# A Measurement of the Time Dependence of $B_{d}-\bar{B}_{d}$ Mixing with Kaon Tagging 

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# A MEASUREMENT OF THE TIME DEPENDENCE OF $B_{D}-\bar{B}_{D}$ MIXING WITH KAON TAGGING 

A Dissertation Presented

by

## JODI L. WITTLIN

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# To my Grandfather Jacob 

whose memory is a blessing.

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ABSTRACT<br>A Measurement of Time Dependence of $B_{d}-\bar{B}_{d}$ Mixing with Kaon Tagging<br>September 2001<br>Jodi L. Wittlin, B.S., N. C. State University<br>M.S., University of Massachusetts Amherst<br>Ph.D., University of Massachusetts Amherst<br>Directed by: Professor Richard R. Kofler

The time dependence of $B_{d}-\bar{B}_{d}$ mixing has been measured in $b \bar{b}$ events containing one or more kaons at the SLD experiment at the Stanford Linear Accelerator Center. A simultaneous measurement of the "right sign production fraction" of kaons from $B_{d}$ decays has also been made. The initial state $B$ hadron flavor was determined using the large forward-backward asymetry provided by the polarized electron beam of the SLC in combination with a jet charge technique and information from the opposite hemisphere. From a sample of $400,000 Z^{0}$ events collected by the SLD experiment at SLC from 1996 to 1998, the kaon right sign production
fraction has been measured to be $0.797 \pm 0.022$ and the mass difference between the two $B_{d}$ eigenstates has been measured to be $\Delta m_{d}=0.503 \pm 0.028 \pm 0.020 \mathrm{ps}^{-1}$.

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## C H A P T ER 1

## INTRODUCTION

Since humankind has been able to ask, we have wondered "What is the world made of? Why is it made in such a way? How does it work?" The Standard Model is physics' most recent, and most successful, theory to answer these age-old questions. This theory, which describes fundamental particles and their interactions, has been verified by experimentalists to a remarkable degree of accuracy. While it is not thought to be the "ultimate theory", it is very good at describing all of the experimental information currently collected by particle physicists.

Of the 18 fundamental parameters in the Standard Model, there is only one which has not been measured at all (the mass of the Higgs Boson), and one aspect which has been poorly measured: CP (Charge Parity) Violation. This dissertation is a contribution to the measurement of CP Violation. In the Standard Model, CP Violation is described in terms of the $3 \times 3$ Cabibbo-Kobayashi-Maskawa mixing matrix, which relates the quark mass eigenstates to the quark weak eigenstates. Two of the important CKM Matrix elements can be measured through the study of $B$ Mixing, and when combined with other measurements, provide a unique opportunity to test the Standard Model.
$B_{d}$ mixing is not a "new" measurement: it has been measured many times and at many experiments, including 3 measurements using the 1993-1995 data set of the SLD experiment at the Stanford Linear Collider (SLC). The measurement in this dissertation however, is still a useful and interesting contribution to the body of knowledge of $B_{d}$ mixing. It is the only measurement to be performed on the larger 1996-1998 SLD data set. The SLD is the only particle collider experiment to have run with polarized $e^{-}$as one of its colliding beams, and the uniqueness of the machine and how that affects analysis techniques will be presented in this dissertation. The SLD is itself a unique detector, and has the world's only CCD vertex detector. This is also the only measurement in the world to use exclusively kaons as a final state tag, which has its own unique power and challenges.

The method of analysis is fairly straightforward. We wish to measure, as a function of time, how often a $b$ quark decays as a $\bar{b}$ quark (or vice versa.) First, we examine the SLD data set for events with hadrons, which are produce in approximately $11 \%$ of $Z^{0}$ collisions. We then use powerful techniques to choose only those events which are like to have $b$ quarks produced from the $Z^{0}$, and therefore contain $B$ hadrons. For a time dependent mixing analysis, we must then determine the initial quark flavor of the $B$ hadron, the flavor of the quark at the decay point of the $B$, and the time it took from creation to decay. To determine the initial flavor, we use the large forward-backward asymmetry provided by the polarized electrons of the SLC and combine it with information from hemisphere opposite that of analysis interest. Kaons which are produced from the $B$ decay are used to
determine the final state with high accuracy. SLD's powerful vertex detector and unique reconstruction techniques are used to reconstruct the point at which the $B$ decays.

This thesis is organized in the following manner. In Chapter 2, we introduce the Standard Model, with an emphasis on CP Violation and the relevance of $B$ mixing. Chapter 3 presents an overview of the SLC accelerator and SLD Detector. Chapter 4 goes into some detail about kaon identification with the Cherenkov Ring Imaging Detector (CRID), which is of fundamental importance to this analysis, and presents investigations of the efficiency and purity of CRID kaon identification. Chapter 5 discusses hadronic and $B$ event selection, and provides the details of the analysis techniques, including initial and final state tagging, vertexing and track attachment. Chapter 6 contains a detailed description of the unbinned maximum log-likelihood fit, results, cross-checks from both data and Monte Carlo, and error analysis.

## CHAPTER 2

## OVERVIEW OF THE STANDARD MODEL

The Standard Model, which is based on Quantum Field Theory, provides a description of the elementary particles and their weak, electromagnetic, and strong interactions. One can find greater detail about the Standard Model in References [1] and [2]. While many precise tests of the Standard Model have been made [3], precision measurements of CP Violation are yet to be done, and the discovery of the Higgs Boson remains elusive. This chapter will give an overview of the Standard Model and will emphasize how a measurement of $\Delta m_{d}$ is an interesting contribution to the exploration of CP Violation.

### 2.1 Electroweak Interactions

Electromagnetic and weak interactions were unified into a single interaction, described by $S U(2)_{L} \otimes U(1)$ gauge symmetry, by Glashow, Salam, and Weinberg, who won the 1979 Nobel Prize in Physics for their ground-breaking work. Leptons and quarks, which each have spin $1 / 2$, make up the elementary matter particles
in the Standard Model, while gauge bosons, with spin 1, are force propagators. The fundamental fermions can be grouped into three families, shown in Table 1. The leptons and quarks are shown arranged in left-handed weak isospin doublets and right-handed weak isospin singlets. The right-handed isospins are singlets because right-handed neutrinos have never been observed. Each doublet has total weak isospin $T=1 / 2$. The upper member of the doublet is assigned $T_{3}=+1 / 2$ while the lower is assigned $T_{3}=-1 / 2$. Each fermion also has a value of weak hypercharge, $Y$, as defined by

$$
\begin{equation*}
Q=T_{3}+\frac{1}{2} Y \tag{2.1}
\end{equation*}
$$

where $Q$ is the charge of the fermion.

Table 1: The fermions of the Standard Model.

$$
\begin{array}{|ccc|}
\binom{\nu_{e}}{e}_{L} & \binom{\nu_{\mu}}{\mu}_{L} & \binom{\nu_{\tau}}{\tau}_{L} \\
\binom{u}{d}_{L} & \binom{c}{s}_{L} & \binom{t}{b}_{L} \\
e_{R} & \mu_{R} & \tau_{R} \\
u_{R} & c_{R} & t_{R} \\
d_{R} & s_{R} & b_{R} \\
\hline
\end{array}
$$

Quarks are the only particles known to interact with all forces; neutral leptons interact only via the weak interaction, and charged leptons interact via the weak and electromagnetic interactions. The gauge bosons which mediate these interactions are listed in Table 2. Photons mediate the electromagnetic force; gluons transmit the strong force, and $W^{ \pm}$and $Z^{0}$ bosons mediate the weak force.

Table 2: The gauge bosons of the Standard Model.

| Boson | Charge $(e)$ | Mass $\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ | Force Mediated |
| :---: | :---: | :---: | :---: |
| $\gamma$ | 0 | 0 | Electromagnetic |
| $g$ | 0 | 0 | Strong |
| $W^{ \pm}$ | $\pm 1$ | $80.37 \pm 0.023$ | Weak |
| $Z^{0}$ | 0 | $91.187 \pm 0.002$ | Weak |

The Standard Model introduces gauge bosons by requiring local gauge invariance, similiar to the $A^{\mu}$ fields of electromagnetism. One can derive Quantum Electrodynamics (QED [1,2]) from the interaction of spin $1 / 2$ particles:

$$
\begin{equation*}
-i e j_{\mu}^{e m} A^{\mu}=-i e\left(\bar{\psi} \gamma_{\mu} Q \psi\right) A^{\mu} \tag{2.2}
\end{equation*}
$$

where $j_{\mu}^{e m}$ is the electromagnetic current, which couples to the $A^{\mu}$ field, $e$ is the electron charge, $Q$ is the charge operator, $\gamma_{\mu}$ are the Dirac matrices, and $\psi$ is the wavefunction.

Weak interactions are included in the Standard Model by introducing a massless $U(1)$ isosinglet $B_{\mu}$ and a massless $S(2)$ isotriplet $\mathbf{W}_{\mu}$, which couple with strengths $g^{\prime}$ and $g$ to the hypercharge and isospin currents respectively. After requiring local gauge invariance, the resulting interaction term is:

$$
\begin{equation*}
-i g\left(J^{i}\right)^{\mu} W_{\mu}^{i}-i \frac{g^{\prime}}{2}\left(j^{Y}\right)^{\mu} B_{\mu} \tag{2.3}
\end{equation*}
$$

where $\mathbf{J}_{\mu}$ is the weak isospin current and $j^{Y}$ is the weak hypercharge current. Thus,
the physical gauge bosons are defined as combinations of the gauge fields:

$$
\begin{align*}
A_{\mu} & =B_{\mu} \cos \theta_{W}+W_{\mu}^{3} \sin \theta_{W} \\
Z_{\mu} & =-B_{\mu} \sin \theta_{W}+W_{\mu}^{3} \cos \theta_{W} \\
W_{\mu}^{ \pm} & =\frac{1}{\sqrt{2}}\left(W_{\mu}^{1} \mp i W_{\mu}^{2}\right) \tag{2.4}
\end{align*}
$$

where $\theta_{W}$ is the experimentally measured electroweak mixing angle. The electroweak neutral current is therefore a sum of electromagnetic and weak neutral terms:

$$
\begin{align*}
& -i g J_{\mu}^{3}\left(W^{3}\right)^{\mu}-i \frac{g \prime}{2} j_{\mu}^{Y} B^{\mu}= \\
& \quad-i\left(g \sin \theta_{W} J_{\mu}^{3}+g \prime \cos \theta_{W} \frac{j_{\mu}^{Y}}{2}\right) A^{\mu}-i\left(g \cos \theta_{W} J_{\mu}^{3}-g \prime \sin \theta_{W} \frac{j_{\mu}^{Y}}{2}\right) Z^{\mu} \tag{2.5}
\end{align*}
$$

As the coefficient on the $A_{\mu}$ term must be $i Q$, by applying equation 2.1, we obtain:

$$
\begin{equation*}
\theta_{W} \equiv \tan ^{-1}\left(\frac{g^{\prime}}{g}\right) \tag{2.6}
\end{equation*}
$$

It is now possible to construct expressions for the charged and neutral electroweak currents:

$$
\begin{align*}
J_{L}^{ \pm \mu} & =\sqrt{2} \bar{\psi} \gamma^{\mu} T_{L}^{ \pm} \psi \\
J_{Z}^{\mu} & =\bar{\psi} \gamma^{\mu}\left[T_{3 L}-Q \sin ^{2} \theta_{W}\right] \psi \tag{2.7}
\end{align*}
$$

where $T^{ \pm}=\left(T^{1} \pm i T^{2}\right) / \sqrt{2} . \psi$ consists of left-handed isospin doublets and righthanded singlets for each of the quark and lepton generations.

For the process $Z^{0} \rightarrow f \bar{f}$ the weak neutral current interaction is given by:

$$
\begin{equation*}
-i \frac{g}{\cos \theta_{W}} J_{\mu}^{N C} Z^{\mu}=-i \frac{g}{\cos \theta_{W}} \bar{\psi}_{f} \gamma^{\mu}\left(\frac{1}{2}\left(1-\gamma^{5}\right) T^{3}-Q \sin ^{2} \theta_{W}\right) \psi_{f} Z_{\mu} \tag{2.8}
\end{equation*}
$$

with $\gamma^{5}=i \gamma^{0} \gamma^{1} \gamma^{2} \gamma^{3}$. Using the expression for the vertex factor for this process:

$$
\begin{equation*}
-i \frac{g}{\cos \theta_{W}} \frac{\gamma^{\mu}}{2}\left(v_{f}-a_{f} \gamma^{5}\right) \tag{2.9}
\end{equation*}
$$

we obtain $v_{f}$ and $a_{f}$, the vector and axial-vector couplings, which are given in the Standard Model by:

$$
\begin{align*}
a_{f} & =T_{f}^{3} \\
v_{f} & =T_{f}^{3}-2 Q_{f} \sin ^{2} \theta_{W} \tag{2.10}
\end{align*}
$$

The Standard Model values for $v_{f}$ and $a_{f}$ for the fermions are given in Table 3.

Table 3 : The neutral vector and axial-vector couplings of the fundamental fermions to the $Z^{0}$.

| Fermion | $a_{f}$ | $v_{f}$ |
| :---: | :---: | :---: |
| $\nu_{e}, \nu_{\mu}, \nu_{\tau}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ |
| $e, \mu, \tau$ | $-\frac{1}{2}$ | $-\frac{1}{2}+2 \sin ^{2} \theta_{W}$ |
| $u, c, t$ | $+\frac{1}{2}$ | $+\frac{1}{2}-\frac{4}{3} \sin ^{2} \theta_{W}$ |
| $d, s, b$ | $-\frac{1}{2}$ | $-\frac{1}{2}-\frac{2}{3} \sin ^{2} \theta_{W}$ |

### 2.1.1 The Higgs Mechanism, Mass Generation, and Symmetry Breaking

In the previous section, the fermions and gauge bosons are all massless. Simply inserting masses would break the gauge invariance of electroweak theory. Instead, this lack of physical reality is partially resolved by the introduction of the Higgs Mechanism.

A weak isospin doublet of a complex scalar field, $\Phi$, which has hypercharge $Y=1 / 2$, is introduced [4]:

$$
\begin{equation*}
\Phi=\binom{\phi^{+}}{\phi^{0}} \tag{2.11}
\end{equation*}
$$

The electroweak Lagrangian then acquires additional terms:

$$
\begin{equation*}
\mathcal{L}_{\Phi}=\left|D_{\mu} \Phi\right|-V\left(|\Phi|^{2} \mid\right)+\mathcal{L}_{\Phi}^{F} \tag{2.12}
\end{equation*}
$$

where $D_{\mu}$ is the electroweak covariant derivative and $\mathcal{L}_{\Phi}^{F}$ is the Yukawa coupling of the fermions to the doublet $\Phi$. The most general renormalizable form for the scalar potential $V$ is:

$$
\begin{equation*}
V=\mu^{2}|\Phi|^{2}+\lambda|\Phi|^{4}=\frac{v^{2}}{2} . \tag{2.13}
\end{equation*}
$$

By minimizing the potential $V$ with respect to $|\Phi|^{2}$, we find the ground state of $|\Phi|^{2}$ occurs at:

$$
\begin{equation*}
|\Phi|^{2}=-\frac{\mu^{2}}{2 \lambda} \tag{2.14}
\end{equation*}
$$

and $\mu^{2}<0$. Thus, the vacuum expectation value of $|\Phi|^{2}$ is non-zero, and there is a continuous set of allowed ground state values. When one of these sets is chosen, the $S U(2) \otimes U(1)$ symmetry is "spontaneously broken" by selecting a preferred direction in weak isospin plus hypercharge space.

It is appropriate to re-express the $\Phi$ doublet relative to its ground state:

$$
\begin{equation*}
\Phi(x)=\exp \left(\frac{i \xi(x) \cdot \tau}{2 v}\right)\binom{0}{(v+H(x)) / \sqrt{2}} \tag{2.15}
\end{equation*}
$$

The two complex fields $\phi^{+}$and $\phi^{0}$ have now been replaced by the four real fields $\xi_{1,2,3}$ and $H$, all of which have a zero vacuum expectation value. By applying an $S U(2)$ gauge transformation with $\alpha(x)=\xi(x) / v$ the phase factor can be removed, and $\Phi(x)$ is expressed in terms of the real scalar field $H$, known as the Higgs field.

The Lagrangian can now be expressed in terms of $H$ :

$$
\begin{equation*}
\mathcal{L}_{\Phi}=\frac{1}{2}(\partial H)^{2}+\frac{1}{4} g^{2} W^{+} W^{-}(v+H)^{2}+\frac{1}{8} g_{Z}^{2} Z Z(v+H)^{2}-V\left[\frac{1}{2}(v+H)^{2}\right]+\mathcal{L}_{\Phi}^{F} \tag{2.16}
\end{equation*}
$$

The $v^{2}$ terms are the $W$ and $Z$ mass terms. While the photon field $A$ has remained massless, the $Z$ and $W^{ \pm}$fields have acquired masses:

$$
\begin{align*}
M_{W} & =\frac{1}{2} g v \\
M_{Z} & =\frac{1}{2} g_{Z} v=\frac{M_{W}}{\cos \theta_{W}} . \tag{2.17}
\end{align*}
$$

While the bosons have now acquired mass, the fermions have not. $\mathcal{L}_{\Phi}^{F}$ describes the Yukawa couplings of the Higgs field $H$ to the fermion fields. The fundamental weak eigenstates for the unbroken gauge theory are assumed to be:

$$
\begin{equation*}
D_{j L} \equiv\binom{u_{j}}{d_{j}}_{L}, \quad u_{j R}, \quad d_{j R} \tag{2.18}
\end{equation*}
$$

where $D_{j L}$ is an $S U(2)$ doublet with $Y=\frac{1}{3}$ and $u_{j R}$ and $d_{j R}$ are $S U(2)_{L}$ singlets with $Y=\frac{4}{3},-\frac{2}{3}$ respectively, representing the up- and down-type quarks. Three generations are assumed, and $j$ is the generation index. The most general $S U(2)_{L} \otimes$ $U(1)$ gauge invariant Yukawa interaction is:

$$
\begin{equation*}
\mathcal{L}=-\sum_{i=1}^{3} \sum_{j=1}^{3}\left[\tilde{G}_{i j} \bar{u}_{i R}\left(\tilde{\Phi}^{\dagger} D_{j L}\right)+G_{i j} \bar{d}_{i R}\left(\Phi^{\dagger} D_{j L}\right)\right]+\text { h.c. } \tag{2.19}
\end{equation*}
$$

where "h. c." stands for the hermitian conjugate, $G_{i j}$ and $\tilde{G}_{i j}$ represent the 18 different coupling constants between quarks and $\tilde{\Phi}=i \tau_{2} \Phi^{*}$.

Using the vacuum expectation values of $\Phi$ and $\tilde{\Phi}$, the quark mass terms which are produced by the Yukawa couplings can be expressed as:

$$
\begin{align*}
& \overline{\left(u_{1}, u_{2}, u_{3}\right)_{R}} \mathcal{M}^{u}\left(\begin{array}{c}
u_{1} \\
u_{2} \\
u_{3}
\end{array}\right)_{L}+h . c . \\
& \overline{\left(d_{1}, d_{2}, d_{3}\right)_{R}} \mathcal{M}^{d}\left(\begin{array}{c}
d_{1} \\
d_{2} \\
d_{3}
\end{array}\right)_{L}+\text { h.c. } \tag{2.20}
\end{align*}
$$

where $\mathcal{M}_{i j}^{u}=\frac{v}{\sqrt{2}} \tilde{G}_{i j}$ and $\mathcal{M}_{i j}^{d}=\frac{v}{\sqrt{2}} G_{i j}$ are the quark weak eigenstate mass matrices. To obtain the quark mass eigenstate matrices, the complex electroweak matrices are transformed to diagonal matrices:

$$
\begin{align*}
U_{R}^{-1} \mathcal{M}^{u} U_{L} & =\left(\begin{array}{ccc}
m_{u} & 0 & 0 \\
0 & m_{c} & 0 \\
0 & 0 & m_{t}
\end{array}\right) \\
D_{R}^{-1} \mathcal{M}^{d} D_{L} & =\left(\begin{array}{ccc}
m_{d} & 0 & 0 \\
0 & m_{s} & 0 \\
0 & 0 & m_{b}
\end{array}\right) \tag{2.21}
\end{align*}
$$

where the diagonal entries are now the quark masses, and $U_{R}, U_{L}, D_{R}$, and $D_{L}$ are
unitary matrices defined such that:

$$
\left(\begin{array}{c}
u_{1}  \tag{2.22}\\
u_{2} \\
u_{3}
\end{array}\right)=U_{L, R}\left(\begin{array}{c}
u \\
c \\
t
\end{array}\right)_{L, R},\left(\begin{array}{c}
d_{1} \\
d_{2} \\
d_{3}
\end{array}\right)=D_{L, R}\left(\begin{array}{c}
d \\
s \\
b
\end{array}\right)_{L, R}
$$

One notes the up (down)-type weak eigenstates are linear superpositions of the up (down)-type quark mass eigenstates.

Previously, we have seen the charged weak current couples the upper and lower members of the $S U(2)$ doublets due to the off-diagonal elements in $T^{ \pm}$. The charged current can also be written as:

$$
\overline{\left(u_{1}, u_{2}, u_{3}\right)_{L}} \gamma_{\mu}\left(\begin{array}{c}
d_{1}  \tag{2.23}\\
d_{2} \\
d_{3}
\end{array}\right)_{L}=\overline{(u, c, t)_{L}} U_{L}^{\dagger} D_{L} \gamma_{\mu}\left(\begin{array}{c}
d \\
s \\
b
\end{array}\right)_{L} .
$$

This allows for generation mixing of mass eigenstates, which can be described by the matrix $V$, defined by $V \equiv U_{L}^{\dagger} D_{L}$ such that

$$
U_{L}^{\dagger} D_{L}\left(\begin{array}{c}
d  \tag{2.24}\\
s \\
b
\end{array}\right)=V\left(\begin{array}{l}
d \\
s \\
b
\end{array}\right) \equiv\left(\begin{array}{c}
d^{\prime} \\
s^{\prime} \\
b^{\prime}
\end{array}\right)
$$

The matrix $V$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes
the extent of the couplings between quark generations due to charged weak interactions. The $W^{ \pm}$couples the up-type mass eigenstates to the rotated down-type states, which are each linear combinations of the down-type mass eigenstates.

One can express the left-handed neutral weak currents in a similiar way:

$$
\overline{\left(u_{1}, u_{2}, u_{3}\right)_{L}} \gamma_{\mu}\left(\begin{array}{c}
u_{1}  \tag{2.25}\\
u_{2} \\
u_{3}
\end{array}\right)_{L}=\overline{(u, c, t)_{L}} U_{L}^{\dagger} U_{L} \gamma_{\mu}\left(\begin{array}{c}
u \\
c \\
t
\end{array}\right)_{L} .
$$

However, $U_{L}^{\dagger} U_{L}=1$, and therefore there is no mixing from the neutral weak interaction.

Now that the bosons and quarks have mass, we turn to the leptons. The situation is simpler as there are no right-handed neutrinos in the Standard Model. This is a situation in experimental flux, as recent evidence suggests neutrinos have mass, and if confirmed, right handed neutrinos must exist. In this case, the derivation follows exactly as that for quarks. For the purpose of this dissertation, it will be assumed right handed neutrinos do NOT exist, and that neutrinos are massless. This results in no intergenerationa mixing in the lepton sector, and a greatly simplified Yukawa coupling Lagrangian:

$$
\begin{equation*}
\mathcal{L}=-G_{e}\left[\bar{e}_{R}\left(\boldsymbol{\Phi}^{\dagger} \mathbf{l}_{\mathbf{L}}\right)+\left(\overline{\mathbf{l}}_{\mathbf{L}} \boldsymbol{\Phi}\right) e_{R}\right], \tag{2.26}
\end{equation*}
$$

where $G_{e}$ is a coupling constant and

$$
\begin{equation*}
\mathbf{l}_{\mathbf{L}}=\binom{\nu_{e}}{e}_{L} \tag{2.27}
\end{equation*}
$$

with equivalent expressions for both the $\mu$ and $\tau$. The Lagrangian can then be reduced to:

$$
\begin{equation*}
\mathcal{L}=-G_{e}(v / \sqrt{2}) \bar{e} e-G_{e}(v / \sqrt{2}) H \bar{e} e \tag{2.28}
\end{equation*}
$$

From this, the mass of the electron is $m_{e}=G_{e} v / \sqrt{2}$ and is coupled to the Higgs Boson.

With the inclusion of the Higgs Boson, the Standard Model accounts for the masses of all the known fundamental fermions and bosons. It also acquires a number of new parameters, such as the Higgs ground state and the Yukawa couplings, which cannot be calculated from the theory. This leaves finding the Higgs boson and obtaining precision measurements of the CKM matrix as two of the most interesting areas of experimental particle physics.

### 2.1.2 The CKM Matrix

In the previous section, the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $V$, was introduced to parameterize the coupling between different quark families and
generations. Written explicitly, and with experimental values from Reference [3]

$$
\begin{align*}
V \equiv U_{L}^{\dagger} D_{L} & =\left(\begin{array}{ccc}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)  \tag{2.29}\\
& =\left(\begin{array}{ccc}
0.9742-0.9757 & 0.219-0.226 & 0.002-0.005 \\
0.219-0.225 & 0.9734-0.9749 & 0.037-0.043 \\
0.004-0.014 & 0.035-0.043 & 0.9990-0.9993
\end{array}\right)
\end{align*}
$$

With three generations of quarks, the CKM matrix is a $3 \times 3$ unitary matrix. While a general $3 \times 3$ complex matrix has 18 free parameters, the unitary constraint of $V^{\dagger} V=1$ provides 9 constraints, leaving 9 free parameters. By utilizing the freedom to choose the phase factors of the quark fields, one can remove an additional 5 parameters. Thus, we are left with four parameters: 3 rotational angles and one complex phase factor. It is this phase factor which will be shown later in this section to be responsible for CP Violation.

A hierarchy in the magnitude of CKM elements inspired the popular Wolfenstein parameterization [5]:

$$
V \approx\left(\begin{array}{ccc}
1-\frac{\lambda^{2}}{2} & \lambda & A \lambda^{3}(\rho-i \eta)  \tag{2.30}\\
-\lambda & 1-\frac{\lambda^{2}}{2} & A \lambda^{2} \\
A \lambda^{3}(1-\rho-i \eta) & -A \lambda^{2} & 1
\end{array}\right)+\mathcal{O}\left(\lambda^{4}\right)
$$

where the three real CKM parameters are $\lambda, A$, and $\rho$, and $\eta$ is the complex phase. $A, \rho$, and $\eta$ are all of order 1 , and by expressing the elements in powers of $\lambda \equiv \sin \theta_{c} \simeq 0.222$, with $\theta_{C}$ being the Cabibbo angle, the relative magnitudes of the matrix elements become very apparent.


Figure 1: The Unitarity Triangle.

The CKM matrix is unitary, and therefore, the elements of the matrix are related by:

$$
\begin{equation*}
\sum_{k} V_{k i} V_{k j}^{*}=\delta_{i j}, \tag{2.31}
\end{equation*}
$$

where $i$ and $j$ are generation indices. One of the most interesting forms of this equation for $B$ physics is the $d b$ matrix element:

$$
\begin{equation*}
V_{u d}^{*} V_{u b}+V_{c d}^{*} V_{c b}+V_{t d}^{*} V_{t b}=0 \tag{2.32}
\end{equation*}
$$

This can be represented as a triangle in the complex plane, as seen in Figure 1, which is known as the unitarity triangle. The apex of the triangle is located at
$(\rho, \eta)$. The side running from the origin to the apex has a length equal to $\frac{\left|V_{d}^{*} V_{u b}\right|}{\left|V_{c d}^{*} V_{c b}\right|}$, and the one running from $(1,0)$ to the apex has length $\frac{\left|V_{t d}^{*} V_{t b}\right|}{\left|V_{c d}^{*} V_{c b}\right|}$. The Standard Model explains CP Violation with a non-zero value for the complex phase factor $\eta$. If $\eta$ is complex, it implies the Standard Model Hamiltonian is also complex, as the CKM matrix is used in the parameterization of the charged weak currents [2]. If the Hamiltonian is complex, the Standard Model is not invariant under time reversal. Since CPT (Charge-Parity-Time) must be a good symmetry for all quantum field theories, a violation of $T$ implies that $C P$ must also be violated.

The unitarity triangle is the subject of much research in experimental particle physics at this time. The BaBar (at SLAC) and Belle (at KEK) experiments have come online and are providing ever-improving measurements of $\sin (2 \beta) . B$ mixing measurements being made at SLD and LEP provide the most precise method of measuring $\left|V_{t d}\right|$, which is the element of the CKM matrix with the greatest amount of uncertainty in its value. How these measurements are related will be discussed in Section 2.3.

### 2.1.3 Production of Fermions in $Z^{0}$ Decays

At the SLC, positrons are collided with longitudinally polarized electrons to study decays of the $Z^{0}$. As the $Z^{0}$ decays to all the Standard Model fermion pairs $f \bar{f}$ with the exception of the top quark, colliding at the $Z^{0}$ provides a unique window into Standard Model interactions, especially the electroweak ones. Some important properties of the $Z^{0}$ can be found in Table 4 [3].

Table 4: Some interesting properties of the $Z^{0}$

| Mass | $91.187 \pm 0.002 \mathrm{Gev} / \mathrm{c}^{2}$ |
| :---: | :---: |
| $\Gamma_{Z^{0}}$ | $2.490 \pm 0.007 \mathrm{GeV}$ |
| Decay Mode | Branching Ratio (\%) |
| $e^{+} e^{-}$ | $3.366 \pm 0.008$ |
| $\mu^{+} \mu^{-}$ | $3.367 \pm 0.013$ |
| $\tau^{+} \tau^{-}$ | $3.360 \pm 0.015$ |
| $\mu^{+} \mu^{-}$ | $3.366 \pm 0.006$ |
| invisible $(\nu \bar{\nu})$ | $20.01 \pm 0.16$ |
| hadrons | $69.90 \pm 0.15$ |
| $(u \bar{u}+c \bar{c}) / 2$ | $10.1 \pm 0.6$ |
| $(d \bar{d}+s \bar{s}+b \bar{b}) / 3$ | $16.6 \pm 0.6$ |
| $c \bar{c}$ | $12.4 \pm 0.6$ |
| $b \bar{b}$ | $15.16 \pm 0.09$ |



Figure 2: The Feynman diagrams for the process $e^{+} e^{-} \rightarrow f \bar{f}$.

The Feynman diagrams for the process $e^{+} e^{-} \rightarrow f \bar{f}$ are shown in Figure 2. The process can be mediated by either a $\gamma$ or a $Z^{0}$. The differential production cross section is therefore given by [1]:

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s} \frac{p_{f}}{p_{e}}\left|\mathcal{M}_{Z^{0}}+\mathcal{M}_{\gamma}\right|^{2} \tag{2.33}
\end{equation*}
$$

where $\mathcal{M}_{Z^{0}}$ and $\mathcal{M}_{\gamma}$ are the matrix elements for the $Z^{0}$ and $\gamma$ exchange, respectively, $\sqrt{s}$ is the total energy, and $p_{e(f)}$ is the momentum of the incoming electron (outgoing fermion). Thus we have a $Z^{0}$ term, a $\gamma$ term, and an interference term.

However, at the $Z^{0}$ resonance, the $Z^{0}$ exchange dominates the $\gamma$ exchange by a factor of nearly 800 , and the interference term is negligible. Thus, the electromagnetic terms can be neglected. Applying the Feynman rules for electroweak interactions, we obtain:

$$
\begin{equation*}
\mathcal{M}=-\frac{g^{2}}{4 \cos ^{2} \theta_{w}}\left[\bar{f} \gamma^{\mu}\left(v_{f}-a_{f} \gamma^{5}\right) f\right] \frac{g_{\mu \nu}-k_{\nu} k_{\mu} M_{Z^{0}}^{2}}{k^{2}-M_{Z^{0}}^{2}}\left[\bar{e} \gamma^{\nu}\left(v_{e}-a_{e} \gamma^{5}\right) e\right] \tag{2.34}
\end{equation*}
$$

where $f$ and $e$ represent the fermion and electron spinors, $k$ is the four-momentum of the virtual $Z^{0}$, and $M_{Z^{0}}$ is the $Z^{0}$ mass.

At this point, the next step would generally be to calculate $|\mathcal{M}|^{2}$, averaging over initial state spins and summing over final state spins. However, SLC produces a highly polarized electron beam, and for the case of longitudinally polarized electrons colliding with unpolarized positrons, the differential cross section can be derived [6]:

$$
\begin{equation*}
\frac{d \sigma^{f}}{d \Omega} \propto\left(v_{e}^{2}+a_{e}^{2}\right)\left(v_{f}^{2}+a_{f}^{2}\right)\left\{\left(1-A_{e} P_{e}\right)\left(1+x^{2}\right)+2 A_{f}\left(A_{e}-P_{e}\right) x\right\} \tag{2.35}
\end{equation*}
$$

where $P_{e}$ is the polarization of the incoming electron beam, and the coupling parameters are defined in terms of the vector and axial-vector couplings:

$$
\begin{equation*}
A_{f}=\frac{2 v_{f} a_{f}}{a_{f}^{2}+v_{f}^{2}} \tag{2.36}
\end{equation*}
$$

### 2.1.4 Electroweak Production Asymmetries and $A_{L R}$

An important feature of equation 2.35 is the large production asymmetry in $\cos \theta$ for negative and positive electron beam polarizations. A $b$ quark is more likely to scatter in the positive $\cos \theta$ direction when $P_{e}<0$ and in the negative $\cos \theta$ direction when $P_{e}>0$. This effect is known as the polarized forward-backward asymmetry $\left(A_{F B}\right)$. For a fermion, $A_{F B}$ is defined as:

$$
\begin{equation*}
A_{F B}^{f}(x)=\frac{\sigma^{f}(x)-\sigma^{f}(-x)}{\sigma^{f}(x)+\sigma^{f}(-x)}=2 A_{f} \frac{A_{e}-P_{e}}{1-A_{e} P_{e}} \frac{x}{1+x^{2}} \tag{2.37}
\end{equation*}
$$

where $x=\cos \theta$. The asymmetry depends on the final and initial state coupling parameters, as well as the polarization of the beam, and is sensitive to space inversion. For a polarized electron beam, one can measure the left-right-forward-backward asymmetry, $\tilde{A}_{F B}^{f}(x)$ :

$$
\begin{equation*}
\tilde{A}_{F B}^{f}(x)=\frac{\left(\sigma_{L}^{f}(x)+\sigma_{R}^{f}(-x)-\left(\sigma_{L}^{f}(-x)+\sigma_{R}^{f}(x)\right)\right.}{\left(\sigma_{L}^{f}(x)+\sigma_{R}^{f}(-x)+\left(\sigma_{L}^{f}(-x)+\sigma_{R}^{f}(x)\right)\right.}=2\left|P_{e}\right| A_{f} \frac{x}{1+x^{2}} \tag{2.38}
\end{equation*}
$$

where the $L(R)$ subscript refers to the negative (positive) beam polarization with magnitude $P_{e}$. This quantity is sensitive to both space and spin inversion, but not the initial state coupling. Thus, the final state coupling is isolated and a direct measurement of $A_{f}$ can be done at SLD.

A second important feature of equation 2.35 is the fact the total cross section is greater for negative polarizations than for positive ones, and therefore, left-handed
fermions are more strongly coupled to the $Z^{0}$. This effect is known as the left-right asymmetry $\left(A_{L R}\right)$. If $\sigma_{L}$ and $\sigma_{R}$ are the total cross sections for $e^{+} e^{-} \rightarrow Z^{0}$ where the electron has left and right handed polarization, respectively, $A_{L R}$ is defined to be:

$$
\begin{equation*}
A_{L R}=\frac{\sigma_{L}-\sigma_{R}}{\sigma_{L}+\sigma_{R}} \tag{2.39}
\end{equation*}
$$

$A_{L R}$ relates to the weak mixing angle $\sin ^{2} \theta_{w}$ by:

$$
\begin{equation*}
A_{L R}=A_{e}=\frac{2\left(1-4 \sin ^{2} \theta_{w}\right)}{1+\left(1-4 \sin ^{2} \theta_{w}\right)^{2}} \tag{2.40}
\end{equation*}
$$

At SLD, this becomes a simple counting experiment of the number of $Z^{0} \mathrm{~s}$ produced with positively vs negatively polarized incoming electrons, excluding the $e^{+} e^{-}$final state (which has a large $\gamma$ contribution):

$$
\begin{equation*}
A_{L R}=\frac{n_{L}-n_{R}}{n_{L}+n_{R}}=\left|P_{e}\right| A_{e} \tag{2.41}
\end{equation*}
$$

The SLD result from the 1992-1998 data is [7]:

$$
\begin{equation*}
A_{L R}=0.15138 \pm 0.00216 \tag{2.42}
\end{equation*}
$$

From equation 2.40, the weak mixing angle is determined to be:

$$
\begin{equation*}
\sin ^{2} \theta_{w}=0.23097 \pm 0.00027 \tag{2.43}
\end{equation*}
$$

This is the world's most precise single determination of the weak mixing angle.

### 2.2 Production of Hadrons in $e^{+} e^{-}$Collisions

While colored quarks can be regarded as free particles during a hard collision, they are subsequently organized into colorless hadrons; an event in which this occurs is called hadronic. Figure 3 shows the stages which occur in the production of a hadron at the $Z^{0}$. Only an outline of the process known as hadronization will be discussed here; further details can be found in References [1] or [4].


Figure 3: Main stages in the $e^{+} e^{-} \rightarrow$ hadrons process: perturbative, hadronization, and decay.

The formation of the $q \bar{q}$ pair in stage i in Figure 3 can be calculated to a high degree of accuracy with perturbative electroweak physics. Pertubative Quantum Chromodynamics (QCD) is used to describe the evolution of the produced parton (quark and gluon) shower to the point where the partons begin to be bound into colorless hadrons (stage ii). The QCD process through which the partons become bound (stage iii) is fundamentally non-perturbative in nature, and a variety of
phenomenological fragmentation models have been utilized as a description. This provides an experimental window to the underlying process, (see, for example, Reference [8]) but means that models, rather than calculation from first principles, must be relied on to describe the production of hadrons in $e^{+} e^{-}$collisions.

### 2.3 Mixing in $B$ Decays

Neutral particle oscillations were first predicted for the $K^{0}$ system in 1955 by Gell-Mann and Pais [9], which led them to predict the existance of a long-lived neutral Kaon, now known as the $K_{L}^{0}$. The existance of the $K_{L}^{0}$ was confirmed at Brookhaven by Lande [10]. Particle oscillations can only occur for the following mesons: $K^{0}, D^{0}, B_{d}^{0}$, and $B_{s}^{0}$. The top quark is too heavy to form stable hadrons, excited meson states decay via strong or electromagnetic interactions before mixing can occur, and the $\pi^{0}$ is its own antiparticle. Here we focus on the $B_{d}^{0}$ system.

### 2.3.1 Phenomenology

$B$ mixing is very similiar to that of the kaon system. $B^{0}$ and $\bar{B}^{0}$ mesons are flavor eigenstates created by the strong interaction production process. When the charged currents of the weak interaction are introduced to the Hamiltonian, flavor changing transitions are introduced. The box diagrams shown in Figure 4 describe how second order weak interactions couple the flavor eigenstates of the $B$. The mass eigenstates (the physical particles whose definite mass and lifetime we can
measure) are no longer the same as the flavor eigenstates, and thus we observe $B$ oscillations.


Figure 4: The second order box diagrams for $B$ oscillations.

One can write the Hamiltonian matrix for the $B^{0} \bar{B}^{0}$ system phenomenologically:

$$
\begin{align*}
H\binom{B^{0}}{\overline{B^{0}}} & =\left(\begin{array}{cc}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{array}\right)\binom{B^{0}}{\overline{B^{0}}} \\
& =\left(\begin{array}{cc}
M-\frac{1}{2} i \Gamma & M_{12}-\frac{1}{2} i \Gamma_{12} \\
M_{12}^{*}-\frac{1}{2} i \Gamma_{12}^{*} & M-\frac{1}{2} i \Gamma
\end{array}\right)\binom{B^{0}}{\bar{B}^{0}} . \tag{2.44}
\end{align*}
$$

$M$ is the mass of the flavor eigenstates and $\Gamma$ their decay width. The diagonal terms are equal because CPT invariance requires a particle and its antiparticle to have the same mass and lifetime. The off-diagonal elements introduce the mixing. $M_{12}$ corresponds to virtual transitions while $\Gamma_{12}$ corresponds to real transitions through common decay modes, and these can be calculated from theory by evaluation of the box diagrams. The real transitions are Cabibbo suppressed and therefore the $\Gamma_{12}$ term can be neglected [12].

By diagonalizing the Hamiltonian and neglecting CP violation, one can obtain the CP eigenstates $B_{1}$ and $B_{2}$ :

$$
\begin{align*}
& \left\lvert\, B_{1}>=\frac{1}{\sqrt{2}}\left(\left|B^{0}>+\right| \overline{B^{0}}>\right)\right. \\
& \left\lvert\, B_{2}>=\frac{1}{\sqrt{2}}\left(\left|B^{0}>-\right| \overline{B^{0}}>\right)\right. \tag{2.45}
\end{align*}
$$

which have masses $M_{1,2}$ and widths $\Gamma_{1,2}$ :

$$
\begin{align*}
& M_{1,2}=M \pm \frac{\Delta M}{2} \\
& \Gamma_{1,2}=\Gamma \pm \frac{\Delta \Gamma}{2} \tag{2.46}
\end{align*}
$$

and the mass and width differences are given by:

$$
\begin{align*}
\Delta m & =2 \operatorname{Re} \sqrt{\left(M_{12}-\frac{\Gamma_{12}}{2}\right)\left(M_{12}^{*}-\frac{\Gamma_{12}^{*}}{2}\right)} \\
\Delta \Gamma & =-4 \operatorname{Im} \sqrt{\left(M_{12}-\frac{\Gamma_{12}}{2}\right)\left(M_{12}^{*}-\frac{\Gamma_{12}^{*}}{2}\right)} \tag{2.47}
\end{align*}
$$

It is now possible to see the time evolution of $\left|B_{1(2)}\right\rangle$ :

$$
\begin{equation*}
\left|B_{1(2)}(t)>=e^{-i M_{1(2)} t} e^{\frac{-\Gamma_{1(2)} t}{2}}\right| B_{1(2)}(0)>. \tag{2.48}
\end{equation*}
$$

If we create a $B^{0}$ at time $t=0$, its time evolution can be written as a linear
combination of $B_{1}$ and $B_{2}$ by inverting equation 2.45:

$$
\begin{equation*}
\left.\left|B^{0}(t)>=\frac{1}{\sqrt{2}} e^{-i M_{1} t} e^{\frac{-\Gamma_{1} t}{2}}\right| B_{1}(0)>+\frac{1}{\sqrt{2}} e^{-i M_{2} t} e^{\frac{-\Gamma_{2} t}{2}} \right\rvert\, B_{2}(0)> \tag{2.49}
\end{equation*}
$$

and then writing $\mid B_{1,2}(0)>$ in terms of $\mid B^{0}>$ and $\mid \overline{B^{0}}>$ :

$$
\begin{equation*}
\left.\left|B^{0}(t)>=\frac{1}{2}\left[e^{-i M_{1} t} e^{\frac{-\Gamma_{1} t}{2}}+e^{-i M_{2} t} e^{-\Gamma_{2} t 2}\right]\right| B^{0}>+\frac{1}{2}\left[e^{-i M_{1} t} e^{\frac{-\Gamma_{1} t}{2}}-e^{-i M_{2} t} e^{\frac{-\Gamma_{2} t}{2}}\right] \right\rvert\, \bar{B}^{0}>. \tag{2.50}
\end{equation*}
$$

It is now apparent that the time evolution has resulted in a $\mid \overline{B^{0}}>$ component for a state that was initially pure $\mid B^{0}>$. It is now straightforward to write the probabilities for a $B^{0}\left(\overline{B^{0}}\right)$ to decay as a $B^{0}\left(\bar{B}^{0}\right)$ (unmixed):

$$
\begin{equation*}
P_{\text {unmixed }}(t)=\frac{1}{4}\left[e^{-\Gamma_{1} t}+e^{-\Gamma_{2} t}+2 e^{-\Gamma t} \cos (\Delta m t)\right], \tag{2.51}
\end{equation*}
$$

and to write the probability it decays into its antiparticle and has therefore mixed:

$$
\begin{equation*}
P_{\text {mixed }}(t)=\frac{1}{4}\left[e^{-\Gamma_{1} t}+e^{-\Gamma_{2} t}-2 e^{-\Gamma_{t}} \cos (\Delta m t)\right] . \tag{2.52}
\end{equation*}
$$

Cabibbo suppression allows us to neglect $\Gamma_{12}$, and then $\Delta m \simeq 2\left|M_{12}\right|$ and $\Delta \Gamma=0$. Thus the probabilities can be further simplified:

$$
\begin{align*}
P_{\text {unmixed }}(t) & =\frac{1}{2} e^{-\Gamma_{t}}(1+\cos (\Delta m t)) \\
P_{\text {mixed }}(t) & =\frac{1}{2} e^{-\Gamma_{t}}(1-\cos (\Delta m t)) \tag{2.53}
\end{align*}
$$

and it is now obvious that the mass difference, $\Delta m$, is in fact the oscillation frequency between the two mass eigenstates.

### 2.3.2 B Mixing in the Standard Model

A theoretical prediction for $\Delta m$ can be obtained by computing the box diagram contributions [11]. Neglecting $\Gamma_{12}$,

$$
\begin{equation*}
\Delta m \approx \frac{G_{F}^{2} M_{W}^{2}}{8 \pi^{2}}<\bar{B}^{0}\left|j_{\mu}^{V-A} j^{V-A, \mu}\right| B^{0}>\sum_{u, c, t} \lambda_{i} \lambda_{j} A_{i j} \tag{2.54}
\end{equation*}
$$

where the valence quark approximation has been used to factorize the transition amplitude into a strong and weak part. The $\lambda_{i=u, c, t}$ terms are products of the CKM matrix elements, defined as $\lambda_{i}=V_{i b}^{*} V_{i d}$. Recalling that the CKM matrix is unitary, $\sum_{u, c, t} \lambda_{i}=0$ and the dependence on the $u$ quark can be removed:

$$
\begin{array}{r}
\sum_{u, c, t} \lambda_{i} \lambda_{j} A_{i j}=\left(\lambda_{c}^{2} U_{c c}+\lambda_{t}^{2} U_{t t}+2 \lambda_{c} \lambda_{t} U_{c t}\right) \\
U_{i j}=A_{u u}+A_{i j}-A_{u i}-A_{u j} \tag{2.55}
\end{array}
$$

Evaluation of the functions $A_{i j}$ are done with loop integrals, resulting in a dependence on the quark masses and $M_{W}$. Because $U_{c c}, U_{c t} \ll U_{t t}$ ( [12]), the top quark transition dominates, with $U_{t t} \approx z F(z), z=\frac{m_{t}^{2}}{M_{W}^{2}}$, and $F(z)$ given by:

$$
\begin{equation*}
F(z)=\frac{1}{4}+\frac{9}{4(1-z)}-\frac{3}{2(1-z)^{2}}-\frac{3 z^{2} \ln (z)}{2(1-z)^{3}} \tag{2.56}
\end{equation*}
$$

The matrix element $<\bar{B}^{0}\left|j_{\mu}^{V-A} j^{V-A, \mu}\right| B^{0}>$ gives the probability that the quarks inside the $B$ meson are close enough together that $W$ exchange between them is possible. One can use the vacuum insertion approximation to calculate the matrix element:

$$
\begin{align*}
<\bar{B}^{0}\left|j_{\mu}^{V-A} j^{V-A, \mu}\right| B^{0}> & =<\bar{B}^{0}\left|\left[\bar{b} \gamma_{\mu}\left(1-\gamma_{5}\right) d\right]\left[\bar{b} \gamma_{\mu}\left(1-\gamma_{5}\right) d\right]\right| B^{0}> \\
& =B_{B_{d}}<\bar{B}^{0}\left|\left[\bar{b} \gamma_{\mu}\left(1-\gamma_{5}\right) d\right]\right| 0><0\left|\left[\bar{b} \gamma_{\mu}\left(1-\gamma_{5}\right) d\right]\right| B^{0}> \\
& =\frac{4}{3} B_{B_{d}} f_{B_{d}}^{2} m_{b} \tag{2.57}
\end{align*}
$$

where $f_{B_{d}} \sim 200 \mathrm{MeV}$ [13] is the decay constant, which can be calculated using QCD sum rules or from lattice QCD, and $B_{B_{d}} \sim 1.3$ [13] is the "bag parameter" due to the vacuum insertion approximation. No experimental information is available for these quantities.

QCD corrections to the box diagram can be taken into account using a multiplicative correction factor, $\eta \sim 1$, and is dependent on the QCD parameter $\lambda_{Q C D}$ and the top and bottom quark masses.

If we put this all together, the final expression becomes:

$$
\begin{equation*}
\Delta m_{d}=\frac{G_{F}^{2}}{6 \pi^{2}} B_{B_{d}} f_{B_{d}}^{2} m_{b}\left|V_{t b}^{*} V_{t d}\right| m_{t}^{2} F\left(\frac{m_{t}^{2}}{M_{W}^{2}}\right) \eta_{Q C D} \tag{2.58}
\end{equation*}
$$

In the ideal world, we could use this equation to calculate $V_{t d}$, one of the least well-measured elements of the CKM matrix, directly. Unfortunately, the error on
the theoretical calculation of $f_{B_{d}} \sqrt{b_{B}}$ is $\sim 20 \%$, much greater than the errors on the measurement of $\Delta m$. One solution is to take the ratio of $\Delta m$ to its companion in the $B_{s}$ system, $\Delta m_{s}$. The Standard Model calculation of $\Delta m_{s}$ follows the same path as that for $\Delta m_{d}$, except substituting $s$ quark terms for $d$ quark terms. Thus, we have:

$$
\begin{equation*}
\Delta m_{s}=\frac{G_{F}^{2}}{6 \pi^{2}} B_{B_{s}} f_{B_{s}}^{2} m_{b}\left|V_{t b}^{*} V_{t s}\right| m_{t}^{2} F\left(\frac{m_{t}^{2}}{M_{W}^{2}}\right) \eta_{Q C D} \tag{2.59}
\end{equation*}
$$

Taking the ratio, we have [13]:

$$
\begin{align*}
\frac{\Delta m_{s}}{\Delta m_{d}} & =\frac{m_{B_{s}} f_{B_{s}}^{2} B_{B_{s}}}{m_{B_{d}} f_{B_{d}}^{2} B_{B_{d}}} \\
& =\frac{m_{B_{s}}}{m_{B_{d}}} \cdot(1.16 \pm 0.05)^{2} \cdot\left|\frac{V_{t s}}{V_{t d}}\right|^{2} \tag{2.60}
\end{align*}
$$

. If the value of $\Delta m_{s}$ were known, it would be possible to measure $V_{t d}$ in this manner. Currently, however, there are only limits on $\Delta m_{s}$ [3]. Fortunately, we may not have long to wait for a measurement, as both CDF and D0 at Fermilab are expected to be able to measure $\Delta m_{s}$ if its value is within the limits predicted by the Standard Model.

## CHAPTER 3

## EXPERIMENTAL APPARATUS

Chapter 3 presents a brief description of the experimental apparatus used to make this measurement. The SLAC Linear Collider (SLC) is a unique machine which accelerates and collides positrons with polarized electrons at the $Z^{0}$ resonance. The interaction point of this collider is located at the geometrical center of the SLC Large Detector (SLD), which is a full-coverage multipurpose particle detector. After an overview of the accelerator and detector, this chapter concludes with brief descriptions of the SLD event trigger and Monte Carlo simulation.

### 3.1 The SLAC Linear Collider (SLC)

The SLC, shown schematically in Figure 5, is unique in being both the only linear accelerator in the world and the only accelerator to use polarized electrons as one of its colliding particles [14]. Compared to other accelerators like LEP, SLC has a low beam crossing rate ( 120 Hz ), but an extraordinarily small collision spot.


Figure 5: Layout of the SLC.

In this section, we describe some of the key features of the SLC, in the order a particle traversing the accelerator would encounter them: first, the polarized electron source and the positron source; second, the linear accelerator (linac) and the damping rings; then the arcs. Finally, we describe two instruments which occur after the interaction point: the Compton Polarimeter and the Energy Spectrometer.

### 3.1.1 Polarized Electron Source

The use of polarized electrons makes the SLC a truly unique machine. The electron source is shown in Figure 6. It consists of a strained-lattice GaAs photocathode in the electron gun at the linac's electron injector. A circularly-polarized Nd:YAGpumped Ti sapphire laser is used to selectively excite transitions of electrons into longitudinally-polarized states in the conduction band of the photocathode. The relevant electron energy level transitions can be seen in Figure 7. A modest voltage is applied to the source to extract the produced electrons.

Before injection into the linac, the electron bunch is accelerated by a 20 kV electric field. For the 1997-8 physics run, a strained lattice photocathode consisting


Figure 6: Schematic layout of the polarized light source.
of a $100 \mu \mathrm{~m}$ GaAs deposited on a GaAsP substrate was used. Because of the difference in lattice spacings of the two materials, a strain is placed on the epitaxials GaAs layer, which breaks the degeneracy in the valence band and allows for a theoretical polarization of $100 \%$. In reality, this system resulted in a measured electron polarization of $73 \%$ for the 1997-1998 physics run.

### 3.1.2 Positron Production

Three particle bunches are accelerated down the linac during each machine cycle: the first two bunches are the positrons and electrons which will be collided at the IP, and the third is a "scavenger" electron bunch. This bunch is diverted off the main linac approximately $2 / 3$ down its length into collision with a tungstenrhenium alloy target. Photons, electrons and positrons are producted in the resulting electromagnetic showers. A yield of approximately one positron per an incident electron is typical. These positrons are filtered from the shower, and in a separate beam line, transported to the start of the linac and injected into the South Damping Ring, where they are stored until the next machine cycle.


Figure 7 : Energy state diagram for (top) bulk GaAs and (bottom) the strained GaAs lattice. Polarization is a result of a preference for certain excitation modes, depending on the polarization and energy of the incident photon; $\sigma^{+(-)}$indicates the transitions for a right (left) -handed polarized incident photon. The circled numbers denote the relative intensities of the transitions. The strained lattice breaks the degeneracy and allows a theoretical polarization of $100 \%$.

The positron beam polarization, expected to be zero, has been checked with the Moeller Polarimeter [15] in SLAC End Station A, with a result of $P_{e^{+}}=(-0.02 \pm$ $0.07) \%$.

### 3.1.3 The Damping Rings and Linear Accerator (Linac)

The produced electrons (positrons) are accelerated to 1.19 GeV in a short section of the linac before being injected into the the North (South) Damping Ring (NDR and SDR).

At the entrance to the NDR, a spin-rotator magnet rotates the electron spin into a direction which is tranverse to the plane of the daming ring orbit, as shown in Figure 5; otherwise, the spins of the electrons would be randomized during the damping process. In both damping rings, the particles experience synchrotron radiation, which reduces both the emittance and longitudinal spread of the beam. This allows for more stable acceleration in the linac, and lower backgrounds, and higher luminosities at the interaction point. The electrons are damped for 1 machine cycle ( 8.3 msec ), during which they settle into stable orbits determined by the ring parameters. After passing through another spin-rotator magnet, they are injected into the linac.

A similiar process in the SDR occurs for the positrons, with a few differences. As the positrons are not polarized, there are no spin-rotator magnets to preserve their polarization. Because they have a larger energy spread due to their production from photoconversion, they are damped for 2 machine cycles, then injected into the linac.

The positron bunch leads the electron bunch and both are accelerated down the straight segment of the linac to 46.7 GeV . At the end of the accelerator, the bunches are separated in the Beam Switchyard by dipole bending magnets, which send the electron bunch into the North arc and the positron bunch to the south one.

### 3.1.4 The Arcs

The North (electron) and South (positron) arcs are each 1 km long, and contain a sequence of dipole magnets, which bend the beams around the arcs, and quadrupole magnets, which keep the beam focused. The bunches lose about 1.1 GeV of energy due to synchrotron radiation as they travel around the arcs to the IP. In order to preserve the longitudinal polarization of the electrons at the IP, the electron beam polarization is rotated to a specific angle before entering the arc.

Before colliding, both beams travel through final focus sections, where they are focused by superconducting quadrupole triplets to a transverse size of about $0.5(2.3) \mu \mathrm{m}$ in the vertical (horizontal). They then collide head-on at the center of SLD, after which they are dumped.

The SLC luminosity history is shown in Figure 8. It shows a continuing improvement in luminosity since SLC saw its first $Z^{0}$ in 1989. In the 1997-98 data run, the luminosity peaked at well above $200 Z^{0}$ /hour.

### 3.1.5 Compton Polarimeter

The Compton Polarimeter [16], located 33 m downstream from the SLD interaction point, is the primary device used to measure the polarization of the electron beam. The polarimeter makes use of the helicity assymetry in the Compton scattering cross section and consists of a laser with polarizing optics and an electron spectrometer, and can be seen in Figure 9. A series of Pockels cells and quarter-

1992-1998 SLD Polarized Beam Running


Figure 8 : SLC/SLD luminosity history, in Z's/week. The solid curve shows the integrated luminosity while the histogram shows the accumulation by week. The arrows indicate the average polarization for that run period. This analysis is based on the 1996-98 run.
wave plates polarize photons from a frequency-doubled YAG laser, which interact with the electron beam. At the Compton polarimeter IP, the electron beam intersects these 2.33 eV circularly-polarized photons, and a small fraction of the electrons Compton scatter off the photons. The scattered electrons are then bent by a precision dipole magnet and then enter the Compton Cherenkov Detector, which functions as an electron spectrometer.


Figure 9: Schematic view of the SLD Compton Polarimeter.

The differential Compton scattering cross section for polarized electrons and polarized photons can be written as:

$$
\begin{equation*}
\frac{d \sigma_{C}}{d E}=\frac{d \sigma_{C}^{u}}{d E}\left[1+A_{C}(E)\right] \tag{3.1}
\end{equation*}
$$

where $E$ is the energy of the scattered electron and $\frac{d \sigma_{C}^{u}}{d E}$ the unpolarized differential Compton cross section. The Compton asymmetry, $A_{C}(E)$, can be written in terms of $P_{e}$, the electron polarization to be measured:

$$
\begin{equation*}
A_{\text {Compton }}(E)=\frac{\sigma_{J_{z}=\frac{3}{2}}-\sigma_{J_{z}=\frac{1}{2}}}{\sigma_{J_{z}=\frac{3}{2}}+\sigma_{J_{z}=\frac{1}{2}}}=a_{d} P_{e} P_{\gamma} a_{C}(E) \tag{3.2}
\end{equation*}
$$

where $P_{\gamma}$ is the known photon polarization, and the assymetry exhibits its depen-
dence on the spin-states of the electron and photon. The "analyzing power" of the detector, $a_{d}$, is determined from a calibration process [17]. As $A_{\text {Compton }}(E)$ can be calculated precisely from QED, the precision of the polarization measurement is limited only by detector systematic uncertainties.

In addition to the Compton Polarimeter, the Polarized Gamma Counter and Quartz Fiber calorimeter [18] are also used to measure the electron beam polarization, and provide a cross check with a precision of better than $1 \%$.

### 3.1.6 Energy Spectrometer

The SLC beam energy is measured by two Wire Imaging Syncrotron Radiation Detectors (WISRDs) [19], which are located between the IP and each of the beam dumps. These enable a measurement of the energies of the electron and positron beams on a pulse-by-pulse basis. The incoming beam is deflected by two horizontal bend magnets, thereby producing a swath of synchrotron radiation which can then be imaged by a multiwise proportional chamber (MWPC). Between the two horizontal magnets is a precisely calibrated vertical bend magnet, which deflects the beam by an angle inversely proportional to its energy. This angle is inferred from the distance between the two swathes imaged on the MWPC. A schematic view of the WISRD can be seen in Figure 10.

A $Z^{0}$ peak scan was performed near the end of the 1997-98 data run to allow the WISRD to be calibrated against the precision $Z^{0}$ peak measurement performed by LEP. The resulting luminosity weighted mean center of mass energy for the 1997-98
data run was found to be $E_{c m}=91.237 \pm 0.029 \mathrm{GeV}[20]$, which is about 46 MeV from the $Z^{0}$-pole. While this requires corrections to be applied to measurements such as $A_{L R}$, it does not impact this analysis.


Figure 10: Schematic layout of the WISRD.

### 3.2 The SLAC Large Detector (SLD)

THE SLC Large Detector (SLD), proposed in 1984 [21], is a general purpose particle detector with nearly $4 \pi$ steradian solid angle coverage. It replaced the less sophisticated Mark II detector and surrounds the sole SLC IP. Figures 11 and 12 respectively show a cut-away and quadrant view of SLD.

The cylindrical design with onion-like layers of detectors is typical of a modern particle detector. The SLD is made up of two main sections: the cylidrical barrel, which is about 10 m in length and has a radius of 4.5 m , and the two endcaps, which mount on large sliders and can be moved for access to the hardware systems.

Each layer is a sub-detector designed to measure specific aspects of the particles produced in the collisions. Constrained by the beampipe and support systems, the detector allows for nearly $98 \%$ solid angle coverage.

The detector technologies are similiar for the barrel and endcap detectors. Moving outwards from the beampipe radially, we first encounter the precision CCD vertex detector (VXD3) and the wire drift chamber (CDC), which allow for precision charged particle tracking and vertex reconstruction. Outside the main tracking system lies the Cherenkov Ring Imaging Detector (CRID), whose particle identification abilities are critical for the tagging used in this analysis, the Liquid Argon Calorimeter (LAC), an 0.6 Tesla conventional solenoidal magnet which also serves as the outer support system for the detector, and the Warm Iron Calorimeter (WIC), used for muon identification. To achieve high bandwidth/low noise transmission of data, all the subsystems use fiber optic transmission cable connections for signal readout.

The standard SLD coordinate system is as follows: the $z$ axis points geographic north along the positron beam direction, and the $x$ and $y$ axes lie in the plane perpendicular to the beam, with the horizontal $x$ axis pointing west and the $y$ azis pointing upwards. In cylindrical coordinates, the radius $r$ is in the $x, y$ plane, the polar angle $\theta$ is defined with respect to the positive $z$ direction and the azimuthal angle $\phi$ defined with respect to the positive $x$ axis.

The remainder of this section discusses each detector subsystem in more detail. As the endcaps are less understood and less important to the analysis in this thesis, the barrel detector will take precedence.


Figure 11 : An isometric view of SLD. The Luminosity Monitor is not shown, and the encaps have been omitted for clarity.


Figure 12: The SLC Large Detector (quadrant view).

### 3.2.1 Vertex Detector (VXD3)

Precision vertexing is provided by the SLD vertex detector [22-24], a unique detector using charged coupled devices (CCD). Two vertex detectors have been used in SLD: VXD2, operational during the 1993-95 run periods, and VXD3, from 1996-1998. As this analysis only uses data from the later period, we will focus on VXD3 in this section.

VXD3 is a three layer device containing 96 CCDs, each of which contain $4,000 \times$ $800(20 \mu \mathrm{~m})^{2}$ pixels. Each layer is constructed of overlapping layers; each ladder carries 2 CCDs, as seen in Figure 13. The CCDs and ladders were optically surveyed to a precision of $\sim 10 \mu \mathrm{~m}$. Single hit spatial resolutions of $3.8 \mu \mathrm{~m}$ in $r \phi$ and $4.2 \mu \mathrm{~m}$ in $r z$ were achieved after correcting for inner and outer hit resolution. As the particles produced are expected to be back to back, $Z^{0} \rightarrow \mu^{+} \mu^{-}$, events can be used to measure the $y z$ miss distance and give a measure of VXD3 resolution. This was measured to be about $7.8 \mu \mathrm{~m}$ in $r \phi$ and $9.7 \mu \mathrm{~m}$ [25].

VXD3 provides 3-hit (1 hit in each layer) acceptance up to $|\cos \theta|<0.85$, which allows for "stand-alone tracking", that is, tracking using only VXD3 hits. The detector is operated at $\sim 190 K$ to limit dark currents. Each CCD is read out serially in 120 ms . The impact parameter resolution of tracks has been studied for individual $\cos \theta$ and momentum regions [23]. The measured VXD3 impact


4-97 8262A11

North End

Figure 13: A VXD3 ladder, showing one CCD on each side.
parameter resolution is:

$$
\begin{align*}
\sigma_{r \phi} & =\sqrt{9.0^{2}+\left(\frac{33.0}{p \sin ^{3 / 2} \theta}\right)^{2}} \mu m \\
\sigma_{r z} & =\sqrt{17.0^{2}+\left(\frac{33.0}{p \sin ^{3 / 2} \theta}\right)^{2}} \mu m \tag{3.3}
\end{align*}
$$

Track impact resolution, comparing data to Monte Carlo as a function of momentum, can be seen in Figure 14 for both VXD2 and VXD3.

### 3.2.2 Drift Chambers (CDC and ECDC)

The SLD Drift Chamber is composed of a Central (barrel) Drift Chamber [26] and Endcap Drift Chamber (ECDC). As only the CDC is used for tracking in this analysis, we shall focus on it.


Figure 14: VXD2 and VXD3 impact resolutions.

The CDC is the primary SLD tracking device, and has a cylindrical shape with an inner radius of 0.2 m and an outer one of 1.0 m , and a total length of 2.0 m . Centered on the SLC IP, it provides tracking coverage for polar angles through $|\cos \theta|<0.85$, the same as the 3 hit coverage for VXD3. The inner and outer walls are constructed of an aluminum sheet-Hexcell fiberboard laminate, with the inner (outer) wall being $1.8 \% X_{0}\left(1.6 \% X_{0}\right)$ thick, where $X_{0}$ is the radiation length of the material. The endplates are constructed of aluminum. The CDC is filled with a gas mixture consisting of $75 \% \mathrm{C0}_{2}, 21 \% \mathrm{AR}, 4 \%$ isobutane, and $0.2 \%$ water. Each element of the mixture is selected for a purpose: the low electron dift velocity and low diffusion of the $\mathrm{C}_{2}$ improves spatial resolution and accurate sampling by SLD electronics; Ar increases the avalanche gain; isobutane assists quenching; and the water helps surpress wire aging. The entire chamber is immersed in the 0.6 Tesla uniform solenoidal field.

The CDC is divided into 10 radial superlayers, as shown in Figure 15. These layers are then divided azimuthally into cells, each measuring about 6 cm wide and 5 cm high. Each cell has 8 sense wires, 18 guard wires, and 25 field wires, with a layout as shown in Figure 16. The guard and field wires are made of $150 \mu \mathrm{~m}$ gold-coated aluminum. The guard wires are held at 3027 V and surround the sense wires to focus the drifting electrons and to provide uniform charge amplification. The voltage on the field wires varies with position and averages 5300 V . This arrangement provies a mean drift field of $0.9 \mathrm{kV} / \mathrm{cm}$. The sense wires, made of $25 \mu \mathrm{~m}$ gold-coated tungsten, are either axial or have a 41 mrad stereo angle with respect to the beam axis.

A charged tracking passing through a cell experiences energy loss due to the ionization of atoms in the material. These electrons are directed by the drift field in the chamber towards the sense wires, as seen in Figure 17. The wire address, drift time information, and charge ratio at the ends of the wire provide a hit position. By combining information from other sense wires in the same cell, a vector hit is formed. As sense wires in a cell are not staggered, an ambiguity in $x y$ arises in the form of a mirror image. Pattern recognition software combines vector hits into track candidates, and then a detailed offline fit is performed using individual wire hit information, more precise $z$ information from the stereo layers, and taking into account fluctuations in the electric and magnetic fields and energy loss of the track. One can find details of the track reconstruction in Reference [27]. The momentum


Figure 15 : The SLD Central Drift Chamber (CDC). An endplate view of the CDC, showing the axial (A) and stereo ( S ) superlayers.
resolution of the CDC has been determined to be:

$$
\begin{equation*}
\frac{\sigma_{p_{\perp}}}{p_{\perp}}=\sqrt{0.010^{2}+\left(0.0050 p_{\perp}\right)^{2}} \tag{3.4}
\end{equation*}
$$

with $p_{\perp}$ the track momentum transverse to the beam, measured in Gev/c. When combined with VXD hit information, the combined momentum resolution improves to [28]:

$$
\begin{equation*}
\frac{\sigma_{p_{\perp}}}{p_{\perp}}=\sqrt{0.0095^{2}+\left(0.0026 p_{\perp}\right)^{2}} . \tag{3.5}
\end{equation*}
$$



Figure 16: Schematic view of the CDC drift cell layout.


Figure 17: The CDC field map and drift path. The field map of a single CDC cell, (left) and the drift paths of electrons produced by a charged particle traversing the cell (right).

### 3.2.3 Cherenkov Ring Imaging Detector (CRID)

The Cherenkov Ring Imaging Detector (CRID) [29] provides SLD with excellent charged particle identification $\left(e^{ \pm}, \mu^{ \pm}, \pi^{ \pm}, K^{ \pm}, p / \bar{p}\right)$ over a wide range of momenta. As kaon identification is fundamental to this analysis, we will go into significantly more detail about the CRID in Chapter 4.

The CRID is based on the Cherenkov effect, hence its name. A charged particle with a velocity, $v$, traverses a dielectric medium whose index of refraction is $n$. If $v$ is greater than the phase velocity of light in that medium, the particle will emit a coherent wave front of Cherenkov photons with an emission angle to the track, $\theta_{C}$ given by:

$$
\begin{equation*}
\cos \left(\theta_{C}\right)=\frac{1}{n \beta} \tag{3.6}
\end{equation*}
$$

where $\beta=v / c$. These photons are imaged by an array of 40 Time Projection Chambers (TPCs), which are filled with a mixture of $\mathrm{C}_{2} \mathrm{H}_{6}$ gas and gaseous tetrakis-dimethylamino-ethylene (TMAE), a photocathode additive. TMAE is ionized by photons with energy greater than 5.4 eV and releases photoelectrons into the $\mathrm{C}_{2} \mathrm{H}_{6}$. These photoelectrons then drift to the instrumented ends of the TPCs. The particle velocity can be determined from the reconstructed ring of photoelectrons, and conversion positions can be inferred from drift time, the wire address, and the charge division on the wire. Combined with momentum, the mass of the particle can be determined, thus giving its identity.

### 3.2.4 Liquid Argon Calorimeter (LAC)

The Liquid Argon Calorimeter (LAC) [30] was designed to measure the energy of charged and neutral particles. Like the CRID and Drift Chamber, it is divided into barrel and endcap sections. The barrel LAC is a 6 m long cylinder with an inner radius of 1.8 m and an outer one of 2.9 m , and provides coverage from $35^{\circ}$ to $145^{\circ}$ in polar angle. As the endcaps extend the coverage to within $8^{\circ}$ of the beamline, the LAC covers about $98 \%$ of the solid angle.

The LAC is a $\mathrm{Pb}-\mathrm{Ar}$ sampling calorimeter, with Pb plates, separated by plastic spacers, immersed in liquid argon. Particles which enter the LAC interact with the Pb plates and produce particle showers which then ionize the argon. The lead plates are held alternately at ground and -2 kV . As the argon does not amplify the charge, the observed charge is proportional to the deposited energy.

The LAC is divided into four radial layers, EM1, EM2, HAD1, and HAD2. The EM sections contain most of the energy from electromagnetic showers (hence the names of the sections), and consist of 2.0 mm lead plates separated by 2.75 mm Ar gaps. The HAD sections consist of 6.0 mm lead plates with the same gap size. A schematic view of a LAC module can be seen in Figure 18. While the EM layers contain most of the electromagnetic shower energy in 21 radiation lengths of material, the HAD sections extend containment of hadronic showers to 2 radiation lengths. In total, the four sections contain 2.8 absorption lengths and 49 radiation lengths of material. The energy resolution $[28,30]$ of the LAC is approximately $60 \% / \sqrt{E}$ for hadronic showers and $15 \% / \sqrt{E}$ for electromagentic showers. For a

45 GeV electron, $99 \%$ of the energy is contained in the EM sections, and $90-95 \%$ of the total energy of a $Z^{0}[31]$ is contained by the LAC.


Figure 18: A schematic view of a barrel LAC module.

### 3.2.5 Warm Iron Calorimeter (WIC)

The Warm Iron Calorimeter (WIC) [32] is the outermost layer of SLD, and is divided into endcap and barrel regions which together provide coverage for almost all of the solid angle. Its primary role is as the flux return for the solenoid and as a support structure for SLD. It is also instrumented to provide additional calorimetry information and muon identification. The original intent of using the WIC for containing the approximately $5 \%$ energy leakage through the LAC by hadronic $Z_{0}$ decays has not been realized due to problems with energy response calibration.

Figure 19 shows a cutaway view of the WIC. The WIC is consists of 18 layers of Iarocci streamer tubes [33] contained in 3.2 cm gaps between 5 cm thick steel plates. The Iarocci streamer tubes are long rectangular plastic tubes with central $100 \mu \mathrm{~m} \mathrm{Be}-\mathrm{Cu}$ wires and are filled with a gaseous combination of $88 \% \mathrm{CO}_{2}, 9.5 \%$ $\mathrm{C}_{4} \mathrm{H}_{1} 0$, and $2.5 \%$ Ar. The tubes have external Cu cathode readouts, square readout pads for calorimetric measurements, and long strips for muon tracking, which are arranged in separate, perpendicular arrays.


Figure 19 : A cutaway view of a WIC section, showing details of single layers and double layers.

### 3.2.6 Luminosity Monitor

The SLD Luminosity Montitor (LUM) [34] was designed for two purposes. It provides a precise measurement of the luminosity deliverd by SLC by measuring small-angle Bhabha scattering, whose cross-section is precisely derived from QED. It was also designed to provide an extension of the electromagnetic calorimetry to small angles. Shown in Figure 20, the LUM is located 1 m downstream from the IP along the beam axis, and consists of two silicon-tungsten calorimeter modules, the Luminosity Monitor/Small Angle Trigger (LMSAT) and the Medium Angle Silicon Calorimeter (MASiC). The LMSATs provide coverage between 28 and 68 mrad from the beamline and while the MASiCs provide coverage from 68 to 200 mrad . The projective towers in both calorimeters are highly segmented and each has a total depth of $21 X_{0}$, containing $99.5 \%$ of a 45.6 GeV electromagnetic shower. Using Bhabha events, an energy resolution of approximately $3 \%$ at 50 GeV has been achieved.


Figure 20 : The SLD luminosity monitor, showing the MASiC, the LMSAT and masks.

### 3.2.7 SLD Event Trigger

The SLD event triggers initiate readout of the detector when a potentially interesting event has occured and determine what events are written to tape for analysis. The design of these triggers is greatly simplified by the 8.3 ms crossing time of the SLC. The triggers are described in detail in [35] and [36]; they are summarized below.

- Bhabha trigger: requires total energy of at least 12.5 GeV simultaneously in both north and south EM2 sections of the LUM.
- Energy trigger: the EM and/or HAD calorimeters of the LAC must have at least 8 Gev of total deposited energy in them. Only towers above the threshold of 246 MeV (1.298 GeV) for EM (HAD) contribute.
- Muon trigger: requires one charged track with 9 CDC superlayers hit and calorimentric counts in the opposite WIC octant.
- Random trigger: This trigger writes out data for background studies every 20 seconds, without regard to the status of other triggers.
- Tracking trigger: requires at least 2 charged tracks, separated by at least $120^{\circ}$ and each passing through at least 9 superlayers of the CDC. A cell hit is recorded if at least 6 of its 8 sense wires record pulses above a threshold. Additionally, the CDC cells hit must match a configuration in a pre-calculated CDC cell pattern map, which contains all the possible trajectories of charged tracks with $p_{\perp}>250 \mathrm{MeV} / \mathrm{c}$.
- WAB (Wide Angle Bhabha) trigger: requires two charged and back-to-back tracks in the CDC, with no regard to length of track.
- HAD trigger: records hadronic events with a combination of the Energy and Tracking triggers. One track must traverse at least 9 CDC superlayers and there must be 2 GeV of energy deposited in the LAC.

All sub-systems are read out for a trigger except for the Bhabha trigger.
The control software for SLD was run on a Digital Equipment Corporation VAX /VMS computer cluster. A typical SLD event is 250-300 kbytes in size, and consists of approximately $40 \%$ CRID information and $25 \%$ each from the CDC and VXD3. Background conditions, dependent on the tuning of the SLC, strongly impact event size. A typical trigger rate for low backgrounds during the 199798 data run was 0.5 Hz . It has been estimated that the combined efficiency for the three hadronic triggers (Energy, Tracking, HAD), exceeded $96 \%$ for hadronic events [35].

### 3.2.8 SLD Monte Carlo

Like most other modern particle physics experiments, SLD data analysis relies on detailed Monte Carlo simulations of both fundamental physical reactions in $e^{+} e^{-}$collisions (an "event") and detector response to those events. Simulation of reconstructed events is a two step process: first, the generation of an $e^{+} e^{-}$reaction and all of its daughter products, and then a full simulation of SLD response to that event.

The simulation of 1996-98 was generated using the JETSET 7.4 [37] event generator, which is based on the LUND fragmentation model with parameters tuned to hadronic $e^{-} e^{+}$annihilation data [38]. To simulate beam-induced backgrounds and electronic hardware noise, random trigger data are overlaid with the results of the simulation.

The GEANT 3.21 [39] software package is used to track particles through the detector, accounting for physical effects such as the magnetic field, multiple scattering, energy loss, and detector response. Calorimeter shower simulations are based on GEANT EGS4 [40] for electromagnetic interactions and GEANT GHEISHA [41] for hadronic ones. Finally, all of the simulated events are processed by the same reconstruction code used to reconstruct data events.

## C H A P TER 4

## KAON IDENTIFICATION USING THE SLD CRID

Kaon identification is an important part of this analysis, and the SLD Cherenkov Ring Imaging Detector (CRID) $[21,29]$ provides SLD with efficient and pure charged particle identification. This chapter provides some general properties of Cherenkov radiation and ring imaging with the CRID. One can find a more thorough discussion of ring imaging detectors in [42]. The results of efficiency and purity studies for kaon identification performed by the author conclude this chapter.

### 4.1 Principles of Cherenkov Ring Imaging

A charged particle which traverses a dielectric medium with a velocity exceeding the phase velocity of light in that medium emits photons in a coherent wave front, similiar to a shock wave in hydrodynamics. This phenomenon is known as Cherenkov radiation, named after its discoverer in 1934 [43]. In 1937, this phenomenon was explained within the context of classical electrodynamics by Frank
and Tamm [44], and in 1940, Ginzburg provided the quantum theoretical calculation, which resulted in only minor modifications to the classical one [46].

For a particle travelling with velocity $v=\beta c$ in a medium with an index of refraction $n$, Cherenkov photons are emitted in a cone with an angle $\theta_{C}$ to the particle's direction such that:

$$
\begin{equation*}
\cos \theta_{C}=\frac{1}{\beta n}, \tag{4.1}
\end{equation*}
$$

and with a uniform azimuthal distribution. From this relation, it is implied that a particle must exceed a threshold velocity $\beta_{t}=\frac{1}{n}$, corresponding to a threshold momentum $p_{t}=\frac{m c}{\sqrt{n^{2}-1}}$, to produce Cherenkov radiation. The Frank-Tamm relation ( $[44,45])$ gives the spectrum of the Cherenkov radiation:

$$
\begin{equation*}
\frac{d N}{d \lambda}=\frac{2 \pi}{\lambda^{2}} \alpha z^{2} L \sin ^{2} \theta_{C} \tag{4.2}
\end{equation*}
$$

where $d N$ is the number of Cherenkov photons in a photon wavelength interval $d \lambda$, $\alpha$ the electromagnetic fine structure constant, $L$ the path length of the particle, and $z e$ is the charge of the particle (with $e$ the charge of the electron). Note that for a constant $n$, this relation shows that Cherenkov photons are predominantly produced at short wavelengths. For a Cherenkov device with detection efficiency $\epsilon(\lambda)$, the number of photons detected is given by:

$$
\begin{equation*}
N_{d e t}=2 \pi \alpha z^{2} L \int_{\Delta \lambda} \frac{\epsilon(\lambda)}{\lambda^{2}}\left[1-\frac{1}{\beta^{2} n^{2}(\lambda)}\right] d \lambda . \tag{4.3}
\end{equation*}
$$

The wavelength dependence of $n$ indicates the medium used in the detection device is dispersive. For the limit of a constant $n$ (i.e. away from any absorption bands), the number of detected photons may be approximated by:

$$
\begin{equation*}
N_{d e t}=N_{0} Z^{2} L \sin ^{2} \theta_{C}, \tag{4.4}
\end{equation*}
$$

with $N_{0}$ defined as:

$$
\begin{equation*}
N_{0}=2 \pi \alpha \int_{\Delta \lambda} \frac{\epsilon(\lambda)}{\lambda^{2}} d \lambda \tag{4.5}
\end{equation*}
$$

which describes the response of a particular Cherenkov detector.
The principle of Cherenkov ring imaging, based on the focusing of the conical Cherenkov surfaces into a focal plane and using the radius of the Cherenkov ring and the measured particle momentum for particle identification, was first propsed by Roberts in 1960 [47]. The first practical detector was built by Séguinot and Ypsilanti in 1977 [48], and utilized an admixture of benzene in a gas-filled multiwire proportional chamber. This technique made it practical to resolve single photons with good spatial resolution. In 1980, Anderson explored a new photocathode material, tetrakis(dimethylamino)ethylene (TMAE), which is sensitive to the dominant UV photon energies [49], and enabled the use of time projection chambers (TPCs) with transparent windows. The Delphi RICH [50] and SLD CRID [51], which are of similiar design, represent the first large scale use of Cherenkov ring imaging in multi-purpose particle detectors.

### 4.2 SLD CRID Design

The SLD CRID consists of a barrel detector, which provides particle identification in the region $|\cos \theta<0.68|$, and an endcap detector, designed to provide coverage for $0.82<|\cos \theta|<0.98$. The barrel CRID was completed two years before the endcap CRID, and due to problems with endcap tracking, the endcap CRID never reached its full physics potential. As it is not used in this analysis, we will focus exclusively on the barrel CRID; details of the hardware performance of the endcap CRID can be found in References [51]- [53].


Figure 21: A sector of the Barrel CRID, shown in an axial view (top) and a transverse view (bottom).

The barrel CRID is shown in Figure 21. It is constructed with two Cherenkov radiators, one liquid and one gas, to provide charged particle identification over a wide momentum range. The liquid $\mathrm{C}_{6} \mathrm{~F}_{14}$ radiator is contained in 40 quartzwindowed trays, each with a 1 cm liquid thickness. The photons from the liquid radiator pass directly into the 40 Time Projection Chambers (TPCs), each of which is filled with $\mathrm{C}_{2} \mathrm{H}_{6}$ and $0.1 \%$ TMAE. The gaseous radiator, filled with $85 \% \mathrm{C}_{5} \mathrm{~F}_{12}$ and $15 \% \mathrm{~N}_{2}$, relies on an array of 400 spherical mirrors to focus the photons into rings on the TPCs. In both cases, the Cherenkov photons ionize the TMAE, and the photoelectrons are drifted to multiwire proportional chambers (MWPC) located at the outer end of the TPCs.

### 4.2.1 Cherenkov Radiators

The SLD barrel CRID was designed to provide particle identification for momenta up to $6 \mathrm{GeV} / \mathrm{c}(e / \pi)$ or $30 \mathrm{GeV} / \mathrm{c}(\pi / K / p)$, thus requiring a combination of liquid $\left(\mathrm{C}_{6} \mathrm{~F}_{14}\right)$ radiators to cover the lower momentum region, and gaseous $\left(\mathrm{C}_{5} \mathrm{~F}_{12}\right)$ radiators to cover the higher momentum region. These fluorocarbon radiators were chosen for their refractive indices (to provide coverage in particle identification with a minimal gap in momentum), their transmission at relevant UV wavelengths, relatively low chromatic dispersion, and compatibility with other CRID materials. For $\lambda=190 \mathrm{~nm}$, the indices of refraction for the liquid and gas radiators, respectively, are $n_{\text {liq }}=1.2723$ [54] and $n_{\text {gas }}=1.0017$ [55]. The Cherenkov angle curves for the most important particles for this analysis, $K^{ \pm}$and $\pi^{ \pm}$, along with curves for $p / \bar{p}$,
are shown in Figure 22. Figure 23 shows the separation ability for the two barrel CRID radiators for the combinations of $e / \pi, \pi / K$, and $K / p$. Separation at the $3 \sigma$ level for the $\pi / K$ combination is available from 0.3 to almost $30 \mathrm{GeV} / \mathrm{c}$.


Figure 22: The Cherenkov angle for the liquid (solid lines) and gas (dashed lines) radiators in the Barrel CRID as a function of momentum for the three hadronic particle hypotheses.

### 4.2.2 Single Electron Sensitive Detector

An array of 40 (TPCs) detects the Cherenkov photons. Each TPC is filled with the $\mathrm{C}_{2} \mathrm{H}_{6}$ drift gas and an $0.1 \%$ admixture of the photocathode TMAE, which is introduced to the drift gas by bubbling the gas through liquid TMAE. UV Cherenkov photons ionize the TMAE and release photoelectrons into the drift gas. A $400 \mathrm{~V} / \mathrm{cm}$ electric field inside the TPC causes the photoelectrons to drift parallel to the SLD


Figure 23 : Separation ability of the two Barrel CRID radiators for various particle combinations. Separation at the $3 \sigma$ level for the $\pi / K$ combination is available over nearly 2 orders of magnitude.


Figure 24: View of a Barrel CRID TPC.
magnetic field to a multiwire plane. Figure 24 shows a single TPC box, which is 126.6 cm in length and 30.7 cm wide. The thickness goes from 9.2 cm at the instrumented detector end to 5.6 cm at the high voltage (HV) end, which prevents electrons from being lost near the TPC readout face due to transverse diffusion. The two windows of the TPC box consist of fused quartz, and the sides are constructed from G-10 fiberglass epoxy.

Figure 25 shows the TMAE quantum efficiency and the transmission of fused quartz. The applied high voltage sweeps the photoelectrons to the TPC anode wire plane, and is set for the best time to distance resolution in order to reconstruct the $z$ coordinate. The $x$ coordinate is determined by the wire address, and the $y$ from the charge division on the wire. Each coordinate has a resolution within


Figure 25 : Quantum efficiency and transmission for several materials, including TMAE and quartz.


Figure 26 : The detector end of a SLD TPC. Field shaping wires and blinds directing the photoelectrons to the anode wire plane are shown.

1 mm , resulting in an approximately 4 mrad resolution on the Cherenkov angle. Figure 26 shows the detector end of a TPC in more detail. The array of field shaping wires and blinds is designed to direct electrons to the anode wires. The blinds help prevent photon feedback ionization. The anodes collect the signal and pass it to the front end electronics.

The preamplifier signals are sampled in 67.2 ns "buckets", stored in Analog Memory Units (AMUs), and then digitized in Analog to Digital Converters (ADCs) when an event trigger occurs. Data Correction Units (DCUs) zero suppress the amplitudes and apply a pedestal correction. Waveform Sampling Units (WSMs) are used to correct the bucket by bucket AMU characteristics and apply a pulse-finding algorithm. The digitized amplitude is then sent to ALEPH Event Builders (AEBs) which format the data for permanent storage on tape. The CRID information stored consists of the pulse leading edge time, the pulse heigh and width, and a 32 bit quality word characterizing the pulse. More detail on CRID readout electronics and processing can be found in Reference [56].

Information about the long-term operational performance of the Barrel CRID, including studies of wire aging and breakage, gas and liquid circulation and purification, electronics performance, CRID hardware monitoring, estimates of the Cherenkov $N_{0}$, and numbers of photoelectrons for gas and liquid rings, can be found in Reference [57].

### 4.3 Particle Identification Using A Maximum Likelihood Method

For particle identification using CRID information, a maximum likelihood method $[58,59]$ is used to distinguish between the five possible particle candidates $\left(e^{ \pm}, \mu^{ \pm}, \pi^{ \pm}, K^{ \pm}, p / \bar{p}\right)$. This method has the advantages of best use of available information, smooth behaviour as a particle's momentum crosses the Cherenkov threshold for a particular particle, and a simple framework for combining gas and liquid radiator information. The sum of the five particle identification likelihoods is normalized to 1. Particle identification is based on the logarithms of the differences between these likelihoods.

For any given hypothesis, a likelihood function $\mathcal{L}$ is the probability of observing the given data distribution for that hypothesis. For particle identification, a hypothesis is a specific set of particle assignments, $\left\{h_{k}\right\}$, for each track $k$ in an event, and using a background model $B(\vec{x})$. Poisson statistics gives the probability to observe $n$ photoelectrons in the CRID when $\bar{n}$ is the expected number for a given hypothesis $\left\{h_{k}\right\}$ :

$$
\begin{equation*}
P(n \mid \bar{n})=\frac{\bar{n}^{n}}{n!} e^{-\bar{n}} \tag{4.6}
\end{equation*}
$$

Information from the spatial distribution of photons is also available. If $P(\vec{x})$ is the probability for a given photoelectron to be in a differential volume $d \vec{x}^{3}$, we can define the expected density of photoelectrons in that volume to be $\rho(\vec{x})=\bar{n} P(\vec{x})$. After taking the permutations of the $n$ photoelectrons into account, the overall
likelihood is given by:

$$
\begin{align*}
\mathcal{L} & =P(n \mid \bar{n}) P\left(\left\{\vec{x}_{i}\right\}\right)  \tag{4.7}\\
& =\bar{n}^{n} e^{-\bar{n}} \prod_{i=1}^{n} P\left(\vec{x}_{i}\right) \\
& =e^{-\bar{n}} \prod_{i=1}^{n} \rho\left(\vec{x}_{i}\right)
\end{align*}
$$

where the index $i$ runs over all observed photoelectrons. The photoelectron density consists of the track-independent background term and a term representing each track's produced Cherenkov ring:

$$
\begin{equation*}
\rho(\vec{x})=B(\vec{x})+\sum_{k} \rho_{k, h_{k}}(\vec{x}), \tag{4.8}
\end{equation*}
$$

where $\rho_{k, h_{k}}(\vec{x})$ represents the density due to track $k$ for the particle hypothesis $h_{k}$, and $B(\vec{x})$ is a function representing background signal.

Theoretically, it is now straightforward to calculate $\mathcal{L}$ for all 5 hypotheses for each track. The set $\left\{h_{k}\right\}$ which maximizes the likelihood function is the best answer. However, it is not practical to compute such a large number of combinations. Instead, it is assumed that the most likely hypothesis $h_{k}$ for track $k$ is largely independent of the hypotheses for other tracks $\left\{h_{j}\right\}_{j \neq k}$. Iteration through the tracks in an event continues until the set $\left\{h_{k}\right\}$ is stable. The likelihood for a hypothesis
$h_{k}$ can be written as:

$$
\begin{equation*}
\mathcal{L}_{k, h_{k}} \equiv e^{-M_{k, h_{k}}} \prod_{i}\left(B_{k}+\rho_{k, h_{k}}\left(\vec{x}_{i}\right)\right), \tag{4.9}
\end{equation*}
$$

where $B_{k}$ is the track-independent background and $M_{k, h_{k}}$ is the number of photoelectrons expected after accounting for total internal reflection. It is the logarithm of the relative likelihood, $\mathcal{L}_{k, h_{k}}^{\prime}$, that is used for physics analysis:

$$
\begin{equation*}
\log \mathcal{L}_{k, h_{k}}^{\prime}=-M_{k, h_{k}}+\sum_{i} \log \left(1+\frac{\rho_{k, h_{k}}\left(\vec{x}_{i}\right)}{B_{k}\left(\vec{x}_{i}\right)}\right) \tag{4.10}
\end{equation*}
$$

The method uses a simplified, uniform, background $B_{k}\left(\vec{x}_{i}\right)$. The iterative procedure begins with the pion hypothesis and converges for most events in only 2-3 iterations.

### 4.4 Particle Identification Performance

As particle identification is a significant factor in this analysis, it is important to understand the performance of the SLD CRID in selecting and identifying particles, particularly kaons and pions. Pions are significant because a pion which is misidentified as a kaon is almost always "wrong-sign" and serves to dilute the kaon tag (discussed in detail in Chapter 5.) The author has performed a number of checks and calculations on CRID performance which are of interest to this analysis and which will be discussed in this section.

### 4.4.1 Quality Kaon Track Selection

Previous to this analysis, quality track selection criteria for CRID identification had been fixed to values resulting from studies performed previous to the installation of the new SLD Vertex Detector (VXD3) in 1996. Tracking has been significantly upgraded since 1996; hence it is interesting to revisit the track quality cuts to verify that they are optimal for use with VXD3, and that they result in the highest possible purity and efficiency for kaon identification.

For a track to be considered for identification in the CRID, a combination of track-related requirements (to ensure good reconstruction of the track in the SLD tracking volume) and CRID-related requirements (to ensure that the CRID could make a quality relationship between the Cherenkov rings and tracks in an event), are used. It is useful to note the previously used cuts to discuss which were updated and the reason. The following are the previously used CRID selection cuts [60]:

- $N_{\text {CDChits }}>40$. There must be more than 40 CDC hits on the track. This ensures there are enough points to have a good momentum measurement.
- $p_{\text {tot }}>0.8 \mathrm{GeV} / \mathrm{c}$. This cut on the total reconstructed momentum of a track reduces the contamination from fragmentation and other tracks which were not produced as part of the hadronic decay of the $Z^{0}$.
- $\chi_{C D C}^{2} / D O F<5$. This requirement on the quality of the CDC track fit excludes poorly found and spurious tracks.
- CRID Status: The barrel CRID High Voltage must be on and the detector operational.
- Track Polar angle $|\cos \theta| \leq 0.68$. This cut was motivated by the acceptance region of the gas radiator and mirrors, as well as to decrease the likelihood of misidentification due to total internal reflection in the liquid radiator at low polar angles.
- CDC last hit radius $r_{\text {last }} \geq 90 \mathrm{~cm}$. Since the CRID is dependent on a reliable extrapolation of the particle trajectory to link a track to its produced Cherenkov ring, it is important to select the highest quality tracks. This cut requires a hit in the outermost layer of the CDC , which minimizes the lever arm for extrapolation. It also reduces the likelihood the track is a particle that scattered or decayed in flight, and therefore unlikely to be identified properly.

Additionally, in the liquid region for CRID particle identification, the following standard CRID reconstruction flags are used:

- BADID: This flag is set if there is no valid liquid identification information. This subsumes the requirements that tracks pass through the liquid radiator trays, the TPC containing the majority of the expected liquid ring to be active and functional, and that the track transverse momentum be above 0.150 GeV/c.
- TPCBAD or TPCSICK: While BADID removes tracks for which the primary TPC is on and functioning, this additional flag is used to indicate whether the two TPCs containing most of the expected liquid ring are functional.
- NOMIP: Because of a concern about mistracked tracks, and because there is no tracking volume outside the radius of the CRID, a requirement that an ionization deposit (saturated hits) be present in a CRID TPC within a loose region of 3 cm from the extrapolated position of the track is imposed. This requirement is in place only if a track has passed through the active volume of the TPC.

All of the flags must be unset for a track using the liquid information for it to be considered a quality CRID track. In the liquid region, a log-likelihood difference of 5 is required for particle identification (i.e. a pion must have $\log \mathcal{L}_{\pi}-\log \mathcal{L}_{K}>5$ and $\log \mathcal{L}_{\pi}-\log \mathcal{L}_{p}>5$. In the region $3<p<5 \mathrm{GeV} / \mathrm{c}$, only protons can be identified and there is no attempt to distinguish between the $\pi$ and $K$ hypotheses.

In the gas region for CRID particle identification, a similiar set of criteria to that of the liquid region are applied:

- BADID: This flag is set if there is no valid gas identification information. This subsumes the requirements that the image not reflect from any known bad mirrors, the TPC onto which the ring image is reflected be active and functional, the track transverse momentum be above $0.150 \mathrm{GeV} / \mathrm{c}$, and that the gas Cherenkov ring be isolated from saturated hits due to minimizing ionizing pulses (MIPs).
- TPCSICK: While BADID removes tracks for which the primary TPC is on and functioning, this additional flag removes TPCs which are known to have reduced detection efficiency.
- NOMIP and NOLIQR: Because the gas momentum regime is above that of the liquid, all gas candidate tracks are above threshold in the liquid and should have liquid rings detected. Thus, one can look for the presence of 4 or more hits with signal weight above background for the liquid Cherenkov angle. This cut can be combined with the NOMIP flag above to obtain a more-efficient selection of CDC tracks which are also present in the CRID.

All of the flags must be unset for a track using the gas information for it to be considered a quality CRID track. For the gas region, a log-likelihood difference of 3 is required, instead of 5. Additionally, in the momentum regime from $2.5<p<$ $10 \mathrm{GeV} / \mathrm{c}$, both the $p$ and $K$ hypotheses are below threshold and there is no ability to discriminate between the two with the gas radiator alone, although combining liquid and gas radiator information can provide some separation.

Using the SLD Monte Carlo, a "base" efficiency, $\epsilon$, and purity, $\pi$, using the cuts above for kaon identification were found to be $37.4 \%$ and $82.1 \%$ respectively. This gives a "sensitivity" of 0.252 , with sensitivity being defined as $S=\epsilon \pi^{2}$. For all possible tracks, the requirements on the number of CDC hits $\left(N_{\text {CDChit }}\right)$, the $\chi_{C D C}^{2} / D O F$ of the CDC track fit, the outermost hit radius in the $\operatorname{CDC}\left(r_{\text {last }}\right)$, track polar angle $(|\cos \theta|)$, minimum total momentum $\left(p_{t o t}\right)$, and log-likelihood differences $\left(\log \mathcal{L}_{K}-\log \mathcal{L}_{\pi}\right.$ and $\left.\log \mathcal{L}_{K}-\log \mathcal{L}_{p}\right)$, were investigated individually. Additionally, the liquid ring requirement for gas tracks (NOLIQR) and the MIP flag for all tracks (NOMIP) were investigated separately as well.

The studies showed that, in fact, most of the requirements were optimal in spite of the improved tracking and significantly different hardware. Two cuts were loosened: the CDC hit requirement, $N_{C D C h i t}$, was relaxed from 40 hits to 23 , and the radius of the outermost hit in the $\mathrm{CDC}, r_{\text {last }}$, was relaxed from 90 cm to 50 . Additionally, the $\chi_{C D C}^{2} / D O F$ of the CDC track fit was loosened, from 5 to 8 . These cuts brought the CRID tracking selection into line with the standard hadronic track cuts used by various $B$ analyses on VXD3 data. The result was a slight increase in efficiency to $39.6 \%$, and a slight decrease in purity, to $81.0 \%$. Combined, a marginal improvement in sensitivity, to 0.260 , was seen for overall kaon identification using the SLD CRID.

### 4.4.2 Identification Efficiencies

An important measurement of CRID performance is the efficiency of identifying kaons. It is particularly important to have direct tests in both data and Monte Carlo. Two studies have been performed to investigate CRID identification efficiencies: kaons from $D^{*}$ decays, and protons from $\Lambda$ decays.

## Efficiency from $D^{*}$ Decays

One pure source of kaons in both data and Monte Carlo is the decay $D^{*} \rightarrow$ $D^{0} \rightarrow \pi \pi K$. These events can be selected by calculating both the $\Delta m_{D^{*}, D^{0}}$ and the $\pi \pi K$ invariant mass peaks, and then placing a cut on $\Delta m$. The $D^{0}$ is reconstructed by searching for a kaon and a pion which, when combined into a single
vertex, have the invariant mass of the $D^{0}$. To reconstruct the $D^{*}$, a slow pion (one with total momentum of greater than $1 \mathrm{GeV} / \mathrm{c}$ ) is combined with the $D^{0}$ vertex, and the invariant mass calculated. Because this is a low statistics test of efficiency, the sample is divided into only two bins based on the tagged kaon momentum: one bin of $2.5-10 \mathrm{GeV} / \mathrm{c}$, and the other for tracks above $10 \mathrm{GeV} / \mathrm{c}$. Distributions for the $\pi \pi K$ invariant mass and $\Delta m$ for the low (high) bin can be seen in Figures 27 and 28 (29 and 30), respectively.

The kaon identification efficiency is extracted by comparing the areas of the background subtracted peak with and without the standard kaon log-likelihood requirements of $\mathcal{L}_{K}-\mathcal{L}_{\pi}>5(3)$ in liquid (gas) and $\mathcal{L}_{K}-\mathcal{L}_{p}>-1$ in both types of radiators. The resulting efficiencies can be found in Table 5. The efficiencies from data and Monte Carlo are in good agreement, with an overall kaon identification efficiency of over $80 \%$ in the data.

Table 5: Kaon efficiencies from $D^{*}$ decays. Given errors are statistical only.

| Momentum Bin $(\mathrm{GeV} / \mathrm{c})$ | Monte Carlo (\%) | Data (\%) |
| :---: | :---: | :---: |
| $2.5-10.0$ | $86.0 \pm 1.7$ | $88.2 \pm 1.5$ |
| Above 10.0 | $88.3 \pm 4.0$ | $82.6 \pm 4.0$ |

## Efficiencies from $\Lambda^{0}$ Decays

A known source of protons is the decay $\Lambda^{0} \rightarrow \pi^{-} p$ and its charge conjugate. These protons can be used to estimate kaon efficiencies for track momentum from 2.5 to $9 \mathrm{GeV} / \mathrm{c}$, as both kaons and protons are separated from pions in this region


Figure 27 : The $D^{*}$ mass peak for $2.5-10 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.


Figure 28 : The $\Delta m_{D^{*}, D^{0}}$ peak for $2.5-10 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.


Figure 29 : The $D^{*}$ mass peak for above $10 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.


Figure 30 : The $\Delta m_{D^{*}, D^{0}}$ peak for above $10 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.
by an absence of a gas ring, but there is no discriminating power between kaons and protons in this momentum range if only the gas radiator is used.
$\Lambda^{0}$ decays can be well-separated from other decays by looking for displaced neutral vertices formed with only 2 tracks (referred to as $V^{0}$ decays) and cutting on the helicity angle $\left|\cos \theta^{*}\right|>0.80$ to eliminate $K_{s}^{0}$ decays. The resulting proton purity, from Monte Carlo studies, is about $85 \%$. The $\Lambda^{0}$ invariant mass peak, showing good agreement between data and Monte Carlo, can be seen in Figures 31 33.

The proton identification efficiency is extracted by comparing the areas of the background subtracted peak with and without the standard proton log-likelihood requirements of $\mathcal{L}_{p}-\mathcal{L}_{\pi}>5$ and $\mathcal{L}_{p}-\mathcal{L}_{K}>-1$. The sample of $\Lambda^{0}$ decays was divided into 3 bins by momentum of the identified proton. The resulting efficiencies can be found in Table 6. The efficiencies from data and Monte Carlo are in agreement, with an overall proton identification efficiency of over $80 \%$ in the data. Additionally, the efficiencies determined from the $\Lambda^{0}$ sample are in good agreement with those determined from the $D^{*}$ sample in the low momentum bin.

Table 6: Proton efficiencies from $\Lambda^{0}$ decays. Given errors are statistical only.

| Momentum Bin (GeV/c) | Monte Carlo (\%) | Data (\%) |
| :---: | :---: | :---: |
| $2.5-4.0$ | $76.2 \pm 0.5$ | $81.4 \pm 0.8$ |
| $4.0-6.0$ | $90.2 \pm 0.4$ | $88.4 \pm 0.9$ |
| $6.0-9.0$ | $88.2 \pm 0.6$ | $82.6 \pm 1.3$ |



Figure 31 : The $\Lambda^{0}$ mass peak for $2.5-4 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.


Figure 32 : The $\Lambda^{0}$ mass peak for $4-6 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.


Figure 33 : The $\Lambda^{0}$ mass peak for $6-9 \mathrm{GeV} / \mathrm{c}$. The dots are data, while the histogram is Monte Carlo simulation.

### 4.4.3 Pion Misidentification as Kaons

A major concern for this analysis is the amount of pion misidentification as kaons. There are two primary reasons for this. First, because a pion which is misidentified as a kaon will be "wrong-sign" half of the time for the final state tag, which will be discussed in the next chapter. Second, since the kaon right-sign fractions for the analysis are parameterized from Monte Carlo, it is important that the data and Monte Carlo agree well.

To study these issues, a high-purity source of pions is required. $K_{s}^{0} \rightarrow \pi^{+} \pi^{-}$ decays are easily identified and are a high purity and copious source of pions in both data and Monte Carlo. $V_{0}$ decays are again used to select events, but the helicity angle cut is instead $\left|\cos \theta^{*}\right|<0.80$. Monte Carlo studies show a pion purity of over $99 \%$ if a cut of $\pm 3 \sigma$ is made on the $K_{s}^{0}$ invariant mass peak after the helicity angle selection is applied. The invariant mass peak for both data and Monte Carlo can be seen in Figure 34, and the pion purity as a function of track momentum (from Monte Carlo) can be seen in Figure 35. The kaon momentum spectra (for kaons used in this analysis) in both data and Monte Carlo can be seen in Figure 36.

The procedure for studying misidentification is similiar to that of studying efficiency: after cutting on the invariant mass of the $K_{s}^{0}$ to obtain a pure sample of pions, the CRID selection cuts are applied both with and without the log-likelihood requirements for separating kaons from pions in the given momentum region. A plot of the data and Monte Carlo misidentification rates as a function of track momentum can be seen in Figure 37. While there is good agreement between data and


Figure 34: The $K_{s}^{0}$ mass peak. The dots are data, while the line is Monte Carlo simulation.


Figure 35: Pion purity in $K_{s}^{0}$ decays, from Monte Carlo.


Figure 36 : Kaon momentum in $B$ decays. The dots are data, while the line is Monte Carlo simulation.

Monte Carlo in the low momentum region, one immediately notes the significant disagreement that occurs as CRID liquid radiator information is lost at higher momenta. This is a known issue for the SLD CRID and has been studied [60]. An ad hoc algorithm to compensate for the difference by smearing the Monte Carlo was developed [60], and has been applied to the 1996-98 Monte Carlo. Misidentification rates after the smear was applied can be seen in Figure 38, and vary from 2.5-10\% as a function of track momentum.


Figure 37 : Pion misidentification as a kaon before applying the Monte Carlo correction. The dots are data, while the line is Monte Carlo simulation.


Figure 38 : Pion misidentification as a kaon after applying the Monte Carlo correction. The dots are data, while the line is Monte Carlo simulation.

## CHAPTER5

## EVENT SELECTION AND TAGGING

This chapter covers an essential part of the analysis: the selection of hadronic events containing $B$ mesons, the tagging of their initial and final state $b$ quark flavor, and reconstruction of the boost and decay length of the presumed $B$ meson. The order of the chapter follows the same progression the analysis does: first, we describe hadronic event selection, and then the $B$ event selection. Then, we provide a detailed description of the SLD Initial State Tag, and an explanation of the Kaon Tag for the final state, with a number of cross-checks of these tags which have been performed on the data. The reconstruction of the boost and decay length of the secondary $(B)$ vertex, resulting in a proper time calculation, are discussed in the last sections of this chapter.

### 5.1 SLD Hadronic Event Selection

The triggers described in Section 3.2.7 are designed to keep data acquisition rates to a manageable level; however, their thresholds are kept low enough that
they accept many events which do not involve the production of $Z^{0}$ bosons. The hadronic event filter is used to eliminate a large fraction of those events before the offline reconstruction. After offline reconstruction, the hadronic event selection requirements are used to provide the best sample of events which are wellreconstructed in the detector.

### 5.1.1 Hadronic Event Filter

The first (PASS 1) filter enhances the selection of hadronic events which have initially passed the HAD trigger. The PASS 1 filter, also know as the Energy Imbalance Trigger (EIT), imposes cuts based on 3 LAC quantities and their combinations:

- NEMHI $\geq 10$, where NEMHI is the number of LAC EM towers above the high threshold of 60 ADC counts, equivalent to $\sim 250 \mathrm{MeV}$ from minimum ionizing particles.
- EHI $>15 \mathrm{GeV}$ of energy from minimum ionizing particles, where EHI is the sum of the energy deposited in all LAC EM (HAD) towers above the high threshold of 60 (120) ADC counts ( $\sim 250(1.3 \mathrm{GeV}) \mathrm{MeV}$.)
- $E L O<140 \mathrm{GeV}$ of energy from minimum ionizing particles, where ELO is the sum of the energy deposited in all LAC EM (HAD) towers above the low energy threshold of 8 (12) ADC counts ( $\sim 33(130) \mathrm{MeV})$.
- $2 \cdot E H I>3 \cdot(E L O-70)$
- $N E M H I>0$ for both hemispheres

The first and third requirements are to reject SLC-based (beam-gas) background events, where one of the incoming beams interacts with residual gas in the hemisphere, yielding tracks which enter the detector and trigger readout. The third and fifth requirements combine to ensure that an event does not satisfy the first two requirements by depositing large amounts of background energy. This removes beam-wall events, which have large numbers of muons.

The EIT filter rejects $\sim 97 \%$ of background events which were written to tape. The combined readout triggers and EIT filter results in a selection efficiency for hadronic $Z^{0}$ events of $\sim 92 \%$ [35]. The events which pass this filter are sent to a second (Pass 2) filter where they are sorted into hadronic, $\mu$-pair, or WAB candidates. The hadronic requirements are discussed next.

### 5.1.2 Hadronic Event Selection

Standard cuts $[27,61]$ are applied to events which pass the EIT filter to determine if they are hadronic events; these cuts are summarized in Table 7, and distributions of some of the selection variables are shown in Figure 39. Together, they provide a hadronic event selection with well reconstructed tracks and well contained within the SLD.


Figure 39 : Distributions of the Hadronic event selection variables after all cuts have been applied. There is reasonable agreement between the Monte Carlo (line), and the data (dots).

Table 7: Hadronic Event Selection Requirements

EIT filter passed
VXD, CDC, and LAC operating
$e^{-}$beam polarization measurement available
Precisely determined interaction point (IP)
At least 7 charged tracks whose $p_{\perp}>0.2$ and distance of closest approach to the IP in the $r z$ plane $<5 \stackrel{\perp}{\mathrm{~cm}}$
At least 3 charged tracks with at least 2 VXD hits each
At least 18 GeV of visible energy, with the $\pi^{ \pm}$mass assumed for each track
$\left|\cos \theta_{\text {thrust }}\right|<0.85$, where $\theta_{\text {thrust }}$, the thrust axis polar angle, is measured with respect to the $e^{+}$beam direction.

The thrust axis, $\hat{T}$, is determined using the LAC energy clusters and is defined as the axis that maximizes the thrust $T$ of an event:

$$
\begin{equation*}
T=\frac{\sum_{\text {clusters }}|\vec{p} \cdot \hat{T}|}{\sum_{\text {clusters }}|\vec{p}|} \tag{5.1}
\end{equation*}
$$

where $\vec{p}$, the momentum of the energy cluster, is determined using the IP as the origin and assuming the particle that caused the energy deposition is a pion. So that the event is well reconstructed in the barrel of the detector, we require $\left|\cos \theta_{\text {thrust }}\right|<0.85 . \hat{T}$ is generally a good indicator of the initial quark direction, and this will be important for the initial state tagging which will be described later in this chapter.

These requirements eliminate all but $0.2 \%$ of possible backgrounds. Leptonic events, which tend to have fewer tracks than hadronic events, are removed by the combination of track requirements; the total energy requirement eliminates most
beam related backgrounds. The remaining background is dominated by higher multiplicity $Z^{0} \rightarrow \tau^{+} \tau^{-}$events. After hadronic event selection, there remain 310,254 events in the 1996-98 data out of an estimated 400,000 hadronic $Z^{0}$ decays.

### 5.2 SLD $B$ Event Selection

The hadronic event sample is a mix of $u d s c$ and $b \bar{b}$ events. A selection process involving three neural network algorithms [62] has been developed: the first allows for cleanly choosing reconstructed seed vertices (other than the IP) for track attachment, and the other two attach tracks to the choosen seed vertex. It can then be determined if that vertex is more likely to be a $b$ or a $c$ decay.

### 5.2.1 Topological Vertexing

The first step to $b \bar{b}$ event selection is to use a method developed at SLD called topological vertexing to reconstruct secondary vertices for analysis. This unique statistical method treats tracks as probability tubes, rather than lines, to perform a more accurate vertex fit by summing the track probabilities in 3-D space, where the vertices will then appear as regions of high overlap probability [63]. This analysis uses the topological vertexing technique with various sets of tracks for $b$ selection, initial state tagging, and reconstruction of the $B$ decay vertex.

The construction of the Gaussian tube $f_{i}(\vec{r})$ can be seen in Figure 40. The width of the tube is the uncertainty in the measured track location near the IP,
and the 3-D trajectory of the tube can be described:

$$
\begin{equation*}
f_{i}(\vec{r})=\exp \left\{-\frac{1}{2}\left[\left(\frac{x^{\prime}-\left(x_{0}^{\prime}+\kappa\left(y^{\prime}\right)^{2}\right)}{\sigma_{T}}\right)^{2}+\left(\frac{z-\left(z_{0}+\tan (\lambda) y^{\prime}\right)}{\sigma_{L}}\right)^{2}\right]\right\} . \tag{5.2}
\end{equation*}
$$

The $z$ axis is defined as the beam axis with the positive direction given by the $e^{+}$beam, and the $x, y$ coordinates of the track have been transformed into $x^{\prime}, y^{\prime}$ for each track such that the momentum of the track is parallel to the positive $y^{\prime}$ coordinate axis in the $x y$ plane at the point of closest approach to the IP; $\sigma_{T}$ and $\sigma_{L}$ are the track position errors in the $x y$ plane and the $z$ direction, respectively; $\kappa$ is a function of the magnetic field, track charge, and track's transverse momentum; $\lambda$ is the angle between the track momentum and the positive $y^{\prime}$ axis, and the $\tan (\lambda) y^{\prime}$ term accounts for track propagation along the $z$ axis. The constructed Gaussian tube is represented by the parallel dotted lines in Figure 40. A figure showing the probability functions projected into the $x y$ plane for a typical Monte Carlo $b \bar{b}$ event can be seen in Figure 41a.

It is reasonable to expect that vertices are most likely to occur in areas where at least two tracks overlap; thus, the first step is to calculate analytically the spatial locations of the maxima of the product $f_{i}(\vec{r}) f_{j}(\vec{r})$. This significantly reduces the 3 -D search area, and we will designate the located maxima $\vec{r}_{i j}$. To further reduce the number of potential vertices, both tracks are required to pass a cut of $\chi^{2}<\chi_{0}^{2}$, where $\chi_{0}^{2}$ is an input parameter from Monte Carlo and $\chi^{2}$ is the square of the distance of the track from $\vec{r}_{i j}$ normalized by the track error. Once these seed


Figure 40 : The construction of the Gaussian tube to be used in topological vertexing for a given track.


Figure 41: The track (a) and vertex (b) functions projected onto the $x y$ plane for a typical $Z^{0} \rightarrow b b$ Monte Carlo event.
vertices have been selected, the total vertex probability is defined as:

$$
\begin{equation*}
V(\vec{r})=\sum_{i=0}^{N} f_{i}(\vec{r})-\frac{\sum_{i=0}^{N} f_{i}^{2}(\vec{r})}{\sum_{i=0}^{N} f_{i}(\vec{r})}, \tag{5.3}
\end{equation*}
$$

where $N$ is the number of tracks in the hemisphere. This function is a smooth, continuous function so the maxima can be found. Note that for a region with a single significant track, $V(\vec{r})=0$. The first term gives a measure of the multiplicity, and the second ensures that the probability is signicant only in areas of high vertex probability. An example of the projection of this function for a $b \bar{b}$ event can be seen in Figure 41b.

The next step is to locate the nearest maximum in $V(\vec{r})$ for each $\vec{r}_{i j}$. These adjusted $\vec{r}_{i j}$ are then clustered to produce the final reconstructed vertices. We begin with the maximum that produces the largest $V(\vec{r})$ and merge in other maxima which are unresolved. Two locations are considered resolved if

$$
\begin{equation*}
\frac{\min \left\{V(\vec{r}): \vec{r} \in \overrightarrow{r_{1}}+\alpha\left(\overrightarrow{r_{2}}-\overrightarrow{r_{1}}\right), 0 \leq \alpha \leq 1\right\}}{\min \left\{V\left(\overrightarrow{r_{1}}\right), V\left(\overrightarrow{r_{2}}\right)\right\}}<R_{0} \tag{5.4}
\end{equation*}
$$

where $\min \left\{V\left(\overrightarrow{r_{1}}\right), V\left(\overrightarrow{r_{2}}\right)\right\}$ is the lower of the two values and the numerator is the minimum of $V(\vec{r})$ on a straight line joining $\overrightarrow{r_{1}}$ and $\overrightarrow{r_{2}}$; in practice, $V(\vec{r})$ is determined for a finite number of points on this line. The value $R_{0}$ is a tunable parameter which determines the number of vertices found and varies between 0 and 1 . If any remaining maxima remain unresolved, the one producing the largest $V(\vec{r})$ is used to begin a new cluster and the clustering process above is repeated. This process is
iterated until all maxima $\vec{r}_{i j}$ are included in a cluster; these clusters then become the reconstructed vertices.

### 5.2.2 Secondary Vertex Selection and Track Attachment

Tracks satisfying minimal quality cuts of $p_{\perp}>0.25 \mathrm{GeV} / \mathrm{c}$ and $N_{\text {hit }}^{V X D} \geq 3$ are fed into the topological vertexing routine to produce seed vertices. Any track with a 3 -D impact parameter relative to the IP $>3 \mathrm{~mm}$, or one consistent with production from a $\gamma, K^{0}$, or $\Lambda^{0}$ decay is removed as well.

Events are divided into two hemispheres using the thrust axis, and the vertexing procedure described above is performed on the tracks in each one separately. Reconstructed vertices are required to be within a 2.3 cm radius of the center of the beam pipe to remove false vertices due to interactions with detector materials. To remove any $K^{0}$ decays which escaped the track cuts, a mass cut of $\left|M_{v t x}-M_{K^{0}}\right|<0.015 \mathrm{GeV} / \mathrm{c}^{2}$ is applied. Vertices are then passed into a neural network to further improve background rejection [62]. Input variables for the neural network are the vertex to IP flight distance, that distance normalized by its error, and the angle between the flight direction and the total momentum vector of the tracks included in the vertex. Distributions of the selection variables and the neural network output can be seen in Figure 42. The output of the neural network is a real number between 0 and 1 , and vertices with a neural network output of $>0.7$ are retained. According to Monte Carlo studies, at least one good vertex, where "good" is defined as being a vertex constructed with only $B$ daughter tracks, is


Figure 42 : Distributions of seed vertex selection variables. (a) distance from IP, (b) normalized distance from IP, (c) angle between flight direction and vertex momentum, (d) neural network output. The arrow indicates the accepted region.
found in $72.7 \%$ of $b$ hemispheres, and more than one good vertex in about $16 \%$ of $b$ hemispheres.


Figure 43: Schematic illustration of the quantities used in the trackattachment procedure described in the text (not to scale).

As $B$ decays often result in tracks displaced from the decay vertex all of the tracks from the heavy hadron may not originate at the same point. To recover this information, a second neural network has been developed to attach tracks to the secondary vertex. Four of the inputs are defined at the point of closest approach (POCA) of the track to the axis joining the secondary vertex to the IP. These are the transverse distance from the track to that axis $(T)$, the distance from the IP along that axis to the POCA $(L)$, that distance divided by the flight distance of the secondary vertex from the IP $(L / D)$, and the angle between the track and the IP-secondary vertex axis $(\alpha)$. The final input is the 3-D impact parameter of the track to the IP normalized by its error $\left(b / \sigma_{b}\right)$. A schematic diagram of these quantities can be seen in Figure 43, and distributions of the variables, along with


Figure 44 : Distributions of cascade track selection variables. (a) $T$, (b) $L$, (c) $L / D$, (d) $\alpha$, (e) $b / \sigma_{b}$, (f) neural network output. The arrow indicates the accepted region.
the neural network output, can be seen in Figure 44. The network is trained to accept only tracks from a $B$ or $D$ decay, and to reject tracks from the IP or strange particle decays. To optimize the charge reconstruction, any tracks not already a part of an accepted secondary vertex and with $N_{\text {hit }}^{V X D} \geq 2$ are tried, including those which were removed from the secondary vertex finding procedure described earlier. Tracks with a neural network output of $>0.6$ are denoted as a secondary vertex track.

Even for a secondary vertex reconstructed with only $B$ decay products, some $B$ decay products will be not found and therefore will be missing track charge and momentum information. From Monte Carlo simulation, we know that about 90 \% of the charged decay products will produce a track with VXD hits that link to a track reconstructed in the CDC. To recover part of the inefficiency, we can use vectors made of VXD hits that are not associated with a CDC track. This raises the SLD tracking efficiency to $\simeq 97 \%$ in Monte Carlo simulation for $B$ decays. We require a VXD-only vector to have at least one hit in each of the 3 VXD3 layers. A helix fit to the VXD hits is performed to construct a trial track. We then use a third neural network to associate these trial tracks with a reconstructed secondary vertex. This neural network uses the first four inputs noted in the paragraph on full reconstructed track attachment. Vectors which have a neural network output of $>0.5$ are designated as a secondary vertex track. Distributions of the selection variables, along with the neural network output, can be seen in Figure 45. Once the VXD-alone vector has been attached to a vertex, the helix fit is performed one


Figure 45 : Distributions of VXD-vector selection variables. (a) $T$, (b) $L$, (c) $L / D$, (d) $\alpha$, (e) neural network output. The arrow indicates the accepted region. The probability to assign the correct charge to a vector based on its fitted curvature is shown in (f), both with and without the secondary vertex as a constraint.
more time, but with the secondary vertex as a fourth helix point to improve the curvature determination. Using this procedure, the charge reconstruction is correct for about $85 \%$ of attached VXD-alone tracks.

### 5.2.3 Selecting $B$ Vertices

After the tracks have been attached using the neural net, it is necessary to determine whether the vertex is a $b$ quark decay or not. The invariant mass of the secondary vertex is calculated, with each track assigned the mass of a $\pi^{ \pm}$. To account for missing neutral particles, this invariant mass is corrected and transformed into what is called the $p_{T}$-corrected mass. It is assumed that the true $B$ meson momentum should be in the same flight direction of the vertex, and therefore a minimum amount of missing transverse momentum is added to the invariant mass. For a given vertex, its invariant mass is simply:

$$
\begin{equation*}
M_{\text {invtotal }}^{2}=\sum_{i} m_{i}^{2}+\sum_{i}\left|\vec{p}_{i}\right|^{2} \tag{5.5}
\end{equation*}
$$

where the sums are over the mass and momentum vectors of each daughter particle in the decay. Assuming all the visible daughter tracks are $\pi^{ \pm} \mathrm{s}$, the visible invariant mass is:

$$
\begin{equation*}
M_{t r k s}=\sqrt{E^{2}-p^{2}} \tag{5.6}
\end{equation*}
$$

where $E, p$ are the reconstructed quantities for the vertex. It is now assumed that all visible energy not associated with a track, as calculated in the center of mass
of the vertex, is from unidentified neutral particles. The invariant mass of the vertex is therefore $M=E_{n}+E_{t r k s}$, where $E_{t r k s}^{2}=M_{t r k s}^{2}+P_{T}^{2}$, is the vertex energy from charged tracks in the track center of mass, and $p_{T}$ is the vertex transverse momentum. In the vertex center of mass, $\vec{p}_{n}$, the momentum of the unseen neutral particles, must be equal and opposite that of the visible charged particles (see Figure 46). In reality, the value used is that which will align the charged momentum to within the errors of the determined IP-vertex flight direction. Thus,

$$
\begin{equation*}
M_{p_{T}}=\sqrt{M_{t r k s}^{2}+p_{T}^{2}}+\left|p_{T}\right| \tag{5.7}
\end{equation*}
$$

where $M_{p_{T}}$ is the minimum mass the secondary vertex could have in order to produce a vertex with an invariant mass $M$. A plot of this quantity for all events passing the hadronic event selection can be seen in Figure 47. By requiring $M_{p_{T}}>$ $2 \mathrm{GeV} / \mathrm{c}^{2}$ a sample of $Z^{0} \rightarrow b \bar{b}$ events which is $98 \%$ pure is acquired with $55 \%$ overall efficiency for hadronic $b$ selection.

In addition to the $M_{p_{T}}$ requirement, we require the reconstructed vertex to have a total track charge of 0 after vx-alone vector attachment. This significantly enhances the $B_{d}$ fraction, as can be seen in the charge distribution shown in Figure 48, and reduces the systematic errors resulting from $B_{u}$ mesons. The distribution of reconstructed secondary vertex charge shows that the data and Monte Carlo are in good agreement.


Figure 46: A sketch of the momentum vectors for calculating the $\mathrm{p}_{T^{-}}$ corrected mass. The sum of the charged particles measured momentum must be equal and opposite that of the unseen (and unmeasured) neutral particles.

A method of comparing charge resolution between data and Monte Carlo is to calculate $A_{F B}$ using the charge of the reconstructed vertex. This has been done using the charge of the vertex combined with the electron polarization to determine the direction of the $b$ quark in the event, although these charged vertices are normally eliminated from the analysis. Figure 49 shows a plot of the fraction $\left(\left(\right.\right.$ events $_{q=-1}-$ events $\left._{q=1}\right) /$ sum $\left.^{\prime}\right)$ for 1997-98 data and Monte Carlo. Additionally, this distribution has been fitted to the Standard Model function for $A_{F B}$ :

$$
\begin{equation*}
A_{F B}=-2 A_{b} P_{e}(1-2 W) \frac{\cos (\theta)}{1+\cos ^{2}(\theta)} \tag{5.8}
\end{equation*}
$$

where the Standard Model value of $A_{b}=0.935$ and the 1997-98 average $e^{-}$polarization of 0.73 have been used; $W$, which is in this case the probability to reconstruct the wrong direction for the $b$ quark, is fit for, with a result of $W=$ $0.197 \pm .011(0.195 \pm .005)$ for data (Monte Carlo). Both the plot and the fit show


Figure 47 : The calculated $p_{T}$-corrected mass for hadronic events. Dots indicate data; the solid line indicates all-flavor SLD Monte Carlo. $b$ events are the dark-shaded histogram, while the white is $u d s c$ background.


Figure 48: The distribution of reconstructed secondary vertex charge.
agreement for data and Monte Carlo, and enhance the confidence in the Monte Carlo charge reconstruction modelling.


Figure 49 : $A_{F B}$ plotted using the charge of the secondary vertex as the $b$ quark tag for charged events which also have a kaon tag. Dots are data, while the solid line is Monte Carlo.

### 5.3 The SLD Initial State Tag (IST)

Once we have selected $b \bar{b}$ events, it is necessary to tag the flavor of the initial state $b$ quark for a given hemisphere. SLD has a number of different methods to do this, which are then combined into a powerful initial state tag.

### 5.3.1 The Polarized Forward-Backward Asymmetry Tag

The polarization IST exploits the large polarized forward-backward asymmetry of the $b$ quark. The differential cross section for the process $e^{+} e^{-} \rightarrow Z^{0} \rightarrow b \bar{b}$ can be written as:

$$
\begin{equation*}
\frac{d \sigma^{b}\left(P_{e}\right)}{d z} \propto\left(1-A_{e} P_{e}\right)\left(1+z^{2}\right)+2 A_{b}\left(A_{e}-P_{e}\right) z \equiv \sigma_{b}(z) \tag{5.9}
\end{equation*}
$$

where $z=\cos \theta, \theta$ is the angle between the incident electron direction and the final state $b$, and $P_{e}$ is the incident electron polarization. ( $P_{e}$ is positive for righthanded polarization and negative for left-handed polarization.) In the Standard Model, $A_{e}$ and $A_{b}$ are determined from the vector and axial coupling constants of the fermion to the $Z^{0}$; they express the degree of parity violation in the coupling between the $Z^{0}$ and the fermions. This parity violation results in an asymmetry in the produced fermion direction. For negative polarization, the $b$ quark is more often emitted in the positive ( $e^{-}$beam) direction, and $\bar{b}$ in the backward direction. The forward-backward asymmetry can be formed using the differential cross-section from above:

$$
\begin{equation*}
A_{F B}^{b}\left(z, P_{e}\right)=\frac{\sigma^{b}(z)-\sigma^{b}(-z)}{\sigma^{b}(z)+\sigma^{b}(-z)}=2 A_{b} \frac{A_{e}-P_{e}}{1-A_{e} P_{e}} \frac{z}{1+z^{2}} . \tag{5.10}
\end{equation*}
$$

The asymmetry is sensitive to space inversion, and that sensitivity is stronger for left-handed electrons.

The event thrust axis is used to estimate the initial direction of the $b \bar{b}$ pair. This thrust axis is signed to point in the direction of the momentum of the reconstructed secondary $B$ vertex in the hemisphere of interest. The probability the initial state quark was a $b$ is therefore:

$$
\begin{equation*}
P_{b}^{p o l}=\frac{1}{2}\left(1+A_{F B}^{b}\right) . \tag{5.11}
\end{equation*}
$$

The calculation uses the Standard Model values of $A_{b}=0.935$ and $A_{e}=0.15$. Figure 50 shows the distribution of $P_{b}^{\text {pol }}$ for both data and Monte Carlo for the events in this analysis. A hemisphere with probability of greater (less) than 0.5 is tagged as a $b(\bar{b})$. The polarization IST is nearly $100 \%$ efficient, and has a purity of $70 \%$ for the events selected for this analysis. It is completely uncorrelated with any of the other ISTs described below.

### 5.3.2 The Jet Charge Tag

The jet charge technique exploits the fact that in a $Z^{0} \rightarrow b \bar{b}$ event, the two quarks are of opposite charge and emerge from the IP back to back, usually forming a jet in each hemisphere of the detector. The $B$ decay products reflect this charge and can be cleanly separated from fragmentation tracks due to their higher momentum. The momentum-weighted jet charge is defined as:

$$
\begin{equation*}
Q_{J e t}=\sum q_{i}\left|\overrightarrow{p_{i}} \cdot \hat{T}\right|^{\kappa} \tag{5.12}
\end{equation*}
$$



Figure 50 : The initial state $b$ probability using only the electron polarization. Dots indicate data; the solid line indicates all-flavor Monte Carlo. The $b$ quark Monte Carlo is the right-leaning hatched histogram, while the $\bar{b}$ is the left-leaning. The hatched histograms show the clear separation of $b$ from $\bar{b}$.
where the sum is over all of the tracks with charge $q_{i}$ and momentum $\overrightarrow{p_{i}}$ in the opposite hemisphere meeting the quality cuts in Table $8 . \hat{T}$ is the thrust axis of the event and $\kappa=0.5$, which is determined from the Monte Carlo to maximize the separation between $b$ and $\bar{b}$. Although this method is called "jet" charge, it would more accurately be called hemisphere charge: $Q_{\text {Jet }}$ is determined for each hemisphere, using the thrust axis (instead of using the jet), and the tag is taken from the hemisphere opposite that of analysis interest. The opposite hemisphere is used so there is no correlation between the final state and initial state tag.

Table 8: $Q_{j e t}$ Track Quality selection cuts

$$
\begin{aligned}
& p<50 \mathrm{GeV} / \mathrm{c} \\
& p_{\perp}>0.15 \mathrm{GeV} / \mathrm{c} \\
& \text { 2-dimensional impact parameter with respect to the IP less than } 2 \mathrm{~cm} \\
& \text { Distance between the IP and distance of closest approach to the beam axis less } \\
& \text { than } 10 \mathrm{~cm} \\
& \left|\cos \theta_{\text {track }}\right|<0.87
\end{aligned}
$$

After calculating the jet charge, the probability that the tagged hemisphere contains an initial $b$ quark can be approximated by:

$$
\begin{equation*}
P_{b}^{J e t Q}=\frac{1}{\left(1+e^{\alpha\left(Q_{j e t}-\delta\right)+\beta\left(Q_{j e t}-\delta\right)^{3}}\right)} \tag{5.13}
\end{equation*}
$$

where $\alpha=-0.328, \beta=0.0014$, and $\delta=0.068$, determined from Monte Carlo. The parameter $\delta$ is an offset to account for an excess of reconstructed positive tracks; these tracks result not from the $b$ decay, but from interactions with detector material which tend to produce more positive tracks than negative ones. Figure 51 shows the distribution of $P_{b}^{J e t Q}$ for both data and Monte Carlo for the analysis events. As with the polarization, a probability of greater (less) than 0.5 is tagged as a $b(\bar{b})$. The jet charge IST is nearly $100 \%$ efficient for this analysis, and has a purity of $66 \%$ as a stand-alone IST.


Figure 51: The initial state $b$ probability using only the quantity of jet charge. Dots indicate data; the solid line indicates all-flavor Monte Carlo. The $b$ quark Monte Carlo is the right-leaning hatched histogram, while the $\bar{b}$ is the left-leaning. The hatched histograms demonstrate the separation of $b$ from $\bar{b}$.

### 5.3.3 Other Initial State Tags

SLD uses four other event quantities to enhance the polarization and jet charge initial state tags. These quantities have high purities, but relatively low efficiencies, as they all require a topological vertex to be reconstructed in the hemisphere opposite that of analysis interest.

The vertex charge IST uses the charge of the reconstructed secondary vertex in the hemisphere opposite the hemisphere of interest. As noted in the jet charge section, the $b$ and $\bar{b}$ quarks are produced in essentially opposite directions; thus, if a positively charged vertex is produced in the opposite hemisphere, it can be assumed to be a $B^{+}$meson, containing a $\bar{b}$ quark, and the analysis hemisphere can be tagged
as a $b$ quark. Conversely, a negative vertex in the opposite hemisphere tags a $\bar{b}$ in the hemisphere of interest. As this tag purity is dependent on the $p_{T}$-added mass of the vertex, it is parameterized as a fourth order polynomial with respect to $M_{p_{T}}$, separately for $\left|Q_{v t x}\right|=1$ and $\left|Q_{v t x}\right|>1$.

The kaon charge IST works exactly like the final state tag for this analysis, except it is applied to the opposite hemisphere, and then that hemisphere is assumed to have the opposite flavor $b$ quark of the analysis hemisphere. By applying it to the opposite hemisphere, it remains uncorrelated from the final state tag used in this analysis. Kaon reconstruction has been described in detail in the previous chapter; details of how a kaon is used to tag events will be described in detail in the next section. A $K^{+}\left(K^{-}\right)$in the opposite hemisphere tags an initial state $b(\bar{b})$ in the analysis hemisphere. If there are multiple kaons, their charges are summed and that sum is used as the tag.

The high $p_{T}$ lepton tag works much the same way as the initial state kaon tag, using a positive (negative) lepton in the opposite hemisphere to tag a $b(\bar{b})$ in the hemisphere of interest, and then the hemisphere of interest is assumed to have the opposite flavor $b$ quark of the analysis hemisphere. Leptons are required to have $p_{T}>2 \mathrm{GeV} / \mathrm{c}$, where $p_{T}$ is the momentum transverse to the reconstructed IP to vertex direction; for multiple leptons in a hemisphere, the lepton with the highest $p_{T}$ is used. The purity of the tag increases as a function of lepton $p_{T}$, and is parameterized with a third order polynomial with respect to that quantity.

The final tag is the charge dipole tag. This tag uses the cascade $b \rightarrow c$ decay topology to tag the $b$ quark flavor. To calculate the dipole, one locates two well separated topological vertices, with the one closer to the IP assumed to be the $B$ decay and the more distant one a $D$ decay. The charge dipole is defined as

$$
\begin{equation*}
\delta q=\operatorname{sign}\left(q_{D}-q_{B}\right) \cdot l_{B D}, \tag{5.14}
\end{equation*}
$$

where $l_{B D}$ is the distance between the reconstructed $B$ and $D$ vertices and $q_{B}$ and $q_{D}$ their respective reconstructed charges. A positive (negative) charge dipole indicates a $\bar{b}(b)$ in the hemisphere of interest. The probability for an initial state $b$ in the analysis hemisphere is parameterized as a third order polynomial, separately for $Q_{v t x}$ neutral and charged.

### 5.3.4 The Combined Tag

All of the initial state tags are combined into one "kitchen sink" tag, with corrections applied to take into account the correlations amongst the various charged ISTs. Figure 52 shows the distribution of $P_{b}^{\text {all }}$ for both data and Monte Carlo for the analysis events. A probability of greater (less) than 0.5 is tagged as a $b(\bar{b})$. The tag is $76.3 \%$ pure and nearly $100 \%$ efficiency for this analysis, and the Figure shows good agreement between data and Monte Carlo.


Figure 52 : The SLD initial state $b$ probability using all if the possible tagging quantities. Dots indicate data; the solid line indicates allflavor Monte Carlo. The $b$ quark Monte Carlo is the right-leaning hatched histogram, while the $\bar{b}$ is the left-leaning. The hatched histograms demonstrate the clear separation of $b$ from $\bar{b}$.

### 5.3.5 Cross Checks on the Charged ISTs: Double tagged events and $A_{F B}$

Two independent cross-checks have been performed on the Charged ISTs: one investigating the correlation between the ISTs in the two hemispheres of a single event, and the other looking at the forward-backward asymmetry calculated in a single hemisphere. For both checks, the usual analysis requirement that the reconstructed vertex be neutral was dropped for the dual purposes of enhancing statistics and checking that charge reconstruction was not a significant factor in the analysis.

As a check on the initial state tagging in data, the correlation between the initial state flavor for events where both hemispheres have final state kaon tags has been investigated. It is expected that the fraction of events where one hemisphere is tagged with the opposite $b$ flavor of the other hemisphere is approximately:

$$
\begin{equation*}
f=R_{q I S T}^{2}+\left(1-R_{q I S T}\right)^{2}, \tag{5.15}
\end{equation*}
$$

where $R_{q I S T}$ is the purity of the combined charged ISTs (that is, the combined probability of all the ISTs excluding polarization. As the polarization tag is correlated for the two hemispheres, it has been excluded from this test.) Using this formula, one can estimate from the data the analyzing power of the IST.

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Figure 53 : The opposite-sign charged IST fraction for all double tagged events. Dots are data, while the solid line is Monte Carlo.


Figure 54 : The opposite-sign charged IST fraction for double tagged events where one hemisphere has a charged reconstructed vertex and the other a neutral one. Dots are data, while the solid line is Monte Carlo.

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Figure 55 : The opposite-sign charged IST fraction for double tagged events where both hemispheres have a charged reconstructed secondary vertex. Dots are data, while the solid line is Monte Carlo.


Figure 56 : The opposite-sign charged IST fraction for double tagged events where both hemispheres have a neutral reconstructed secondary vertex. Dots are data, while the solid line is Monte Carlo.

The distribution of "opposite-tag" double tags with respect to $\cos (\theta)$, where $\theta$ is the angle between the incident electron and the thrust axis, can be seen in Figures 53-56. Table 9 shows the integrated fraction for each distribution for data and Monte Carlo. The sample where both hemispheres are charged has a slightly greater purity due to the availability of the vertex charge IST, which is not usable when the reconstructed vertex in the opposite hemisphere is neutral. The results are in good agreement between data and Monte Carlo, thus giving confidence to the results of the initial state tagging parameterizations in the Monte Carlo. The extrapolated correct tag probability for the overall possible sample is $68 \%(71 \%)$ for data (Monte Carlo). It should be noted that these results slightly over-estimate the purity of the charged tags, as the SLD code does not take all of the correlations
between the tags into account for this quantity, which it does for the Combined Initital State Tag described in the previous section.

Table 9: The Integrated Charged IST Double-tag Fraction

| Type of Event | Monte Carlo | Data |
| :---: | :---: | :---: |
| Both reconstructed vertices neutral | $0.565 \pm 0.013$ | $0.517 \pm 0.031$ |
| Both reconstructed vertices charged | $0.616 \pm 0.010$ | $0.633 \pm 0.021$ |
| Mixed reconstructed vertices | $0.583 \pm 0.008$ | $0.542 \pm 0.020$ |
| All possible events | $0.590 \pm 0.006$ | $0.567 \pm 0.012$ |

A second check on the charged ISTs is to calculate $A_{F B}$ using the charged IST to calculate the asymmetry. Figures 57-59 show plots of the fraction ((events $s_{b t a g}-$ events $\left._{\bar{b} t a g}\right) /$ sum) for $^{1997-98}$ data and Monte Carlo. Additionally, these distributions have been fitted to the Standard Model function for $A_{F B}$, given in equation 5.8, where the Standard Model value of $A_{b}=0.935$ and the 1997-98 average SLD $e^{-}$polarization of 0.73 have been used. The fit result is $W$, the misidentification rate of the tag; the results for both data and Monte Carlo can be seen in Table 10. Both the plots and $t$ he fits show agreement for data and Monte Carlo, and enhance the confidence in the SLD Initial State Tag.

Table 10: Results of Charged IST $A_{F B}$ Fit to Misidentification Rates

| Type of Event | Monte Carlo | Data |
| :---: | :---: | :---: |
| Neutral Secondary Vertex | $0.351 \pm 0.0025$ | $0.377 \pm 0.0064$ |
| Charged Secondary Vertex | $0.355 \pm 0.0030$ | $0.368 \pm 0.0054$ |
| All Reconstructed Vertices | $0.344 \pm 0.0004$ | $0.358 \pm 0.0008$ |



Figure 57 : $A_{F B}$ plotted using the charged IST for all events with a kaon tag. Dots are data, while the solid line is Monte Carlo.


Figure 58: $A_{F B}$ plotted using the charged IST for charged vertices with a kaon tag. Dots are data, while the solid line is Monte Carlo.


Figure 59: $A_{F B}$ plotted using the charged IST for neutral vertices with a kaon tag. Dots are data, while the solid line is Monte Carlo.

### 5.4 Kaon Tagging

This analysis uses charged kaons to tag the final state $b$ quark flavor. SLD is fortunate to have excellent charged particle identification using the CRID subsystem; the previous chapter has gone into the CRID and kaon identification in great detail. This section will discuss the details of how we tag using the kaons, and the numerous checks we have done to ensure the parameterizations of the kaons in the likelihood fit (described in the next chapter) are well-modelled for the data in the SLD Monte Carlo. These checks are particularly important as the kaon right sign fractions (the fractions of hemispheres for which the $B$ flavor tag are correct) are the least well-measured input parameters for this analysis; therefore, we have used three completely independent methods of looking at the agreement of the kaon tag
between data and Monte Carlo. For all of these methods, we have waived the analysis requirement that the secondary vertex be neutral; this both increases statistics and allows a check on the charge resolution between data and Monte Carlo.

### 5.4.1 Principles of Kaon-Tagging

The Kaon Final State Tag (FST) exploits the dominant $b \rightarrow c \rightarrow s$ decay chain of the $B$ meson. This chain results in a correlation between the charge of the $b$ quark and the charge of the decay kaon. Wrong sign $K$ 's can be produced via the popping of $s \bar{s}$ pairs from the vacuum or the decay of charm mesons produced by the virtual $W^{ \pm}$boson by the transition $b \rightarrow c \bar{c} s$ with $\bar{c} \rightarrow \bar{s}$. The $B_{d}$ and $B_{u}$ right sign fractions for kaons have been measured to be $82 \pm 5 \%$ and $85 \pm 5 \%$, respectively, thus making them tags [64] to discriminate between the $B$ and $\bar{B}$ decays. The $B_{s}$ and $\Lambda_{b}$ right sign fractions are not so well known.

Kaons used in the tag are required to be attached to either a secondary or tertiary vertex in the hemisphere of interest and must be well-identified in the CRID, as described in the previous chapter. Tertiary kaons are used preferentially, and if multiple kaons are attached to the same vertex, their charges are summed and that sum is used as the tag. A position total kaon charge tags a $B$ meson and a negative one tags a $\bar{B}$. The efficiency and purity of the tag varies by $B$ type; overall efficiencies will be discussed in Section 6.1.3, as they are parameterized with respect to proper time and implemented in an integral table; however, the fraction of neutral events with kaons (kaon efficiency) and the right sign fractions, parameterized from Monte Carlo, can be seen in Figure 11.

Table 11: Monte Carlo Kaon Efficiencies and Right Sign Fractions

| B Type | Kaon Efficiency | Right Sign Fraction $\left(R_{\text {type }}\right)$ |
| :---: | :---: | :---: |
| $B_{u}$ | 0.247 | 0.776 |
| $B_{d}$ | 0.272 | 0.797 |
| $B_{s}$ | 0.263 | 0.497 |
| b-baryons | 0.215 | 0.614 |
| $u d s c$ | N/A | 0.50 |

### 5.4.2 Double-Tagged Events Using Kaons

As a cross-check on the final state tagging in data, the correlation between the final state flavor for events where both hemispheres have final state kaon tags has been investigated. Naively, it is expected that the fraction of events where one hemisphere is tagged with the opposite $b$ flavor of the other hemisphere is approximately:

$$
\begin{equation*}
f=R_{k F S T}^{2}+\left(1-R_{k F S T}\right)^{2}, \tag{5.16}
\end{equation*}
$$

where $R_{k F S T}$ is the purity of the kaon FST. However, this does not take into account that different $B$ types have different kaon analyzing powers, nor does it take into account $B_{d}$ and $B_{s}$ mixing. Even so, investigating this fraction can give insight into whether the Monte Carlo models the data well.

The fraction of "opposite-tag" double tags with respect to $\cos (\theta)$, where $\theta$ is the angle between the incident electron and the thrust axis, can be seen in Figures 60 63. Table 12 shows the integrated fraction for each distribution for data and Monte Carlo. The charged sample shows a larger fraction of opposite tags due to there


Figure 60 : The opposite-sign kaon FST fraction for all double tagged events. Dots are data, while the solid line is Monte Carlo.

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Figure 61: The opposite-sign kaon FST fraction for double tagged events where one hemisphere has a charged reconstructed vertex and the other a neutral one. Dots are data, while the solid line is Monte Carlo.


Figure 62 : The opposite-sign kaon FST fraction for double tagged events where both hemispheres have a charged reconstructed secondary vertex. Dots are data, while the solid line is Monte Carlo.

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Figure 63 : The opposite-sign kaon FST fraction for double tagged events where both hemispheres have a neutral reconstructed secondary vertex. Dots are data, while the solid line is Monte Carlo.
being significantly fewer $B_{d}$ and $B_{s}$ decays, which dilute the tag due to mixing. The data and Monte Carlo show good agreement and lend confidence to the kaon tagging method.

Table 12: The Integrated Kaon FST Double-tag Fraction

| Type of Event | Monte Carlo | Data |
| :---: | :---: | :---: |
| Both reconstructed vertices neutral | $0.555 \pm 0.013$ | $0.521 \pm 0.031$ |
| Both reconstructed vertices charged | $0.651 \pm 0.010$ | $0.612 \pm 0.021$ |
| Mixed reconstructed vertices | $0.586 \pm 0.008$ | $0.584 \pm 0.017$ |
| All possible events | $0.601 \pm 0.006$ | $0.582 \pm 0.012$ |

### 5.4.3 Kaon and Lepton Tag Correlations

A second check on the Kaon FST is to compare the final state tag generated by the kaon method to one generated by a lepton attached to the secondary vertex. The high- $p_{T}$ lepton IST is discussed in Section 5.3.3. The use of the lepton here is similiar; it is particularly useful that the lepton tags the final state of the $B$ in the hemisphere of interest, and they have a high right sign fraction across most of the momentum spectrum, as seen in Figure 64. As the lepton tags the final state, this provides a check on the kaon tagging independent of mixing, and is the only check we have that does so.

The agreement fraction between the lepton and kaon FSTs is shown in Figures 65-67. Although the plots show a bin from $0-0.8 \mathrm{GeV} / \mathrm{c}$, these events are not included in the integrated fractions given in Table 13 due to the low lepton right sign fraction in this bin. The data and Monte Carlo show good agreement for the neutral, charged, and combined reconstructed vertices.


Figure 64 : The lepton right sign tag fraction with respect to lepton transverse momentum to the reconstructed $B$ vertexas determined from the Monte Carlo. The high right sign fraction across the spectrum makes the leptons ideal for checking the kaon FST.

Table 13: Kaon-Lepton Integrated Agreement Fractions

| Type of Event | Monte Carlo | Data |
| :---: | :---: | :---: |
| Neutral Secondary Vertex | $0.779 \pm 0.009$ | $0.752 \pm 0.022$ |
| Charged Secondary Vertex | $0.871 \pm 0.006$ | $0.877 \pm 0.014$ |
| All Reconstructed Vertices | $0.830 \pm 0.005$ | $0.824 \pm 0.013$ |

### 5.4.4 $\quad A_{F B}$ Using the Final State Kaon Tag

A third check on the Kaon FST is to calculate $A_{F B}$ using the kaon charge tag to calculate the asymmetry. Figures 68-70 show plots of the fraction ((events $s_{b t a g}-$ event $\left._{\bar{b} \text { tag }}\right) /$ sum , where events $_{b(\bar{b}) \text { tag }}$ is the number of events tagged as having a $b(\bar{b})$ quark in the final state, for 1997-98 data and Monte Carlo. Additionally, these


Figure 65 : The kaon-lepton agreement fraction for all events. Dots are data, while the solid line is Monte Carlo.

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Figure 66 : The kaon-lepton agreement fraction for charged vertices. Dots are data, while the solid line is Monte Carlo.


Figure 67 : The kaon-lepton agreement fraction for neutral vertices. Dots are data, while the solid line is Monte Carlo.
distributions have been fitted to the Standard Model function for $A_{F B}$, given in equation 5.8, where once again the Standard Model value of $A_{b}=0.935$ and the 1997-98 average SLD $e^{-}$polarization of 0.73 have been used. $W$, the misidentification rate, is fit for; the results for both data and Monte Carlo can be seen in Table 14; errors given are fit statistical errors only. One sees evidence for the dilution from mixing in the neutral sample in the large difference between the mis-tag rates for the neutral and charged only samples. Both the plots and the fits show good agreement for data and Monte Carlo, and enhance the confidence in the kaon Final State Tag.


Figure 68 : $A_{F B}$ plotted using the kaon FST for all events. Dots are data, while the solid line is Monte Carlo.

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Figure 69 : $A_{F B}$ plotted using the kaon FST for charged vertices. Dots are data, while the solid line is Monte Carlo.


Figure 70 : $A_{F B}$ plotted using the kaon FST for neutral vertices. Dots are data, while the solid line is Monte Carlo.

Table 14: Kaon FST $A_{F B}$ Fit Wrong Tag Fractions

| Type of Event | Monte Carlo | Data |
| :---: | :---: | :---: |
| Neutral Secondary Vertex | $0.362 \pm 0.006$ | $0.378 \pm 0.013$ |
| Charged Secondary Vertex | $0.238 \pm 0.006$ | $0.265 \pm 0.012$ |
| All Reconstructed Vertices | $0.290 \pm 0.004$ | $0.313 \pm 0.009$ |

### 5.5 The Proper Time Calculation

As noted in Chapter 2, one must know the proper time of the $B$ decay to study time-dependent mixing. In principle, the proper time calculation is quite simple:

$$
\begin{equation*}
t=\frac{l}{c b}, \tag{5.17}
\end{equation*}
$$

where $l$ is the decay length, $c$ is the speed of light, and $b$ is the boost. The boost of a $B$ meson with mass $m_{B}$, total energy $E_{B}$, and momentum $p_{\text {total }}$ respectively is:

$$
\begin{equation*}
b=\frac{p_{\text {total }}}{m_{B}}=\sqrt{\left(\frac{E_{B}}{m_{B}}\right)^{2}-1} . \tag{5.18}
\end{equation*}
$$

The error on the proper time measurement is therefore:

$$
\begin{equation*}
\sigma_{t}^{2}=\left(\frac{\sigma_{l}}{c b}\right)^{2}+\left(\frac{t \sigma_{b}}{b}\right)^{2} \tag{5.19}
\end{equation*}
$$

where $\sigma_{l(b)}$ is the resolution of the decay length (boost) measurement. The boost and decay length reconstruction methods, resolutions, and comparisons of Monte Carlo results with data are described in the next sections.

### 5.5.1 Boost Reconstruction and Resolution

SLD utilizes two different methods to achieve the most accurate possible reconstruction of the boost. Both methods of reconstruction focus on reconstructing the energy of the $B$ meson in order to calculate the boost. The two boost results are then combined, taking into account the correlations between the methods, to give a single boost result for the reconstructed $B$ vertex.

The Mass Method

The first method looks to calculate the energy of the $B$ by using the known invariant mass of the $B$ vertex. The energy of the $B, E_{B}$ is:

$$
\begin{equation*}
E_{B}=E_{c h}+E_{\nu}+E_{\text {neut }} . \tag{5.20}
\end{equation*}
$$

$E_{c h}$ is the total energy of all charged tracks attached to the secondary vertex (assuming they are all pions), and $E_{\text {neut }}$ is the energy taken away by neutral particles which are seen in the calorimeter but are not tracked, and $E_{\nu}$ is the unseen energy in the event due to neutrinos.

To determine $E_{\nu}$, the total energy in the jet containing the $B$ meson, $E_{j e t}$, must be calculated [65]. At the $Z_{0}$ resonance, the total energy in the event is $m_{Z}$. For events with two jets, it is assumed the $b \bar{b}$ pair is produced back to back, and therefore we simply have $E_{j e t}=m_{Z} / 2$. For a three jet event, the jet direction can be used to estimate the energy in each jet by using the conservation of energy and momentum. The energy is therefore:

$$
\begin{equation*}
E_{j e t}^{1,2,3}=m_{Z} \frac{\sin \theta_{23,13,12}}{\sin \theta_{12}+\sin \theta_{13}+\sin \theta_{23}}, \tag{5.21}
\end{equation*}
$$

where $\theta_{i j}$ is the angle between jets $i$ and $j$ [65]. For the small fraction of events with four jets, the event is divided into two hemispheres and treated as a two jet event. The unseen energy in the event is therefore $E_{\nu}=E_{j e t}-E_{c h}-E_{c a l}$, where
$E_{\text {cal }}$ is all of the energy which is measured in the calorimeter for the hemisphere of interest but is not associated with a charged track.

The final task in determining $E_{B}$ via the mass method is to determine which energy clusters in the calorimeter are associated with $B$ decay products but are not associated with one of the charged tracks. To do this, each unattached cluster is added to the secondary vertex, while keeping the direction of the vertex momentum unchanged, starting with the cluster which is closest to the secondary vertex flight direction. The invariant mass of the vertex is then recalculated. If the mass remains below $5.3 \mathrm{GeV} / \mathrm{c}^{2}$, the "test" cluster is considered associated with the $B$. If it is above $5.3 \mathrm{GeV} / \mathrm{c}^{2}$, the "test" cluster is removed from the vertex and is considered unassociated. This process continues until all of the clusters in the hemisphere have been tested.

## The Maximum Missing Mass Method

This method seeks to determine the $E_{\nu}$ and $E_{\text {neut }}$ by determining the missing invariant mass, $M_{0}$, longitudinal and transverse momentum, $p_{0 l}^{2}$ and $p_{0 t}^{2}$, to reconstuct the neutral energy:

$$
\begin{equation*}
E_{0}=E_{\nu}+E_{\text {neut }}=M_{0}^{2}+p_{0 t}^{2}+p_{0 l}^{2} \tag{5.22}
\end{equation*}
$$

As SLD has excellent resolution for transverse momentum, the challenge is to determine the other two elements [8]. One kinematic constraint on the missing quan-
tities can be obtained by requiring the $B$ decay vertex mass to be equal to the $B$ hadron mass, $M_{B}^{2}=E_{B}^{2}-p_{B}^{2}$, where the total momentum of the secondary vertex is $p_{B}=p_{c h l}+p_{0 l}$, and $p_{c h l}$ is the longitudinal momentum of the vertex-associated charged tracks (i.e., the component along the vertex flight direction). This reduces the problem of determining the longitudinal momentum and missing invariant mass to simply determining the missing invariant mass.

It is possible to further constrain the missing invariant mass to

$$
\begin{equation*}
\sqrt{M_{c h}^{2}+p_{T}^{2}}+\sqrt{M_{0}^{2}+p_{T}^{2}} \leq M_{B} \tag{5.23}
\end{equation*}
$$

where the inequality is valid in the limit where both $p_{0 l}$ and $p_{c h l}$ vanish in the $B$ rest frame. This effectively sets an upper bound on the missing mass, with the lower bound being 0 , and the upper bound is

$$
\begin{equation*}
M_{0 \max }^{2}=M_{B}^{2}-2 M_{B} \sqrt{M_{c h}^{2}+p_{T}^{2}}+M_{c h}^{2} \tag{5.24}
\end{equation*}
$$

It has been shown that for small $M_{0 \max }^{2}$, one can approximate $M_{0}^{2}$ to be simply equal to $M_{0 \max }^{2}$ for $M_{0 \max }^{2} \geq 0$, and to be 0 for $M_{0 \max }^{2} \leq 0$ [8]. It is then possible to calculate $p_{0 l}$ using:

$$
\begin{equation*}
p_{0 l}=\frac{M_{B}-M_{c h \perp}}{M_{c h \perp}} p_{c h l}, \tag{5.25}
\end{equation*}
$$

where $M_{c h \perp}^{2}=M_{c h}^{2}+p_{T}^{2}$. It is now possible to calculate $E_{0}$, and hence $E_{B}$.

## Reconstruction Results and Resolutions



Figure 71 : The reconstructed secondary vertex boost for analysis events. Dots indicate data; the solid line indicates the Monte Carlo.

The distribution of reconstructed boost measurements for data and Monte Carlo analysis events can be seen in Figure 71. Rather than fit for the boost resolution, $\sigma_{b}$, we instead fit for the relative boost resolution, $\sigma_{b} / b$, and the distributions of this quantity, separated by $B$ type, can be seen in Figure 72. The distributions of the relative boost residuals, where the relative residual $=\frac{b_{\text {reconstructed }}-b_{\text {generated }}}{b_{\text {generated }}}$ is fit with the sum of two gaussian functions, with a constraint that $55 \%$ of the total area is in the core gaussian and $45 \%$ of the area is in the tail gaussian. The fitted resolutions are shown in Table 15. One notes that the resolutions of the $B_{u}$ relative boost is worse than that of the other $B$ 's due to the neutral charge requirement discussed earlier; any $B_{u}$ that remains in the analysis sample has been misreconstructed, as the $B_{u}$ is a charged hadron. This results in a degraded reconstruction of the boost.


Figure 72 : Distributions of the relative boost residuals by $B$ type. The fit is a double gaussian with a $55 \%$ core fraction.

Table 15: Fitted Relative Boost Resolutions by $B$ Type.

| $B$ Type | Core Resolution (\%) | Tail Resolution (\%) |
| :---: | :---: | :---: |
| $B_{u}$ | 10.2 | 26.8 |
| $B_{d}$ | 6.80 | 20.6 |
| $B_{s}$ | 7.34 | 23.0 |
| b-baryons | 8.87 | 22.4 |

### 5.5.2 Decay Length Reconstruction and Resolution

While it is not straightforward to measure decay length, it is straighforward to calculate it:

$$
\begin{equation*}
l=\sum_{i=1}^{3}\left(x_{i}-x_{i}^{0}\right), \tag{5.26}
\end{equation*}
$$

where $x_{i}$ denotes the coordinates of the decay vertex and $x_{i}^{0}$ denotes those of the origin of the particle. Thus, one must determine both the origin, which at SLD can be assumed to be the interaction point (IP) and the decay point of the $B$. Fortunately, the extremely small IP and high precision of the Vertex Detector (VXD3) allow for good determination of the decay length of a secondary vertex.

## Interaction Point Determination

The IP determination is a two step process: one step to determine the location of the IP in the $x y$ plane, and another to determine the $z$-coordinate of the IP.

For the $x-y$ coordinate, we first determine a single event IP position by extrapolating all charged tracks in a hadronic event to the center of the beam pipe, and fitting them to a common vertex. However, due to the extremely small and stable

SLC interaction region, it is possible to make a more accurate determination of the IP by averaging over a set of 30 sequential hadronic events. A seed IP position is chosen and all tracks with VXD hits which pass within $3 \sigma$ of the seed are fit to a common vertex. This vertex is taken as the new seed and the fit is iterated until it converges and the $\chi^{2} / d o f<1.3$. This method leads to an IP precision for a single event of $\sigma_{x y} \simeq 3.5 \mu \mathrm{~m}$ for VXD3 data. It has been cross-checked using the $x y$ impact parameters from tracks in $Z^{0} \rightarrow \mu^{+} \mu^{-}$events, shown in Figure 73. The two muon tracks were extrapolated to the IP and their extrapolation errors subtracted, resulting in a fitted width of $3.5 \pm 2 \mu \mathrm{~m}$.


Figure 73 : $x y$ impact parameter of tracks in $\mu^{+} \mu^{-}$events. A resolution of $\simeq 3.5 \mu \mathrm{~m}$ is obtained using VXD3 data after track extrapolation corrections.

The longitudinal position cannot be determined as accurately and must be done on an event-by-event basis. For each track with a VXD hit, the point of closest
approach to the transverse IP is determined. Tracks which pass within $3 \sigma$ of the IP and have an $x y$ impact parameter of less than $500 \mu \mathrm{~m}$ are then selected, and the median of the $z_{P O C A}$ coordinate distributions is used as the $z$ position of the IP. The resolution on the $z$ position of the IP is about $17 \mu \mathrm{~m}$ for $b \bar{b}$ events.

## Secondary Vertex Reconstruction

The topological vertexing method described in Section 5.2.1 is used to reconstruct all of the vertices in a hemisphere of interest. If only primary (IP) and secondary (presumed to be the $B$ ) vertices are present, the secondary vertex given by this method is used as the decay vertex. If a tertiary vertex is also found, a second iteration of vertexing using a Kalman Filter method [66] is performed. This refines the knowledge of the position of the secondary vertex by assuming the tertiary vertex is from the decay of a daughter particle of the secondary vertex. Thus, one can create a "track" by drawing a vector from the secondary to tertiary vertex, giving that track the reconstructed momentum of the tertiary vertex, and then refitting the secondary vertex with that "track" included. This allows for significant improvement in the resolution of the secondary vertex, especially for low track multiplicity vertices.

## Decay Length Resolution

The distribution of reconstructed decay length for data and Monte Carlo for the analysis events can be seen in Figure 74. The two distributions are in good
agreement, and lends confidence to parameterizing the decay length resolutions from Monte Carlo. The decay length residual $\left(=l_{\text {reconstructed }}-l_{\text {generated }}\right)$, separated by $B$ type, can be seen in Figure 75. The decay length residual distributions are fit with the sum of two gaussian functions, with a constraint of having $67 \%$ of the area in the core gaussian and $33 \%$ of the area in the tail. The fitted resolutions are shown in Table 16. One notes that the resolutions of the $B_{u}$ decay length is worse than that of the other $B$ 's due to the neutral charge requirement discussed earlier; any $B_{u}$ that remains in the analysis sample has been misreconstructed, as the $B_{u}$ is a charged hadron.


Figure 74: The reconstructed secondary vertex decay length for analysis events. Dots indicate data; the solid line indicates SLD Monte Carlo.


Figure 75 : Distributions of the reconstructed decay length residuals by $B$ type. The fit is a double gaussian with $67 \%$ in the core.

Table 16: Fitted Decay Length Reslutions by $B$ Type

| $B$ Type | Core Resolution $(\mu \mathrm{m})$ | Tail Resolution $(\mu \mathrm{m})$ |
| :---: | :---: | :---: |
| $B_{u}$ | 90.0 | 533.3 |
| $B_{d}$ | 79.9 | 426.0 |
| $B_{s}$ | 85.9 | 394.4 |
| b-baryons | 68.6 | 342.4 |

Proper Time

Combining the computed decay length and boost, it is possible to calculate the proper time as described in the introduction to this section. The distribution of reconstructed proper time for data and Monte Carlo for events in the analysis can be seen in Figure 76, and there is good agreement between the two.


Figure 76 : The reconstructed secondary vertex proper time for analysis events. Dots indicate data; the solid line indicates SLD Monte Carlo.

## CHAPTER 6

## LIKELIHOOD FITTING AND RESULTS

This chapter covers the heart of this dissertation: it describes the analysis used to measure $\Delta m_{d}$, the $B_{d}$ oscillation frequency. We begin with an in-depth discussion of the likelihood fit function and the fitting process. The results of the fit for both data and Monte Carlo are presented, as are a number of cross-checks to the fit results. We conclude with a discussion of potential systematic errors.

### 6.1 The Likelihood Function

In Chapter 2, we saw that, theoretically, the mixing probability for a $B_{d}$ measured at time $t$ after its creation is quite straightforward:

$$
\begin{equation*}
P_{m i x}=\frac{\Gamma_{d}}{2} e^{-\Gamma_{d} t}\left(1-\cos \left(\Delta m_{d} t\right)\right) \tag{6.1}
\end{equation*}
$$

where $\Gamma_{d}=1 / \tau_{d}, \tau_{d}$ is the $B_{d}$ lifetime, and $P_{u n m i x}=1-P_{m i x}$. In a real detector, of course, life is not so simple:

- $B_{d}$ mesons are produced along with other $b$ hadrons ( $B_{u}, B_{s}$, and b-baryons), which have similiar topological and kinematical properties as the $B_{d}$ mesons.
- The short lifetime of the $B$ hadron makes difficult the task of identifying its decay vertex and determing its time of decay.
- Detector effects, mostly due to the analysis requirement to have separated vertices from the IP with some level of significance, can cause measurement efficiences to vary with decay time.
- Finite resolutions that must be measured and included in the analysis.
- Both the initial and final state tags are correct less than $100 \%$ of the time.

To perform the $B_{d}$ mixing analysis, we use an unbinned maximum log-likelihood fit which provides us with the ability to write a straightforward function to measure the mixing while explicitly accounting for the various detector and physics effects mentioned above. It also allows us to use each event independently to get the most information out of the SLD data.

The probability density function (pdf), $\mathcal{P}_{\text {mix }}$, for a mixed event can be written as the sum of five terms, one for each of the $B$ types present at the $Z^{0}$ resonance, and one to describe the non- $B$ background, which consists of $u d s c$ events:
$\mathcal{P}_{m i x}=f_{B_{d}} P_{m i x, B_{d}}+f_{B_{s}} P_{m i x, B_{s}}+f_{B_{u}} P_{m i x, B_{u}}+f_{b-b a r y o n s} P_{m i x, b-b a r y o n s}+f_{u d s c} P_{m i x, u d s c}$
where $f_{x}$ represents the fraction of events in the sample which is of the particular $B$ type (or the fraction of events which are background $u d s c$ events which fake a $B_{d}$ event). The condition that $\sum f_{x}=1$ is imposed. These fractions are determined in the SLD Monte Carlo, and can be seen in Table 17. All of the parameterizations from Monte Carlo are treated as possible sources of systematic error later in the analysis. Once the pdf has been defined, the likelihood function is a product of individual event probabilities, $\prod_{i=1}^{N_{\text {mix }}} \mathcal{P}_{\text {mix }}^{i}$.

Table 17: Monte Carlo parameterizations used in the Likelihood Fit

| B Type | Right Sign Fraction $\left(R_{\text {type }}\right)$ | Fraction $\left(f_{\text {type }}\right)$ |
| :---: | :---: | :---: |
| $B_{u}$ | 0.776 | 0.146 |
| $B_{d}$ | 0.797 | 0.607 |
| $B_{s}$ | 0.497 | 0.170 |
| b-baryons | 0.614 | 0.067 |
| $u d s c$ | 0.50 | 0.012 |

In the following two subsections, we write the likelihood function in terms of true proper time, $t^{\prime}$. Reconstructed proper time relies on measuring the decay length and boost, and has a finite resolution. The efficiency of the detector is a function of both $B$ type and proper time and these functional dependences are determined from the SLD Monte Carlo, and will be described later. To accommodate all of the possible detector effects, we parameterize all of the time elements of the fit, with the exception of the background term, as an integral table which takes into account the finite detector resolutions and time-dependent efficiencies for each $B$ type. The efficiency functions used in the integral table will be discussed in Section 6.1.3, and the construction of the complete integral table will be discussed in Section 6.1.4.

In this section, we will describe only the mixed likelihood, as the pdf for unmixed events is simply $\mathcal{P}_{\text {unmix }}=1-\mathcal{P}_{\text {mix }}$. An event is called "mixed" if the measured final state quark is the anti-particle of the measured initial state quark. One sees that due to mis-tagging, it is possible to have "mixed" $B_{u}$ and b-baryon events, even though these particles are not known to mix.

### 6.1.1 The $B_{d}$ and $B_{s}$ Terms

The $B_{d}$ and $B_{s}$ terms are similiar in nature, as both these hadrons are known to mix. Taking into account the uncertainty in the initial and final state tags, these terms are of the form:

$$
\begin{aligned}
P_{m i x}^{B_{d(s)}} & =\frac{\Gamma_{d(s)}}{2} e^{-\Gamma_{d(s)} t^{\prime}}\left(i R_{d(s)}+(1-i)\left(1-R_{d(s)}\right)\right)\left(1-\cos \left(\Delta m_{d} t\right)\right)+\left((1-i) R_{d(s)}+i\left(1-R_{d(s)}\right)\right)( \\
& =\frac{\Gamma_{d(s)}}{2} e^{-\Gamma_{d(s)} t^{\prime}}\left(1-\left(1-2 R_{d(s)}-2 i+4 i R_{d(s)}\right) \cos \left(\Delta m_{d(s)} t^{\prime}\right)\right)
\end{aligned}
$$

where $R_{d(s)}$ is the kaon right sign fraction for the $B_{d(s)}$, determined as a single value for each $B$ type from the Monte Carlo (noted in Table 17), and $i$ is the initial state right sign probability on an event by event basis, given by the initial state tag described in Section 5.3. The likelihood fit is used to determine both $\Delta m_{d}$ and $R_{d}$. Although the frequency for $B_{s}$ mixing is unmeasured, we use a value of $\Delta m_{s}=10 \mathrm{ps}^{-1}$. This matches the SLD Monte Carlo, and due to the fact $R_{s} \simeq 0.5$, it does not affect the analysis; we are fundamentally insensitive to $B_{s}$ mixing. Even so, we treat the uncertainty in $\Delta m_{s}$ as a possible systematic error.

We have compared the fraction of events tagged as mixed in SLD Monte Carlo with that generated by the probability parameterization for $B_{d}$ and $B_{s}$ separately, and the results can be seen in Figure 77. The fraction of events tagged as "mixed" is simply the fraction of events which have a final state $b$ tag which is the opposite flavor of the initial state $b$ tag as a function of reconstructed proper time. The $B_{d}$ fraction probability is generated using the fitted 2-D Minuit fit which is described later in the chapter. The fraction probability describes the Monte Carlo fraction well for both mixed $B$ types and gives confidence that the parameterizations are appropriate.

### 6.1.2 The $B_{u}$ and b-baryon Terms

The b-baryon and $B_{u}$ terms are similiar to one another, as b-baryons are not known to mix, and $B_{u}$ are charged and cannot mix. As noted earlier in this section, all $B_{u}$ and b-baryon events that are tagged as "mixed" must have either the initial or final state tag incorrect. Using the subscript of "a" to label the b-baryon parameterization, the terms used for these species in the likelihood function are of the form:

$$
\begin{align*}
P_{m i x}^{B_{u(a)}} & =\Gamma_{u(a)}\left(R_{u(a)}(1-i)+i\left(1-R_{u(a)}\right)\right) e^{-\Gamma_{u(a)} t^{\prime}}  \tag{6.5}\\
& =\Gamma_{u(a)}\left(R_{u(a)}+i-2 i R_{u(a)}\right) e^{-\Gamma_{u(a)^{\prime}}}, \tag{6.6}
\end{align*}
$$



Figure 77 : Individual $B$ mixed fractions. The Monte Carlo fractions of decays tagged as "mixed" as a function of proper time for each separate $B$ type with the parameterized likelihood term. Monte Carlo mixed fractions are represented by open circles while the line represents the fraction produced by the probabilities described in Sections 6.1.1 and 6.1.2.
where $R_{u(a)}$ is the kaon right sign fraction for the $B_{u(a)}$, determined as a single value for each $B$ type from the Monte Carlo (noted in Table 17), and $i$ is the initial state right sign probability on an event by event basis, given by the initial state tag described in Section 5.3.

We have compared the fraction of events tagged as mixed in Monte Carlo with the fraction generated by the probability parameterization for $B_{u}$ and b-baryons separately, and the results can be seen in Figure 77. The fraction probability describes the Monte Carlo fraction well for both unmixed $B$ types and gives confidence that the parameterizations are appropriate.

### 6.1.3 Selection Efficiency

The efficiency to select events for the analysis is dependent on two important variables: first, the effects that our selection cuts have on the event pool, and second, the ability of SLD to reconstruct a secondary vertex at a given proper time. This must be taken into account in the fit; to do so, we use the Monte Carlo to measure the event selection efficiency as a function of Monte Carlo true proper time, $t^{\prime}$ of decay, with a separate distribution for each $B$ type. These distributions are fit with a function of the form:

$$
\begin{equation*}
\epsilon\left(t^{\prime}\right)=p_{1} \frac{1-e^{p_{2} t^{\prime}}}{1+e^{p_{2} t^{\prime}}}+p_{3}+p_{4} t^{\prime}+p_{5} t^{\prime 2}+p_{6} t^{\prime 3} \tag{6.7}
\end{equation*}
$$

where the $p_{i}$ represent the variables determined by the fit.


Figure 78 : Event selection efficiency vs Monte Carlo true proper time separated by $B$ type. Monte Carlo points are represented by crosses, while the smooth curves represent the fit described in the text. The $x$-axes are in units of ps.

The efficiencies overlaid with the fitted functions can be seen in Figure 78. True proper time is used because we will integrate over it in the integral table discussed in the next section; four efficiency functions are used because the requirement of a neutral secondary vertex greatly reduces the efficiency for accepting contaminating $B_{u}$ events, which are actually charged, and to obtain a more accurate description of each of the neutral $B$ hadrons. The function includes a sharp dropoff at low proper time due to the inability to identify secondary vertices that are close to the SLC interaction point.

To check how well the fitted efficiency functions describe the Monte Carlo they are fitted to, we have compared the true proper time distribution of individual $B$ types to the function

$$
\begin{equation*}
f\left(t^{\prime}\right)=\frac{1}{\tau} e^{-\frac{t^{\prime}}{\tau}} \epsilon\left(t^{\prime}\right) \tag{6.8}
\end{equation*}
$$

where $\tau$ represents the particular $B$ lifetime that the Monte Carlo is generated with. These comparsions are shown in Figure 79.

### 6.1.4 The Integral Table for Parameterizing Time-related Quantities

We have written the physics parameterizations above in terms of true, not reconstructed, proper time. Now we turn to transforming them into what we measure: the reconstructed time of the decay. This parameterization takes into account the efficiency functions discussed in the previous section, and the finite decay length and boost resolutions discussed in Section 5.5.


Figure 79 : The lifetime distribution predicted by the parameterized efficiency functions compared to Monte Carlo true proper lifetimes separated by $B$ type. Monte Carlo points are represented by asterisks, while the smooth curves represent the modelled lifetime distributions. The $x$-axes are in units of ps .

In Section 5.5, we showed the double gaussian fits to the relative boost (Figure 72 and Table 15) and decay length (Figure 75 and Table 16) residuals, which resulted in a total of four fitted resolutions: $\sigma_{\text {relboost }}^{\text {core }}, \sigma_{\text {relboost }}^{\text {tail }}, \sigma_{\text {length }}^{\text {core }}$, and $\sigma_{\text {length }}^{\text {tail }}$. In this section, for simplicity, we will refer to these as $\sigma_{(L, B)^{\alpha}}$, where the $L$ and $B$ subscripts refer to decay length and relative boost, respectively, and superscript $i, j=1$ for the core and 2 for the tail. By propagating errors, we can write the resolution for the proper time measurement:

$$
\begin{align*}
\sigma_{t^{\prime}}^{i, j} & =\sqrt{\left(\frac{\sigma_{L}^{i}}{c \gamma \beta}\right)^{2}+\left(\frac{\sigma_{B, a b s o l u t e}^{j} t^{\prime}}{\gamma \beta}\right)^{2}}  \tag{6.9}\\
& =\sqrt{\left(\frac{\sigma_{L}^{i}}{c \gamma \beta}\right)^{2}+\left(\sigma_{B}^{j} t^{\prime}\right)^{2}} \tag{6.10}
\end{align*}
$$

where $\gamma \beta$ is the reconstructed boost and $t$ the reconstructed time. One notes that as proper time increases, the resolution on the measurement becomes worse. We also see that this results in four different components to $\sigma_{t}$, one resulting from each possible combination of core and tail resolutions. With this, we can now construct a gaussian resolution function for the reconstructed proper time:

$$
\begin{equation*}
G\left(t^{\prime}, t\right)=\sum_{i=1}^{2} \sum_{j=1}^{2} f_{L}^{i} f_{B}^{j} \frac{1}{\sqrt{2 \pi} \sigma_{t}^{i j}} e^{-\frac{1}{2}\left(\frac{\left(t^{\prime}-t\right)}{\sigma_{t}^{i j}}\right)^{2}} \tag{6.11}
\end{equation*}
$$

where $t^{\prime}$ is the true proper time, $t$ the reconstructed time, and $f_{L}^{i}$ and $f_{B}^{j}$ are the core and tail fractions of the decay length and boost residual fits. This function reproduces the proper time residual distribution reasonably well; an overlay of
the function with the residuals from the Monte Carlo event sample can be seen in Figure 80, divided into six bins of true proper time. The solid histogram is a representation of $G\left(t^{\prime}=0, t\right)$ from equation 6.11, and the points represent a histogram for $t-t^{\prime}$ (i.e. (Monte Carlo true- Monte Carlo reconstructed)) proper time values.

The slight offset between the resolution function and the distribution of the residuals is due to the fact there are slight biases in the proper time reconstruction, which are not explicitly described by the resolution function. This will be addressed in the study of systematic errors.

By summing over all of the possible core/tail combinations and integrating over true proper time, we can now write $P_{m i x}$ in terms of reconstructed proper time $t$ :

$$
\begin{equation*}
P_{m i x}(t)=\sum_{x} f_{x} \int_{0}^{\infty} P_{m i x, x}\left(t^{\prime}\right) \epsilon_{x}\left(t^{\prime}\right) G_{x}\left(t^{\prime}, t\right) d t^{\prime} \tag{6.12}
\end{equation*}
$$

where the $x$ subscript represents the four types of $B$ hadrons in the analysis, and $\epsilon_{x}\left(t^{\prime}\right)$ are the efficiency functions described in the previous section. To complete the parameterization, we ensure normalization of the function by dividing by the normalization integral, $I_{x}^{n}$ :

$$
\begin{equation*}
I_{x}^{n}=\int_{0}^{10 p s}\left[P_{u n m i x, x}(t)+P_{m i x, x}(t)\right] d t \tag{6.13}
\end{equation*}
$$

For a small quantity of events, it is possible to perform these calculations during analysis on an event by event basis. However, due to the relatively large quantity of


Figure 80 : The proper time residuals (dots) shown with the resolution function (smooth curve) described in the text, showing all $b$ hadrons combined. The $x$-axis is in units of ps .
events in this analysis, we instead construct a table which contains the values of the integrals over the necessary range in reconstructed proper time. To provide a check on the construction, we can calculate a "lifetime" distribution of the proper time of decay from the integral table, and compare it to the distribution of the values in both data and Monte Carlo. To obtain the "lifetime" in the integral table, we take a proper time $t^{\prime}$ and the four typical resolution measurements of the time, $\sigma_{\alpha \beta}\left(t^{\prime}\right)$, extract the value of the integral tables at those points, and sum over $\alpha$ and $\beta$. These distributions are shown in Figure 81, and we see reasonable agreement between data and the integral table and Monte Carlo and the integral table.


Figure 81 : Proper time distributions from the integral table (the histogram in each plot) shown with Monte Carlo generated events (left points) and data events (right points).

### 6.1.5 Background Parameterization

Due to the excellent $B$ event selection described in Section 5.2, background events, consisting of light flavor udsc decays, are less than $1.5 \%$ of the selected analysis sample. The background events term in the mixed pdf is:

$$
\begin{equation*}
P_{m i x, u d s c}=\left(1-R_{u d s c}\right) P_{u d s c}(t) . \tag{6.14}
\end{equation*}
$$

As background events are equally likely to be tagged as mixed as they are to be tagged as unmixed, we use $R_{u d s c}=0.5$, as determined from the Monte Carlo simulation. Instead of using a complex integral table, we parameterize the $u d s c$ distribution in reconstructed proper time, using a gaussian function with power law tails:

$$
F_{u d s c}(t)=p_{1} \frac{p_{1} e^{-\frac{1}{2}\left(\frac{t-p_{2}}{p_{3}}\right)^{2}}}{l_{1} \leq t \leq l_{2}} \begin{align*}
& e^{p_{5} \ln \left(\frac{p_{3} p_{5}}{p_{4}} e^{-\frac{1}{2} p_{4}^{2}}\right.} \\
& e_{5} \ln \left(p_{2}+\frac{p_{3} p_{4}-p_{3}}{\left.p_{4}-p_{4}-t\right)}\right. \tag{6.15}
\end{align*} t \leq l_{1}
$$

where $l_{1}=p_{2}-p_{3} p_{4}, l_{2}=p_{2}+p_{3} p_{6}$ and $p_{i}$ represent the values of the fit. The function can be seen overlaid on the Monte Carlo distribution in Figure 82. While the function is quite complicated, it has the benefits of being smooth and is inherently normalized. The normalization constant is fit as $p_{1}$, and therefore the probability which goes into the likelihood function is simply $P_{u d s c}(t)=F_{u d s c}(t) / p_{1}$. Additionally, by parameterizing in reconstructed proper time, we take into account detector resolution and efficiency.


Figure 82 : The Monte Carlo udsc background distribution as a function of reconstructed proper time. Monte Carlo is represented by the histogram, while the smooth curve shows the fit described in the text.

### 6.1.6 The Complete Mixed PDF For A Single Event

To summarize this section, we present the complete probability distribution function for a single event which has been tagged as mixed:

$$
\begin{align*}
\mathcal{P}_{m i x}^{i}(t) & =f_{u d s c} F_{u d s c}(t) \\
& +f_{B_{d}} \Gamma_{d} \frac{\int_{0}^{\infty} \frac{1}{2} e^{-\Gamma_{d} t^{\prime}}\left(1-\left(1-2 R_{d}-2 i+4 i R_{d}\right) \cos \left(\Delta m_{d} t^{\prime}\right)\right) \epsilon_{B_{d}}\left(t^{\prime}\right) G_{B_{d}}\left(t^{\prime}, t\right) d t^{\prime}}{\int_{0}^{10 p s}\left[P_{u n m i x, B_{d}}(t)+P_{m i x, B_{d}}(t)\right] d t} \\
& +f_{B_{s}} \Gamma_{s} \frac{\int_{0}^{\infty} \frac{1}{2} e^{-\Gamma_{s} t^{\prime}}\left(1-\left(1-2 R_{s}-2 i+4 i R_{s}\right) \cos \left(\Delta m_{s} t^{\prime}\right)\right) \epsilon_{B_{s}}\left(t^{\prime}\right) G_{B_{s}}\left(t^{\prime}, t\right) d t^{\prime}}{\int_{0}^{10 p s}\left[P_{u m m i x, B_{s}}(t)+P_{m i x, B_{s}}(t)\right] d t} \\
& +f_{B_{u}} \Gamma_{u} \frac{\int_{0}^{\infty}\left(R_{u}+i-2 i R_{u}\right) e^{-\Gamma_{u} t^{\prime}} \epsilon_{B_{u}}\left(t^{\prime}\right) G_{B_{u}}\left(t^{\prime}, t\right) d t^{\prime}}{\int_{0}^{10 p s}\left[P_{u n m i x, B_{u}}(t)+P_{m i x, B_{u}}(t)\right] d t} \\
& +f_{B_{a}} \Gamma_{a} \frac{\int_{0}^{\infty}\left(R_{a}+i-2 i R_{a}\right) e^{-\Gamma_{a} t^{\prime}} \epsilon_{B_{a}}\left(t^{\prime}\right) G_{B_{a}}\left(t^{\prime}, t\right) d t^{\prime}}{\int_{0}^{10 p s}\left[P_{\text {unmix, } B_{a}}(t)+P_{m i x, B_{a}}(t)\right] d t} \tag{6.16}
\end{align*}
$$

### 6.2 Results of the Fit

We use the likelihood function described previously, with constants determined either from world average measurements [3] or from the Monte Carlo, to fit to $\Delta m_{d}$ and, in the two-dimensional case, to $R_{d}$ as well. There are three types of fits which have been implemented; the main mode of analysis is the two-dimensional MINUIT [67] fit, which returns values for both $\Delta m_{d}$ and $R_{d}$. To ensure the integrity of the fit, two other types of fits have been performed: a one-dimensional fit in MINUIT to $\Delta m_{d}$, and a "scan" fit, which calculates the value of the log-likelihood at 80 different values of $\Delta m_{d}$ from 0.40 to $0.60 \mathrm{ps}^{-1}$. These two one-dimensional fits use the Monte Carlo value for $R_{d}$, and are expected to agree exactly. They are reported used as cross-checks on the two-dimensional fit.

### 6.2.1 Monte Carlo

After all selection requirements are applied, there remains a total of 35796 reconstructed secondary vertices. The three $\Delta m_{d}$ fits were performed on this Monte Carlo analysis set, and the results, with statistical errors only, can be seen in Table 18. The generator level Monte Carlo values for $\Delta m_{d}$ and $R_{d}$ are given in the Table for comparison. We see good agreement between all four values for $\Delta m_{d}$ and the two values for the RSF, giving confidence that the parameterization of the fit adequately describes the Monte Carlo.

As a further check on the parameterization, we have performed a fit using all three methods to only the $B_{d}$ portion of the sample using only the $B_{d}$ terms in

Table 18: Results of the Likelihood Fit in Monte Carlo

| Fit Description | $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ | $R_{d}$ |
| :---: | :---: | :---: |
| 1-D Scan | $0.465 \pm 0.015$ | N/A |
| 1-D Minuit | $0.464 \pm 0.015$ | N/A |
| 2-D Minuit | $0.462 \pm 0.015$ | $0.790 \pm .010$ |
| MC Generator | 0.484 | 0.797 |

the likelihood. The results, with statistical errors only, can be seen in Table 19. The generator level Monte Carlo values for $\Delta m_{d}$ and $R_{d}$ are given in the Table for comparison. We see excellent agreement between all four values for $\Delta m_{d}$ and the two values for the RSF, giving additional confidence that the parameterization of the fit describes the Monte Carlo well.

Table 19 : Results of the Likelihood Fit in Monte Carlo for an Exclusively $B_{d}$ Sample.

| Fit Description | $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ | $R_{d}$ |
| :---: | :---: | :---: |
| 1-D Scan | $0.480 \pm 0.010$ | N/A |
| 1-D Minuit | $0.480 \pm 0.011$ | N/A |
| 2-D Minuit | $0.481 \pm 0.011$ | $0.797 \pm .007$ |
| MC Generator | 0.484 | 0.797 |

### 6.2.2 Data

After all selection requirements are applied, there remains a total of 7844 reconstructed secondary vertices for analysis purposes. The three $\Delta m_{d}$ fits were performed on this data analysis set, and the results, with statistical errors only, can be seen in Table 20. The data vs Monte Carlo fraction of events tagged as "mixed" can be seen in Figure 83; the Monte Carlo fraction is shown with the

Monte Carlo generated value of $\Delta m_{d}=0.484 \mathrm{ps}^{-1}$. The plot shows reasonable agreement between data and Monte Carlo. A plot of the fitted two-dimensional likelihood function with the data fraction of events tagged as "mixed" can be seen in Figure 84. The fit is in good agreement with the data with a $\chi^{2}=18.9$ for 18 degrees of freedom.

We see excellent agreement between the three values for $\Delta m_{d}$ and note that the statistical error has scaled appropriately from the larger Monte Carlo analysis set to the fata set. Systematic errors will be discussed in the final section of this chapter.

Table 20: Results of the Likelihood Fit for the Selected Data Sample

| Fit Description | $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ | $R_{d}$ |
| :---: | :---: | :---: |
| 1-D Scan | $0.505 \pm 0.030$ | N/A |
| 1-D Minuit | $0.504 \pm 0.030$ | N/A |
| 2-D Minuit | $0.503 \pm 0.028$ | $0.797 \pm .022$ |

### 6.2.3 Cross-Checks of the Fit

In order to enhance confidence in the fitted results given in the previous two subsections, we have performed a number of checks involving smaller samples of data and/or Monte Carlo to verify that there are no biases due to event selection, or tagging. These investigations and their results are given in this section.


Figure 83 : The data and Monte Carlo mixed fraction plots. Monte Carlo is represented by the histogram, while data are points.


Figure 84 : The data and two-dimensional likelihood fit mixed fraction plots. The mixed fraction from the two-dimensional likelihood fit is represented by the histogram, while data are points.

## Stability Checks

To investigate possible biases in the fit due to detector resolution and performance, we have performed fits on data and Monte Carlo sets with varying selection cuts. These selections isolate physical sections of the detector, tag performance, or areas of reconstructed time. The various cuts and their two-dimensional fit results for $\Delta m_{d}$ for both data and Monte Carlo can be seen in Table 21. There are no significant discrepancies between these fits and the full sample fits.

Table 21 : Results of Data and Monte Carlo Fits with Various Cuts Applied to Samples.

| Selection Applied | $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ <br> Data | $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ <br> Monte Carlo |
| :---: | :---: | :---: |
| Only use polarization initial state tag | $0.536 \pm 0.040$ | $0.431 \pm 0.020$ |
| Only use charged initial state tags | $0.531 \pm 0.046$ | $0.483 \pm 0.021$ |
| Only use events with $b$ initial state tag | $0.487 \pm 0.041$ | $0.463 \pm 0.018$ |
| Only use events with $\bar{b}$ initial state tag | $0.523 \pm 0.034$ | $0.509 \pm 0.023$ |
| Only use events with $b$ final state tag | $0.545 \pm 0.045$ | $0.497 \pm 0.019$ |
| Only use events with $\bar{b}$ final state tag | $0.468 \pm 0.040$ | $0.436 \pm 0.017$ |
| Only events with no VXD alone tracks | $0.491 \pm 0.031$ | $0.473 \pm 0.014$ |
| Only events with kaons from the tertiary vertex | $0.496 \pm 0.030$ | $0.478 \pm 0.017$ |
| Events with reconstructed proper time $<6 \mathrm{ps}$ | $0.496 \pm 0.031$ | $0.442 \pm 0.014$ |
| Events with reconstructed proper time $<8 \mathrm{ps}$ | $0.494 \pm 0.036$ | $0.439 \pm 0.015$ |
| Events with reconstructed proper time $>0.25 \mathrm{ps}$ | $0.504 \pm 0.032$ | $0.464 \pm 0.015$ |
| Events with reconstructed proper time $>0.5 \mathrm{ps}$ | $0.504 \pm 0.034$ | $0.464 \pm 0.019$ |
| Events with $\cos \theta_{\text {thrust }}>0$ | $0.525 \pm 0.048$ | $0.481 \pm 0.020$ |
| Events with $\cos \theta_{\text {thrust }}<0$ | $0.495 \pm 0.034$ | $0.447 \pm 0.018$ |
| Events with $\left\|\cos \theta_{\text {thrust }}\right\|<0.7$ | $0.495 \pm 0.030$ | $0.465 \pm 0.016$ |
| Nominal 2-D Minuit Fit | $0.503 \pm 0.028$ | $0.462 \pm 0.015$ |

To investigate the statistical error returned by the two-dimensional fit, the Monte Carlo was divided into 25 data-sized subsamples. The individual $\Delta m_{d}$ results and their statistical errors can be seen in Table 22. The resulting distribution of the fit results, and the "pull plot" (where pull $=\frac{\Delta m_{d, r e c o n}-\Delta m_{d, m c t r u e}}{\sigma_{\Delta m_{d}}}$ ) for the errors can be seen in Figure 85. The width of the "pull" plot is expected to be $R M S=1$ if the errors are properly determined; we see that in Figure 85 that the width is in reasonable agreement with 1, implying that the statistical error is properly calculated. The mean should be at 0 , and we see reasonable agreement with that, indicating no bias towards either high or low values of $\Delta m_{d}$ in the fit.


Figure 85 : The distribution of fitted $\Delta m_{d}$ for the 25 mini-fits (left) and the "pull" distribution (right).

Table 22: Results of Monte Carlo fits for statistically independent data-sized samples

| Sample | $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ | Fit Statistical Error $\left(\mathrm{ps}^{-1}\right)$ |
| :---: | :---: | :---: |
| 1 | 0.482 | 0.030 |
| 2 | 0.424 | 0.033 |
| 3 | 0.540 | 0.041 |
| 4 | 0.537 | 0.041 |
| 5 | 0.463 | 0.035 |
| 6 | 0.483 | 0.038 |
| 7 | 0.460 | 0.023 |
| 8 | 0.414 | 0.030 |
| 9 | 0.477 | 0.024 |
| 10 | 0.437 | 0.027 |
| 11 | 0.436 | 0.034 |
| 12 | 0.490 | 0.031 |
| 13 | 0.491 | 0.033 |
| 14 | 0.528 | 0.031 |
| 15 | 0.456 | 0.030 |
| 16 | 0.514 | 0.029 |
| 17 | 0.443 | 0.023 |
| 18 | 0.485 | 0.028 |
| 19 | 0.475 | 0.033 |
| 20 | 0.462 | 0.032 |
| 21 | 0.463 | 0.035 |
| 22 | 0.493 | 0.030 |
| 23 | 0.503 | 0.031 |
| 24 | 0.428 | 0.024 |
| 25 | 0.488 | 0.033 |
| Mean | 0.478 | 0.0312 |
| RMS | 0.0315 | 0.0048 |
|  |  |  |

Table 23: Monte Carlo Generator and World Average $B$ Fractions.

| B Type | World Average Fraction | SLD Monte Carlo Generator |
| :---: | :---: | :---: |
| $B_{u}$ | $0.4005 \pm 0.010$ | 0.4064 |
| $B_{d}$ | $0.4005 \pm 0.010$ | 0.4064 |
| $B_{s}$ | $0.100 \pm 0.012$ | 0.1148 |
| b-baryons | $0.099 \pm 0.017$ | 0.0724 |

### 6.3 Potential Systematic Errors

We have studied systematic uncertainties due to physics effects and modelling, detector resolution, and reconstruction effects. In Table 24, each possible source of error considered is shown, along with the variation used and the resulting error on the data value of $\Delta m_{d}$. Here we will briefly discuss each possible error:

- B Hadron and udsc Fractions: The amount of contamination from non- $B_{d}$ events affects the evolution of the oscillation as well as the overall background of the measurement. The input fractions are determined from the Monte Carlo analysis sample. For the $B$ hadrons, the central value of each fraction is varied independently by the error on the measured world average values scaled to the Monte Carlo generated values. Both the Monte Carlo generator values and the world averages, taken from Reference [3], can be seen in Table 23. For the udsc background, we use a conservative variance of $\pm 50 \%$ on the central value.
- B Hadron Lifetimes: As the lifetime affects the evolution of the data sample, it is a possible source of systematic error. The input values are taken from the
world average [3]. We have varied the central value of each lifetime separately by the error on the world average.
- Hadron Right Sign Fractions $\left(R_{d}, R_{u}, R_{s}\right.$, and $\left.R_{a}\right)$ : The hadron right sign fractions are determined from the Monte Carlo analysis sample. We use a variance of $\pm 0.05$, which is the error on the only experimental determinations of the $B$ right sign fractions [64].
- $\Delta m_{s}$ : The oscillation frequency of the $B_{s}$ has yet to be measured, but is known to be higher than the Monte Carlo value of $10 \mathrm{ps}^{-1}$. As a conservative estimate of the error, we double the value of $\Delta m_{s}$ to $20 \mathrm{ps}^{-1}$. Fortunately, as the RSF for $B_{s}$ is essentially 0.50 , this results in no variance on the $\Delta m_{d}$ result, and so is not included in Table 24.
- Relative Boost Resolution: To estimate the systematic error due to the finite resolution of the calorimeter, we use a very conservative variance of $\pm 20 \%$ on the values of $\sigma_{b} / b$ discussed in Chapter 5. The core and tail $\sigma_{b} / b$ are varied at the same time and in the same direction. Due to the large central values of the relative resolutions and the linear dependence of $\sigma_{t}$ on $\sigma_{b} / b$, this results in a significant systematic error for the analysis.
- Decay Length Resolution: We use a very conservative variance of $\pm 20 \%$ on the values of $\sigma_{l}$ discussed in Chapter 5. The core and tail $\sigma_{l}$ are varied at the same time and in the same direction.
- Initial State Tag: Although SLD has excellent right sign probability resolution for the initial state tag, there are possible errors on it from the calculation of $A_{F B}$ and measurement of the $e^{-}$polarization for the polarization tag, and from Monte Carlo parameterizations and data measurements in the various charged tags. We use a conservative variance of $\pm 0.02$ on the IST correct tag probability, applied on an event by event basis.
- Track Multiplicities: Tracking studies have shown the SLD reconstruction to be more efficient at reconstructing tracks in Monte Carlo compared to data. A correction has been developed to determine the systematic error on SLD analyses due to this effect, with the nominal result having the correction on, and the systematic being the analysis result with it off.
- Track Resolutions: Tracking studies have shown the SLD reconstruction to be better at measuring the momentum of tracks in Monte Carlo compared to data. A correction has been developed to determine the systematic error on SLD analyses due to this effect, with the nominal result having the correction on, and the systematic being the analysis result with it off.
- $\pi$ Misidentification: As discussed in Chapter 4, there is a discrepency in the amount of $\pi \rightarrow K$ misidentification between the data and Monte Carlo. The calibration applied to Monte Carlo statistics was derived from the $K_{s}^{0} \rightarrow$ $\pi^{+} \pi^{-}$data samples, and thus the calibration is affected by the limited statistics available in these samples. We treat this as a systematic error. The
misidentification is increased and decreased by $1 \sigma_{\text {stat }}$ across the entire momentum region, and the analysis is again performed, with the difference being treated as a systematic error.

After the calculation of all systematic errors, they are combined in quadrature for a result with an overall error due to systematic effects of $\pm 0.020 \mathrm{ps}^{-1}$.

Table 24: Systematic Errors in the measurement of $\Delta m_{d}$

| Error | Central Value | Variation | $\delta\left(\Delta m_{d}\right)\left(\mathrm{ps}^{-1}\right)$ |
| :---: | :---: | :---: | :---: |
| $B_{u}$ Fraction | 0.146 | $\pm 0.004$ | $\pm 0.005$ |
| $B_{d}$ Fraction | 0.607 | $\pm 0.015$ | $\mp 0.003$ |
| $B_{s}$ Fraction | 0.170 | $\pm 0.014$ | $\mp 0.002$ |
| b-baryon Fraction | 0.067 | $\pm 0.011$ | $\pm 0.001$ |
| udsc Fraction | 0.012 | $\pm 0.006$ | $\pm 0.001$ |
| $B_{u}$ Lifetime | 1.640 ps | $\pm 0.027 \mathrm{ps}$ | $\pm 0.002$ |
| $B_{d}$ Lifetime | 1.550 ps | $\pm 0.024 \mathrm{ps}$ | $\mp 0.002$ |
| $B_{s}$ Lifetime | 1.47 ps | $\pm 0.057 \mathrm{ps}$ | $\pm 0.000$ |
| b-baryon Lifetime | 1.22 ps | $\pm 0.051 \mathrm{ps}$ | $\pm 0.001$ |
| $B_{u}$ RSF | 0.776 | $\pm 0.005$ | $\pm 0.012$ |
| b-baryon RSF | 0.614 | $\pm 0.005$ | $\pm 0.006$ |
| $\Delta m_{s}$ | $0.010 \mathrm{ps}{ }^{-1}$ | $+10 \mathrm{ps}{ }^{-1}$ | $\pm 0.000$ |
| Initial State Tag | Event by Event | $\pm 0.020$ | $\pm 0.004$ |
| Boost Resolution | See Table 15 | $\pm 20 \%$ | $\pm 0.006$ |
| Decay Length Resolution | See Table 16 | $\pm 20 \%$ | $\pm 0.001$ |
| Track Resolution | N/A | on/off | $\pm 0.005$ |
| Track Efficiency | N/A | on/off | $\pm 0.004$ |
| $\pi$ Misidentification | N/A | Statistics | $\pm 0.003$ |
| Detector Modelling | N/A | on/off | $\pm 0.007$ |
| Proper Time Residuals | N/A | on/off | $\pm 0.000$ |

## CHAPTER 7

## CONCLUSIONS

In this dissertation, we have presented a measurement of $\Delta m_{d}$ performed on a sample of 7844 events selected from the 400,000 hadronic $Z^{0}$ decays collected by the SLD at SLC in 1996-98, with a result of

$$
\Delta m_{d}=0.503 \pm 0.028 \text { (statistical) } \pm 0.020 \text { (systematic) } \mathrm{ps}^{-1} .
$$

$B_{d}$ mixing is not a "new" measurement: it has been measured many times and by many experiments, including 4 measurements using the 1993-1995 data set of the SLD experiment at the Stanford Linear Collider (SLC). The measurement in this dissertation (labelled "SLD K/Qjet+pol (1996-98)" in Figure 86), however, is still a useful and interesting contribution to the body of knowledge of $B_{d}$ mixing, and it is one of the most precise measurement to come from accelerators running at the $Z^{0}$ resonance. It is the only measurement to be performed on the larger 19961998 SLD data set. The SLD is the only particle collider experiment to have run with polarized $e^{-}$as one of its colliding beams; this feature allowed an initial state tag that is novel among all of the other experiments. Another distinctive feature
of the SLC was the small size of the interaction region $\left(\sigma_{x} \simeq 1.5 \mu \mathrm{~m}, \sigma_{y} \simeq 0.7 \mu \mathrm{~m}\right)$, whose location was stable over time to better than $5 \mu \mathrm{~m}$. The SLD is itself a unique detector, and has the world's only CCD vertex detector. The exceptional tracking resolution of the vertex detector allowed the identification of secondary vertices with an accuracy better than any other experiment. This feature, along with the accurate primary interaction point determination, gave superior resolution for the decay time of the $B$ meson. This is also the only measurement in the world to use exclusively kaons as a final state tag, which has its own unique power and challenges. The vast majority of other $\Delta m_{d}$ measurements use semileptonic decays, whereas this measurement is fully inclusive.

The experimental constraints on the unitarity triangle first presented in Chapter 2 can be seen in Figure 87, with this result highlighted. $B_{d}$ mixing is an important constraint as it measures the $V_{t d} V_{t b}^{*}$ side of the triangle.

To truly constrain the unitarity triangle of Chapter 2 and enhance our understanding of CP Violation in the Standard Model, it is crucial that $B_{s}$ mixing be measured. With experiments at the Fermilab Tevatron beginning their new data runs, which will enable us to determine whether $B_{s}$ oscillations are properly handled in the Standard Model, the next few years promise to hold interesting and exciting measurements in the study of CP Violation.


Figure 86 : Current Measurements of $\Delta m_{d}$, as of July, 2001. The measurement in this analysis is the dot labelled "SLD K/Qjet+pol (199698)".


Figure 87: Current experimental constraints on the unitarity triangle. This analysis is labelled " $\Delta m_{d}$ measurement".

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