DARK MATTER CANDIDATES: FRONT RUNNERS AND DARK HORSES

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ABSTRACT

A good case can be made for believing that there is a substantial amount of nonbaryonic dark matter in the universe, most likely composed of an as-yet-undiscovered elementary particle. The various candidates for this particle, both front runners and dark horses, are reviewed.

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1 Introduction

Other lecturers at this school[1–3] have discussed some of the many cosmological and astrophysical issues associated with dark matter. My charge is to explain the possible implications for particle physics.

Of course, the most exciting possible implication is that some or all of the dark matter is composed of a species of an elementary particle which has not yet been discovered in the laboratory. What might this particle be? What properties must it have in order to be (some or all of) the dark matter? What implications do these properties have for astrophysics? Cosmology? Laboratory particle physics? We will discuss these issues, but one must keep in mind that this is a vast subject[4] and we cannot possibly cover it all. Also, since full and detailed analyses are generally widely available already,[5,6] I will concentrate on developing and explaining the qualitative picture with easy-to-understand, "rule of thumb" estimates.

If an elementary particle is to comprise (some or all of) the dark matter, it must have a few obvious properties. First of all, the average total mass density must have the right value, corresponding to

$$0.05 \lesssim 1 - \Omega_{\rm DM} \leq \Omega_{\rm B} \ , \tag{1}$$

where we have defined the mass density of dark matter $\rho_{\rm DM}$ in units of the critical density $\Omega_{\rm DM} = \rho_{\rm DM}/\rho_{\rm crit}$ and the mass density of baryons (that is, ordinary matter) in units of the critical density $\Omega_{\rm B} = \rho_{\rm B}/\rho_{\rm crit}$. The critical density is given in terms of Newton's constant and the present value of the Hubble parameter by $\rho_{\rm crit} = 3H_0^2/8\pi G$. Defining $h = H_0/(100\,{\rm km\,s^{-1}\,Mpc^{-1}})$, where one megaparsec is $3.086 \times 10^{19}\,$km, we have

$$\begin{aligned}
\rho_{\rm crit} &= 1.879 \times 10^{-29}\, h^2\, {\rm g\,cm^{-3}} \\
&= 1.054 \times 10^{-5}\, h^2\, {\rm GeV\,cm^{-3}} \\
&= 2.775 \times 10^{-7}\, h^2\, M_\odot\, {\rm pc}^{-3} \ ,
\end{aligned} \tag{2}$$

where $M_\odot = 1.116 \times 10^{57}\,$GeV is the mass of the sun. From here on, we will (usually) use the standard units of particle physics, setting $c = 2.998 \times 10^{10}\,{\rm cm/s} = 1$, $\hbar c = 1.973 \times 10^{-14}\,{\rm GeV\,cm} = 1$, and $k_{\rm Boltzmann} = 8.617 \times 10^{-5}\,{\rm eV/Kelvin} = 1$. Newton's constant can then be expressed as $G = 1/M_{\rm Pl}^2$ with $M_{\rm Pl} = 1.221 \times 10^{19}\,$GeV. Secondly, our candidate DM particles must not be disappearing as we speak; thus the particle's lifetime (if it is not absolutely stable) must be at least comparable to the age of the universe.

Having made these two simple observations, we can begin to talk about the properties of our hypothetical new elementary particle. I find it useful to categorize candidate particles according to four dichotomies:

Hot vs. Cold

Thermal vs. Nonthermal

Easy vs. Hard to detect

Expected vs. Unexpected

Let us discuss each of these in turn.

The first, hot vs. cold, is an astrophysical classification that has been discussed by other lecturers.[2,3] It refers to whether the dark matter particles are relativistic ("hot") or nonrelativistic ("cold") at the time that galaxy-sized fluctuations first came within the horizon. For completeness, I will explain this briefly here.

The size of the horizon $L_{\rm hor}$ at time t is given roughly by $L_{\rm hor}(t) \sim 1/H(t)$, where $H(t)$ is the value of the Hubble parameter at time t (see Mike Turner's lectures[3] for more details). According to Einstein's equations, if the universe is at critical density ($\Omega = 1$), then $H^2(t) = 8\pi G\rho(t)/3$, where ρ is the total mass-energy density. For a universe whose mass-energy density is dominated by relativistic particles in thermal equilibrium, $\rho(t) = \sigma T^4(t)$, where $T(t)$ is the temperature at time t, and σ is a constant of order one (in units with $\hbar = c = k_{\rm Boltzmann}$) that depends on the number of species of particles in equilibrium. Thus, $L_{\rm hor}(t) \sim G^{-1/2}T^{-2}(t)$. Now, if the dark matter particles are in thermal equilibrium (see the discussion of thermal vs. nonthermal below), then they become nonrelativistic when the temperature drops below the mass m of one dark matter particle ($k_{\rm B}T \lesssim mc^2$). So the dark matter particles are relativistic when the horizon is smaller than $\sim G^{-1/2}m^{-2}$, and nonrelativistic when the horizon is larger than $\sim G^{-1/2}m^{-2}$. This is important because relativistic particles will not clump gravitationally; a positive density fluctuation will dissipate or "free stream" away. Also, fluctuations which are bigger than the horizon cannot begin to collapse, because this is forbidden by causality. Thus, clumps which are smaller than $\sim G^{-1/2}m^{-2}$ dissipate, and only larger clumps can collapse. To see how big $G^{-1/2}m^{-2}$ is, we must first remember that this is the instantaneous size of the horizon when $T \sim m$, and the universe has grown since then by a factor of $T/T_0 \sim m/T_0$, where $T_0 = 2.75\,\mathrm{K}$ is the present microwave background temperature. Thus, the relevant length scale is

$$
\begin{aligned}
L &\sim \frac{1}{G^{1/2} T_0^{1/2} m} \\
&\sim (15\,\mathrm{Mpc}) \left(\frac{100\,\mathrm{eV}}{m} \right) \ ,
\end{aligned}
\tag{3}
$$

where I have restored appropriate numerical factors in the last line. We see that light neutrinos ($m \lesssim 100\,\mathrm{eV}$) produce minimum fluctuations of a size measured in megaparsecs, much bigger than a galaxy (a few kiloparsecs). Instead, we need a dark matter particle mass of a few keV in order to have produced galaxy-sized fluctuations. Thus, for dark matter particles which were once in thermal equilibrium, a mass of a few keV is the dividing line between "cold" dark matter (which can produce galaxy-sized fluctuations) and "hot" dark matter (which can only produce fluctuations much larger than galaxies).

What if the particles were not in thermal equilibrium? This brings us to the next dichotomy, thermal vs. nonthermal. A "thermal" dark matter particle is one which was once in thermal equilibrium with the rest of the matter and radiation in the hot universe. Any particle which is not extremely weakly coupled will fulfill this condition. However, it is possible to invent particles which *are* sufficiently weakly coupled to avoid it ("invisible" axions are the best-known example). In this case, the conclusion that a dark matter particle must weigh more than a few keV does not apply, and we must consider the detailed history of the dark matter particles. On the other hand, for thermal dark matter, all we need to be able to do is compute the annihilation cross section of two dark matter particles; then we can use the Boltzmann equation to calculate their abundance today. This fact, known and used for 30 years,[7,8] is a key ingredient in understanding thermal dark matter and constraining its properties.

The third dichotomy, easy vs. hard to detect, is one which is more sociological than physical. We would like to discover the nature of the dark matter particles experimentally. This may be impossible for the foreseeable future: there is no argument against the depressing possibility that the dark matter particle's interactions with ordinary matter are *solely* gravitational, or otherwise simply too weak for any known or foreseeable detection methods. Alas, this is not all that unlikely: ordinary neutrinos, endowed with small masses, make an excellent candidate for at least some of the dark matter, and no one has ever thought of a way to detect these neutrinos directly. Of course, in this case, the mass of the neutrino(s) can potentially be inferred from other experiments, solar physics, etc., and then (since neutrinos are thermal particles) we can compute their relic mass density. Axions are another candidate for dark matter, and their interactions are even weaker than neutrinos; in this case, however, clever detection schemes (which take advantage of coherent effects) can be used.[9] This points to the need for more serious thought about novel detection schemes, in particular, anything which can take advantage of purely gravitational effects.

More optimistically though, many proposed dark matter candidates *are* detectable with present or foreseeable technology,[10] and many experiments are already proceeding. That they may succeed is an exciting possibility. In discussing dark matter candidates then, it is important to note whether they are "easy" to detect (by which I really mean "possible") or "hard" to detect (by which I really mean "impossible" by presently known methods).

This leads us to the last dichotomy, "expected" vs. "unexpected." A great many hypothetical particles have been proposed as dark matter candidates. The existence of some of these had already been suggested for other reasons, and this gives them an added air of plausibility. However, we do not really know what the underlying great principles of particle physics are. Thus, the popular dark matter candidates reflect current theoretical prejudice, but theoretical prejudice has been wrong before and undoubtedly will be again at some times in the future. Is this one of those times? Of course, this is an unanswerable question. Thus, while current direct-search experiments concentrate primarily on the expected candidates, we must always be on the lookout for unexpected ones. In practice, this means doing experiments which can expand the bounds on mass and interaction cross section (with some target material), irrespective of whether or not we have dreamed up any particle which is within these bounds. Of course, no one is going to mount a difficult, expensive, time-consuming experiment for a marginal improvement on current bounds if there is not presently a good reason for suspecting a particle lurking just outside these bounds. What is really needed are as many ideas as possible for novel search techniques which can expand present bounds, even if this expansion is in a currently unfashionable direction.

2 Thermal Relics

In this section, I will review the basic formalism for calculating the mass density of a stable particle, given its annihilation cross section. Many excellent presentations of the details of this are already available, and therefore, I will concentrate on qualitative features and rule-of-thumb results.

If everything was in thermal equilibrium at temperature T, the number of type-i particles with momentum in a range of d^3p about φ would be $f_i^{\rm eq}(\varphi)d^3p$, where

$$f_i^{\rm eq}(\varphi) = \{\exp[(\varphi^2 + m_i^2)^{1/2}/T] - \varepsilon_i\}^{-1} \ , \tag{4}$$

where m_i is the mass of a type-i particle, and $\varepsilon_i = +1$ if type-i particles are bosons and $\varepsilon_i = -1$ if type-i particles are fermions. Here, "type" means all distinguishing characteristics except momentum and energy; thus, for example, there are two types of photons (one for each helicity), four types of electrons and positrons, and six types of neutrinos. The total mass-energy density in type-i particles is then

$$\rho_i^{\rm eq} = \int \frac{d^3p}{(2\pi)^3} f^{\rm eq}(\varphi)(\varphi^2 + m_i^2)^{1/2} \ . \tag{5}$$

The contribution to the total entropy density of type-i particles is

$$s_i^{\rm eq} = -\int \frac{d^3p}{(2\pi)^3} f^{\rm eq}(\varphi) \ln f^{\rm eq}(\varphi) \ . \tag{6}$$

These formulas simplify considerably in either the extreme relativistic ($T \gg m_i$) or extreme nonrelativistic ($T \ll m_i$) limits. In the former case, we have

$$\rho_i^{\rm eq} \simeq \frac{\pi^2}{30} \begin{pmatrix} 1 \\ \frac{7}{8} \end{pmatrix} T^4 \qquad \text{for} \qquad T \gg m_i \tag{7}$$

and

$$s_i^{\rm eq} \simeq \frac{2\pi^2}{45} \begin{pmatrix} 1 \\ \frac{7}{8} \end{pmatrix} T^3 \qquad \text{for} \qquad T \gg m_i \ , \tag{8}$$

where the upper (lower) number is for bosons (fermions), while in the latter we get

$$\rho_i^{\rm eq} \simeq (2\pi m_i T)^{3/2} \exp(-m_i/T) \qquad \text{for} \qquad T \ll m_i \tag{9}$$

and

$$s_i^{\rm eq} \simeq 0 \qquad \text{for} \qquad T \ll m_i \ , \tag{10}$$

where we really mean that $s_i^{\rm eq}$ is small enough to ignore in all cases of interest.

The total mass-energy density in all particle types is $\rho = \sum_i \rho_i$, and the total entropy density is $s = \sum_i s_i$. Entropy is conserved in all cases which will be of interest to us; what does this mean? In a homogeneous, isotropic, expanding universe, all lengths must be multiplied by the "scale factor" $R(t)$. Thus, the entropy in a particular volume V is constant, but we consider V itself to change with time; if we use a subscript zero to denote quantities at the present time, and take $R(t_0) = 1$, then $V(t) = R^3(t)V_0$, and so $s(t) = s_0/R^3(t)$. Therefore, during a time period for which there is no change in the number of *relativistic* particle types (i.e., the temperature is never comparable to the mass of any particle type), we see from Eq. (8) that $T \propto 1/R(t)$. To find $R(t)$, we need Einstein's equation:

$$(\dot{R}/R)^2 = (8\pi/3)G\rho \; , \tag{11}$$

where the dot denotes d/dt as usual. Again, if the temperature is never near the mass of any particle, and if the mass-energy is dominated by the contributions of *relativistic* particles in thermal equilibrium ("radiation dominated"), then we can combine $T \propto 1/R(t)$ with Eq. (7) to get

$$(\dot{T}/T)^2 = (4\pi^3/45)N_{\rm S}T^4/M_{\rm Pl}^2 \; , \tag{12}$$

where $N_{\rm S} = N_{\rm B} + \frac{7}{8}N_{\rm F}$; $N_{\rm B}$ is the number of boson particle types which are relativistic at temperature T (e.g., $N_{\rm B} = 2$ for photons), and $N_{\rm F}$ is the number of fermion particle types which are relativistic at temperature T (e.g., $N_{\rm F} = 4$ for electrons and positrons when $T \gg m_e$, $N_{\rm F} = 6$ for three families of neutrinos). The solution to Eq. (12) is

$$T(t) = (45/16\pi^3)^{1/4}N_{\rm S}^{-1/4}M_{\rm Pl}^{1/2}t^{-1/2} \; , \tag{13}$$

so the temperature drops like the inverse square root of the time.

Now we need to find out what happens when equilibrium conditions are not maintained. The particles will drop out of equilibrium if the reaction rates are not fast enough to maintain it. Thus, the actual distribution of momentum $f_i(\varphi)$ will not be $f_i^{\rm eq}(\varphi)$. In all situations which will be of interest to us though, $f_i(\varphi)$ retains the same functional form and only changes by an overall constant. This is because it is always the case that there are still reactions which redistribute energy among the type-i particles, even though the reactions which change the total *number* of type-i particles have become too slow to be effective.

This is governed by a form of the Boltzmann equation, heuristically derived as follows. Let

$$n_i = \int d^3p \, f_i(p) \tag{14}$$

be the number density of type-i particles. The number of particles in an expanding volume of size R^3, where R is the time-dependent scale factor, is $n_i R^3$. If there are no microscopic processes changing the number of type-i particles, then N_i remains constant. However, if type-i particles can annihilate with type-j particles (and note that we are not excluding the possibility that $j = i$), and we know the cross section σ_{ij} for this process, then we can write a schematic equation

$$\frac{d}{dt}(n_i R^3) = -(n_i R^3) \sum_j \sigma_{ij} F_j + \text{inverse process} \ , \tag{15}$$

where F_j is the flux of type-j particles into the region of volume R^3. This flux is simply $n_i v_{ij}$, where v_{ij} is the velocity of the type-j particle in the frame of the type-i particle with which it is colliding. The right-hand side of Eq. (13) must be averaged over the momentum distribution of each of the two incoming particles; this distribution is assumed to be thermal with temperature T. The inverse process must also be included and must result in zero on the right-hand side if $n_i = n_i^{\mathrm{eq}}$. All of this gives us

$$\frac{d}{dt}(n_i R^3) = -\sum_j \langle \sigma_{ij} v_{ij} \rangle R^3 (n_i n_j - n_i^{\mathrm{eq}} n_j^{\mathrm{eq}}) \ , \tag{16}$$

where $\langle \ldots \rangle_T$ denotes appropriate thermal averaging for temperature T.

We can now combine Eqs. (11), (16), and the constancy of the entropy in an expanding volume, to get

$$\frac{d}{dT}\left(\frac{n_i}{s}\right) = \frac{M_{\mathrm{Pl}}}{(24\pi)^{1/2}} \left(\frac{s'}{\rho^{1/2}}\right) \sum_j \langle \sigma_{ij} v_{ij} \rangle_T \left(\frac{n_i n_j}{s^2} - \frac{n_i^{\mathrm{eq}} n_j^{\mathrm{eq}}}{s^2}\right) \ , \tag{17}$$

where a prime denotes a derivative with respect to temperature: $s' = ds/dT$. If T is not near the mass of any particle, we can apply Eqs. (7)–(10) to get

$$\frac{s'}{\rho^{1/2}} = \left(\frac{8\pi^2}{15}\right)^{1/2} N_{\mathrm{S}}^{1/2} \ . \tag{18}$$

That is, $s'/\rho^{1/2}$ is constant when the temperature is far from any mass thresholds.

Equation (17) can be analyzed heuristically as follows. We begin at high temperatures, assuming that $n_i = n_i^{\rm eq}$. Then, as long as

$$\left| \frac{d}{dT}\left(\frac{n_i^{\rm eq}}{s}\right) \right| \ll \frac{M_{\rm Pl}}{(24\pi)^{1/2}} \left(\frac{s'}{\rho^{1/2}}\right) \sum_j \langle \sigma_{ij} v_{ij} \rangle_T \frac{n_i^{\rm eq} n_j^{\rm eq}}{s^2} \ , \tag{19}$$

the differential equation is "stiff," and we will have $n_i \simeq n_i^{\rm eq}$. Eventually, however, the condition expressed in Eq. (19) will cease to hold, and n_i will not drop as fast as $n_i^{\rm eq}$. The temperature where this occurs [for some precise definition of "cease to hold," such as equality of the left and right sides of Eq. (19)] is called the freeze-out temperature. Below the freeze-out temperature, the evolution of n_i is governed approximately by Eq. (17), but with the second term on the right-hand side dropped (since $n_i \gg n_i^{\rm eq}$).

Let us consider Eq. (17) in a little more detail. First, consider the case where the particle in question is relativistic at the freeze-out temperature. (This is the case for ordinary neutrinos with masses less than a few keV.) For relativistic bosons (fermions), we have

$$n_i^{\rm eq} \simeq \frac{\zeta(3)}{\pi^2} N_i \begin{pmatrix} 1 \\ \frac{3}{4} \end{pmatrix} T^3 \ , \tag{20}$$

where N_i is the number of spin states of the particle in question, and $\zeta(x)$ is the Riemann zeta function. Combining this with Eq. (8), we get

$$\frac{n_i^{\rm eq}}{s} = \frac{45\zeta(3)}{2\pi^4} \frac{N_i}{N_{\rm S}} \begin{pmatrix} 1 \\ \frac{3}{4} \end{pmatrix} \ , \tag{21}$$

which is independent of T. Hence, the condition in Eq. (19) is automatically satisfied. Thus, we have $n_i = n_i^{\rm eq}$ until we cross a mass threshold; during such a crossing, Eq. (19) might cease to hold. Then, n_i/s will not be able to follow $n_i^{\rm eq}/s$, and instead will remain "stuck" at the value that $n_i^{\rm eq}/s$ had before the threshold. Let us see how this works for the case of ordinary neutrinos. For neutrinos, the dominant processes entering Eq. (17) are the "neutral-current" processes $\nu_i \bar{\nu}_i \to e^+ e^-$, $\nu_i e^- \to \nu_i e^-$, etc. (with cross sections $\langle \sigma v \rangle_T \sim G_F^2 T^2$, where G_F is the Fermi constant), and for electron-neutrinos only, the "charged-current" processes $\nu p \to e^+ n$, etc. (with cross sections $\langle \sigma v \rangle_T \sim G_F^2 m_p T$, where m_p is the proton mass). Equation (19) fails when the temperature drops past the electron-positron mass m_e. Before crossing this threshold, we have $N_{\rm B} = 2$ for photons and $N_{\rm F} = 10$ for electrons, positrons, and neutrinos. For *each species* of neutrino, $N_i = 2$

in Eq. (21), and of course, we use the factor of $\frac{3}{4}$; this gives $n_\nu/s = \frac{1}{25.8}$ for *each species* of neutrino. Then, after freeze-out, this number is fixed at this value right up to the present. So, to get the present number density of each species of neutrino, we must compute the entropy density s today. This is given by Eq. (8), summed over particle types which are present in significant numbers today: photons and neutrinos. One complication, though, is that these no longer have the same temperature. The entropy of neutrinos alone is conserved after they freeze-out. Then entropy of photons, electrons, and positrons is separately conserved, since there are no longer effective interactions with neutrinos. Thus, when electrons and positrons annihilate (to photons, and not to neutrinos), the photons get all of the entropy of the electrons and positrons. Before annihilation, this entropy is

$$s_{e^+e^-\gamma} = \frac{2\pi^2}{45}\left(\frac{7}{8}\cdot 4 + 2\right) T_\nu^3 \ , \tag{22}$$

where T_ν is the temperature of the neutrinos, the electrons and positrons, and the photons; after annihilation, this entropy is

$$s_\gamma = \frac{2\pi^2}{45}\,(2)\,T_\gamma^3 \ , \tag{23}$$

where T_γ is the new temperature of the photons. Setting $s_{e^+e^-\gamma} = s_\gamma$, we find $T_\nu/T_\gamma = (4/11)^{1/3}$. Now, the photon temperature is $T_\gamma = T_0 = 2.73\,\mathrm{K}$. Thus, the total entropy today is

$$\begin{aligned} s_0 &= \frac{2\pi^2}{45}\left(2 + \frac{7}{8}\cdot 6\cdot\frac{4}{11}\right) T_\gamma^3 \\ &= 1.71\,T_\gamma^3 \\ &= 2970\,\mathrm{cm}^{-3} \ . \end{aligned} \tag{24}$$

This gives us, for each species of neutrino today,

$$n_{\nu 0} = \frac{2970\,\mathrm{cm}^{-3}}{25.8} = 115\,\mathrm{cm}^{-3} \ . \tag{25}$$

If we assume a mass m_ν for a particular neutrino species, this species contributes

$$\begin{aligned}\Omega_\nu &= \frac{m_\nu n_{\nu 0}}{\rho_{\rm crit}} \\ &= 0.56\,\frac{m_\nu T_0^3}{H_0^2 M_{\rm Pl}^2} \\ &= \frac{m_\nu}{(92\,{\rm eV})\,h^2} \end{aligned} \tag{26}$$

to the present mass density of the universe.

Let us now turn our attention to more hypothetical particles, stable (or nearly so) particles which are *nonrelativistic* at the time of freeze-out. The nonrelativistic analog of Eq. (20) for particles of mass m with N_i spin states is

$$n_i^{\rm eq} \simeq N_i (mT/2\pi)^{3/2} e^{-m/T} \ , \tag{27}$$

for either bosons or fermions, and so

$$\frac{n_i^{\rm eq}}{s} = \frac{45}{2\pi^2}\frac{1}{(2\pi)^{3/2}}\frac{N_i}{N_{\rm S}}\left(\frac{m}{T}\right) e^{-m/T} \ . \tag{28}$$

We assume that the only process which can change the numbers of our stable particle is self-annihilation (and its inverse), so that $j = i$ in Eq. (17). The freeze-out condition [equality of the left and right sides of Eq. (19)] then gives

$$x_{\rm fr}\exp(x_{\rm fr}) = \left(\frac{45}{32\pi}\right)^{1/2}\left(\frac{N_i}{N_{\rm S\,fr}}\right)^{1/2} M_{\rm Pl}\, m\, \langle\sigma v\rangle_{\rm fr} \ , \tag{29}$$

where $\langle\sigma v\rangle_{\rm fr}$ is the thermally averaged annihilation cross section at the freeze-out temperature, $N_{\rm S\,fr}$ is the number of relativistic particle types at the freeze-out temperature (with fermions counting $\frac{7}{8}$), and we have defined

$$x_{\rm fr} = \frac{m}{T_{\rm fr}} \ . \tag{30}$$

If we now suppose that $\langle\sigma v\rangle_T$ is independent of temperature (usually a good first approximation for nonrelativistic particles), then the solution of Eq. (29) is

$$x_{\rm fr} = \ln R_{\rm fr} - \frac{1}{2}\ln\ln R_{\rm fr} + \ldots \ , \tag{31}$$

where $R_{\rm fr}$ is the right-hand side of Eq. (29). So to get an approximate solution to Eq. (17), we assume that $n_i = n_i^{\rm eq}$ until $T = T_{\rm fr}$, and then afterward, that Eq. (17) holds with the second term on the right-hand side set to zero (since n_i is increasingly larger than $n_i^{\rm eq}$). This modified form of Eq. (19) can be integrated to give

$$\left(\frac{s}{n_i}\right)_0 = \left(\frac{s}{n_i}\right)_{\rm fr} + K_{\rm fr}(T_{\rm fr} - T_0) \ , \tag{32}$$

where $K_{\rm fr}$ is the coefficient on the right-hand side of Eq. (17), evaluated at the freeze-out temperature:

$$K_{\rm fr} = \frac{M_{\rm Pl}}{(24\pi)^{1/2}} \left(\frac{s'}{\rho^{1/2}}\right)_{\rm fr} \langle\sigma v\rangle_{\rm fr} \ . \tag{33}$$

In Eq. (32), T_0 can be neglected compared to $T_{\rm fr}$, and $(s/n_i)_{\rm fr}$ compared to $(s/n_i)_0$, so we simply get

$$n_i = \frac{1}{K_{\rm fr}T_{\rm fr}}\, s_0 = \frac{x_{\rm fr}}{mK_{\rm fr}}\, s_0 \ , \tag{34}$$

with $x_{\rm fr}$ given by Eq. (30); $x_{\rm fr}$ depends only logarithmically on the particle's mass and annihilation cross section, and therefore, will not be very large or very small for any reasonable ranges of these parameters. We can now compute $\Omega_i = mn_i/\rho_{\rm crit}$. Since Eq. (34) tells us that $n_i \sim 1/m$, we obtain the surprising result that Ω_i has no explicit dependence on the particle mass m:

$$\begin{aligned} \Omega_i &= \frac{m\, n_i}{\rho_{\rm crit}} \\ &= 54.2 \left(\frac{x_{\rm fr}}{N_{\rm S}^{1/2}}\right) \frac{T_0^3}{M_{\rm Pl}^3\, H_0^2\, \langle\sigma v\rangle_{\rm fr}} \\ &= \left(\frac{x_{\rm fr}}{N_{\rm S}^{1/2}}\right) \frac{8.7\times 10^{-11}\,{\rm GeV}^{-2}}{\langle\sigma v\rangle_{\rm fr} h^2} \ . \end{aligned} \tag{35}$$

Thus, we see that a particle which is nonrelativistic at its freeze-out temperature, and has an annihilation cross section of order $\sigma v \sim M_{\rm Pl}^3 H_0^2/T_0^3 \sim 10^{-10}\,{\rm GeV}^{-2}$, will contribute significantly to the mass of the universe. Amazingly, this value for σv is in the range of what we expect for particles with masses of tens to hundreds of GeV, and couplings to ordinary particles of roughly the same size as electroweak gauge couplings. This is either a profound clue as to the nature of dark matter or a tantalizing false lead; time will tell.

3 Nonthermal Relics

A simple example of a nonthermal relic is an oscillating scalar field. In an expanding universe, a spatially uniform scalar field obeys the equation

$$\ddot\varphi + 3\frac{\dot R}{R}\dot\varphi + m^2\varphi = 0 \ , \tag{36}$$

where a dot denotes a time derivative and m is the mass of the corresponding scalar particles. Given an initial value φ_i of φ, φ remains frozen at this value until $t \gtrsim 1/m$; then φ begins oscillating. The energy density stored in φ is

$$\rho_\varphi = \tfrac{1}{2}\dot{\varphi}^2 + \tfrac{1}{2}m^2\varphi^2 \ , \tag{37}$$

and it follows from Eq. (36) that ρ_φ decreases from its initial value like $1/R^3 \sim T^3$, so that its energy density today is

$$\rho_{\varphi 0} \simeq \tfrac{1}{2}m^2\varphi_i^2 \left(\frac{T_0}{T_1}\right)^3 \ , \tag{38}$$

where T_1 is the temperature at the time t_1 given by $t_1 = 1/m$. Working out the numbers and dividing by $\rho_{\rm crit}$ gives[11]

$$\Omega_\varphi h^2 \sim 0.7 \left(\frac{\varphi_i}{3 \times 10^8 \text{ GeV}}\right)^2 \left(\frac{m}{100 \text{ GeV}}\right)^{1/2} \ . \tag{39}$$

This formula assumes that m is constant. A more interesting case arises if m is temperature-dependent. At first glance, this does not seem possible, because if thermal effects were important, we would expect them to thermalize the scalar field. The axion, however, is a very special particle whose mass arises from QCD instanton effects, and these are altered by finite temperatures. Roughly,

$$m_a(T) \sim m_a\Lambda^4/T^4 \ , \tag{40}$$

where m_a is the axion mass at zero temperature, Λ is the QCD scale, and we have assumed $T \gg \Lambda$. In this case, we find, instead of Eq. (39), the very different formula[12–14]

$$\Omega_a h^2 \sim 0.3 \left(\frac{\varphi_i}{10^{12}\,\text{GeV}}\right)^2 \left(\frac{m}{10^{-5}\,\text{eV}}\right) \ . \tag{41}$$

We will return to discuss axions more fully later.

Another nonthermal relic particle is one which is prevented from annihilating completely by a conserved quantum number. This quantum number should correspond to a global (rather than a local or gauge) symmetry. In this case, it is possible for the universe as a whole to have a nonzero "charge." Then there would have to be some particles around today which carry this charge. The mass density of these particles would depend on the value of the charge of the universe. This is the explanation for the present mass density of baryons, except that we would like to go farther here; assume that the baryon number is not *exactly* conserved, and use the precise form of its nonconservation to compute the present baryon number density. The problem for new, hypothetical particles is that there are too many possible models, with no good way to choose among them. Thus, it is not possible to say anything at all about the expected value of the new quantum number in our universe. Nevertheless, this is a real possibility which cannot be dismissed.

4 Unexpected Dark Matter Candidates

Since an unexpected candidate is just that, they are hard to write about. Nevertheless, it is important to remember that surprise has been a constant in the history of physics, and dark matter may well provide another one.

I classify a candidate dark matter particle as unexpected if it is not predicted by an extension of the Standard Model which seems necessary for other reasons. A simple example is the scalar phantom of Silveira and Zee.[15] They suggested that a new real scalar field χ be added to the Standard Model, and that a discrete global symmetry $\chi \to -\chi$ be imposed. In this case, the only possible renormalizable interaction with any other field in the Standard Model is

$$\mathcal{L}_{\rm int} = \lambda \chi^2 H^\dagger H \ , \tag{42}$$

where H is the usual Higgs doublet of the Standard Model. The $\chi \to -\chi$ symmetry guarantees that the χ particles are stable. If they are heavier than the Higgs boson, then two χ's can annihilate into two Higgs bosons, with a cross section given roughly by $\sigma v \sim \lambda^2/16\pi^2 m_\chi^2$. We then get $\sigma v \sim 10^{-10}\,{\rm GeV}^{-2}$ if $m_\chi \sim 8\lambda\,{\rm TeV}$, which is not too unreasonable. Such a particle would be extremely hard to detect, either in an accelerator or as dark matter, and is certainly not ruled out experimentally. Furthermore, its existence cannot be excluded on the basis of any theoretical principles that we presently understand.

Another unexpected dark matter candidate is a stable, electrically charged particle.[16,17] Such particles would be extremely *easy* to detect, and their nondetection so far could be explained only if they were extremely heavy, with a mass in excess of 10 TeV. Such particles would have a number of astrophysical consequences, however,[18,19] the most serious of which appears to be that they would congregate in the centers of neutron stars, where they would form black holes that would quickly destroy the neutron stars![19] Stable charged particles are not, therefore, a currently viable candidate, but it was important to examine this possibility.

Yet another unexpected candidate is a strongly interacting particle.[20] We have seen that a heavy particle comes with about the right annihilation cross section if its interactions are of roughly electroweak strength. However, a more strongly interacting particle could be protected by the global-symmetry mechanism, and so must be considered. Restrictions on the allowed regions of the candidates' mass and scattering cross section (with different types of nuclear targets, depending mainly on whether the scattering is coherent or spin-dependent) can then be deduced. Here, some of the direct detection experiments were able to reduce the allowed regions by some relatively simple reconfigurations.[21] All such possibilities for exploring parameter space of possible dark-matter candidates must be explored.

5 Expected Dark Matter Candidates

A dark matter candidate must be stable or have a lifetime as long as the age of the universe. The list of *known* particles meeting this requirement is short: the photon, the graviton, the electron, and the lightest spin-$\frac{1}{2}$ particle (which is probably, but not necessarily, one or more of the known neutrinos). They are all absolutely stable (we believe) for very good reasons: electromagnetic gauge invariance, general coordinate invariance, electric charge conservation, and angular momentum conservation, respectively. It is very difficult to imagine that one or more of these hallowed principles is in error, and that one or more of these particles could therefore be unstable (but, of course, experimental limits must always be improved whenever possible). The last known particle which is stable or long-lived is the proton. There is no really good reason for this that we understand at present, certainly nothing comparable to an entry on our previous list. The best we can do for the proton is to note that, within the Standard Model, it is impossible to write down a renormalizable interaction which would allow it to decay. Any physics beyond the Standard Model, however, could result in proton decay. Such new physics would appear at some mass scale M, and we would then expect the proton decay amplitude to be inversely proportional to some (integer) power of M, most likely to M^{-1} (if proton decay is mediated by a new fermion of mass M) or to M^{-2} (if proton decay is mediated by a new boson of mass M);

this leads to a decay rate proportional to m_p^3/M^2 or m_p^5/M^4, where the factors of m_p come from phase space integrals, and we also need some coupling constants. Experimental limits on the proton lifetime lead to the conclusion that M must be very large, even if the interaction is boson mediated, unless there are some big extra suppression factors (from very small couplings, for example). The lesson I wish to draw for hypothetical particles is that a good way to make them long-lived is to arrange for their decay rates to be inversely proportional to some mass scale which is much larger than the mass of the particle in question.

A good example of this is the axion. The axion is expected, nonthermal, cold, and potentially detectable—an ideal candidate. The axion is an oscillating scalar field, as discussed earlier, but its temperature-dependent mass leads to Eq. (41) for its relic density. Furthermore, the axion field a actually lives on a circle, and so must be periodically identified. (This circle is the "bottom of a wine bottle" in the potential for a complex scalar field.) The period of a is $2\pi f$, where f is the "axion decay constant." In terms of f, the mass of the axion is $m_a = \frac{(m_u m_d)^{1/2}}{m_u + m_d}\frac{f_\pi m_\pi}{f}$, where $m_u \simeq 4\,\mathrm{MeV}$ and $m_d \simeq 7\,\mathrm{MeV}$ are quark masses, $m_\pi = 130\,\mathrm{MeV}$ is the pion mass, and $f_\varphi = 93\,\mathrm{MeV}$ is the pion decay constant. Since the axion field is periodic, it is reasonable to say its initial value is $a_i = f\theta_i$ with $-\pi < \theta_i < \pi$. Since $f \sim 1/m_a$, we then see from Eq. (41) that $\Omega_a h^2 \sim \theta_i^2/m_a$. A refined estimate is[22]

$$\Omega_a h^2 \simeq 0.13 \times 10^{\pm 0.4} \;, \left(\frac{200\,\mathrm{MeV}}{\Lambda_{\mathrm{QCD}}}\right)^{0.7} \theta_i^2 \;, \left(\frac{10^{-5}\,\mathrm{eV}}{m_a}\right)^{1.18} \;, \tag{43}$$

so that we need $m_a \sim 10^{-5}\,\mathrm{eV}$, corresponding to $f \sim 10^{12}\,\mathrm{GeV}$. This large value of f is what we want. The axion couples to ordinary particles, but all its couplings are suppressed by inverse powers of f. For example, the $a\gamma\gamma$ interaction Lagrangian is

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a \vec{E}\cdot\vec{B} \;, \tag{44}$$

in Heaviside-Lorentz units, with[23,24]

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi}\frac{1}{f}\left(\frac{E}{N} - 1.92\right) ; \tag{45}$$

here $\alpha = 1/137$ is the fine structure constant, E/N is a number characterizing the fermion content of the axion model, and 1.92 is a ratio of quark masses. Alas, in any theory where all fermions come in complete multiplets of SU(5) (as they do in the Standard Model), $E/N = +8/3$, and so there is an unfortunate cancellation.

Still, if they comprise the dark matter, axions with $E/N = +8/3$ are in principle detectable. If we are lucky, and $E/N - 1.92$ is bigger, they will be detected soon.[9]

Another expected dark matter candidate is the lightest supersymmetric particle, or LSP.[25] Supersymmetry has been considered seriously for a number of reasons. It represents a profound extension of the symmetry structure of our fundamental theories; it helps to explain the discrepancy between the weak interaction scale and much higher mass scales (of grand unification, quantum gravity, etc.); and it is part of string theory, the only self-consistent quantum theory incorporating gravity. In supersymmetric theories, there is a new particle for every known one. However, in a supersymmetric version of the Standard Model, it is possible to write down renormalizable interactions which violate lepton and baryon number conservation. These must be forbidden in some way, most simply by imposing an extra discrete symmetry ("R-parity") which R-parity $+1$ to all known particles, and -1 to all new ones. This turns out to forbid all the new lepton and baryon number violating interactions. Of course, it also means that the lightest particle carrying $R = -1$ must be stable, and hence, an excellent dark matter candidate.

The LSP must have a mass of some tens to thousands of GeV, and so would cold dark matter. Since it must be electrically neutral and not have strong interactions, it must be a sneutrino (spin-0 partner of a neutrino) or a linear combination of the four neutral spin-$\frac{1}{2}$ particles: the photino, the zino, and two higgsinos.[26] Such a particle would have been in thermal equilibrium in the early universe, with a relic abundance that can be reliably computed (as discussed in Sec. 2). We would like to obtain the magic number $\sigma\ v\ \sim 10^{-10}\,\mathrm{GeV}^{-2}$ for the annihilation cross section. This is not too difficult, since there are many parameters (the mass matrices for all the superparticles!) to play with. Still, some general features emerge. First, it is very difficult to have a sneutrino as dark matter, though perhaps not completely impossible.[27] The main problem is that the sneutrino scatters coherently off nuclei, and so would have appeared in direct-detection experiments unless it is very heavy (several TeV). If it is very heavy, its annihilation cross section sinks below the magic number. The four neutralinos can be divided into gauginos (the photino and the zino) and higgsinos. The gauginos are better thought of as the neutral wino and the bino (superpartners of the neutral SU(2) gauge boson W^0 and the hypercharge gauge boson B) because the $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariant mass terms given to them are typically large enough (in interesting regions of parameter space) to dwarf the effects of electroweak symmetry breaking, which on their own would mix the neutral wino and bino into the photino and the zino. The LSP is more likely to be a gaugino than a higgsino.[28] This is because, when a higgsino is the LSP, the annihilation cross section is too large. If the mass of the higgsino is less than the mass of the Z^0, the dominant process is s-channel annihilation through the Z^0 and $W^{\pm}$ with the other higgsinos (charged and neutral) which are nearly degenerate in mass, and therefore, present with nearly the same number density.[29] If the mass of the higgsino is greater than the mass of the Z^0, the dominant process is annihilation into the gauge boson pairs Z^0Z^0 and W^+W^-.[28] On the other hand, the bino does not couple to the Z^0 or $W^{\pm}$ at all; if the bino is the LSP, it annihilates primarily through t-channel exchange of squarks and sleptons (the spin-0 partners of quarks and leptons), and so its annihilation cross section depends crucially on the unknown masses of these new particles. This adjustability means that there is a wide range of bino masses for which it can be the LSP and the dark matter. The neutral wino could also be the LSP, but the theoretical prejudice (explained below) is that the bino is lighter than the winos.

The real problem with coming to definite conclusions is the great number of new particle masses and mixings one must choose. Two general approaches have been taken. One is to try to set limits. For example, it is difficult to have a bino as the LSP if its mass is greater than about 350 GeV; above this mass, the annihilation cross section becomes too small, and the relic density of binos is too big ($\Omega_{\widetilde{B}}h^2 > 1$). But there are loopholes, for example, including CP violating phases in the neutralino mass matrix at the maximum (experimentally allowed) level pushes the limit up to about 650 GeV (Ref. 30). Completely generic statements, with *all* loopholes closed, allow LSP masses of several TeV (if the LSP is a higgsino).

Another approach is to assume maximal symmetry of the mass matrices at a high energy scale; this follows from grand unification, for example. Then one uses renormalization group equations to find the physical masses and computes relic densities of the LSP (which always turns out to be a neutralino and never the sneutrino) on this basis.[31] The advantage of this approach is that the number of unknown parameters is greatly reduced, and more specific predictions can be made. The disadvantage is that it is not clear how reasonable these assumptions are. They do not necessarily hold in string models, for example.

Still, the LSP is a terrific dark matter candidate. It is predicted by a beautiful general principle (supersymmetry) applied to the real world (invoking R-parity to conserve baryon and lepton numbers), and getting the magic number $\sigma v \sim 10^{-10}\,\mathrm{GeV}^{-2}$ is not at all difficult. It provides cold dark matter, which seems to be a necessity for galaxy formation. A search for LSP-comprised dark matter is, therefore, extremely important.

Direct detection experiments[32] are discussed in these proceedings by Sadoulet.[10] These represent the best hope for demonstrating that at least some of the dark matter is comprised of heavy elementary particles which scatter off nuclei. Pinning down the properties of this dark matter particle will undoubtedly require accelerator-based experiments as well. Unfortunately, some accelerator experiments have recently dampened the highest direct detection hopes.[33] CLEO has found a small value for the branching ratio of $b \to s\gamma$, precluding a light charged Higgs boson. Direct detection experiments benefit greatly from a light *neutral* Higgs boson (as a mediator of the scattering of a dark matter particle off a nucleus). But if there is no light charged Higgs, then (in supersymmetric theories) there is no light neutral Higgs either.

Another potential source of information arises from methods based on detection of annihilation products of the dark matter particles. Annihilation is ongoing, and we know what the annihilation cross section must be. (Actually, we know what it must be at the freeze-out temperature, and it could be substantially less at the near-zero temperature which prevails today.) In order for present-day annihilation to be significant, an enhanced density of dark matter is required. Of course, this is exactly what we get in galactic halos! The enhancement is about a factor of 10^5 near our solar system. This is not enough to do anything dramatic, but dark matter annihilations can produce a variety of unusual products: antiprotons,[34] positrons,[35] gamma ray lines,[36] etc. None of these predictions, however, are dramatic enough to be of immediate use.

Another source of density enhancement is trapping by the sun or earth.[37] A dark matter particle can scatter off of a nucleus in the sun or earth, lose some kinetic energy, and become gravitationally bound. This can yield a huge enhancement of the number density and hence, the annihilation rate; the observable signature which results is a flux of high-energy neutrinos.[38] These neutrinos can be seen by detectors such as Kamiokande. In fact, Kamiokande has already set some modest limits on the parameter space of the LSP,[39] and large neutrino telescopes like AMANDA and DUMAND could do much better.[40]

Thus, all in all, there is reason to be optimistic about eventual experimental detection of the LSP as dark matter, assuming that this is indeed the case!

6 Some Dark Horses

Even within the context of supersymmetric theories, there are a few dark horse candidates for the LSP and dark matter. One of these, a very heavy sneutrino, has already been mentioned. Here, I would like to discuss two more.

One is a relatively light ($\sim 15\,\mathrm{GeV}$) photino, along with an only slightly heavier "stop," the scalar partner of the top quark.[41] Each quark actually has two scalar partners, one corresponding to the left-handed quark and one corresponding to the right-handed quark. It is easy to see why this is necessary: left- and right-handed quarks can (and do) have different gauge quantum numbers (we have left-handed doublets under the SU(2) of weak interactions, and right-handed singlets); both sets of quantum numbers must be represented among the superpartners. These two scalars can mix, so that the mass eigenstates are linear combinations of the original superpartners of the left- and right-handed pieces of the corresponding quark. It turns out that this mixing is proportional to the mass of the *quark* and so is negligibly small except for the stops. If we call the stop mixing angle θ_t, then the coupling of the Z^0 to the lightest of the two stop mass eigenstates is proportional to $3\cos^2\theta_t - 4\sin^2\theta_w$, where θ_w is the usual weak mixing angle. The lighter stop, thus, does not couple to the Z^0 if $\theta_t = 0.98$ and would have escaped detection so far if $0.8 < \theta_t < 1.2$, even if its mass is only $20\,\mathrm{GeV}$! In this case, the LSP can be the photino, provided that its mass is no more than $5\,\mathrm{GeV}$ less than that of the stop. In this case, there are enough stops in thermal equilibrium in the early universe to make it necessary to take photino-stop annihilation into account, and this can be of the right size to produce $\Omega = 1$ in photinos, even if all the other squarks and sleptons weigh several hundred GeV.

Another interesting dark horse is the axino, superpartner of the axion.[42,43] After all, if one believes in axions and in supersymmetry, then there must be an axino. The axino's properties are unfortunately quite model dependent. However, an interesting scenario[43] has an axino with a mass of a few to a few hundred keV; thermal axinos then act as cold (or warm) dark matter. However, there is another source of axinos: decays of what would have been the LSP (e.g., the bino). This decay rate is typically quite slow and so occurs late in the history of the universe. Also, because the axino is so much lighter than the bino, bino decay results in relativistic axinos which act as hot dark matter!

7 Conclusions

There are a great many candidates for nonbaryonic dark matter. Some are cold, some are hot; some are thermal, some are nonthermal; some can be detected in the near future, some cannot; some are expected, many are dark horses. It seems clear that the future of this subject is experimental. We need some data to point us in the right direction. However, it seems equally clear that a clever new idea on how to acquire this data is not impossible to imagine and would be of immense importance.

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