RECENT ADVANCES IN HEAVY QUARK THEORY

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ABSTRACT

Some recent developments in heavy quark theory are reviewed. Particular emphasis is given to inclusive weak decays of hadrons containing a b quark. The isospin violating hadronic decay $D_s^* \to D_s\pi^0$ is also discussed.

1 Introduction

In this lecture, I describe some of the developments in heavy quark theory that have occurred recently. Those aspects of heavy quark theory that impact the determination of parameters in the Standard Model like $|V_{ub}|, |V_{cb}|$, and m_b are the most important. A precise determination of m_b may play a role in testing ideas about unification of the strong, weak, and electromagnetic interactions. Many unified theories predict that $m_b = m_\tau$ at the unification scale. (Later, I will distinguish between various definitions of the heavy quark mass, e.g., the pole mass or the $\overline{MS}$ mass. It is the $\overline{MS}$ mass that is approximately equal to the tau mass at the GUT scale.) In the Standard Model, the couplings of the W bosons to the quarks are given in terms of the elements of the Cabibbo-Kobayashi-Maskawa matrix, V_{ij}, which arises from diagonalizing the quark mass matrices. In the minimal Standard Model (i.e., one Higgs doublet), it is this matrix that is responsible

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for the CP nonconservation observed in weak kaon decays. (The limit on the electric dipole moment of the neutron means that the QCD vacuum angle is too small, $\bar{\theta} < 10^{-9}$, to have a measurable impact on weak decays.) A precise determination of the elements $|V_{ub}|$ and $|V_{cb}|$ will play an important role in testing this picture for the origin of CP violation and will constrain extensions of the Standard Model that make predictions for the form of the quark mass matrices.

While most of this lecture is directed towards aspects of heavy quark theory that impact the determination of $|V_{ub}|, |V_{cb}|$, and m_b, I will also spend some time discussing the implications of a recent measurement of the branching ratio for the isospin violating decay $D_s^* \to D_s \pi^0$. Heavy quark theory is still a rapidly developing field of study, and in this lecture, I only review a small part of the recent activity in this subject area.

2 Inclusive Semileptonic B and Λ_b Decay

The theory of inclusive B decay has developed rapidly over the last few years.[1–3] In this lecture, I consider inclusive semileptonic B decay in some detail and then generalize this discussion to other cases. The inclusive B semileptonic decay rate is equal to the b-quark decay rate with corrections suppressed by powers of Λ_{QCD}/m_b. Over the past few years, it has been shown how to express these nonperturbative QCD corrections in terms of the matrix elements of local operators in the heavy quark effective theory. The method used involves an operator product expansion and a transition to the heavy quark effective theory.

As far as hadronic physics is concerned, the basic quantity needed in inclusive semileptonic B decay is the second-rank hadronic tensor

$$W^{\mu\nu} = (2\pi)^3 \sum_X \delta^4(p_B - q - p_X) \langle B(v)| J_j^{\mu\dagger} |X\rangle \langle X| J_j^\nu |B(v)\rangle. \qquad (1)$$

In Eq. (1), J_j^μ is the weak current

$$J_j^\mu = \bar{q}_j \gamma^\mu \frac{(1-\gamma_5)}{2} b, \qquad j = u, c,$$

for either $b \to c$ or $b \to u$ transitions, v is the four velocity of the B meson, $p_B = m_B v$, and the sum goes over all possible final hadronic states X. The tensor $W_{\mu\nu}$ can be expanded in terms of scalars, $W_a(q^2, v\cdot q)$, $a = 1, \ldots 5$, as follows:

$$W^{\mu\nu} = -g^{\mu\nu}W_1 + v^\mu v^\nu W_2 - i\varepsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta W_3$$
$$+ q^\mu q^\nu W_4 + (q^\mu v^\nu + q^\nu v^\mu)W_5. \tag{2}$$

The form factors W_4 and W_5 give effects proportional to the lepton mass and can be neglected in $B \to Xe\bar{\nu}_e$ decay (they are important for $B \to X\tau\bar{\nu}_\tau$ decay).[4] In terms of the scalar form factors W_a, the inclusive semileptonic $B \to Xe\bar{\nu}_e$ differential decay rate is

$$\frac{d\Gamma}{dq^2 dE_e dE_\nu} = \frac{|V_{jb}|^2 G_F^2}{2\pi^3}[W_1 q^2 + W_2(2E_e E_\nu - \frac{1}{2}q^2) + W_3 q^2(E_e - E_\nu)]$$
$$\times\, \theta(E_\nu - q^2/4E_e). \tag{3}$$

Here, E_e and E_ν are the electron and neutrino energies in the B rest frame. The limit over neutrino energies given by the theta function comes from

$$q^2 = (p_e + p_{\bar{\nu}})^2 = 2E_e E_\nu(1 - \cos\theta_{e\nu}) < 4E_e E_\nu. \tag{4}$$

The form factors W_a are proportional to the discontinuity across a cut in the analogous form factors for the time-ordered product of currents

$$T^{\mu\nu} = -i\int d^4x e^{-iq\cdot x}\langle B(v)|\{T(J_j^{\mu\dagger}(x)J_j^\nu(0)\}|B(v)\rangle$$
$$= -g^{\mu\nu}T_1 + v^\mu v^\nu T_2 - i\varepsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta T_3 + q^\mu q^\nu T_4 + (q^\mu v^\nu + q^\nu v^\mu)T_5, \tag{5}$$

where $T_a = T_a(q^2, v\cdot q), a = 1, ..., 5$. Viewing q^2 as fixed, T_a has cuts in the complex $v\cdot q$ plane along the real axis. The discontinuity across the cut associated with $B \to Xe\bar{\nu}_e$ semileptonic decay gives the W_a. (There are other cuts not associated with this process. For example, along the positive real axis, there is a cut corresponding to $\nu_e B \to Xe$, where X contains two b quarks. This cut arises from $J_j^{\mu\dagger}$ acting on the B meson to produce X.) It is possible to express weighted averages (over $v\cdot q$) of the form factors W_a as contour integrals of the analytically continued $T_a(q^2, v\cdot q)$, where, for the most part, the contour is not close to the cuts.

We perform an operator product expansion on the time-ordered product of the two currents. Because this expansion holds at the operator level, we can identify the operators and their coefficients by taking matrix elements of the time-ordered product between b-quark "states" and comparing that with b-quark matrix elements of local operators. The momentum of the incoming b quark is written as $p_b = m_b v + k$ and the residual momentum, k, is expanded in, with higher powers of k being associated with higher dimensional operators in the heavy quark effective theory. The leading operators encountered are $\bar{b}\gamma_\lambda b$ and $\bar{b}\gamma_\lambda\gamma_5 b$. In a B meson, the second of these has a zero matrix element because of the parity invariance of the strong interactions. The first operator has a known forward matrix element because it is the conserved b-quark number current

$$\langle B(v)|\bar{b}\gamma_\lambda b|B(v)\rangle = 2v_\lambda. \tag{6}$$

(Hadronic B-meson states are normalized to $2v_0$ instead of $2m_B v_0$.) At this level, the operator-produced expansion reproduces the b-quark decay rate.

At zeroth order in the residual momentum k, there is no reason to make a transition to the heavy quark effective theory. However, at linear order in k, it is convenient, for keeping track of the m_b dependence of matrix elements, to make the transition to the heavy quark effective theory (HQET) defining

$$b(x) = e^{-im_b v\cdot x} h_v^{(b)}(x), \tag{7}$$

where the HQET b-quark field, $h_v^{(b)}(x)$, satisfies

$$\not{v} h_v^{(b)}(x) = h_v^{(b)}(x). \tag{8}$$

The operators that are encountered at linear order have dimension four, and the only ones are $\bar{h}_v^{(b)} iD^\tau \gamma_5 h_v^{(b)}$ and $\bar{h}_v^{(b)} iD^\tau h_v^{(b)}$, where D^τ denotes a covariant derivative. But these operators have a zero forward matrix element. For example, the first vanishes by parity, while for the second, Lorentz invariance implies that

$$\langle B(v)|\bar{h}_v^{(b)} iD^\tau h_v^{(b)}|B(v)\rangle = Y v^\tau. \tag{9}$$

Contracting the above with v^τ and using $v^2 = 1$ gives

$$\langle B(v)|\bar{h}_v^{(b)} iv \cdot D h_v^{(b)}|B(v)\rangle = Y, \tag{10}$$

and the equation of motion in HQET,

$$iv \cdot D h_v^{(b)} = 0, \tag{11}$$

implies that $Y = 0$. This means that there are no Λ_{QCD}/m_b corrections to the b-quark decay picture! Nonperturbative strong interaction corrections first arise at order $(\Lambda_{QCD}/m_b)^2$ and are parametrized by the two matrix elements[2,3]

$$\lambda_1 = \frac{1}{2}\langle B(v)|\bar{h}_v^{(b)}(iD)^2 h_v^{(b)}|B(v)\rangle, \tag{12a}$$

and

$$\lambda_2 = \frac{1}{6}\langle B(v)|\frac{g}{2}\bar{h}_v^{(b)}\sigma_{\alpha\beta}G^{\alpha\beta}h_v^{(b)}|B(v)\rangle. \tag{12b}$$

The first of these is related to the kinetic energy of the b quark in the B meson, and the second is related to the chromomagnetic energy arising from the b-quark spin. λ_2 is determined by the $B^* - B$ mass splitting to be

$$\lambda_2 = m_b\left(\frac{m_{B^*} - m_B}{2}\right) = 0.12\ GeV^2, \tag{13}$$

and we expect (at the order of magnitude level) $\lambda_1 \approx -\lambda_2$.

The operator product expansion gives the decay rate in terms of quark kinematics with the phase space set by the heavy quark pole masses m_b and m_c. However, we can reexpress the differential decay rate in terms of hadron masses using

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b}, \tag{14a}$$

$$m_D = m_c + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_c}. \tag{14b}$$

The differential decay rate depends on $\bar{\Lambda}$ and λ_1 (λ_2 is fixed by experiment) and may be used to determine these quantities. At the present time, such an analysis[5] (including perturbative QCD corrections at order α_s) for semileptonic $B \to X_c e\bar{\nu}_e$ decay gives the lower bound

$\bar{\Lambda} > \left[0.33 - 0.07\left(\frac{\lambda_1}{0.1\ GeV^2}\right)\right] GeV$. It is $\bar{\Lambda}$ that determines the pole mass m_b. The pole mass is not a physical quantity,[6] and the perturbative expression for the $\overline{MS}$ mass $\overline{m_b}(m_b)$ in terms of the pole mass, m_b, is not Borel summable giving rise to what is sometimes called a "renormalon ambiguity" in the pole mass. However, when the differential semileptonic decay rate is expressed in terms of hadron masses and $\bar{\Lambda}$, the perturbative QCD corrections to the decay rate are also not Borel summable. If $\bar{\Lambda}$ (or equivalently, the b-quark pole mass) extracted from the differential semileptonic decay rate is used to get the $\overline{MS}$ mass, these ambiguities cancel so one can arrive at a meaningful prediction for the $\overline{MS}$ b-quark mass. The basic lesson here is that it is fine to introduce unphysical quantities like the heavy quark pole mass or $\bar{\Lambda}$ as long as one works consistently to a given order of QCD perturbation theory. Since the final relations one considers always involve relations between physically measurable quantities, any "renormalon ambiguities" resulting from the bad behavior of the QCD perturbation series at large orders will cancel out.[7]

The method I have outlined for semileptonic B decay has been extended to polarized Λ_b decay and to the rare decay, $B \to X_s\gamma$ (Ref. 8). The latter may play a particularly important role[9] in extracting the parameter $\bar{\Lambda}$. Study of the exclusive decay $\Lambda_b \to \Lambda_c e\bar{\nu}_e$ can also lead to a determination of $\bar{\Lambda}$ (Ref. 10).

We have seen that the electron spectrum in semileptonic B decay, $d\Gamma/dE_e$, can be predicted, including nonperturbative strong interactions effects, in terms of $\bar{\Lambda}$ and two matrix elements λ_1 and λ_2. Over most of the phase space, this description is adequate with λ_1 and λ_2 giving only modest corrections $\sim 5\%$. However, for extracting $|V_{ub}|$, it is necessary to focus on the endpoint region of electron energies $(m_B^2 - m_D^2)/2m_B < E_e < (m_B^2 - m_\pi^2)/2m_B$, where $b \to c$ transitions are forbidden kinematically. For electron energies very near their maximal value, only low-mass final hadronic states are allowed and a description in terms of the operator product expansion is inappropriate. For $B \to X_u e\bar{\nu}_e$ decay, the nonperturbative QCD corrections proportional to λ_1 and λ_2 are singular at $E_e^{max} = m_b/2$. They must be averaged over a region of electron energies ΔE_e before a comparison with experiment can be made. It is sufficient to stop the operator product expansion at dimension five operators provided $\Delta E_e \gg \Lambda_{QCD}$. This is too large a region of electron energies to be useful for getting $|V_{ub}|$ from the endpoint region of the electron spectrum in semileptonic B decay. If a particular infinite class of operators is included, the resolution with which the electron spectrum can be examined near maximal electron energies is improved to $\Delta E_e \sim \Lambda_{QCD}$. This may be small enough to allow a comparison with experimental data in the endpoint region. However, there is now a loss of predictive power because an infinite number of nonperturbative matrix elements are needed to characterize the electron energy spectrum. Fortunately, it has been shown that the same infinite class of operators (occurring in the same linear combination) determines the photon energy spectrum for the inclusive decays $B \to X_s\gamma$ near maximal photon energy. In principle, experimental information on $B \to X_s\gamma$ can be used to predict the electron spectrum in B decay, in a region near the maximal electron energy[11] that may be small enough to allow a model-independent extraction of $|V_{ub}|$. A comparison between exclusive B and D decays can also lead to a model-independent determination of $|V_{ub}|$ (Ref. 12).

In addition to the nonperturbative QCD corrections suppressed by powers of Λ_{QCD}/m_b, there are perturbative α_s corrections to the b-quark semileptonic decay rate that must be included to make an accurate prediction for the B or Λ_b semileptonic decay rate and the electron energy spectrum. These have been calculated at order[13] $(\alpha_s(m_b)/\pi)$, and recently, the corrections of order $(\alpha_s(m_b)/\pi)^2$ that are proportional to the QCD beta function (these are tagged by computing the part of the order $(\alpha_s(m_b)/\pi)^2$ correction proportional to the number of light quark flavors) have also been computed.[14] Typically, these are the most important order $(\alpha_s(m_b)/\pi)^2$ corrections and Brodsky, Lepage, and Mackenzie[15] have advocated choosing the argument of α_s in the leading perturbative QCD correction to remove this "two loop" correction.

For definiteness, consider the case of $b \to u$ transitions. Then

$$\Gamma(B \to X_u e \bar{\nu}_e) = \frac{G_F^2 |V_{ub}|^2 m_B^5}{192\pi^3} \Bigg[1 - 2.41 \frac{\bar{\alpha}_s(m_b)}{\pi} - 2.98 \beta_0 \left(\frac{\bar{\alpha}_s(m_b)}{\pi} \right)^2 - \frac{5\bar{\Lambda}}{m_B} + ... \Bigg], \tag{15}$$

where

$$\beta_0 = 11 - \frac{2}{3} n_f,$$

with n_f the number of light quark flavors. If the subtraction point used for the strong coupling in the order α_s term is changed from m_b to μ_{BLM} (μ_{BLM} is chosen so that the two-loop term proportional to β_0 is removed), one finds $\mu_{BLM} \simeq 0.08\ m_b$. The two-loop term proportional to β_0 can be reduced in comparison to the order α_s term if one eliminates $\bar{\Lambda}$ in favor of a physically measurable quantity characteristic of these decays. For example, the average final hadronic mass squared is[5]

$$< m_{X_u}^2 > = m_B^2 \left[0.20 \frac{\alpha_s(m_b)}{\pi} + 0.35 \beta_0 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 + \frac{7}{10} \frac{\bar{\Lambda}}{m_B^2} + ... \right], \tag{16}$$

and reexpressing the semileptonic decay rate in terms of this quantity gives

$$\Gamma(B \to X_u \ell \bar{\nu}_e) = \frac{G_F^2 |V_{ub}|^2 m_B^5}{192\pi^3} \Bigg[1 - 0.98 \frac{\alpha_s(m_b)}{\pi} - 0.48 \beta_0 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 - 7.14 \frac{\langle m_{X_u}^2 \rangle}{m_B^2} \Bigg]. \tag{17}$$

Now the BLM scale is considerably larger, $\mu_{BLM} = 0.38\ m_b$. Note that expressing the decay rate in terms of a physical quantity free of renormalon ambiguities does not guarantee a reasonably large BLM scale. For example, if one expresses the semileptonic decay rate in terms of the $\overline{MS}$ quark mass $\bar{m}_b(m_b)$, the BLM scale is still quite low.[14]

Perturbative QCD corrections to the electron spectrum (like the nonperturbative ones) become large in the endpoint region, and careful consideration of their effects is necessary for an extraction of $|V_{ub}|$ from the endpoint region of the electron spectrum.[16]

3 Inclusive Nonleptonic B and Λ_b Decay

The ideas I have outlined for inclusive semileptonic decay of hadrons containing a b quark have also been applied to nonleptonic decays.[17] Now there is no analog to $v \cdot q$ to analytically continue. Nonetheless, we expect to be able to express the total decay rate as b-quark decay plus nonperturbative QCD corrections given by forward matrix elements of local operators. This is because the energy release in B decay is large enough that threshold effects which spoil the applicability of local duality are probably negligibly small. (A similar argument is used to compare $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ with data at a fixed, but large, s.) The general structure of the nonperturbative QCD corrections to the nonleptonic decay rate is (schematically)

$$\Gamma = \Gamma_0 \Bigg[1 + \frac{c}{m_b^2} \langle B(v) | \bar{h}_v^{(b)} (iD)^2 h_v^{(b)} | B(v) \rangle + \frac{d}{m_b^2} \langle B(v) | \frac{g}{2} \bar{h}_v^{(b)} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(b)} | B(v) \rangle + \frac{e_\Gamma}{m_b^3} \langle B(v) | \bar{h}_v^{(b)} \Gamma q \bar{q} \Gamma h_v^{(b)} | B(v) \rangle + \ldots \Bigg], \tag{18}$$

where the ellipsis denote terms of order higher than $1/m_b^3$, and Γ_0 is the b-quark decay rate. A similar formula holds for the nonleptonic Λ_b decay rate. An interesting aspect of the order $1/m_b^3$ corrections is that they correspond to contributions to the nonleptonic decay rate where the phase space (at the quark level) is basically two body, and so the coefficients, e_Γ, are enhanced by a factor of 16 π^2 over c and d. Phenomenological models suggest that the contributions of the four quark operators are the most important for lifetime differences between hadrons containing a b quark.

Experimentally, the Λ_b lifetime is about 20% shorter than the B lifetime. This is a smaller lifetime than can be accommodated by quark model estimates of the matrix elements of the four quark operators. The charm quark mass dependence of the perturbative order α_s corrections to the nonleptonic decay rate increase the $b \rightarrow c\bar{c}s$ contribution[18] leading to an expected charm multiplicity of $n_c \sim 1.3$. Experimentally, the charm multiplicity is only $n_c = 1.17 \pm 0.04$.

At the present time, it is difficult to interpret these conflicts between the theory of inclusive nonleptonic decay and the experimental data. One possibility is that the matrix elements of the four quark operators are unusually large and the experimental value of the charm multiplicity (which relies on absolute branching ratios) is mismeasured. Another possibility is that both the charm multiplicity and the Λ_b lifetime are correctly measured and one has an unusually large violation of local duality in inclusive nonleptonic B decay. In any case, it seems prudent given these problems to use the semileptonic decay width for precision extractions of $|V_{cb}|$ from experiment rather than the B lifetime. Inclusive semileptonic B decay should give a determination of $|V_{cb}|$ with a precision comparable to the extraction of $|V_{cb}|$ from exclusive $B \to D^* e \bar{\nu}_e$ decay.[19] Some recent work that uses dispersion relations[20] to reduce uncertainties associated with the extrapolation of the Isgur–Wise function to zero recoil may improve the accuracy of extractions of $|V_{cb}|$ from this exclusive decay.

4 Decays of D^* Mesons

The ground state multiplet of charm mesons has spin of the light degrees of freedom, $s_\ell = 1/2$, and negative parity. This gives a doublet of mesons with total spins zero and one. The heavier, spin-one mesons decay to the lower mass spin-zero mesons by emission of either a photon or a pion. The measured branching ratios are shown in the table below.

Decay Mode	**Branching Ratio %**
$D^{*0} \to D^0 \pi^0$	$63.6 \pm 2.3 \pm 3.3$
$D^{*0} \to D^0 \gamma$	$36.4 \pm 2.3 \pm 3.3$
$D^{*+} \to D^0 \pi^+$	$68.1 \pm 1.0 \pm 1.3$
$D^{*+} \to D^+ \pi^0$	$30.8 \pm 0.4 \pm 0.8$
$D^{*+} \to D^+ \gamma$	$1.1 \pm 1.4 \pm 1.6$

In the nonrelativistic constituent quark model, the invariant matrix elements for radiative D^* decay are determined in terms of the *constituent* quark masses

$$M(D^{*0} \to D^0\gamma) \propto \left[\frac{2}{3m_c} + \frac{2}{3m_u}\right], \tag{19a}$$

$$M(D^{*+} \to D^+\gamma) \propto \left[\frac{2}{3m_c} - \frac{1}{3m_d}\right], \tag{19b}$$

$$M(D_s^{*0} \to D_s^0\gamma) \propto \left[\frac{2}{3m_c} - \frac{1}{3m_s}\right]. \tag{19c}$$

For $m_u = m_d = 350\ MeV$, $m_s = 550\ MeV$, and $m_c = 1.7\ GeV$, these are in the ratio

$$M(D^{*0} \to D^0\gamma) : M(D^{*+} \to D^+\gamma) : M(D_s^{*0} \to D_s^0\gamma)$$

$$= 1 \quad : \quad -0.25 \quad : \quad -0.1. \tag{20}$$

Presumably, the smallness of the radiative D^{*+} decay rate is due to the cancellation between down and charm quark magnetic moments in Eq. (19b). Notice that this cancellation is even stronger for the D_s^* decay because the constituent strange quark is heavier than the down quark. But how can we verify experimentally that this decay rate is very small? After all, the D_s^* is too narrow for its width to be measured. The answer is through measurement of the $D_s^* \to D_s\pi^0$ branching ratio. At leading order in chiral perturbation theory,[21] $D_s^* \to D_s\pi^0$ decay arises from isospin violating $\eta - \pi^0$ mixing which gives the rate

$$\Gamma(D_s^* \to D_s\pi^0) = \frac{g^2}{48\pi f^2}\left[\frac{m_u - m_d}{m_s - (m_u + m_d)/2}\right]^2 |\vec{p}_\pi|^3. \tag{21}$$

The factor in square brackets is $1/43.7$ (since this is greater than α/π, electromagnetic contributions to isospin violation can be neglected). In Eq. (21), g is the $D^*D\pi$ coupling. Equation (21) implies that

$$Br(D_s^* \to D_s\pi) = \frac{\Gamma(D_s^* \to D_s\pi^0)}{\Gamma(D_s^* \to D_s\gamma)}$$

$$= [8 \times 10^{-5}/Br(D^{*+} \to D^{*+}\gamma)]\left[\frac{\Gamma(D^{*+} \to D^{*+}\gamma)}{\Gamma(D_s^* \to D_s\gamma)}\right]. \tag{22}$$

Here, we have used the theoretical expression for the $D^{*+} \to D\pi$ decay rate to eliminate g. We expect $Br(D^{*+} \to D^+\gamma)$ to be around 1%. Then the branching ratio for $D_s^* \to D_s\pi^0$ should be significantly greater than 10^{-2} if the constituent quark model suppression of the $D_s^* \to D_s\gamma$ amplitude occurs in nature. The recent CLEO measurement,[22] $Br(D_s \to D_s\pi^0) = 0.062^{+0.020}_{-0.018} \pm 0.022$, indicates that, at least at some level, this suppression does occur.

References

[1] M. Shifman and M. Voloshin, Sov. J. Nucl. Phys. **41**, 120 (1985).

[2] J. Chay *et al.*, Phys. Lett. B **247**, 399 (1990); I. I. Bigi *et al.*, Phys. Lett. B **293**, 430 (1992); Phys. Lett. B **297**(E), 477 (1993); I. I. Bigi *et al.*, Phys. Rev. Lett. **71**, 496 (1993).

[3] A. V. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994); B. Blok *et al.*, Phys. Rev. D **49**, 3356 (1994); T. Mannel, Nucl. Phys. B **413**, 396 (1994).

[4] A. F. Falk *et al.*, Phys. Lett. B **326**, 145 (1994); L. Koyrakh, Phys. Rev. D **49**, 3379 (1994); S. Balk *et al.*, Z. Phys. C **64**, 37 (1994).

[5] A. F. Falk *et al.*, UT PT-95-11 (hep-ph/9507284), 1995 (unpublished); UTPT-95-24 (hep-ph/9511454), 1995 (unpublished).

[6] M. Beneke and V. M. Braun, Nucl. Phys. B **426**, 301 (1994); I. Bigi *et al.*, Phys. Rev. D **50**, 2234 (1994).

[7] M. Beneke *et al.*, Phys. Rev. Lett. **73**, 3058 (1994); M. Luke *et al.*, Phys. Rev. D **51**, 4924 (1994); M. Neubert and C. Sachrajda, Nucl. Phys. B **438**, 235 (1995).

[8] A. F. Falk *et al.*, Phys. Rev. D **49**, 3367 (1994); M. Neubert, Phys. Rev. D **49**, 4623 (1994); I. Bigi *et al.*, in the Fermilab Meeting, *Proceedings of the Annual Meeting of the DPF of the APS*, edited by C. Albright *et al.* (World Scientific, Singapore, 1993), p. 610.

[9] A. Kapustin and Z. Ligeti, Phys. Lett. B **355**, 318 (1995); R. D. Dikeman *et al.*, TPT-MINN-95/9-T hep-ph/9505397, 1995 (unpublished).

[10] H. Georgi *et al.*, Phys. Lett. B **252**, 456 (1990).

[11] M. Neubert, Phys. Rev. D **49**, 3392 (1994); ibid 4623; I. I. Bigi *et al.*, Int. J. Mod. Phys. A **9**, 2467 (1994).

[12] For a recent discussion, see Z. Ligeti and M. B. Wise, CALT-68-2029, hep-ph/9512225, 1995 (unpublished).

[13] M. Jezabek and J. H. Kühn, Nucl. Phys. B **314**, 1 (1989).

[14] M. Luke *et al.*, Phys. Lett. B **343**, 329 (1995); Phys. Lett. B **345**, 301 (1995).

[15] S. J. Brodsky, P. Lepage, and P. B. Mackenzie, Phys. Rev. D **28**, 228 (1983).

[16] A. F. Falk *et al.*, Phys. Rev. D **49**, 4553 (1994); G. P. Korchemsky and G. Sterman, Phys. Lett. B **340**, 108 (1994).

[17] For a recent review, see I. I. Bigi, UND-HEP-PH-BIG02, hep-ph/9508408, 1995 (unpublished).

[18] Q. Hokin and X-Y. Pham, Ann. of Phys. **155**, 202 (1984); E. Bagen *et al.*, Phys. Lett. B **351**, 546 (1995).

[19] For a review, see M. Neubert, Phys. Rev. D **45**, 250 (1994).

[20] C. G. Boyd *et al.*, UCSD-PTH-95-11, hep-ph/9508211, 1995 (unpublished).

[21] P. Cho and M. B. Wise, Phys. Rev. D **49**, 6228 (1994).

[22] J. Gronberg *et al.*, (CLEO Collaboration) Phys. Rev. Lett. **75**, 3232 (1995).