The Strong Coupling Constant in Grand Unified Theories*

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ABSTRACT

The prediction for the strong coupling constant in grand unified theories is reviewed, first in the Standard Model, then in the supersymmetric version. Various corrections are considered. The predictions in both supergravity-induced and gauge-mediated supersymmetry breaking models are discussed. In the region of parameter space without large fine tuning, the strong coupling is predicted to be $\alpha_s(M_Z) \gtrsim 0.13$. Imposing $\alpha_s(M_Z) = 0.118$, we require a unification scale threshold correction of typically -2%, which is accommodated by some GUT models but in conflict with others.

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1 bf Introduction

In the Standard Model, given values of the three gauge couplings, the Yukawa couplings, the Higgs boson self-coupling, and a dimensionful observable (e.g., the muon lifetime, a gauge boson mass, or a quark mass), any other observable can then be computed. In a grand-unified theory (GUT), one need only input two of the gauge couplings, then the third one can be predicted, as well as other observables.

Thus, GUT theories are more predictive. Because the SU(2) and U(1) couplings are measured quite precisely, we will examine the prediction of the strong coupling constant in grand-unified theories, both in the Standard Model and in supersymmetric models. We start with the one-loop results and then proceed to discuss the next higher-order corrections. The inclusion of these corrections leads to a precise prediction of the strong coupling constant.

2 bf Renormalization Group Equations

We are interested in measuring gauge couplings at the weak-scale or below, and running the gauge couplings up to higher scales. The solution of the renormalization group equations (RGE's) accurately describes the evolution of the couplings, even as they are evolved over many decades. At one-loop, the renormalization group equations for the gauge couplings are

$$\frac{dg_i}{dt} = \frac{b_i}{16\pi^2} g_i^3, \qquad t = \ln \frac{Q}{Q_0} \ , \tag{1}$$

where the b_i are the one-loop beta-constants. These constants receive contributions from every particle which circulates in the one-loop gauge-boson self-energy diagram. For example, for a fermion doublet (ν, e), $\Delta b_2 = 1/3$.

The one-loop renormalization group equation is easily solved. The inverse of the gauge coupling evolves linearly with the log of the scale,

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(Q_0) - \frac{b_i}{2\pi} \ln \frac{Q}{Q_0} \ . \tag{2}$$

If we assume that the couplings do unify at some scale, we have that

$$\frac{b_1 - b_2}{\alpha_3(Q)} + \frac{b_2 - b_3}{\alpha_1(Q)} + \frac{b_3 - b_1}{\alpha_2(Q)} = 0 \ . \tag{3}$$

This equation is valid for any scale Q between the weak scale and the unification scale. Next, we deduce the $\overline{MS}$ values of the U(1) and SU(2) gauge couplings from

the quantities $\hat{\alpha}^{-1} = 127.90 \pm 0.09$ and $\hat{s}^2 = 0.2315 \pm 0.0004$ (Ref.[1]) ($\hat{\alpha}$ and $\hat{s}^2$ are the $\overline{MS}$ values of the electromagnetic coupling and sine-squared weak mixing angle evaluated at M_Z)

$$\alpha_1^{-1}(M_Z) = 58.97 \pm 0.05 \ , \qquad \alpha_2^{-1}(M_Z) = 29.61 \pm 0.05 \ . \tag{4}$$

Given the Standard Model values of the one-loop beta-constants

$$b_1 = \frac{41}{10} \ , \qquad b_2 = -\frac{19}{6} \ , \qquad b_3 = -7 \ , \tag{5}$$

we determine the prediction of the strong coupling constant in the Standard Model

$$\alpha_s(M_Z) = 0.07 \qquad \text{(Standard Model, one-loop)} \tag{6}$$

with negligible error. Comparing with the measured value as quoted by the Particle Data Group,[1] $\alpha_s(M_Z) = 0.118 \pm 0.003$, we see that the Standard Model prediction of the strong coupling constant is about 16 standard deviations too small. Including higher order corrections cannot help repair this situation by more than a few standard deviations. In order to remedy this huge discrepancy, we need to go beyond the Standard Model, adding matter to change the beta-constants b_i. We also need to specify the mass scale of the new matter.

Rather than determining what additional matter content will lead to successful gauge coupling unification in an ad hoc fashion, we will examine the implications of a motivated model. It is widely accepted that the Standard Model is an effective theory. Some new physics must become manifest at scales below the multi-TeV scale. A promising model of new physics is supersymmetry,[2] which, among other virtues, explains how a theory with a weak-scale elementary Higgs boson is stable with respect to the Planck scale. Supersymmetry also naturally explains the breaking of electroweak symmetry,[3] and it typically contains a natural dark matter candidate.[4]

In order to make the Standard Model supersymmetric, we are required to add a specific set of new particles, the superpartners. For every Standard Model gauge boson, we must add a spin-1/2 majorana particle, a gaugino. And for every Standard Model fermion we must add a spin-0 partner, a squark or a slepton. In order to give both up- and down-type fermions a mass, and to ensure anomaly cancellation, we must also add an additional Higgs doublet. We refer to the two Higgs doublets as "up-type" and "down-type." This additional particle content defines the minimal supersymmetric standard model (MSSM). In the simplest and most often considered versions of the MSSM with high-energy inputs, the entire supersymmetric spectrum scales with one or two parameters which must be near the weak scale by naturalness considerations.

Given this new matter content and the superpartner mass scale, which we take to be M_Z for the moment, we can take a first-order look at the prediction of the strong coupling constant in the MSSM. As before, we use Eq. (3) to solve for $\alpha_s(M_Z)$ as a function of $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$. Now the beta-constants are different. Because of the new matter content, we have the following changes from the Standard Model values

$$b_1 = 4.1 \to 6.6 \ , \qquad b_2 = -3.2 \to 1 \ , \qquad b_3 = -7 \to -3 \ . \tag{7}$$

We use the central values and errors in Eq. (4) to find

$$\alpha_s(M_Z) = 0.116 \pm 0.001 \qquad \text{(MSSM, one-loop)}. \tag{8}$$

The new matter content mandated by supersymmetry brings the prediction for the strong coupling constant to the measured value (0.118 ± 0.003) within one standard deviation! This could be a coincidence, or it could be a hint that we are on the right track by considering minimal low-energy supersymmetry. Next we will consider corrections to the prediction of α_s in the MSSM.

3 bf Corrections to the prediction of α_s

In this section, we consider two sources of corrections to the prediction of the strong coupling constant in the MSSM. The first involves improving the evolution of the couplings from their initial values at the grand unification scale to the weak scale (or, the supersymmetry breaking scale). We simply extend the renormalization group equations to one higher loop order. This means that in addition to resumming logarithms of the form $[\alpha/4\pi \ln(M_{GUT}/M_Z)]^n$ from the one-loop RGE, we now resum logarithms of the form $[(\alpha/4\pi)^2 \ln(M_{GUT}/M_Z)]^n$ from the two-loop terms. At two loops, it is necessary to run both the gauge and Yukawa couplings together, as they form a coupled set of differential equations. The general form for either a gauge or Yukawa coupling RGE at two-loops is[5]

$$\frac{dg_i}{dt} = g_i \left\{ \frac{b_{ij}}{16\pi^2} g_j^2 + \frac{b_{ijk}}{(16\pi^2)^2} g_j^2 g_k^2 \right\} \ . \tag{9}$$

Since we can safely ignore the small Yukawa couplings of the first two generations, the set g_i includes $\{g_1,\ g_2,\ g_3,\ \lambda_t,\ \lambda_b,\ \lambda_\tau\}$. The equations are readily solved numerically. The correction due to the two-loop RGE's is large and positive. The prediction of $\alpha_s(M_Z)$ increases by about 0.01.

The second source of corrections we will consider in this section are the supersymmetric threshold corrections. These are divided into two types, the logarithmic

corrections and the finite (i.e., nonlogarithmic) corrections. We will discuss these in turn.

When we arrived at prediction (8), we assumed that the masses of the superpartners were equal to M_Z. However, we know from collider searches that many of the superpartner masses must be heavier than M_Z, and in general, they could be an order of magnitude heavier without straining naturalness considerations too much. As we evolve the gauge couplings down from high scales, we must decouple each superpartner contribution in turn as we cross the mass threshold. Hence, we arrive at logarithmic corrections of the form $\Delta b_i \ln(M_{susy}/M_Z)$. From these corrections, we determine the shift in the prediction of the strong coupling and find, in general, for a particle of mass $M > M_Z$ with Δb_i contributions to the beta-constants

$$\Delta\alpha_s^{-1}(M_Z) = -\frac{1}{2\pi(b_1-b_2)}\Big[(b_2-b_3)\Delta b_1+(b_3-b_1)\Delta b_2+(b_1-b_2)\Delta b_3\Big]\ln\frac{M}{M_Z}. \quad (10)$$

Note that the beta-constants b_i in this equation *include* the entire superpartner spectrum. Plugging in all the superpartners, we arrive at the following supersymmetric threshold correction

$$\begin{aligned}\Delta\alpha_s^{-1}(M_Z) &= \frac{1}{28\pi}\Big[3\ln\frac{M_L^3M_Q^7}{M_D^3M_E^2M_U^5}+32\ln\frac{M_2}{M_Z}-28\ln\frac{M_3}{M_Z} \\ &\quad + \ 3\ln\frac{M_H}{M_Z}\ +12\ln\frac{|\mu|}{M_Z}\Big]\ .\end{aligned} \quad (11)$$

In the first term, M_Q and M_L (M_U, M_D, and M_E) denote the left-handed (right-handed) squark and slepton masses. This term is easily modified to account for generation-dependent masses.

There are a few aspects of this equation worth pointing out. First, notice that the first term does not contain M_Z. This is because the squarks and sleptons are in complete SU(5) multiplets, and degenerate complete multiplets do not affect α_s at one loop. It happens that this particular combination of soft squark and slepton masses is near unity in the entire parameter space of both of the models that we consider in the following section. Hence, the squarks and sleptons do not significantly affect the prediction of $\alpha_s(M_Z)$.

Another point is that the SU(2) and SU(3) gauginos contribute corrections which largely combine into the term $(1/\pi)\ln M_2/M_3$. In GUT models, the gaugino masses are degenerate above the unification scale. At one-loop, they are renormalized proportional to the corresponding gauge couplings, so the ratio $M_2/M_3 = \alpha_2/\alpha_3$. At the weak scale, this ratio is about $8/30 \simeq 0.27$. Hence, this term contributes about $+0.006$ to $\alpha_s(M_Z)$, independent of parameter space.

The remaining terms include the residual term from the SU(2) gaugino mass, and the contributions of the heavy Higgs bosons and Higgsinos. If we take these

three masses to be characterized by a single scale M_{susy}, then these terms combine to give

$$\Delta\alpha_s^{-1}(M_Z) = \frac{19}{28\pi}\ln\frac{M_{susy}}{M_Z} \ . \tag{12}$$

Hence, as we increase the supersymmetric mass scale, the prediction of the strong coupling constant decreases. This makes sense, since in the limit that the entire supersymmetric spectrum is raised above M_{GUT}, we should recover the prediction of the Standard Model.]In fact $(0.116)^{-1} + (19/28\pi)\ln M_{GUT}/M_Z = (0.064)^{-1}$.] If $M_{susy} = 1$ TeV, this correction yields $\Delta\alpha_s(M_Z) = -0.007$. Note that the largest contribution to Eq. (12) is due to the $\ln(|\mu|/M_Z)$ term. Because we impose electroweak symmetry breaking, $|\mu|/M_Z$ is a measure of the fine tuning necessary to obtain the Z-mass from the input mass parameters. The predicted strong coupling is larger in the region of no fine tuning ($|\mu|/M_Z \lesssim 2$) than in the region of appreciable fine tuning ($|\mu|/M_Z \gtrsim 10$).

There is one last point about Eq. (11). Because of the particle content of the supersymmetric standard model, the particles which are charged under SU(2) always come into the expression for $\Delta\alpha_s^{-1}$ with a positive coefficient in front of the logarithm of the mass. The SU(2) singlets always come with a negative coefficient. Hence, heavy SU(2) doublets, for example, $\tilde{W}$, H, or $\tilde{H}$, will decrease α_s, and heavy SU(2) singlets, e.g., the gluino, will increase the prediction of the strong coupling.

Besides the logarithmic corrections of Eq. (11), there are also finite corrections. These arise when the full one-loop correction to $\hat{s}^2$ is taken into account. Taking the electromagnetic constant, the Z-boson mass, and the muon lifetime as inputs, we determine the $\overline{DR}$ (Ref.[6]) renormalized weak mixing angle,[7]

$$\hat{s}^2\hat{c}^2 = \frac{\pi\hat{\alpha}}{\sqrt{2}G_\mu M_Z^2(1-\Delta\hat{r})} \ , \qquad \Delta\hat{r} = \frac{\hat{\Pi}_{WW}(0)}{M_W^2} - \frac{\hat{\Pi}_{ZZ}(M_Z^2)}{M_Z^2} + \delta_{vb} \ . \tag{13}$$

The correction $\Delta\hat{r}$ is comprised of the real and transverse $\overline{DR}$ gauge-boson self-energy contribution (the oblique correction), and the vertex and box diagram contributions, δ_{vb} (the non-universal part).[8] The oblique correction contains both logarithmic and finite contributions, while δ_{vb} is purely finite. The finite contributions decouple as M_Z^2/M_{susy}^2 for large supersymmetric masses.

In the regions of parameter space where some SU(2) nonsinglet particles are of order M_Z, the finite corrections to the weak mixing angle can substantially increase the prediction of the strong coupling constant.[9,10] We illustrate this in Fig. 1. We show the prediction of the strong coupling with and without taking the finite corrections into account in the supergravity model described below. If all the supersymmetric particles are above a few hundred GeV, the finite cor-

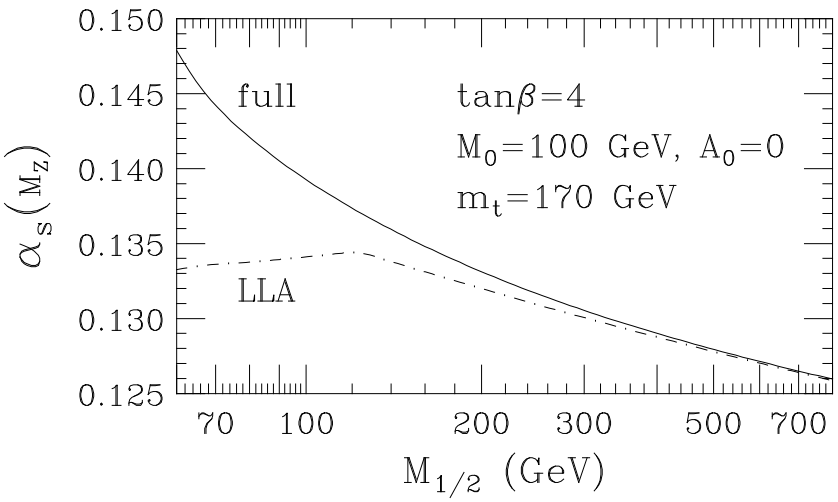

Fig. 1. The prediction of $\alpha_s(M_Z)$ with (solid) and without (dot-dashed) including the finite corrections. From Ref..[10]

rections are negligible. In that case, the low-energy threshold corrections are well-approximated by the logarithms in Eq. (11).

4 bf $\alpha_s(M_Z)$ at Two Loops in Two Supersymmetric Models

Utilizing the two-loop renormalization group equations and the weak-scale threshold corrections, we are now in a position to present the improved prediction of the strong coupling constant in the supersymmetric case. We still need to specify the supersymmetric particle spectrum. In what follows, we consider two models in turn, first a minimal supergravity model, then a simple model with gauge-mediated supersymmetry breaking.

4.1 Minimal Supergravity Model

In the minimal supergravity model, there are universal soft-supersymmetry breaking parameters induced at the grand unification scale. These include a gaugino mass $M_{1/2}$, a scalar mass M_0, and a trilinear scalar coupling A_0. Also, we require that electroweak symmetry is broken radiatively.[3] This happens naturally, as the large top-quark Yukawa coupling drives the up-type Higgs mass-squared negative near the weak scale. Given the Z-boson mass and $\tan\beta$ [the ratio of

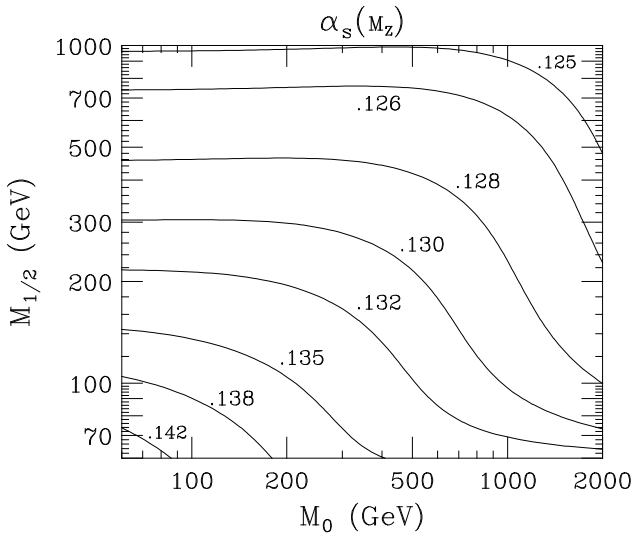

Fig. 2. The prediction of $\alpha_s(M_Z)$ in the M_0, $M_{1/2}$ plane, with $\tan\beta = 2$ and $A_0 = 0$. From Ref..[10]

up-type Higgs boson vacuum expectation value (vev) to down-type], we impose electroweak symmetry breaking, and this allows us to solve for the masses of the heavy Higgs bosons and their superpartners, M_H and $|\mu|$.

In Fig. 2, we show the prediction of the strong coupling in the M_0, $M_{1/2}$ plane. After including the corrections, the supersymmetric prediction is not as consistent with the measured value 0.118 ± 0.003. We find that for squark masses below about 1 TeV (where the model is more natural), α_s is predicted to be greater than 0.127. For the smallest supersymmetric masses, we find numbers larger than 0.140. The predicted value is three to seven standard deviations larger than the measured value.

The prediction of α_s depends slightly on m_t. It changes by about 0.001 if m_t changes by 10 GeV. Of the three input parameters M_Z, G_μ, and $\hat{\alpha}$, only $\hat{\alpha}$ has an appreciable error. Changing $\hat{\alpha}$ by 1-σ changes α_s by about 0.001. The prediction weakly depends on $\tan\beta$ because at small (~ 1) or large ($\gtrsim 30$) $\tan\beta$, the top, bottom, and/or tau-Yukawa couplings become large, and they enter into the two-loop renormalization group equations of the gauge couplings. The prediction for $\alpha_s(M_Z)$ can be lowered by about 0.002 at the extreme values of $\tan\beta$.[†] The strong coupling is also insensitive to the sign of the Higgsino mass term μ. In Fig. 2 and the following figures, we set $\mu > 0$.

Hence, we find that we cannot avoid the large values of α_s shown in Fig. 2.

[†]The change -0.002 is due solely to the Yukawa couplings entering into the two-loop RGE's. There may be an additional dependence because the particle spectrum depends on $\tan\beta$.

We will give an interpretation of these large numbers after discussing the gauge-mediated case.

4.2 Minimal Gauge-Mediated Model

Models with gauge-mediated supersymmetry breaking are an attractive alternative to the supergravity models. In the supergravity model we considered, we chose universal boundary conditions, which suppress dangerous squark- and slepton-mediated flavor changing neutral currents. However, there is no symmetry which protects this choice of boundary conditions, and hence it is an artificial imposition. The advantage of the gauge-mediated models is that flavor changing neutral currents are automatically suppressed, since the soft supersymmetry breaking masses which are induced by the gauge interactions are flavor diagonal and generation independent.

In the simplest models with gauge-mediated supersymmetry breaking,[11] there is a messenger sector with the superpotential interaction

$$W = \lambda S M \overline{M} \ , \tag{14}$$

where S is a Standard Model singlet superfield, M and $\overline{M}$ are a pair of messenger fields which are vector-like under the Standard Model gauge group, and λ is the messenger Yukawa coupling. In a grand unified theory, M and $\overline{M}$ come in full SU(5) representations. We will consider n_5 $5+\overline{5}$ pairs and n_{10} $10+\overline{10}$ representations. Perturbative unification of the gauge couplings is ensured if we allow at most $(n_5, n_{10}) = (1,1)$ or $(4,0)$.

In these models, the singlet superfield S couples to a sector in which supersymmetry is dynamically broken, and as a result, it acquires both a vev (S) and an F-term (F). This in turn generates supersymmetry-conserving diagonal entries and supersymmetry-violating off diagonal entries in the M, $\overline{M}$ scalar mass matrix. The M and $\overline{M}$ fields enter into loop diagrams with Standard Model fields on the external legs, thereby generating soft supersymmetry breaking masses for the superpartners and the Higgs bosons. The gaugino and scalar masses are generated at one- and two-loops, respectively, and in the limit $F \ll \lambda S^2$ are given by[11]

$$M_i(M) = (n_5 + 3n_{10}) \frac{\alpha_i(M)}{4\pi} \Lambda \ , \tag{15}$$

$$\tilde{m}^2(M) = 2(n_5 + 3n_{10}) \sum_{i=1}^{3} C_i \left(\frac{\alpha_i(M)}{4\pi} \right)^2 \Lambda^2 \ , \tag{16}$$

where $M = \lambda S$ is the messenger mass scale, $\Lambda = F/S$, and $C_i = 3Y^2/5, 3/4, 4/3$ for fundamental representations and zero for singlet representations.

The mass parameters (15,16) serve as boundary conditions for the renormalization group equations at the messenger scale. In order to determine the superparticle spectrum, we run these parameters from the messenger scale to the weak scale using the two-loop renormalization group equations. As before, we impose radiative electroweak symmetry breaking and determine the Higgs boson and Higgsino masses M_H and $|\mu|$ for a given $\tan\beta$. Hence, the parameter space of the model under consideration is

$$\tan\beta, \quad M, \quad \Lambda, \quad n_5, \quad n_{10}, \quad \lambda, \quad sgn(\mu),$$

where λ is the value of the messenger Yukawa coupling at the grand unification scale (we assume a single Yukawa coupling). Note that a physical messenger spectrum requires $M > \Lambda$.

After we determine the superpartner spectrum at the weak scale, we apply the same weak-scale gauge coupling threshold corrections as in the supergravity model. However, the messenger sector gives rise to additional corrections. Not only are the one- and two-loop renormalization group equations altered above the messenger scale,[12] but there are also new threshold corrections at the scale M. A degenerate SU(5) multiplet does not affect the prediction of α_s at one loop. However, the evolution of the messenger Yukawa couplings from the grand unification scale down to the messenger scale splits the messenger multiplets. If we decompose a $5+\overline{5}$ pair of messenger fields into their doublet and triplet components, respectively denoted L and D, then the messenger sector superpotential below the grand unification scale becomes

$$W = \lambda_L S M_L \overline{M}_L + \lambda_D S M_D \overline{M}_D \ . \tag{17}$$

Thus, the messenger doublet (triplet) ends up with mass $\lambda_L S$ ($\lambda_D S$). According to Eq. (10), this mass splitting leads to the threshold correction (the $10+\overline{10}$ fields decompose into (SU(2),SU(3)) representations Q (2,3), U (1,3), and E (1,1)) [‡]

$$\Delta\alpha_s^{-1}(M) = \frac{9n_5}{14\pi}\ln\frac{\lambda_L}{\lambda_D} \ + \ \frac{n_{10}}{14\pi}\left(15\ln\frac{\lambda_Q}{\lambda_U} + 6\ln\frac{\lambda_Q}{\lambda_E}\right) \ . \tag{18}$$

The Yukawa couplings are evaluated at the messenger scale, either $\lambda_L S \simeq \lambda_D S$ or $\lambda_Q S \simeq \lambda_U S \simeq \lambda_E S$. Note that at one loop order $\Delta\alpha_s^{-1}(M_Z) = \Delta\alpha_s^{-1}(M)$.

To determine the messenger Yukawa couplings at the messenger scale, we solve the renormalization group equations which are of the form[12]

$$\frac{d\lambda_i}{dt} = \frac{\lambda_i}{16\pi^2}\left(\sum_j c_j\lambda_j^2 - \sum_k a_k g_k^2\right) + \cdots, \tag{19}$$

[‡]There is an additional negligible correction due to the splitting within each U(1), SU(2), or SU(3) multiplet. Each logarithm in Eq. (18) is replaced according to $\ln(\lambda_1/\lambda_2) \to \ln(\lambda_1/\lambda_2) + (1/12)\ln[1-(F/\lambda_1 S^2)^2]/[1-(F/\lambda_2 S^2)^2]$.

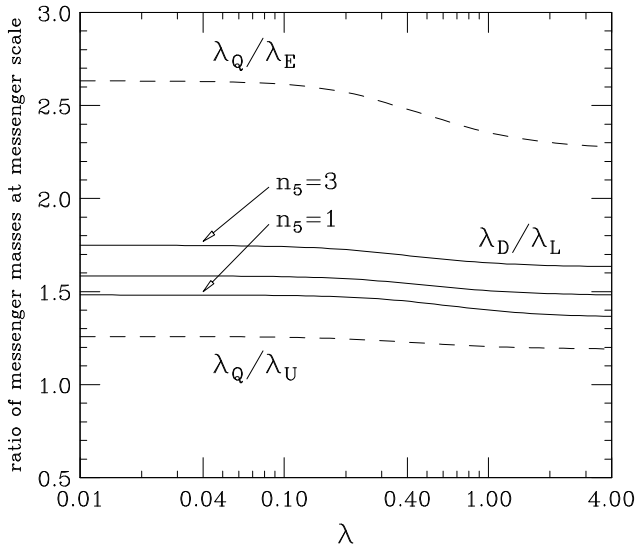

Fig. 3. The ratios of the messenger masses at the messenger scale vs. the value of the Yukawa coupling at the unification scale. Both the $n_{10} = 1$ (dashed) and $n_5 = 1, 2, 3$ (solid) cases and are shown.

where the dots indicate that there may be extra contributions due to interactions of the singlet with the supersymmetry breaking sector fields. These extra contributions are the same for all the messenger fields so they do not affect the ratios of Yukawa couplings. Note that for a large initial value of the messenger Yukawa coupling, the renormalization group evolution is initially dominated by the Yukawa term which leads to the same evolution for the various messenger fields. Hence, there will be less splitting if the initial value is large. We illustrate this in Fig. 3 where we show the ratio of messenger Yukawa couplings at the messenger scale vs. the starting value at the grand unification scale. In the $10+\overline{10}$ case, we see that λ_Q/λ_U (λ_Q/λ_E) varies from 1.2 to 1.3 (2.3 to 2.6). Hence, the splitting is small and confined to a narrow range of values. Plugging this splitting into Eq. (18), we find that the messenger threshold correction results in at most $\Delta\alpha_s(M_Z) = -0.003$ for $n_{10} = 1$, $+0.001$ for $n_5 = 1$, and $+0.005$ for $n_5 = 3$.

Combining all the effects together, we show the full result for the prediction of $\alpha_s(M_Z)$ in the gauge-mediated model in the M/Λ, Λ plane in Fig. 4, with $\tan\beta = 4$, $n_5 = 1$, and $\lambda = 3$. We see slightly smaller numbers than in the supergravity case (Fig. 2). Relative to other choices of n_5, n_{10}, and λ, this case results in the smallest values of $\alpha_s(M_Z)$. As before, the result is not very sensitive to the value of $\tan\beta$. Changing the value of the messenger Yukawa coupling does not change the value of the strong coupling significantly.

The dashed lines in Fig. 4 indicate that the region of parameter space with the

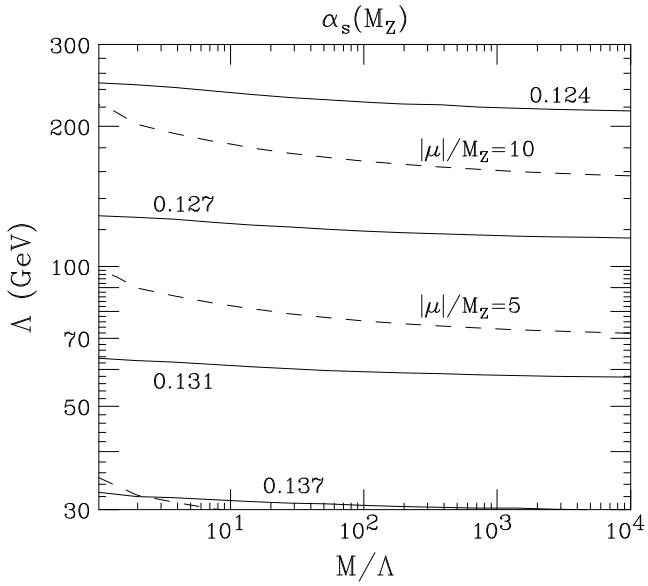

Fig. 4. Contours of $\alpha_s(M_Z)$ in the gauge mediated model with $n_5 = 1$, $\tan\beta = 4$, and $\lambda = 3$. The dashed lines show contours of $|\mu|/M_Z = 2$, 5, and 10.

	$\|\mu\| < 10M_Z$		$\|\mu\| < 2M_Z$	
	$\lambda = 0.01$	$\lambda = 3$	$\lambda = 0.01$	$\lambda = 3$
$n_5 = 1$	> 0.125	> 0.124	> 0.137	> 0.136
$n_5 = 3$	> 0.126	> 0.125	> 0.139	> 0.136
$n_{10} = 1$	> 0.127	> 0.128	> 0.137	> 0.140

Table 1. Summary of predictions for $\alpha_s(M_Z)$ in th

least fine tuning ($|\mu|/M_Z \simeq 2$) corresponds to the largest value of $\alpha_s(M_Z) \simeq 0.137$. The strong coupling can be reduced significantly at the cost of fine tuning. In the fine tuned region $|\mu|/M_Z \simeq 10$, $\alpha_s(M_Z) \simeq 0.125$.

We summarize the predicted values of α_s in the gauge-mediated models in Table 1. Here we set $m_t = 175$ GeV, $\tan\beta = 4$, and $\mu > 0$. We find that if $|\mu| < 10M_Z$, $\alpha_s(M_Z)$ is greater than 0.124. This is two standard deviations larger than the measured value. Hence, we are faced with the fact that, like the Standard Model, supersymmetric models do not predict that the gauge couplings meet. However, the discrepancy in the Standard Model case is enormous. The small discrepancy in the MSSM can be accounted for, and is required by, threshold corrections at the grand unification scale.

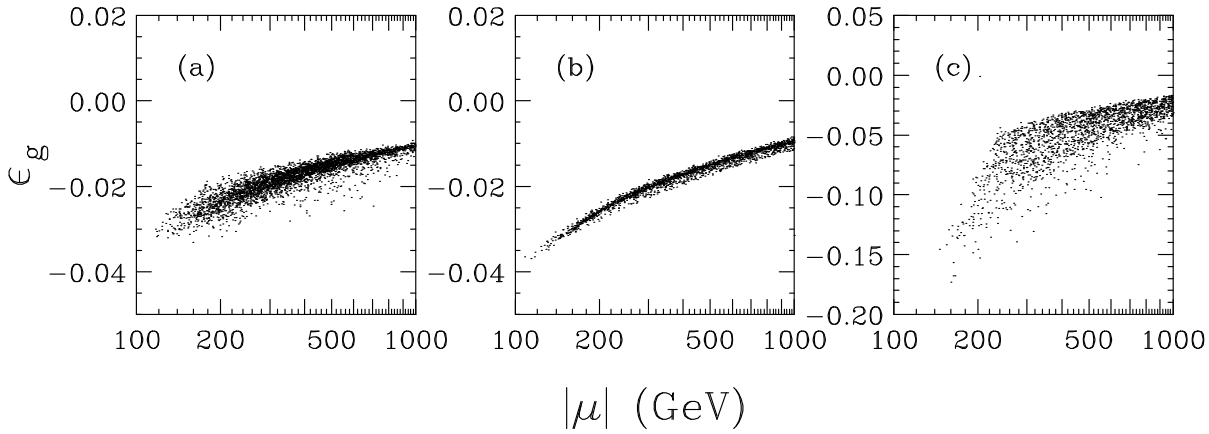

Fig. 5. The discrepancy ε_g with $\alpha_s(M_Z) = 0.118$ vs. $|\mu|$ in (a) the supergravity model, and the messenger model with (b) $n_5 = 1$ and (c) $n_5 = n_{10} = 1$.

5 GUT Threshold Corrections

At the GUT scale, there are incomplete SU(5) multiplets (most notably the color triplet Higgs bosons) and there can be split multiplets as well. These fields give rise to threshold corrections which are expected to be of order a couple of percent (or larger if α_{GUT} is larger).

If we take all three gauge couplings as input at the weak scale and run them up to the grand unification scale M_{GUT} (which is defined to be the scale where g_1 and g_2 meet), we find a discrepancy between the value of g_3 and $g_1 = g_2$. We define the discrepancy ε_g as

$$g_3(M_{GUT}) = g_2(M_{GUT})(1 + \varepsilon_g) \ . \tag{20}$$

If we fix $\alpha_s(M_Z) = 0.118$, the discrepancy is negative. We show a scatter plot of the discrepancy in Fig. 5 for three different models: the supergravity model, the messenger model with $n_5 = 1$, and the messenger model with $n_5 = n_{10} = 1$ (Ref.[13]). For the supergravity model and the messenger model with $n_5 = 1$, ε_g varies from about -3 to -1% as $|\mu|$ varies from 100 to 1000 GeV. For the messenger model with $n_5 = n_{10} = 1$, ε_g is larger because α_{GUT} is larger.

In a grand unified theory, there will be a discrepancy due to the grand unification threshold corrections.[§]. In any particular model of grand unification, we can calculate ε_g as a function of the GUT model parameter space. By varying the parameters over their allowed range, we can see whether the model is consistent

[§]There are possibly additional corrections due to higher dimension operators suppressed by M_{GUT}/M_{Planck} See Ref.[18] for a discussion.

with the "measured" values of ε_g shown in Fig. 5. Here we give two examples, the minimal SU(5) model[14] and the missing doublet SU(5) model.[15]

In the minimal SU(5) model, we find the correction[10,16]

$$\varepsilon_g = \frac{3\alpha_{GUT}}{10\pi} \ln \frac{M_{H_3}}{M_{GUT}} \ , \tag{21}$$

where M_{H_3} is the triplet Higgs boson mass. It is constrained to be larger than about 10^{16} GeV by the lower limits on the nucleon lifetimes.[17] Both M_{GUT} and the bound on M_{H_3} are functions of the supersymmetric parameter space. The bound on M_{H_3} is typically such that $M_{H_3} > M_{GUT}$, so that in most of the minimal SU(5) model parameter space, ε_g is positive. From Fig. 5, we know that in order to be compatible with gauge coupling unification, ε_g must be negative. Hence, the minimal SU(5) model is not compatible with coupling constant unification.

The minimal SU(5) model contains a $5+\overline{5}$ of Higgs fields. The doublet parts are the MSSM Higgs fields with order M_Z masses. The triplet parts mediate nucleon decay via dimension five operators. In order to be compatible with the lower bound on the nucleon lifetimes, the triplet Higgs particles must have GUT scale masses. In general, it is problematic to find a GUT model which naturally yields light Higgs doublets and heavy triplets. In the minimal SU(5) model, this doublet-triplet splitting is imposed by fine tuning superpotential parameters. The missing doublet model elegantly solves the doublet-triplet splitting problem by a judicious choice of Higgs representations. The 75, 50, and $\overline{50}$ representations are employed, and when the 75 gets a vev, the superpotential term 75 50 5 generates a mass for the triplet, but not for the doublet.

In the missing doublet model, we find[10,19]

$$\varepsilon_g = \frac{3\alpha_{GUT}}{10\pi} \left\{ \ln \frac{M_{H_3}^{eff}}{M_{GUT}} - 9.72 \right\} \ . \tag{22}$$

We have defined an effective triplet Higgs mass $M_{H_3}^{eff}$ which enters into the nucleon decay amplitude in the same way as in the minimal SU(5) model, so the same bounds apply. However, the splitting in the 75-dimensional representation gives rise to a negative correction, such that in the missing doublet model, the discrepancy ε_g is negative, just as it must be in order to be consistent with the measured values of $g_1(M_Z)$, $g_2(M_Z)$, and $g_3(M_Z)$. In fact, in each of the three models shown in Fig. 5, the allowed range of ε_g in the missing doublet model just about overlaps the required values. Hence, the missing doublet model is consistent with gauge coupling unification.

6 Conclusion

In the end, what we really do when we investigate gauge coupling unification is constrain the physics at the grand-unification scale. The weak-scale measurements of the gauge couplings imply that ε_g is negative. We can calculate ε_g in various grand unified models to see whether the grand unified model parameter space can accommodate the required value. Here we showed that this is not possible in the minimal SU(5) model, but that it is possible in the missing doublet SU(5) model. In other words, the missing doublet model requires that ε_g is negative, which corresponds with the measured values of the gauge couplings. Similarly, there are SO(10) models which can accommodate the "measured" ε_g, and others which cannot. Gauge coupling unification remains an effective constraint on model building.

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