CP Violation in the Standard Model

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The violation of the CP quantum numbers has been observed in the neutral kaon system. Nonzero values for the quantities η_{+-} and η_{00}, defined by

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H_{\text{eff}}^{|\Delta S|=1} | K_L \rangle}{\langle \pi^+\pi^- | H_{\text{eff}}^{|\Delta S|=1} | K_S \rangle}, \quad \eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H_{\text{eff}}^{|\Delta S|=1} | K_L \rangle}{\langle \pi^0\pi^0 | H_{\text{eff}}^{|\Delta S|=1} | K_S \rangle}, \tag{1}$$

are an indication of CP violation. Experimentally[1] $|\eta_{+-}|$ and $|\eta_{00}|$ are about 2×10^{-3}. In this paper the predictions that the standard model makes for CP violation in the neutral kaon system are discussed.

The standard model for strong, weak and electromagnetic interactions[2] is based on the gauge group SU(3) × SU(2) × U(1). There is experimental evidence for three generations of quarks and leptons. The charged W-bosons couple to the quarks through the weak current

$$J_\mu^{(+)} = \frac{g_2}{2\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \gamma_\mu (1 - \gamma_5) U \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{2}$$

Here U is a 3×3 unitary matrix that arises from the diagonalization of the quark mass matrices. By adjusting the phases of the quark fields, U can be written in the form

$$U = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{3}$$

where $c_i \equiv \cos\vartheta_i$, $s_i \equiv \sin\vartheta_i$, $i \in \{1, 2, 3\}$ and the angles ϑ_1, ϑ_2, and ϑ_3 lie in the first quadrant. Provided $s_i \neq 0$ for $i \in \{1, 2, 3\}$ and no quarks of the same charge are degenerate in mass, the phase δ cannot be removed from U by a redefinition of quark fields and δ is a source of CP violation.[3]

The weak interaction phase δ is not the only source of CP violation in the standard model. A nonzero value for the strong interaction parameter $\bar{\vartheta}$ also violates CP. However, the stringent experimental upper limit on the electric dipole moment of the neutron,[4]

$$d_n \lesssim 10^{-25}\, e-cm \tag{4}$$

implies that[5]

$$\bar{\vartheta} \lesssim 10^{-9}\,. \tag{5}$$

Strong interaction CP violation is too small to be responsible for the CP violation observed in the neutral kaon system.

Experimental information on neutron β decay and semileptonic hyperon decays give[6] (for small s_3)

$$s_1 \simeq 0.22\,. \tag{6}$$

The weak mixing angles ϑ_2 and ϑ_3 are constrained by experimental information on B-meson decays. Since the b-quark is heavy compared with the typical strong interaction scale, the rate for semileptonic B-meson decay can be approximated by the decay rate of a free b-quark. Thus

$$\Gamma(B \to e\bar{\nu}x) \simeq \Gamma(b \to c\, e\bar{\nu}) + \Gamma(b \to u e\bar{\nu}) \tag{7}$$

where

$$\Gamma(b \to c\, e\bar{\nu}) = |U_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3} f(m_c/m_b), \tag{8a}$$

$$\Gamma(b \to u e\bar{\nu}) = |U_{ub}|^2 \frac{G_F^2 m_b^5}{192\pi^3} \tag{8b}$$

and

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x\,. \tag{9}$$

With $m_b = 4.8\ GeV$, and $m_c = 1.5\ GeV$ Eqs. (8a & b) imply

$$\Gamma(b \to c\, e\bar{\nu}) = 4.3\, |U_{cb}|^2 \times 10^{13}\ s.^{-1}\,, \tag{10a}$$

$$\Gamma(b \to u e\bar{\nu}) = 8.7\, |U_{ub}|^2 \times 10^{13}\ s.^{-1}\,. \tag{10b}$$

Comparing Eqs. (10a & b) with

$$\Gamma(b \to c\, e\bar{\nu}) = Br(b \to c)\, \frac{Br(B \to e\bar{\nu}x)}{\tau_B}\,, \tag{11a}$$

$$\Gamma(b \to u e\bar{\nu}) = Br(b \to u)\, \frac{Br(B \to e\bar{\nu}x)}{\tau_B}\,, \tag{11b}$$

using the experimental values[7]

$$Br(b \to u) < 0.04\,, \tag{12}$$

$$Br(B \to e\bar{\nu}x) = 11.6\%\,, \tag{13}$$

gives

$$|U_{cb}|^2 \simeq 3 \times 10^{-3} (10^{-12}\, s./\tau_B) \tag{14a}$$

$$|U_{ub}|^2 \simeq 5 \times 10^{-5} (10^{-12} s./\tau_B)\,. \tag{14b}$$

Recent measurements[8] of the B-meson lifetime imply τ_B is about 10^{-12} sec and constrains the angles ϑ_2 and ϑ_3 to be very small. To leading nontrivial order in the small angles Eqs. (14a & b) become

$$\{s_2^2 + s_3^2 + 2 s_2 s_3 c_\delta\} = 3 \times 10^{-3} (10^{-12} s./\tau_B) \tag{15a}$$

$$\{s_3^2\} \qquad \leq 1 \times 10^{-5} (10^{-12} s./\tau_B)\,. \tag{15b}$$

Since $-1 < c_\delta < 1$ Eq. (15a) gives

$$s_2 \lessgtr (\sqrt{3} \pm 1) 10^{-3/2} (10^{-12} s/\tau_B)^{1/2} . \tag{16}$$

The CP violation parameters η_{+-} and η_{00} depend on CP violation in the kaon decay amplitudes A_0 and A_2 defined by

$${}_{out}\langle \pi\pi(I=0) | H_{eff}^{|\Delta S|=1} | K^0 \rangle = i A_0 e^{i\delta_0} , \tag{17}$$

$${}_{out}\langle \pi\pi(I=2) | H_{eff}^{|\Delta S|=1} | K^0 \rangle = i A_2 e^{i\delta_2} . \tag{18}$$

Here δ_0 and δ_2 are the $I = 0$ and $I = 2$ $\pi\pi$ phase shifts. It is convenient to choose the phase of the kaon states so that A_0 is real. Then an imaginary part for A_2 violates CP. CP violation in the decay amplitudes A_0 and A_2 is usually described by the parameter

$$\varepsilon' \equiv \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \left[\frac{\mathrm{Im} A_2}{A_0} \right] . \tag{19}$$

The CP violation parameters η_{+-} and η_{00} also depend on CP violation in second order weak $(\sim G_F^2)$ $K^0 - \bar{K}^0$ mixing. The eigenstates K_S and K_L are given by

$$K_S = \frac{1}{[2(1 + |\varepsilon|^2)]^{1/2}} [(1+\varepsilon) K^0 - (1-\varepsilon)\bar{K}^0] \tag{20a}$$

$$K_L = \frac{1}{[2(1 + |\varepsilon|^2)]^{1/2}} [(1+\varepsilon) K^0 + (1-\varepsilon)\bar{K}^0] , \tag{20b}$$

where

$$\varepsilon = \frac{-i \,\mathrm{Im}\, M_{12} - \mathrm{Im}\Gamma_{12}/2}{[(M_{12}^* - i\Gamma_{12}^*)(M_{12} - i\Gamma_{12})]^{1/2} + \mathrm{Re}\, M_{12} - i \mathrm{Re}\Gamma_{12}/2} . \tag{21}$$

In Eq. (21)

$$M_{12} \simeq \langle \bar{K}^0 | H_{eff}^{|\Delta S|=2} | K^0 \rangle / \langle K^0 | K^0 \rangle , \tag{22a}$$

$$\Gamma_{12} \simeq 2\pi \sum_F \rho_F \langle \bar{K}^0 | H_{eff}^{|\Delta S|=1} | F \rangle \langle F | H_{eff}^{|\Delta S|=1} | K^0 \rangle / \langle K^0 | K^0 \rangle , \tag{22b}$$

where ρ_F is the density of final states. The parameter ε is a measure of CP violation in $K^0 - \bar{K}^0$ mixing.

Using the experimental values $\delta_0 \simeq 46°$, $\delta_2 \simeq -7.2°$, $A_2/A_0 = +1/20$, $-(m_{K_S} - m_{K_L}) \simeq (\Gamma_S - \Gamma_L)/2$ and $\mathrm{Im}\Gamma_{12}/\mathrm{Im}M_{12} < 1/10$ we have that

$$\varepsilon \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\mathrm{Im} M_{12}}{(m_{K_S} - m_{K_L})} , \tag{23a}$$

$$\varepsilon' \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\mathrm{Im} A_2}{A_0} . \tag{23b}$$

The CP violation parameters η_{+-} and η_{00} can be expressed in terms of ε and ε'

$$\eta_{+-} \simeq \varepsilon + \varepsilon' , \tag{24a}$$

$$\eta_{00} \simeq \varepsilon - 2\varepsilon' . \tag{24b}$$

Experimentally[1]

$$|\varepsilon| \simeq 2.3 \times 10^{-3} \tag{25}$$

$$\varepsilon'/\varepsilon \simeq (-4.6 \pm 5.3 \pm 2.4) \times 10^{-3} . \tag{26}$$

In order to compare the experimental result for ε with the prediction of the standard model we need to know the effective Hamiltonian for $K^0 - \bar{K}^0$ mixing (or at least its imaginary part) and the matrix element of this Hamiltonian between K^0 and $\bar{K}^0$ states. The effective Hamiltonian density for $K^0 - \bar{K}^0$ mixing has been computed in the leading logarithmic approximation by successively treating the W-boson, t-quark, b-quark, and c-quark as heavy and integrating them out of the theory.[9] The result, to leading nontrivial order in the large masses, is

$$
\mathrm{H}_{\text{eff}}^{|\Delta S|=2} = \frac{G_F^2}{16\pi^2} s_1^2 (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{s}_\beta \gamma^\mu (1-\gamma_5) d_\beta) m_c^2
$$

$$
\cdot \{ \eta_1 c_2^2 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})^2 + \eta_2 s_2^2 (c_1 s_2 c_3 + c_2 s_3 e^{-i\delta})^2 (m_t / m_c)^2
$$

$$
+ 2\eta_3 s_2 c_2 (c_1 c_2 c_3 - s_2 s_3 e^{-i\delta})(c_1 s_2 c_3 + c_2 s_3 e^{-i\delta}) \ln(m_t^2 / m_c^2) \} + h.c. \tag{27}
$$

Here η_1, η_2 and η_3 are QCD correction factors. They are roughly independent of the top quark mass and have the values $\eta_1 \simeq 0.7$, $\eta_2 \simeq 0.6$ and $\eta_3 \simeq 0.4$ (using $m_c = 1.5\ GeV$, $m_b = 4.8\ GeV$, $M_W \simeq 80\ GeV$, $\Lambda_{QCD} = 0.1\ GeV$ and $\alpha_s(\mu^2) = 1$).

In Eq. (27) long distance contributions to the effective Hamiltonian have been neglected since they do not contain a factor of a heavy quark mass squared. A typical long distance contribution is the time ordered product of two effective Hamiltonians for $|\Delta S| = 1$ nonleptonic kaon decays. It is likely that the $K^0 - \bar{K}^0$ matrix element of this operator dominates $\mathrm{Re}M_{12}$. The contribution of the matrix element of this operator to $\mathrm{Im}M_{12}$ involves CP violation in $|\Delta S| = 1$ amplitudes.[10] Since these amplitudes are not necessarily suppressed by the $\Delta I = 1/2$ rule, their contribution to ε is expected to be of order $20\varepsilon'$. Anticipating a modest improvement in the experimental limit on ε' I shall ignore long distance contributions to $\mathrm{Im}M_{12}$. Then using Eqs.(27) and (23a)[f1]

$$
\varepsilon \simeq \frac{-s_1^2 B G_F^2 f m_K^2 m_c^2}{16\sqrt{2}\pi^2 (m_{K_S} - m_{K_L})} s_2 s_3 s_\delta [-\eta_1 + \eta_3 \ln(m_t^2/m_c^2)
$$

$$
+ \eta_2 (m_t^2/m_c^2)(s_2^2 + s_2 s_3 c_\delta)] e^{i\pi/4} , \tag{28}
$$

to leading nontrivial order in small angles. Here f is the pion decay constant and the dimensionless parameter B is defined by[f2]

$$
\langle \bar{K}^0 | \int d^3x\, O'(x) | K^0 \rangle = B f m_K^2 , \tag{29}
$$

where

$$
O' = (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{s}_\beta \gamma^\mu (1-\gamma_5) d_\beta) . \tag{30}
$$

The magnitude of the parameter B is determined in chiral perturbation theory.[11] In chiral perturbation theory the up, down and strange quark masses are treated as small compared with the typical hadronic scale ($\sim 4\pi f$). The operator O' transforms as $(27_L, 1_R)$ under $SU(3)_L \times SU(3)_R$. The operator

$$
O = (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{u}_\beta \gamma^\mu (1-\gamma_5) u_\beta) + (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha)(\bar{u}_\beta \gamma^\mu (1-\gamma_5) d_\beta)
$$

$$
- (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{d}_\beta \gamma^\mu (1-\gamma_5) d_\beta) , \tag{31}
$$

also transforms this way. The effective Hamiltonian density for $|\Delta S| = 1$, $|\Delta I| = 3/2$ weak nonleptonic decays is

$$
\mathrm{H}_{\text{eff}} = \frac{-G_F}{2\sqrt{2}} s_1 c_1 c_3 C O . \tag{32}
$$

Here C is a factor that takes into account strong interaction corrections. With the same parameters used previously $C \simeq 0.4$. To leading order in derivatives and quark masses there is a unique operator involving the pseudo-Goldstone boson fields that transforms as $(27_L, 1_R)$. Incorporating these fields in the special unitary matrix

$f1$ There is a short distance contribution from $|\varepsilon|$ arising from the redefinition of kaon fields to make A_0 real. It has magnitude $40|\varepsilon'|\, |\mathrm{Re}M_{12}^{box}/m_{K_S} - m_{K_L}|$, where $\mathrm{Re}M_{12}^{box}$ is the short distance contribution to the $K^0 - \bar{K}^0$ mass mixing. Anticipating a modest improvement in the limit on ε' this term will be neglected.

$f2$ One particle states are normalized to $2EV$, where V is the volume of space and E is their energy. Factors of V are set to unity.

$$\Sigma = \exp\left\{\frac{2i}{f}\begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{bmatrix}\right\}, \tag{33}$$

that transforms under $SU(3)_L \times SU(3)_R$ as

$$\Sigma \to L\Sigma R^+ , \tag{34}$$

the leading operator that transforms as $(27_L, 1_R)$ is

$$-\alpha\, T^{ij}_{kl}(\Sigma\partial_\mu\Sigma^+)^k_i(\Sigma\partial^\mu\Sigma^+)^l_j . \tag{35}$$

Here T^{ij}_{kl} is a traceless symmetric tensor and α is a constant. For the operator O T^{ij}_{kl} has nonzero components

$$T^{13}_{12} = T^{31}_{12} = T^{13}_{21} = T^{31}_{21} = 1/2, \quad T^{23}_{22} = T^{32}_{22} = -1/2 , \tag{36}$$

while for O' the only nonzero component is

$$T^{33}_{22} = 1 . \tag{37}$$

Expanding the operator in Eq. (35) gives

$$\langle \bar{K}^0 | \int d^3x\, O'(x) | K^0 \rangle = 8\alpha m_K^2/f^2 , \tag{38}$$

$$\langle \pi^+\pi^0 | \int d^3x\, O(x) | K^+ \rangle = \frac{12 i\alpha}{\sqrt{2} f^3}(m_K^2 - m_\pi^2) . \tag{39}$$

Thus

$$B = 8\alpha/f^3 m_K \tag{40}$$

and

$$\langle \pi^+\pi^0 | H_{eff}^{|\Delta S|=1} | K^+ \rangle = -\frac{3iG_F}{8} B s_1 c_1 c_3 C m_K (m_K^2 - m_\pi^2) . \tag{41}$$

Therefore from the measured $K^+ \to \pi^+\pi^0$ decay rate we extract

$$|B| = 0.37 . \tag{42}$$

Here a small correction, arising from the fact that isospin violating quark masses allow the $(\Delta I = 1/2)$ weak Hamiltonian to contribute to the $K^+ \to \pi^+\pi^0$ decay amplitude, has been included. Unfortunately, the order $m_K^4 \ln m_K^2$ corrections to Eq. (38) are large indicating a breakdown of chiral perturbation theory for B.[12] Therefore B shall be kept as a free parameter.

The long B meson lifetime implies that the angles ϑ_2 and ϑ_3 are small. Then the top quark mass may have to be heavy to have Eq. (28) accommodate the measured value of ε. Therefore there may be a useful lower bound on the top quark mass as a function of the B meson lifetime.[13] Substituting the numerical values for some of the quantities that appear in Eq. (28) for ε gives[f3]

$$0.11 = (B/0.37)s_2 s_3 s_\delta\{-0.7 + 0.4\ln(m_t^2/m_c^2) + 0.6(m_t^2/m_c^2)(s_2^2 + s_2 s_3 c_\delta)\} . \tag{43}$$

Since $s_3 < s_2$ it is clear that $B s_\delta > 0$. Using the identities

$$s_2^2 s_\delta^2 \le \{s_2^2 + s_3^2 + 2 s_2 s_3 c_\delta\} \tag{44a}$$

and

$$|s_\delta c_\delta| < 1/2 , \tag{44b}$$

together with Eqs. (15) and (16) the bound for $c_\delta < 0$ is[f4]

[f3] The subtraction point dependence of B cancels against the subtraction point dependence of the η_i in the expression for ε.
[f4] The constraint $c_\delta > 0$ gives a more stringent bound on the top quark mass.

$$\left(\frac{m_t}{m_c}\right)^2 > \frac{(0.01)\cdot|0.37/B| - \sqrt{3}\times 10^{-3}(10^{-12}s./\tau_B)\{-0.7+0.4\ln(m_t^2/m_c)\}}{0.6(2+4\sqrt{3})\times 10^{-6}(10^{-12}s./\tau_B)^2}. \quad (45)$$

Note that if the second term in the numerator is negligible this give a lower bound on m_t that is linear in the B meson lifetime

$$m_t \gtrsim 70(\tau_B/10^{-12}s.)|B/0.37|^{-1/2}\,GeV. \quad (46)$$

However, the second term in the numerator is not completely negligible. For $\tau_B = 10^{-12}s$ and $|B| = 0.37$ the second term in the numerator is about 40% of the first,and the lower bound is $m_t \gtrsim 55\ GeV$. Therefore, if the prediction for B based on lowest order chiral perturbation theory is off by more than a factor of two,the term proportional to m_t^2 is not needed to account for the measured value of ε.

The expression for ε can be used to derive a lower bound on the combination of angles $|6s_2s_3s_\delta|$ as a function of the B-meson lifetime and the top quark mass.[14] For $c_\delta < 0$ it is[f5]

$$|6s_2s_3s_\delta| > \frac{0.06\,|0.37/B|}{\{-0.7+0.4\ln(m_t^2/m_c^2)+0.6(\sqrt{3}+1)^2\times 10^{-3}(10^{-12}s/\tau_B)(m_t^2/m_c^2)\}}. \quad (47)$$

Even for $|0.37/B| = 1/2$, if $\tau_B = 10^{-12}s$ and $m_t = 45\,GeV$, we have $|6s_2s_3s_\delta| \gtrsim 5\times 10^{-3}$.

The bound on $|6s_2s_3s_\delta|$ coming from ε has implications for the CP violation parameter ε'. This is a measure of CP violation in kaon decay amplitudes and arises from "Penguin-type" diagrams involving a heavy quark loop. Here the electromagnetic contribution will be included.[15] Electromagnetic "Penguin-type" diagrams give rise to local operators that transform under $SU(3)_L \times SU(3)_R$ as $(8_L, 8_R)$ and their $K \to \pi\pi$ matrix elements do not vanish in the chiral limit $m_{u,d,s} \to 0$. Therefore they can make a significant contribution to ε' despite their suppression by the factor $\alpha_{em}/\alpha_s(m_t^2)$. The effective Hamiltonian density for $|\Delta S| = 1$ weak nonleptonic decays is

$$H_{eff}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \sum_{i=2}^{8}{}' C_i Q_i + h.c. \quad (48)$$

where

$$Q_1 = (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha)(\bar{u}_\beta\gamma^\mu(1-\gamma_5)u_\beta), \quad (49a)$$

$$Q_2 = (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\beta)(\bar{u}_\beta\gamma^\mu(1-\gamma_5)u_\alpha), \quad (49b)$$

$$Q_3 = (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha)[(\bar{u}_\beta\gamma^\mu(1-\gamma_5)u_\beta) + (\bar{d}_\beta\gamma^\mu(1-\gamma_5)d_\beta) + (\bar{s}_\beta\gamma^\mu(1-\gamma_5)s_\beta)], \quad (49c)$$

$$Q_4 = (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\beta)[(\bar{u}_\beta\gamma^\mu(1-\gamma_5)u_\alpha) + (\bar{d}_\beta\gamma^\mu(1-\gamma_5)d_\alpha) + (\bar{s}_\beta\gamma^\mu(1-\gamma_5)s_\alpha)], \quad (49d)$$

$$Q_5 = (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha)[\bar{u}_\beta\gamma^\mu(1+\gamma_5)u_\beta) + (\bar{d}_\beta\gamma^\mu(1+\gamma_5)d_\beta) + (\bar{s}_\beta\gamma^\mu(1+\gamma_5)s_\beta)], \quad (49e)$$

$$Q_6 = (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\beta)[(\bar{u}_\beta\gamma^\mu(1+\gamma_5)u_\alpha) + (\bar{d}_\beta\gamma^\mu(1+\gamma_5)d_\alpha) + (\bar{s}_\beta\gamma^\mu(1+\gamma_5)s_\alpha)], \quad (49f)$$

[f5] The constraint $c_\delta > 0$ gives a more stringent bound on $(6s_2s_3s_\delta)$.

$$Q_7 = (\bar{s}_\alpha \gamma^\mu (1-\gamma_5) d_\alpha)[(\bar{u}_\beta \gamma_\mu (1+\gamma_5) u_\beta) - \tfrac{1}{2}(\bar{d}_\beta \gamma_\mu (1+\gamma_5) d_\beta) - \tfrac{1}{2}(\bar{s}_\beta \gamma_\mu (1+\gamma_5) s_\beta)] \text{ , and} \tag{49g}$$

$$Q_8 = (\bar{s}_\alpha \gamma^\mu (1-\gamma_5) d_\beta)[(\bar{u}_\beta \gamma_\mu (1+\gamma_5) u_\alpha) - \tfrac{1}{2}(\bar{d}_\beta \gamma_\mu (1+\gamma_5) d_\alpha) - \tfrac{1}{2}(\bar{s}_\beta \gamma_\mu (1+\gamma_5) s_\alpha)] . \tag{49h}$$

The prime on the sum in Eq. (48) means the Q_4 is not included since it is linearly dependent on the other operators. The operators Q_4, Q_5 and Q_6 arise from strong interaction, "Penguin-type" diagrams. They transform as $(8_L, 1_R)$ under $SU(3)_L \times SU(3)_R$ and give an imaginary part to A_0. The redefinition of kaon fields to comply with the phase convention A_0 real induces an imaginary part to A_2. The operators Q_7 and Q_8 arise from electromagnetic "Penguin-type" diagrams. They transform as $(8_L, 8_R)$ under $SU(3)_L \times SU(3)_R$ and contribute an imaginary part directly to A_2.[f6] It follows from Eqs. (23b), (48) and (49) that

$$|\varepsilon'/\varepsilon| \simeq |6 s_2 s_3 s_\delta| \cdot \left| \frac{{}_{\text{in}}\langle \pi\pi(I=0) | \sum_{i=1}^{6}{}' \tilde{C}_i \int d^3x\, Q_i(x) | K^0\rangle - 20\alpha_{em}\, {}_{\text{in}}\langle \pi\pi(I=2) | \sum_{i=7}^{8} \tilde{C}_i \int d^3x\, Q_i(x) | K^0\rangle}{0.1\ GeV^3} \right| . \tag{50}$$

Here the dependence of $\mathrm{Im}\, C_j$ on the angles and on α_{em} has been made explicit by writing

$$\mathrm{Im}\, C_j \equiv \tilde{C}_j\, c_2 s_2 s_3 s_\delta / c_1 c_3 \qquad j = 1, \cdots, 6 \tag{51a}$$

$$\mathrm{Im}\, C_j \equiv \alpha_{em} \tilde{C}_j\, c_2 s_2 s_3 s_\delta / c_1 c_3 \qquad j = 7, 8 . \tag{51b}$$

With the strong interaction parameters used previously[15,16] (and $m_t = 30\ GeV$):

$$\tilde{C}_1 = -\tilde{C}_2 = -0.03,\quad \tilde{C}_3 = 0.006,\quad \tilde{C}_5 = -0.004,\quad \tilde{C}_6 = 0.1,\quad \tilde{C}_7 = 0.007,\quad \tilde{C}_8 = 0.1 .$$

The operators $Q_1, \cdots, Q_6$ transform under chiral $SU(3)_L \times SU(3)_R$ as $(8_L, 1_R)$, and $(27_L, 1_R)$. It follows that their $K \to \pi\pi$ matrix elements are proportional to the strange quark current algebra mass and are expected to be of order $0.1\ GeV^3$. The operators Q_7 and Q_8 transform as $(8_L, 8_R)$ under chiral $SU(3)_L \times SU(3)_R$. Their $K \to \pi\pi$ matrix elements do not vanish as the current algebra masses go to zero and are expected to be of order $1\ GeV^3$.

It has been suggested[17] that the $(V-A) \times (V+A)$ chiral structure of Q_5 and Q_6 leads to an enhancement of their $K \to \pi\pi$ matrix elements by an order of magnitude or more[f7] and that this is responsible for the $\Delta I = 1/2$ rule. This is supported by bag model calculations[18] which predict that the magnitude of the $K \to \pi\pi(I=0)$ matrix element of $\int d^3x\, Q_6(x)$ is about $1\ GeV^3$. If this is the case then[19]

$$|\varepsilon'/\varepsilon| \simeq |6 s_2 s_3 s_\delta| \left| \frac{\langle \pi\pi(I=0) | \int d^3x\, Q_6(x) | K^0\rangle}{1\ GeV^3} \right| |\tilde{C}_6 / 0.1| . \tag{52}$$

Given the bound on $|6 s_2 s_3 s_\delta|$ from ε, the value of $|\varepsilon'|$ should be near the present experimental limit. If Q_6 is responsible for the $\Delta I = 1/2$ rule then ε'/ε has the same sign as s_δ provided the sign of $\mathrm{Re}\, C_6$ computed in the leading logarithmic approximation is reliable. The sign of s_δ is positive if B has the same sign as in the "vacuum insertion" approximation.[f8]

f6 The $(8_L, 8_R)$ piece of the effective Hamiltonian also gives an imaginary part to A_0. However this is suppressed by a factor of $|A_2/A_0|$ in ε', while the imaginary part of A_2 is not.

f7 The value of the Wilson coefficient $\mathrm{Re}\, C_6$ is very uncertain and therefore one cannot say precisely how much enhancement of the matrix elements of Q_6 is needed for the local $(V-A) \times (V+A)$ operators to be the source of the $\Delta I = 1/2$ rule.

f8 Here long distance contributions to ε are neglected.

An improvement in the experimental limit on ε'/ε could either rule out the order of magnitude or more enhancement of the matrix elements of $\int d^3x\, Q_6(x)$ needed to explain the $\Delta I = 1/2$ rule, or perhaps signal a failure of the standard six quark model for CP violation. In order for ε'/ε to serve as a quantitative test of the standard model it is necessary to have reliable computations of hadronic matrix elements of local 4-quark operators. It is not inconceivable that lattice Monte Carlo methods could provide these.[20] If chiral perturbation theory is valid it is not necessary to compute the matrix element $\langle \pi\pi(I=0) | \int d^3x\, Q_6(x) | K^0\rangle$. It is sufficient to calculate both $\langle \pi | \int d^3x\, Q_6(x) | K^0\rangle$ and $\langle O | \int d^3x\, Q_6(x) | K^0\rangle$, where these matrix elements conserve 3-momentum but not energy. Because energy is not conserved, operators that are total derivatives must be included in the chiral Lagrangian. To leading order in derivatives and masses there are only two operators involving Σ that transform under $SU(3)_L \times SU(3)_R$ as $(8_L, 1_R)$ and are invariant under (CP)$\bullet D$, where D is a discrete transformation that interchanges strange and down quarks. Matrix elements of Q_6 between kaon and pion states can be replaced by matrix elements of[21]

$$a\, Tr(O\partial_\mu \Sigma\, \partial^\mu \Sigma^+) + b\{Tr(mO\Sigma) + Tr(Om\Sigma^+)\}\,, \tag{53}$$

to leading order in chiral perturbation theory. Here

$$m = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix}, \tag{54}$$

is the quark mass matrix,

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{55}$$

and a and b are constants. The term proportional to b is a total derivative (provided the masses m_u, m_d and m_s are not all equal) and doesn't contribute to $\langle \pi\pi(I=0) | \int d^3x Q_6(x) | K^0\rangle$. However

$$\langle O | \int d^3x\, Q_6(x) | K^0\rangle = \frac{2i}{f} b\{m_d - m_s\} \tag{56a}$$

and

$$\langle \pi^0 | \int d^3x\, Q_6(x) | K^0\rangle = \frac{4a}{f^2} m_K m_\pi - \frac{2b}{f^2}(m_d + m_s)\,. \tag{56b}$$

Therefore a computation of the two matrix elements above allows one to extract a and hence predict the $K^0 \to \pi\pi(I = 0)$ matrix element of $\int d^3x\, Q_6(x)$.

In summary, the standard model makes predictions for ε and ε' that, at the present time, are compatible with experiment. All the CP violation observed so far is consistent with it arising from $K^0 - \bar{K}^0$ mixing. Since this is a second order weak ($\sim G_F^2$) process, very short distance physics beyond the standard model could be responsible for most of the CP violation observed there. Measurement of a nonzero value for ε'/ε is very important, since it would eliminate this possibility. Reliable computations of the matrix elements $\langle \pi\pi(I=0) | \int d^3x\, Q_6(x) | K^0\rangle$ and $\langle \bar{K}^0 | \int d^3x O'(x) | K^0\rangle$ would make ε and ε' sensitive tests of the standard six quark model for CP violation.

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