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TRANSIENTS IN JUNCTION DIODE CIRCUITS

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ABSTRACT

Switching properties of junction diodes are analyzed with emphasis on the effects of conductivity modulation resulting in inductive behavior. Transients are computed including conductivity modulation, diffusion capacitance, transition capacitance, and finite generator impedance.

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I. INTRODUCTION

Junction diodes have been finding increasing application in high-speed switching circuits. Technological advances have resulted in increased operating speed, improved reliability, and lower cost.

Switching properties of junction diodes are analyzed in this report, with particular emphasis on the effects of conductivity modulation resulting in inductive behavior. A simple diode model is presented incorporating dc junction properties, diffusion capacitance, transition capacitance, and conductivity modulation. Transient responses are computed with this model driven by a generator with a finite source impedance.

II. THE DIODE MODEL

A. DC Characteristics of the Junction

The dc characteristics of the junction can be represented by

$$I = I_0 \left(e^{v_j/V_T} - 1 \right). \quad (1)$$

Here I is the current through the junction, v_j is the voltage across it, I_0 is the saturation current (a constant for a given diode at a fixed temperature), and V_T is given by

$$V_T = n \frac{kT}{q}, \quad (2)$$

where k is the Boltzmann constant $k = 1.38 \times 10^{-23}$ $\text{Ws}/^\circ\text{K}$, T is the absolute temperature in $^\circ\text{K}$, and $q = 1.6 \times 10^{-19}$ Coul is the charge of the electron. The multiplier n is 1 to 1.5 for germanium, 1.5 to 2 for silicon diodes. The value of kt/q is in the vicinity of 25 mV at room temperature, resulting in a V_T of between 25 mV and 50 mV.

From (1), for large negative v_j one would expect the diode current I to approach $-I_0$, typically a few nanoamperes. In reality, to this current one has to add a surface leakage current and a current resulting from breakdown at some negative v_j . In the following, however, these corrections will be omitted and (1) will be utilized throughout.

A small signal characteristic of the diode, the incremental resistance

$$r_i = \left(\frac{dI}{dv_j} \right)^{-1}, \quad (3)$$

can be expressed from (1) as

$$r_i = \frac{V_T}{I + I_0}. \quad (4)$$

B. Stored Charge and Capacitance.

The charge Q stored in a junction diode, and the resulting capacitance C , can be divided into two parts:*

$$Q = Q_d(I) + Q_t(v_j) \quad (5)$$

and

$$C = C_d(I) + C_t(v_j) \quad (6)$$

where

$$C_d = \frac{dQ_d}{dv_j} \quad (7a)$$

$$C_t = \frac{dQ_t}{dv_j} \quad (7b)$$

hence also

$$C = \frac{dQ}{dv_j} \quad (7c)$$

The charge Q_d is, to a very good approximation, proportional to the junction current I :

$$Q_d = \tau_0 I, \quad (8)$$

where the factor of proportionality, τ_0 , is the minority carrier lifetime. The diffusion capacitance C_d can be written as

$$C_d = \frac{dQ_d}{dv_j} = \tau_0 \frac{dI}{dv_j} \quad (9)$$

By utilizing (1) and (8),

$$C_d = \tau_0 \frac{I + I_0}{V_T}, \quad (10)$$

which can be also written as

$$C_d = \frac{\tau_0 I_0}{V_T} e^{v_j/V_T} \quad (11)$$

*The assumption is made that Q_d and Q_t are each stored in a single storage element. For more accurate models, see J. G. Linvill, Models of Transistors and Diodes, (McGraw Hill, New York, 1963).

The charge Q_t is that stored in the transition capacitance of the junction, which depends on the effective width of the junction as

$$Q_t = \frac{C_0 V_0^m}{1-m} \left[V_0^{(1-m)} - (V_0 - v_j)^{(1-m)} \right] \quad (12)$$

Also

$$C_t = \frac{dQ_t}{dv_j} = \frac{C_0}{\left(1 - \frac{v_j}{V_0}\right)^m} \quad (13)$$

Here m is between $1/3$ and $1/2$, V_0 is in the vicinity of 0.7 volts.

In addition to the above, there is also a stray capacitance associated with the diode; this will be neglected in what follows.

C. Body Resistance and Conductivity Modulation

The junction described above is made between two materials, at least one of which is a semiconductor. In reality, these are of finite size resulting in a series ohmic body resistance which at low currents can be approximated by a constant. At high current levels, however, the stored charge may substantially increase the conductivity of the body material: this is the phenomenon of conductivity modulation. The resulting conductance $1/r_s$ can be approximated by the sum of two terms: a constant g_0 and one proportional to the stored charge:

$$\frac{1}{r_s} = g_0 + \text{constant} \times Q_d \quad (14)$$

Utilizing (1) and (8), this becomes

$$\frac{1}{r_s} = g_0 + \frac{I}{V_s} \quad (15)$$

which can be written as

$$\frac{1}{r_s} = g_0 + \frac{I_0}{V_s} \left(e^{v_j/V_T} - 1 \right) \quad (16)$$

Here g_0 and V_s are constants, characteristic of the diode.

D. The Complete Diode Model

The complete diode model is shown in Fig. 1. For convenience, the expressions for I , r_i , C_d , C_t , and r_s are collected here.

$$I = I_0 \left(e^{v_j/V_T} - 1 \right) \quad (17a)$$

$$v_j = V_T \ln \left(1 + \frac{I}{I_0} \right) , \quad (17b)$$

$$r_i = \frac{V_T}{I + I_0} , \quad (18)$$

$$C_d = \tau_0 \frac{I + I_0}{V_T} = \frac{\tau_0 I_0}{V_T} e^{v_j/V_T} , \quad (19)$$

$$C_t = \frac{C_0}{\left(1 - \frac{v_j}{V_0} \right)^m} , \quad (20)$$

$$\frac{1}{r_s} = g_0 + \frac{I}{V_s} = g_0 + \frac{I_0}{V_s} \left(e^{v_j/V_T} - 1 \right) . \quad (21)$$

III. JUNCTION DIODE TRANSIENTS

A. The Circuit

Transients of junction diodes in the circuit of Fig. 2(a) will be analyzed. The circuit with the diode model of Fig. 1 substituted is shown in Fig. 2(b); the generator current waveform $i_g(t)$ is shown in Fig. 2(c).

Several special cases will be considered. The simplest case, when C_t and r_s are zero, and $R_g \rightarrow \infty$, is analyzed in Section III. B. The analysis is extended in Section III. C for the case of $r_s \neq 0$. Section III. D gives results for finite R_g and $C_t \neq 0$. Transients at very high forward current levels when (15) is not valid are discussed in Section III. E.

B. Transients with $C_t = 0$, $r_s = 0$, and $R_g \rightarrow \infty$.

In this case, in the circuit of Fig. 2(b) $v_j = v_{in}$. Resistor R_g can be chosen to be arbitrarily large; it cannot, however, be omitted entirely since then all of $i_g(t)$ would be forced into the junction, and (1) may be violated.

Neglecting the current through R_g , one can write the equations:

$$i_g = I + i_c \quad , \quad (22)$$

$$i_c = C_d \frac{dv_j}{dt} \quad , \quad (23)$$

and

$$I = I_0 \left(e^{v_j/V_T} - 1 \right) \quad , \quad (24a)$$

the latter of which can also be written as

$$v_j = V_T \ln \left(1 + \frac{I}{I_0} \right) \quad . \quad (24b)$$

Expanding (23), and utilizing (4) and (10), one obtains

$$i_c = C_d \frac{dv_j}{dt} = C_d \frac{dv_j}{dI} \frac{dI}{dt} = C_d r_i \frac{dI}{dt} = \tau_0 \frac{dI}{dt} \quad , \quad (25)$$

as a result (22) becomes

$$i_g = I + \tau_0 \frac{dI}{dt} \quad . \quad (26)$$

Thus, although neither r_i nor C_d is constant, the resulting differential equation is linear in I .

From (26), for the $i_g(t)$ of Fig. 2(c) with $-I_{g^-} < -I_0$, one can write for the turn-on transient

$$I = -I_0 + (I_{g^+} + I_0) \left(1 - e^{-t/\tau_0}\right) \quad . \quad (27)$$

Also, by utilizing (24b)

$$v_j = V_T \ln \left[\left(\frac{I_{g^+}}{I_0} + 1 \right) \left(1 - e^{-t/\tau_0} \right) \right] \quad . \quad (28)$$

For $I_{g^+} \gg I_0$, this becomes

$$v_j \approx V_T \left[\ln \frac{I_{g^+}}{I_0} + \ln \left(1 - e^{-t/\tau_0} \right) \right] \quad . \quad (29)$$

The turn-off transient, assuming that equilibrium conditions are established by the time t_{off} , is given by

$$I = I_{g^+} - (I_{g^+} + I_{g^-}) \left(1 - e^{-(t-t_{\text{off}})/\tau_0} \right) \quad , \quad (30)$$

valid as long as $I \geq -I_0$, i. e., as long as $t - t_{\text{off}} \leq \tau_0 \ln \frac{I_{g^+} + I_{g^-}}{I_{g^-} - I_0}$; $I = -I_0$

otherwise. The voltage v_j can be obtained by substituting (30) into (24b):

$$v_j = V_T \ln \left\{ 1 + \frac{1}{I_0} \left[-I_{g^-} + (I_{g^+} + I_{g^-}) e^{-(t-t_{\text{off}})/\tau_0} \right] \right\} \quad (31)$$

The voltage v_j becomes zero at a time $t_{\text{off}} + t_s$, where from (31) with $v_j = 0$,

$$t_s = \tau_0 \ln \left(1 + \frac{I_{g^+}}{I_{g^-}} \right) . \quad (32)$$

The time t_s is defined as the storage time of the diode, which is a function of τ_0 and I_{g^+}/I_{g^-} .

Representative transients of (28) and (31) are shown in Fig. 3 and Fig. 4.

C. Transients with $C_t = 0$, $r_s \neq 0$, and $R \rightarrow \infty$.

With a non-zero ohmic body resistance r_s , the input voltage can be written with (21) as

$$v_{\text{in}} = v_j + \frac{i_{\text{in}}}{g_0 + \frac{I}{V_s}} . \quad (33)$$

For $t < t_{\text{off}}$ and $I \gg I_0$, utilizing Eqs. (27) and (29) for I and v_j , respectively, the input voltage becomes

$$v_{\text{in}} \cong V_T \left[\ln \frac{I_{g^+}}{I_0} + \ln \left(1 - e^{-t/\tau_0} \right) + \frac{V_s/V_T}{\frac{g_0 V_s}{I_{g^+}} + 1 - e^{-t/\tau_0}} \right] \quad (34)$$

It can be shown that the nature of the forward transient is determined by the value of $\frac{I_{g^+}}{g_0 V_s}$. For $\frac{I_{g^+}}{g_0 V_s} < 4$, the turn-on transient is monotonic. For $4 < \frac{I_{g^+}}{g_0 V_s} < 4.536$ the transient has a local maximum followed by a local minimum, but the value of the maximum is below the final value of the voltage. For $\frac{I_{g^+}}{g_0 V_s} > 4.536$, the transient has an overshoot followed by an undershoot. As $\frac{I_{g^+}}{g_0 V_s}$ is increased more and more above 4.536, the magnitude of the overshoot increases, and that of the undershoot decreases. These various possibilities are shown in Fig. 5.

Figure 5 also shows the turn-off transient obtained by substituting (30) and (31) into (33). As a result of non-zero r_b , there is an instantaneous drop in the terminal voltage followed by a storage time close to that of the case with $r_s = 0$.

D. Transients with $C_t \neq 0$ and $R_g \neq \infty$.

If R_g is finite or C_t is not zero, the transient analysis of the circuit of Fig. 2(b) becomes considerably more involved. The following equations can be written:

$$C_d = \frac{\tau_0 I_0}{V_T} e^{v_j/V_T} \quad (35)$$

$$C_t = \frac{C_0}{\left(1 - \frac{v_j}{V_0}\right)^m} \quad (36)$$

$$I = I_0 \left(e^{v_j/V_T} - 1 \right) \quad (37)$$

$$r_s = \frac{1}{g_0 + I/V_s} \quad (38)$$

$$i_{in} = \frac{i_g R_g - v_j}{R_g + r_s} \quad (39)$$

$$v_{in} = (i_g - i_{in}) R_g \quad (40)$$

$$i_c = i_{in} - I \quad (41)$$

Also

$$v_j = \int \frac{i_c}{C_d + C_t} dt$$

which can be approximated by a finite sum:

$$v_j(t + \Delta t) = v_j(t) + \Delta v \quad (42a)$$

where

$$\Delta v = \frac{i_c \Delta t}{C_d + C_t} \quad (42b)$$

Equations (35) through (43) are solved by a digital computer using the flow-chart of Fig. 6 with $\Delta t_{max} = 0.1\tau_0$, $\Delta t_{min} = 10^{-6}\tau_0$, and $\Delta v_{max} = 0.1V_T$.

The Fortran - IV computer program is shown in Fig. 7.

Representative transients are shown in Fig. 8 and Fig. 9. Storage times are summarized in Fig. 10. Transients for $C_t \neq 0$ with $V_0 = 25 V_T$ and $m = \frac{1}{2}$ are shown in Fig. 11 and Fig. 12.

E. Transients at Very High Forward Currents.

In general, the body resistance r_s can be written as*

$$r_s = \frac{1}{g_0} \left[1 - \frac{1}{X_n} \ln \left(\frac{1 + \frac{I}{g_0 V_s}}{1 + \frac{I}{g_0 V_s} e^{-X_n}} \right) \right] \quad (43)$$

Here g_0 and V_s are constants and $X_n = W/L$, where W is the dimension of the diode in the direction of the current flow, and L is the diffusion length of the minority carriers.

It can be shown that r_s is always positive; also that in the limit $X_n \rightarrow 0$, r_s of (43) becomes

$$r_s \xrightarrow{X_n \rightarrow 0} \frac{1}{g_0 + \frac{I}{V_s}} \quad (44)$$

by L'Hospital's rule.

If the diode is "thin" as it was assumed in Section III. C, (44) is applicable; in general (43) must be used. The effect of finite X_n on the transient is shown in Fig. 13.

* W. H. Ko, "The forward transient behavior of semiconductor function diodes," *Solid-State Electronics* 3, 59-69 (1961).

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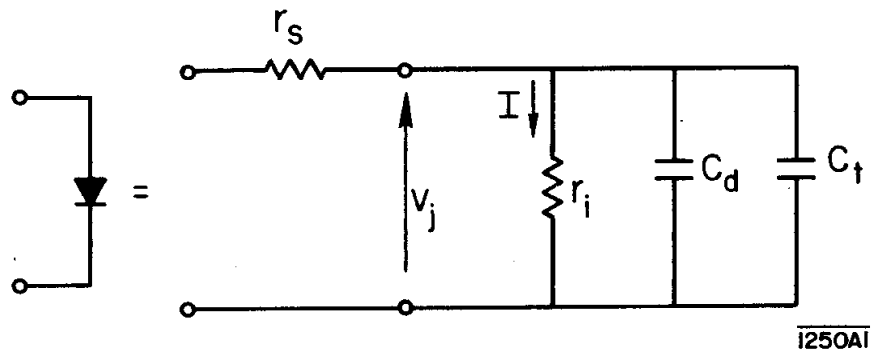
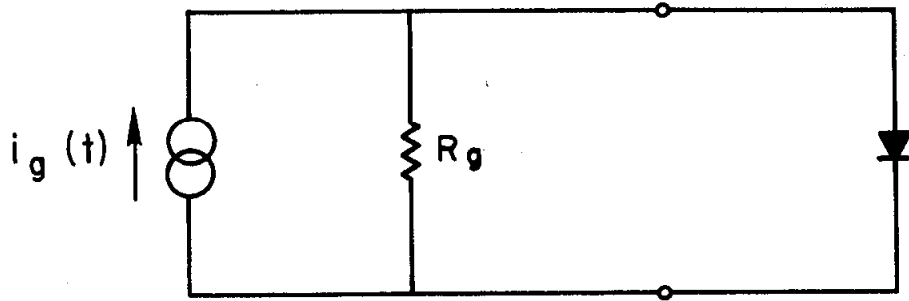
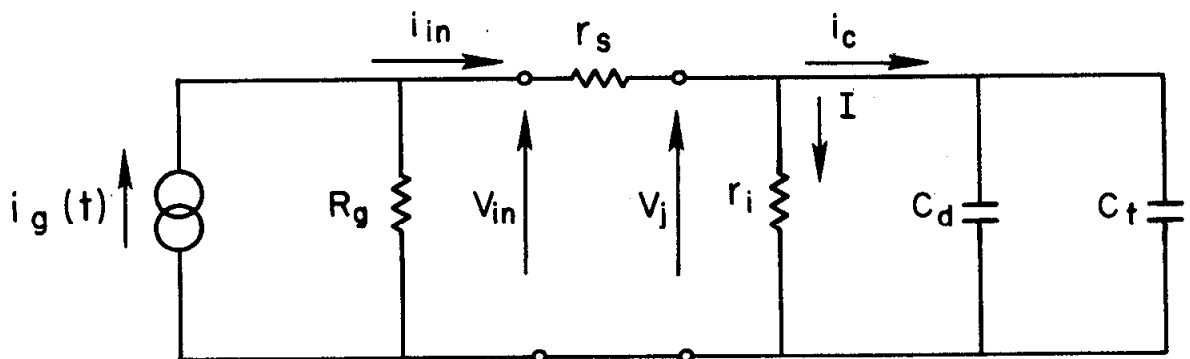


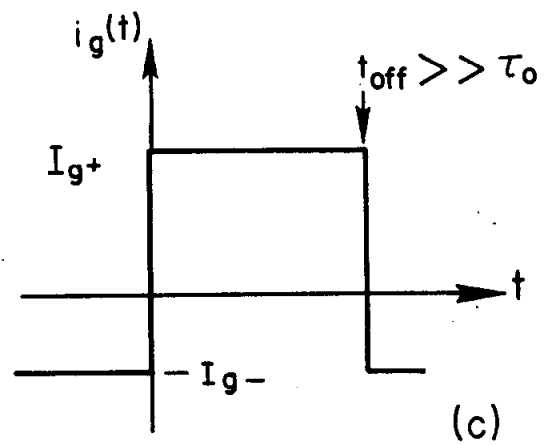
Fig. 1



(a)



(b)



(c)

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Fig. 2

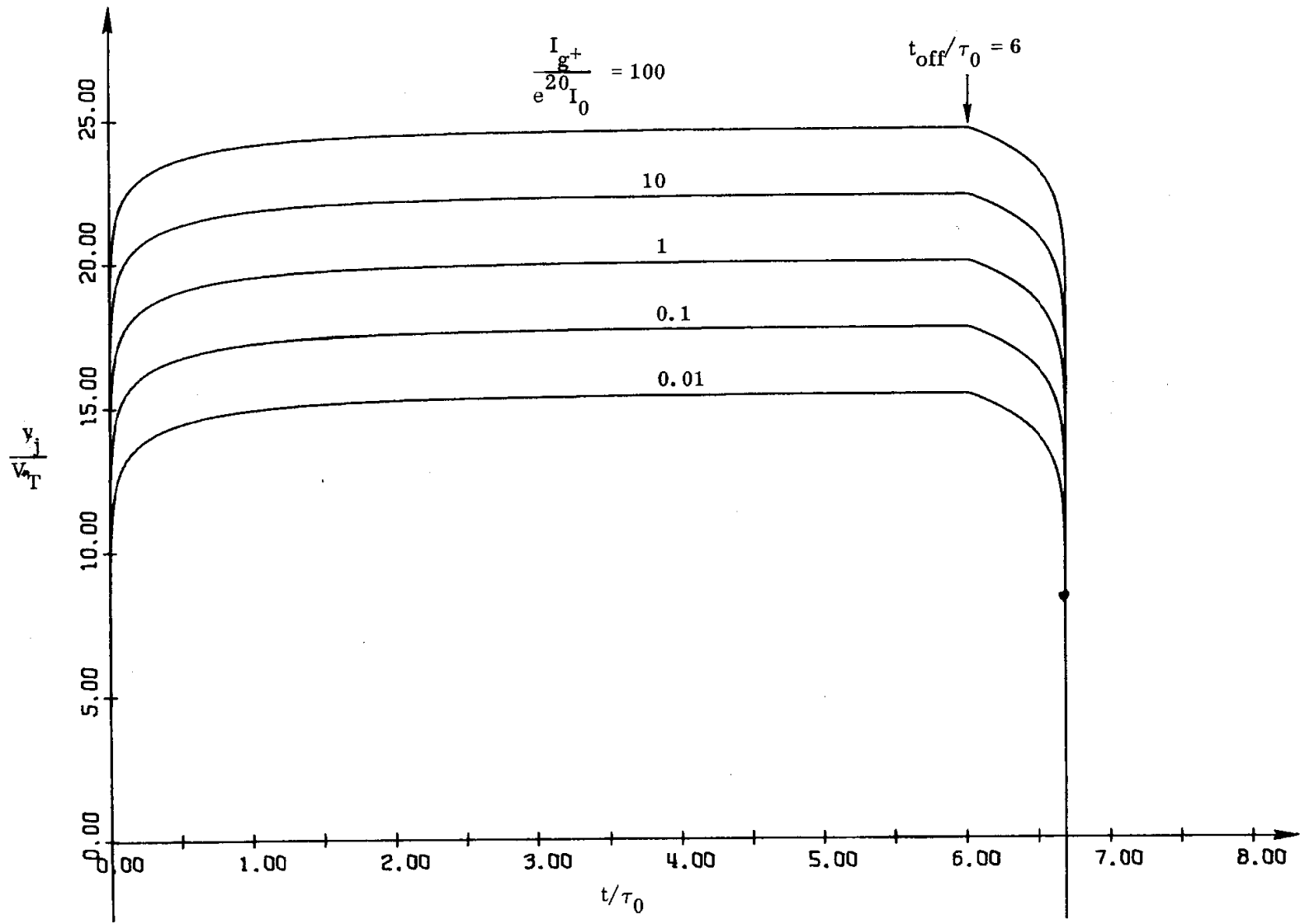


Fig. 3a

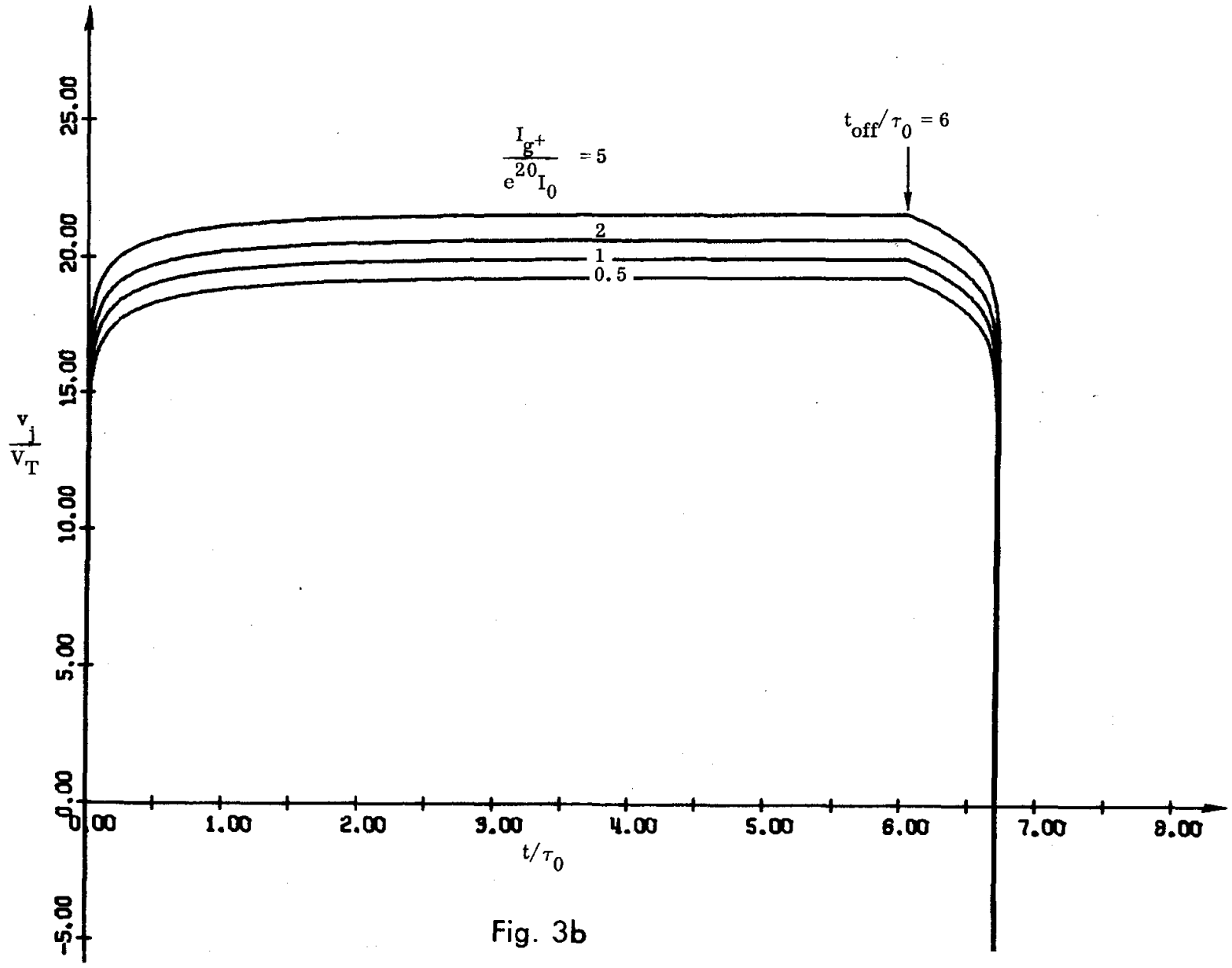


Fig. 3b

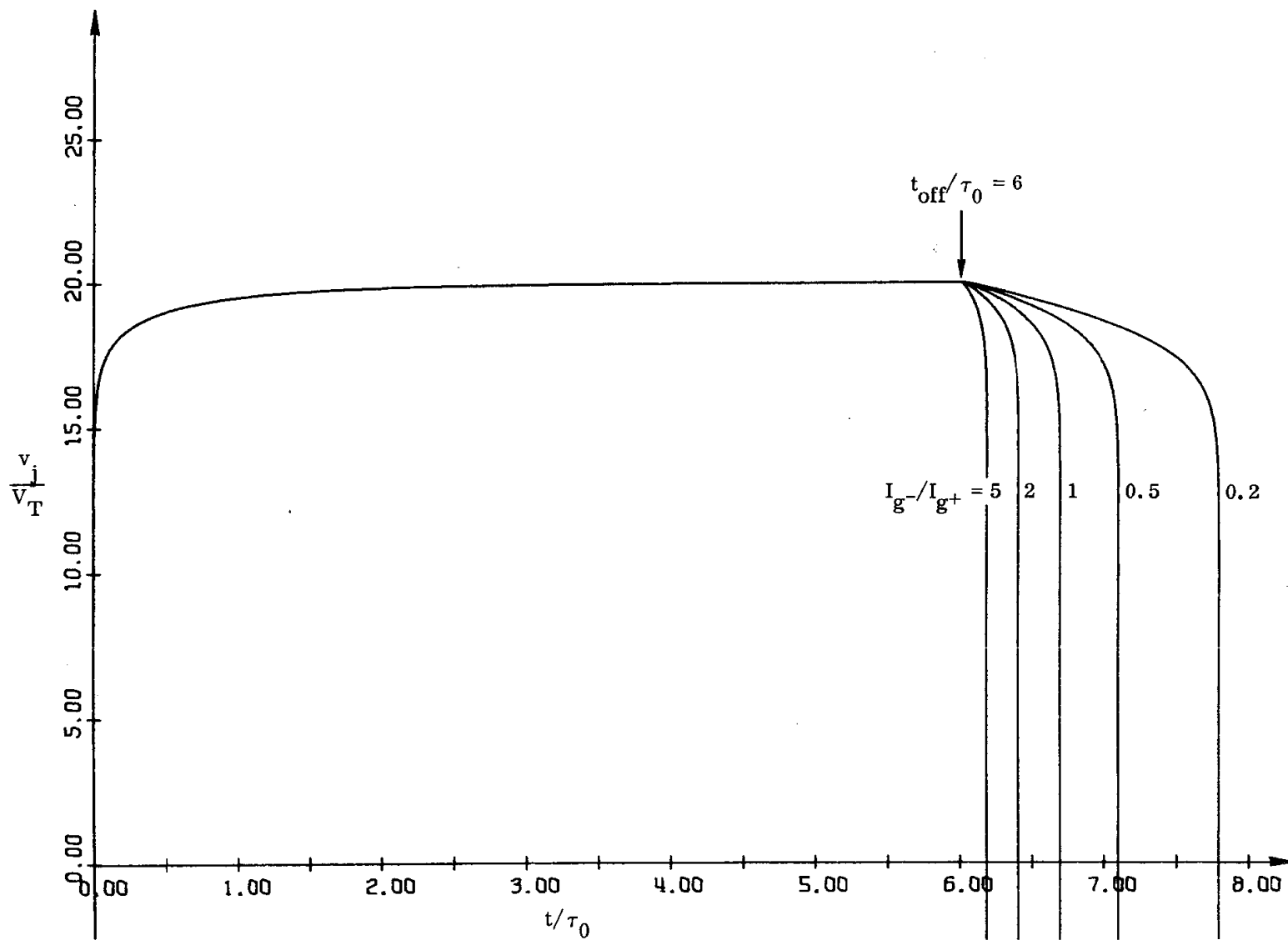


Fig. 4

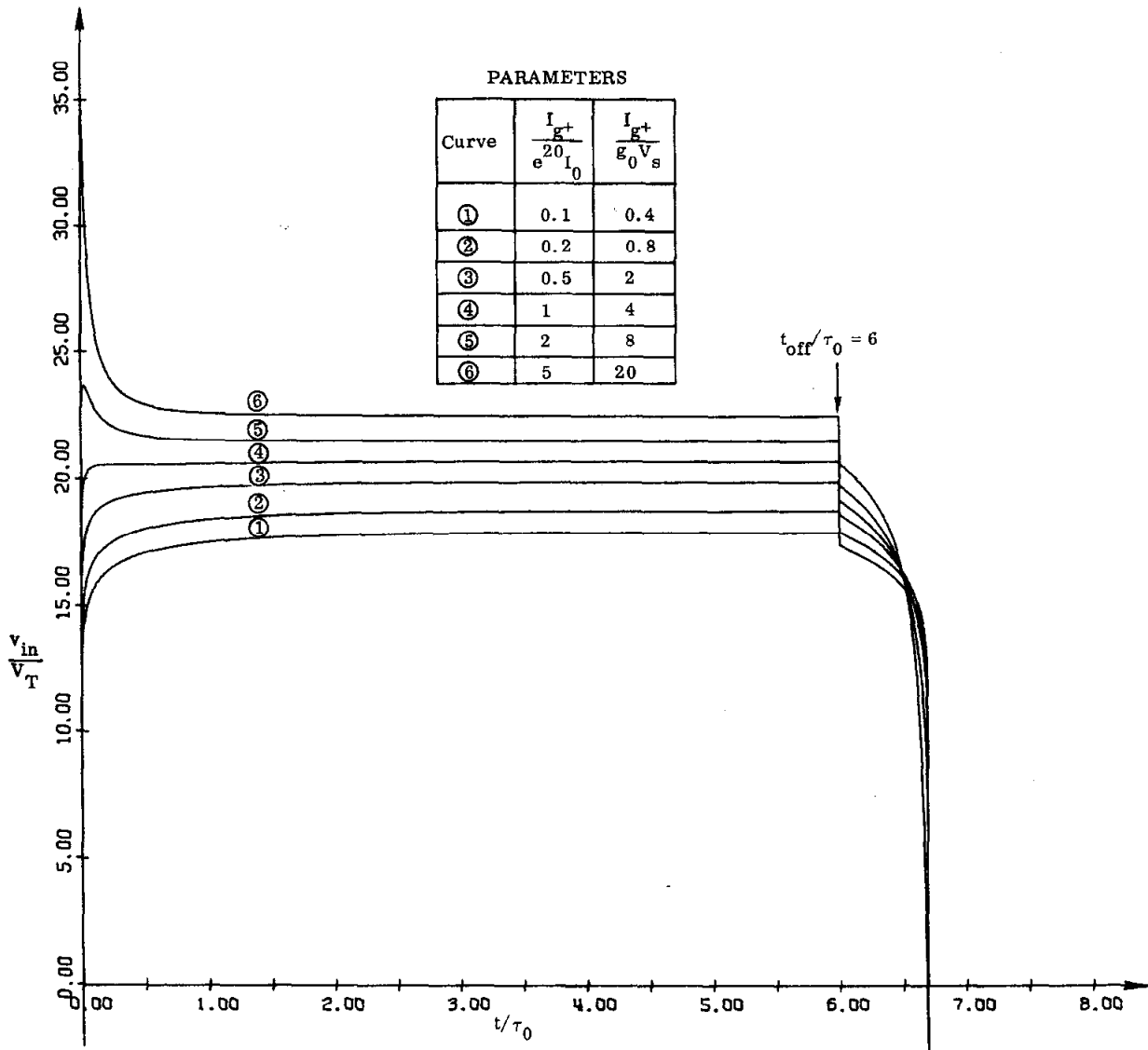


Fig. 5a

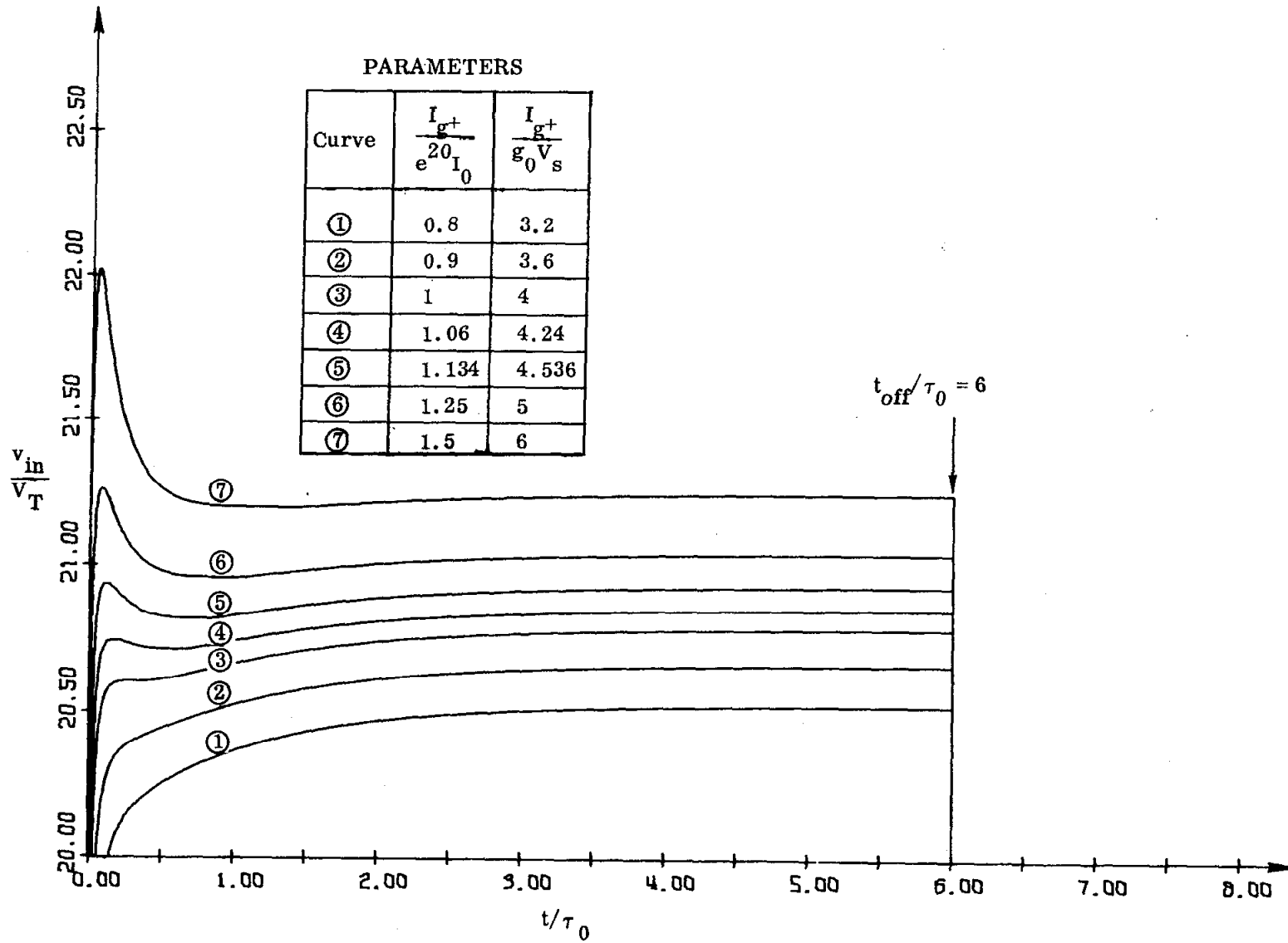
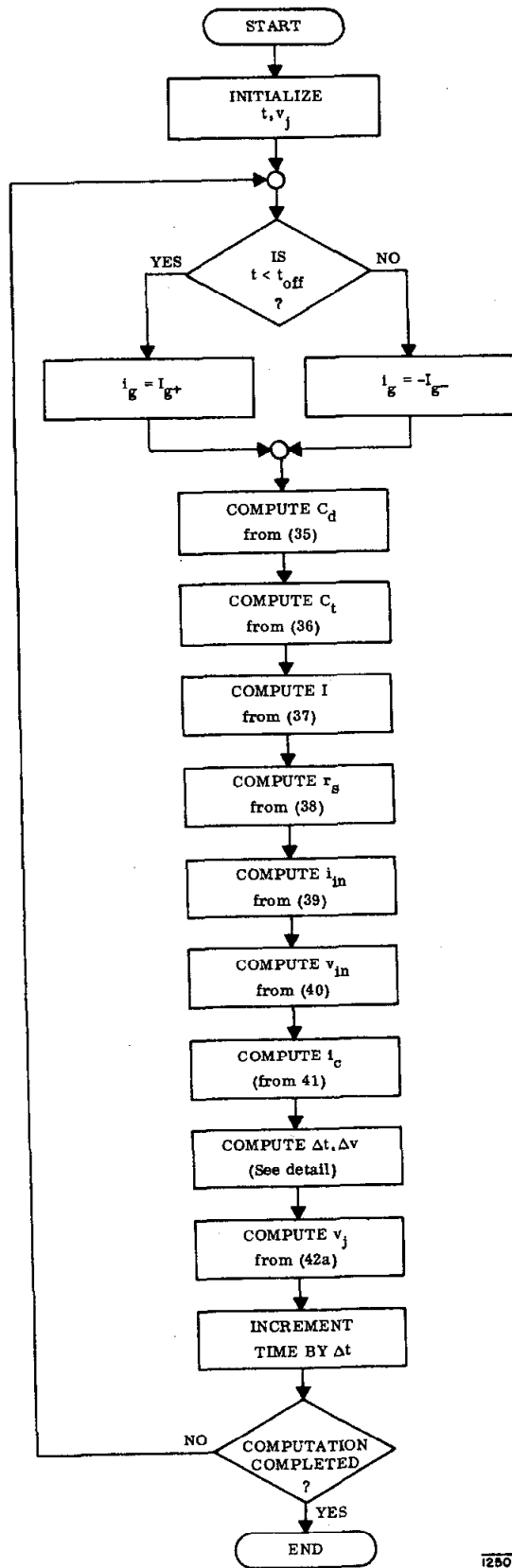
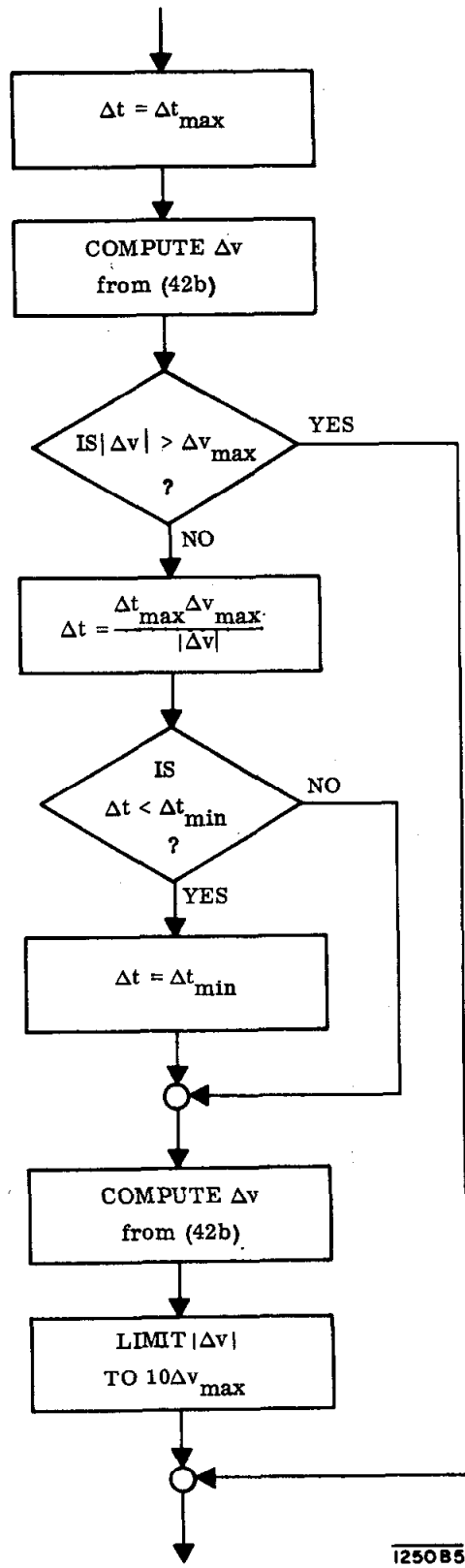


Fig. 5b



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Fig. 6a



125085

Fig. 6b


```

C *** DIODE TRANSIENT***
C
FUNCTION CQ(V)
IF(V.LT.-100.0)V=-100.0
IF(V.GT.+140.0)V=+140.0
CQ=EXP(V-20.0)
RETURN
END

FUNCTION DIODE(V)
IF(V.LT.-100.0)V=-100.0
IF(V.GT.+120.0)V=+120.0
DIODE=EXP(V)-1.0
RETURN
END

REAL ID,IC,IG,L
DOUBLE PRECISION V
CALL STRP1(29)
CALL PLOT1(0.0,-30.0,23)
CALL PLOT1(0.0,0.5,23)
1 FORMAT(6F10.4,11)
2 FORMAT(' ',RISETIME=' ',1PE10.3, ' TURNOFFTIME=' ',1PE10.3)
3 FORMAT(' ',7F10.4,11)
4 FORMAT('1')
5 FORMAT(' ')
6 FORMAT(F10.4)
ARG1=-20.0
EXP1=EXP(ARG1)
7 CONTINUE
READ(5,6)CTR
IF(CTR.LE.0.0)GO TO 100
IF(CTR.LT.0.001)CTR=0.0
WRITE(6,4)
11 CONTINUE
READ(5,1)G,GN,R1,DELTA,L,C1,NEWPLT
IF(G.LE.0.0)GO TO 7
WRITE(6,5)
WRITE(6,3)G,GN,R1,DELTA,L,C1,CTR,NEWPLT
IF(C1.GT.100.0)C1=1E20
DELTA2=1E-6
CALL PLOT1(0.0,0.0,+3)
IF(NEWPLT.EQ.0)GO TO 12
CALL PLOT1(15.0,0.0,-3)
CALL AXIS1(0.0,2.0,'T',-1,10.0,0.0,0.0,1.0,20.0)
CALL AXIS1(0.0,0.0,'V',+1,10.0,90.0,-10.0,5.0,10.0)
12 CONTINUE
TRISE=0.0
TTOFF=0.0
XPLOTM=-0.02
V=-GN
VL=-GN
IC=0.0
T=0.0
XPLOT=0.0
YPLT=2.0+0.2*VL
IF(YPLT.LE.0.0)YPLT=0.0
CALL PLOT1(XPLOT,YPLT,+3)
13 CONTINUE
VI=V
IF(CTR.LE.0.0)IC=CQ(VI)
IF(CTR.GT.0.0)IC=CQ(VI)+CTR/SQRT(1.0-VI*0.04)
IF(T.LT.6.0)VG=G
IF(T.GE.6.0)VG=-GN
ID=DIODE(V)*EXP1
GS=G+L*ID
GS1=1.0/GS
IG=(VG-V)/(GS1+R1)
VL=VG-IG*R1
IC=IG-ID
DELTA1=DELTA
DV=IC*DELTA1/C
ABSDV=ABS(DV)
IF(ABSDV.LT.0.1)GO TO 16
DELTA1=0.1*DELTA1/ABSDV
IF(DELTA1.LT.DELTA2)DELTA1=DELTA2
DV=IC*DELTA1/C
IF(DV.GT.0.1)DV=0.1
IF(DV.LT.-0.1)DV=-0.1
16 CONTINUE
V=V+DV
T=T+DELTA1
IF(VL.GT.0.0.AND.TRISE.EQ.0.0)TRISE=T
IF(VL.LT.0.0.AND.TRISE.NE.0.0.AND.TTOFF.EQ.0.0)TTOFF=T-6.0
XPLOT=T
IF(XPLOT-XPLOTM.LT.0.01)GO TO 15
XPLOTM=XPLOT
YPLT=2.0+0.2*VL
IF(YPLT.LT.0.0)YPLT=0.0
IF(YPLT.GT.10.0)YPLT=10.0
14 CONTINUE
CALL PLOT1(XPLOT,YPLT,+2)
IF(YPLT.LE.0.0.AND.T.GT.6.0)GO TO 17
15 CONTINUE
IF(XPLOT.LE.10.0)GO TO 13
17 CONTINUE
WRITE(6,2)TRISE,TTOFF
WRITE(6,5)
GO TO 11
100 CONTINUE
CALL PLOT1(15.0,0.0,-3)
CALL ENDP1
STOP
END

```

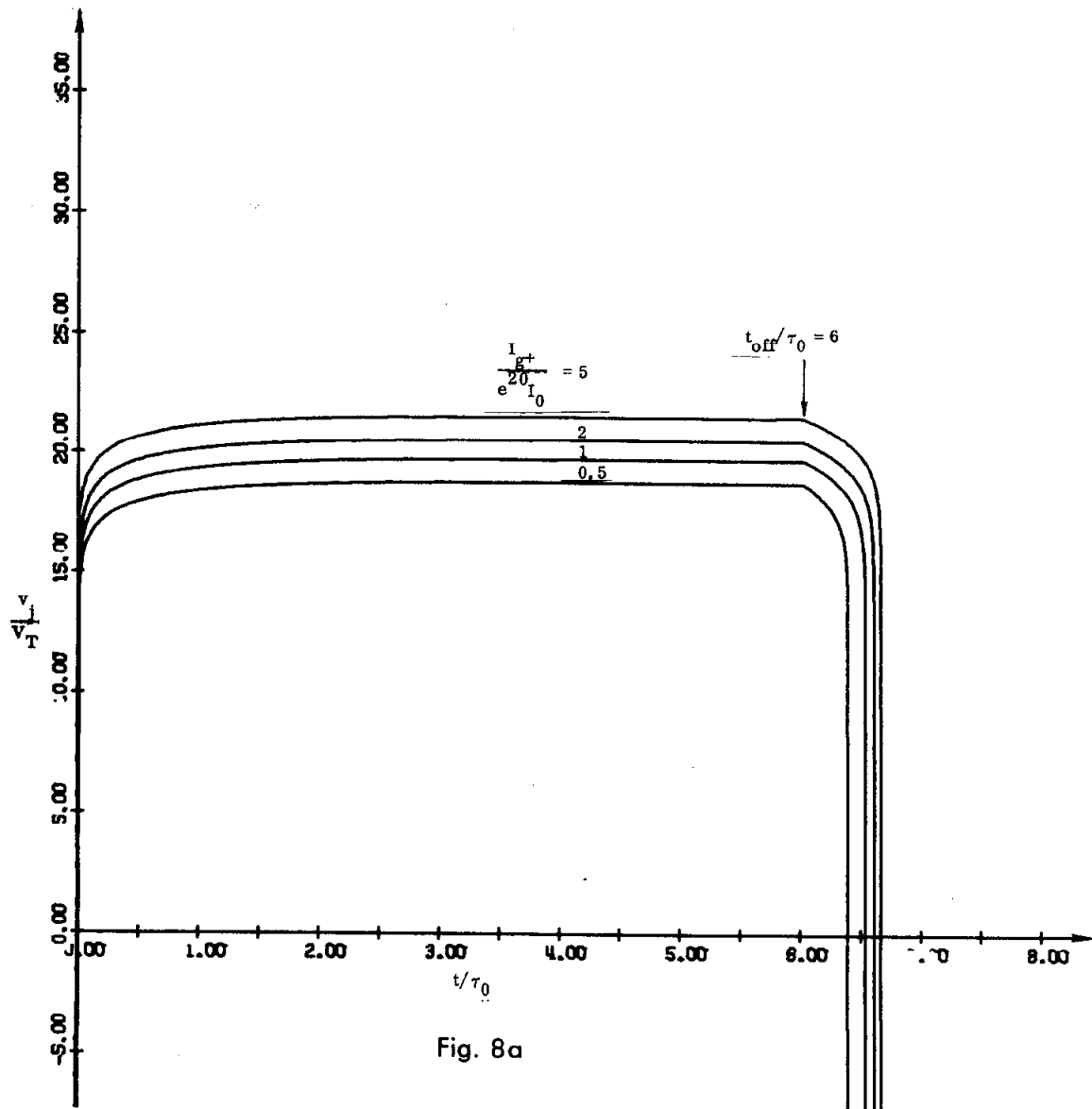


Fig. 8a

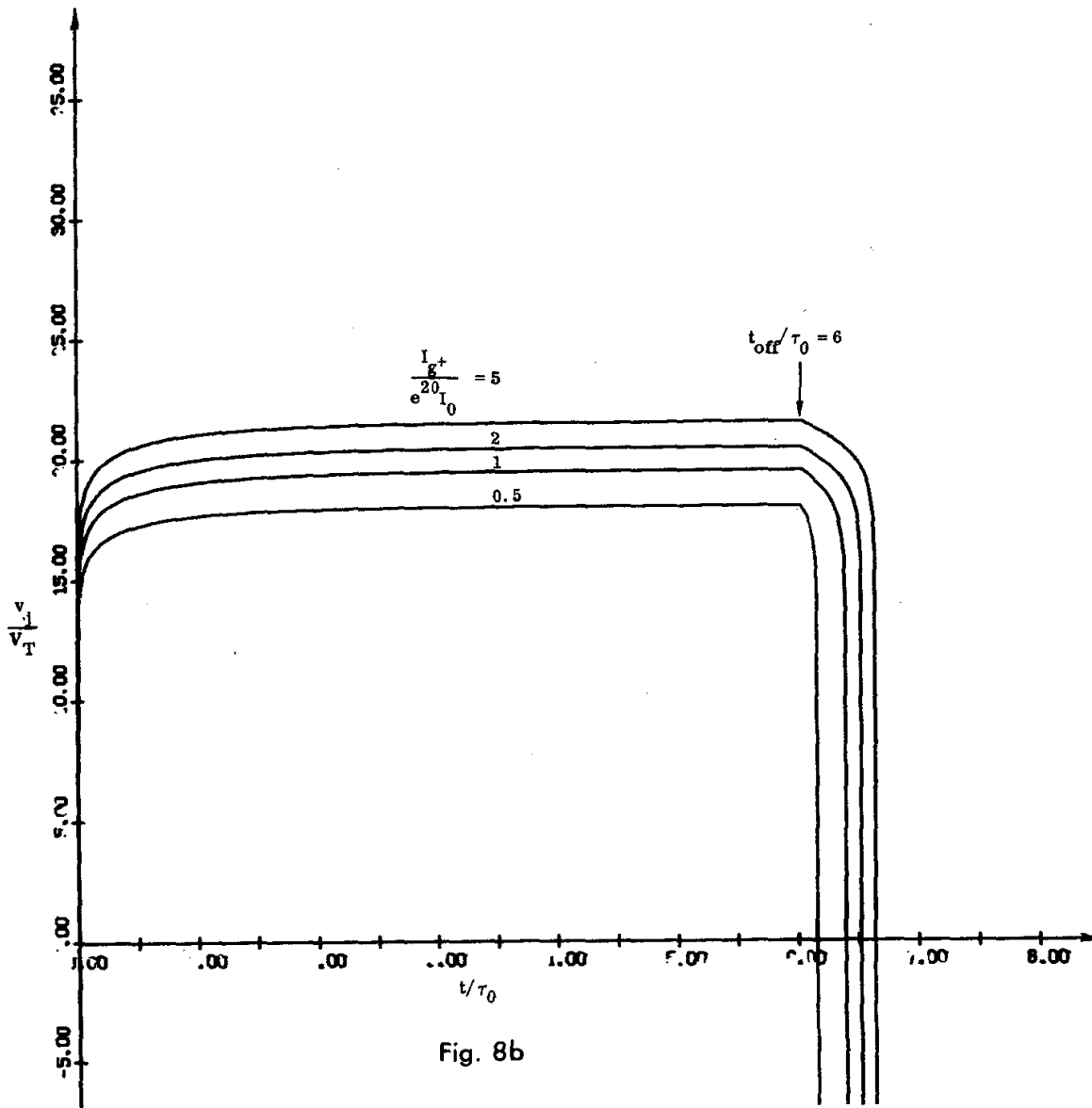


Fig. 8b

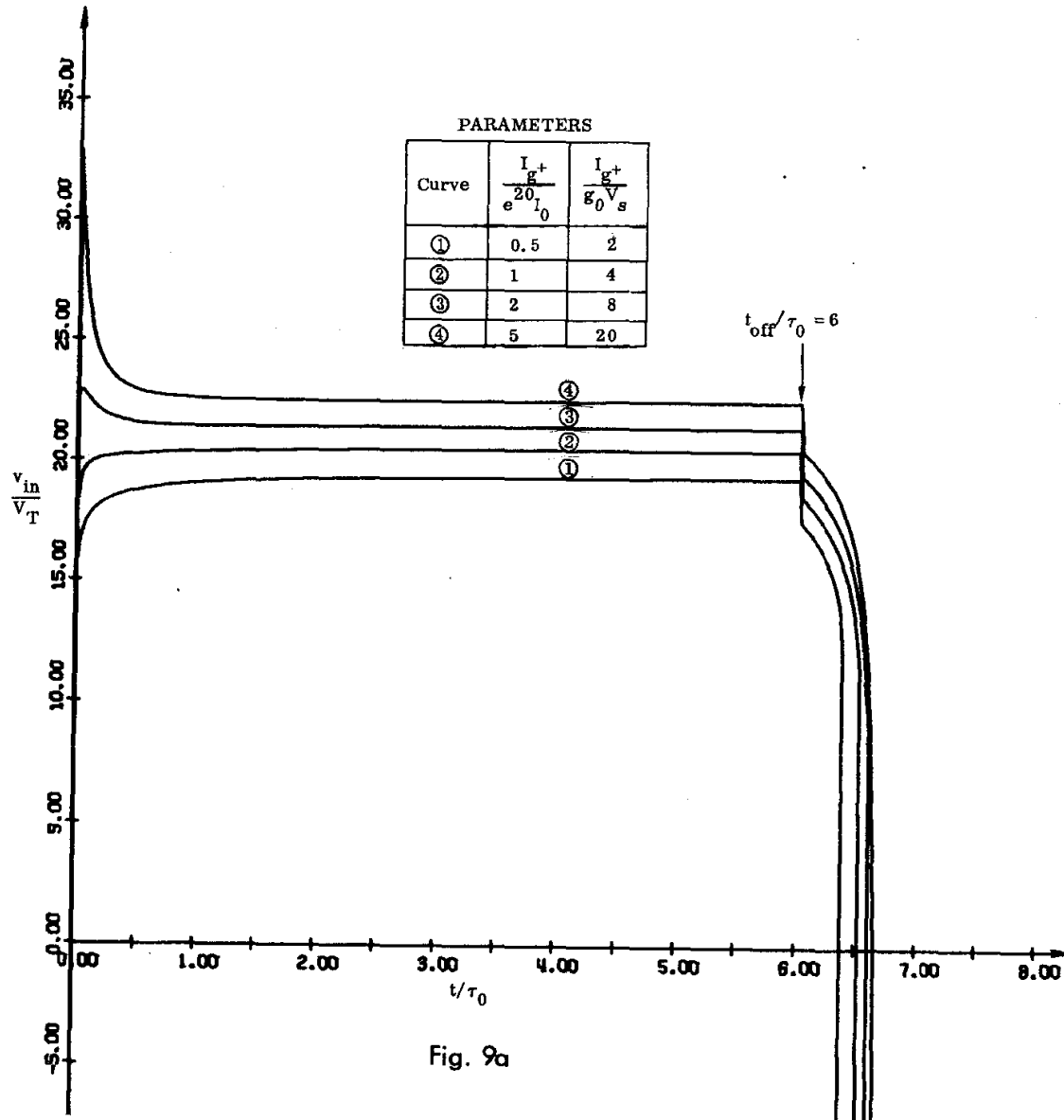


Fig. 9a

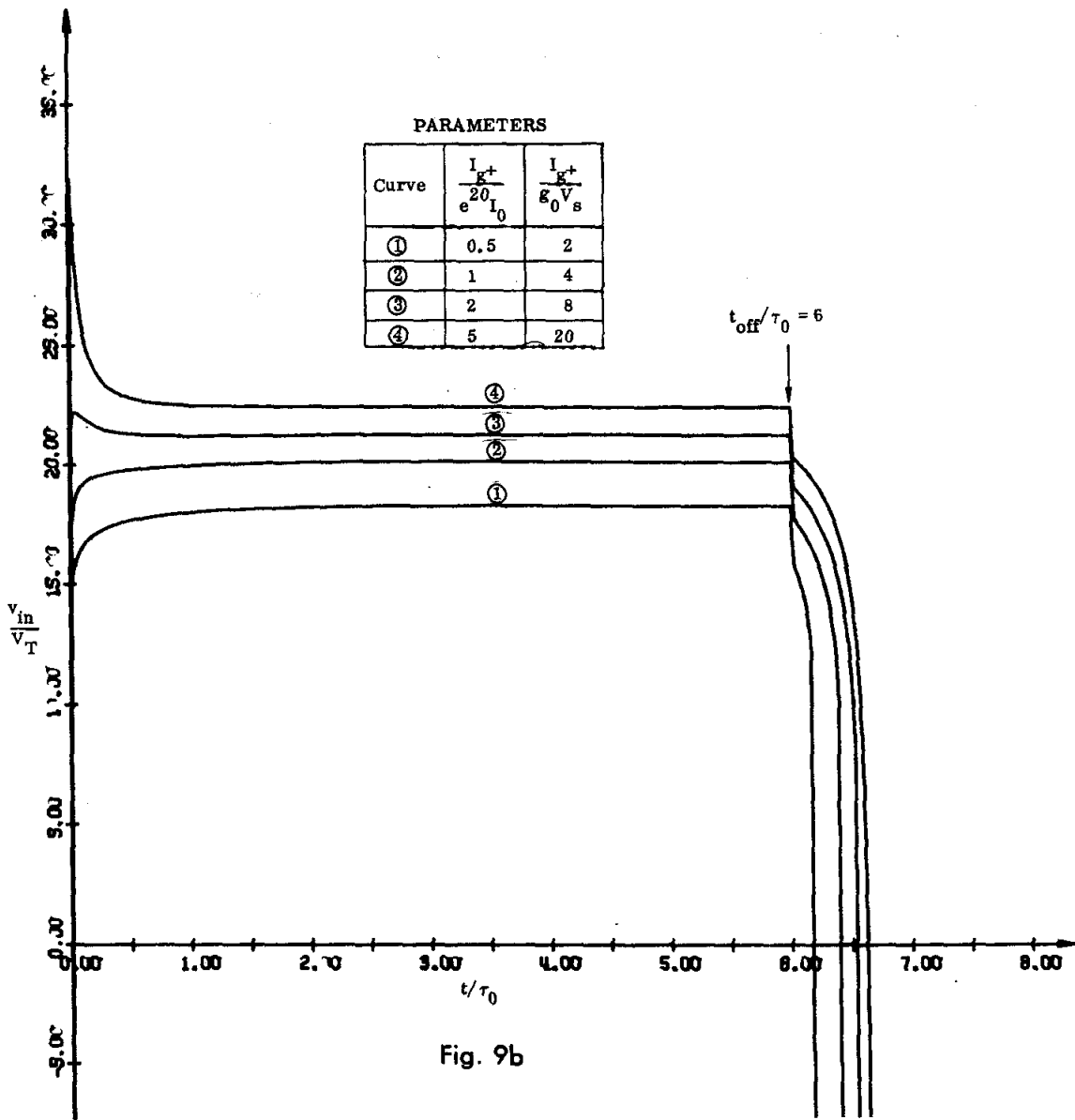


Fig. 9b

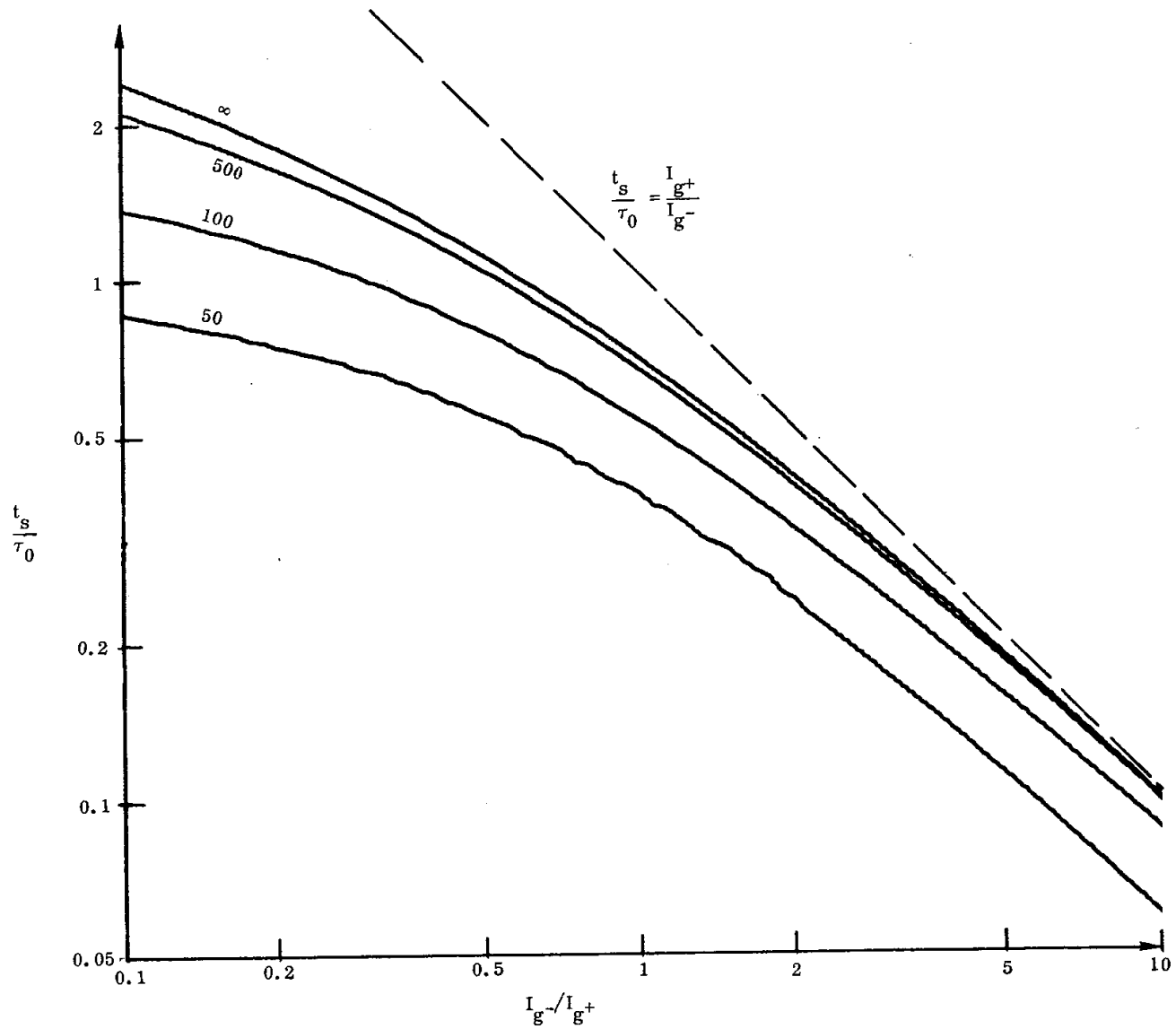


Fig. 10

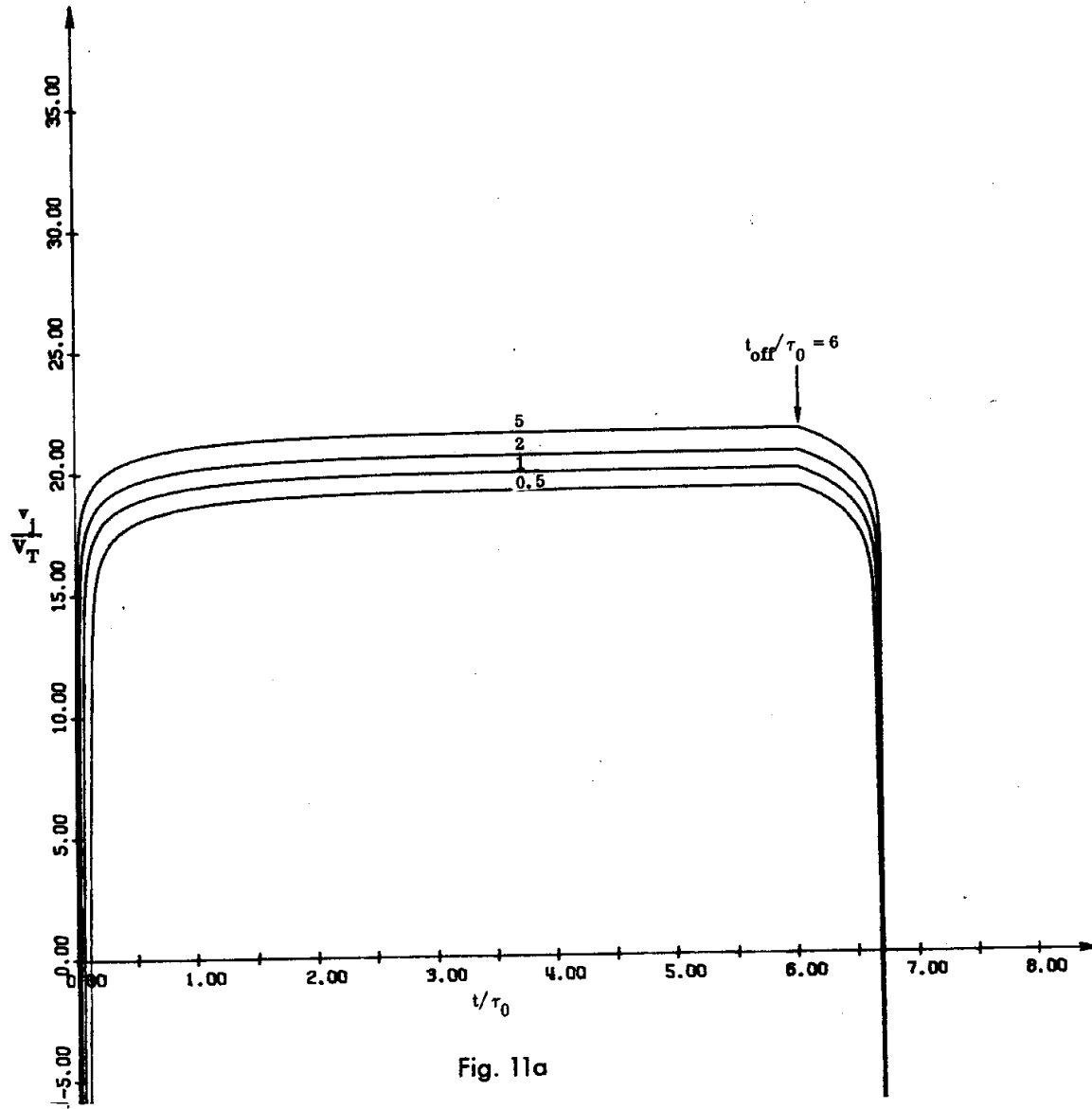


Fig. 11a

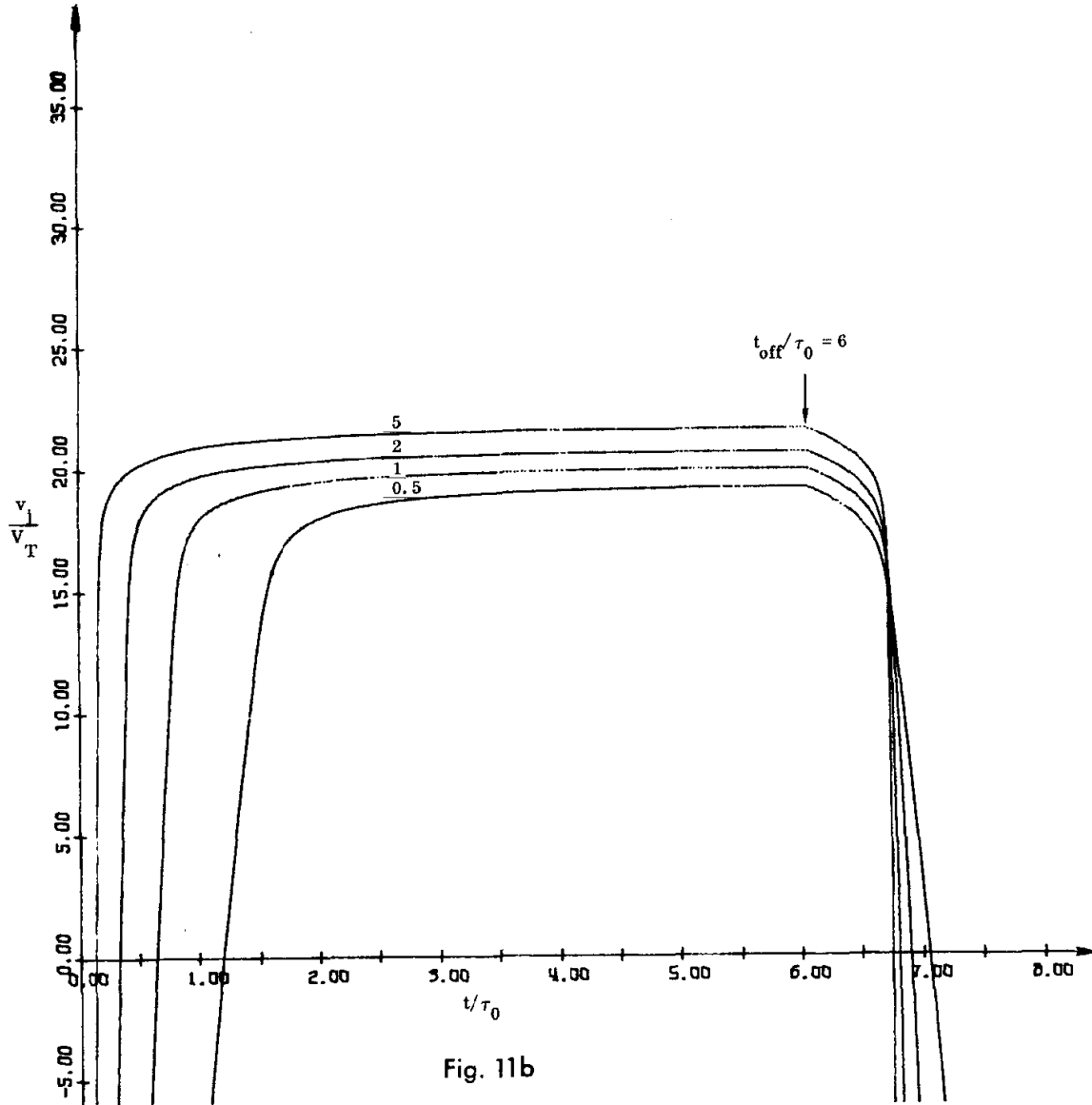
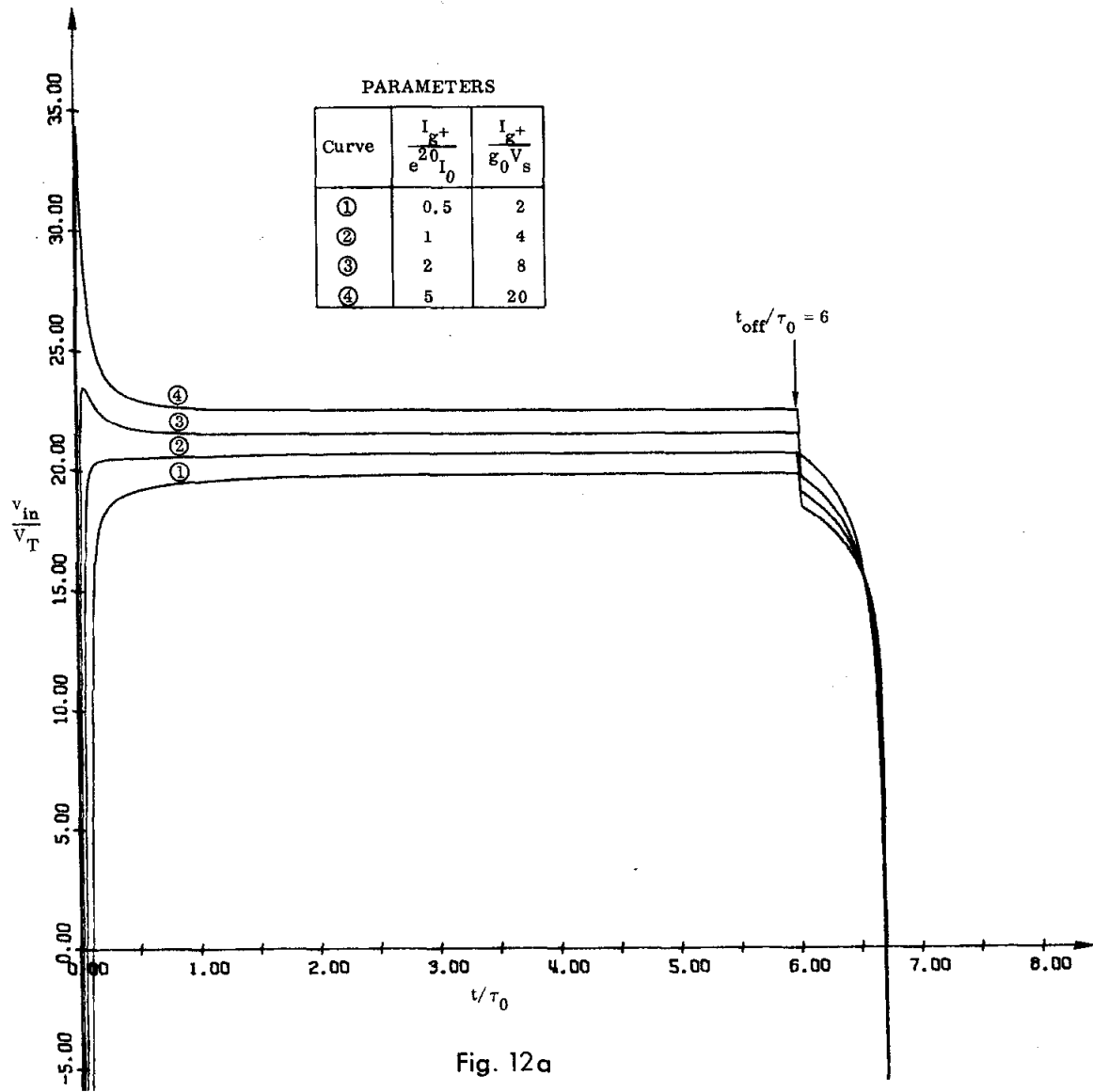


Fig. 11b



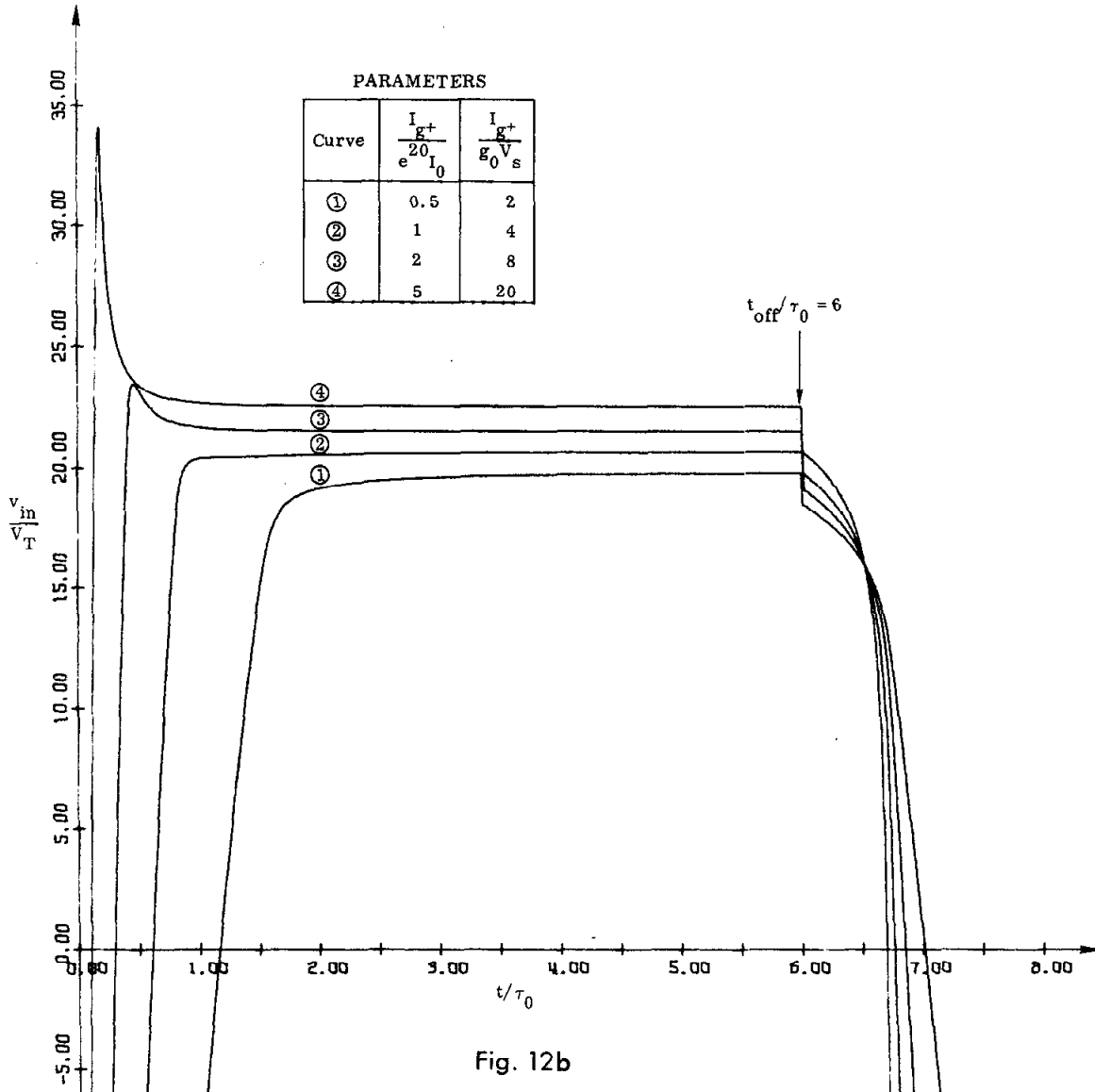


Fig. 12b

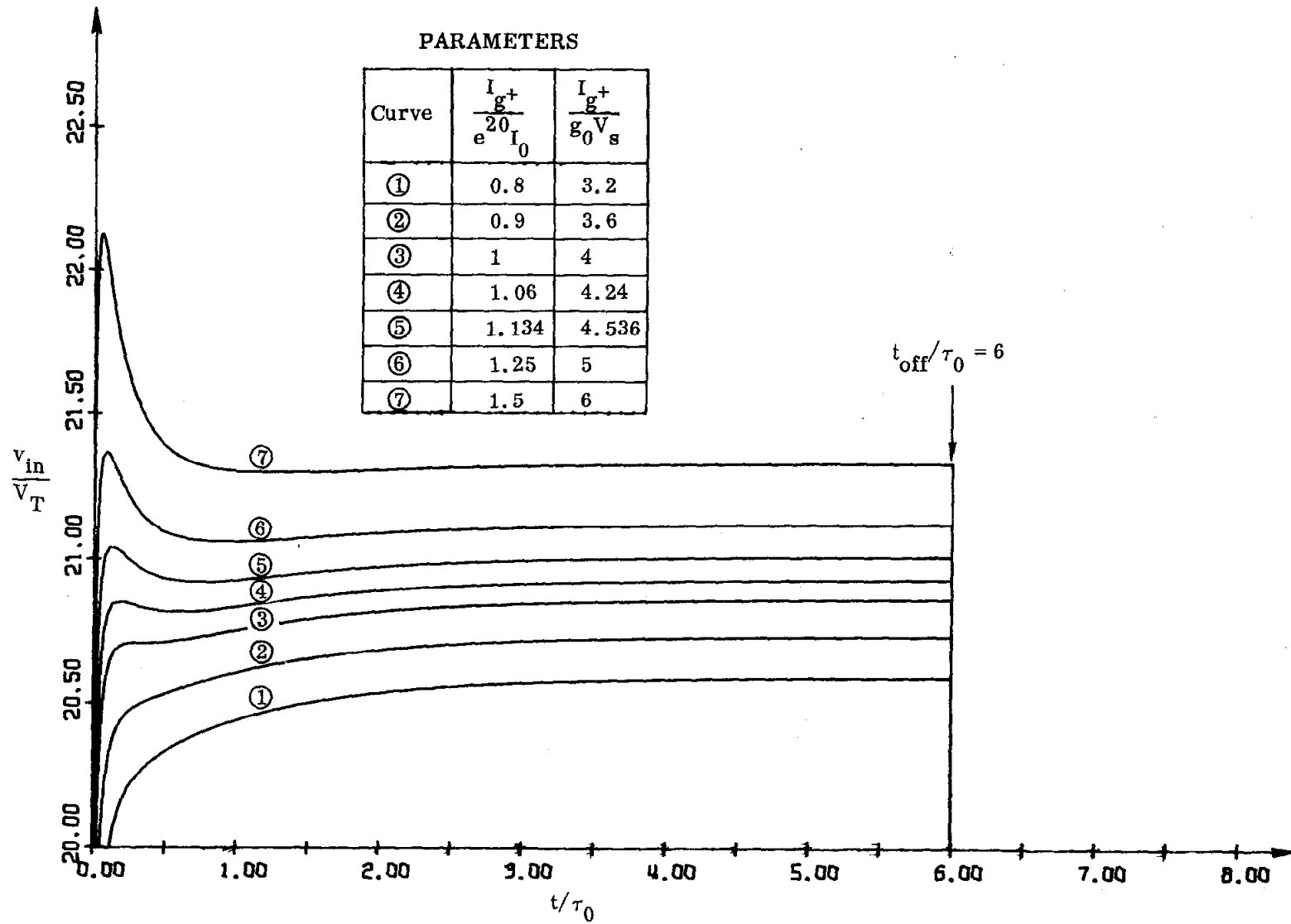


Fig. 13a

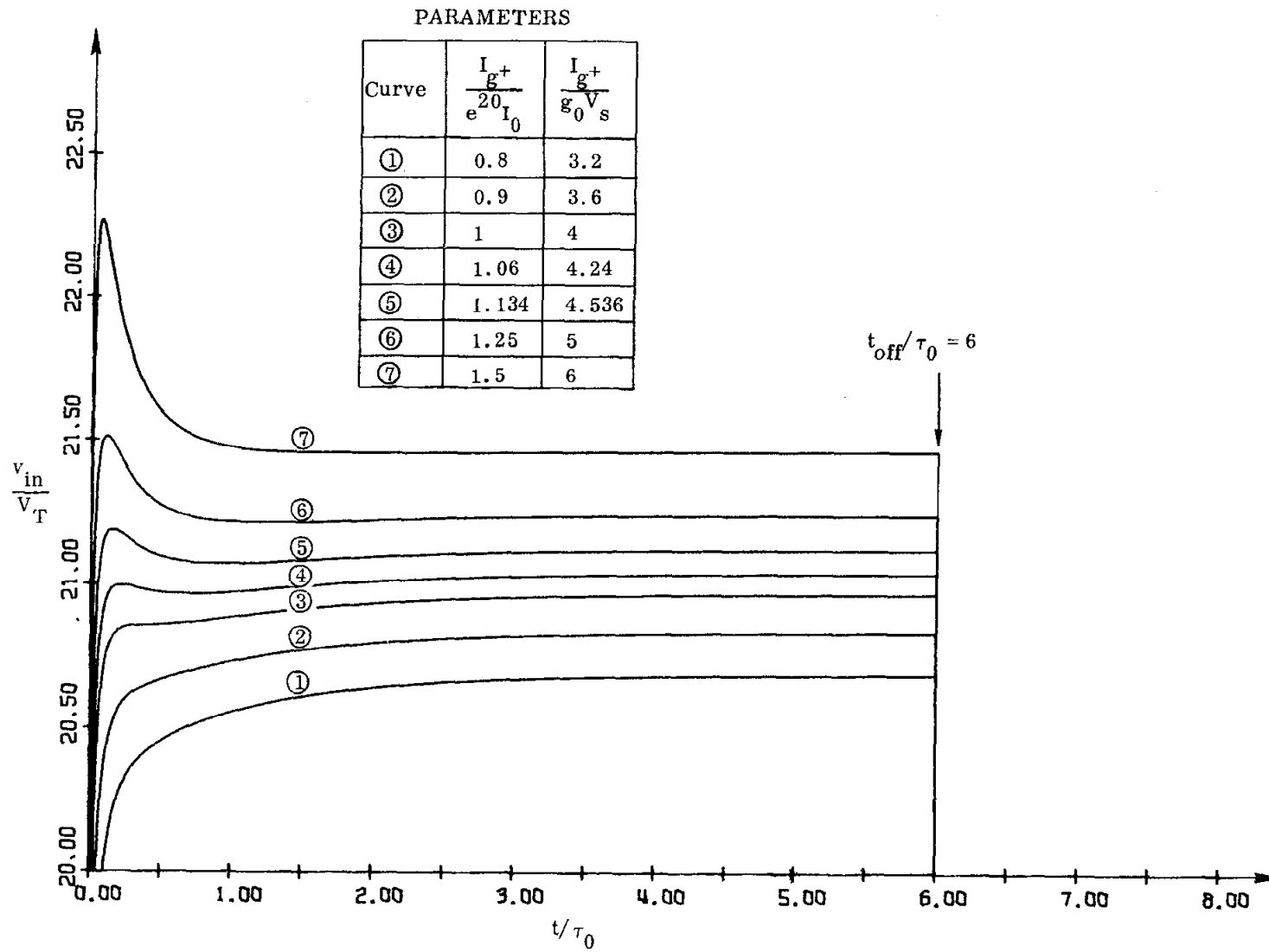


Fig. 13b

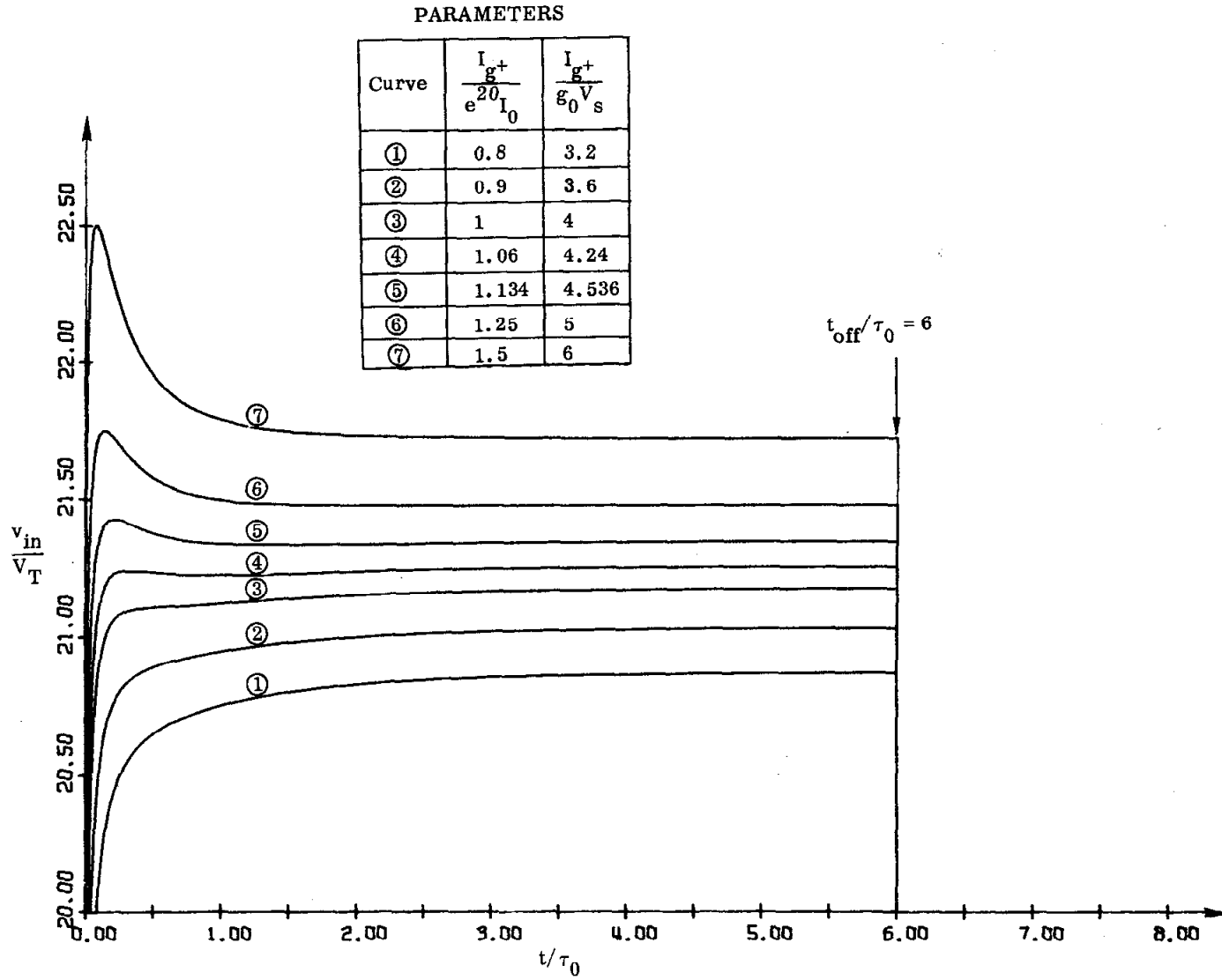


Fig. 13c