DARK MATTER AND LARGE-SCALE STRUCTURE

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ABSTRACT

The three parts of this paper cover the following topics: (1) status of the cosmological parameters H_0, t_0, Λ, and especially Ω; (2) how to compare cosmological theories of structure formation and cosmological simulations to observational data; and (3) structure formation in Cold Dark Matter (CDM) and related cosmological models, especially Cold + Hot Dark Matter (CHDM). I will also briefly discuss experimental data suggesting nonzero neutrino masses.

*In my lectures at SLAC, I covered more elementary material than is included in the written version. Most of this elementary material can be found in my lectures at earlier schools, starting with the 1984 Enrico Fermi School at Varenna, edited by N. Cabibbo (1987). I wrote up a very long version for the proceedings of the BCSPIN school in Puri, India, which is to appear soon edited by J. Pati (World Scientific). There are, of course, several excellent textbooks as well. So in these written notes, I covered mostly current topics.

1 Part I. Status of the Cosmological Parameters

The cosmological parameters that I will discuss are the traditional ones: the Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}, the age of the universe t_0, the average density $\Omega_0 \equiv \bar{\rho}/\rho_c$ in units of critical density ρ_c, and the cosmological constant Λ. To focus the discussion, I will concentrate on the issue of the value of the density Ω_0 in currently popular cosmological models in which most of the dark matter is cold, especially Cold + Hot Dark Matter (CHDM) and flat low-Ω CDM with a Cosmological Constant (ΛCDM). The evidence would favor a small $\Omega_0 \approx 0.3$ if (1) the Hubble parameter actually has the high value $h \approx 0.8$ favored by many observers, and the age of the universe $t_0 \geq 13$ Gy; or (2) the baryonic/total mass ratio in clusters is actually $\sim 20\%$, about three to four times larger than expected for standard Big Bang Nucleosynthesis (BBN) in an $\Omega = 1$ universe, and standard BBN is actually right in predicting that the density of ordinary matter Ω_b lies in the range $0.009 \leq \Omega_b h^2 \leq 0.02$. The evidence would favor $\Omega = 1$ if (1) the POTENT analysis of galaxy-peculiar velocity data is right, in particular regarding outflows from voids or the inability to obtain the present-epoch non-Gaussian density distribution from Gaussian initial fluctuations in a low-Ω universe; or (2) the preliminary report from LSND indicating a neutrino mass ≥ 2.4 eV is right, since that would be too much hot dark matter to allow significant structure formation in a low-Ω ΛCDM model. Statistics on gravitational lensing of quasars provide a strong upper limit on Λ. The era of structure formation is another important discriminant between these alternatives, low Ω favoring earlier structure formation, and $\Omega = 1$ favoring later formation with many clusters and larger-scale structures still forming today. Reliable data on all of these issues is becoming available so rapidly today that there is reason to hope that a clear decision between these alternatives will be possible within the next few years.

As I write this in early 1995, shortly after publication of the first article[1] using HST observations of Cepheid variable stars to determine a distance to a relatively distant galaxy (17.1 ± 1.8 Mpc for M100), articles in the popular news media are full of talk about a crisis in cosmology: "Big Bang Threatened...." The reason is, of course, that with the additional assumptions that M100 lies in the core of the Virgo cluster and that the recession velocity of Virgo corrected for infall is about 1400 km s^{-1}, the value obtained for the Hubble parameter is at the high end of recent estimates: $H_0 = 80 \pm 17\,\mathrm{km\,s^{-1}\,Mpc}$. Using $h = 0.8$ gives, for $\Omega = 1$ and a vanishing cosmological constant $\Lambda = 0$, a very short age for the universe $t_0 = 8.15$ Gy, almost certainly younger than the ages of Milky Way globular clusters and even some nearby white dwarfs. Even with $\Omega_0 = 0.3$, about as low as permitted by observations, and with $\Omega_\Lambda \equiv \Lambda/(3H_0^2) = 0.7$, as high as permitted by observations, $t_0 = 11.8$ Gy for $h = 0.8$, which is also uncomfortably short. Is this a crisis? Does it undermine the strong evidence for the standard Big Bang? I do not think so. Given the considerable uncertainties reflected in the large quoted error on H_0, I think even $\Omega = 1$ models are not excluded. But this Cepheid measurement of the distance to M100 bodes well for the success of the HST Key Project on the Extragalactic Distance Scale, which seeks to measure H_0 to 10% within a few years. The expectation that accurate measurements of the key cosmological parameters will soon be available is great news for theorists trying to construct a fundamental theory of cosmology, and helps motivate the present summary.

In addition to the Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}, I will discuss the age of the universe t_0, the average density Ω_0, and the cosmological constant Λ. But there are several additional cosmological parameters whose values are critical for modern theories: the densities of ordinary matter Ω_b, cold dark matter Ω_c, and hot dark matter Ω_ν, and, for primordial fluctuation spectra $P(k) = Ak^{n_p}$, the index n_p and the amplitude A, or equivalently (for a given model) the bias parameter $b \equiv 1/\sigma_8$, where $\sigma_8 \equiv (\delta M/M)_{rms}$ on a scale of $8\,h^{-1}\,\mathrm{Mpc}$. A full treatment of these parameters would take a much longer article than this one, so to focus the discussion, I will concentrate on the issue of the value of the density Ω_0 in currently popular cosmological models in which most of the dark matter is cold. Although much of the following discussion will be quite general, it will be helpful to focus on two specific cosmological models which are perhaps the most popular today of the potentially realistic models: low-Ω CDM with a Cosmological Constant (ΛCDM, discussed as an alternative to $\Omega = 1$ CDM since the beginning of CDM[2,3] and worked out in greatest detail in Ref. 4), and $\Omega = 1$ CHDM (proposed in 1984 [Ref. 5], and first worked out in detail in 1992-93 [Refs. 6 and 7]). I will begin by summarizing the rationale for these models.

2 Models with Mostly Cold Dark Matter

Let me begin here by recalling the definitions of "hot" and "cold" dark matter. These terms describe the astrophysically relevant aspects of candidate dark matter particles. The fact that the observational lower bound on Ω_0—namely $0.3 \lesssim \Omega_0$—exceeds the upper limit on baryonic mass $\Omega_b \lesssim 0.02\ h^{-2}$ from Big Bang Nucleosynthesis[8] is the main evidence that there must be such nonbaryonic dark matter particles.

About a year after the Big Bang, the horizon surrounding any point encompassed a mass of about $10^{12}\ M_\odot$, the mass now in the dark matter halo of a large galaxy like the Milky Way. The temperature then was about a kilovolt. We define *cold* dark matter as particles that were moving sluggishly, and *hot* dark matter as particles that were still relativistic at that time. The lightest superpartner particle (LSP neutralino) and the axion remain the best motivated cold dark matter candidates, although, of course, many other possibilities have been suggested.

The three known neutrino species ν_e, ν_μ, and ν_τ are the standard hot dark matter candidates. Their contribution to the cosmological density today is

$$\Omega_\nu = \frac{\sum_i m(\nu_i)}{94 h^2\,\mathrm{eV}}.$$

Since $\Omega_\nu < \Omega_0 \lesssim 2$, each neutrino's mass must be much less than a keV, so they were certainly moving at relativistic speeds a year after the Big Bang. Any of these neutrinos that has a cosmologically significant mass ($\gtrsim 1$ eV) is therefore a hot dark matter particle.

If a horizon-sized region has slightly higher than average density at this time, cold dark matter—moving sluggishly—will preserve such a fluctuation. But neutrinos—moving at nearly the speed of light—will damp such fluctuations by "free streaming." For example, two years after the Big Bang, the extra neutrinos will have spread out over the now-larger horizon. The smallest fluctuations that will not suffer this fate are those that come into the horizon when the neutrinos become nonrelativistic, i.e., when the temperature drops to the neutrino mass. In a universe in which most of the dark matter is hot, primordial fluctuations will damp on all scales up to superclusters (with mass $\sim 10^{16}\ M_\odot$), leading to a sequence of cosmogony (cosmological structure formation) in which galaxies form only after superclusters. But this is contrary to observations, which show galaxies to be old but with superclusters still forming. Indeed, with fluctuations on large scales consistent with COBE, pure HDM models (i.e., with the dark matter being mostly neutrinos, and a Zel'dovich spectrum of Gaussian adiabatic fluctuations) cannot have formed any significant number of galaxies by the present. Thus, most current comparisons of cosmological models with observations have focused on models in which most of the dark matter is cold.

The standard CDM model[2] assumed a Zel'dovich (i.e., $n_p = 1$) spectrum of primordial Gaussian adiabatic fluctuations with $\Omega = 1$. It had the great virtues of simplicity and predictive power, since it had only one free parameter, the amplitude or bias b. Moreover, for a while it even looked like it agreed with all available data, with $b \approx 2.5$. One early warning that all was not well for CDM was the cosmic background dipole anisotropy, indicating a large velocity of the local group with respect to the cosmic background radiation rest frame, about 600 km s^{-1}. I must say, many other theorists and I did not immediately appreciate its possibly devastating impact. However, as evidence began to accumulate, starting in 1986, that such velocities were common on large scales—indeed, that there were large-scale flows of galaxies with such velocities[9]—it became clear that standard CDM could fit these large-scale, galaxy-peculiar velocities (i.e., motions in addition to the general Hubble expansion) only for $b \approx 1$. Standard CDM had various problems for any value of b; for example, the CDM matter correlation function, and hence also the galaxy and cluster correlations, are negative on scales larger than about $30\,h^{-1}\,$Mpc, while observations on these large scales show that the cluster correlations are at least $\sim 3\sigma$ positive. A low value of the bias parameter subsequently also turned out to be required by the COBE DMR data, which was first announced in April 1992. But for such a small $b \lesssim 1$, CDM produces far too many clusters and predicts small-scale galaxy velocities that are much too large.[10] Thus, standard CDM does not look like a very good match to the now-abundant observational data. But it did not miss by much—if the bias parameter b is adjusted to fit the COBE data, the fluctuation amplitude is too large on small scales by perhaps a factor of approximately two to three.

In the wake of the discovery of the existence of large-scale, galaxy-peculiar velocities, I suggested that Jon Holtzman (then a UCSC graduate student whose planned Ph. D. research based on HST observations had been indefinitely postponed by the Challenger explosion) improve the program that George Blumenthal and I had written to do linear CDM calculations, and use it to investigate a variety of models in which the dark matter was mostly cold. He ultimately worked out a total of 94 such models, about half of them including some hot dark matter, and (since this was the largest such suite of interesting models all worked out the same way) his thesis[11] provided the basis for the COBE-DMR interpretation paper.[12] Meanwhile, in a follow-up paper,[13] we showed that of all these CDM-like models, the ones that best fit the available data—especially the cluster correlations—were $\Omega = 1$ CHDM, and low-Ω CDM with a ΛCDM. Since both of these models turned out to fit all available data rather well when their fluctuation amplitudes were normalized to COBE observations, they remain perhaps the most popular models for galaxy formation and large-scale structure. Moreover, since CHDM works best for $h \approx 0.5$ while ΛCDM works best for higher h, they will serve nicely for this review as representatives of these two opposing alternatives.

3 Age of the Universe t_0

The strongest lower limits for t_0 come from studies of the stellar populations of globular clusters (GC's). Standard estimates of the ages of the oldest GC's are 14–18 Gy, and a conservative lower limit on the age of GC's is 13 ± 2 Gy, which is then a lower limit on t_0. The main uncertainty in the GC age estimates comes from the uncertain distance to the GC's: a 0.25 magnitude error in the distance modulus translates to a 22% error in the derived cluster age.[14] Stellar mass loss is the latest idea for lowering the GC t_0,[15] but observations constrain the reduction in t_0 to be less than ~ 1 Gy. Allowing $\sim$ 1–2 Gy for galaxy and GC formation, we conclude that $t_0 \gtrsim 11$ Gy from GC's, with $t_0 \approx 13$ Gy a "likely" lower limit on t_0, obtained by pushing most but not all the parameters to their limits.

The GC age estimates are, of course, based on standard stellar evolution calculations. New calculations using new stellar opacities now underway are not expected to change the estimates by more than a few percent. But the solar neutrino problem reminds us that we are not really sure that we understand how even our nearest star operates; and the sun plays an important role in calibrating stellar evolution, since it is the only star whose age we know independently (from radioactive dating of early solar system material). What if the GC age estimates are wrong for some unknown reason?

The only independent estimates of the age of the universe come from cosmochronometry—the chemical evolution of the galaxy—and white dwarf cooling. Cosmochronometry age estimates are sensitive to a number of uncertain effects such as the formation history of the disk and its stars, and possible actinide destruction in stars.[16] Age estimates also come from the cooling of white dwarfs in the neighborhood of the sun. The key observation is that there is a lower limit to the temperature of nearby white dwarfs; although cooler ones could have been seen, none have been found. The only plausible explanation is that the white dwarfs have not had sufficient time to cool to lower temperatures, which initially led to an estimate of 9.3 ± 2 Gy for the age of the Galactic disk.[17] Since there is evidence that the stellar disk of our Galaxy is about 2 Gy younger than the oldest GC's,[18] this in turn gave an estimate of the age of the universe of $t_0 \sim 11 \pm 2$ Gy. However, more recent analyses[19] conclude that sensitivity to disk-star formation history, and to effects on the white dwarf cooling rates due to C/O separation at crystallization and possible presence of trace elements such as ^{22}Ne, allow a rather wide range of ages for the disk of about 10 ± 4 Gy.

Figure 1: Age of the universe t_0 as a function of Hubble parameter H_0 in inflation-inspired models with $\Omega_0 + \Omega_\Lambda = 1$, for several values of the present-epoch cosmological density parameter Ω_0.

Suppose that the GC stellar age estimates that $t_0 \gtrsim 13$ Gy are right. Figure 1 shows that $t_0 > 13$ Gy implies that $H_0 \leq 50$ km s^{-1} Mpc^{-1} for $\Omega = 1$, and that $H_0 \leq 81$ km s^{-1} Mpc^{-1} even for Ω_0 as small as 0.2 (in flat cosmologies with $\Omega_0 + \Omega_\Lambda = 1$).

4 Hubble Parameter H_0

The Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1} remains uncertain by about a factor of two: $0.4 \lesssim h \lesssim 1$. Sandage has long contended that $h \approx 0.5$, and he still concludes[20] that the latest data are consistent with this. De Vaucouleurs has long contended that $h \approx 1$. A majority of observers currently favor a value intermediate between these two extremes (recent reviews include Refs. 21-23).

The Hubble parameter has been measured in two basic ways: (A) Measuring the distance to some nearby galaxies, typically by measuring the periods and luminosities of Cepheid variables in them, and then using these "calibrator galaxies" to set the zero point in any of the several methods of measuring the relative distances to galaxies. (B) Using fundamental physics to measure the distance to some distant object directly, thereby avoiding at least some of the uncertainties of the cosmic distance ladder.[24] The difficulty with method (A) is that there are so far only a handful of calibrator galaxies close enough for Cepheids to be resolved in them. However, the success of the HST Cepheid measurement of the distance to M100[1] shows that the HST Key Project on the Extragalactic Distance Scale can significantly increase the set of calibrator galaxies within a few years. Adaptive optics from the ground may also be able to contribute to this effort, although I am not very impressed by the first published result of this approach.[25] The difficulty with method (B) is that in every case studied so far, some aspect of the observed system or the underlying physics remains somewhat uncertain. It is nevertheless remarkable that the results of several different methods of type (B) are rather similar, and indeed not very far from those of method (A). This gives reason to hope for convergence.

4.1 (A) Relative-Distance Methods

One piece of good news is that the several methods of measuring the relative distances to galaxies now mostly seem to be consistent with each other.[22,23] These methods use either (1) "standard candles" or (2) empirical relations between two measurable properties of a galaxy, one distance independent and the other distance dependent. The old favorite standard candle is type Ia supernovae; a new one is the apparent maximum luminosity of planetary nebulae.[22] Sandage and others still get low values of $h \approx 0.4$–0.5 from HST Cepheid distances to SN Ia host galaxies.[26] There are claims that taking account of an empirical relationship between the SN Ia light curve shape and maximum luminosity leads to higher h (Ref. 27), but Sandage and Tammann counter that any such effect is small.[28] The old favorite empirical relation used as a relative-distance indicator is the Tully-Fisher relation between the rotation velocity and luminosity of spiral galaxies (and the related Faber-Jackson or D_n–σ relation); a new one is based on the decrease in the fluctuations in elliptical galaxy surface brightness on a given angular scale as galaxies are seen at greater distances.[29]

4.2 (B) Fundamental Physics Approaches

The fundamental physics approaches involve either type Ia or type II supernovae, the Sunyaev-Zel'dovich (S-Z) effect, or gravitational lensing. This type Ia SN method for determining H_0 avoids the uncertainties of the distance ladder by calculating the absolute luminosity of Type Ia supernovae from first principles using a plausible but as yet unproven physical model. The result obtained is that $h = 61 \pm 10$ (Ref. 30); however, another study[31] finds that uncertainties in extinction (i.e., light absorption) toward each supernova increases the range of allowed h. The type II SN method compares the expansion rate of the SN envelope measured by redshift with its size increase inferred from its temperature and magnitude; the 1992 result $h = 0.6 \pm 0.1$ (Ref. 32) has since been revised upward by the same authors to $h = 0.73 \pm 0.06 \pm 0.07$ (Ref. 33). However, there are various complications with the physics of the expanding envelope.[34]

The S-Z effect is the Compton scattering of microwave background photons from the hot electrons in a foreground galaxy cluster. This can be used to measure H_0 since properties of the cluster gas measured via the S-Z effect and from x-ray observations have different dependences on H_0. The result from the first cluster for which sufficiently detailed data was available, A665 (at $z = 0.182$), was $h = (0.4$–$0.5) \pm 0.12$ (Ref. 35); combining this with data on A2218 ($z = 0.171$) raises this somewhat to $h = 0.55 \pm 0.17$ (Ref. 36). Early results from the ASCA x-ray satellite gave $h = 0.47 \pm 0.17$ for A665 ($z = 0.182$) and $h = 0.41^{+0.15}_{-0.12}$ for CL0016+16 ($z = 0.545$) (Ref. 37). A few S-Z results have been obtained using millimeter-wave observations, and this promising method should allow many more such measurements soon.[38] Corrections for the near-relativistic electron motions will raise these estimates for H_0 a little,[39] but it seems clear that the S-Z results favor a smaller value than many optical astronomers obtain. However, since the S-Z measurement of H_0 is affected by the orientation of the cluster ellipticity with respect to the line of sight, this will only become convincing if it agrees with results from observations of a significant number of additional clusters. Fortunately, this now appears to be possible within the next several years.

Several quasars have been observed to have multiple images separated by a few arc seconds; this phenomenon is interpreted as arising from gravitational lensing of the source quasar by a galaxy along the line of sight. In the first such system discovered, QSO $0957+561$ ($z = 1.41$), the time delay, Δt, between arrival at the earth of variations in the quasar's luminosity in the two images has been measured to be 409 ± 23 days[40] (although other authors found a value of 540 ± 12 days[41]). Since $\Delta t \approx \theta^2 H_0^{-1}$, this observation allows an estimate of the Hubble parameter, with the results $h = 0.50 \pm 0.17$ (Ref. 42), or $h = 0.63 \pm 0.21$ ($h = 0.42 \pm 0.14$) including (neglecting) dark matter in the lensing galaxy,[43] with additional uncertainties associated with possible microlensing and unknown matter distribution in the lensing galaxy. However, recent deep images have allowed mapping of the gravitational potential of the lensing cluster (at $z = 0.36$) using weak gravitational lensing, which leads to the conclusion that $h \leq 0.70$ if $\Delta t \geq 1.1$ y (Ref. 44). Although the allowed range for H_0 remains rather large, it is reassuring that this method gives results consistent with the other determinations. The time-delay method is promising, and when delays are reliably measured in several other multiple-image quasar systems, that should lead to a reliable value for h.

4.3 Correcting for Virgocentric Infall

What about the recent HST Cepheid measurement of H_0, giving $h \approx 0.8$? (Ref. 1) This calculated value is based on neither of the two methods (A) or (B) above, and I do not regard it as being very reliable. Instead, this result is obtained by assuming that M100 is at the core of the Virgo cluster and dividing the sum of the recession velocity of Virgo, about 1100 km s^{-1}, plus the calculated "infall velocity" of the local group toward Virgo, about 300 km s^{-1}, by the measured distance to M100 of 17.1 Mpc. (These recession and infall velocities are both a little on the high side, compared to other values one finds in the literature.) Adding the "infall velocity" is necessary in this method in order to correct the Virgo recession velocity to what it would be if it were not for the gravitational attraction of Virgo for the Local Group of galaxies, but the problem with this is that the net motion of the Local Group with respect to Virgo is undoubtedly affected by much besides the Virgo cluster—e.g., the "Great Attractor." For example, in our CHDM supercomputer simulations (which appear to be a rather realistic match to observations) Anatoly Klypin and I have found that galaxies and groups at about 20 Mpc from a Virgo-sized cluster often have net outflowing rather than infalling velocities. Note that if there were no net "infall," or if M100 were in the foreground of the Virgo cluster (in which case the actual distance to Virgo would be larger than 17.1 Mpc), then the indicated H_0 would be smaller.

The authors of Ref. 1 gave an alternative argument that avoids the "infall velocity" uncertainty. The relative galaxy luminosities indicate that the Coma cluster is about six times farther away than the Virgo cluster, and peculiar motions of the Local Group and the Coma cluster are much smaller corrections to the much larger recession velocity of Coma; dividing the recession velocity of the Coma cluster by six times the distance to M100 again gives $H_0 \approx 80$. However, this approach still assumes that M100 is in the core rather than the foreground of the Virgo cluster; and in deducing the relative distance of the Coma and Virgo clusters, it assumes that the galaxy luminosity functions in each are comparable, which is dubious in view of the very different environments.

To summarize, many observers, using mainly method (A), favor a value $h \approx 0.6$–0.8 although Sandage and collaborators continue to get $h \approx 0.4$–0.6, while the methods I have grouped together as (B) typically lead to $h \approx 0.4$–0.7. The fact that the latter measurements are mostly of more distant objects has suggested[45] that the local universe may actually be underdense and therefore be expanding faster than is typical. But in reasonable models where structure forms from Gaussian fluctuations via gravitational instability, it is extremely unlikely that a sufficiently large region has a density sufficiently smaller than average to make more than a rather small difference in the measured value of h (Ref. 46).

There has been recent observational progress in both methods (A) and (B), and I think it likely that the Hubble parameter will be known reliably to 10% within a few years. But until then, we must keep an open mind.

5 Cosmological Constant Λ and t_0 Again

Inflation is the only known solution to the horizon and flatness problems, and the avoidance of too many GUT monopoles. And inflation has the added bonus that with no extra charge (except the perhaps implausibly fine-tuned adjustment of the self-coupling of the inflation field to be adequately small), simple inflationary models predict a near-Zel'dovich spectrum (i.e., with $n_p \approx 1$) of adiabatic Gaussian primordial fluctuations—which seems to be consistent with observations. All simple inflationary models predict that the curvature constant k is vanishingly small, although inflationary models that are extremely contrived (at least, to my mind) can be constructed with negative curvature and therefore $\Omega_0 \lesssim 1$ without a cosmological constant.[47] Thus, most authors who consider inflationary models impose the condition $k = 0$, or $\Omega_0 + \Omega_\Lambda = 1$, where $\Omega_\Lambda \equiv \Lambda/(3\ H_0^2)$. This is what is assumed in ΛCDM models, and it is what was assumed in Fig. 1. (I hope it has been clear from the foregoing that I use Ω to refer only to the density of matter and energy, not including the cosmological constant, whose contribution in the Ω units is Ω_Λ.)

I know of no one who actually finds the idea of a nonvanishing Λ intrinsically attractive. There is no known physical reason why Λ should be so small (from the viewpoint of particle physics), though there is also no known reason why it should vanish. The most unattractive features of $\Lambda \neq 0$ cosmologies are the fact that Λ must become important only at a relatively low redshift—why not much earlier or much later?—and also that $\Omega_\Lambda \gtrsim \Omega_0$ implies that the universe has recently entered an inflationary epoch (with a de Sitter horizon comparable to the present horizon). The main motivations for $\Lambda > 0$ cosmologies are (1) reconciling inflation with observations that seem to imply $\Omega \lesssim 1$, and (2) avoiding a contradiction between the lower limit $t_0 \gtrsim 13$ Gy from globular clusters and $t_0 = (2/3)H_0^{-1} = 6.52h^{-1}$ Gy for the standard $\Omega = 1$, $\Lambda = 0$ Einstein-de Sitter cosmology, if it is really true that $h > 0.5$.

The cosmological effects of a cosmological constant are not difficult to understand.[48,49] With a positive Λ, there is a repulsion of space by space. In the early universe, the density of energy and matter is far more important than Λ on the r.h.s. of the Friedmann equation. But the average matter density decreases as the universe expands, and at a rather low redshift $z \sim 1$, the Λ term finally becomes significant. If it has been adjusted just right, Λ can almost balance the attraction of the matter, and the expansion nearly stops. For a long time, the scale factor $a \equiv (1+z)^{-1}$ increases very slowly, although it ultimately starts increasing exponentially as the universe starts inflating under the influence of the increasingly dominant Λ term. The existence of a period during which expansion slows while the clock runs explains why t_0 can be greater than $\Lambda = 0$, but this also shows that there is an increased likelihood of finding galaxies at the redshift interval when the expansion slowed and correspondingly increased opportunity for lensing of quasars at higher redshift $z \gtrsim 2$ by these galaxies.

The frequency of such lensed quasars is about what would be expected in a standard $\Omega = 1$, $\Lambda = 0$ cosmology, so this data sets fairly stringent upper limits: $\Omega_\Lambda \leq 0.70$ at 90% C.L.,[50,51] with more recent data likely to give even tighter constraints.[52]

A weaker but independent constraint comes from the cosmic background radiation data. In standard $\Omega = 1$ models, the quantity $\ell(\ell+1)C_\ell$ (where $C_\ell =< a^2_{\ell,m} >_m$ is the average of squared coefficients of the spherical harmonic expansion of the CMB data) is predicted to be roughly constant for $2 \leq \ell \lesssim 10$ (with an increase for higher multiples toward the Doppler peak at $\ell \sim 200$), while in models with $\Lambda > 0$ $\ell(\ell+1), C_\ell$ is predicted to dip before rising toward the Doppler peak. Comparison with the two-year COBE data, in which such a dip is not seen, implies that $\Omega_\Lambda \leq 0.78$ at the 90% C.L.[53]

Figure 1 shows that with $\Omega_\Lambda \leq 0.7$, the cosmological constant does not lead to a very large increase in t_0 compared to the Einstein-de Sitter case, although it may still be enough to be significant. For example, the constraint that $t_0 \geq 13$ Gy requires $h \leq 0.5$ for $\Omega = 1$ and $\Lambda = 0$, but this becomes $h \leq 0.73$ for $\Omega_\Lambda \leq 0.7$.

6 Measuring Ω_0

Although it would be desirable to measure Ω_0 and Λ through their effects on the large-scale geometry of space-time, this has proved difficult in practice since it requires comparing objects at higher and lower redshift, and it is hard to separate the effects of the evolution of the objects from those of the evolution of the universe. For example, in "redshift-volume" tests involving number counts of galaxies per redshift interval, how can we tell whether the galaxies at redshift $z \sim 1$ correspond to those at $z \sim 0$? Several galaxies at higher redshift might have merged, and galaxies might have formed or changed luminosity at lower redshift. Eventually, with extensive surveys of galaxy properties as a function of redshift using the largest telescopes such as Keck, it should be possible to perform these classical cosmological tests at least on a particular class of galaxies—that is one of the goals of the Keck DEEP project. At present, perhaps the most promising technique involves searching for Type Ia supernovae at high redshift, since these are the brightest supernovae and the spread in their intrinsic brightness appears to be relatively small. Gerson Goldhaber, Saul Perlmutter, and collaborators have recently demonstrated the feasibility of finding significant numbers of such supernovae,[54] but a dedicated campaign of follow-up observations of each one will be required in order to measure Ω_0 by determining how the apparent brightness of the supernovae depends on their redshift. This is therefore a project that will take at least several years.

6.1 Large-Scale Measurements

The largest scales on which Ω_0 has been measured with some precision today are about $\sim 50\,h^{-1}$ Mpc, using the data on peculiar velocities of galaxies, and on a somewhat larger scale using redshift surveys based on the IRAS galaxy catalog. Since the results of all such measurements to date have recently been summarized in an excellent review article,[55] I will only comment briefly on them. The analyses such as "POTENT" that try to recover the scalar velocity potential from the galaxy-peculiar velocities are looking increasingly reliable, since they reproduce the observed large-scale distribution of galaxies—that is, many galaxies are found where the converging velocities indicate that there is a lot of matter, and there are voids in the galaxy distribution where the diverging velocities indicate that the density is lower than average. The comparison of the IRAS redshift surveys with POTENT and related analyses typically give fairly large values for the parameter $\beta_I \equiv \Omega_0^{0.6}/b_I$ (where b_I is the biasing parameter for IRAS galaxies), corresponding to $0.3 \lesssim \Omega_0 \lesssim 3$ (for an assumed $b_I = 1.15$). It is not clear whether it will be possible to reduce the spread in these values significantly in the near future—probably both additional data and a better understanding of systematic and statistical effects will be required.

A particularly simple way to deduce a lower limit on Ω_0 from the POTENT-peculiar velocity data has recently been proposed,[56] based on the fact that high-velocity outflows from voids are not expected in low-Ω models. Data on just one void indicates that $\Omega_0 \geq 0.3$ at the 97% C.L. This argument is independent of assumptions about Λ, the initial fluctuations, or galaxy formation, but, of course, it does depend on the success of POTENT in recovering the peculiar velocities of galaxies.

However, for the particular cosmological models that I am focusing on in this review—CHDM and ΛCDM—stronger constraints are available. This is because these models, in common with almost all CDM variants, assume that the probability distribution function (PDF) of the primordial fluctuations was Gaussian. The PDF deduced by POTENT from observed velocities (i.e., the PDF of the mass, not that of the galaxies) is far from Gaussian today. It agrees with a Gaussian initial PDF if and only if Ω is about unity or larger: $\Omega_0 < 1$ is rejected at the 2σ level, and $\Omega_0 \leq 0.3$ is ruled out at $\geq 4\sigma$ (Ref. 57). Evolution from a Gaussian initial PDF to the non-Gaussian mass distribution observed today requires considerable gravitational nonlinearity, i.e., large Ω.

6.2 Measurements on Scales of a Few Mpc

On smaller length scales, there are many measurements that are consistent with a smaller value of Ω_0 (Ref. 58). For example, the cosmic virial theorem gives $\Omega(\sim\ 1\ h^{-1}\,\mathrm{Mpc}) \approx 0.15\ [\sigma(1\ h^{-1}\,\mathrm{Mpc})/(300\,\mathrm{km\ s^{-1}})]^2$, where $\sigma(1\ h^{-1}\,\mathrm{Mpc})$ here represents the relative velocity dispersion of galaxy pairs at a separation of 1 h^{-1} Mpc. Although the classic paper[59] which first measured $\sigma(1\ h^{-1}\,\mathrm{Mpc})$ using a large redshift survey (CfA1) got a value of 340 km s^{-1}, this result is now known to be in error since the entire core of the Virgo cluster was inadvertently omitted;[60] if Virgo is included, the result is $\sim$ 500–600 km s^{-1} (Refs. 60 and 61), corresponding to $\Omega(\sim 1\ h^{-1}\,\mathrm{Mpc}) \approx 0.4$–$0.6$. Various redshift surveys give a wide range of values for $\sigma(1\ h^{-1}\,\mathrm{Mpc}) \sim 300$–$750$ km s^{-1}, with the most salient feature being the presence or absence of rich clusters of galaxies; for example, the IRAS galaxies, which are not found in clusters, have $\sigma(1\ h^{-1}\,\mathrm{Mpc}) \approx 320$ km s^{-1} (Ref. 62), while the northern CfA2 sample, with several rich clusters, has much larger σ than the SSRS2 sample, with only a few relatively poor clusters. It is evident that the $\sigma(1\ h^{-1}\,\mathrm{Mpc})$ statistic is not a very robust one.

A standard method for estimating Ω on scales of a few Mpc is based on applying virial estimates to groups and clusters of galaxies to try to deduce the total mass of the galaxies including their dark matter halos from the velocities and radii of the groups; roughly, $GM \sim rv^2$. (What one actually does is to assume that all galaxies have the same mass-to-light ratio M/L, given by the median M/L of the groups, and integrate over the luminosity function to get the mass density.)[63,64,65] The typical result is that $\Omega(\sim 1\,h^{-1}\,\mathrm{Mpc}) \sim 0.1$–$0.2$. However, such estimates are at best lower limits, since they can only include the mass within the region where the galaxies in each group can act as test particles. In CHDM simulations, my colleagues and I[66] have found that the effective radius of the dark matter distribution associated with galaxy groups is typically two to three times larger than that of the galaxy distribution. Moreover, we find a velocity biasing[67] factor in CHDM groups $b_v^{grp} \equiv v_{\mathrm{gal,rms}}/v_{\mathrm{DM,rms}} \approx 0.75$, whose inverse squared enters in the Ω estimate. Finally, we find that groups and clusters are typically elongated, so only part of the mass is included in spherical estimators. These factors explain how it can be that our $\Omega = 1$ CHDM simulations produce group velocities that are fully consistent with those of observed groups, even with sophisticated robust and discriminatory statistical tests such as the median rms group velocity vs. the fraction of galaxies grouped.[66,68] This emphasizes the point that local estimates of Ω are at best lower limits on its true value.

Another approach to estimating Ω from information on relatively small scales has been pioneered by Peebles.[69] It is based on using the least action principle (LAP) to reconstruct the trajectories of the Local Group galaxies, and the assumption that the mass is concentrated around the galaxies. This is a reasonable assumption in a low-Ω universe, but it is not at all what must occur in an $\Omega = 1$ universe where most of the mass must lie between the galaxies. Although comparison with $\Omega = 1$ N-body simulations showed that the LAP often succeeds in qualitatively reconstructing the trajectories, the mass is systematically underestimated by a large factor by the LAP method.[70] Unexpectedly, a different study[71] found that the LAP method underestimates Ω by a factor of four to five even in an $\Omega_0 = 0.2$ simulation; the authors say that this discrepancy is due to the LAP neglecting the effect of "orphans"—dark matter particles that are not members of any halo.

6.3 Estimates on Galaxy Halo Scales

Recent work by Zaritsky and White[72] and collaborators has shown that spiral galaxies have massive halos. A classic paper by Little and Tremaine[73] argued that the available data on the Milky Way satellite galaxies required that the Galaxy's halo terminate at about 50 kpc, with a total mass of only about $2.5 \times 10^{11}\ M_\odot$. But by 1991, new data on local satellite galaxies, especially Leo I, became available, and the Little-Tremaine estimator increased to $1.25 \times 10^{12}\ M_\odot$. Zaritsky and collaborators have collected data on satellites of other spiral galaxies, and conclude that the fact that the relative velocities do not fall off out to a separation of at least 200 kpc shows that massive halos are the norm. The typical rotation velocity of $\sim$ 200–250 km s^{-1} implies a mass within 200 kpc of $2 \times 10^{12}\ M_\odot$. A careful analysis taking into account selection effects and satellite orbit uncertainties concluded that the indicated value of Ω_0 exceeds 0.13 at 90% confidence, with preferred values exceeding 0.3 (Ref. 72).

7 Clusters

7.1 Cluster Baryons vs. Big Bang Nucleosynthesis

A recent review[8] of Big Bang Nucleosynthesis (BBN) and observations indicating primordial abundances of the light isotopes concludes that $0.009\ h^{-2} \leq \Omega_b \leq 0.02\ h^{-2}$ for concordance with all the abundances, and $0.006\ h^{-2} \leq \Omega_b \leq 0.03\ h^{-2}$ if only deuterium is used. For $h = 0.5$, the corresponding upper limits on Ω_b are 0.08 and 0.12, respectively. The recent observations[74] of a possible deuterium line in a hydrogen cloud at redshift $z = 3.32$ indicating a deuterium abundance of $\sim 2 \times 10^{-4}$ (and therefore $\Omega_b \leq 0.006\ h^{-2}$) are contradicted by a similar observation[75] in a system at $z = 3.58$ but with a deuterium abundance about ten times lower, consistent with solar system measurements of D and ^{3}He and the higher upper limit on Ω_b. (The earlier observations[74] were most probably of a Lyα forest line.)

White *et al.*[76] have emphasized that recent x-ray observations of clusters, especially Coma, show that the abundance of baryons, mostly in the form of gas (which typically amounts to several times the total mass of the cluster galaxies), is as much as 20% if h is as low as 0.5. For the Coma cluster, they find that the baryon fraction within the Abell radius is

$$f_b \equiv \frac{M_b}{M_{tot}} \geq 0.009 + 0.050\ h^{-3/2},$$

where the first term comes from the galaxies and the second from gas. If clusters are a fair sample of both baryons and dark matter, as they are expected to be based on simulations, then this is two to three times the amount of baryonic mass expected on the basis of BBN in an $\Omega = 1$, $h \approx 0.5$ universe, though it is just what one would expect in a universe with $\Omega_0 \approx 0.3$. The fair sample hypothesis implies that

$$\Omega_0 = \frac{\Omega_b}{f_b} = 0.3 \left(\frac{\Omega_b}{0.06}\right)\left(\frac{0.2}{f_b}\right).$$

A recent review of gas in a sample of clusters[77] finds that the baryon mass fraction within about 1 Mpc lies between 10% and 22%, and argues that it is unlikely that (a) the gas could be clumped enough to lead to significant overestimates of the total gas mass—the main escape route considered in Ref. 76. If $\Omega = 1$, the alternatives are then either (b) that clusters have more mass than virial estimates based on the cluster galaxy velocities or estimates based on hydrostatic equilibrium of the gas at the measured x-ray temperature (which is surprising since they agree[78]), or (c) that the BBN upper limit on Ω_b is wrong. It is interesting that there are indications from weak lensing[79] and galaxy velocities[80] that at least some clusters may actually have extended halos of dark matter—something that is expected to a greater extent if the dark matter is a mixture of cold and hot components, since the hot component clusters less than the cold.[81,118] If so, the number density of clusters as a function of mass is higher than usually estimated, which has interesting cosmological implications (e.g., σ_8 is higher than usually estimated). It is, of course, possible that the solution is some combination of alternatives (a), (b), and (c). If none of the alternatives is right, then the only conclusion left is that $\Omega_0 \approx 0.3$. The cluster baryon problem is clearly an issue that deserves very careful examination.

7.2 Cluster Morphology

Richstone, Loeb, and Turner[82] showed that clusters are expected to be evolved—i.e., rather spherical and featureless—in low-Ω cosmologies, in which structures form at relatively high redshift, and that clusters should be more irregular in $\Omega = 1$

cosmologies, where they have formed relatively recently and are still undergoing significant merger activity. There are very few known clusters that seem to be highly evolved and relaxed, and many which are irregular—some of which are obviously undergoing mergers now or have recently done so (see e.g., Ref. 83). This disfavors low-Ω models, but it remains to be seen just how low. Recent papers have addressed this. In one,[84] a total of 24 CDM simulations with $\Omega = 1$ or 0.2, the latter with $\Omega_\Lambda = 0$ or 0.8, were compared with data on a sample of 57 clusters. The conclusion was that clusters with the observed range of x-ray morphologies are very unlikely in the low-Ω cosmologies. However, these simulations have been criticized because the $\Omega_0 = 0.2$ ones included a rather large amount of ordinary matter: $\Omega_b = 0.1$. [This is unrealistic both because $h \approx 0.8$ provides the best fit for $\Omega_0 = 0.2$, but then the standard BBN upper limit is $\Omega_b < 0.02\ h^{-2} = 0.03$, and also because observed clusters have a gas fraction of $\sim 0.15\ (h/0.5)^{-3/2}$.] Another study[85] using dissipationless simulations and not comparing directly to observational data found that ΛCDM with $\Omega_0 = 0.3$ and $h = 0.75$ produced clusters with some substructure, perhaps enough to be observationally acceptable. Clearly, this important issue deserves study with higher resolution hydrodynamic simulations, with a range of assumed Ω_b, and possibly including at least some of the additional physics associated with the galaxies which must produce the metallicity observed in clusters, and perhaps some of the heat as well. Better statistics for comparing simulations to data may also be useful.[86]

7.3 Cluster Evolution

There is evidence for strong evolution of clusters at relatively low redshift, both in their x-ray properties[87] and in the properties of their galaxies. In particular, there is a strong increase in the fraction of blue galaxies with increasing redshift (the "Butcher-Oemler effect"), which may be difficult to explain in a low-density universe.[88] Field galaxies do not appear to show such strong evolution; indeed, a recent study concludes that over the redshift range $0.2 \leq z \leq 1.0$, there is no significant evolution in the number density of "normal" galaxies.[89] This is compatible with the predictions of CHDM with two neutrinos sharing a total mass of about 5 eV [Ref. 90] (see below).

8 Early Structure Formation

In linear theory, adiabatic density fluctuations grow linearly with the scale factor in an $\Omega = 1$ universe, but more slowly if $\Omega < 1$ with or without a cosmological constant.[58] As a result, if fluctuations of a certain size in an $\Omega = 1$ and an $\Omega_0 = 0.3$ theory are equal in amplitude at the present epoch ($z = 0$), then at higher redshift, the fluctuations in the low-Ω model had higher amplitude. Thus, structures typically form earlier in low-Ω models than in $\Omega = 1$ models.

Since quasars are seen at the highest redshifts, they have been used to try to constrain $\Omega = 1$ theories, especially CHDM, which because of the hot component has additional suppression of small-scale fluctuations that are presumably required to make early structure (e.g., Ref. 91). The difficulty is that dissipationless simulations predict the number density of halos of a given mass as a function of redshift, but not enough is known about the nature of quasars—for example, the mass of the host galaxy—to allow a simple prediction of the number of quasars as a function of redshift in any given cosmological model. A recent study[92] concludes that very efficient cooling of the gas in early structures, and angular momentum transfer from it to the dark halo, allows for formation of *at least* the observed number of quasars even in models where most galaxy formation occurs late.

Another sort of high redshift object which holds more promise for constraining theories is damped Lyman α systems (DLAS). DLAS are dense clouds of neutral hydrogen, generally thought to be protogalactic disks, which are observed as wide absorption features in quasar spectra.[93] They are relatively common, seen in roughly a third of all quasar spectra, so statistical inferences about DLAS are possible. At the highest redshift for which data is published, $z = 3$–3.4, the density of neutral gas in such systems in units of critical density is $\Omega_{gas} \approx 0.6\%$, comparable to the total density of visible matter in the universe today.[94] Several recent papers[95] pointed out that the CHDM model with $\Omega_\nu = 0.3$ could not produce such a high Ω_{gas}. However, my colleagues and I showed that CHDM with $\Omega_\nu = 0.2$ could do so,[96] since the power spectrum on small scales is a very sensitive function of the total neutrino mass in CHDM models. This theory makes two crucial predictions:[96] Ω_{gas} must fall off at higher redshifts z, and the DLAS at $z \gtrsim 3$ correspond to systems of internal rotation velocity or velocity dispersion less than about 100 km s^{-1} (this can be measured from the Doppler widths of the metal line systems associated with the DLAS). Preliminary reports regarding the latest data at redshifts above 3.5 appear to be consistent with these predictions.[97]

One of the best ways of probing early structure formation would be to look at the main light output of the stars of the earliest galaxies, which is redshifted by the expansion of the universe to wavelengths beyond about five microns today. Unfortunately, it is not possible to make such observations with existing telescopes; since the atmosphere blocks almost all such infrared radiation, what is required is a large infrared telescope in space. The Space Infrared Telescope Facility (SIRTF) has long been a high priority, and it would be great to have access to the data such an instrument would produce. But even if NASA started such a mission immediately, it would not be available until the next millennium. In the meantime, an alternative method is to look for the starlight from the earliest stars as extragalactic background infrared light (EBL). Although it is difficult to see this background light directly because our galaxy is so bright in the near infrared, it may be possible to detect it indirectly through its absorption of TeV gamma rays (via the process $\gamma\gamma \to e^+ e^-$). Of the more than 20 AGN's that have been seen at ~ 10 GeV by the EGRET detector on the Compton Gamma Ray Observatory, only one, the nearest, Mk421, has also been clearly detected in TeV gamma rays by the Whipple Atmospheric Cherenkov Telescope. Absorption of TeV gamma rays from active galactic nuclei (AGN's) at redshifts $z \sim 0.2$ has been shown to be a sensitive probe of the era of galaxy formation.[98]

9 Neutrino Mass

There are several experiments which suggest that neutrinos have mass. In particular, the recent announcement of the observation of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations at the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos suggests that $\delta m^2 \equiv |m(\nu_\mu)^2 - m(\nu_e)^2| \approx 6$ eV^2 (see Ref. 99), and the observation of the angular dependence of the atmospheric muon neutrino deficit at Kamiokande[100] suggests $\nu_\mu \to \nu_\tau$ oscillations are occurring with an oscillation length comparable to the depth of the atmosphere, which requires that the muon and tau neutrinos have approximately the same mass. If, for example, $m(\nu_e) \ll m(\nu_\mu)$, then this means that $m(\nu_\mu) \approx m(\nu_\tau) \approx 2.4$ eV (Ref. 101). Clearly, discovery of neutrino mass in the few eV range favors CHDM; and, as I mentioned above, this total neutrino mass of about 5 eV is just what seems to be necessary to fit the large scale structure observations.[96] Dividing the mass between two neutrinos results in a somewhat lower fluctuation amplitude on the scale of clusters of galaxies because of the longer neutrino free-streaming length, which improves agreement between CHDM normalized to COBE and observations of cluster abundance.[101]

Of course, one cannot prove a theory since contrary evidence may always turn up. But one can certainly disprove theories. The minimum neutrino mass required by the preliminary LSND result[99] $\delta m^2 = 6$ eV^2 is 2.4 eV. This is too much hot dark matter to permit significant structure formation in a low-Ω universe; for example, in a ΛCDM model with $\Omega_0 = 0.3$, the cluster number density is more than two orders of magnitude lower than observations indicate.[101] Thus if this preliminary LSND result is correct, it implies a strong lower limit on Ω_0, and a corresponding upper bound on Λ, in ΛCDM models.

10 Conclusions on Ω

The main issue that I have tried to address is the value of the cosmological density parameter Ω. Strong arguments can be made for $\Omega_0 \approx 0.3$ (and models such as ΛCDM) or for $\Omega = 1$ (for which the best class of models that I know about is CHDM), but it is too early to tell for sure which is right.

The evidence would favor a small $\Omega_0 \approx 0.3$ if (1) the Hubble parameter actually has the high value $H_0 \approx 80$ favored by many observers, and the age of the universe $t_0 \geq 13$ Gy; or (2) the baryonic fraction $f_b = M_b/M_{tot}$ in clusters is actually $\sim 20\%$, about three to four times larger than expected for standard Big Bang Nucleosynthesis in an $\Omega = 1$ universe. This assumes that standard BBN is actually right in predicting that the density of ordinary matter Ω_b lies in the range $0.009 \leq \Omega_b\, h^2 \leq 0.02$; if the systematic errors in the ^{4}He data are larger than currently estimated, using the deuterium upper limit $\Omega_b h^2 \leq 0.03$ lessens the discrepancy between f_b and Ω_b somewhat. High-resolution, high-redshift spectra are now providing important new data on primordial abundances of the light isotopes that should clarify the reliability of the BBN limits on Ω_b. Another important constraint on Ω_b will come from the new data on small angle CMB anisotropies—in particular, the height of the Doppler peaks.[102]

The evidence would favor $\Omega = 1$ if (1) the POTENT analysis of galaxy-peculiar velocity data is right, in particular, regarding outflows from voids or the inability to obtain the present-epoch non-Gaussian density distribution from Gaussian initial fluctuations in a low-Ω universe; or (2) the preliminary report from LSND indicating a neutrino mass ≥ 2.4 eV is right, since that would be too much hot dark matter to allow significant structure formation in a low-Ω ΛCDM model.

The statistics of gravitational lensing of quasars is incompatible with large cosmological constant Λ and low cosmological density Ω_0. Discrimination between models will improve fairly rapidly as additional examples of lensed quasars are sought.

The era of structure formation is another important discriminant between these alternatives, low Ω favoring earlier structure formation, and $\Omega = 1$ favoring later formation with many clusters and larger-scale structures still forming today. A particularly critical test for models like CHDM is the evolution as a function of redshift of Ω_{gas} in damped Lyα systems.

Reliable data on all of these issues is becoming available so rapidly today that there is reason to hope that a clear decision between these alternatives will be possible within the next few years.

What if the data ends up supporting what appear to be contradictory possibilities, e.g., large Ω_0 *and* large H_0? Exotic initial conditions (e.g., "designer" primordial fluctuation spectra) or exotic dark matter particles beyond the simple "cold" vs. "hot" alternatives (e.g., decaying intermediate mass neutrinos) could increase the space of possible inflationary theories somewhat. But it may ultimately be necessary to go outside the framework of inflationary cosmological models and consider models with large scale spatial curvature, with a fairly large Λ as well as large Ω_0. This seems particularly unattractive, since in addition to implying that the universe is now entering a final inflationary period, it means that inflation did not happen at the beginning of the universe, when it would solve the flatness, horizon, monopole, and structure generation problems. Therefore, along with most cosmologists, I am rooting for the success of inflation-inspired cosmologies, with $\Omega_0 + \Omega_\Lambda = 1$. But the universe is under no obligation to live up to our expectations.

11 Part II. Improved Ways to Compare Simulations to Data

Theoretical models for structure formation with Gaussian initial fluctuations have been worked out in considerable detail and compared with observations on various scales. It is on nonlinear scales $\lesssim 10\ h^{-1}$ Mpc that the greatest differences exist between $\Omega = 1$ models that have been normalized to agree on the largest scales with the COBE data; here especially, there is a need for better statistical tests which are simultaneously *robust*, *discriminatory*, and *interpretable*. The era at which galaxy and cluster formation occurs is also a critical test of some models. Needs for the future include faster and more clever codes, better control of cosmic variance in simulations, better understanding of processes leading to galaxy formation, better ways of comparing observational data with models, and better access to observational and simulation data.

Although many cosmological models have been considered by various authors, I propose to concentrate here on a particular class of such models, namely those inspired by the hypotheses of inflation (hence with $\Omega = 1$, or at least curvature $k = 0$, and a near-Zel'dovich primordial spectrum of adiabatic fluctuations) and (all or mostly) cold dark matter. I do this not only because I believe that such models still have the best prospects of ultimately being found to be consistent with the data. My main motivation for concentrating on these models is that they are *well-specified*, in the sense that they are described by a small number of adjustable parameters (unlike general non-Gaussian models, for example), and very *predictive*, in the sense that many of their consequences can be worked out fairly easily with relatively few uncontrolled approximations. Thus, they can be confronted rather directly with the observational data, and eventually most (or all) such theories can actually be ruled out—as standard CDM already has been (or is close to being) ruled out. If a small class of such models survive, they may actually have something to do with the real universe. Even if not, this research should help to develop better statistical methods for comparison of cosmological theories with observational data.

The great advantage of keeping a tentative theory in mind as one thinks about data is that it helps in organizing the facts. If it is a good theory, it will also call attention to particularly important facts—especially those that may contradict it! CDM stimulated the creation of better models of the origin and evolution of galaxies and large scale structure, it helped motivate the acquisition and analysis of crucial data, and it has been a valuable test bed for data analysis tools—allowing, for example, development and testing of the POTENT algorithm[103] for reconstructing the total density field from measured peculiar velocities. Comparison of the original standard $\Omega = 1$ CDM model and its variants (cf. e.g., Ref. 13) with the observational data has certainly been useful during the past decade.

12 Testing Models

It is useful to divide the discussion of how to confront models with the observational data according to the scale of the observations: (a) greater than 100 Mpc, (b) 10–100 Mpc, (c) less than 10 Mpc, and (d) early structure formation. If we restrict attention to CDM-like $\Omega = 1$ models, the data on the largest scales (a) is probably useful mainly for establishing the normalization of the fluctuations, measuring the contribution of gravity waves, and testing inflation. For low-Ω CDM models with a cosmological constant Λ such that the curvature vanishes (i.e., with $\Omega_\Lambda \equiv \Lambda/(3\ H_0^2) = 1 - \Omega_0$), data on the power spectrum $P(k)$ of galaxies and especially of the mass will be a crucial test, since if normalized to COBE[104] these models predict much higher $P(k)$ than $\Omega = 1$ CDM; however, the available data on the largest scales (mainly from the APM angular correlations and from the CfA2 + SSRS2 data[105]) is not yet a powerful constraint on theories. Comparing with the data on smaller scales tests the gravitational clustering hypothesis, and (assuming gravitational clustering is valid) the spectral shape and other features of the cosmological model, including the cosmological parameters H_0, Λ, and Ω, and the nature of the dark matter—e.g., whether it is a mixture of cold and hot dark matter. With a given normalization of the spectrum, the smaller scale data also tests the shape of the primordial spectrum (e.g., whether there is "tilt"). But there are many problems with actually carrying out this program even when large-scale redshift surveys become available, perhaps the worst of which is the uncertainties about galaxy formation which make identification of "galaxies" in the simulations uncertain. Fortunately, there are several ingenious new techniques that promise to improve this situation. For example, weak gravitational lensing[106] or extension of peculiar velocity surveys to larger scales (which requires new ways of measuring distance independent of redshift, or of measuring the peculiar velocity directly, e.g., by the Sunyaev-Zel'dovich velocity term) may allow direct determination of the distribution of mass on large or even very large scales.

12.1 Very Large Scales

The shape of the power spectrum deduced from the two-year COBE data and other large-angle CBR measurements is consistent with a power-law primordial power spectrum $P(k) = Ak^{n_p}$ with $n_p \approx 1.1 \pm 0.3$, and with the rms amplitude quoted as the quadrupole for an $n_p = 1$ spectrum given by $Q_{\rm rms-ps} \approx 20\,\mu$K (Ref. 104). If a CDM spectrum is normalized to this amplitude for $h = 0.5$, it appears to be consistent with all the available data on large scale structure (LSS) down to scales of approximately 100 h^{-1} Mpc (e.g., Refs. 103, 107, and 108). But the amplitude of the LSS is presently known at best to about $\pm$ 25%—that is roughly the uncertainty on the large-scale bulk velocity (e.g., Ref. 103), which at least measures the mass-power spectrum. The amplitude of the galaxy or cluster power spectrum has the further uncertainty that the ratio of the rms fluctuation amplitudes of these objects to that of the mass, i.e., the "bias" $b_{\rm gal} = (\frac{\delta\rho}{\rho})_{\rm gal}/(\frac{\delta\rho}{\rho})_{\rm mass}$, is known rather poorly. Indeed, the extent to which the bias of a given category of astronomical objects can be regarded as a constant even on a given scale is not very well-tested.

Within these fairly large uncertainties, the consistency between the CMB data and the LSS data supports the validity of the gravitational clustering hypothesis. In order to really test both this and the hypothesis of cosmic inflation, it will be necessary to do better. Perhaps the most important issue is the contribution of gravity waves. These tensor modes can contribute to the low spherical harmonics $\ell \lesssim 20$ of the CMB temperature fluctuations, but hardly at all to the higher ℓ ones; and they are, of course, irrelevant to the formation of structure, to which only the scalar mode density fluctuations contribute. In principle, the tensor contribution can be determined by comparing the large and small angle CMB anisotropies, but in practice, this will require more accurate CMB measurements on small scales than are presently available and also knowledge of Ω_b and the extent of ionization of the universe after the recombination era, both of which have a strong influence on the amplitude of the CMB fluctuations near the first Doppler peak, $\ell \sim 200$. For the time being, it is perhaps best to regard the COBE normalization $Q_{\rm rms-ps} \approx 20$ as an *upper* limit to the density fluctuation amplitude, since the tensor and scalar modes add in quadrature. One of the most urgent issues for CMB studies is to determine a *lower* limit on the density fluctuation amplitude by constraining the tensor mode amplitude. This will allow improved tests of the gravitational clustering hypothesis and measurement of the "tilt" of the primordial fluctuation amplitude.

All the nearby surveys, such as the CfA2 survey with an effective depth of about 15,000 km/s, have found structures as large as the surveys themselves. This left open the possibility that still larger structures would be found by even larger surveys, which would contradict the gravitational clustering hypothesis (e.g., Ref. 109). Although the very large scale periodicity of peaks in the galaxy distribution with a length scale of $\sim 135\,h^{-1}$ Mpc seen in the BEKS[110] pencil beam redshift survey was unexpected in any cosmological model,[111] it is significant that no indications of still larger scales were seen in this data (preliminary reports indicate that pencil beams in different directions also have peaks with such separations but not such strong periodicity). Large-scale redshift surveys are now in progress, notably the KOSS southern sky redshift survey and the ESO Key Project. Preliminary reports suggest that no larger structures have in fact been found (Kirshner characterizes this as "the end of greatness"), again supporting the gravitational clustering hypothesis. The much larger scale surveys just beginning—the Two Degree Survey at the AAT and the Sloan Digital Sky Survey at the Apache Point Observatory—will be able to measure the sizes of these large structures and characterize their correlations, shapes, and other statistical properties. These will provide essential constraints on models of cosmic structure formation. These statistical properties appear on the basis of the data available at present to be consistent with the expectations from CDM-type models, but it remains to be seen whether this is true for topological defect models.

12.2 Large-Scale Structure ($\sim$ 10–100 Mpc)

On these scales, a great deal of galaxy redshift data[112] and peculiar-velocity data[103] is already available, although much of it remains unpublished. There are also several redshift surveys for optically selected clusters, and large-scale redshift surveys for x-ray selected clusters (which are likely to be less affected by projection effects and galactic obscuration) are now in progress. Comparison of the spectrum of fluctuations measured with this data and with the small-angle CMB data when it is available will eventually provide a test of the gravitational clustering hypothesis.

Comparison with specific theories must be done on the basis of nonlinear simulations since on these scales, linear theory is no longer reliable. All the available tests—power spectra of galaxies and mass (a preliminary POTENT result), galaxy and cluster correlations, skewness, and higher moments of the pdf—suggest that CDM, normalized to fit on scales of 100 Mpc and above, fits increasingly poorly on smaller scales; it has too much power. For example, CDM predicts far too many clusters,[10] and it predicts that mass autocorrelations become negative for separations beyond about 30 h^{-1} Mpc, while all measurements of cluster correlations show that they remain significantly positive for separations out to ~ 50 h^{-1} Mpc. These correlations are sensitive to the slope of the power spectrum and indicate a steeper decline with increasing k than CDM.

12.3 Intermediate Scales (Less Than 10 Mpc)

It is on scales less than about 10 Mpc that the greatest differences exist between $\Omega = 1$ CDM variants that have all been normalized to agree on the largest scales with the COBE data. It is on these scales that galaxy locations and velocities, as revealed by relatively dense redshift surveys (i.e., with fainter galaxies included), have the greatest potential to help discriminate between cosmological models, for example, those containing more or less of various mixtures of cold and hot dark matter, with or without a cosmological constant. (Someday, there may be enough galaxy-peculiar velocities based on accurate distance measurements independent of redshift to allow these to be used to discriminate between theories on small scales, but for the time being, it remains necessary to smooth peculiar-velocity data over scales of at least 5 Mpc to overcome the large uncertainty in each such measurement.) The statistics that have been used for this purpose include N-point functions, the void probability function VPF and related functions, skewness and kurtosis coefficients S_3 and S_4, multifractal analyses, the genus density, etc. These statistics indicate that galaxies exhibit hierarchical scaling as expected in gravitational clustering models, but most of these statistics (with the possible exception of the VPF, see e.g., Ref. 113) do not appear to be able to discriminate very efficiently in redshift space between alternative Gaussian models—although they may discriminate between these and non-Gaussian (e.g., Ref. 114) or scale-dependent-bias models (e.g., Ref. 115).

Simulations are of random patches of the universe, so comparisons with observational data must be statistical. There are broadly two different approaches to making such comparisons; one can work either in the "theoretical plane"— i.e., attempt to "correct" the data for selection effects, redshift space effects, etc.—or in the "observational plane"—i.e., "observe" the simulations. As computational power has grown, it has become increasingly advisable to observe the simulations rather than attempt to correct the data, since simulations have much more information—for example, the velocity as well as location of each object identified as a galaxy. Thus, it is possible to construct magnitude-limited redshift surveys from simulations with no ambiguity in the conversion to redshift space, but it is more difficult to recover real space information from only redshift space data.

On the other hand, one should not underestimate the difficulties of simulating observational data. Perhaps the greatest problem is determining which objects in the simulations are to be identified with observed galaxies. In dissipationless simulations, with only dark matter included, perhaps the worst problem is overmerging. Nearby dark-matter halos merge into large blobs, even though in the real universe the individual galaxies within a group or cluster can retain their separate identities since the gas can condense a great deal within the larger dark-matter halos. Even in hydrodynamical simulations (e.g., Ref. 116) there are serious limitations; only limited spatial resolution is available with even the largest supercomputers, and many relevant physical processes such as energy input from stars and supernovae are neglected or treated superficially. Although the main strengths and limitations of the several different approaches to hydrodynamical simulations seem to be reasonably well-understood (see e.g., Ref. 117), the accuracy and resolution currently available is limited.

The necessary art, at the present stage of cosmology, is to invent statistical tests that are both *robust* and *discriminatory*. Robust means that the difficulties of the sort mentioned above—e.g., in galaxy identification or "illumination" (assignment of luminosity to objects identified as galaxies)—do not significantly affect the statistical results. And discriminatory means that the statistical tests give significantly different results for the various cosmological models that are of interest. That a given statistical test is actually robust and discriminatory can be checked by trying a wide variety of different approaches to galaxy identification and illumination of simulations of a number of different cosmological models. A further desirable feature of cosmological statistics is *interpretability* in terms of the physical assumptions or observational consequences of the cosmological model in question. For example, the matter two-point correlation function is just the Fourier transform of the (nonlinear, i.e., evolved) power spectrum $P(k)$, which is of fundamental theoretical importance.

For examples of such statistics and tests by my collaborators and students, see e.g., Refs. 66 and 68 (galaxy group statistics),[132] (void probability function),[119,120] (shape statistics). To avoid the problem of cosmic variance, discussed in more detail in the next section, we should ideally have compared many cosmological models by running simulations of them with the same random numbers (producing, for example, the same random phases). In the test-bed of simulations that we had available, this was only possible to a limited extent, but we are improving on this.

13 Large-Scale Constrained Realizations

A great deal of effort is being devoted to creating improved methods of doing dissipationless and hydrodynamical cosmological simulations. In a new research project with Avishai Dekel and our students and other collaborators, we propose to complement this by developing more efficient methods for setting up such simulations, for comparing the results to observational data.

The distribution and velocities of galaxies on scales of $\sim$ 1–10 h^{-1} Mpc as revealed by redshift surveys are particularly sensitive to the nature of the dark matter, but discriminatory statistics such as relative galaxy velocities are also sensitive to the largest waves in the simulation volume. Because there are only a few such waves, they cannot fairly represent the Gaussian distribution assumed in models. Moreover, the dominant structures—rich clusters, "great walls"—in the largest dense redshift surveys such as PPS, CfA2, and SSRS2 also strongly affect statistics such as velocities. The solution we propose is to do simulations to be compared to **specific** redshift surveys, using the technique of constrained Gaussian realizations to set up initial conditions that will produce these dominant structures, with smaller waves put in and the simulation evolved to the present using the mixture of cosmological parameters (H_0, Ω, and Λ), and dark-matter types according to each model to be tested. This "Large Scale Constrained Realizations" (LSCR) approach certainly needs much development and testing; but assuming that it works as well as we hope, we anticipate that it could grow into a major addition to the technology of observational cosmology.

Cosmic variance is perhaps the most serious problem in comparing simulations such as ours to redshift data. The cosmic scatter between random realizations is artificially large because the perturbations of the largest scales are represented by only a few waves and they therefore do not represent properly a Gaussian field. Each such realization is therefore typically dominated by one or a few large-scale waves, with strong systematic effects on the small-scale structure of interest, and especially on the velocities. A brute force way to beat this cosmic scatter is by averaging over many random realizations, but this can be quite expensive and impractical with full N-body simulations, although it is quite practical with the truncated Zel'dovich approximation (e.g., Ref. 121).

A much more economical way would be to fix the large-scale structure in the initial conditions at its true pattern and generate random realizations of the relevant small-scale structure only. The large waves on scales $> 20\ h^{-1}$ Mpc in regions of our cosmological neighborhood covered by dense redshift surveys can be extracted from the IRAS redshift surveys or from peculiar velocity data (e.g., using the POTENT reconstruction).[103] These large-scale constraints will be imposed on a random Gaussian realization of smaller waves representing each of the models of interest using the technique of Hoffman.[122]

We also need to test the effectiveness of the overall LSCR procedure. For example, we plan to impose LS constraints from both CHDM and ΛCDM simulations, use the LSCR procedure to set up initial conditions for each model with the same parameters, and then evolve all four models—CHDM-CHDM, ΛCDM-ΛCDM, and the two-crossed models—to the present. We can then see how well we can recover the same statistical results on various tests as in the original CHDM and ΛCDM models, and understand the nature of the biases, if any, in the crossed models.

14 Early Structure Formation

A major difference between cosmological models is in their predictions for the origins of galaxies, clusters, and large-scale structure, and the evolution of these with redshift. Detection of large-mass collapsed objects at high redshift would certainly be contrary to the predictions of models with $\Omega = 1$, especially models such as CHDM in which small-scale fluctuation power is significantly suppressed compared to standard CDM. For example, a possible detection of a large HI cloud was reported but not confirmed; the upgraded Arecibo telescope and the Giant Meter-Wave Radio Telescope will soon provide sensitive tools for searching for such clouds at high redshift. Detections of galaxy clusters and quasar superclusters at redshifts $z \sim 2$ have also been reported, and there are some remarkable HST WFPC2 pictures of clusters at redshifts $z \sim 1$. Although the striking differences between the galaxies in such clusters and those at lower redshift support earlier indications that cluster galaxies have evolved significantly since redshift 1, galaxies in the field appear to have evolved less dramatically since the universe was half its present age (e.g., Ref. 123). The most useful data would provide indications of the number densities of the objects (e.g., galaxies or clusters) considered at various redshifts as a function of their mass (indicated, for example, by internal velocity dispersion or gas temperature), since that is what simulations predict most directly.

One of the most useful data sets for comparison with theories of structure formation is provided by absorption lines in the spectra of high-redshift quasars. The absorption systems with the highest density of neutral gas—known as damped Lyman α absorption systems—are presumably protogalaxies, and the quantity of gas in such systems at redshift $z \sim 3$ is roughly the same as the amount of ordinary matter in all the stars and gas that we can see in the universe at the present epoch (see e.g., Ref. 124). An important question that will discriminate strongly between various cosmological models is whether the quantity of gas in such systems peaks at redshifts about three to four, as expected in models where structure forms relatively late, such as CHDM (see e.g., Ref. 125), or increases to still higher redshift, as expected in models such as ΛCDM where structure forms significantly earlier.

15 Needs for the Future

Perhaps the most important information needed as a basis for constructing the first fundamental theory of cosmology is the values of the fundamental cosmological parameters, especially H_0, Ω, and Λ. Below, we summarize a number of other areas in which progress is needed.

Bigger computers, faster and more clever codes, shared software. The greatest dynamical range in force resolution currently available is only a little better than three orders of magnitude in dissipationless simulations, worse in hydrodynamical simulations. This means that in a simulation with a 100 Mpc box, not large enough to simulate large surveys and large structures such as the Great Wall, the resolution is not much better than 100 kpc, an order of magnitude larger than the visible parts of galaxies. Moreover, there is a tradeoff between mass and force resolution; codes that permit better force resolution use fewer particles and thus have poorer mass resolution. A few groups such as the Grand Challenge Cosmology Consortium (GC^3) have recently devoted a great deal of effort to developing new and improved codes that exploit the new generation of massively parallel supercomputers that is now becoming available. It is very desirable that these codes become widely available so that the entire cosmology community can benefit. New technologies for visualization of the results of supercomputer calculations also hold considerable promise.

Cosmic variance. One of the worst problems with all simulations is cosmic variance; since only a random patch of universe is simulated, a number of such simulations must be compared to a number of regions for which galaxy survey data is available, and it is not clear that even the largest redshift surveys yet completed provide fair samples. This problem is exacerbated by the fact that, as many calculations have shown, there is a feed-down of effects from large structures to small; for example, rms pairwise velocities are strongly influenced by the presence of relatively rare rich clusters of galaxies (see e.g., Ref. 105). Controlling cosmic variance is one of the most important challenges for current theory, until really large simulations can be compared to really large data sets such as the SDSS.

Better understanding of processes leading to galaxy formation. Since galaxy formation includes a number of generations of stars, including supernovae and the resulting chemical evolution, and perhaps often also involves more exotic objects such as the massive black holes thought to power active galactic nuclei, developing a secure understanding of these formation processes is likely to take a long time. It is even possible that a general theory of cosmology, including at least a general outline of the initial conditions and the nature of the dark matter, will precede rather than follow a detailed understanding of galaxy formation. No doubt a great deal of data on galaxies in both early and intermediate stages of formation will be necessary. Fortunately, such data is now coming from the new generation of great telescopes in space and on the ground. But present-epoch galaxies are brightest in the near infrared, and higher-redshift galaxies are expected to be brightest in the several micron band which can only be accessed from space. A large space infrared telescope such as SIRTF has long been seen as a high priority for astronomy, and the need for it was reiterated several times during the Snowmass workshop. Meanwhile, we will need to make use of even indirect data such as the amount of extragalactic background infrared light from early galaxies, which can perhaps be probed by its absorption of TeV photons from AGN's via pair production.[125]

Better ways of comparing observational data with models. We need new and better statistical tests, which are both *robust* against the difficulties of galaxy identification in simulations, and the biases and selection effects always present in survey data, and *discriminatory* between the classes of cosmological models of interest. On the whole, it is probably better to compare theory with data by "observing" simulations rather than "correcting" data. It is very desirable that standard software becomes available to theorists and observers, so that standard versions of various statistics can be tried on many datasets from simulations and observations.

Better access to observational and simulation data. It is unfortunate that the only dense redshift survey covering a reasonably large volume which is publicly available is the 1982 CfA1 survey. Many papers have been published analyzing data from newer and larger redshift surveys in the years since then, but the redshift data remains largely unavailable. It is also desirable that simulation data (e.g., catalogs of objects identified as galaxies) be made available. The journals and funding agencies should ask a committee of observers and theorists to establish reasonable rules regarding access to such data—for example, all data used for a given paper must be made publicly accessible within one year of the publication of the paper—and ask referees to help enforce these rules. The POTENT group has set a good example of the sort of public access advocated here, by making their peculiar velocity dataset available in a timely way and in convenient form.

16 Part III. Structure Formation in CHDM Cosmology

In this part, I consider the formation of galaxies and large-scale structures in the universe in cosmological models that are spatially flat and in which most of the dark matter is cold. I particularly emphasize the consequences of light neutrinos, with mass in the few eV range. The CHDM cosmological model appears to require about 5 eV of neutrino mass in order to produce early enough galaxy formation. These neutrinos would constitute hot dark matter accounting for a fraction $\Omega_\nu = 0.2\ (0.5/h)^2$ of critical density, where $h \equiv H_0/(100\ \mathrm{km\ s^{-1}\ Mpc^{-1}})$ is the Hubble parameter. Recent experimental data suggests that this neutrino mass may be divided between two or more species of neutrinos. The linear calculations and N-body simulations that I discuss here indicate that an $\Omega = 1$ CHDM cosmological model with two neutrinos each of mass ≈ 2.4 eV (we will call this model Cν^2DM) agrees well with all available observations. However, my collaborators and I find that this is true only if the Hubble parameter $h \approx 0.5$. I also consider CDM models with a cosmological constant Λ and show that evidence for hot dark matter raises serious difficulties for low-Ω ΛCDM models.

The standard CDM cosmological model has too much power on small scales when normalized to COBE. Because of the large velocities of the light neutrinos that make up the hot component of CHDM, these neutrinos cluster less on small scales than the cold component of CHDM, thereby producing a lower abundance of clusters and smaller pairwise galaxy velocities in better agreement with observations than standard CDM with the same large-scale normalization. Predictions of a CHDM model with a single massive neutrino species and $\Omega_\nu = 0.3$ (corresponding to $m_\nu \approx 7$ eV for Hubble parameter $h = 0.5$) have been shown[68,126,127] to agree well with observations, with the possible exception that galaxies may form too late to account for the observations of quasars and damped Lyα systems[128,129] at high redshifts $z \gtrsim 3$ (Refs. 95 and 131). The latter observations can be accommodated[96,131] if the assumed neutrino mass in CHDM is lowered from ~ 7 eV to ~ 5 eV. Lowering the neutrino mass in CHDM also gives a better account of the Void Probability Function[113] and of the properties of galaxy groups.[66,68] With one ~ 5/eV neutrino, COBE-normalized CHDM probably overproduces clusters, as we show below, but this can be avoided if the neutrino mass is shared between two or three species of neutrinos.

As I explain in more detail in the following section, current experimental hints regarding neutrino masses suggest that the net neutrino mass of ~ 5 eV required for CHDM is shared among two or three species of neutrinos. In particular, if the deficit of atmospheric ν_μ relative to ν_e is due to $\nu_\mu \to \nu_\tau$ oscillations, then the hot component must involve more than one species of neutrinos; because of the long baseline, the ν_μ–ν_τ mass-squared difference must then be rather small, $\sim 10^{-2}$ eV^2. This is consistent with the possible detection of $\bar{\nu}_m u \to \bar{\nu}_e$ oscillations reported by LSND, which, if valid, requires neutrino mass in the range relevant for hot dark matter. The theory of r-process nucleosynthesis in type II supernovae that currently seems most promising imposes constraints on neutrino mass and mixing patterns, but an inverted neutrino mass hierarchy with ν_e being the heaviest meets these constraints.

I will first summarize the experimental hints of neutrino masses from the (a) solar and (b) atmospheric neutrino deficits, and from (c) LSND. We also summarize recent work[101] showing that if we take seriously CHDM and all the hints (a)–(c) of neutrino mass, then the r-process nucleosynthesis constraint leads to an essentially unique pattern of neutrino masses and mixings. Then we will consider in more detail, the consequences of such neutrino masses for the formation of galaxies and large-scale structure in the universe, for cosmological models in which $\Omega = 1$ [or $\Omega + \Omega_\Lambda = 1$, where $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$] in which most of the dark matter is cold. We show that $\Omega_\nu = 0.2$ CHDM with the mass evenly shared between two neutrino species—Cν^2DM—agrees better with observations than the one-neutrino version, better indeed than any other variant of CDM that we have considered. We also discuss other variants of CDM and show in particular that low-Ω ΛCDM is incompatible with light neutrinos with $m(\nu) \gtrsim 2$ eV. The material presented here is an updated version of that in Ref. 134; among other things, I now use the latest COBE normalization (corresponding to $Q_{\rm rms-ps} = 20\,\mu$K) for the larger set of cosmological models that we consider.

17 Experimental Data on Neutrino Masses

Evidence for a neutrino mass explanation of the solar ν_e deficit is now fairly convincing, since at least two of the three types of experiments have to be wrong to be compatible with some nonstandard solar models.[134] If the solar ν_e deficit is due to MSW $\nu_e \to \nu_\mu$ or $\nu_e \to \nu_s$ neutrino oscillations in the sun, the mass-squared difference between either pair of particles is $\Delta m_{ei}^2 \equiv |m(\nu_e)^2 - m(\nu_i)^2| \approx 10^{-5}\ \mathrm{eV}^2$. (Here ν_s denotes a "sterile" neutrino, one that contributes negligibly to the width of the Z^o. An example is any right-handed neutrino, which would not participate in standard $SU(2) \times U(1)$ electroweak interactions.)

Similarly, evidence for a neutrino mass explanation of the deficit of ν_μ's relative to ν_e's in atmospheric secondary cosmic rays is also increasing, with compatible results from three experiments,[135] and especially, new information from Kamiokande.[136] The latter includes accelerator confirmation of the ability to separate ν_e and ν_μ events, as well as an independent higher energy data set giving not only a ν_μ/ν_e ratio agreeing with the lower energy data, but also a zenith-angle (hence, source-to-detector) dependence compatible with $\nu_\mu \to \nu_e$ or $\nu_\mu \to \nu_\tau$ oscillations with $\Delta m_{\mu i}^2 \approx 10^{-2}\ \mathrm{eV}^2$. Since almost the entire region of $\Delta m_{\mu e}^2 - \sin^2 2\theta_{\mu e}$ allowed by the Kamiokande data is excluded by data from the Bugey and Krasnoyarsk reactor neutrino oscillation experiments, $\nu_\mu \to \nu_\tau$ oscillations are favored as an explanation of the atmospheric ν_μ deficit. Moreover, the absolute calculated ν_e and ν_μ fluxes—backed by measurements of μ fluxes—agree with ν_e data but show a ν_μ deficit.[137] (Because the mixing angle $\theta_{\mu i}$ must be large to account for the near 50% deficit of atmospheric ν_μ, $\nu_\mu \to \nu_s$, oscillation is disfavored because such large mixing would populate a fourth neutrino species in the early universe, contrary to Big Bang Nucleosynthesis constraints.[138])

The LSND experiment at Los Alamos has detected an excess of nine beam-on events of a type for which the most plausible interpretation appears to be $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations, with a background of only $\lesssim 2.1 \pm 0.3$ events mimicking a $\bar{\nu}_e$, so the probability that the excess is a statistical fluke is $< 10^{-3}$ (Ref. 141). These events have both a positron track and a correlated γ-ray consistent with $\bar{\nu}_e + p \to e^+ + n$, followed by capture of the neutron by a proton in the mineral oil filling the LSND tank to form a deuteron. If the LSND events are interpreted as $\bar{\nu}_\mu \to \bar{\nu}_e$, the indicated mixing angle is $\sin^2 2\theta_{\mu e} \approx 3 \times 10^{-3}$. There are several ranges of mass-squared difference $\Delta m_{\mu e}^2 \equiv |m(\nu_\mu)^2 - m(\nu_e)^2|$ which are compatible with the KARMEN[142] and BNL E776[143] experiments, $\Delta m^2_{\mu e} \sim 2$, 6, and 10 eV^2, of which $\Delta m^2_{\mu e} \sim 6$ eV^2 appears to be favored, especially if the excess events LSND has detected of the $\nu_\mu \to \nu_e$ type are also considered.[141] If the ν_e mass is relatively small ($\lesssim 1$ eV, as indicated for Majorana neutrino mass from neutrinoless double-beta decay experiments), then $\Delta m^2_{\mu e} \sim 6$ eV^2 implies that the ν_μ mass is $\sim\ 6^{1/2} \approx$ 2.4 eV. This and the $\nu_\mu \to \nu_\tau$ explanation of the atmospheric ν_μ deficit then makes $m(\nu_\mu) \approx m(\nu_\tau) \approx 2.4$ eV. It is this scenario for the hot dark matter in a CHDM cosmology which we will show below gives predictions that appear to be in good agreement with astronomical observations.

However, a possibly disturbing consequence of taking all three hints of neutrino mass seriously is that the three incompatible Δm^2 require a minimum of *four* neutrino species, i.e., a sterile neutrino ν_s in addition to ν_e, ν_μ, and ν_τ[139,101] The LSND limit $\Delta m^2_{e\mu} > 0.2$ eV^2 implies that atmospheric ν_μ oscillations cannot be to ν_e so they must be $\nu_\mu \to \nu_\tau$; then MSW solar ν_e oscillations cannot be to ν_μ or ν_τ, so they must be $\nu_e \to \nu_s$.

An additional constraint that should perhaps be imposed on neutrino masses and mixings comes from r-process nucleosynthesis,[144] which produces all the heavy chemical elements (e.g., gold). The favored site for this process is in a neutrino-heated "hot bubble" well above the neutron star remnant; this model produces the observed abundance of r-process nuclei without any *ad hoc* parameters or dependence on the messy details of the type II supernova mechanism. However, matter-enhanced (MSW) neutrino oscillations $\nu_m u$ or $\nu_\tau \to \nu_e$ will lead to a hardening of the ν_e spectrum and too much neutron depletion via $\nu_e + n \ \ \to \ \ e^- + p$ for successful r-process nucleosynthesis for the LSND-suggested neutrino mass $\delta m^2_{e\mu} \approx 6$ eV^2 and $\sin^2 2\theta \approx 3 \times 10^{-3}$—unless the mass of the ν_e is higher than that of ν_μ and ν_τ, so that no level crossing can occur. (Level crossing and MSW oscillation then will occur for the antineutrinos, but this appears to be consistent with the SN87A neutrino signal.[145]) With the r-process constraint leading to an inverted neutrino mass spectrum, taken together with the previous three experimental hints of neutrino mass and the need to have about 5 eV of neutrino mass for CHDM cosmological models, the neutrino masses and mixings are determined essentially uniquely: $m(\nu_e) \approx 2.7$ eV and $m(\nu_\mu) \approx m(\nu_\tau) \approx 1.1$ eV (Ref. 101). While it is remarkable that there actually is a consistent solution, we should also keep in mind the likelihood that not all these hints are right. For the purposes of the rest of this paper, we will consider CHDM with either one or two massive neutrinos; if the same total mass were shared by three rather than two neutrinos, the cosmological implications would be very similar.

18 Comparison of Cosmological Model Predictions with Observations

COBE observations[104] of fluctuations in the microwave background radiation provide an upper limit (since they include possible tensor gravity wave as well as scalar density wave contributions) on the normalization of the spectrum of density fluctuations in models of structure formation in the universe. When COBE normalization is used for the standard CDM model[146] in a critical density ($\Omega = 1$) universe with a Zel'dovich primordial power spectrum [$P(k) = Ak^{n_p}$ with $n_p = 1$] as predicted by simple inflationary models, this fits large-scale data but produces too much structure on smaller scales.

We report (quasi)linear estimates for CDM and CDM variants in Table 1. All models in the table are normalized to COBE[104,147] except for the two models marked with an asterisk (*). The first two lines of numbers give our estimates of a variety of observational quantities and the uncertainties in them, from large to small scales. The bulk velocity at $r = 50\ h^{-1}$ Mpc is derived from the latest POTENT analysis;[148] the error includes the error from the analysis but not cosmic variance. However, similar constraints come from other data on large scales such as power spectra that may be less affected by cosmic variance since they probe a larger volume of the universe. We have estimated the current number density of clusters ($N_{\rm clust}$) from comparison of data on the cluster temperature function from x-ray observations with hydrodynamic simulations[149] as well as from number counts of clusters.[150] All recent estimates of the cluster correlation function give fairly large values at 30 h^{-1} Mpc;[151] this also suggests that the zero crossing of the correlation function must exceed $\sim$ 40 h^{-1} Mpc. The linear estimate of pairwise velocities (σ_v) is not an observed value, since pairwise velocities are strongly influenced by nonlinear evolution. However, from previous experience with N-body simulations for various models, we have found that the results from simulations are about a factor of three or four larger than the linear estimate. The limit we quote here is our estimate of the maximum allowed linear value to match numbers derived from redshift surveys, although the statistics on velocities derived from these surveys may not be very robust[152] since they are heavily influenced by the presence of (relatively rare) clusters.[153] To get a better estimate of pairwise velocities in our preferred Cν^2DM model, we have performed N-body simulations, as discussed below. The final column gives the observed density in cold hydrogen and helium gas at $z = 3.0$–3.5 from the latest observations of damped Lyman α systems.[129]

The next two lines present predictions from the CDM model and illustrate its problems. The cluster correlation function at 30 h^{-1} Mpc is smaller than observations indicate regardless of CDM normalization, reflecting the fact that the matter correlation function becomes negative beyond $\sim 40\ h^{-1}$ Mpc. In addition, CDM normalized to COBE produces more than an order of magnitude too many rich clusters (this problem was emphasized by Ref. 155 when the COBE DMR data first became available) and excessive small-scale pairwise velocities. If CDM is normalized to $\sigma_8 = 0.7$ (or equivalently to linear bias $b \equiv \sigma_8^{-1} = 1.43$), the cluster density problem is avoided, but small-scale velocities are still too large[66,68,126] and bulk velocities on a scale of 50 h^{-1} Mpc are probably too low. Even biased CDM is able to account for observations of damped Lyα systems, judging from $\Omega_{\rm gas}$, our Press-Schechter estimate of the amount of gas in collapsed halos at redshift $z = 3$–3.5.

CDM is attractive because of its simplicity and the existence of well-motivated particle candidates (lightest superpartner particle and axion[156]) for the cold dark matter; moreover, CDM came remarkably close to predicting the COBE signal. So several variations have been tried to patch up the CDM model. Lowering the normalization (introducing a lot of "bias") or "tilting" the primordial spectrum (assuming $n_p \approx 0.7$) improves agreement somewhat with data on intermediate ($\sim$ 10 Mpc, e.g., cluster) scales and small ($\sim$ 1 h^{-1} Mpc, e.g., galaxy pairwise velocity) scales, but leads to serious disagreement with larger-scale (30–100 h^{-1} Mpc) measurements of galaxy bulk velocities and power spectra, and galaxy and cluster correlations. Less tilt will lead to serious overproduction of clusters and large galaxy pairwise velocities—e.g., $n_p = 0.9$ with $h = 0.45$ and no gravity waves, as advocated by Ref. 158, predicts $N_{clust} = 2 \times 10^{-6}$ and $\sigma_v = 279$, both calculated as in Table 1.

From the viewpoint of agreeing with observations, the best variants of CDM that have been discussed[158] add either a cosmological constant (ΛCDM) or a little hot (neutrino) dark matter (CHDM). The former assumes $\Omega \approx 0.3$ and adds a cosmological constant Λ such that $\Omega_\Lambda \equiv \Lambda/(3H_0^2) = 1 - \Omega$ to preserve flatness (predicted by inflation) as well as improve agreement with data. ΛCDM works best for a larger Hubble parameter $h \approx 0.7$ favored by many observers. It predicts relatively early galaxy formation since by late times, structure formation ceases as the universe goes into inflation caused by the positive cosmological constant.

The problem with CDM is that it has too much power on small scales relative to power at large scales. Since the presence of light neutrinos reduces small scale power (because neutrino free streaming causes neutrino perturbations to damp on smaller scales, and this in turn leads to a slower growth rate for the fluctuations in the cold component of CHDM), including a neutrino component improves the agreement of model predictions with observations.

The first version of CHDM to be studied in detail[126,127] assumed 60% cold, 30% hot (corresponding to a neutrino of mass 94 $h^2\Omega_\nu \approx 7$ eV), and 10% baryonic matter, with $\Omega = 1$ and $h = 0.5$. This version of CHDM fits galaxy and larger scale structures in the present-epoch universe quite well. The small-scale velocities in this model are almost small enough[126] to agree with the old result $\sigma(1\ h^{-1}\ \mathrm{Mpc}) = 340\ \mathrm{km\ s}^{-1}$ from the CfA1 survey.[155] However, this result is now known to be in error because of the accidental omission of the Virgo cluster;[159] as we mentioned above, this is not a very robust statistic. A direct comparison of galaxy groups in "observed" CDM and CHDM simulations with identically selected CfA1 groups shows that CDM velocities are much too high, even with biasing, while the velocities in the $\Omega_\nu = 0.3$ CHDM model are in reasonable agreement.[66,68] However, the fraction of galaxies in groups is slightly too high for $\Omega_\nu = 0.3$ CHDM, while it is significantly too low for CDM. Thus, agreement is improved for a lower Ω_ν.

CHDM with $\Omega_\nu = 0.3$ has Ω_{gas} too small[95,96] to account for the observed damped Lyα absorption systems.[128,129] This model forms galaxies too late since the large fraction of free-streaming neutrinos washes out small-scale density fluctuations too effectively. But the small-scale power in CHDM models is a very sensitive function of Ω_ν, and lowering the hot fraction to about 20% solves this problem.[96] However, this model (called 1ν in Table 1) may have too much power at intermediate scales and overproduces clusters, especially with the new COBE normalization. In order to avoid this, it should probably be normalized lower—which we might imagine could reflect some tilt and gravity waves—but the danger is that this would result in too little early structure formation (because $\Omega_{\rm gas}$ is exponentially sensitive to the power spectrum).

These CHDM models have placed the needed neutrino mass in one flavor of neutrino, presumably the ν_τ, whereas if the atmospheric ν_μ deficit is due to $\nu_\mu \rightarrow \nu_\tau$ oscillations, this cannot be correct. If we take the evidence for atmospheric neutrino oscillations *or* the LSND indications seriously, then the ~ 5 eV mass ought to be shared about equally between the ν_μ and ν_τ. Having two neutrinos of 2.4 eV each, which we call the Cν^2DM model, produces an interesting effect: the ratio of the power spectrum for Cν^2DM compared to that for CHDM with the same total neutrino mass in one species is essentially unity at large and small scales, but it has a dip of about 30% centered at $\sim 10\ h^{-1}$ Mpc. The larger neutrino free-streaming length, resulting from a neutrino mass of 2.4 eV instead of twice that, lowers the abundance of clusters and gives better agreement with observations (cf. also Ref. 161).

The first Cν^2DM line in Table 1 gives the results for the first-year COBE normalization $Q_{\rm rms-ps} = 17\mu$K, for which all quantities are in good agreement with the astronomical data. The extra free streaming due to the smaller neutrino mass suppresses cluster formation. The small-scale power in this model is nearly identical to that in the 1ν version, so Cν^2 also produces enough $\Omega_{\rm gas}$. Raising the normalization to the COBE two-year value[104,147] leads to overproduction of clusters—though it is not as bad as for the 1ν version. But this could be counteracted by introducing a little tilt as shown by the next two lines in Table 1, which correspond to COBE-normalized[147] chaotic inflation models with inflation potentials $V(\phi) = m^2\phi^2$ and $\lambda\phi^4$ respectively, which lead to tilts $n = 0.960$, 0.939 with quadrupole tensor-to-scalar power ratios $(T/S)_2 = 0.126$ and 0.255, which are reduced by about 15% for the $\ell = 11$ multipole.[161] These models are in good agreement with observations, except possibly for the small-scale velocities which must be tested by comparing simulations with data. We note that σ_v is a fairly sensitive decreasing function of both Ω_b and Ω_ν, decreasing nearly 10% if Ω_b is increased from 7.5% to 10% *or* if Ω_ν is increased from 20% to 22%.

We are now in the process of analyzing results from new N-body simulations of Cν^2DM (high resolution 800^3 PM mesh in a 50 h^{-1} Mpc box with 256^3 cold and 2×256^3 hot particles). The case simulated was the first of the Cν^2DM ones in Table 1, with the lower first-year COBE normalization ($Q = 17$); although, as just discussed, we expect that the results will not be very different from those for the higher normalization with a little tilt. We find that the hot particles are much more spread out than the cold ones, because the lower amplitude of the fluctuations in the hot component and their higher velocities even at late times [at $z = 0$, $v_{rms} = 75$ km s$^{-1}(m_\nu/2.4$ eV$)^{-1}$ from the Fermi-Dirac distribution[126]]. This implies that the usual growth rates for an $\Omega = 1$ cosmology should be lowered when velocities are estimated at $z = 0$ for $\Omega = 1$ CHDM, since the hot component clusters so much less. Projected pairwise velocities can be estimated from the simulation results by placing an "observer" in the box and measuring relative velocities along the line of sight for a given projected separation.[126] The dark matter particle pairwise velocity calculated in this way is $\sigma_v(\text{projected, dark matter}) = 560$ km s^{-1} at 1 h^{-1} Mpc separation. If we conservatively estimate that the velocity bias (the ratio of the rms velocity of the dark matter halos to that of the dark matter particles[162]) is 0.8, this corresponds to 450 km s^{-1} for galaxies, consistent with current observations (and, as expected, about a factor of three larger than the linear estimate in Table 1). As already mentioned, the Void Probability Function from these simulations is in excellent agreement with the PPS and CfA2 data.[132]

It is remarkable that, with the experimentally suggested neutrino masses, only cosmological models with $h \approx 0.5$ match observations. As we discussed in Ref. 134, for $h = 0.7$—favored by many observers—CDM (CDM$_{0.7}$) is an even worse fit to the data than for $h = 0.5$ because the larger h makes matter-dominance ($\propto \Omega h^2$) occur earlier and thus moves the bend in the CDM spectrum to smaller scales, giving more intermediate and small scale power for a given large scale normalization. Adding two 2.4 eV neutrinos only slightly improves the situation, because this only gives $\Omega_\nu \sim 0.1$ for $h = 0.7$, so the spectrum is not modified very much. Of course, with large h, $\Omega = 1$ models also lead to too short a time since the Big Bang: $t_0 = 2/(3H_0) = 6.52\,\text{Gy}/h = 9.3$ Gy for $h = 0.7$.

A larger age is obtained for an open universe; in order to be consistent with inflation, we assume a positive cosmological constant. The maximum value of Λ allowed by the COBE data is $\Omega_\Lambda \equiv \Lambda/(3H_0^2) \approx 0.78$,[163] and the maximum allowed by quasar lensing statistics is $\Omega_\Lambda \approx 0.7$.[164] For a flat ($k = 0$) universe with $\Omega_\Lambda = 0.7$ and $\Omega = 0.3$, $h = 0.7$ corresponds to $t_0 = 13.5$ Gy. Here, ΛCDM with these parameters is a good fit[165] to the data. However, this model becomes much worse if even one neutrino of 2.4 eV is added, seriously underproducing clusters and $\Omega_{\rm gas}$ because of the excessive fraction of hot dark matter suppressing small-scale structure. Consequently, low-Ω_0 models have serious problems if any neutrinos have significant mass. Raising Ω_0 to 0.5 gives enough cold dark matter to counteract the poisoning of structure formation by a single neutrino species of 2.44 eV mass, but this model must have a lower Hubble parameter to be consistent with $t_0 \geq 13$ Gy ($h \geq 62.5$ for $\Omega_0 = 0.5$).

A similar situation occurs for $\Omega = 1$ Cν^2DM with $h = 0.4$, for which $\Omega_\nu = 0.32$ with two 2.4 eV neutrinos. (Recall that for given $m(\nu)$, Ω_ν scales as h^{-2} since critical density is $\propto h^2$.) Because the bend in the CDM spectrum moves to larger scales as h decreases, there is less intermediate and small scale power for given large scale normalization; adding hot dark matter further decreases small scale power. We find that even with only one 2.4 eV neutrino, there is just not enough power to generate the observed number of clusters or high-redshift objects.

19 Conclusions on Structure Formation with CHDM

Ever since the early 1980s, there have been hints[166] that features on small and large scales may require a hybrid scenario in which there are two different kinds of dark matter. Preliminary studies of the CHDM scenario were carried out in 1984 (Ref. 168), and it was first worked out in detail only in the last two years[68,126,127] with one massive neutrino. We have shown here that the Cν^2DM model, with Hubble parameter $h = 0.5$ and both neutrinos having a mass of 2.4 eV as suggested by ongoing experiments, gives a remarkably good account of all presently available astronomical data. New data on CMB, large-scale structure, and structure formation will severely test this highly predictive model. Results expected soon from ν-oscillation experiments will clarify whether indeed $m(\nu_\mu) \approx m(\nu_\tau) \approx 2.4$ eV. Table 1 shows the implications of such neutrino masses for a variety of popular CDM-type cosmological models. If even just the ν_μ has a mass of 2.4 eV, as suggested by preliminary results from the LSND experiment, flat low-Ω CDM models are disfavored.

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Table 1: Comparison of Models — COBE normalization (except for models marked *): $Q_{\mathrm{rms-ps}} = 20\mu$K, or $a_{11} = 7.15$ and $a_8 = 9.5$ [28].

Model	Ω_{bar} (%)	Ω_ν (%)	N_ν [a]	m_ν [a]	σ_8 [b]	V [c] 50Mpc	N_{clust} [d] (10^{-7})	ξ_{cc} [e] 30Mpc	r [f] $\xi = 0$	σ_v [g] 1Mpc	Ω_{gas} [h] (10^{-3})
OBSERVATIONS						335	4.0	0.30	> 40	< 200	3.0
uncertainties						80	2.0	0.15			1.0
CDM models, h=0.5 ($t_0 = 13.0$ Gy)											
COBE (Q_{20})	7.5	0	0	0.00	1.28	422	100.	0.12.	41	479	39
biased*	7.5	0	0	0.00	0.70	231	1.2	0.08	36	262	14
CHDM models, h=0.5 ($t_0 = 13.0$ Gy)											
KHPR (Q_{20})	10.0	30	1	7.04	0.78	425	16.	0.40	51	117	2.4
1ν (Q_{20})	7.5	20	1	4.69	0.89	423	27.	0.30	52	184	12.
Cν^2DM (Q_{17}*)	7.5	20	2	2.35	0.67	347	2.4	0.35	70	144	4.6
Cν^2DM (a_{11})	7.5	20	2	2.35	0.78	408	11.	0.38	70	169	11.
Cν^2DM$_{n0.96}$	7.5	20	2	2.35	0.69	374	3.7	0.39	71	142	5.5
Cν^2DM$_{n0.94}$	7.5	20	2	2.35	0.63	350	1.5	0.39	72	128	3.3
49	199	4.4									
ΛCDM/ΛCHDM models, $h = 0.7$, $\Omega_0 = 0.3$, and $\Omega_\Lambda = 0.7$ ($t_0 = 13.5$ Gy)											
ΛCDM (a_8)	2.6	0	0	0.00	1.02	328	1.6	0.23	125	133	21.
ΛCHDM (a_8)	2.6	5.3	1	2.44	0.69	342	0.08	0.43	135	60	0.0004
ΛCDM/ΛCHDM models, $h = 0.6$, $\Omega_0 = 0.5$, and $\Omega_\Lambda = 0.5$ ($t_0 = 13.5$ Gy)											
ΛCDM (a_8)	3.5	0	0	0.00	1.25	403	22.	0.29	66	253	27.
ΛCHDM (a_8)	3.5	7.2	1	2.43	0.86	390	3.2	0.50	100	122	3.5

[a] N_ν is the number of ν species with mass. If $N_\nu \geq 1$, each species has the same mass m_ν.

[b] $(\Delta M/M)_{\mathrm{rms}}$ for $R_{\mathrm{top-hat}} = 8h^{-1}$ Mpc.

[c] Bulk velocity in top-hat sphere of radius 50 h^{-1} Mpc.

[d] Number density of clusters $N(>M)$ in units of $10^{-7}\,h^3\mathrm{Mpc}^{-3}$ above the mass $M = 10^{15}h^{-1}M_\odot$, calculated using a Press-Schechter approximation with a Gaussian filter and $\delta_c = 1.50$.

[e] The cluster-cluster correlation function amplitude at $30h^{-1}$ Mpc, computed using linear theory [HP93] and assuming a unit bias factor for the dynamical contribution.

[f] Zero crossing ($\xi(r) = 0$) of the correlation function in units of h^{-1} Mpc.

[g] Linear estimate of pairwise velocity at $r = 1h^{-1}$ Mpc scale: $\sigma_v^2 = 2H_0^2 \int dk\, P(k)(1 - \sin kr)/kr$.

[h] Mean density of collapsed baryons at $z = 3 - 3.5$ in units of 10^{-3} of critical density, calculated using $\Omega_{\mathrm{gas}} = (\Omega_b/\Omega_c)\,\mathrm{erfc}(\delta_c/\sqrt{2}\sigma)$, with $\delta_c = 1.4$ [8], and σ computed for mass $5 \times 10^{10}h^{-1}\ M_\odot$ using Gaussian smoothing and assuming all gas is neutral. Since some gas may be ionized or removed by star formation, Ω_{gas} for the various models should be at least as high as the observations.