



# Optics of a Proton Driver\*

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## Abstract

In a four month study, a design for a Proton Driver was developed as a possible replacement for Fermilab's Booster. Its optical properties are summarized briefly.

## 1 OVERVIEW

A Proton Driver design (PD2) was developed as a possible replacement for Fermilab's Booster in a four months' study involving more than sixty participants. Constraints that influenced the design, as well as achieved values, are summarized in Table 1; constraints are identified with an asterisk. Based primarily on considerations of available space and the size of lattice functions, a superperiod 2 race-track configuration was chosen: two 75.44 m straight sections connected by 161.66 m, 180° arcs.<sup>1</sup> Transition is avoided by requiring  $\gamma_t$  to be beyond the reach of the extraction energy.

The racetrack's straight sections will be used for injection, extraction, tuning, and acceleration; collimation and (possibly) tuning will be done in the arcs. Trim quads should be powered symmetrically so as to maintain superperiodicity. It is intended to reuse RF cavities from the current Fermilab Booster in the Proton Driver's straight sections. At least 21 will be needed. The phase advance between kickers and septa as well as lattice functions at various extraction devices were chosen so as to avoid excessive demands on magnets.

In order that quadrupoles and dipoles track well during the ramp, while avoiding both saturation and excessive power loss, it was decided early to limit the maximum field in dipoles to 1.5 T and the peak gradient in quadrupoles to 10 T/m. Because of fabrication requirements for magnet ends and bellows, the minimum space between quadrupoles (in a doublet) was set to 47 cm; the minimum space between a quadrupole and a dipole, to 85 cm.

Phase space painting will flatten the transverse charge distribution, reducing space charge tune spread. The invariant transverse beam emittance, after painting and including space charge effects, is to be no larger than  $40 \pi$  mm-mr (95%). Dynamic aperture should be *at least* three times that number, for the entire momentum range of  $\pm 1\%$ . Emphasis was placed on minimizing the maximum values of lattice functions,  $\beta_x$  and  $\beta_y$ , and horizontal dispersion,  $D$ . To satisfy physical aperture requirements, a criterion was infor-

length	474.2 m *
kinetic energy	600 MeV * to 8 GeV *
revolution frequency	$501 \text{ kHz} \leq f \leq 629 \text{ kHz}$
RF frequency	53 MHz *
RF cavity length	2.35 m *
protons per bunch	$\approx 3 \times 10^{11}$ *
cycling frequency	15 Hz *
transition	$\gamma_t = 13.8 > 9.5$ *
natural chromaticity	(-13.6, -11.9)
momentum acceptance	$ \delta p/p  < 0.01$ *
phase advance	$\Delta\psi_{\text{arc}}/2\pi = (n_x, n_y)$ * = (8, 6)
tunes	(11.747, 8.684)
dynamic aperture	$> 120\pi$ mm-mr *
magnetic fields	$\leq 1.5$ T, dipoles, * $\leq 10$ T/m, quadrupoles *
lattice functions	$\max(\beta_x, \beta_y)$ = (15.1 m, 20.3 m)
max dispersion	2.5 m
max dipole correction	5 mrad *
element spacing	$\geq 47$ cm, quad-quad, * $\geq 85$ cm, quad-dipole *

Table 1: Parameters for a Proton Driver. Asterisks indicate design constraints.

mally established that set upper bounds of  $\beta \leq 20$  m and  $D \leq 2.5$  m. Together, these assure a maximum horizontal excursion of  $x \approx \pm 5$  cm from the closed orbit, evenly divided between dispersion and emittance, and  $|y| < 2.2$  cm.

To avoid any possibility of synchro-betatron coupling resonances [1], the PD2 design set to zero (a) chromaticity, using sextupoles in the arcs, and (b) dispersion in the straight sections, where RF cavities will be placed. To assure the latter, horizontal phase advance through an arc was required to be an integer multiple of  $2\pi$ .

In what follows, we will summarize the optics of the PD2 lattice. Necessarily, this presentation is short, but more information, in greater detail, can be found in the Proton Driver Design Reports [2, 3].

## 2 LATTICE DESCRIPTION

The PD2 racetrack is partitioned into forty-four 10.777 m cells, 15 in each arc and 7 in a straight section. Each cell contains a defocusing (D) and focusing (F) quadrupole doublet on the main bus and a corresponding pair of independently powered trim quads. The F quad is 1.262 m long, the D quad, 1.126 m; trim quads, positioned just outside the doublet, are 20 cm long. This leaves about 7.1 m of empty space in each cell. In the arcs, it will be

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<sup>1</sup> Several possible designs were considered before settling upon one. While the decisions that were made may not be final, they are, nonetheless, the ones under which this study was conducted.

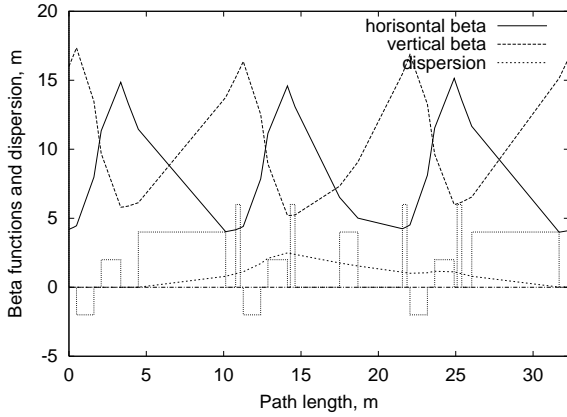


Figure 1: Lattice functions of an arc module, comprising three arc cells, treated as a periodic unit.

filled with dipoles and collimation hardware; in the straight sections, with hardware for injection, extraction, and acceleration. Additional diagnostic and control devices – beam position monitors, orbit correctors, dampers, and the like – must be squeezed into whatever space remains. (PD2’s length of 474.2 m, equal to that of the Booster, severely limits space available for utility hardware.)

## 2.1 Arc Module

Each arc is organized into 5 modules of 3 cells, 15 cells in all. Quadrupole lengths were chosen so as to produce a phase advance per module of  $(\Delta\psi_x, \Delta\psi_y)|_{\text{module}} = (8\pi/5, 6\pi/5)$  which sets the average phase advance per cell to be  $(\Delta\psi_x, \Delta\psi_y)|_{\text{cell}} = (8\pi/15, 2\pi/5) = (96^\circ, 72^\circ)$ . A horizontal phase advance close to  $90^\circ$  is convenient for injection and extraction. Because the total phase advance across an arc is  $(\Delta\psi_x, \Delta\psi_y)|_{\text{arc}} = (8\pi, 6\pi)$ , it is, to first order, optically transparent: its  $4 \times 4$  transfer matrix is the identity. Thus, the arcs will preserve lattice functions, including zero dispersion, across the straights. If the phase advance per cell were exactly repeated throughout the racetrack, the tunes would be  $(\nu_x, \nu_y) = (11.73, 8.80)$ .

The two outer cells of an arc module contain a large dipole (5.646 m,  $16.2^\circ$  bend) and the inner cell a small one (1.188 m,  $3.4^\circ$  bend). Their lengths were chosen so as to create a first order achromatic bend, thus zeroing the dispersion between modules. As a stand-alone periodic unit, the lattice functions of a single arc module would be as shown in Figure 1

Four chromaticity correcting sextupoles are placed in each arc module. To conserve space, they replace the four trim quads closest to the short dipole. (They are shown as tall thin elements in Figure 1.) Alternatively, it may be possible, and perhaps preferable, to build a correction package consisting of quadrupole and sextupole. If not, then we must rely on two trim quads (not shown) in the remaining cell to control the phase advance through each module.

## 2.2 Straight Section

The main bus quadrupoles in a straight section cell are identical to their counterparts in the arc cell; dipoles are removed, and trim quads replace the sextupoles. The absence of dipole edge focusing distorts lattice functions (esp.,  $\beta_y$ ) slightly. Phase advance across a single cell is  $(\Delta\psi_x, \Delta\psi_y)|_{\text{cell}} = (8\pi/15, 0.96 \cdot (2\pi/5))$ .

## 3 ANALYSIS

When the straight sections and arcs are attached, a small vertical beta wave is established *but is confined to the arcs*, because of their optical transparency. The tunes associated with the base configuration (trim quads off) are shifted from (11.73, 8.80) to (11.747, 8.684). Space charge will reduce tunes in the core of the beam by an amount that will depend on painting. Protons undergoing large amplitude oscillations will be less affected by space charge, but their tunes will increase (slightly) due to the presence of chromaticity correcting sextupoles. The combined effects of space charge and sextupole fields (and octupole error fields) will spread the tunes away from that point in opposite directions. The single particle “optical tune,” the “working point,” acts as a reference for this distribution. As the beam’s energy increases, the distribution will collapse into it: space charge forces decrease, as  $v/c \rightarrow 1$ , shrinking the distribution from below, and the sextupole/octupole tune spread decrease, as emittances become smaller, shrinking the distribution from above.

Chapter 3 of the Design Study Report [2] contains an optics analysis, including consideration of chromatic properties, tune footprint, field errors, steering magnets, tune adjustment, coupling, resonances, and dynamic aperture. In the following sections we will summarize only the discussions of the  $\nu_x + 2\nu_y$  sextupole resonance and dynamic aperture.

### 3.1 Sextupole Resonance: $\nu_x + 2\nu_y$

A phase advance of  $(\Delta\psi_x, \Delta\psi_y) = (8\pi/5, 6\pi/5)$  across an arc module means that the sextupoles’ contribution to the  $\nu_x + 2\nu_y = 29$  resonance driving term will add in phase from one module to the next. This is mitigated by the fact that 29 is an odd number, so that whatever resonance driving term is produced by one of the arcs should be cancelled by the other. Nonetheless, trusting in cancellations across opposite sides of a ring is risky. Superperiodicity of the lattice can be broken by the tuning quads in the straight sections, or simply because of field errors. A phase error between the two arcs will certainly arise. Estimating the effect of this resonance must take that into account.

A (moderately) “safe region” for the  $\nu_x + 2\nu_y$  resonance is bounded by the curves [4, pp.233-238]

$$\frac{1}{8} \left( \frac{\delta}{g} \right)^2 = \frac{1}{4} \frac{\epsilon_y}{\pi} + 2 \left( \sqrt{\frac{\epsilon_x}{2\pi}} - \frac{1}{4} \frac{|\delta|}{g} \right)^2,$$

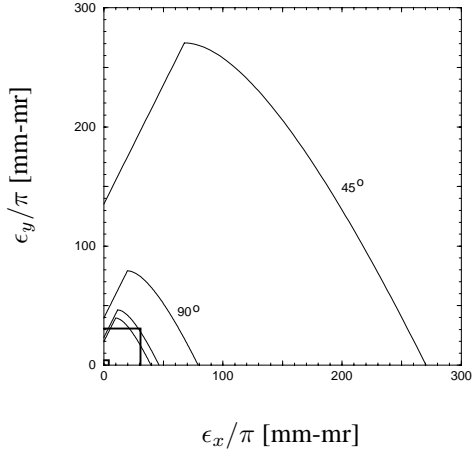


Figure 2: “Safe regions” for the  $\nu_x + 2\nu_y$  resonance, in the base configuration, when  $\Phi \in \{45^\circ, 90^\circ, 135^\circ, 180^\circ\}$ , with invariant emittance of  $40\pi$  mm-mr shown at injection and extraction. Ignoring space charge and second order sextupole effects, the injected beam is within the boundary provided that  $\Phi \ll 105^\circ$ .

$$\text{and } -\frac{1}{4} \left( \frac{\delta}{g} \right)^2 = 2 \frac{\epsilon_x}{\pi} - \frac{\epsilon_y}{\pi},$$

where  $\delta = \nu_x + 2\nu_y - 29 = 0.12$  and  $g = (\sqrt{2}/8\pi) \times |\sum (B''l/B\rho) \sqrt{\beta_x \beta_y} \exp[i(\psi_x + 2\psi_y - \delta \cdot \theta)]|$ . If sextupoles existed in all three of an arc module’s cells, the value of  $g$  for one module would (almost) vanish, because  $\langle \Delta(\psi_x + 2\psi_y) |_{\text{cell}} \rangle = (2/3) \cdot 2\pi$ . However, with sextupoles in only two of the three cells, the value of  $g$  for one arc module is  $\approx 1.36 \text{ m}^{-1/2}$ . Let the phase error between the two arcs be  $\Phi \equiv \Delta\psi_x + 2\Delta\psi_y$ . Then,

$$g_{\text{racetrack}} = 2 |\sin(\Phi/2)| g_{\text{arc}} \approx 14 \text{ m}^{-1/2} |\sin(\Phi/2)|.$$

The “safe regions” are plotted in Figure 2 for  $\Phi \in \{45^\circ, 90^\circ, 135^\circ, 180^\circ\}$ . Two squares in the lower left corner show the beam emittances at injection and extraction. The boundary crosses the injected beam at  $\Phi \approx 105^\circ$ . Superperiodicity must be preserved to a value much smaller than that.<sup>2</sup>

### 3.2 Dynamic Aperture

Dynamic aperture was estimated by tracking a single particle within the static magnetic environment produced by dipoles, quadrupoles, and chromaticity sextupoles. Among the effects not considered were space charge, synchrotron oscillations, acceleration, and magnetic field errors. (Details of the procedures used can be found in the Design Study Report [2].) Thus, results obtained should be considered an upper bound on the actual dynamic aperture. They are shown in Figure 3 at the injection energy

<sup>2</sup>This optimistic statement does not take into account the effects of tune spread. A more conservative estimate would restrict  $\Phi$  to a value four or five times smaller. (See reference [2])

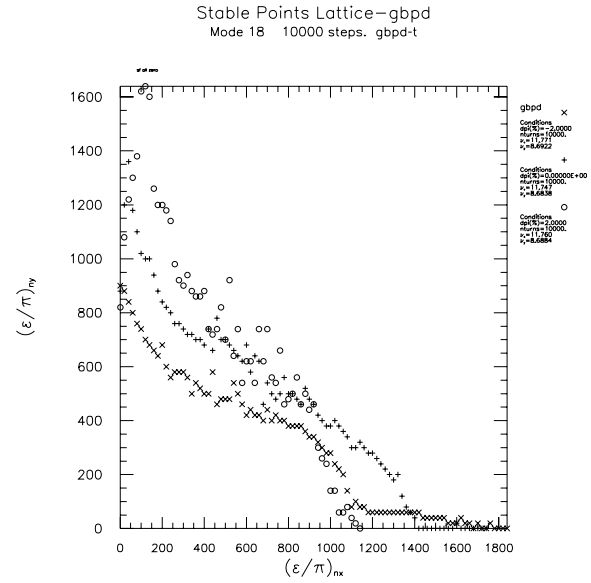


Figure 3: Dynamic aperture: Scatter plot of largest amplitude stable orbits at  $\Delta p/p = 0$  and  $\pm 2\%$ .

of 600 MeV, where dynamic aperture is smallest. Along the diagonal,  $\max \epsilon_{\text{inv}} \approx 10 \times 40\pi$  mm-mr. For purely horizontal orbits it increases to  $\epsilon_{\text{inv}} \approx 25 \times 40\pi$  mm-mr, and for mostly vertical orbits it is slightly less,  $\epsilon_{\text{inv}} \approx 20 \times 40\pi$  mm-mr.

Peaks of the tune spectra for orbits just inside the dynamic aperture were calculated. All but four cluster on the resonance line  $4\nu_y = 35$ . Chromaticity sextupoles both excite this resonance and provide the necessary tune spread to put it within the reach of very large amplitude orbits.

## 4 REFERENCES

- [1] A. Piwinski. Handbook of Accelerator Physics and Engineering, p.72. Edited by A. Chao and M. Tigner.
- [2] Proton Driver Design Study II, Part 1. Technical Report, Fermilab, May, 2002. Fermilab-TM-2169. Available electronically at <http://fnalpubs.fnal.gov/archive/2002/tm/TM-2169.html>
- [3] The Proton Driver Design Study. Technical Report, Fermilab, December, 2000. Fermilab-TM-2136. Available electronically at <http://library.fnal.gov/archive/test-tm/2000/fermilab-tm-2136.shtml>
- [4] Leo Michelotti. *Intermediate Classical Dynamics with Applications to Beam Physics*. John Wiley & Sons, Inc., New York, 1995.