While these predictions were not specifically tested, several cases were observed in which flux jumps in hollow samples occurred when the flux fronts were approaching the sample's hollow central cavity.

Changes in the shape which change the demagnetization factor, nd, are also expected to affect stability. Decreases in nd should increase stability by reducing $d\Phi/dHa$, since for a diamagnetic body the applied field is enhanced by as much as a factor of 1/(1-nd) over portions of its surface. However, a careful study of the effect of the demagnetization factor on stability has not yet been done. In the experiments here, the effect of the demagnetization factor on stability was observed by measuring the flux jumping behavior of samples in configurations with differing values of nd. In one case, nd was gradually changed by machining away portions of a Nb sample. Flux jumps occurred in this sample only when nd was at its maximum value (section V-B-4). In Pb-Bi eutectic alloys, flux jumps were observed in cylinders in transverse fields, nd≃1/2, but not in spherical samples, nd≃1/3 (sections V-C-2,3). Nb-Ti strip samples were more prone to flux jumping in a flat configuration in a normal field, nd~1, than when rolled into cylinders and tested in transverse fields, nd≃1/2 (section V~E-2).

The thickness of a sample affects the maximum field that can be shielded or trapped and the energy per unit volume associated with the trapping

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or shielding currents. Finer wire thicknesses have been used to increase the stability of wires used in large superconducting magnets, and changes in stability have been observed with changes in the diameter of fairly large cylindrical samples and in the thicknesses of disks. Chikaba et al. (1968) found that the stability of Nb-Ti rods (tested at dHa/dt = .2kDe/sec) decreased with increasing rod diameters. Wertheimer and Gilchrist (1967) were able to eliminate flux jumps entirely from Nb, Nb-Zr, and Nb-Ti-disks (tested in axial fields) by reducing their thicknesses below .010-.001 cm. In the case of long cylinders or wires, sample sizes can be changed without appreciably changing nd. With disks, the reduction in thickness is accompanied by unavoidable increases in nd, although the increased stability due to the sample size reduction can evidently more than offset the effect of the increased demagnetization factor. A further complicating factor is that changes in size and shape also affect the ease with which heat is conducted out of the superconductor and into the bath. The relation between sample thickness and stability was not confirmed here; flux jumps were observed in 0.010 cm-thick Nb-Ti flat disk and plate samples (section V-E-2), and in .0006 cm-thick Nb₃Sn samples (section V-D-2).

4. Composite Materials

Increases in the fields at which flux jumps occur, and decreases in the severity of the flux jumps that do occur, have been achieved by forming composite materials from high field superconductors and normal metals. The presence of normal metals of high electrical conductivity close to superconducting materials influences the flux jumping process through

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eddy current damping of changes in the magnetic field. In cases in which normal metals are directly bonded to superconductors, improvements in stability also result from the relatively high thermal conductivity and heat capacity of the normal materials, and from improved mechanical constraint. The magnetic damping action of the normal metal reduces perturbations in the applied field that could otherwise trigger flux jumps, and decreases the potential speed of flux jumps. For example, Harrison et al.-(1975) observed an increase in τ m from 1.4 to 2.1 msec when a brass or aluminum plug was placed under (but insulated from) a Nb-Zr disk. Greatest stability was obtained with a high electrical conductivity Al plug. The experiments of Ando and Nozawa (1974) on a sample consisting of a stack of V+V3Ga disks with and without interleaved aluminum and copper disks also demonstrated the stabilizing effect of magnetic damping. When Al or Cu disks were included in the stack, the value of 7m increased from .032 ms to 23.8 ms, while Hfj changed from ≈ 11 kOe to complete stability in applied fields as high as 50k0e. In the present experiments, a measurable improvement in the stability of a Pb-Bi sample was achieved by surrounding the sample with a copper tube (section V-C-2); in other cases (sections V-D-1,2,3, V-E-2) samples incorporating copper in their structure proved to have good flux trapping and shielding abilities.

The stability of composite wires or strips carrying transport currents is also improved by the presence of the low resistance shunt provided by thick coatings of high conductivity metals. However, this mechanism does not apply to situations where shielding or trapping currents are induced in bulk samples, since there is no mechanism to replenish the energy dissipated in the normal metal shunts during flux jumps.

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5. Physical Properties

Of the several physical properties which influence flux jumping, the heat capacity, C, and the temperature dependence of the critical current, $Td=Jc(dJc/dT)^{-1}$, are thought to be the most important. Experiments indicate that the pinning and critical current level also affect flux jumping.

The exact effect of heat capacity changes on flux jumping is difficult to determine, since the heat capacity of a given material is essentially fixed, and the use of different materials to vary C usually also changes a variety of other material properties. The usual approach is to vary the heat capacity by varying the temperature of the sample (C is approximately proportional to T^3 for many superconducting materials). At best however, this can only show a correlation between changes in Hfj and changes in C. Experimentally, changes in Hfj have been observed that roughly parallel changes in C_2^1 (Ogasawara et al. 1970, Wright et al. 1974, Boyer et al. 1972).

The exact effect of changes in the temperature dependence of the pinning is also uncertain. Jc(T) is often approximated by Jc(T)=Jc(O)(1-T/To), where the parameter To is usually slightly less than Tc. In this approximation Td = To - T, and is therefore only weakly dependent on a particular sample's pinning structure. Other slightly more sophisticated forms for Jc(T) imply that there is a small field dependence in Td. Unfortunately, the actual form of Jc(T) is rarely measured in samples on which flux jumping behavior is being observed.

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Consequently, until more carefully controlled experiments are performed, in which both Jc(T) and the flux jumping behavior are measured, the exact dependence of Hfj on Jc(T) will remain uncertain.

The situation is similar with respect to the effects of the critical current density level. Although there are several systems in which metallurgical treatments may be used to vary pinning and critical currents without-affecting most of the other relevant properties, not enough systematic studies have been done to allow firm conclusions to be drawn on the effect of changes in pinning on stability. In one case, Sutton (1973) indicated that a heat-treatment which increased the critical current density of Nb-Ti rods by a factor of five left Hfj unchanged. However, an earlier study by Baker and Sutton (1969) with a smaller diameter sample of a different Nb-Ti alloy, reported an increase in flux jumping activity with increasing heat-treatment times. In another case, Lange (1965) found that flux jumping increased with increasing critical current density. The present experiments with Nb-Ti strip samples (section V-E-2) showed that flux jumping increased with increases in the pinning and critical currents caused by longer heat treatment times. Experiments with Nb₃Sn tubes (section V-D-2) also provided some evidence for a positive correlation between critical current levels and flux jumping activity. A comparison of a number of measurements made by diverse investigators reveals large variations among values of Hfj for materials with similar heat capacities and critical temperatures. These variations may be due to differences in the pinning strengths of the materials, cooling conditions, or measurement techniques. The further question of whether different

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substructures that result in similar overall pinning forces might have differences in stability remains unanswered.

B. Flux Motion

A number of different resistivities have been used to characterize the flow of magnetic flux through a material, including: the normal state resistivity, the flux flow resistivity, the flux oreep resistivity, and resistivities characterizing the orderly flow of flux into or out of superconducting samples due to changes in applied field. The rapid flow of magnetic flux during a flux jump is similar in some respects to the less catastrophic flow that occurs in these other situations. Knowledge of the effective resistivity of a superconductor during a flux jump is usually required in theories that attempt to model the dynamics of flux jumping. A differentiation among the possible mechanisms involved in flux motion during a flux jump can be made by comparing the value and functional dependence of the effective resistivity that characterizes the flow of flux during a flux jump to resistivities that characterize the other states of flux motion.

1. Flux Flow

Flux flow refers to the viscous flow of flux lines through a type-II superconductor (see for example Parks, p. 1107). The concept was developed to explain the voltage vs current behavior of superconducting

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wires in transverse magnetic fields. The observed linear relation between changes in I and V at applied currents above I=Ic is consistent with a state in which flux lines are driven by the force imbalance developed when the driving force exceeds the pinning force, and are impeded by a viscous force, ηv .

where η is the viscosity coefficient of the superconductor, v is the velocity of the -vortices, and $JX \oint 10$ (J > Jc) is the driving force on the flux lines. The energy dissipated through the viscous motion of the vortices is supplied by the action of the transport current against the electric field E=vB/10⁸, generated by the motion of the flux. The flux flow resistivity is defined by

$\rho f = dE/dJ = \Phi_0 B/10^9 \eta \qquad 3-2$

and is equal to the slope of the V(I) curve in the flux flow region. Experimentally, ρf , and thus η , are determined by measuring the slope of this curve above J=Jc in various applied fields and at various temperatures. The values of ρf and η seem to represent a basic property of the material which is insensitive to changes in pinning and temperature and which at low temperature varies with field approximately as

2. Flux Creep

Flux creep is a much slower process than flux flow, and is believed to result from the thermally activated motion of individual (or bundles of) flux lines. Flux creep is used to describe the extremely slow decay of trapped fields that occurs as flux lines jump out of the potential wells developed by the pinning centers. Changes in field due to flux creep are characterized experimentally by a logarithmic dependence on time rather than by the exponential dependence characteristic of flux flow. The longer time scales for flux creep imply that the process has a very low effective resistivity (ρ (creep)<(ρ n).

3. Normal State_

The effective resistivity and magnetic diffusivity of a material can be determined from the details of the magnetic flux's motion into or out of a sample.

Maxwell's equations, curlE=-(1/10⁸)dB/dt, curlB=(4 π /10)J, and a linear relation between J and E E=peffJ, 3-4 lead to dB/dt = 10⁸(10/4 π) peff curlcurlB, a diffusion-type equation for the magnetic field. The equation shows that there is a characterisitic magnetic diffusivity

 $Dm=(10^9/4\pi)\rho eff (cm^2/sec)$

time constant $\tau m = (4/\pi^2) L^2 / Dm$ (sec) 3-5

and effective resistivity, for changes in flux through a sample of characteristic size L. For normal materials the effective resistivity is just the normal state

resistivity (peff=pn).

4. Flux Motion during Flux Jumps

Flux motion during flux jumps usually evolves in a manner characteristic of a diffusion process. In experiments in which the demagnetization factor is small (long cylinders or slabs) the observed duration of a flux jump can be directly converted into an effective resistivity and magnetic diffusivity using relations 3-5. Resistivities and magnetic diffusivities derived in this manner (values of τ m were obtained from the works of Neuringer & Shapira 1966, Iwasa et al. 1969, Chikaba 1970, LeBlanc & Vernon 1964, and Smith et al. 1965) generally have values that range from on the order of, to a factor of 10-100 times less than, the expected magnitude of the flux flow resistivity ρ nH/Hc₂*. If flux flow is the appropriate mechanism for flux motion during flux jumps, then the effective resistivity should increase with increasing fields approximately as H/Hc₂. Unfortunately, while many investigators have observed flux jumping over a wide range of applied fields, the dependence (if any) of τ m on Ha is not usually indicated.

An additional complication arises in disks or plates in fields parallel to their normals, since the enhanced driving field behind the flux jump affects the effective diffusivity of the material. Wertheimer and Gilchrist (1967) have estimated that in such cases the diffusivity and effective resistivity are enhanced by a factor of 1/1-nd.

*The results of Onishi (1974) suggest that resistivities may be somewhat higher than those implied by equations 3-5 and that therefore the magnitude of the flux jump resistivities may actually be somewhat closer to flux flow resistivities.

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Magnetooptical studies (Wertheimer & Gilchrist 1967, Harrison et al. 1975) of the growth of flux jumps in disk-shaped samples have generally found that the extent of the jump grows with time approximately as $(1-\exp(-t/\tau m))$, a dependence consistent with a diffusion-type process, and that the speed increases and the duration τm decreases with decreasing disk thickness in a manner consistent with the proposed dependence of the diffusivity on the demagnetization factor. The speed and duration are_essentially independent of the pinning strength of the material and the ramping rate of the applied field.

5. Flux Motion due to Changes in the Applied Field

Several investigators have measured the resistivity associated with the non-catastrophic flow of flux into an irreversible superconductor that ordinarily occurs when the applied field is changed (Lubell & Wipf 1966, Gandolfo et al.1968, 1969). These measurements differ from conventional flux flow measurements in that the current density in the superconductor is kept approximately constant at J=Jc, while an electric field is induced by the changing flux linking the currents. The measured . resistivity is approximately equal to E/J, and is generally substantially less than either the typical flux jump resistivities or the differential resistivity dE/dJ measured in most flux flow experiments. The resistivity, associated with the orderly flow of flux into or out of the material, decreases with decreasing dHa/dt, and at very slow ramping rates is on the order of flux creep resistivities. In such cases, the pinning strength of the material has also been shown to affect the time constants for flow of flux into the samples. The

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movement of the flux in such cases seems to be more closely related to flux creep than to viscous flux flow. It is possible that the flux jump resistivity arises in a similar manner, with the much higher rates of change of flux during a flux jump resulting in the higher observed resistivities. However, some flux jump experiments do find good agreement with predictions based on flux flow concepts. Therefore, at this time there appears to be some uncertainty in the form of the relation describing the motion of flux during a flux jump.

C. Theoretical Analysis

A full theoretical description of the flux jumping process involves the heat equation

CdT/dt-div(KgradT)=dw/dt, with w=JE 3-6 and Maxwell's equations

curlH=($4\pi/10$)J curlE=-($1/10^8$)dB/dt, 3-7 together with equations which describe the specific properties of superconductors, such as the relations appropriate to the flux flow state

 $1/10JX\Phi_0=fp+\eta v$, $\eta=\Phi_0B/10^9\rho f$ $\rho f=\rho nB/Bc_2$. 3-8 An exact analysis of the process governed by these coupled differential equations is a formidable problem. It is however, possible to simplify the problem in cases in which the time scales for magnetic and thermal processes differ. When the magnetic field changes on a time scale which is short compared to τ th (Dm>>Dth), the flux jump process is nearly

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adiabatic and the div(KgradT) term in the heat equation (3-6) is neglected. When τm is of the order of, or greater than τth , and $D th \ge Dm$ (usually in the case of composite materials), a dynamic theory is used, which includes the effects of heat conduction out of the sample. If τ m>> τ th, it may be reasonable to neglect the CdT/dt term of the heat equation (3-6) when carrying out the analysis in the dynamic case. In either the adiabatic or dynamic case it is possible, with the help of simplifying assumptions, to estimate the conditions under which a small field or temperature perturbation leads to a runaway situation, a flux jump. However, a jump occurring at this field, H=Hfj, may be only a partial flux jump. A second field, Hffj, is defined as the applied field above which a complete flux jump is expected. A complete jump is a jump which results in a uniform field, equal to the applied field, throughout the sample. A shortcoming of the simpler theories is that they only indicate the conditions at which a disturbance is initially expected to propagate, but do not treat the subsequent evolution of the flux jump. When a more complete analysis is used, or when descriptions of the field profiles during or just after a flux jump are required, the complexity of the problem increases. It becomes difficult to develop a closed form expression for Hfj or for the forms of the flux profiles, although progress can be made in individual cases with the help of computers.

1. Adiabatic Flux Jump Criteria

Most early theories of flux jumping were based on the assumption that changes in the magnetic field occur in time spans which are short

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compared to the thermal diffusion time. In this adiabatic approximation, heat flow out of the area of the flux jump is ignored. As a result, straightforward analyses lead directly to simple closed form expressions for estimates of Hffj and Hfj.

A full flux jump is expected to occur when the total energy difference between the state of the superconductor before a flux jump and the normal state of the material is sufficient to drive the sample normal. For the case of a sample shielding a field from most of its volume, the expressions determining Hffj are (Swartz & Bean 1968):

T.f $Hffj^2(Ti)/8\pi = 10^7 \int C(T) dT$, and $Hffj(Ti)=Hc_2(Tf)$, Ti where Ti is the initial temperature of the sample, and Tf is its final temperature, assuming there is no heat flow. These equations lead to an expression relating Ti and Tf:

Ti⁴ = Tf⁴ - $(10^{-7}/2\pi)$ (Hc₂(0))²(Tc²-Tf²) / (C(Tc) Tc) 3-9 Values of Hffj derived by substituting values of Tf into the expression for Hffj are listed in Table III-1.

The derivation of Hfj given by Swartz & Bean (1968) illustrates in more detail the general procedure used to derive flux jumping criteria in the adiabatic approximation. They considered the problem of a semi-infinite slab cooled in an applied field directed parallel to its surface. This is a convenient arrangement since curlH= $(4\pi/10)$ J reduces to dH/dx= $(4\pi/10)$ J (here x measures distance into the slab). The arrangement also approximates a large cylinder into which an axial field has penetrated a relatively small distance. The outer surface of the slab is brought into the critical state by raising the applied field by

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TABLE III-1

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ESTIMATES OF FLUX JUMPING FIELDS IN THE ADIABATIC APPROXIMATION

	ß (J∕cm ³ -K⁴)	C(mJ/cm ³ -K)		Td(K)		Hfj(Oe)		Hffj(Oe)	
		2. 0K	4.2K	2.OK	4.2K	2.OK	4.2K	2. 0K	4.2K
Pb-Bi	2.4×10-4	1.9	17.8	9.4	3.4	2400	4300	6200	6000
Nb-Ti	0.6X10-4	0.5	4.4	6.0	3.0	1000	2000	3800	3700
Nb	-	-	2.5-4.8	-	2.8	-	1500-2000	-	- '
NbaSn	-	0.15	1.4	70	30	1750	3700	_	-

an amount Hs above the frozen-in field. A magnetic test pulse, ΔH , is then applied in a time which is short compared to τ th but long compared to τm . The test pulse causes, in turn, a change in the flux through the slab, power dissipation, heating, and a decrease in the pinning and critical current density (assuming that dJ/dT < 0). The decrease in pinning allows a further change in flux, $\Delta \Phi$, to occur. In the analysis the current density is taken as Jc(B,T)=Jc(0)+(dJc/dT)dT, that is, independent of field, and thermal conduction is ignored. With these assumptions* the power dissipation, temperature change, and change in Jc, all as a function of x, are implicitly included in the analysis. At a field

Hfj=Hs= $(-\pi^3 \ 10^7 \ C \ Jc/(dJc/dT))\frac{1}{2} = (\pi^3 \ 10^7 \ C \ Td)\frac{1}{2}$ 3-10 the ratio of the change in flux through the slab to the size of the test pulse, $\Delta \Phi/\Delta H$, becomes unbounded, and the development of a flux jump from a small perturbation becomes favorable. Therefore in the adiabatic approximation Hfj depends only on the heat capacity, C, of the material, and on Td, a parameter weakly related to the material's irreversible superconducting properties. Table III-1 includes estimates of the values of C, Td, and the resulting values of Hfj and Hffj for the materials used in this study. The sources of the figures used in the table are discussed in the sections which follow.

In some simpler treatments, an average power dissipation and temperature rise is used to compute the changes in Jc, resulting in a slightly different numerical factor in the final expression for Hfj, $12\pi \times 10^7$ in the case of Wilson et al (1970), and $8\pi \times 10^7$ in the case of Hancox (1965). A detailed derivation by Wipf (1967), which used $Jc=\alpha(T)/(Bo+B)$ rather than a value of Jc independent of field, resulted in an equation essentially identical to 3-10.

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Values of Hfj and Hffj for Pb-Bi alloys can be estimated from published data on the heat capacity and pinning of this material. These data (Hultgren et al. 1963, 1968) indicate that, in the temperature range of interest, the electronic contribution to the heat capacity of elemental Pb and Bi is much smaller than the lattice contribution. The results of the harmonic lattice dynamic theory (summarized for example in Phillips 1971) can be used to estimate the lattice specific heat:

 $\beta = 12\pi^4 \text{Rg} / 58 \text{d}^3$ 3-11 C(lattice)=BT³+B'T⁵+..., where Rg is the gas constant and θd is the Debye temperature. Estimates of the heat capacities of the elements Pb and Bi are then obtained by substituting appropriate values for 0d into eq. 3-11. In his review of the low temperature heat capacities of metals, Phillips gives $\theta d(Pb) = 105K$ and $\theta d(Bi) = 120K$, which yield $\beta (Pb) = .85 \times 10^{-9}$ and ß(Bi)=.56X10⁻⁺ Jcm⁻³K⁻⁺. Shiffman and Neighbor (1972) studied the effect of the alloy composition and ambient temperature on 0d of Pb-Bi alloys containing up to 15%Bi. They found that 0d decreased with increasing Bi content and with increasing temperature. At T=4K and with 15%Bi, θd was equal to 92K, implying a ß of 1.26X10⁻⁴ Jcm⁻³K⁻⁴. Hultgren et al. present the results of measurements by Shubnikov and Chotkevich (1934) of the superconducting heat capacity of Pb-34.8%Bi at 5,6, and 8 K. The data are consistent with a β of ~2.4X10⁻⁴ Jcm⁻³K⁻⁴. Ewert and Sander(1971) measured the heat capacity of Pb-30%Bi films at 5.2 and 8.5 K. Their results are consistent with B = 1.85X10⁻⁴ Jcm⁻³K⁻⁴. The Pb-Bi samples tested in this study, which were susceptible to flux jumping, had compositions in the 50-56%Bi range. A

Pb-Bi

heat capacity $C = 2.4 \times 10^{-4} T^3 J cm^{-3} K^{-1}$ represents a conservative estimate for the heat capacity of these higher Bi-content alloys.

The temperature dependence of Fp in Pb-Bi alloys has been measured. Campbell et al. (1968) measured Jc(B) at 4.2K and 2.1K and found that the change in Jc with temperature was proportional to the change in Me (the change in Me with temperature was inferred from observed changes in Hc₂). Coote et al. (1972) measured the pinning at four temperatures between 4.45 and 8.1 K, and found a similar form for the temperature dependence of the pinning. A log-log plot of Fp(at constant h) vs Hc₂ indicated that Fp was proportional to $(Hc_2(T))^2$, which is in turn proportional to Me at fields much greater than Hc₁. Using $Hc_2(T)=Hc_2(0)(1-t^2)$, Coote's expression for Fp (equation 2-3), $Fp(T,h) \propto h_2^2(1-h)Hc_2(T)^2$, reduces to Fp(T,H) $\propto H_2^2(1-t^2-h_0)$, where $h_0=H/Hc_2(0)$. This expression in turn implies that:

 $(1/Jc)dJc/dT = 1/Td = (t/1-t^2) (3(1-t^2)-h_0)/(1-t^2-h_0).$ 3-12 With Tc=8.6K, equation 3-12 results in Td(H=0,T=4.2) = 4.47, Td(h_0=.25,T=4.2) = 3.38, and Td(h_0=.25,T=2) = 9.39. By comparison, the usual assumption that the pinning and critical currents are proportional to (1-T/To) with To=Tc leads to $-(1/JcdJc/dT)^{-1}=(To-T)$ which is independent of field and equal to 4.4K at T=4.2K and 6.6K at T=2K. With B=2.4X10⁻⁺Jcm⁻³K⁻⁺ and Td given by equation 3-12, the predicted flux jump fields for Pb-Bi are: Hfj(4.2)=4330 and Hfj(2.0)=2364 Oe.

It is possible to check the applicability of the adiabatic approximation by comparing the magnetic diffusivity $Dm=(10^9/4\pi)\rho$ to the thermal diffusivity Dth=K/C. Pb-Bi at 4.2K has a large heat-capacity, C~18mJ/cm³-K, and a normal state resistivity $\rho n\simeq 3-5\mu\Omega$ -cm (Levy et al.

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1966, Voigt 1968). This resistivity implies a value of Dth of ~3000 cm²/sec. Data is available on the thermal conductivity of low Bi-content Pb-Bi alloys at 4.2K (Powell & Blanpied 1954). The data shows a rapid decrease in K with increasing Bi-content. With Pb-10%Bi K(superconducting)~100mW/cm-K, and K(normal)~25mW/cm-K. Using 100 mW/cm-K as an upper limit on K for the eutectic alloy, results in Dth(5 cm²/sec. Even if the actual flux jump resistivity is a factor of 100 smaller than $\rho_{n,r}$ -Dm will still be much larger than Dth. Therefore, the adiabatic approximation should be quite good in this material.

Nb-Ti

In superconducting Nb-Ti the lattice specific heat is considerably smaller than the electronic portion of the specific heat (values for θd for elemental Nb and Ti are in the 275-425 K range). Therefore, predictions of C(T) based on estimates of θd and the harmonic lattice dynamic theory are not useful. However, Sukharevskii et al. (1968) and Zbasnik (as quoted in Iwasa et al. 1969) have made detailed measurements of the low temperature heat capacity of Nb-75%Ti (Tc~7.5k) and Nb-65%Ti (Tc~9.5k) respectively. The measured heat capacity increased roughly as T³ at low temperatures, passed through a peak a degree or two below Tc, decreased somewhat as Tc was approached, and then increased as T³ again at temperatures above Tc. In the 4-6 K range Sukharevskii's data are consistent with a B of 7.3-8.4X10⁻⁵ Jcm⁻³K⁻⁴ and Zbasnik's with a B of 4.8-6.0x10⁻⁵ Jcm⁻³K⁻⁴*. It is therefore reasonable to use a ß of ~6X10⁻⁵ Jcm⁻³K⁻⁴ in eq 3-10 to estimate the flux jump field for our Nb-64%Ti sample.

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The temperature dependence of Fp and Jc for Nb-Ti alloys has been measured in a number of cases. Hampshire et al. (1969) and Noel et al. (1971) have published detailed plots of Jc(T) for 60 and 56-%Ti respectively, at several values of applied field. The plots are generally linear, with the slope decreasing slighly near T=Tc. Near T=4.2, Jc(T) can be reasonably well fitted by the function $J_{C}(T)=J_{C}(0)(1-T/T_{O})$ with To approximately 1-2 K less than Tc. The parameter Td=Jc/dJc/dT at T=4.2K ranges from 3.85 to 5.05 K for fields of 24 kOe to O kOe in Hampshire's data. The data of Noel et al. (1971) imply a value Td(4.2)=3.2K for fields in the 2.5-6.0 k0e range. Less detailed measurements reported by Wipf (1969) and Mathur et al. (1974) imply similar values for Td(4.2) (Td=3-5 K at low fields and 2-3 K at higher fields) but are not detailed enough to accurately determine the functional form of Jc(T). Several other authors (Kramer 1973, Brand 1972, Hampshire & Taylor 1972, Hillmann & Best 1977) have fitted Jc or Fp data to expressions of the form $Jc=C(b)Hc_2**n$. Using the form $Hc_2=Hc_2(0)(1-t^2)$, which is a good approximation for Nb-Ti (Hillmann & Best 1977), these expressions can be reduced to functions of $h_0=H/Hc_2(0)$ and T/Tc. Since the field appears only in factors of the form $H/Hc_2(0)$, the effect of H on Td is small at the fields of interest in this work. Near H=0 the expressions reduce to the form $J_{c}(T)/J_{c}(0)=(1-t^{2})**m$, where

*Bussiere and LeBlanc have fitted Zbasnik's data to the function $C=3.816T^{3.19}$ J/cm³K, which implies a $\beta(4.2)$ of $~5X10^{-5}$ J/cm³K⁴. An additional reference to the heat capacity of Nb-Ti has been made by Ogasawara, who, during a discussion of flux jumping in Nb-50%Ti, stated that the heat capacity is described by $C=5.5X10^{-5}T^3$ J/cm³-K. However, Ogasawara did not indicate the source of his figure. It should also be noted that both Brechna and Stekley have quoted values of C(4.2) for Nb-Ti that are approximately a factor of four lower than the values reported by Sukharevsikii and Zbasnik.

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1 \leq m \leq 2 (see fig 3-1). The slope of the functions $(1-t^2)$ **m, with m \geq 1.5, decrease as Tc is approached; therefore these functions would presumably provide a good fit to the experimental data of Hampshire and Noel. At intermediate temperatures, the slopes of the Jc(T) curves are all quite similar, and the values of Td(4.2) implied by the scaling laws are thus generally close to values derived from the direct measurements of Jc(T). For our purposes the value Td(4.2) and not the exact functional form of Jc(T) is required, and a low field value Td(4.2) \geq 3 K is consistent with most of the published data on Nb-Ti. With B=6X10⁻⁵ Jcm⁻³K⁻⁴ and Td(4.2)=3K, Td(2)=6K, equation 3-10 predicts for Nb-Ti that Hfj(4.2)=2033, and Hfj(2)=945 De.

Unfortunately, from the available data it is difficult to determine if the adiabatic approximation is valid for Nb-Ti. Values of ρ n for Nb-Ti at 4.2K are generally in the 50-100 μ Ω-cm range (Grigsby 1968). This results in values of Dm of ~4000-8000 cm²/sec. However, since values of ρ eff derived from the period of flux jumps, τ m, are often factors of 100 smaller than ρ n, the actual value of Dm during a flux jump could easily be as small as ~50 cm²/sec. The value of Dth is also uncertain since measured values of K range from 10 to 400 mW/cm³-K (Dubeck & Setty 1968, Morton et al. 1975). This gives a range in Dth (using C=4.4X10⁻³) of 2-100 cm²/sec. Therefore, without further measurements of the relative magnitudes of Dm and Dth in a given sample, the applicability of the adiabatic analyses remains uncertain.

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Niobium

The low temperature heat capacity of Nb has been measured by numerous investigators. A reasonable value (Hultgren et al.) is $C(4.2) \approx 2.5$ mJ/cm³-K. Since Hc₂ for Nb is only a few kOe, magnetic fields of a kOe or more have a strong effect on the heat capacity. In Ferreira's measurements, C(4.2) increased from 2.66 mJ/cm³-K at H=O-600 G to 4.25 mJ/cm³-K at H=1480 Oe, and to 4.86 mJ/cm³-K at H=2488 Oe. At higher fields C decreased somewhat.

The temperature dependence of Jc in sintered Nb tubes was measured by Kim et al. (1962). The Jc(T) data were fitted reasonably well by Jc=Jc(0)(1-T/To) with To=.75Tc, implying Td(4.2)=2.75K.

Using the heat capacity in zero field and Td=2.75K, equation 3-10 predicts Hfj=1460 Oe for Nb. Including the effects of the magnetic field on C by using C=4.8 mJ/cm³-K causes the calculated value of HFj to increase to 2000 De.

The heat-capacity and thermal conductivity of Nb is similar to that of Nb-Ti. K is of the order of $\simeq 30$ mW/cm-K (Powell & Blanpied 1954). Therefore, Dth for Nb is less than 10 cm²/sec. The normal state resistivity of Nb depends strongly on its metallurgical condition. However, values of the order of 1 $\mu\Omega$ -cm are commonly reported (Weber 1970, Van Der Klein et al. 1970). This gives a value of Dm of ~100 cm²/sec. Therefore, depending on the actual value of ρ n, and its relationship to ρ eff, the adiabatic approximation may be appropriate.

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Dirty samples with good pinning tend to have higher values of the resistivity, and therefore would be more likely to have Dm >> Dth.

Nb₃Sn

Vieland and Wicklund (1968) measured the heat capacity of sintered Nb₃Sn; the value for C(4.2) was ~1.4 mJ/cm³-K. Harper (1975) measured the specific heat of a dense single crystal of Nb₃Sn and found generally similar, although slightly higher, values for C(T) (~1.8 mJ/cm³-K at 4.2K). Unfortunately, there is little detailed data available on C(T) below 4.2K. However, Hfj at 2K can be estimated by assuming that the ~T³ dependence of above 4.2K continues at the lower temperatures. Other investigators (Bussiere et al. 1974, Schwall et al. 1975, Howard 1976) have measured Jc(T) of Nb₃Sn prepared in several different manners. The function Jc(T)=Jc(0)(1-T²/To²) with To 1-2 K less than Tc generally describes the experimental data quite well. This form of Jc(T) implies that Td = $(1-T^2/To^2)x(To^2/2T)$ which with To=16.6K results in Td(4.2)~30, and Td(2)~70. Therefore, Hfj(4.2)=3670 0e, and Hfj(2)=1750 0e.

The heat capacity of Nb₃Sn is slightly smaller than in Nb, and the thermal conductivity is very low (\simeq .4mW/cm-K (Weger & Goldberg 1973)). As a result Dth is less than 1 cm²/sec, smaller than any of the other materials considered here. The resistivity of Nb₃Sn is roughly of the order of 10 μ Ω-cm (Weger & Goldberg 1973), so Dm is on the order of 1000 cm²/sec. Thus Nb₃Sn should approach the adiabatic limit. However, as with Nb-Ti, there can be large variation in the the thermal conductivity and the effective resistivity among different samples.

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Discussion

The simple adiabatic flux jump criterion does not indicate anything about the size of an ensuing flux jump. In an attempt to deal with this problem Swartz and Bean developed the concept of the adiabatic critical state. In this state each region of the superconductor carries a current density appropriate to its local temperature. The local temperature is assumed to be consistent with preceding adiabatic flux changes. Computer calculations of the flux profiles which followed a flux jump were made using equations consistent with the adiabatic critical state. The calculations showed that complete flux jumps began to occur at a field Hffj which turned out to be equal to the field calculated in eq. 3-9. At fields between Hfj and Hffj partial flux jumps were possible. Unfortunately, the analysis did not describe the actual evolution of a flux jump, consequently there is no guarantee that the computed adiabatic critical states will actually be reached. In fact, work by Noel et al. (1971) shows that the flux and temperature profiles in a Nb-Ti cylinder just after a jump do not agree with the predictions of adiabatic theories.

Some of the shortcomings of the adiabatic model may be due to the failure of the assumption that Dm>>Dth. A review of the data on the relevant physical properties shows that this assumption may not be valid for several of the materials tested. The situation in Nb-Ti is particulary uncertain due to the large possible variation in the thermal conductivity and effective resistivity. Sizable variations in both K and ρ eff among different samples are also likely to occur with Nb and

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Nb₃Sn. In all cases some uncertainty in the values of Dm arises because of the lack of information on the effective resistivity of a superconductor during a flux jump. However, the theory should apply in at least some cases, and does provide insights into the flux jumpimg process. The discussion here also indicates several types of experiments that can be used to verify the theory. While attempts have already been made to find materials with dJc/dT > 0 (these materials should be stable in all fields according to the adiabatic theory), systematic measurements of the dependence of Hfj on Jc(T) have yet to be done. The measurements of the heat capacity of Nb-Ti show that there is a region with dC/dT < 0. According to the adiabatic theory, stability should be reduced in this temperature range. Measurements of Hfj(T) in this region would be very interesting.

2. Dynamic Flux Jump Criteria

As noted in section A-4, high conductivity normal metals in composite materials decrease τm considerably. In such composite materials (or superconductors with low effective resistivities), where $\tau m \ge \tau th$, the thermal conduction term in the heat equation (3-6), rather than the heat capacity term is used in the analysis of temperature changes caused by the power dissipation in the superconductor. As in the adiabatic model, a flux jump criterion can be developed by examining the response of the material to small thermal or magnetic perturbations.

Wilson et al. (1970) present a rough estimate of the flux jump criteria for a composite conductor consisting of alternating layers of Cu and

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superconductor, perturbed by a small temperature change ΔTi . The perturbation should die out and the material remain stable as long as the increase in temperature due to the flux change in the superconductor caused by the initial temperature pulse is smaller than the initial pulse. The temperature pulse ΔTi causes a change in Jc and a movement of the flux front further into the material:

 $\Delta Jc = (dJc/dT)\Delta Ti$, $\Delta x = (10/4\pi)(H/VsJc)\Delta Jc/Jc$, 3-13 where Vs is the volume fraction of the superconducting material in the composite, and H is the field difference maintained across the portion of the sample in the critical state. The energy dissipation due to the field change is proportional to the product of the induced E-field and the critical current density Jc. The average energy dissipated per unit volume (averaged over the total depth of penetration of the field, x) is given by

3 - 14

 $w=10^{-8}$ JcH $\Delta x/3$ (J/cm³)

The characteristic time over which this energy dissipation takes place is determined by the time constant for changes in flux (equations 3-5). Wilson uses x, the distance into the material that the field had penetrated before the temperature pulse was applied, for the characteristic dimension, L, in eq. 3-5. Since the resistivity of high-purity Cu or Al is equal to or lower than the flux jump resistivity of most superconductors, the effective resistivity of the composite material is approximately equal to $\rho/(1-Vs)$ where ρ is the resistivity of the normal metal. The additional temperature rise resulting from the flux motion, assuming uniform power dissipation across all the superconducting layers (each layer d(cm) thick, with thermal conductivity K(W/cm-K)), is approximately given by (see Carslaw & Jaeger 1959, p.130):

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$\Delta T = (dw/dt)d^2/12K$.

Combining the equations, with dw/dt=w/(τ m/2), and requiring ΔT to be less than or equal to the initial perturbation, ΔT i, results in an expression that can be written as:

d² ≤ 8KTd(1-Vs)/pVsJc². 3-16

The expression indicates that for layers below a certain thickness, the composite should be stable. The level of the applied field does not appear in this expression. The expression is at best a rough estimate, since the derivation embodies several questionable approximations, including that of uniform energy dissipation in the superconducting layers and a critical current density independent of field. Aside from demagnetization effects, the derivation should apply equally well to the case in which a field is directed perpendicular to the layers of the composite.

Equation 3-16 can be used to estimate the critical thicknesses for composite superconductors. For Nb-Ti (Td=3 K, Jc=2X10⁵ A/cm², K=10-400 mW/cm-K) with Vs=.5 and with interleaved layers of copper with ρ =2X10⁻⁸ ohm-cm, equation 3-16 predicts a critical thickness of .010-.080 cm. This thickness corresponds to a field difference of 2500-20000 G across each superconducting layer. The effects of heat conduction out of the superconducting Nb-Ti layer and the flux damping effect of the copper layers has led to an increase in the estimate of the maximum field difference that can be stably maintained across a superconducting slab by a factor of between 1.25 and 10 over the strictly adiabatic case. The critical thickness in Nb₃Sn is somewhat smaller, since Jc is a factor of 10 larger and K somewhat smaller than for Nb-Ti. With

3-15

Jc= 3×10^6 A/cm², K= 4×10^{-4} W/cm-K, Td=31, and Vs=.5, equation 3-12predicts dNb₃Sn=.00074 cm. This thickness corresponds to a maximum field difference across one layer of 2800 Oe, which is a field difference somewhat less than Hfj calculated in the adiabatic approximation. The calculation shows that because of the low thermal conductivity of Nb₃Sn, the stabilizing effect of the heat capacity is of the same order as the heat conduction out of the Nb₃Sn. Therefore a more detailed analysis is required to obtain a meaningful dynamic stability criterion for Nb₃Sn.

Since the maximum energy dissipation may be much larger than the averaged value given in equation 3-14 above, a better approximation can be achieved by focusing more closely on the behavior of the region of the material experiencing the largest dissipation. An improved stability criteria is obtained by applying a small hypothetical field perturbation, calculating the resultant energy dissipation in the outermost superconducting layer, and carrying through with an analysis of the succeeding field changes in the material. The energy dissipation caused by a field perturbation AHi is:

w=10⁻⁸Jc∆Hix.

3-17

The rate of this dissipation is determined by τm (equation 3-5). If the length L in the expression for τm is taken as x, the macroscopic field penetration depth (as in Wilson's analysis), then the expression for the power dissipation in the outermost layer,

 $dw/dt=(10/4\pi)(Jc\Delta Hip)/(x(1-Vs)),$ 3-18

is a factor of 3/Vs larger than in Wilson's analysis. This will lead to a corresponding decrease in the predicted critical thickness of the material.

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Chester (1967) has presented a very terse derivation of the flux jump criterion for composite materials. He apparently used an expression for the energy dissipation in the outermost superconducting layer similar to equation 3-17. He also apparently assumed that, while the energy dissipation was determined by the distance, x, into which the field had penetrated, im was determined by the thickness of the outer copper layer. Chester also apparently used a less rigorous form for the temperature rise AT caused by the power dissipation dw/dt. His result, in the limit of small Vs, was expressed as a limitation on the field, Hs, that can be maintained across the composite, rather than a limitation on the layer thickness, d. His expression is more restrictive than Wilson's, since the energy dissipation in the outer layer is higher, and the time constant, determined by the copper layer thickness rather than the penetration depth, x, is shorter than in Wilson's analysis. Chester also presents, without derivation, a criterion for the case of the field perpendicular to the composite. ľn this case the condition is independent of Hs and stability is predicted for superconducting layer thicknesses less than a critical size.

Chester's analysis is overly restrictive since he assumes that $\Delta \Phi$ results from a change in field throughout the macroscopic penetration depth, x, while using a τm which is determined only by the thickness of the outer copper layer. Accounting more accurately for the effects of the copper laminations on the field changes shows that if a field perturbation is applied to a composite material the outermost copper layer will limit dB/dt in the first layer to " $\Delta Hi/\tau m$, and that the succeeding layers of copper will effectively prevent the flux change

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from appreciably affecting the field farther into the material during this initial time period τm . Therefore, the maximum value of $\Delta \Phi$, which determines the energy dissipated, is actually much smaller than is assumed by either Chester or Wilson. However, the time constant, τm , is relatively short since it is determined by the thickness of one copper layer (as in Chester's analysis). In this case, the rate of energy dissipation in an element of the outermost superconducting layer d cm thick is:

dw/dt=JcAHd/10⁻⁸ Tm. Inserting the expression for Tm, and combining with the expression for the temperature rise due to power dissipation dw/dt (eq. 3-15), and the expression for the change in the shielding ability of the layer due to an increase in temperature ΔT , results in $\Delta Hs = (\pi^2/24)$ (ρ/K) Jc dJc/dt d² ΔHi . Requiring that $\Delta Hs/\Delta H \le 1$ results in an expression which may be written as:

 $d^2 \leq (24/\pi^2) (Td K)/(Jc^2 \rho).$

3-19

This represents a critical thickness that differs by a factor of $((3/\pi^2)(Vs/(1-Vs)))^{\frac{1}{2}}$ from the d in equation 3-16.

D. Discussion

The most obvious problems with most flux jump theories are: they produce a necessary but not sufficient condition for the occurrence of flux jumps, they are not useful in predicting the extent of a flux jump once it is under way, they fail to do more than qualitatively explain the extremely varied flux jumping behavior that has been observed, and

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they are often based on questionable approximations. In order to overcome the first two problems the theories have to be extended past the initial moments of the flux jump to the period during which it propagates through the sample. The development of the adiabatic critical state is one approach to this problem. Computer solution of the full set of equations describing the process is also feasible, although it must be based on a definite theory of the flux motion in a superconductor before and during a jump. The disparity between experimental observations and the simple theories may not be due simply to the level of approximation used in most theories, but may indicate that the theories are in some way neglecting important factors. Experimental evidence favors the latter interpretation since the two factors that seem to affect Hfj most strongly, the rate of change of the applied field and the thermal environment, are essentially absent from most of the basic theoretical expressions for Hfj.

Experimental and theoretical work indicates that the properties of the surface region of the sample are particlularly important. Despite the fact that flux jumps in unclad bulk materials often occur on time scales that are short compared to the thermal diffusion time for the whole sample, the cooling environment does affect the value of Hfj. Usually the power dissipation due to the flow of flux into or out of the sample is highest near its outer surface where the flux linked by the critical currents and therefore the E-field are the highest. This is particularly true of flux trapping situations where the magnetic field is lowest and the critical currents are highest near the outer edge of the sample. In addition, it should be noted that the surface is the

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first portion of the sample to feel a change in applied field, and the first portion to experience the power dissipation due to the propagation of a change in applied field into the material. Although the more detailed derivations of Hfj implicitly include the spatial dependence of the power dissipation, the final expressions do not seem to reflect the particular importance of the surface. In unclad bulk materials it may therefore be necessary to focus more closely on the behavior of a small surface layer when discussing the initial development of flux jumps. Some progress has been made in this direction with flux jumping theories for samples with surface barriers (S. Kim et al. 1977) and for samples with decreased pinning in a surface layer (Sutton 1973). In both cases an adiabatic analysis showed that increased pinning in the surface layer can cause decreases in stability. However, purely adiabatic theories may remain insufficient since the conduction of heat out of the surface layer seems to be important even when the observed value of auth is much greater than τm . The problem of determining whether a flux jump will continue to grow once it leaves the nucleation region must also be solved. Although computer solutions of the flux jumping process (Morton and Darby 1973) should implicitly include most of these factors, not enough solutions to specific experimental situations have been published to verify their utility. In materials clad with normal metals these considerations are somewhat less important since the superconductor is both thermally and magnetically separated from the external environment. However, there is still a need for a more detailed analysis of the flux jump process in sheet composite materials.

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In some respects the present understanding of these instabilities is similar to the situation with flux pinning. A good qualitative understanding of the roles of the major factors affecting flux jumping behavior has emerged from the numerous experimental and theoretical studies of the problem, and while many theoretical analyses embody different simplifying assumptions, there is again a tendency for theories to result in similar final expressions. As in the cases of most flux pinning theories, the present flux jumping theories also seem to be in good qualitative agreement with many experiments. However, at this point a full calculation of the flux jump process using a computer seems to be possible, while a complete calculation of the flux pinning in any but the simplest structures is not. Experimentally, more systematic studies of the affect of parameters such as the pinning and heat capacity on flux jumping behavior will also be useful.

A. Experimental Method

Experiments were usually carried out inside a 6.3 cm ID glass dewar system at 4.2K with the samples immersed in liquid helium, although temperatures down to 1.8K could be reached by reducing the pressure above the liquid He. The dewar was inserted into various conventional magnets at SLAC, including a small 5kG dipole magnet, large 10 and 20 kG transverse dipole magnets, and transverse guadrupole and sextupole magnets. In most cases, tubular samples held within the dewar by a long thin-walled stainless steel tube were aligned parallel to the axis of the dewar and perpendicular to the field direction. In general the samples did not extend past the end of the magnet gap and therefore relatively high field levels were present at the ends of the samples. With one of the magnets, the 10kG dipole, the dewar reached only into the upper portion of the gap and consequently there was an appreciable field gradient present at the sample location. The smaller 5kG dipole magnet in which some of the earlier measurements were made had 15 cm diameter pole faces, and therefore samples tested in it experienced some nonuniformities in the field.

The magnetic fields were measured by inexpensive Hall effect probes (Bell FH-301-020 and FH-301-040). Although the probes were not specifically designed for low temperature applications, they proved to be convenient. They were relatively small, .318 cm square X .051 cm

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thick, with an active area, .102 cm X .203 cm, sensitive to the normal component of the field. The probes were mounted in phenolic holders attached to a .635 cm diameter stainless steel tube concentric with the larger sample support tube. The tubes passed through simple 0-ring seals in the dewar cap. Thus the probes could be either translated axially or rotated from above, while their position was monitored by potentiometers coupled to the driving mechanism. The probes were fairly reliable, they failed only occasionally due to delamination. With sensitivities on the order of 10-20 mV/kG (for currents of tens of milliamps) the probes produced easily measurable signals at the field levels of interest. The response of the probes was fairly linear out to 5-10 kG; at higher fields there was a gradual falloff in sensitivity. Calibration of the probes was accomplished by measuring the field produced by the magnet with a Bell gaussmeter and then recording the output of the Hall probes as a function of magnet current at either temperatures above Tc, or with the probes moved away from the samples.

Flux trapping experiments were carried out by establishing a magnetic field in the sample while its temperature was greater than Tc, cooling the sample to 4.2K (or lower) while the applied field remained constant, and then reducing the applied field to zero (Fig. 1-1). Shielding experiments were carried out by cooling the sample in zero field and then gradually applying a field. Additional information on sample behavior was obtained by cycling the field to full field and back to zero after a trapping or shielding test was completed. Information on the field distribution inside the sample was obtained by either moving Hall probes across a sample at successive field levels, or by

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leaving probes in a fixed position adjacent to or within a sample as the applied field was varied. Curves of the Hall probe voltage vs applied field or position were plotted by a Moseley/HP 163A dual-pen X-Y recorder. Flux trapping and shielding ability was directly reflected in either type of curve. Flux jumping was indicated by radical changes in the shape of successive field profiles, and by vertical steps in the B vs Ha curves.

The nature of the experiments did not call for high precision measurements. The aim of the work was to investigate the general properties of and limitations on the flux trapping process. One major limitation on flux trapping, flux pinning, differs from sample to sample even for similar materials; the other major limiting feature, flux jumping, is not a particularly repeatable process even with the same sample. Therefore, it did not make sense to attempt to achieve a very high degree of accuracy in the absolute field determinations. In practice, the precision of the field determinations varied considerably since several different magnets, experimental setups, and procedures were used. The potential sources of uncertainty included errors in the calibration of the Hall probe driving mechanism (1 1%), misalignment of the Hall probes (variable, but generally \leq 1-2%), variations in the Hall current (negligble except in the early experiments), the hysteresis and remanent magnetism of the steel pole faces (varied with magnet used and experimental procedure), and the limited response of the pen recorder to rapid field changes. Typical overall uncertainties in the absolute field determinations were on the order of a few percent, although the relative accuracy of sets of measurements taken by a particular Hall

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probe during one run was better since most of the factors introducing uncertainties into absolute field determinations remained constant.

B. Computer Modeling

Analysis of the field profiles generated in the geometries used in this study is somewhat more difficult than for the usual case of a long cylinder in an axial field. Because of the high degree of symmetry in the latter case, the form of the critical state is particularly simple. Except near the ends of the sample, all currents flow azimuthaly, Jtot=Jø, the field is essentially axial, B=Bz, the equation Jtot=(10/4 π)curlB simplifies to Jtot=(10/4 π)dBz/dr, and critical currents flow in regions where there is a gradient in B. As a result, the field distributions caused by various magnetic histories can be easily estimated. A full solution of the problem of a thin flat disk or plate in an axial field is not nearly so straightforward. However, the problem does retain some of the symmetries of the long cylinder in an axial field, and can be reduced to an essentially one-dimensional problem with some additional approximations. A computer model of trapping and shielding by a thin one-dimensional disk was developed. The model was able to account adequately for the experimental behavior of a Nb-Ti sample, and highlighted some interesting aspects of the critical state in this geometry. The problem of a cylinder in a transverse field has less symmetry, and as a result determining the form of the critical state is a difficult problem. Nevertheless, progress

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was made in describing the nature of the critical state and in accounting for experimental observations through the use of another simple computer model.

1. Thin Disk in Axial Fields

Complications arise in the case of a flat disk in an axial field because the cylinder length is no longer greater than its diameter. Critical state currents still flow in the azimuthal direction, $Jtot=J\phi$, but curlB=($4\pi/10$)J only reduces to ($4\pi/10$)J ϕ =(∂ Br/ ∂ z- ∂ Bz/ ∂ r). This equation combines with divB=0 and some form for the Jc(B) relation to form a nontrivial second order differential equation that describes the behavior of an irreversible superconducting disk. The solution of the equation for a disk of radius A and thickness q, involves various combinations of Bessel, sine, and exponential functions in the three regions; r(A, $|z| \langle q/2$ (inside the disk); and r)A, $|z| \langle t/2$; $|z| \rangle t/2$ (outside the disk). The solutions appropriate to the various regions have to be matched to each other at the boundaries of the regions and to the applied field at infinity. As an alternative to solving this rather difficult boundary value problem, an iterative approach to a simplified version of the problem was used. The three-dimensional disk, which in practice was 4.56 cm in diameter, and was made up of three .005 cm layers of copper interleaved with two .005 cm layers of Nb-Ti superconductor, was modeled by an array of coaxial current loops lying in the z=O plane (Fig. 4-1). A closed form expression for the z-component of the field at an arbitrary point above a current loop:

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Fig. 4-1 Critical current model for a thin disk in an axial field.

 $Bz(Gauss) = (0.2I / ((ri+r)^2 + z^2)^{\frac{1}{2}}) (K + E(ri^2 - r^2 - z^2)/((ri-r)^2 + z^2))$

where ri=radius of a current loop (cm)
(r,z)=location of the observation point
I=current in a loop (amps)
K,E=complete elliptic integrals of the first & second
 kind with argument k given by
 k = 4rxri 7 (ri+r)²+z²

was used to calculate the field generated by currents flowing in the loops. The sums of the contributions from the nested loops to the field at a series of points above the model disk were plotted, resulting in Bz(r) profiles along a line above a diameter of the model disk.

The disk was subdivided into N2180 current loops, each with an effective width of $\Delta r=2.286/180=.0127$ cm or less, and thickness $\Delta z=.010$ cm. The current I(ri) flowing in the ith loop was equal to $J \neq (ri) \times \Delta r \times \Delta z$, where $J \neq (ri)$ was the current density at the radius ri of the ith loop. The field was calculated at points which were 0.0762cm above the plane of the current loops. These points coincided with the approximate location of the active area of the Hall probe used to measure the experimental field profiles. The computer-generated profiles were insensitive to changes in N, for N > 100, as long as z was large compared to the spacing of the loops. The profiles were sensitive to changes in z only near the center and edge of the disk; in these areas the magnitude of Bz increased somewhat with decreasing z.

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In its simplest form, the model was used to compute field profiles due to a uniform current distribution, $J\phi(ri)$ =constant for ri < A, and other simple, arbitrarily specified forms of $J\phi(ri)$. The field profiles generated by uniform azimuthal currents agreed well with experimental measurements at higher fields (see Fig. 4-2). At lower fields where Jc, dJc/dB, and the total field difference across the disk face were larger, the agreement was poorer.

A better fit to the data was achieved with a more sophisticated form of the model (Fig. 4-1) in which the current distribution in the disk was adjusted in an iterative fashion until $J\phi(ri)$ was equal to Jc(Bz(ri)), where Bz(ri) was the z-component of the field at r=ri, and Jc(B) was an assumed form for the critical current vs field relation. The computer used Bz(ri) values calculated in the nth iteration to determine the values of $J \neq (ri)$ used to calculate Bz(ri) in the n+1th iteration. Computations were stopped when sucessive field profiles became indistinguishable. Starting with an initial guess of $J \neq (ri)$ =constant, the final self-consistent solution was usually obtained after fewer than 10 iterations. Flatter Jc(B) relations allowed faster convergence than more rapidly varying forms of Jc(B). Behavior at different field levels was modeled by adding a uniform bias field Ba to the field generated by the circulating currents. In these cases, the J ϕ values at each iteration were determined by $J\phi(ri)=Jc(\Delta Bz(ri)+Ba)$, where $\Delta Bz(ri)$ was the field generated by the currents J¢ from the previous iteration. New $\Delta Bz(ri)$ values were then calculated with the new set of $J\phi(ri)$ values. After sufficient iteration a self-consistent solution in which $J\phi(ri)=Jc(\Delta Bz(ri)+Ba)$ at every current loop was obtained. Simulation of

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Fig. 4-2 Experimental and model curves for the field profile above a thin superconducting disk. Solid curve - field profile from a shielding test of a Nb-Ti/Cu disk. Dotted curve - field profile generated with model by assuming a uniform current density of .76 x 10⁵ A/cm² flows throughout the disk. either trapping or shielding currents was obtained by choosing the appropriate direction ($\pm z$) of the bias field in relation to the direction of J ϕ . The model allowed any analytic form for Jc(B) to be used (or even a completely arbitrary form of Jc(B) specifield by a small table of Jc,B values), although for convience Jc= α /B+Bo (Kim model) was generally used in the calculations described here.

The parameters in the Jc(B) expression were varied until good agreement between experimental and theoretical field profiles at several field levels was obtained. Comparisons between a series of measured and calculated profiles for the Nb-Ti disk are shown in figures 4-3 and 4-4. With the simple Kim model expression for Jc(B), close agreement could not be achieved over the full 0-20kG field range, indicating that the actual Jc(B) relation for the material was more complicated than $Jc=\alpha/B+Bo$. However, as shown in the figures, the model was able to account for the general form of the trapping and shielding profiles which were observed while the sample was in the full critical state.

Further refinements in the model are possible, such as using the total field $(Bz^2+Br^2)^{\frac{1}{2}}$ calculated at points closer to the disk to determine the J ϕ (ri) values, although the additional computations entail added complications and increases in computer time. In cases where trapping currents are flowing, the effect of using Bz at z=.076 cm rather than z=0 is to slightly underestimate the field used to determine J ϕ near the center, and slightly overestimate it near the edge. Where shielding currents are flowing, the reverse occurs. The overall effect of such refinements on the generated field profiles is not expected to be large.

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Fig. 4-3 Field trapping curves from a Nb-Ti/Cu thin-disk sample. Solid curves - series of field profiles taken in decreasing applied fields (trapping). Dashed curves - curves generated with critical state model with $Jc=2.2x10^5/(1+B/2700) A/cm^2$. Dotted curves - points generated with currents reduced by a factor of (r/.25) at r \leq .25 cm.



Fig. 4-4 Shielding curves from a Nb-Ti/Cu thin disk sample. Solid curves - series of field profiles taken in increasing applied fields (shielding). Dashed curves - curves generated with critical state model with Jc = $2.2 \times 10^5 / (1+B/2700)$ A/cm². Dotted curves - points generated with currents reduced by a factor of (r/.25) at r $\le .25$ cm. Crosses - points generated with currents reduced by a factor of (r/.38) at r $\le .38$ cm.

Modeling of curves taken before field changes had penetrated to the center of the disk (Fig. 4-5a) required further assumptions about the critical current pattern in the disk. It was evident that the whole sample did not enter the critical state until the flux front penetrated to the center of the disk. However, attempting to model the behavior by allowing currents to flow only in regions where there was large scale flux penetration was not successful. Reducing $J\neq$ (ri) to zero at some cutoff radius invariably leads to negative field peaks in Bz(r) near the cutoff radius. Further investigation with the model indicated that the extra field peak could be eliminated, and a flat Bz(r) profile at small r generated, only by allowing some current to flow over the central portion of the disk.

One attempt to determine the required current distributions was made by using a procedure in which the computer generated field profile was compared at every iteration to the experimental one. The difference between the field calculated in the nth iteration and the experimental field was used to determine the adjusted values of the currents used to calculate the field profile in the n+1th iteration. Unfortunately, the solution would not converge to a stable form, even when field differences over a range of radii near a loop were used to compute the modified value of the current in that loop. The nonlocal nature of the problem evidently prevented the procedure from working. However, a relatively good fit to the data was obtained after surprisingly few iterations by manually modifying the current distribution after each iteration. The resultant fits to the data (Fig. 4-5a) indicated that

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Fig. 4-5 Field profiles just above a Nb-Ti/Cu thin disk sample, obtained after the sample was cooled in zero field, but before field changes had penetrated to the center of the sample. Dashed curves were calculated with critical state model using Jc=6x10⁸/(B+2700), and the current distributions shown in (b).

I(ri) was a steadily decreasing nonzero function of r across the inner portion of the disk. The postulates of the critical state require that currents flow at a current density $J=\pm Jc(B)$ or not at all. The two constraints can be met by assuming that the shielding currents flow at J=Jc(B) in a thickness of material that decreases with decreasing radius. This thickness can be readily estimated by calculating the thickness of material needed to conduct the current I(ri) at a current density Jc(Bz(ri)+Ba): $z=I(ri)/\Delta rJc(Bz(ri)+Ba)$. As shown in Fig. 4-5b, reasonable forms for the depth of penetration of the critical state currents vs radius were obtained. These current distributions are similar to the current distribution required to generate a uniform field inside an extremely oblate ellipsoid of revolution. Such a body can be uniformly magnetized by uniform azimuthal currents flowing on its surface.

Reasonably good fits to experimental field profiles were also obtained with the same basic model for cases in which profiles were recorded shortly after the direction of dHa/dt had been changed. In these cases the experimental curves were accounted for by assuming that all the critical currents flowed in the reverse direction at radii greater than the distance into which the field change had penetrated, and that some proportion of the current flowing at smaller radii also reversed direction.

Although most of the features of the critical state are retained in the disk geometry, the usual arrangement in which critical state critical currents flow only in regions where a large scale flux change has been

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felt, is not sufficient. In the disk geometry critical currents flow in regions where Bz 0.075 cm. above the disk is less than 5% of the applied field and where presumably the z-component of the field actually threading the material is even less. The currents cannot be wholly attributed to reversible magnetization currents since similar behavior is observed when an applied field is decreased after the disk is cooled in a 3400 Oe field (Fig.4-6). Apparently the small fraction of dHa/dt felt near the center of the disk induces the critical currents. One other interesting feature of these measurements is that critical currents apparently flow over a sizable region in which Bz is nearly constant. The expression $J\neq=10/4\pi(\partial Br/\partial z-\partial bZ/\partial r)$ can therefore be satisfied only if there is a large change in Br, on the order of a few hundred gauss, in the material. The features observed here with superconducting disks should also occur near the ordinarily neglected ends of long cylinders in axial fields.

2. Tubular Samples in Transverse Fields

Applying a transverse field to a cylindrical sample eliminates much of the symmetry present in the axial field case, and additional complications arise when a sample's length/diameter ratio is not large. A rough idea of the critical current pattern for this geometry can be obtained by deforming the current pattern of the thin disk to fit a semi-cylinder surface. The currents evidently flow in approximately saddle-shaped paths over the cylinder surface. The equations which determine the actual form of the critical state, $(10/4\pi)$ curlH=J, divJ=O, J=Jc(H), plus the proper boundary conditions and the magnetic history,

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Fig. 4-6 Flux trapping with a Nb-Ti/Cu thin disk in an axial field.

are quite complicated for this geometry. Again a simplified form for the critical state was assumed and used to compute field profiles that could be compared with experimental observations.

The model developed was one in which the critical currents flowed axially down the sides of the tubular samples and followed circular arcs across their ends (Fig. 4-7). (The current paths are similar to ones in the circular-head coils used to deflect the electron beam in some simple CRT tubes.) The transverse field along a sample's axis due to these currents was computed by summing field contributions from a series of pairs of nested circular-head coils arrayed on the sample surface. An expression for the transverse field at a point along the axis a distance l_1 from one end and a distance l_2 from the other end of a pair of circular-head coils can be derived from the Biot-Savart Law (Harris 1934). The result is:

Bx = .4I/R ($c(2+c^2)/(1+c^2)^{3/2} + d(2+d^2)/(1+d^2)^{3/2}$) sin(\$) where: $c=l_1/R$, $d=l_2/R$,

> L=l₁+l₂=length of the straight segments R=radius of the curved end segments 2¢=angle included by a curved end segment.

The surface of a sample was typically subdivided into N=15-31 current paths. The effective width of the straight sections, Δs , was equal to $\pi R/4N$. The width of the circular sections was between $\pi R/4N$ and .5L/(N-1). A width of $R\pi/4N$ for the circular segments resulted in a uniform current density on the sample surface, and a current pattern (for $2\pi R$ (L) in which the straight sections of the current paths

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Fig. 4-7 Critical current model for tubular samples in transverse fields. The lower portion of the figure shows the type of field distribution obtained by summing the field contributions from a series of nested current loops similar to the one shown in the upper portion of the figure.



Fig. 4-7 Critical current model for tubular samples in transverse fields. The lower portion of the figure shows the type of field distribution obtained by summing the field contributions from a series of nested current loops similar to the one shown in the upper portion of the figure. reached the center of the surface before the circular sections. A width of L/2(N-1) resulted in lower current density in the circular-end segments than along the straight sections, and a current pattern in which all the current paths reach the center of the tube surface at once. The latter pattern resulted in profiles with steadily increasing values of Bx towards the center of the tube, while the uniform current density pattern resulted in steeper field gradients near the ends of the tube and a flatter profile near its center. The latter pattern also resulted in small field peaks at the sample ends due to the concentration of current in that region. Thin-walled samples were modeled by setting the effective thickness of the current paths, Δr , equal to the thickness of the sample wall. Thicker walled samples were modeled by adding contributions from a series of current sheets located at decreasing values of the radius. The current per loop was set equal to the product of an assumed current density and the cross-sectional area of the current paths, $I=Jc\Delta s\Delta r$. The field due to currents flowing over a limited portion of the sample surface could be calculated by restricting the values of ϕ to between $\pi/2$ and some angle greater than zero. However, the field profiles along the axis of the tube were found to be insensitive to such changes as long as the total circulating current was kept constant. It was also possible to have the current density vary with angular position in a given manner. With $J \propto \sin(\phi)$ (a current distribution expected to produce a roughly uniform transverse field inside the tube), Bx(z) curves with a form similar to ones generated with J=constant, but with approximately half the peak field intensity were obtained. As with the disk model, trapping or shielding at different field levels was modeled by adding a uniform dipole field of the appropriate polarity.

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The model was reasonably successful in accounting for the field profiles along the axis of a variety of samples. Several examples of the fits obtained are shown in chapter 5 (see Figs. 5-19,37,38,39,42,43,44,45). However, since similar field profiles along the axis could be generated by routing the same net current in different patterns on the surface, the model was not able to determine precisely the current pattern of the critical state in these samples. Since only the field along the axis and not the field near the superconducting material was calculated, the model was limited to a simple Bean-type form for Jc(B).

CHAPTER V-EXPERIMENTAL RESULTS

A. Lead Samples

Lead cylinders were tested in both axial and transverse fields. All measurements were made at 4.2K with the samples immersed in LHe. The samples did not exhibit a strong Meissner effect, and even annealed samples of this type-I material were able to trap transverse dipole fields of over Hc/2 (Hc(Pb)=528 Oe). Transverse quadrupole fields with a gradient of ~140 G/cm, corresponding to a peak field of ~160G in the sample wall, were trapped in a hollow sample. The effective trapping currents, which decreased with increasing field, were on the order of $1X10^3 \text{ A/cm}^2$. No flux jumps were observed with any of the lead samples. The measurement techniques used with the lead samples are described in section 1, and the experimental results are summarized in table V-1 and discussed in more detail in sections 2~5.

1. Experimental Setup for Dipole Field Measurements

The hollow samples had one Hall probe located inside and near the center of the cylinder and a second probe on the outer surface nearly in line with the first. Transverse or axial fields were applied as an X-Y recorder plotted the local field vs applied field. Starting with the sample cooled in zero field, four characteristic fields could be determined: H_1 , the shielded field at which the internal field began to change appreciably with further increases in the applied field; H_2 , the field at which evidence of superconductivity vanished, that is, when the local field became approximately equal to the applied field; H_3 , the field at which evidence for superconductivity reappeared as the field was reduced from above, i.e., when the local field no longer equaled the applied field; and Btf, the magnitude of the field trapped in the sample when the applied field was reduced to zero. In general there was a large spread among the values of these fields, particularly in transverse fields. H_3 was less than H_2 , and the field measured by the probes was always greater than the applied field when the applied field was reduced from H_2 .

The Pb samples were tested in a magnet whose pole diameter was approximately equal to the sample length. Therefore, in the transverse field measurements the sample experienced a nonuniform field, and the ends of the sample may have remained superconducting after the center portion had gone normal. The field nonuniformity contributed to a smearing out of the transition and the increased spread among the various fields in the transverse field measurements.

With most samples the exact form of the B vs Ha curve depended to some extent on the rate of change of the applied field. In the case of the hollow lead cylinder, flux was observed to leak out of the sample for some time after the applied field had been turned off. Other investigators have also found long relaxation times in lead samples. For example, a lead sample tested by Voigt required up to tens of minutes to come into equilibrium with the applied field. No systematic

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study of this effect was undertaken with the lead samples measured here, However rate effects which prevent the sample from being in equilibrium with the applied field can account in part for the large values of H_2 , H_3 , and Btf that were observed. In addition, the field ramp in these initial experiments was not particularly smooth. The irregular ramp coupled with the limited response time of the pen recorder led to somewhat irregular B vs Ha curves from which it was often difficult to accurately determine values of the characteristic fields.

2. Results in Transverse Dipole Fields

In its as-formed state the hollow cylinder trapped a transverse field of over 3006 in its center and approx. 1006 at the outer surface. A larger difference between the internal field and the applied field (over 5006) was observed when a reverse field was applied to the sample with a trapped field in it. In this case the field in the material was everywhere less than 3006. The measurement therefore indicated that the pinning and effective critical currents were higher at low fields than near Hc. The computer program (described in section IV-B-2) indicated that an average current density of approximately $.86 \times 10^3$ A/cm² is required to maintain a 5006 field difference in the transverse case. Little change in behavior was observed after the cylinder was annealed.

The cylinder was then bored to an ID of 1.90 cm, slipped over an Al tube and annealed J 200°C for 65 hrs. As a result of the reduction in wall thickness from .77 cm to .21 cm, Btf was reduced from approximately 3006

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Pb-A	Pb-A1	Pb-A:	2	Pb-C
.79 cm	ID	1.90 cr	n ID	solid sample
2.34 cm	OD	2.34 cm	n OD	1.9 cm OD
15.24 cm	Long	15.24 cr	n long	15.60 cm long
As-Received 5-nines Lead	Annealed 79 Hrs. at 140°C	Annealed another 65 hrs. at 200°C		As-Formed
Ha-Transverse	Ha-Transverse	Ha-Transverse	Ha-Axial	Ha-Transverse
-	-	320 Oe	300-480	-
-	595-620 Oe	550-560	586-607	595-610
490-500 Oe	490-510	510-550	523-545	507-514
310 G	290	160	480-544	-
100-115 G	90-95	150	-	80
	Pb-A .79 cm 2.34 cm 15.24 cm As-Received 5-nines Lead Ha-Transverse - 490-500 0e 310 G 100-115 G	Pb-A Pb-A1 .79 cm ID 2.34 cm 0D 15.24 cm Long 15.24 cm Long As-Received Annealed 79 Hrs. 5-nines Lead at 140°C Ha-Transverse Ha-Transverse - - - 595-620 0e 490-500 0e 490-510 310 6 290 100-115 6 90-95	Pb-A Pb-A1 Pb-A3 .79 cm ID 1.90 cm 2.34 cm OD 2.34 cm 15.24 cm Long 15.24 cm As-Received Annealed 79 Hrs. As-Received Annealed 79 Hrs. S-nines Lead at 140°C Ha-Transverse Ha-Transverse - - - 320 0e - 595-620 0e 490-500 0e 490-510 310 6 290 160 100-115 6 90-95 150	Pb-A Pb-A1 Pb-A2

TABLE V-1 RESULTS OF EXPERIMENTS WITH LEAD SAMPLES

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to 160G, the spread in H-values decreased, and the area of the B vs Ha curve was reduced. The computer program indicated that an average current density of approximately .9X10³ A/cm² is required to trap a 150G transverse field in this sample. With the thinner wall the trapped field at the center of the outer surface was approximately equal to the field trapped inside the center of the cylinder. A scan of the transverse field along the axis of the cylinder showed that the field was effectively_trapped over approximately 2/3 of the length of the cylinder. The sample effectively shielded a field approximately twice as large as it trapped. This result may have been due in part to increases in shielding or trapping currents with decreasing fields. In a shielding situation most of the material is in a low or zero field while in a trapping situation most of the material is in a field close to Btf. The asymmetry may also have been partially due to the reversible magnetization of the material assisting flux shielding and opposing flux trapping. (An ideal reversible lead sample would be expected to shield out fields of up to Hc/(1-nd)=160 Oe in the transverse case, but would oppose any flux trapping.)

3. Results in Uniform Axial Fields

The characteristic fields H_1 , H_2 , and H_3 , were more sharply defined in the B vs Ha curves taken in the axial field than in those taken in the transverse field. H_3 was close to values measured in the transverse case and agreed well with the value of Hc(4.2)=5286 for pure lead (Lynton 1962). The cylinder trapped axial fields of 480-5446 with some apparent loss of trapped flux with time from the higher trapped fields. The axial field shielding increased over the value for transverse fields, consistent with the reduction in demagnetization factor from ~1/2 in the latter case to ~0 in the former. A field difference of 500G across the 0.21 cm wall implies an average current density on the order of 1.9×10^3 A/cm2 in the cylinder wall, although theoretically the trapping and shielding could have been accomplished by Meissner currents flowing within a penetration depth of the sample surface.

4. Results in Transverse Quadrupole Fields

As shown in Fig. 5-1, the sample was cooled in a transvere quadrupole field with a gradient of 1656/cm. Hall probes mounted on a rotatable phenolic holder suspended near the midpoint of the hollow cylinder sensed Br(\$\neq\$) at r=.43 cm, and r=.83 cm. The peak field intensity increased by ~2% as the sample was cooled with liquid He. After the applied field was reduced to zero, a slightly distorted quadrupole field with peak values some 10% smaller than the original applied field remained trapped in the interior of the hollow sample. The gradient of the trapped field, ~1426/cm, implied a peak trapped field of 1656 in the outer portion of the cylinder wall. This peak field level was approximately equal to the largest transverse dipole field that could be trapped in the cylinder. An ideal 2-dimensional quadrupole field can be generated by a cylindrical current distribution described by:

 $\sigma(\phi) = (10/2\pi) \ G \ r \ \cos(2\phi)$

where σ is the linear current density (A/cm) on the cylinder surface, G

5-1

is the gradient of the quadrupole field generated, r is the radius of the cylindrical surface, and ϕ is the azimuthal angle (Beth 1966). This



Fig. 5-1 Br(\$\not p) at r = .43 cm (upper curves), and r = .83 cm (lower curves), as sample Pb-A2 was cooled in a 165 G/cm quadrupole field. Solid lines - T > Tc, Ha-on; dotted lines - T=4.2K, Ha-on; dashed lines - T=4.2K, Ha-off. expression predicts that a 142G/cm field requires currents of up to 1.1×10^3 A/cm² flowing in the .21 cm thick wall of the lead sample.

5. Solid Sample

A solid lead cylinder of approximately the same size as the hollow sample was tested in a transverse dipole field. A probe on the outer surface measured_trapped fields nearly equal to those measured on the outer surface of the hollow cylinder. Again there was a lack of a strong Meissner effect.

6. Discussion

The interpretation of the data on the Pb samples presents some difficulties because of the potentially large contribution from the reversible magnetization, and the problem of quantifying the reversible magnetization in a sample which clearly has nonuniform fields inside. In the transverse case the sample shielded a field at least 60 G in excess of what an ideal reversible Pb sample could achieve, and trapped fields of 160-300 G (a reversible Sample cannot trap transverse fields at all). Trapping currents of at least 0.9X10³ A/cm² were implied by the trapping results, although the actual magnitude of the irreversible currents must be somewhat higher since the currents due to the reversible magnetization act in opposition to the trapping currents.

The results in axial fields do not provide any additional information on the irreversible currents since even a fully reversible hollow cylinder is capable of trapping and shielding fields of approximately Hc.

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The size of the trapped quadrupole field is significant since the quadrupole field trapping ability is consistent with irreversible currents flowing at roughly the same level as was demonstrated with the dipole field measurements.

B. Niobium Samples

Variously shaped niobium samples were tested in transverse dipole, quadrupole, and sextupole fields. One sample was also tested in an axial field. Relatively large trapped fields were attained with several of the samples in transverse dipole fields; in some cases Btf was close to Hc₂. Quadrupole fields of moderate strength and with good fidelity to initial applied fields were trapped inside two hollow cylinders. A small sextupole field was also trapped in one sample. Flux jumping limited trapped fields in some of the samples to lower levels in some runs. Table V-2 summarizes the results with the Nb samples. The behavior of the individual samples is discussed in more detail in the sections below.

1. Nb-B

4.95 cm OD X 4.29 cm ID X 13.46 cm long Total impurities < 1000 ppm

Transverse dipole fields of 2500-2600G were trapped in the center of the cylinder, although flux jumps often limited trapped fields to the

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TABLE V-2 MAXIMUM FIELDS TRAPPED IN THE NIOBIUM SAMPLES

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	Nb-B,B1	Nb-E	Nb-F, F1, F2	Nb-G			
Sample:	Hollow cylinder	Hollow cylinder	Sample with rectangular central cavity	Solid sample			
Heat treatments:	Heated to T>2000°C and vacuum outgassed	- Si	strain annealed at T<1000°C ome recrystallizati	As-received			
Grain size	1 cm	-	.009	.007			
KNOOP hardness:	55	-	65	100			
Maximum field trapped:							
Dipole	2500-2600 G	-	2900	3400			
Quadrupole	660 G/cm	460	-	-			
Sextupole	42 G/cm ²	-	-	-			

1350-1550 G range. The flux jumps were observed only in decreasing applied fields or when a reverse field was applied after the sample had trapped a field. The critical current level required to trap a 2500G transverse field was estimated by applying the computer program outlined in section IV-B-2 to this sample. The calculation implied that the average current density in the sample was approximately 1.0×10^4 A/cm². ł

2. Nb-B1

4.88 cm OD X 4.37 cm ID X 13.46 cm long Sample Nb-B with slightly reduced wall thickness

A sextupole field with $\partial^2 B/\partial r^2 = 42 \ G/cm^2$ was trapped. The trapped field retained its basic sextupole nature, although some distortion was evident, particularly near the inner wall of the sample. The peak values of the trapped field were up to 5% greater than the initial applied field, due to flux redistribution. The magnitude of the trapped field was limited to a peak value of approximately 100G by the capabilities of the magnet and power supply.

A 66 G/cm quadrupole field, corresponding to a peak field of ~150G, was trapped with only slight distortion and an increase of $\leq 8\%$ in peak field strength from the original applied field. A 660 G/cm field with peak trapped fields of over 1300G and somewhat more distortion was also trapped (Fig. 5-2). A slightly larger quadrupole field could be trapped by cycling the magnet to full field and back to zero. Equation 5.1 implies that current densities on the order of 1.0X10⁴ A/cm² are required to produce these quadrupole fields.

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Fig. 5-2 $Br(\phi)$ at r = 1.55 cm (upper curves), and r = 2.06 cm (lower curves), before and after sample Nb-B1 was cooled in a 660 G/cm quadrupole field. Solid curve - T > Tc, Ha-on; dashed curves - T=4.2K, Ha-off.

Nb-B1 was also used in a test with a transverse dipole field in which a rotating coil was used to sense the field. A harmonic analyzer measured the harmonic content of the applied field inside the hollow sample with T > Tc, the field inside the sample with the external magnet still on and the sample cooled to 4.2K, and the trapped field remaining after the applied field was reduced to zero. The results of the measurements are shown in table V-3 and figures 5-3, 5-4, and 5-5. The figures show the amplitudes of dipole, quadrupole, and sextupole components of the field when the sample is normal and the applied field on, and of the trapped field. The figures show that the quadupole and sextupole components of the applied field (as well as the dipole component) increase approximately linearly with magnet current. For fields less than 1200G, the multipole content of the trapped field was nearly identical to the original field. At higher fields the dipole component is sharply reduced, and the quadrupole and sextupole components were much larger than in the original undisturbed applied field. The reduced value and increased distortion of the trapped field were caused by a flux jump or jumps occurring as the applied field was reduced to zero. As noted above, experiments with Nb-B showed that the sample could occasionally trap fields of 2500-2600G but that these fields often decayed via flux jumps to the 1400G level. The presence of the spinning coil with its attendant vibration and turbulence in the liquid helium, or the slightly different field ramping function of the power supply-magnet combination used in this test, was apparently limiting trapped fields to the lower values by increasing the probability of a flux jump occurring. Cycling the magnet to over 6000G and back to zero also resulted in a distorted trapped field of ~1400G. The magnitude of these trapped dipole fields



Fig. 5-3 Dipole component of spinning coil output as a function of the field in which sample Nb-B1 was cooled to 4.2K. x - sample above Tc, Ha-on; + - sample at 4.2K, Ha-off.

0.5 Harmonic Analyzer Output (Volts) 0.4 Quadrupole Component of the 0.3 0.2 0.1 ¥ 0.0 0.5 1 2 1.5 2.5 0 Magnetic Field Sample was Cooled in (kG)

> Fig. 5-4 Quadrupole component of spinning coil voltage as a function of the field in which sample Nb-B1 was cooled to 4.2K. x - sample above Tc, Ha-on; + - sample at 4.2K, Ha-off.

0.3 Harmonic Analyzer Output (Volts) Sextupole Component of the 0.2 0.1 0.0 0.5 1 1.5 2 2.5 0 Magnetic Field Sample was Cooled in (kG)



	+	\$	3	. #
т(к)	Ha(Oe)	B(G)	Vquad(mv)	Vsex(mv)
) Tc	755	755	12.5	18.2
4.2	755	753	13.6	19.9
4.2	0	752	13.6	19.8
> Tc	980	980	13.0	23.9
4.2	980	980	13.0	25.0
4.2	0	980	13.2	25.0
> To	1143	1143	14.9	27.9
4.2	1137	1139	15.0	28.0
4.2	0	1125	17.0	22.3
4.2	0	1125	17.0	22.3 **
> Tc	1207	1207	16.0	29.3
4.2	1207	1198	16.0	30.0
4.2	0	927	262.0	133.1
> Tc	1444	1444	23.0	35.0
4.2	1436	1444	24.0	34.0
4.2	0	1305	118.0	85.0
> Tc	1636	1636	26.7	39.0
4.2	1636	1636	26.7	39.0
4.2	0	1305	305.0	182.0
> Tc	1809	1809	19.2	44.5
4.2	1839	1853	25.0	45.0
4.2	0	1418	380.0	225.0
> Tc	2160	2160	36.0	52.0
4.2	2250	2250	38.0	56.0
4.2	0	1440	440.0	280.0

TABLE V-3 RESULTS OF MEASUREMENTS WITH THE SPINNING COIL AND HARMONIC ANALYZER ON SAMPLE NB-B1

+ Ha = applied transverse dipole field

\$ B = amplitude of the dipole field within the hollow sample, derived from the dipole component of the spinning coil voltage

&,# Vquad and Vsex = amplitudes of the quadrupole and sextupole components of the spinning coil voltage ** This measurement was made after waiting 1 hr with the

sample maintained at 4.2K with Ha=O.

was approximately equal to the magnitude of the peak quadrupole fields trapped in the same sample.

3. Nb~E

5.99 cm OD X 3.66 cm ID X 8.51 cm long

The sample was cooled in quadrupole fields with gradients of up to 327 G/cm (~560G at the inner wall of the sample). As the sample was cooled to 4.2K, the peak fields increased by 3-6% near the center of the hollow cylinder and 15-22% near the cylinder's inner wall, due to flux redistribution within the relatively thick sample walls. This flux concentration effect was most likely due to the partial expulsion (Meissner effect) of flux from the portions of the sample wall in low fields. The field then changed only slightly as the applied field was reduced to zero, leaving a trapped field with a basic quadrupole form and a magnitude that ranged from slightly less than the original applied field to 10-20% more than the original applied field. Figure 5-6 shows the form of $Br(\phi)$ before and after the sample was cooled in a 275 G/cm field, and after the applied field was removed from the cooled sample. The magnet was then cycled to full field and back to zero leaving a trapped guadrupole field with a gradient of ~460 G/cm and peak fields of ~750G near the cylinder wall. Equation 5-1 implies that the peak current density must be at least 1.25×10^4 A/cm² in order to trap this field.

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Fig. 5-6 Br(≠) at r = .43 cm (upper curves), and r = 1.7 cm (lower curves), as sample Nb-E was cooled in a 275 G/cm quadrupole field. solid curves - T > Tc, Ha-on; dotted curves - T=4.2K, Ha-on; dashed curves - T=4.2K, Ha-off.

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This sample started out as a 6.18 cm OD X 5.44 cm long cylinder with a rectangular 2.57 cm X .74 cm axial cavity. The cylinder wall along the long side of the cavity was reduced in two steps, first to 1.81 cm to form a sample with equal wall thickness around the central cavity (Nb-F1), then to .56 cm so that the wall thickness along the long side of the cavity was much thinner than along the short side (Nb-F2). A rotatable Hall probe was located at the center of the cavity and stationary probes were located at the center of the outer surface in line with the axes of the cavity. A variety of effects was observed with this sample.

The effect of flux penetrating into the cavity through the open ends was observed. With the long side of the cavity parallel to the applied field, the size of the opening was of the same order as the length of the opening. The assumption (discussed in Appendix I) that the field penetrates into the opening approximately as (Ha/1.5)exp(-1.8z/r)(z=distance down into the cavity, r \simeq half-size of the opening parallel to the field direction, and Ha/1.5 \simeq field at the opening of the cavity) and does not penetrate through the superconducting wall, implies that the ratio of the change in the field at the center of the sample to the change in the applied field is:

2X(1/1.5)exp(-1.8(2.72/1.29)) = .030, or a fringing effect of a few percent at the center and much larger effects nearer the ends of the sample. With the long side of the cavity perpendicular to the applied field, this effect is important only very close to the ends of the cylinder, since r is now the half-width of the cavity rather than the half-length.

The demagnetization coefficients of the sample can be estimated on the basis of the lengths of its three semi-axes through the use of graphs published by Osborn (1945). Table V-4 shows the coefficients for the three states of the sample for the field aligned parallel and perpendicular to the long side of the cavity, as well as data on the maximum fields trapped in the various configurations. With the cylinder in its original form (Nb-F) the demagnetization factors for the parallel and perpendicular orientations were essentially equal. As the wall along the long axis was reduced, the demagnetization factor for the perpendicular case increased, and the factor for the parallel case decreased. To the extent that the sample aproximates an ellipsoid and acts as a perfect diamagnet the applied field near the equator of the sample is enhanced by the factor 1/(1-nd).

Nb-F

6.18cm OD X 5.44cm long with a 2.57 X 0.74 cm axial cavity

The sample was cooled in a 2300G transverse dipole field perpendicular to the long side of the cavity. The central field remained essentially constant, and the field measured by the probe on the outer surface decreased by up to 400G as the applied field was reduced to zero. Cycling the applied field to above Hc_2 and back to zero, resulted in trapped fields of ~2900G in the center of the cavity, ~2775G at one end

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TABLE V-4 DEMAGNETIZATION COEFFICIENTS AND MAXIMUM TRAPPED FIELDS FOR SAMPLES NB-F-F2

of the cavity, and ~2150G at the center of the outer surface. The apparent value of Hc_2 was approximately 3200G.

When the sample was cooled in a field parallel to the long side of the cavity, larger decreases in the interior field were observed as the applied field was decreased to zero. Fields of only ~1900G in the

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center of the cavity, 21506 at one end of the cavity, and approx. 22506 at the center of the outer surface were trapped by cycling the field. The differences in these three fields are evidently due to bowing of the flux lines as they pass through the relatively long empty space of the cavity. However, this effect does not directly account for the large difference between the maximum fields trapped in the parallel and perpendicular orientations.

Nb-F1

Equal wall thickness (1.81 cm) around the cavity

When tested in a field applied perpendicular to the long side of the cavity, Nb-F1 trapped a field of essentially the same magnitude (2900±1006) in the center of the cavity as Nb-F. The trapped field at the outer face of Nb-F1, 2700±1006, was substantially larger than the field trapped with Nb-F, due to a reduction in the spreading of the field lines near the outer periphery, caused by the reduced wall thickness.

When tested in the parallel configuration Nb-F1 trapped 400G more than Nb-F did in the center of the cavity (2300G total), and 200G more at the end of the cavity (2430G total) than Nb-F. The reason for the increase is probably related to the change in the demagnetization factor, which for this orientation is lower in Nb-F1 than in Nb-F.

Measurements of the field inside the cavity vs the applied field were made at three different distances from the opening. The results,

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together with estimates based on an exponential field falloff into the cavity, are shown in Table A-I in Appendix I.

Nb-F2

wall thickness along the long side of the cavity = .56 cm wall thickness along the short side of the cavity= 1.81 cm

In the parallel orientation there was a small increase in the maximum trapped field in the cavity compared to that trapped in Nb-F. In this orientation, the trapped field at the surface of Nb-F2 was slightly smaller than the one in Nb-F1.

Flux jumping was observed for the first time with this sample when it was tested in the perpendicular orientation. Flux jumps occurred in the 2400<u>S</u>Ha<u>S</u>29006 range of the shielding portion of the B vs Ha curve, and in the trapping portion of the curve at low fields, where the field differences were on the same order. Three Hall probes, located on the long surface of this sample, showed that the flux jumps in decreasing applied fields were causing large changes in the field near the edge of the sample and only small losses in flux from the central portion of the sample. The appearance of flux jumps in this sample was evidently related to the large increase in the field enhancement factor over that of Nb-F1. By cooling the sample in a smaller field (~12006) a field was trapped without any flux jumps occurring. In this case there was little loss of flux, as the applied field was reduced to zero even near the edge of the sample.

Summarizing the results on Nb-F-F2

This Niobium sample trapped fields of up to 29006. This value was quite close to the apparent value of Hc_2 =32006, which was deduced by observing the applied field at which the B(Ha) curve met the B=Ha line. The flux jump field was evidently above 29006, although flux jumps could be induced by increasing the demagnetization factor. Behavior was observed which was consistent with flux lines bending and penetrating into the open ended cavity approximatly as (Ha/1.5)exp(-1.8z/r), with r the appropriate opening size. The poorer trapping ability of the sample in parallel fields may have been related to the increased leakage of flux out of the cavity in this orientation. Changes in flux trapping ability that were partially attributable to changes in the demagnetization factor were also observed.

5. Nb-G



Hall probes were located on the surface as shown and also inside the hole at the center of the sample. The sample was tested in uniform transverse dipole fields directed parallel and perpendicular to the large face. An estimate of Hc_2 was obtained by noting the field level at which B=Ha, for Nb-G this appeared to be of the order of 3500 Oe. Because of the gradual approach of the B(Ha) curve to B=Ha near Hc_2 , only a rough estimate of Hc_2 was obtained.

With the large face perpendicular to the field, the sample was cooled in a 16006 field. After removing the field, the trapped field remaining in the center of the sample and at the centers of the large faces, differed by $\leq 2\%$ from the original applied field. The field near the edges of the face decreased by a factor of approximately 2 as the applied field was reduced to zero. Similar behavior was observed with the applied field directed at the narrow face.

With the sample in either orientation, fields of over 3400G in the center and somewhat less at the surface were trapped by cycling the field up to 7000G and back to zero. Flux jumps of varying magnitude on the ramp-down reduced the trapped fields by as much as 33% in some cases.

6. Discussion

The superconducting properties of Nb depend strongly on its chemical and metallurgical state. Reported values of $Hc_2(4.2)$ range from 3000 to over 6000G, and values for $Hc_1(4.2)$ usually fall in the 1000-1500G range. Annealed material with low levels of impurities can have an essentially reversible magnetization curve, while worked or dirty

material can produce extremely hysteretic curves. Studies have shown that heat treatments above the recrystallization temperature cause reductions in pinning which are reflected as magnetization curves with reduced hysteresis and smaller remanent magnetization (trapped flux). Analysis of the heat-treated materials shows that grain growth and reductions in the dislocation density with longer heating times or higher heating temperatures generally parallel the reductions in magnetic hysteresis. Increased pinning in heat treated Nb with impurities such as Y or Gd added has been attributed to the inhibition of grain growth and the tendency toward a more random grain structure caused by the presence of the impurities (Koch & Kroeger 1974). A direct correlation between grain boundary area and pinning was also noted. In worked niobium, pinning is attributed to interactions between flux lines and dislocations, as well as interactions with impurity concentrations. In heat treated material pinning due to dislocations is reduced as the dislocation density decreases and pinning is mainly due to flux line interactions with grain boundaries or impurities.

In most cases, previous investigators have not been specifically interested in trapping magnetic fields. However, values of remanent magnetization left after a magnetization measurement, or estimates of trapped fields left in samples studied with magnetooptical techniques, have often been reported. Values of the remanent magnetization usually represent the average value of the trapped field across the sample cross section. Hence they may underestimate peak trapped fields in cases where nonuniform fields are trapped. Reported values range from hundreds of gauss for disk or plate samples (De Sorbo & Healy 1964,

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Carroll 1966), to 1-3kG for larger cylindrical samples (Weber et al. 1971), and up to 5kG in the case of a heavily deformed Nb ellipsoid (Cline et al. 1966). The higher values of trapped fields are generally for unannealed cold-worked samples.

In our experiments the trapped fields were measured directly. Transverse fields in the 1.0-3.4kG range were trapped either by exceeding the critical field of the material and then reducing the applied field to zero, or by cooling the sample in an applied field and subsequently reducing it. The latter method permitted the trapping within hollow cylinders of multipole fields that closely approximated initial applied fields. The experiments with Nb-B,B1 indicated that expression 5-1, together with estimates of Jc based on dipole field trapping ability could be used to estimate the maximum guadrupole field that could be trapped. The experiments with Nb-E showed that problems can arise when trapping field configurations (such as transverse guadrupole fields) which go to zero within a material which exhibits a significant Meissner effect. The highest dipole fields (~3400G) were trapped in Nb-G, which was in the as-received state and had the smallest grain size and highest hardness value of all the Nb samples tested. Nb-F, which had a low temperature heat treatment that resulted in an average grain size slighly larger than Nb-G and a reduced hardness, trapped fields of up to 29006. Nb-B,B1, which had a high temperature anneal which produced large grains and low hardness values, trapped fields of up to 26000.

In niobium, some experimenters observe flux jumps in nearly all experimental runs, others in some limited situations, and others do not observe jumps at all. We have observed similar variability in our experiments with Nb samples in transverse fields. We generally observed O-3 flux jumps in one sweep (up or down) of the field. The flux jumping behavior varied somewhat from run to run, with some samples being on the edge of stability; flux jumping on some runs and not on others. The experiments with Nb-F-F2 demonstrated that flux jumping can be induced in an otherwise stable sample merely by changing its shape.

The adiabatic stability criterion (section III-C-1, equation 3-10) predicts that flux instabilities can begin to occur when a sample traps or shields fields greater than ~2000G. The results obtained with the Nb samples are only in rough agreement with this prediction. Flux jumps did occasionally occur at field differences on the order of the predicted value of Hfj. However most of the Nb samples tested were able to trap fields considerably larger than Hfj (more than 1.5Hfj in the case of Nb-G).

C. Pb-Bi Samples

As part of the series of measurements on Pb-Bi alloys, flux trapping experiments were carried out on large, thick-walled, hollow cylinders, spheres of three different sizes, and a disk-shaped sample. Magnetization curves and critical temperatures for the alloys were

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obtained from measurements on smaller solid cylinders. The structure of the material was studied with light and electron microscopes. Samples with compositions near the eutectic (Pb-56%Bi) were able to trap and shield transverse dipole fields in the 5000-70006 range. Samples with a higher Pb concentration had poor flux trapping abilities. A treatment which cold-worked the outer surface of the samples increased flux trapping and shielding by approximately 200-400 G. Estimates of the critical current density in the materials were derived from both the magnetization curves and the flux trapping and shielding data. Critical current levels were consistent with expressions discussed in section 3-E-1 and with measurements by other investigators. Although flux jumping limited trapping and shielding in two of the large cylindrical samples, the material was somewhat more stable than predicted by the adiabatic flux jump criteria (equation 3-10, section III-C-1).

1. Pb-Bi-N1,N2 Pb-30%Bi

Pb-Bi-N1 Annealed 25 hrs. a 150-160°C

The critical temperature of a 1.4cm diam X 5.1 cm long solid cylinder was 8.61 \pm .06 K, in agreement with previous determinations (Adler & Ng 1965, King et al. 1966). The width of the transition was approximately .4K. The magnetization curve was hysteretic; as a result values of Hc derived from the area under the curve, and values for Hc₁ derived from the location of the first deviation of the curve from linearity, represented upper limits on the values of these parameters. The area under the curve corresponded to a field of ~12256 and the first

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deviation from linearity appeared to occur at 2006, approximately 1506 before the peak in the magnetization curve. The accuracy of the magnetization curves was not sufficient to allow a precise determination of Hc₂ to be made. The values were consistent with the values Hc₁=146±5 Ge, Hc=776±240e, Hc₂=14600±1000e, κ_2 =12.9±.1, which Campbell et al. (1968) found for a nearly reversible 35%Bi sample.

A larger hollow cylinder, 5.08 cm OD X 1.45 cm ID X 10.06 cm long, trapped transverse fields of ~6206. The fields could be trapped by either cooling the sample in a field greater than or equal to 6206 and reducing the field, or cycling the applied field to well over this value and returning it to zero. The field flowed smoothly into or out of the cylinder, in response to changes in the applied field, once the threshold field difference of ~6006 (less at higher fields) was exceeded.

A disk-shaped sample 5.08 cm OD X 1.17 cm long trapped axial fields with a maximum value of 620-6406, and could shield nearly 6006 from its center. A movable Hall probe was used to obtain curves of the axial field just above the sample surface as a function of radius. Figure 5-7 shows a family of shielding curves obtained at a series of increasing values of the applied field. The figure shows the gradual penetration of the flux front into the disk at low fields, and a reduction in shielding ability at higher fields, caused by decreases in Fp and Jc. At (1), Ha=1070 Oe, the field was decreased to 910 Oe, a scan taken (2), and then the increases in applied field resumed. The effect of the field decrease was felt only above the outer portion of the sample.

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Fig. 5-7 Field profiles ;

Fig. 5-7 Field profiles just above the surface of a Pb-Bi disk (Pb-Bi-N1) after the sample was cooled to 4.2K in zero field and a uniform axial field was applied. Radius of sample = 2.54 cm.

Pb-Bi-N2 Bead-Blasted

The magnetization of the small solid cylinder (Fig. 5-8) was measured after the bead blasting treatment with .005-.010 cm glass beads. Micrographs of a polished cross section of a bead-blasted sample (samples were polished at liquid nitrogen temperature) indicated that the treatment affected a surface layer approximately 0.0005 cm thick. As a result of this treatment the apparent value of Hc₁ increased to approx. 2506, the remanent magnetization increased by 306 to approx. 2806, and the total area under the magnetization curve increased. The location of the peak in the magnetization curve also shifted to a higher field (4506) after the bead blasting treatment.

After the bead-blasting treatment the disk-shaped sample trapped fields with maximum values of up to 8256 and shielded the center of the disk from applied fields of up to ~7506. Figure 5-9 shows field profiles obtained during a shielding test. The increase in pinning near the surface of the sample caused by the bead-blasting treatment is reflected in the increase in the slope of the field profiles above the outer half of the disk over those in the untreated sample. The increased pinning also resulted in a decrease in the depth to which a small reduction in the applied field was felt (curves 1 & 2 in Fig. 5-9) compared to the untreated sample (curves 1 & 2 in Fig. 5-7). Hc₂ was greater than 9700 De in this alloy since small perturbations in the applied field were observed in field scans taken at applied fields up through the 97006 limit of the magnet.



Fig. 5-8 Magnetization of sample Pb-B1-N2.

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Fig. 5-9 Field profiles just above the surface of a Pb-Bi disk (Pb-Bi-N2) after the sample was cooled to 4.2K in zero field and a uniform axial field was applied. Radius of sample = 2.54 cm. No flux jumps were observed in any of the experiments on the 30%Bi samples. This observation is consistent with the results of the adiabatic stability theory, since the low pinning in this alloy prevented field differences of the order of Hfj(Pb-Bi) from being attained with the samples used. That is, the critical size (the size required to maintain a field difference equal to Hfj) was larger than the size of any of the samples tested.

2. Pb-Bi-L,L1,L3 Pb-49.4%Bi

Pb-Bi-L Room temperature aged three months

The flux trapping ability of a 5.31 cm OD X 1.45 cm ID X 9.83 cm long hollow cylinder was tested by cooling the sample in a transverse field and reducing the field to zero. Transverse dipole fields of up to 48006 were trapped. A flux jump was triggered after the sample trapped the highest field by the field transient that occurred as the power supply was shut down (at $Ha\simeq0$). An attempt to trap a field of 56706 failed when a flux jump occurred at Ha=3506 (Hi-Ha=53006), and an attempt to trap a 74506 field failed due to a flux jump at Ha=15806 (Hi-Ha=5870). These results indicated that the sample was slightly more stable at higher fields. The increased stability was probably due to decreased Jc-values at the higher fields. Although the flux jumps were large, they were not complete. Nonuniform fields with peak values of up to several k6 remained trapped in the sample after the jumps. The sample (when tested in fields of up to 10kG) never reached a state in which Hi changed smoothly and at approximately the same rate as Ha; the Hi vs Ha curves were generally horizontal (Hi independent of Ha) except for occasional vertical steps caused by the flux jumps. Evidently the flux jumps occurred before, or at the same time as the leading edge of the flux front reached the hole in the center of the sample. A typical experimental curve from Pb-Bi-L is shown in Fig. 5-10.

Pb-Bi-L1 Annealed 28 hrs. a 120°C

The 28 hr anneal, at just below the melting point of the alloy, increased the sample's stability slightly, allowing fields of up to 58006 to be trapped. Flux jumps were still triggered by the perturbations that occurred when the power supply was shut down, although the stability with respect to these perturbations was improved by surrounding the sample with an OFHC copper tube (0.19 cm wall) with an estimated magnetic time constant of a few hundred milliseconds at 4.2K. At low fields flux jumps still occurred before or as the flux front reached the central hole. Even with the copper tube in place, attempts to trap large fields failed via flux jumps when [Hi-Ha] exceeded approximately 6kG. In increasing fields above approximately 8kG, the sample was able to enter the full critical state and flux jumping ceased as the changes in Hi started to parallel the changes in Ha. Fig. 5-11 shows two plots of Hi vs Ha for this sample. The decrease in pinning strength and Jc at higher fields lead to decreases in [Ka-Hi]; at Ha=9700G, [Hi-Ha] was less than 3000G, while near Ha=0

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Fig. 5-11 Two curves of the transverse field inside hollow cylindrical sample Pb-Bi-L1, as a function of applied transverse field. Dotted lines indicate flux jumps.

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field differences of over 5800G were seen. The annealing treatment was expected to decrease the pinning strength and Jc-values of the material, but since Pb-Bi-L never reached the full critical state, and since its Hi vs Ha curves fell within those taken for Pb-Bi-L1, there was no way of determining the exact effect of annealing on the critical currents. However, the small increase in stability suggests that some change in Fp(h,T) was caused by annealing.

The magnetization of an annealed 1.4 cm diam X 5.1 cm long solid cylinder was measured. The hysteretic nature of the magnetization curve again prevented the determination of Hc1 and Hc. However, estimates of the parameters (α, Bo) in the Kim-model expression for the critical current density were obtained from the curves. A fit of the expression for the magnetization of a long cylinder to the experimental magnetization curve resulted in parameter values of: α=5.5-7.7×10⁷ A/cm²-G, Bo=6000-10000 G. Figure 5-12 shows the experimental curve together with the curves generated from the Kim-model expression for the magnetization. The fit is quite good for increasing applied fields but only fair for decreasing fields. The inclusion of the reversible magnetization would improve the fit somewhat by raising the calculated curve slightly. However, since the maximum value of the magnetization, Hc, is on the order of 150G, the inclusion of the reversible magnetization would have only a small effect on the parameters α and Bo, and would not appreciably improve the match in decreasing fields. The lack of agreement between the experimental and theoretical curves is most likely due to an asymetric effect, such as the presence of a surface barrier to flux entry but not to flux exit. Such a barrier

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Fig. 5-12 Magnetization of sample Pb-Bi-L1.

would cause the magnetization in decreasing fields to be less than that expected on the basis of a fit of the expression for M(H) to increasing field data, since M(H) in increasing fields would be anomalously large. The expression for M(Ha) with $Jc=\alpha/B\frac{1}{2}$ was also fitted to the data with similar success. Figure 5-13 shows a plot of $Jc(B)=\alpha/B+Bo$ obtained from the fit of the magnetization curve to the Kim-model expression. The values implied by the magnetization measurements are on the order of those generally Teported for Pb-Bi eutectic alloys, although the fit to the magnetization measurements implies a less rapid variation in Jc with B than is usually found. For example, Voigt measured peak values of $Jc=1-2X10^4$ A/cm² at H=0 and values of $1-5X10^3$ A/cm² in the 5000-9000 Oe range on his eutectic composition samples. In general, the data from Voigt, Campbell, and Litomisky & Sirovatka can be reasonably well fitted by Kim parameters of $2X10^7 < \alpha < 6X10^7$, and 500 < Bo < 1500 Oe.

Pb-Bi-L3 Bead-Blasted

The bead blasting treatment increased the flux trapping in the large hollow cylinder by 400-500G, with most of this improvement being lost after aging at room temperature for several months. The flux jumping behavior was similar to the behavior observed with Pb-Bi-L1. The full critical state was reached only at high fields.

The flux trapping ability of the large cylinder was also tested at temperatures between 1.9-2.0 K. Although higher fields, up to 6650G, could be trapped at the lower temperatures, the stability was poor. Trapped fields of 5360-6650G decayed via flux jumps triggered when the

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power supply was shut down. A field of 4500G was stably trapped. All the Hi vs Ha curves taken at the lower temperatures were horizontal (except for the vertical steps due to flux jumps); the flux jumping once more prevented the full critical state from being reached. Since even at the higher field differences there was no evidence that the flux front had reached the center of the sample before a flux jump occurred, it was not possible to estimate the change in Jc caused by the cooling.

The magnetization of the small solid cylinder after the bead-blasting treatment was similar to the magnetization of the annealed specimen. Fitting the magnetization curve to the model expression resulted in Kim-model parameters α =5.8-8.0×10⁷, Bo=6000-10000 G. Figure 5-14 shows the data and the calculated magnetization curves, and Fig. 5-15 shows the form of Jc(B) implied by the fits. The effect of the bead-blasting treatment was reflected in small increases in the pinning and critical currents. No flux jumps were observed during any of the magnetization measurements on the smaller solid cylinder.

Although the transverse geometry, together with the relatively thick walls of these samples, made the determination of critical current levels from the flux trapping data difficult, an estimate of the current level was obtained by assuming that the critical currents flowed in saddle-shaped paths throughout the cylinder wall. A computer calculation showed that the ratio of the field at the center of the hollow cylindrical sample to the density of the critical current (assumed to be independent of field) was approximately 15006/(A/cm²). A field difference on the order of 60006 implies critical currents on the

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Fig. 5-14 Magnetization of sample Pb-Bi-L3.



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Jc(B) for sample Pb-Bi-L3.

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order of 4×10^{3} A/cm², somewhat smaller than the Jc's implied by the magnetization measurements.

3. Pb-56%Bi spheres

The field profiles inside spheres of three different diameters were measured. The spheres had narrow slits through their centers enabling the field profile to be measured along a diameter perpendicular to the applied field. Although of the same nominal composition, the spheres were cast separately and so may have had slightly different compositions and structures. Table V-5 shows the maximum fields trapped and shielded by the spheres. No flux jumps were observed during testing of the spheres at 4.2K or during one run at 2K with the 4 cm sphere. Figure 5-16 shows field profiles taken during a typical shielding test on the 5.7 cm sphere.

As in the case of the large cylinder, assumptions about the form of the current distribution must be made in order to simplify the computation of the critical current levels in these samples. A simple analytical expression for the field at the center of a sphere, assuming uniform azimuthal currents flowing throughout is:

 $Bz = (\pi^2/10) JcR,$ or $Jc=(10/\pi^2) Bz/R$ where R=radius of the sphere. Table V-5 includes estimates of the critical current density derived from this expression. The differences in Jc values are evidently due to differences in composition and structure among the spheres. The critical current levels in the spheres

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Fig. 5-16 Field profiles from a shielding test of a 5.72 cmdiameter Pb-56% Bi sphere. Curves show the transverse field vs radius, along a slit through center of the sphere.

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TABLE V-5 FLUX TRAPPING AND SHIELDING WITH PB-56%Bi SPHERES

Average Jc (A/cm²):	4.4X10 ³	3.0X10 ³	2.8X10 ³
field (G):	6350	5350	7760
Maximum shielded			
field (G):	6310	5940	7040
Maximum trapped			
Diameter(cm)	2.90	3.96	5.72

were similar to those in the cylindrical 50.6%Bi samples. A slightly improved current model was developed by introducing field-dependent current densities into the model in an approximate manner. In the model the critical current level at a radius r' was determined by substituting the magnitude of the field at the center of the sphere, generated by all currents outside the radius r', into a Jc(B) expression. The trapped field at the center of the sphere could then be calculated by summing the contributions from a series of spherical shells of current. This approximate model indicated that peak Jc-values on the order of twice the values indicated in Table V-5 were required to account for the trapped fields.

4. Large Pb-Bi Trapping Tube Pb-50%Bi As-cast

12.19 cm OD X 10.16 cm ID X 33 cm long hollow cylinder

In order to test flux trapping by a large tube, a sample of the maximum practicable size was cast and tested in an 18kG dipole magnet with a 15.2 cm gap, using a simple double-walled glass helium dewar without a

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nitrogen shield. As a result, the helium surrounding the sample was rapidly boiling during the experiment, and the maximum duration of a run was 20-30 minutes. The sample was cooled in fields of 40006 and 30006. In both these cases a large flux jump prevented the full field from being trapped, although remanent fields with peak values of up to ~18006 remained trapped in the material. Upon cooling the sample in a 25006 field, a field of up to 25006 near the inner wall, and approximately 15006 along the axis, was trapped in the central 12-15 cm of the tube. The low field value along the axis is puzzling since the field at larger radii was between 2200-25006 and did not show any evidence of large scale distortions.

The computer program which calculated the field due to saddle-shaped current elements on the surface of a cylinder was applied to this sample. It showed that a uniform current density of 1.9×10^3 A/cm² flowing throughout the cylinder wall would generate a field of approximately 1500G along the axis of the sample. Since the critical current of the material is approximately twice this value, the sample should have been able to trap fields approximately twice as large if flux jumping could have been prevented. The susceptibility of this large hollow sample to flux jumping while maintaining field differences much smaller than those obtained in Pb-Bi-L1, indicates that samples may become more unstable as the ratio of the volume occupied by the trapped field to the volume of the superconductor increases (this ratio is 3.3 for this sample, compared to approximately 1 for Pb-Bi-L1). 5. Discussion

Pb-Bi is a particularly easy alloy to work with. Material of the minimum melting point (the eutectic composition melts at 125°C) has reasonably good superconducting properties (Tc~8.6K, Hc₂~15 kOe). The constituent materials, Pb and Bi, are not particularly expensive, and because of their relatively low melting points can be easily combined and cast into almost any desired shape. The properties of the alloy change only slowly with composition, so that precise control over the proportions of the starting material is not needed. Since the pinning in this material is due to precipitates of normal material (mostly Bi) and is enhanced by fine grain structure, it is not necessary (and probably is counterproductive) to maintain a high degree of purity in the starting material.

In our case we were able to trap and shield moderate fields, 5000-7000 G, with samples prepared in a very simple manner. Although annealing and room temperature aging have been shown to reduce pinning in these alloys (Campbell 1968), our annealed and aged samples were still able to trap and shield sizable fields. The quantitative results were consistent with models based on the critical state concept. Estimates of the critical currents derived from critical state models of the trapping and shielding process agreed with values derived from fittings to magnetization curves, and were generally consistent with previous measurements of Jc in this material. Flux instabilities did occur in the large cylindrical samples. However, the experiments indicated that liberal use of high purity copper around the samples, and possibly changes in geometry, could reduce the occurrence of flux jumping. The predictions of the adiabatic stability model were qualitatively correct. As expected, samples which were too small to maintain field differences of the order of Hfj were totally stable. Some of the larger samples flux jumped at field differences of the order of Hfj and exhibited some decrease in stability at lower temperatures. However, fields of nearly twice Hfj were trapped and shielded with the spherical samples without any flux jumps occuring at all, and no decrease in stability was observed when the 4 cm sphere was cooled to 2K.

D. Nb₃Sn Samples

A number of layered Nb₃Sn samples were tested. Plasma-sprayed samples with large amounts of copper incorporated into their structures (prepared by the Linde division of Union Carbide) trapped transverse quadrupole fields, and a dipole field of over 17kG. Several smaller samples of electron-beam vapor-deposited material were also tested in transverse dipole fields. Although these samples were on the edge of stability at 4.2K, they were able to trap and shield modest fields. Flux jumping data on the samples showed a decrease in stability at lower temperature, and suggested an inverse correlation between stability, and the product of the critical current with the thickness of the

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superconductor. The stabilizing effect of copper was also demonstrated with one of the smaller samples.

1. Plasma-Sprayed Samples

Nb₃Sn-D

Hollow cylinder 2.41 cm OD X 1.67 cm ID X 24.6 cm long 4 layers Nb_3Sn_i interspersed with layers of copper

This sample was tested in a transverse quadrupole field. Fig. 5-17 shows curves of $Br(\phi)$ that were obtained when the sample was cooled in a field with a gradient of approximately 2360G/cm. The field inside the tube remained essentially unchanged as the applied field was reduced to zero. Maximum values of the trapped field differed by $\leq 3\%$ from the original applied field. $Br(\phi)$ curves of the trapped field, measured at points up to 10 cm apart axially, were essentially the same (maximum values of the trapped field differed by $\leq 1\%$).

Nb3sn-H

Split cylinder 2.24 cm OD X 1.45 cm ID X 15.2 cm long 12 outer layers + 2 inner layers of Nb₃Sn interspersed with copper

The sample was tested in the 10kG and 18kG dipole magnets. The fields were measured by Hall-probes imbedded in a cylindrical glass-epoxy plug that could be rotated inside the center of the hollow sample. Probes were arranged to sense $Br(\phi)$ and $B\phi(\phi)$ near the inner wall of the sample and B at the axis of the sample. With the field direction aligned perpendicular to the line joining the two slits the cylinder easily



Fig. 5-17 Br(\$\u03c6) at r = .43 cm (upper curves), and r = .83 cm (lower curves), before and after sample Nb₃Sn-D was cooled in a 2360 G/cm transverse quadrupole field. Solid curves - T > Tc, Ha-on; Dashed curves - T=4.2K, Ha-off.

trapped the largest dipole fields generated. With the slits aligned parallel to the field the trapping ability of the cylinder was degraded.

In the perpendicular orientation the sample trapped fields of up to 10400G with a < 1% change in field from the original applied field. A 17250G field, with a <4% change in field from the original applied field of 17650G, was also trapped. In the case of a 2360G field, $Br(\phi)$ and $B\phi(\phi)$ curves taken with the external magnet on and T > Tc were virtually indistinguishable from curves taken after the sample was cooled to 4.2K and the external field ramped down to zero.

In the parallel orientation the two halves of the cylinder acted separately and some flux was able to flow through the slits either into or out of the center of the tube. The field at the center of the tube decreased by approximately 15-30% as the applied field was ramped down from the field in which the sample was cooled. The $B(\neq)$ curves showed that the trapped fields decreased as the openings were approached (Fig. 5-18). This form of trapped field is similar to the field distribution that would be generated by two pairs of critical state current loops circulating on either side of the slits.

2. Nb₃Sn Vapor Deposited Samples.

Several Nb₃Sn samples were kindly loaned by R. Howard of Stanford University. Most of them consisted of alternate layers of Nb₃Sn and Nb-Sn alloy or Nb₃Sn and Y, vapor deposited onto .64 cm diameter, 7.56 cm long Hastelloy tubes. Total thickness of the Nb₃Sn in the layered

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samples was 6-7 microns with individual layers on the order of 2000 Å thick. Howard (1976) gives a more complete description of these samples.

The shielding ability of the tubular samples was tested in transverse dipole fields. The transverse field along the axis of the tube Bx(r=0,z) was scanned at a series of applied fields, and the internal field as a function of the applied field Bx(Ha) was recorded at fixed positions inside the tubes. Differences of more than 2000G between the fields inside the tubular samples and the applied transverse fields were observed in some cases. Uniform fields of approximately 1200-1600G could generally be trapped or shielded by the tubes. Below Ha=3000-6000 Oe, field changes occurred through flux jumps. The jumps usually affected only a portion of the tube at a time and occurred in the regions of the tube supporting the largest field difference. At higher fields, flux jumps ceased to occur and the internal field, while still different from the applied field, changed continously with the applied field. At lower temperatures (1.8-2.0 K) the samples became more prone to flux jumps and the changeover to smooth behavior occurred at higher fields. Stability was improved somewhat when a sample was coated with a layer of electroplated copper.

The relation of flux jumping behavior to the adiabatic stability model is described in the discussion section below, and summarized in Table V-7. The change in stability of a given sample with changes in temperature, and differences in stability among different samples were qualitatively accounted for by this stability theory. However, the

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quantitative predictions of the theory were again off by substantial factors. The behavior of the samples was also compared with predictions based on the critical state model for thin-walled cylinders (section IV-B-2). The model adequately described the shape of typical shielding and trapping profiles, and gave critical current values of $3-4\times10^6$ A/cm² for the tubes, in agreement with values measured in axial fields (Howard). The model also indicated that most flux jumps took place at field differences of roughly 50-100% of the maximum possible for the thickness and critical current density of the material. Table V-6 summarizes the results obtained with the tubular samples, the details of which are presented below.

Nb3Sn 74-72

20 $Nb_{3}Sn$ layers interspersed with 19 Nb-6-10%Sn layers Tc=17.1 K

Fig. 5-19 shows a series of field profiles generated during a typical shielding test. The sample was able to effectively shield the interior of the tube from fields of 1500-1600 Oe and was able to trap comparable uniform fields. Flux jumps involving part or all of the tube disturbed the shielding action at higher field levels. Field differences of up to approximately 2000G were observed between Ha and the field inside the tube, although in these instances the field inside the tube was nonuniform. The dashed curves were field profiles generated with the critical current model described in section IV-B-2. (The approximate paths of the critical currents are shown in the inset). The dashed line for the Ha \approx 1500 Oe field profile was obtained with currents of 3×10^6 A/cm² flowing over the whole surface of the tube. The dashed lines for

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Sample	Maximum Uniform Field Trapped and Shielded	Ha-Hi Max	Applied fie increasing 4.2K	ld at which fields 1.9K	Last Flux Jur decreasing 4.2K	np occurs fields 1.9K	Jc (Howard)
74 -72 uncoated coated	1500-1600 G 1600-1750	2300 Oe 2100	3000-5000 Oe 2300-2400	>7000 0e >7000	2600-4500 Oe 1100-1200	>7000 0e >7000	-
74-87	1500	1800	3300-3500	13000	3100-4100	13200	3.9X10 ⁶ A/cm ²
74-92	1600	-	7000	14400	2300	10300	4.4X10 ⁶
75-94	1300-1400	1400	7300-8000	-	5500-5800	<u> </u>	4.2X10 ⁶

TABLE V-6 RESULTS OF MEASUREMENTS WITH .64 cm DIAM Nb $_3$ Sn-COATED TUBES

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Fig. 5-19 field profiles from a shielding test of a hollow .64 cmdiameter Nb_3Sn -coated tube (Nb_3Sn 74-72). Curves show the transverse component of the field along the axis of the tube for a series of increasing values of applied transverse field.

the lower applied fields were obtained either with currents of less than 3×10^{6} A/cm² flowing over the whole tube, or with currents of 3×10^{6} A/cm² (or more) flowing over limited portions of the tube. The calculated profiles along the sample axis for these two cases were indistinguishable. Figure 5-20 shows a plot of Bx vs Ha obtained by cycling the applied field up to 7000 Oe and back to zero with the Hall probe located near the center of the tube. The figure also includes a plot of the profile of the trapped field remaining after cycling the field. In increasing fields, above 2500-3000 Oe, the sample became more stable. The internal field changes started occurring smoothly rather than through flux jumps, and flux jumping ceased altogether above Ha=5000 Oe. Similarly flux jumps did not occur in decreasing fields until the applied field was below 4200-4500 Oe, and some flux flow was observed in applied fields as low as 1140 Oe. Typically between two and four flux jumps were observed in the unstable region as the field was cycled up from Ha=O, and four to six jumps were seen as it was returned to zero from above 5000G. The B vs Ha curves showed increasing differences between B and Ha at lower fields, implying higher critical currents at these fields (R. Howard et al. found that low field critical currents of similar tubes could be fitted with Jc=a/B+Bo, with Bo on the order of 5kG).

The tube was also tested at T=2K. Several features related to a general reduction in stability were observed. Flux jumps were triggered by smaller field differences and as a result occurred more often. Up through applied fields of 7000 Oe changes in the internal field occurred only through flux jumping; no smooth flow of flux into or out of the

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- Fig. 5-20 (a) Transverse field inside Nb₃Sn sample 74-72 as a function of the applied transverse field. The curve was generated with the probe located near the center of the tubular sample, and T = 4.2K.
 - (b) Trapped field along the axis of sample 74-72 at conclusion of the cycle shown in (a).



Fig. 5-21 (a) Internal field vs applied transverse field, for Nb₃Sn sample 74-72 at 2K.

(b) Trapped field remaining after cycle shown in (a). Arrow indicates position of Hall probe during cycle shown in (a).
sample was observed. Figure 5-21 shows a typical Bx vs Ha plot for sample 74-72 at 2K, as well as the trapped field left after the field was cycled. At 2K, the tube shielded out approximately 900G and the maximum observed field difference was only 1400G, despite the fact that the critical currents should have been higher at this temperature. The maximum remanent field after a field cycle was 600-700 G. As shown in Fig. 5-22, the shape of the field profiles also changed; the slopes of the Bx(z) curves at 2K were less steep than at 4.2K (Fig. 5-19).

In an effort to increase the stabilty at low fields a layer of copper was electroplated onto the tube and the sample was retested. The shielding and trapping ability at 4.2K was not significantly different after the plating, flux jumps still prevented the full critical state from being established at low fields. However, the region of stability increased somewhat, flux jumps were only observed below Ha=2400 Oe, and the number of flux jumps on a complete field cycle was about halved.

The copper-coated sample was also tested at 2K. At this temperature, the stabilizing effect of the copper was more pronounced, although the sample was still somewhat less stable than at 4.2K. At 2K, the trapping and shielding ability of the plated sample was significantly better than that of the uncoated material at the same temperature, though still no better than the latter at 4.2K. The appearance of the field profiles of the coated sample (see Fig. 5-23a) reverted to the form typical of the profiles found at 4.2K, and the B vs Ha curve took on a new appearance (Fig. 5-23b). The curve consisted of a succession of very small flux jumps with ΔHa≃ΔB≃50G (no large jumps were observed). The jumps

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Fig. 5-22 Field profiles from a shielding test of sample 74-72 at 2K. Note reduced slope of the field profiles compared to data taken at 4.2K (Fig. 5-19).

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Fig. 5-23 Field profiles, and Bx vs Ha curves for sample Nb₃Sn 74-72 with copper coating. Curves were taken at T = 2K.

occurred when field differences between the internal field and the applied field were slightly less than the corresponding differences obtained at 4.2K. The copper coating allowed larger field differences to build up, and severely limited the size of the flux jumps that did occur.

In order to determine whether end effects were responsible for the flux jumping, the plated sample was also tested in a short field generated by a pair of small superconducting coils arranged with their common axis perpendicular to the sample axis. A further decrease in flux jumping was observed, and as a result most of the critical state B vs Ha curve could be traced out without flux jumps interfering. Only one spontaneous flux jump occurred in decreasing fields, and in increasing fields the sample was on the edge of stability, flux jumping in some cases as the field scanning process was initiated. Since nearly the full critical state curve was traced out (see Fig. 5-24), it was possible to evaluate the limits of the trapping and shielding ability of the central portion of the sample. Flux first penetrated into the center of the sample at Ha=1620 Oe, and an extrapolation of the decreasing field B vs Ha curve indicated that if the spontaneous flux jump had been prevented, the sample would have trapped approximately 1700 G. These data were used to estimate the critical current of the material by applying the computer program written to calculate the field on the axis of an array of saddle-shaped current paths. The computer calculations indicated that a current density of 3.0×10^6 A/cm² in the 6.6 microns of Nb₃Sn would be needed to generate a 1600G transverse field along the axis of the tube. A simple check on this calculation

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Fig. 5-24 Bx vs Ha curve for sample 74-72 with copper coating. Applied field was generated with a pair of small superconducting coils. Hall probe was located approximately at the crossing point of the sample axis and the axis of the magnet coils. Dashed curve is derived from field profile measurements taken at applied fields equal to the fields at the locations of the dots. \$

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was obtained from the maximum shielding ability of the tube. Until the flux front penetrated through the sample wall, the sample was essentially a diamagnetic body, and the maximum field at the outer surface of the tube was approximately Ha/(1-nd). When the flux front reached the inner wall of the sample the maximum field difference maintained by the tube was 1620/(1-nd)=(1.5)1620=2430 Oe, which across a 6.6 micron wall corresponds to a current density of 2.90×10^6 A/cm².

The behavior of the sample in the short magnet was quite similar to the behavior observed in the large dipole magnet. The small observed decrease in flux jumping may have been due to the elimination of end effects, or to the different magnet power-supply and field-ramp controller. In either case the experiment showed that flux jumps could not be totally eliminated by having the ends of the sample extend out of the field. The experiment also indicated that inhomogeneities existed in the pinning strength along the length of the tube since slightly larger fields were trapped and shielded in a region closer to one end of the tube than in the central portion.

Nb₃Sn 74-87

20 Nb₃Sn layers interspersed with 19 layers of Nb-5at%Sn

The behavior of sample 74-87 was very similar to that of sample 74-72. As shown in Fig. 5-25a, 74-87 was prone to flux jumping at low fields, but became stable at fields above approximately 3500G. The hysteresis in the B vs Ha curves above 3500 Oe was as large or larger than the hysteresis for 74-72, since the critical current density in 74-87 was



Fig. 5-25 Bx vs Ha curves for sample 74-87 at 4.2K and 2K.

slightly higher than in 74-72. However, because of flux jumping the shielding and trapping ability of the sample was somewhat poorer than for 74-72. A series of field profiles from a typical shielding test at 4.2K are shown in the left portion of Fig. 5-26.

As shown in the right portion of Fig. 5-26, the flux profiles obtained at 1.9K exhibited reduced slopes compared to the 4.2K curves, similar to the change in slopes observed with sample 74-72. The 1.9K curves in the right portion of Fig. 5-26 are actually mirror images of the field profiles, and are thus taken from the same half of the tube as the 4.2K profiles shown in the left portion of the figure. The B vs Ha curve taken at 1.9K (Fig. 5-25b) showed that flux jumps were occurring every 150-300 Oe, somewhat more frequently than in 74-72(uncoated) at the same temperature. The flux jumps occurred less frequently at higher fields and ceased completely at fields above 13350 Oe.

Nb₃Sn 74-92

20 Nb₃Sn layers interspersed with 19 layers of Nb-5at%Sn

This sample had the highest value of Jc among the vapor-deposited samples tested. In most respects 74-92 behaved similarly to 74-87. Its only distinguishing feature was an asymmetry in the field limits that defined its region of stability (Fig. 5-27). Flux jumping persisted to considerably higher fields in increasing applied fields than in decreasing fields. This behavior was observed at both 4.2K and 2K.



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Fig. 5-26 Field profiles along the axis of sample 74-87 during shielding tests at 4.2 and 2K.



Fig. 5-27 Bx vs Ha curves for sample 74-92 at 4.2 and 2K.

Nb₃Sn 75-94

Alternate layers of .4 microns of Nb₃Sn and 90Å of Y Tc=16.4-17.5 K

This sample, which had the highest potential trapping ability (given by Hd=(4m/10)Jcx x=Nb₃Sn thickness) and a high Jc achieved with a modified layering structure, exhibited a further decrease in stability. At 4.2K flux jumps were more closely spaced (Δ Ha=300-600 G) than with any of the other tubes (Fig. 5-28), and persisted through applied fields of 5500-8000 Oe. The sample was stable in higher fields. Despite the high value of Hd, the maximum field that could be effectively shielded was limited to 1300-1400 G (Fig. 5-29) by flux jumping. Figure 5-30 shows an example of a trapping test in which the sample was cooled in a uniform transverse field of ~1150 Oe, and scans of the field were taken at five intermediate field values as the applied field was reduced to zero. A partial flux jump occuring between Ha=160 Oe and Ha=0, resulted in the loss of some trapped flux from parts of the tube. Similar flux trapping behavior was observed with the other vapor-deposited tubular samples.

Discussion

The experiments demonstrated that measures must be taken to stabilize Nb_3Sn samples if they are to be useful flux trapping devices at 4.2K or below. When properly stabilized they can trap and shield large fields (up to the 18kG limit of the external magnet in the case of Nb_3Sn-D). The experiments also demonstrated that higher order multipole fields could be trapped in Nb_3Sn tubes.



Fig. 5-28 Bx vs Ha curve for Nb₃Sn sample 75-74, at T=4.2K.



Fig. 5-29 Field profiles from a shielding test of a hollow .64 cm-diameter Nb_3Sn -coated tube (Nb_3Sn 75-94). Curves show the transverse component of the field along the axis of the tube for a series of increasing values of applied transverse field. Sample was originally cooled in zero field.



Fig. 5-30 Field profiles for a trapping test on sample 74-92. A large flux jump occurred at an applied field between 0 and 160 0e.

TABLE V-7 COMPARISON OF FLUX JUMPING BEHAVIOR OF Nb₃Sn SAMPLES TO PREDICTIONS BASED ON THE ADIABATIC STABILITY CRITERION

Sample	Thickness Jc of NbaSn Hd			Maximum field Calculated		for instabilities Observed	
	(β/cm^2)	(cm)	(00)	4.2K	1.9K	4.2K	1.9K
74 70	2 01106		2500	1550	0000	2600-5000	17000
74-72	3. UX 10°	0.00X10 ·	2500	1000	9900	2000-3000	//000
74-87	3.9X10*	6.33X10**	3100	3200	13500	3100-4100	13100
74-92	4.4X10 ⁶	6.33X10-4	3500	4200	15900	2300-7000	10300-14400
75-94	4.2X106	6.90X10-4	3600	4600	16700	5500-8000	-

The behavior of the smaller tubes was interesting in several respects. At both 2 and 4.2 K they exhibited flux jumps at low fields but were stable at higher fields. At 2K the unstable region extended to higher fields than at 4.2K. The samples with higher values of Hd were somewhat more unstable than the lower Hd samples. The results suggested that the change in stability was related both to changes in Hd, and to changes in a property such as the heat capacity which varies rapidly in the 2-4.2K range.

Most of the flux jumping behavior in the vapor deposited samples can be reasonably well accounted for by the simple adiabatic stability model if it is assumed that a sample will become unstable and exhibit flux jumps if a field difference greater than Hfj is present across the sample wall, and will be stable if field differences smaller than Hfj are maintained. The predicted value of Hfj(4.2) for Nb₃Sn is approximately 3700 Oe (section II-C-1). At 1.9K the large decrease in heat capacity coupled with an increase in Td reduce Hfj by a factor of 2.23 to

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Hfj(1.9)=1700 Oe (assuming C= β T³, and Td=(1-T²/To²)(To²/2T)).

Unfortunately, these estimates of Hfj are a little too high for the vapor-deposited material since at 4.2K samples with Hd(3700 0e do flux jump, and flux jump when |Ha-Hil ≤ 2300 0e. However, the general features of the theory can still be tested by choosing a more reasonable value for Hfj and comparing the resulting predictions of sample behavior to the experimental observations. A value Hfj(4.2)=1900 0e is close to the maximum values of |Ha-Hi| observed in the more stable samples (74-72, 74-87), and as will be shown, accounts fairly well for the behavior of the other samples as well. This value is not inconsistent with the predicted value of Hfj, since the demagnetization factor can account for a reduction in the theoretical value, Hfj=3700 0e, by a factor of 1.5 to Hfj=2400 0e. As an example of the method of analysis the case of sample 74-87 will be treated in detail:

74-87 has a Nb₃Sn thickness of 6.33×10^{-9} cm. In order to maintain a field difference of 19006 or more (and therefore be potentially unstable at 4.2K) critical currents of J \geq 1900/1.257X6.33X10⁻⁹ = 2.39X10⁶ A/cm² must flow. Since Jc(B) \simeq 3.9X10⁶ / (1+B/5000) *, critical currents of this magnitude only flow below 32006. At 1.9K the theory predicts that Hfj should be reduced by a factor of 2.23 to 850 0e. Therefore at 1.9K flux jumps should occur only when J > 1.12X10⁶ A/cm² and this occurs only when the ambient field is less than 135006 (Jc(B, 1.9K)=Jc(B=0, 1.9K)/(1+B/5000),

*Howard has measured $Jc(B=0) = 3.9X10^6$ and found that many of his samples follow the Kim model with Bo on the order of 5000G.

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 $Jc(T)=Jc(0)(1-T^2/To^2)$ with To=15.4K have been assumed).

These results and the results of analogous calculations for the other samples are shown in Table V-7. While the model accounts for the gross variations in stability between samples, there are significant deviations between experiment and theory, and the theory does no account for all the details of the flux jumping behavior. For instance, the model does not indicate why there are variations in the size and frequency of flux jumps among the samples (although it seems clear that the samples with higher values of Hd tend to undergo more, smaller flux jumps), and it does not explain in detail why the slope of the Bx(z)curves is reduced at 1.9K, although the reduced slope is generally in accord with the concepts of the theory. (The measured field profiles reflect the state of the sample when flux jumps are brought to a halt, and at the lower temperatures the propagation of the flux jump is more favored since the heat capacity is decreased and Jc is increased.) The effect of the copper coating in reducing the size of flux jumps at 1.9K and in allowing steeper profiles at 2K is consistent with the general features of the dynamic stability criterion.

The concept of the critical state accounted for most aspects of the observed shielding and trapping profiles. A simple model which assumed that currents flow uniformly over the surface of the tube in saddle-shaped paths gave values of a field-independent critical current density of $\sim 3X10^6$ A/cm², in good agreement with values obtained by Howard. The shape of the calculated profiles is also in fairly good agreement with the measured profiles, except near the ends of the tube, where some discrepancies arise (Fig. 5-19). If the current density

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flowing around the ends is assumed to be the same as is flowing down the sides of the tube, the field gradient near the end, dBx/dz, becomes too large and has an overshoot which is not observed in the experiments. If the current density is reduced somewhat, most of the overshoot can be eliminated, but dBx/dz is then smaller than the measured values.

E. Nb-Ti Samples

1. Bulk Nb-Ti

6.1 cm OD X .69 cm ID X 10.6 cm long As received Nb-64at%Ti

A bulk sample of Nb-64at%Ti was machined to 6.1 cm dia. and a .69 cm dia. axial hole was drilled. The as-machined sample was able to trap and shield transverse dipole fields of approximately 35006. The trapping and shielding was limited by flux jumps which occurred when differences between the applied and interior field exceeded 2500-35006. Fig. 5-31 shows examples of the field profiles obtained during a test in which the sample trapped a 26006 transverse dipole field. The magnetization-like B vs Ha curves (Fig. 5-32) showed that the field at the center of the cylinder axis changed only through flux jumps, up to applied fields of 15 kOe. However, smooth penetration of the flux front near the ends of the sample was observed between flux jumps (Fig. 5-33). The data suggests that the flux jumps may have occured as flux fronts penetrating radially inward through the 2.7 cm sample wall, reached the hollow center of the sample. The field profiles indicated that the flux



Fig. 5-31 Field profiles for a trapping test on a Nb-Ti thickwalled hollow cylinder. The sample was originally cooled in a 2600 Oe uniform transverse dipole field.



Fig. 5-32 Bx vs Ha curves for a thick-walled Nb-Ti cylinder at 4.2K and 2K. Approximate position of the Hall probe within the hollow sample is indicated by the arrow in Fig. 5-33. Ramping rate was the same for plots (a) and (b).

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Fig. 5-33 Field profiles for a shielding test of a thick-walled Nb-Ti cylinder. Curves show that a combination of smooth flux penetration and flux jumping occurs in this sample.

н 188 г jumps were usually partial, involving only one section of the sample at a time, although nearly complete flux jumps were observed in some cases.

The adiabatic stability criterion, equation 3-10, predicts a flux jump field of approximately 2000 Oe for Nb-Ti at 4.2K. This result is somewhat lower than the flux jump field observed here of 2500-3500 Oe in a transverse geometry. Although it is possible that C and Td for our sample were a little higher than the values used to predict Hfj=2000 Oe, the underestimate of Hf by the adiabatic theory was similar to the results obtained with most of the other materials tested.

At T=2K the agreement with the adiabatic theory is much poorer. No significant change in stability was observed compared to the 4.2K data. The experimental field profiles obtained at 2K closely resembled profiles obtained at 4.2K. Flux jumps occurred about as frequently at 2K as at 4.2K, and the sample was still able to maintain field differences of approximately 3000G between the applied transverse field and the central field, far in excess of the predicted value of Hfj(2)=1000 De.

2. Nb-Ti/Cu strip composite

A promising composite material containing Nb-70at%Ti and Cu in the form of a sandwich of three layers of Cu interleaved with 2 layers of Nb-Ti was obtained from Teledyne-Wah Chang Albany (Albany, Oregon). The sandwich had been roll reduced until the layers were 0.005 cm thick and metallurgically bonded to each other. The magnetic behavior of this

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.025 cm thick sheet composite material was observed in two different arrangements. In one, a rectangular or circular piece of the strip with characteristic dimensions of a few cm was oriented normal to the applied field. Movable Hall probes sensed the field just above the strip. In the second arrangement one or more strips approximately 5 cm wide by up to 30 cm long were wound into a spiral on a copper tube. The inner ends of the strips were soldered to the copper tube, and the strips were held in place by either fitting a larger copper tube snugly over the spirals or by tightly winding stainless steel wire around the spirals. The magnetic field was sensed by Hall probes mounted in a holder that could move axially inside the tube. The whole assembly was suspended in a transverse magnetic field.

In shielding tests (Fig. 5-34), the field above the center of the flat strip typically remained a few percent of the applied field for applied fields up to a certain level. When larger fields were applied, they began to penetrate to the center of the strip in a spatially nonuniform but repeatable manner. The greatest penetration was probably occurring in areas where the superconducting layer of the composite was the thinnest (micrographs of the composite showed that the thickness of a superconducting layer was less than .0005 cm in places). As the applied field was increased further, the field near the strip retained its nonuniform shape while increasing in magnitude, and generally maintained its difference (with some decreases at higher field levels) from the applied field. Subsequent decreases in the applied field initially caused changes only above the outer portions of the samples, with larger and larger fractions of the sample subsequently being affected. In some

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Fig. 5-34 Field profiles for a shielding test on Nb-Ti/Cu flatstrip composite heat treated for 4 1/2 hr. The 4.8 cm x 5.56 cm flat strip was tested in a nearly uniform dipole field directed normal to the strip. The applied field decreased somewhat at large z since the sample was located near the end of the magnet gap.

cases the orderly evolution of the critical state was interrupted by flux jumps. In experiments with the flat composite, the jumps affected the whole sample and tended to result in nearly uniform fields approximately equal to the field applied at the time of the jump (Fig. 5-35). During experiments with spiral configurations, some flux jumps occurred in the outer layers without affecting the interior field at all, while at other times massive flux jumps involving the whole assembly were observed. Samples prone to flux jumps during shielding tests at low fields often became more stable at higher fields.

The behavior in trapping tests was similar: if the initial field in which the sample was cooled was not too large, the sample trapped the field with only small decreases near the center of the strip or tube. If the field was too large, decreases and distortions of the trapped field near weak spots in the strip, or flux jumps, occurred before the applied field reached zero.

Effect of Low temperature Heat Treatments

The effect of the pinning strength on flux trapping and shielding was studied by giving a flat strip sample a series of heat treatments, and observing the resultant variations in the trapping and shielding behavior.

Maximum differences between the applied field and the field just above the center of the strip of 100-200 G were observed with the sample in the as-received state (no heat treatments after rolling to final thickness).

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Fig. 5-35 Field profiles for a shielding test of a Nb-Ti/Cu flatstrip composite (same sample as in Fig. 5-34) after being heat-treated for 61 hr.

Heating the sample for 4 1/2 to 24 hours (in vacuum) at 355°C improved the trapping and shielding ability by a factor of more than 10. After heating 4 1/2 hours, field differences of 1000-1500 G were observed (Fig. 5-34), and in one run a nonuniform field with a peak value of approximately 1800G was trapped. Only one flux jump (which occurred in a decreasing field) was observed with the sample at this stage (after 20 hours of heating).

When the sample was heated for longer than ~24 hours, it became prone to flux jumping in decreasing applied fields. After heating for 43 or more hours, flux jumping started limiting the trapped fields to lower levels (14006 after 60 hours and less than 10006 after 130 hours). Flux jumping in increasing fields began to occur as heat treatment times exceeded 60 hours (see figure 5-35). In shielding runs, samples aged 48-59 hours maintained field differences of over 35006, but were limited to 1800-2400 G by flux jumping after 130 hours of aging.

The precipitation of α -Ti (hcp) has been correlated with increases in flux pinning and critical current density values upon heat treatment of cold worked Ti-rich Nb-Ti alloys (section II-E-2). Analysis of X-ray data showed that the proportion of α -Ti in our material increased with increasing heat treatment time. The measured increase in α -Ti content paralleled increases in the trapping and shielding ability of the composite, until the material became limited by flux jumps (Fig. 5-36).

These experiments indicated that while the optimum heat treatment time (at 355°C) depends to some extent on the application, heat-treatment

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Fig. 5-36 Relation between amount of precipitated Ti and [Ha-Hi]max and heat treatment time. For heat treatment times longer than 60 hr, |Ba-Bi| was limited by flux jumping rather than by pinning. Solid circles - amount of α -Ti present; open circles - measured values of |Ba-Bilmax.

times of 20-60 hours represent a good compromise between critical current level and stability. Since most applications require the use of hollow tubular samples, further experiments were carried out on samples fabricated by winding the flat strip material around tubular forms. In addition, a flat disk-shaped sample was used to study the shape of the field profiles and the form of Jc(B) in more detail.

Spiral Configurations

A Nb-Ti/Cu strip approximately 25 by 4.8 cm was given an initial 43 hour heat treatment, and subsequently another heat treatment of 18 hours. The strip was wound into a 4-layer spiral on a 1.91 cm diameter copper tube, and tested in transverse fields.

Typical field profiles from a trapping test are shown in figure 5-37. The sample was able to maintain field differences of 7-10 kG and trap fields of nearly 10 kG. Large flux jumps occurred before higher fields could be shielded or trapped. Also shown in the figure is a curve generated with the computer program described in section IV-B-2. Analysis of such curves indicates that the critical current level was at least 2-3X10⁵ A/cm² for fields in the 0-15k6 range. (An analysis of field profiles generated with a flat strip sample with the same heat-treatment implied Jc-values of at least 1.5-2.0X10⁵ A/cm².) The additional 18 hour heat treatment caused a decrease in the stability of the sample which resulted in lower remanent fields and a lower field for the first flux jump in shielding runs.



Fig. 5-37 Field profiles obtained in decreasing applied fields along the axis of a sample consisting of a four-layer spiral of 4.78 cm wide Nb-Ti/Cu strip composite wound on a 1.91 cm diam copper tube. The sample had been heat-treated for 43 hr at 355°C. Final trapped field is compared with a computer-generated curve (dashed line) obtained by assuming Jc=2.85x10⁵ A/cm². Mineral oil was added to the space between the two concentric Cu tubes constraining the sample spiral in order to ascertain whether immobilizing the strip would improve trapping and shielding behavior. The oil increased susceptibility to flux jumps and decreased the maximum attainable field differences by 0-50% depending on the type of test. Any advantage gained by immobilization with the solidified oil was evidently more than offset by the elimination of direct thermal contact between the sample and the liquid helium.

Figures 5-38 and 5-39 show examples of the results obtained with a slightly larger sample assembly. In this case the three-layer spiral assembly, heat-treated for 48 hrs, was able to shield fields of ~9k0e, and trap slightly less. A specimen consisting of the above 4-layer spiral assembly (heated 59 hours) surrounded by this 3-layer assembly, flux jumped for the first time in a shielding test at Ha=16000 Oe (see figures 5-40,41) and retained a peak field of 120006 after cycling the field.

Several strips 4.3-4.5 cm wide by approximately 30 cm long were were used to further investigate the feasibility of building up a long sample tube from shorter spirals. The strips were heat-treated for 24 hours at 350° C in order to obtain good stability along with moderate current densities (1-2X10⁵ A/cm²). Pairs of strips were placed end to end, and were wound into two 4-layer spirals on a 4.57 cm diameter copper tube. The two 4.45 cm long spirals were butted together. Figures 5-42,5-43 show typical shielding and trapping profiles for this sample assembly. In this arrangement the spirals act nearly independently and the field

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Fig. 5-38 Field profile obtained during a trapping test of a Nb-Ti/Cu 3-layer 5.44 cm wide spiral wound on a 2.32 cm Cu tube. The sample had been heat-treated for 48 hr a 355°C. In this case the sample was cooled in a field of 70006. The dashed curve was generated by the computer model with Jc=2.55x10⁵ A/cm².



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Fig. 5-39 Field profiles obtained during a shielding test with the same sample assembly as in Fig. 5-38. The dashed curve was generated by the model with Jc=3.1x10⁵ A/cm².

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Fig. 5-40 Trapping curves generated with Nb-Ti/Cu strips wound into coaxial 3- and 4-layer tubes. Inset shows the sample assembly.



Fig. 5-41 Shielding curves generated with Nb-Ti/Cu strips wound into coaxial 3-layer and 4-layer tubes.



Fig. 5-42 Shielding curves generated with two Nb-Ti/Cu four-layer spirals butted together, total width 8.89 cm, diam = 4.57 cm, heat-treated for 24 hr d 350°C. Dashed curve generated by the model with Jc=1X10⁵ A/cm².


Fig. 5-43 Field profiles from a trapping test of the same assembly as in Fig. 5-42. Sample was initially cooled to 4.2K in a field of 6700G. The dashed curve was generated by the computer model with $Jc=1.23\times10^5$ A/cm².

easily penetrates through the gap between the two spirals. The dotted curves in figures 5-42,5-43 were generated by superimposing the calculated shielding or trapping effect of two adjacent spirals. The agreement between the curves generated from this simple model and the experimental curves is fairly good, although the field penetration due to the separation of the spirals was somewhat more severe in practice than indicated by the model calculation. The analysis also indicated that the critical current density in material heat-treated for 24 hrs. was approximately 1×10^5 A/cm², about half that of the material heated for 43 hrs. Figures 5-44,5-45 show curves for the same sample with the two spirals overlapping by 0.318 cm. The field penetration is only slightly reduced in this case. The computer generated curves also show a corresponding decrease in field penetration although the actual effect is still more severe than that indicated by the simple model. Even with the overlap, the assembly acts as two nearly independent spirals rather than one longer tube. The field penetrates through any available opening or break in the superconductor and as a result the shielding currents in each section of the assembly flow in the manner required to shield the field from that section. Therefore when building up a large device from smaller subsections sufficient overlap must be provided to attenuate the field penetrating through the breaks in the superconductor.

Flat Disk Configuration

The flat disk-shaped sample, which was also heat-treated 24 hr d 350 C, was used to study the critical current density and critical state of the



Fig. 5-44 Shielding curves generated with the two Nb-Ti/Cu four-layer spirals overlapped .318 cm. The dashed curve was generated with the model with $Jc=1.0x10^5$ A/cm².



Fig. 5-45 Field trapping profiles from the same assembly as in Fig. 5-44. The dashed curve was generated by the model with $Jc=1.15x10^5$ A/cm².

Nb-Ti/Cu material in more detail. Field profiles above the thin disk specimen were measured in applied axial fields of 0-12 kG (Figs. 4-2 through 4-6). The critical current model described in section IV-B-1 was used to analyze the field profiles. As described in detail in chapter 4, the results were found to be consistent with a somewhat modified form of the basic critical state model. The model allowed the form of Jc(B) in the 0-12 kG range to be estimated. The resultant values of Jc(B), along with relevant theoretical forms (from section II-E-2), are shown in Fig. 5-46. The model indicated that the critical currents in the 0-12 kG range were $0.5-2X10^5$ A/cm². Fitting the data to the Kim-model expression for Jc(B) resulted in $\alpha=6.0\times10^8$, Bo=2700G. The form of Jc(B) was such that the Kim-model expression with Bo held constant could only fit the data over the range 0 \leq B \leq 7000 G, since at higher fields the actual critical currents decreased more slowly than $\alpha/(B+2700)$. Fig. 5-46 shows that the theoretical forms of Jc(B) given by Hampshire & Taylor, Dew-Hughes, and Bychkov, change much more slowly at low fields than the experimentally determined form of Jc(B). Better agreement with the data is obtained with expressions from theories such as Campbell's and Kramer's with Jc \propto B⁻¹. Unfortunately there are too many unknown parameters and constants in Kramer's low field expression to compare the magnitude of Jc with the theory, and Campbell's theory is not designed to account for pinning in the type of microstructure present in the Nb-Ti sample. Brand's model, which features an expression for Jc that decreases with decreasing field, is clearly at variance with these measurements.





Stability of the Nb-Ti/Cu Composite

Comparison of flux jumping behavior with the dynamic stability model is difficult without actually measuring K and peff, which can vary considerably (section III-B,C). As shown in section III-C-2, Wilson's analysis predicts a critical thickness of .010-.080 cm, depending in the actual value of K. This thickness corresponds to a field difference of 2.5-20kG. The slightly more detailed analysis (eq. 3-19) leads to a reduction in the predicted critical thickness by a factor of:

 $(3/\pi^2 V_S/1-V_S)_{1}^{\frac{1}{2}} \approx 1/2$ in this case. The dynamic theories do indicate that the critical thickness should decrease with increasing Jc, although in their simple form they do not indicate how the flux jumping behavior will change with further increases in Jc once the stability criterion has been exceeded. The results with the Nb-Ti/Cu composite are in qualatative agreement with the dynamic theories. The composite, which contained two layers of .005 cm thick Nb-Ti was stable with low values of Jc, and became unstable as Jc was increased. The presence of the copper in this composite material enabled it to far exceed the capabilities of the bulk Nb-Ti sample.

CHAPTER VI-CONCLUSIONS

The conclusions drawn from this work fall into four major categories: (1) the actual flux trapping and shielding levels achieved with the samples tested, (2) the ability of the basic critical state model to account for the observed trapping and shielding behavior, (3) the practical limits imposed by flux jumping and how they relate to theories of instabilities, and (4) the extrapolations and predictions that can be made on the basis of the experimental results and their relations to the critical state and stability models. Each of these four categories will be briefly summarized below.

A. Trapping and Shielding Levels Attained

The experiments with the lead samples showed that even a type-I material can trap fields in a simply connected geometry. The lead samples trapped transverse dipole fields of up to 300G and transverse quadrupole fields with gradients of up to 140G/cm. Instabilities were not a problem at the low field levels present in the experiments with lead.

Experiments with niobium samples demonstrated that fields quite close to Hc_2 could be trapped in this material. A sample with a fairly small grain size and high hardness values trapped fields of up to 3400G; annealed samples with larger grains trapped somewhat less. A 5 cm

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diameter tubular sample trapped transverse quadrupole and sextupole fields, as well as sizable dipole fields. The fidelity of the fields trapped in this sample was quite good for fields of up to approximately 11006. Experiments with a fairly thick-walled tubular sample showed the potentially disrupting effect a partial Meissner effect can have on trapped multipole fields. The flux trapping in the niobium samples was in some cases limited by flux jumps and in other cases by sample size and pinning strength.

Samples of Pb-Bi alloys with compositions near the eutectic trapped and shielded fields in the 5-7 kG range. These levels were achieved in samples which were formed by making castings from a Pb-Bi melt. Since the pinning and critical current level in these alloys is not particularly high (1-10 x 10^3 A/cm^2), adequate wall thickness (on the order of cm) must be provided. Hc₂ is on the order of 15 kOe in these alloys, hence the material can potentially trap and shield larger fields if the pinning can be increased by suitable treatments, or the wall thickness increased. Instabilities rather than pinning strength limited flux trapping in some of the Pb-Bi samples. Flux jumping was particularly severe in a sample with a large ratio of trapped field volume to sample wall volume, and was totally absent in several nearly solid spherical samples.

Tubular plasma-sprayed Nb₃Sn samples, incorporating large amounts of Cu in their structures, trapped both dipole fields of up to 17kG and transverse quadrupole fields with gradients of up to 23606/cm. In both cases the trapping was limited by the capabilities of the magnet and not

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by the material, or by instabilities. Trapping and shielding of transverse dipole fields in smaller .64 cm diameter unstabilized tubular samples was limited to the 1500-2000G level by flux jumping.

Large fields were trapped with a Nb-Ti/Cu-composite material. With proper heat treatment flat strips of this material tested in fields perpendicular to the plane of the strips trapped and shielded over 30006. Multilayer tubular assemblies of the same material trapped and shielded transverse dipole fields of over 12kG. Flux jumping limited the performance of this material to some extent, although the presence of copper in the composite and the ability to vary the critical current level through heat treatment, allowed the material to exceed by far the capabilities of untreated bulk Nb-Ti.

B. Critical State Models

Essentially all the field profiles measured in the various samples could be well accounted for with a critical state model. Because of the transverse-geometries used, the model differed in some respects from the conventional form used with cylindrical samples in axial fields. However, the measurement supported the basic postulate that currents flow at levels of ±Jc in whatever portion of the material is required to shield out changes in the applied field. The only modification required in the model was in the manner in which the trapping or shielding currents were distributed in the sample. Once adapted to the other

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geometries, the critical state model was useful in estimating critical current levels in the various samples. The calculated critical currents were generally similar to critical current levels measured by others in similar materials. Pb-Bi alloys with compositions near the eutectic had critical currents in the $4-10\times10^3$ A/cm² range in fields of 0-10 kG. An annealed Nb sample had critical currents of the order of 1X10³ A/cm² and vapor-deposited Nb₃Sn samples had Jc's of 3-4x10⁶ A/cm². Cold-worked Nb-Ti samples had relatively low values of Jc in their as-received form, and achieved values of well over 10^5 A/cm² after heat treatment of 20 or more hrs at 355 °C. Flux trapping experiments with quadrupole fields indicated that a simple current model (eq. 5-1) could be used to predict the ability to trap higher order multipole fields from critical current levels based on dipole field measurements. Current models were also useful in accounting for the behavior of butted and overlapped tubular samples in transverse fields. The critical state model developed for the flat disk sample may prove to be particularly useful in determining the critical currents of relatively small samples. The method, which has the advantages of other induced current methods, allows the critical currents of small flat samples to be easily and accurately determined. The measurements can also be used to detect defective regions in superconducting material, before it is assembled into a device.

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C. Instabilities

The possibility of flux instabilities remains an important consideration in the discussion of the flux trapping and shielding process. In several cases flux trapping and shielding proved to be limited by flux jumping rather than by pinning strengths. The present experiments confirmed many of the empirical relations between flux jumping and experimental parameters. Higher rates of change of the applied field, perturbations in the applied field, decreases in temperature, decreases in thermal contact with the bath, and increases in the demagnetization factor were all observed to increase flux jumping activity in at least some cases. Much of the observed behavior was consistent with the basic theories of flux jumping, although quantitative agreement with the theories was not always good. The behavior of the vapor-deposited Nb₃Sn tubes was in guite close agreement with the adiabatic stability theory. Differences in the stability of these samples between 1.9K and 4.2K, and differences in stability among different samples, were fairly well accounted for by the theory, although the observed value of Hfj was somewhat smaller than expected. Some of the bulk samples of Pb-Bi, Nb, and Nb-Ti flux-jumped at fields on the order of Hfj, although the actual values of the field at which these samples usually flux jumped were somewhat higher than predicted by the adiabatic theory. The behavior of samples incorporating copper into their structures was also in qualitative agreement with theory. Copper coatings were able to reduce or eliminate flux jumping in several cases, and stability was observed to decrease with increasing Jc-values as predicted by the dynamic theory.

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In some other respects the stability theories failed to account for the observed behavior. The simple theories were not able to explain the detailed variations in the flux jumping behavior of the Nb₃Sn tubes. In most cases samples with higher Jc's were more unstable and experienced more (and more closely spaced) flux jumps than samples with lower Jc's. The small change in stability of Pb-Bi-L after annealing was not accounted for either, although here again the material was slightly less stable when its bulk pinning was higher. The lack of a change in the stability of the bulk Nb-Ti sample with a reduction in temperature was also at variance with the simple theory.

Although the dearth of accurate information on the relevant properties leads to difficulties in applying the various stability theories, this study showed that past empirical observations and the stability theories can be used to predict qualitative changes in stability with respect to changes in the basic experimental parameters. In most cases the theories could not be relied upon to predict detailed quantitative results. Thus the accurate a priori prediction of flux jumping behavior in new situations remains problematical, although in any given case we can predict which directions can lead to increased stability and which directions lead toward more severe flux jumping problems. Additional systematic studies of instabilities in bulk and composite materials should provide a better basis for quantitative predictions of flux jumping behavior.

D. Extrapolations

It can now be seen that with respect to overall flux trapping and shielding abilities, the critical state model, together with information on flux pinning, can be used to predict potential flux trapping and shielding behavior, but that the full trapping and shielding potential may not always be reached, due to the occurrence of flux instabilities.

The critical state models developed here can make projections of variations in shielding and trappping ability with size. Calculations show that if the wall thickness of a tubular sample is held constant, and if its length:diameter ratio remains > 1, a tube can be increased in size without a decrease in the magnitude of the peak trapped field. However, it can be expected that flux jumping will increase in severity as sample size increases, so it is unlikely that samples can be increased in size indefinitely without losses in stability. Transverse multipole field trapping has been modeled with critical state current patterns that vary with azimuthal angle as $sin(n\neq)$. A limit on the complexity and magnitude of such fields may be imposed by the maximum allowable value of $(1/r)dJc/d\neq$. However, little research has been done on the maximum possible rate of change of Jc with distance, and the experiments here have shown that moderately large (1-2k6) and complex fields can be trapped with distortions of less than a few percent.

The trapping levels already demonstrated have several potential applications in high energy physics. For example, SLAC employs large 20

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cm diameter X 100-150 cm long transverse quadrupole magnets to focus charged particle beams. In many cases the actual beam diameter is less than 10 cm, and there is adequate room to insert a dewar containing a superconducting field trappping tube. The present magnets typcally generate fields with gradients of ~1.3kG/cm, or a maximum field of ~7kG at r=5 cm. This field level is well within the capabilities of both the Nb-Ti and Nb₃Sn composite material. Estimates based on the critical current models indicate that several layers of the Nb-Ti/Cu material would be sufficient to trap such quadrupole fields. Numerous large dipole magnets are also employed at SLAC to bend the particle beams. These magnets typically have gaps of ~15 cm and generate 10-20kG fields. Properly designed flux trapping devices may also prove useful with these magnets. Both the quadrupole and dipole magnets are operated for long periods of time, and consume rather large quantities of energy. The flux trapping process can reduce this power consumption considerably.

Outside of this study, relatively little work has been done to improve overall flux trapping and shielding abilities of bulk superconductors, while good progress has been made in improving the properties of superconducting wires. Further efforts to improve flux trapping and shielding appear to have good potential for success. To date there does not appear to be a fundamental limit to the magnitude of trapped fields, below the limit Hc_2 imposes. Transverse fields of nearly Hc_2 have been trapped in Nb samples, and fields of ~ $Hc_2/2$ have been trapped in Pb-Bi. Higher fields, in the 10-20kG range, have been trapped in dense high-field materials (Nb₃Sn, and Nb-Ti) in the present study, and others have trapped and shielded fields of up to 50kG in porous sintered-Nb₃Sn.

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APPENDIX I-MAGNETIC SHIELDING BY SUPERCONDUCTING CYLINDERS

IN TRANSVERSE FIELDS, LOW-FIELD MEASUREMENTS

A. Theory

At sufficiently low fields superconductors act essentially as perfect diamagnets. The field configuration outside the material is then determined only by the shape of the superconductor, and not by the details of the critical state, the pinning forces, or other material parameters. This type of behavior should be observable with type-II materials in fields below Hc_1 , and to a certain extent in irreversible materials at fields above Hc_1 , as long as the characteristic field penetration distance Ha/(1.257 Jc) is much smaller than characteristic sample dimensions, such as the wall thickness or aperture size. At higher fields, macroscopic changes in the form of the critical state currents, and field penetration directly through the superconductor, will affect the field configuration, although in portions of the sample still screened from the high fields the field configuration remains largely determined by the diamagnetic properties of the superconductor and the configuration of the applied field.

Several authors (Bondarenko et al. 1974, Cabrera 1975, Grohmann & Hechtfischer 1977) have considered the theoretical problem of the shielding effect of a hollow diamagnetic cylinder, therefore only a sketch of the method of solution will be given here. The situation is a rather simply stated boundary value problem. Inside the hollow cylinder curl H = 0 and a scalar potential Φ m can be defined such that H = - grad

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 (Φm) and div grad $(\Phi m) = 0$. At the surfaces of the sample the normal component of H =0, and therefore $d\Phi m/dr = 0$. The general solution of div grad $(\Phi m) = 0$ in a cylindrical region with $d\Phi m/dr = 0$ at r = a =radius of the cylinder is a summation over an infinite series of terms, each one a product of a Bessel function of the radius, an exponential function of z, and a trigonometric function of the azimuthal angle. The particular properties of Bessel functions along with the symmetries of the problem allow several simplifications to be made in the series. If only transverse fields are allowed, the final expression for Hr along the axis is given by:

Hr =
$$-\sum_{n=1}^{\infty} k'_{1n} B_{1n} J_{1}'(0) \exp(-k'_{1n} z/a) \cos(\phi)$$

 $Hr = (-1.84/2 B_{11} \exp(-1.84 z/a) - 5.33/2 B_{12} \exp(-5.33 z/a) - ...)\cos(\phi)$

where k'₁n are the zeros of the Bessel functions Jn', and B₁n are coefficients to be determined. Since the second and succeeding terms fall off much more rapidly than the first term, the solution predicts that sufficiently far into the shielding tube there will be a simple exp(-1.84 z/a) falloff of the field with increasing distance into the tube. The coefficient B₁₁ which determines the value of the field at the mouth of the tube must be obtained by matching this solution to a solution valid outside the tube which embodies the factor Ha - the applied transverse field. Outside the tube the general solution will involve Fourier-Bessel integrals rather than series. As a result the matching procedure is rather complicated, and to date no published solution to this matching problem has been located, and no simple analytic solutions have been found. A solution involving numerical analysis methods and a computer appears to be complicated, but possible.

B. Results

A variety of tubular samples were tested in uniform transverse fields. The samples were first cooled to 4.2K in zero field, then scans of the transverse field along the axis of the tubes Bx(z) were taken at a series of increasing values of the applied field Ha. In general Bx(z)started decreasing outside the tube, was reduced to a value Ha/co at the tube opening ($c_0 \approx 1.5$), and dropped off in an essentially exponential fashion inward from the opening. At higher applied fields the exponential dropoff commenced at a small distance into the tube.

The rapid drop in the field near the opening of the tube, coupled with the difficulty of precisely locating the tube-opening on the field profiles, lead to rather large uncertainties in the factor c_0 . However, the further dropoff of the field with increasing z was fitted in nearly all cases by the simple exponential function Ho $exp(c_1 z/a)$, where a = radius of the opening of the sample, z = distance into the tube (measured from the opening). The parameters Ho and c_1 were determined by a computer- calculated least-squares fit of the exponential function to data points obtained from the experimental field profiles. Data points starting at the nominal location of the tube end were used with low values of Ha, and points past which the approximately exponential dropoff began were used at higher fields. Semilog plots of the field vs axial location confirmed that the nearly exponential decline of the field started very near the nominal location of the opening in most cases (Figs. A-1,2). Measurements of the slope of a straight line drawn through the points on the semilog plots gave values of the parameter c_1 similar to those obtained from the computer fit. The plots showed some deviations from straight-line fits, indicating the possible presence of higher order terms and some uncertainty in the value of c_1 . Table A-I summarizes the results of the analysis of the shielding properties of several of the tubular samples.

Figure A-1 shows an example of a semilog plot of Bx(z) vs z for field profiles generated at Ha=70 Oe, and Ha=670 Oe. The sample in this case consisted of a number of Nb-Ti/Cu composite strips wound around a 4.39 cm OD copper tube. The overall length of the superconducting portion of the tube was 13.72 cm. The curves are typical of data obtained with several other similar Nb-Ti/Cu assemblies with diameters ranging from 1.9- 5.08 cm. The largest diameter sample had a very small length/diameter ratio and it was therefore necessary to include the effect of the field penetrating from both ends of the tube. A

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	IC FUNCTION	0 0 0	navco exp(-c.	2/a/ IU	DATA ON THE
LOW FIELD	SHIELDING	OF TRANSVE	RSE FIELDS BY	SUPERCOND	UCTING TUBE
Sample Material	Inner Radius a (cm)	Length (cm)	length/diam	C 1	CO
Nb-Ti/Cu	2.54	4.83	. 95	1.6-1.75*	1.3-1.5
Nb-Ti∕Cu	2.2	13.72	3.12	1.41-1.49	1.45-1.54
Nb-Ti/Cu	1.17	5.44	2.33	1.47-1.61	1.21-1.35
Nb-Ti∕Cu	. 95	4.78	2.51	1.34-1.35	1.3-1.35

= Ha/ca evo(-o FITS OF THE EUNCTION 7 () TO DATA ON THE S

*derived from a measurement of dBz/dHa, assuming $c_0 = 1.3-1.5$

30.9

24

24

1.52-1.74

1.46-1.68

1.6-1.7

not

not

not

determined

determined

determined

measurement of dBx(center)/dHa allowed a rapid estimate of c1 to be made in this case, since at the center

Bx = 2 Ha/c₀ exp(-c₁ zo/a)

and

Bulk

Nb-Ti

Nb₃Sn

74-72

Nb₃Sn

75-94

.34

. 32

. 32

10.60

7.62

7.62

```
dBx/dHa = 2/c_0 \exp(c_1 z_0/a).
```

Figure A-2 shows semilog plots of Bx(z) vs z for field profiles generated with a smaller diameter Nb_3Sn sample at Ha=310 and 1520 0e. With the smaller diameter tubes, the field falls off very rapidly with distance into the tube, and fewer points are available for fitting the

Table A-I



t





Exponential dependence of Bx(z) for a .64 cm-diameter sample tube.

exponential function. The value of c_0 is very sensitive to variations in the position of the opening of the tube on the plots. The actual position of the opening was not known with sufficient accuracy to enable c_0 to be estimated with any confidence in the cases of the smaller diameter samples.

The measurements confirmed that transverse fields penetrated into a hollow cylinder approximately as $exp(-c_1 z/a)$. The measured values of c_1 were generally 10-15% less than the expected value of 1.8. This result may be due to the presence of higher order terms, although until values of B_1n are known, the expected magnitude of the higher order terms remains uncertain. Uncertainty in the fit and errors in the calibration of the driving mechanism of the Hall probe are the only major sources of uncertainty that could affect the calculated values of c_1 . Errors in these factors were probably no greater than several percent each.

The exponential penetration of flux into openings was also demonstrated with the niobium sample Nb-F1 (described in Section V-B-4). This sample had a rectangular, 2.58 cm x 0.74 cm, central cavity, rather than a circular opening. Measurements of the transverse field on the axis of the sample as a function of the applied field were made at three distances from the opening. The measurements were made with the applied field directed parallel and perpendicular to the long side of the cavity. The data, shown in Table A-II, are consistent with a roughly exponential falloff into the cavity, with the parameter 'a' approximately equal to the half width of the cavity parallel to the

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field direction. Considering the many complicating factors, including the fact that the opening was not circular, the factor of 1/1.5 is only approximate, and measurements were made with a field already trapped in the sample, the agreement with the theory is reasonable. In a shielding test the ratio of the change in the central field to changes in the applied field for sample Nb-F2 was .061, which is slightly larger than the value of .052 for Nb-F1.

TABLE A-II

RATE OF FIELD PENETRATION INTO THE CAVITY OF SAMPLE NB-F1

z(cm)	Ha parallel to long side of the cavity		Ha perpendicular to long side of the cavity		
	dB/dHa (measured)	exp(-1.8z/1.29)/1.5	dB/dHa (measured)	exp(-1.8z/.37)/1.5	
0.50	. 57	. 33	0.051	0.059	
1.13	. 19	. 14	0	.003	
2.72	.052	.015	0	1.1×10-6	

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