Duality and other Exotic Gauge Dynamics in Softly Broken Supersymmetric QCD

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ABSTRACT

We analyze the theory of softly broken supersymmetric QCD. Exotic behavior like spontaneously broken baryon number, massless composite fermions and Seiberg's duality seems to persist also in the presence of (small) soft supersymmetry breaking. We argue that certain, specially tailored, lattice simulations may be able to detect the novel phenomena. Most of the exotic behavior does not survive the decoupling limit of large SUSY breaking parameters.

Based on a talk presented by J. S in SUSY 95.

1 Introduction and summary

* (1) We study multi-flavored four dimensional QCD by adding soft supersymmetry breaking (SSB) terms to supersymmetric QCD (SQCD)

* (2) Certain exotic phenomena discovered in SQCD persist also in the regime of small, compared to Λ_{QCD}, SSB parameters.

* (3) The exotic behavior includes (i) a baryon number violating vacuum, (ii) massless composite fermions and (iii) Seiberg's duality[1].

* (4) In the decoupling limit most of the "exotic" phenomena disappear.

* (5) We argue that all the exotic behavior of the region close to the SUSY limit should be detected in lattice simulations.

2 On the effective action of SQCD

$SU(N_c)$ SQCD contains a gauge superfield W_α, a chiral quark and an anti quark superfields $Q^i_a, \overline{Q}^a_i$ where $i = 1, \ldots, N_f$ is a flavor index and $a = 1, \ldots, N_c{}^6$ is a color index. These superfields transform under the quantum global symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R, \tag{2.1}$$

in the following way

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
Q	N_c	N_f	1	$\frac{1}{N_c}$	$(N_f - N_c)/N_f$
$\overline{Q}$	$\overline{N}_c$	1	$\overline{N}_f$	$\frac{-1}{N_c}$	$(N_f - N_c)/N_f$
W_α	$N_c^2 - 1$	1	1	0	1

The low energy effective action of SQCD is expressed, for small values of N_f, in terms of the meson superfield

$$T^i{}_j = Q^i \cdot \overline{Q}_j. \tag{2.2}$$

Beginning at $N_f = N_c$, S_{eff} is built also from chiral baryon and anti-baryon superfields. The baryon

$$B_{i_1 \cdots i_{(N_f - N_c)}} = \epsilon^{a_1 \cdots a_{N_c}} \epsilon_{j_1 \cdots j_{N_c} i_1 \cdots i_{(N_f - N_c)}} Q^{j_1}_{a_1} \cdots Q^{j_{N_c}}_{a_{N_c}}, \tag{2.3}$$

transforms in the $(N_f - N_c)$-index antisymmetric tensor representation of $SU(N_f)_L$, and, similarly, an antibaryon chiral superfield $\overline{B}^{i_1 \cdots i_{(N_f - N_c)}}$ is built from N_c powers of the field $\overline{Q}$.

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[6]We take here $N_c \geq 3$. The special case of $N_c = 2$ is discussed in ref.[2].

Using the gauge supermultiplet, it is possible to build the (massive) "glue ball" chiral superfield

$$S = -\text{tr}[W^\alpha W_\alpha] = \text{tr}[\lambda \cdot \lambda] + \cdots . \tag{2.4}$$

The Kähler potential which determines the kinetic energy terms of the fields T, B, and $\overline{B}$ was hypothesized in the work of Masiero and Veneziano [**?**, 4] to take the following form

$$K[T, B, \overline{B}] = A_T \text{tr}[T^\dagger T] + A_B(B^\dagger B + \overline{B}^\dagger \overline{B}). \tag{2.5}$$

Our main results will rely on weaker assumptions about the Kähler potential, in particular, that it is nonsingular on the space of supersymmetric vacuum states. However, we will support our general remarks by explicit calculations using this simple model. We expect (2.5) to be the correct form of the Kähler potential near the origin of moduli space, in the cases for which the mesons and baryons give an effective infrared description of the theory. This is not the case for $N_c > N_f$[5].

2.1   Soft Supersymmetry Breaking

We break supersymmetry by adding mass terms for the squarks and gaugino fields,

$$\Delta \mathcal{L} = -m_Q^2\big(|Q|^2 + |\overline{Q}|^2\big) + (m_g S + \text{h.c.}), \tag{2.6}$$

where, in (2.6), Q, $\overline{Q}$, and S are the scalar component fields of the superfields. The squark mass term is the unique soft supersymmetry breaking term which does not break any of the global symmetries (2.1) of the original model. The gaugino mass term breaks only the $U(1)_R$ symmetry, and thus breaks the global symmetry of the supersymmetric model down to that of ordinary QCD with N_f massless flavors. Actually, there are two consistent possibilities for SSB theories which we denote by R and $\not{R}$

$$R: \quad m_Q^2 \neq 0 \; m_g = 0 \quad \Longrightarrow_{m_Q^2 \to \infty} \; standard \; QCD \; with \; N_f \; quarks \; + \; adjoint \; fermions$$

$$\not{R}: \quad \quad m_Q^2 \neq 0 \; m_g \neq 0 \quad \Longrightarrow_{m_Q^2, m_g \to \infty} \; standard \; QCD \; with \; N_f \; quarks$$

Since we will be working in the language of the low-energy effective Lagrangian, we must ask how the supersymmetry breaking term (2.6) shows up in this Lagrangian. To work this out, rewrite (2.6) in the superfield form

$$\Delta \mathcal{L} = \int d^4\theta M_Q(Q^\dagger e^V Q + \overline{Q}^\dagger e^{-V^T}\overline{Q}) + \int d^2\theta M_g S + \text{h.c.} \tag{2.7}$$

where M_Q is a vector superfield whose D component equals $(-m_Q^2)$ and M_g is a chiral superfield whose F component equals m_g. It is straightforward to see that these superfields are gauge-invariant and neutral under all of the global symmetries.

The effective Lagrangian description of $\Delta \mathcal{L}$ for $N_f \leq N_c + 1$ is then given by writing the most general Lagrangian built from T, B, $\overline{B}$ and a fixed number of factors of M_Q and

M_g. The supersymmetry breaking terms have an ambiguity related to that of the Kähler potential, because many possible invariant structures can be built from T B, and $\overline{B}$. In our explicit calculations, we will assume that the coefficient of M_Q is quadratic in these fields. This assumption holds near the origin of moduli space. Then the first order soft supersymmetry breaking terms in the effective Lagrangian are

$$\Delta\mathcal{L} = \int d^4\theta\Big(B_T M_Q \mathrm{tr}[T^\dagger T] + B_B M_Q \{B^\dagger B + \overline{B}^\dagger \overline{B}\} + \overline{M_g}\mathcal{M}(T,B,\overline{B}) + \mathrm{h.c.}\Big)$$
$$+ \int d^2\theta M_g \langle S\rangle + \mathrm{h.c.}, \tag{2.8}$$

clarifications and remarks:

(i) $\mathcal{M}(T,B,\overline{B})$ is a function of the effective Lagrangian superfields which is neutral under the global symmetries.

(ii) The quantity $\langle S\rangle$ in (2.8) should be a combination of the effective Lagrangian chiral superfields which has the quantum numbers of S. In general, this condition restricts that function to be proportional to the expectation value of S as determined from the effective Lagrangian of refs. [6, 7] which includes S as a basic field.

(iii) The squark mass terms in (2.8) are not the most general terms that can be written down. Higher order terms in the fields, suppressed by powers of Λ, may appear. However, we expect (2.8) to be approximately true near the origin of moduli space $T = B = \overline{B} = 0$.

(iv) We have assumed throughout this work that the coefficients B_B and B_T are positive. This is not an innocent assumption since had they been negative chiral symmetry breaking would have taken place instead of several of the exotic features. At present we do not have a proof of the positivity of those coefficients, but the picture we derive assuming that they are positive seems to be consistent.

(v) The ratio of coefficients B_B/B_T will be important to our later analysis, but this ratio cannot be determined from the effective Lagrangian viewpoint. At best, we can argue naively that the coefficient of the mass term of a composite field should be roughly proportional to the sum of the coefficients of the mass terms of the constituents. This would give the relation $B_B \approx \frac{N_c}{2}B_T$ which the reader might take as qualitative guidance.

(vi) It is important for our analysis that the behavior of the theory is non–singular when adding the squark and gluino masses, i.e. that no new non–perturbative effects occur. In general it is not possible to prove this in non–supersymmetric theories, but a proof of this is possible in softly broken supersymmetric theories, when the soft breaking can be viewed as arising via spontaneous breaking of supersymmetry.[8]

3 Baryon number violation ($N_f = N_c$)

The cases of $N_c > N_f$ are discussed in ref.[2], here we start with the case $N_f = N_c$. The low-energy effective Lagrangian of the supersymmetric limit contains both meson and baryon superfields. The quantum numbers of the $T^i{}_j$, B and $\overline{B}$ are summarized in the following table.

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
T^i_j	1	N_f	$\overline{N}_f$	0	0
B	1	1	1	1	0
$\overline{B}$	1	1	1	-1	0

This model has a manifold of quantum mechanical supersymmetric ground states, in which the meson and baryon fields satisfy the relation (in units where $\Lambda = 1$)

$$\det T - B\overline{B} = 1 \ . \tag{3.9}$$

Several forms for the superpotential are consistent with this relation. The S-dependent superpotential, for example, has the form $W = S\log(\det T - B\overline{B})$. Note that this superpotential leads to conditions for a supersymmetric vacuum state which imply not only (3.9) but also the constraint $\langle S \rangle = 0$, so that the $U(1)_R$ symmetry is not spontaneously broken.

The presence of a manifold of degenerate vacuum states not related by a global symmetry is necessarily accidental unless it is a consequence of supersymmetry. Thus, any such degeneracy should be broken as soon as supersymmetry breaking terms are added to the Lagrangian. To first order, this is the main effect of the soft supersymmetry breaking perturbation. To analyze this effect, we should restrict our attention to the values of T, B, and $\overline{B}$ obeying the constraint (3.9), for which the vacuum energy vanishes in the supersymmetric limit, and study the behavior of the supersymmetry breaking potential over this space.

For the *R models*, the soft supersymmetry breaking terms (2.8) lead to the potential

$$\Delta V = B_T m_Q^2 \text{tr}[T^\dagger T] + B_B m_Q^2 (B^\dagger B + \overline{B}^\dagger \overline{B}). \tag{3.10}$$

Using $SU(N_f) \times SU(N_f)$, we can diagonalize T to complex eigenvalues t_i. Parameterize the baryon fields as $B = xb$, $\overline{B} = -\frac{1}{x}b$ with x and b complex. This potential has three types of
stationary points
$\swarrow$ $\quad \Downarrow \quad$ $\searrow$

$b = 0$ $\forall_i$ $\|t_i\| = 1$	$T \neq 0$, $b \neq 0$ $\|t_i\|^{(N_f-2)} = (B_T/B_B)$ unstable	$T = 0$, $b = \pm 1$ for $B_B > (N_f/2)B_T$ globally stable

with $x = 1$ in all cases. The method of effective Lagrangians cannot tell us which of the two vacuum states at $b = 0$ and $T = 0$ is the preferred one. This depends on the ratio B_B/B_T, which is a phenomenological input to the effective Lagrangian analysis. The vacuum at $T = 0$ is locally stable if $B_B > B_T$ and is globally stable if $B_B > (N_f/2)B_T$. Our naive estimate puts the theory just at the boundary at which the two vacuum states have equal energy.

The structure of the **spectrum** in the $b = 0$ and $T = 0$ cases is summarized as follows (for details see [2]).

	$b = 0$ vacuum	$T = 0$ vacuum
Symmetry	$SU(N_f)_D \times U(1)_B \times U(1)_R$	$SU(N_f)_L \times SU(N_f)_R \times U(1)_R$
parametrize	$B = b + c ,\qquad \overline{B} = -(b - c)$	$B = (1 + b + c) , \overline{B} = -(1 + b - c)$
constraint	$t_V + it_A = -\sqrt{\frac{2}{N_f}}(b^2 - c^2)$	$b = -\frac{1}{2}\det T + \frac{1}{2}c^2$
bosons	t_{A_I}- NG bosons t_A massive	$Im(c) - NG boson$
	$m_{Vi}^2 = \frac{2}{A_T}B_T m_Q^2$	$m_T^2 = \frac{B_T}{A_T}m_Q^2$
	$m_-^2 = \frac{2}{A_B}(B_B - B_T)m_Q^2$	$m_{cR}^2 = \frac{B_B}{A_B}m_Q^2$
	$m_+^2 = \frac{2}{A_B}(B_B + B_T)m_Q^2$	
fermions	$m_{\psi_T} = m_{\psi_B} = m_{\psi_{\overline{B}}} = 0$	$m_{\psi_T} = m_{\psi_B} = m_{\psi_{\overline{B}}} = 0$

Notice that since (3.9) is a superfield constraint, it also removes one fermion from the theory, specifically, the fermionic partner of $det[T]$ or of $B + \overline{B}$.

For the $\not{R}$ *models* with small m_g the qualitative picture remains the same. We showed earlier that the superpotential implies that, in the manifold of supersymmetric vacuum states about which we are perturbing, $\langle S \rangle = 0$. Thus, the superpotential term proportional to M_g does not contribute to the vacuum energy. More generally, since M_g, T, B, and $\overline{B}$ are all invariant under $U(1)_R$, while a superpotential has R charge 2, this term does not contribute to the superpotential to any order in m_g. There are possible Kähler potential terms involving M_g. However, near the vacuum with $b = 0$, these will be polynomials in B and $\overline{B}$ of order at least 2, and near the vacuum with $T = 0$ they will be polynomials in T of order at least 2. Thus, these terms will not affect the presence of stationary points of the vacuum energy at these positions in the field space. Examples of D-term terms that can be induced are

$$\Delta\mathcal{L} = \int d^4\theta \overline{M_g}(C_T \det T + C_B B\overline{B}) + \text{h.c.} , \qquad (3.11)$$

where C_T and C_B are some constants. If one begins from the effective Lagrangian including S, with the canonical superpotential and Kähler terms,

$$\mathcal{L} = \int d^4\theta S^* S + \int d^2\theta (S \log(\det T - B\overline{B}) + M_g S) + \text{h.c.} , \qquad (3.12)$$

and integrates out S, one finds a breaking term

$$\Delta\mathcal{L} = \int d^4\theta \overline{M_g} \log(\det T - B\overline{B}) + \text{h.c.}, \qquad (3.13)$$

which gives qualitatively similar results. In the following discussion, we will work with (3.11). Once the $U(1)_R$ is broken it does not protect fermions from acquiring mass. Around $b = 0$ the flavor adjoint and baryonic fermions, acquire the following masses $m_{\psi i} = \frac{1}{2}\frac{C_T}{A_T}m_g$ and $m_{\psi B} = \frac{C_B}{A_B}m_g$ and no zero-mass fermions remain. Around the $T = 0$ vacuum ψ_{T_I} and ψ_T remain massless but the baryonic fermion acquires a mass $m_{\psi B} = \frac{2C_B}{A_B}m_g$.

3.1 Toward the Decoupling Limit

The transition from the region of weak supersymmetry breaking to the decoupling limit $m_Q^2, m_g \to \infty$ is very different in the four models of R and $\not{R}$ with vacua at $b = 0$ and $T = 0$.

(i) $\not{R}$ **b=0**- In this case (like in the cases of $N_c > N_f$) the global group $SU(N_f) \times SU(N_f) \times U(1)_B$ is broken spontaneously to $SU(N_f) \times U(1)_B$, leaving no massless particles except for the required Goldstone bosons. It is reasonable to expect that there is a smooth transition from the situation of weak supersymmetry breaking to the decoupling limit $m_Q^2, m_g \to \infty$. In QCD, chiral symmetry breaking is characterized by a nonzero vacuum expectation value of the quark-antiquark bilinear, $\psi_Q^i \psi_{\overline{Q}j}$ in our present notation. In the language of the supersymmetric effective Lagrangian, this operator is a part of the F term of the superfield $T^i{}_j$. The expectation value of this term may easily be found[2] to be proportional to $m_Q^{\frac{N_c}{2N_c - N_f}}$. Thus, the F term of T does obtain an expectation value in the vacuum state that we have found. This expectation value naturally becomes a nonzero expectation value for the quark bilinear in the decoupling limit. As m_Q increases, the quark bilinear becomes larger while the squark bilinear becomes smaller, in exact accord with our expectations.

(ii) R **b=0**- As m_Q^2 is taken to infinity, the squarks decouple, and the model becomes a purely fermionic QCD theory with N_f quark flavors plus one fermion flavor in the adjoint representation of the gauge group. For small values of the supersymmetry breaking mass m_Q^2, this vacuum contains massless fermions corresponding to the fermionic components of the superfields T, B, and $\overline{B}$. We might think of these as being built out of scalars, with one squark replaced by a quark to give the composite spin $\frac{1}{2}$. But it is also possible to build composite bound state with the same quantum numbers purely out of fermions, by replacing

$$Q^i \to \lambda^\alpha \psi_{Q\alpha}^i , \qquad \overline{Q}_j \to \lambda^\alpha \psi_{\overline{Q}\alpha j} , \qquad (3.14)$$

where α is a two-component spinor index and the gauge indices are implicit. Notice that this combination has the same quantum numbers as the squark, including zero R charge. Then, for example, the fermion created by $T^i{}_j$ could be constructed as

$$\psi_{T\alpha}{}^i{}_j \to \psi_{Q\alpha}^i \lambda^\beta \psi_{\overline{Q}\beta j} . \qquad (3.15)$$

With this replacement, the composite fermions are built only out of constituents which remain massless as the squarks are decoupled. Thus, it is apriori reasonable that the $b = 0$ vacuum of the R model could go smoothly into a vacuum of the purely fermionic QCD theory described above. This vacuum would have broken $SU(N_f) \times SU(N_f)$ but unbroken chiral $U(1)_R$, zero values for the vacuum expectation values of quark-antiquark bilinears, massless composite fermions in the adjoint representation of flavor $SU(N_f)$, and massless baryons. We will refer to this scenario as 'option 1'.

The other possibility for this model is that, after the squarks decouple, the gluino fields pair-condense, in a second-order phase transition at some value of m_Q^2, and the nonzero value

of the condensate $\langle \lambda \cdot \lambda \rangle$ spontaneously breaks $U(1)_R$. In this case, the physics would revert to the usual symmetry-breaking pattern of QCD, and the composite fermions would become massive. The gluino condensate would make itself felt only by providing an extra $SU(N_f)$-singlet Goldstone boson. We will refer to this scenario as 'option 2'.

(iii) $\not{R}$ **T=0**- In this model, in the supersymmetric limit, the massless composite fermions belong to the $(N_f, \overline{N_f})$ representation of an unbroken flavor group $SU(N_f) \times SU(N_f)$. There are no constraints from the $U(1)_R$ symmetry, which is explicitly broken, or from baryon number, which is spontaneously broken. With this freedom, can we build these fermionic composites out of fields that survive in the decoupling limit $m_Q^2, m_g \to \infty$? For N_c even, it is impossible, because the only constituents available are the quarks $\psi_Q^i, \psi_{\overline{Q}j}$, and gauge-invariant states must contain an even number of these. For N_c odd, however, it is possible to build composites with the correct quantum numbers, as follows:

$$\psi_{T\alpha}{}^i{}_j \to \epsilon^{ab\cdots d} \psi_{Q\alpha a}^i \, \epsilon_{jk\cdots \ell} \overline{\psi}_{\overline{Q}b}^{\dot{\beta}k} \cdots \overline{\psi}_{\overline{Q}\dot{\gamma}d}^{\ell} \, , \tag{3.16}$$

where $\overline{\psi}_{\overline{Q}}$ is the right-handed fermion field in $(\overline{Q})^*$. The $(N_c - 1)$ right-handed fermion fields must be contracted into a Lorentz scalar combination. For the case $N_f = N_c = 3$, eight of the nine fermions in (3.16) have the quantum numbers of the baryon octet in QCD.

However, in this case, there are two compelling arguments that the spectrum which we find cannot survive to the decoupling limit. In the limit $m_Q^2 \to \infty$, even without introducing m_g, we have a vectorlike gauge theory of fermions. For such theories, the QCD inequalities of Weingarten [10] and Vafa and Witten [11] apply. In [2] we use Weingarten's method to prove that, in the decoupling limit, flavor nonsinglet composite fermions must be heavier than the pions, which are massive in the $T = 0$ vacuum. Alternatively, we can apply the theorem of Vafa and Witten in the decoupling limit to show that vectorlike global symmetries, in particular, baryon number, cannot be spontaneously broken.

By either argument, the $T = 0$ vacuum state must disappear via a second order phase transition at a finite value of m_Q^2. Most likely, this vacuum becomes locally unstable with respect to a decrease in the expectation value of b, driving the theory back to the more familiar vacuum at $b = 0$.

(iv) R **T=0**- In the supersymmetric limit we have massless fermions in the following representations $(N_f, \overline{N_f}, -1) + (1, 1, -1)$ of the unbroken symmetry group. corresponding to the fermions in T and a linear combination of the fermions in B and $\overline{B}$. Both multiplets are necessary to satisfy the anomaly conditions involving $U(1)_R$. The arguments that we have just presented for the $T = 0$ vacuum in the $\not{R}$ models apply equally well to the R case. Again, we must have a second-order transition, probably with an instability to the $b = 0$ vacuum. There are then two possible endpoints, depending on which option is chosen for the $b = 0$ vacuum. If the option 1 for the $b = 0$ vacuum is correct, it is not necessary that $U(1)_R$ be spontaneously broken in this transition.

4   No χSB, massless fermions ($N_f = N_c + 1$)

In the case of $N_f = N_c + 1$, the low-energy effective Lagrangian in the supersymmetric limit is expressed in terms of the baryon, anti-baryon and meson superfields which transform under the global symmetry as follows (2.1)

$$B \ : \ (\overline{N_f}, 1)_{1,1-\frac{1}{N_f}} \qquad \overline{B} \ : \ (1, N_f)_{-1,1-\frac{1}{N_f}} \qquad T \ : \ (N_f, \overline{N_f})_{0,\frac{2}{N_f}} \qquad (4.17)$$

where the second subscript is the R charge of the scalar component of the superfield. In the supersymmetric theory the low energy effective theory is described (at least near the origin of moduli space) by the Kähler potential given by (2.5), and by the following superpotential [13]:

$$W = B_i T_j^i \overline{B}^j - \det T. \qquad (4.18)$$

The supersymmetric vacuum is, thus, described by a moduli space characterized by

$$B_i T_j^i \ = \ 0 \qquad T_i^j \overline{B}^i \ = \ 0 \qquad \frac{1}{N_c!} \epsilon_{i_1, \dots, i_{N_f}} \epsilon^{j_1, \dots, j_{N_f}} T_{j_1}^{i_1} \dots T_{j_{N_c}}^{i_{N_c}} - B_{i_{N_f}} \overline{B}^{j_{N_f}} = 0. \qquad (4.19)$$

As was argued in [13], these equations correctly describe the moduli space of vacuum states in the full quantum theory. At the origin of the moduli space, $< T > = < B > = < \overline{B} > = 0$, where the full global symmetry (2.1) remains unbroken, there is a further consistency check for the low energy behavior. The fermionic components of the low-energy superfields (4.17) match the global anomalies of the underlying theory.

When we break supersymmetry by squark and gluino masses, we add to the effective Lagrangian the mass terms for T, B and $\overline{B}$ indicated in (2.8). Since we are adding terms to the potential which are positive and vanish at the origin of moduli space, it is obvious that the origin becomes the only vacuum state of the theory. All of the scalar particles in the effective theory obtain mass terms proportional to $B_T m_Q^2$ or $B_B m_Q^2$.

Though all of the scalars obtain mass, all of the fermions remain massless. The superpotential (4.18) is a least cubic in the fields, thus, any mass term derived from this superpotential vanishes at the origin. Similarly, in the $\not{R}$ case, the M_g term in (2.8) requires a function of T, B, and $\overline{B}$ which is neutral with respect to the global group; the only such functions, quadratic in fields, are $\text{tr}[T^\dagger T]$, $B^\dagger B$, and $\overline{B}^\dagger \overline{B}$, and these do not give rise to fermion masses when multiplied with M_g. In fact, it is required that no fermions should obtain mass, since the full multiplet of fermions in T, B, and $\overline{B}$ is needed to satisfy the 't Hooft anomaly conditions for the global symmetry group $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$.

5   Seiberg's duality in SSB models ($N_f \geq N_c + 2$)

No solution of the 't Hooft anomaly matching conditions for SQCD involving gauge invariant bound states is known for $N_f > (N_c + 1)$. However, Seiberg has suggested a compelling

solution to these constraints in terms of a novel type of *duality* in which new gauge degrees of freedom are dual to the original quarks and gluons [1]. The pair of dual theories are

"Electric theory"	"Magnetic theory"
$SU(N_c)$ gauge theory	$SU(N_f - N_c)$ gauge theory
$a = 1, \ldots, N_c$	$\hat{a} = 1, \ldots, (N_f - N_c)$
$Q_a^i, (N_f, \ 1, \ \frac{1}{N_c}, \ (N_f - N_c)/N_f)$	$q_i^{\hat{a}}, (\overline{N_f}, \ 1, \ \frac{1}{N_f - N_c}, \ N_c/N_f)$
$\overline{Q}_i^a (1, \ \overline{N_f}, \ -\frac{1}{N_c}, \ (N_f - N_c)/N_f)$	$\overline{q}_{\hat{a}}^i, (1, \ N_f, \ -\frac{1}{N_f - N_c}, \ N_c/N_f \ )$
	$T_j^i (N_f, \ \overline{N_f}, \ 0, \ 2(N_f - N_c)/N_f)$
$W = 0$	$W = T^i_{\ j} q_i^{\hat{a}} \overline{q}_{\hat{a}}^j$

On top of the fact that these theories have the same 't Hooft anomalies, they were shown to be equivalent in the infrared also by analyzing their "chiral rings", flat directions and mass perturbations. In the supersymmetric limit of the magnetic theory the scalar potential has a moduli space of vacua, which includes the point $< T >=< q >=< \overline{q} >= 0$ at which the chiral symmetry is unbroken [1].

Now add squark masses to the theory. Their effect should be seen in the effective Lagrangian. In case that the latter is given by the dual theory we should add to the effective Lagrangian of the dual theory the term

$$\Delta V = B_T m_Q^2 \mathrm{tr}(T^\dagger T) + B_q m_Q^2 (|q|^2 + |\overline{q}|^2), \tag{5.20}$$

at least near the origin of moduli space. After we add this perturbation, the only minimum of the potential is at $< T >=< q >=< \overline{q} >= 0$. Thus, adding a squark mass leaves the theory in the phase in which the chiral symmetry is unbroken. All scalars get masses (originating only from ΔV, since the original scalar potential is quartic in the fields), while all fermions remain massless. As in the original supersymmetric theory, this complement of massless fermions has just the right quantum numbers to satisfy the 't Hooft anomaly conditions for completely unbroken chiral symmetry.
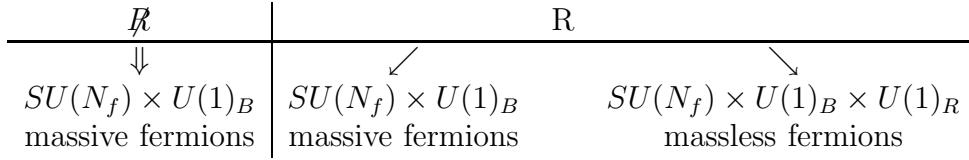
The glueball operator $\mathrm{tr}(W_\alpha^2)$ is identified (up to a sign) between the original and the dual theory [12]. Thus, to leading order in m_g, a gluino mass in the original theory is just equal to a gluino mass in the dual theory. Adding this term breaks the $U(1)_R$ symmetry, but the $SU(N_f) \times SU(N_f)$ global symmetry still remains and protects the dual quarks from getting a mass. Thus, we find the same spectrum in the R and $\not{R}$ cases, except that in the latter case the dual gluino, which can be an asymptotic particle, becomes massive.

Let us discuss now the infrared description of the theory. For small m_Q (and small m_g, in the $\not{R}$ case) the situation is schematically described in the following diagram

electric theory				**AS free** *		:	**IR free**
$\|$	$\Downarrow$	:	*	*		:	$\Uparrow$
N_c+2	$\Downarrow$	:	*	*		:	$\Uparrow$
	$\Downarrow$	$\frac{11}{9}N_c$	*	**IR**	*	$\frac{11}{2}N_c$	$\Uparrow$
	$\Downarrow$	:	$\frac{9}{7}N_c$	**fixed**	$\frac{9}{2}N_c$	:	$\Uparrow$
	$\Downarrow$	:	*	**point**	*	:	$\Uparrow$
$\|$	$\Downarrow$	:	*	*		:	$\Uparrow$
magnetic theory	**IR free**	:	*	**AS free**			

The picture describes a scenario where in the range between the : signs, namely $\frac{11}{9}N_c < N_f < \frac{11}{2}N_c$ for the R case and between the $*$ signs, namely, $\frac{9}{7}N_c < N_f < \frac{9}{2}N_c$ for the $\not{R}$ case, the two theories flow to a common fixed point in the IR. In the range $N_f < \frac{11}{9}N_c$ ($N_f < \frac{9}{7}N_c$) the R ($\not{R}$) system flows to the free magnetic theory in the IR, whereas for $N_f > \frac{11}{2}N_c$ ($N_f > \frac{9}{2}N_c$) the picture in the infrared for R ($\not{R}$) is that of the free electric theory.

The discussion of the decoupling limit for these theories is very similar to that for the $N_f = N_c$ theory. Thus, for those values of N_f which have massless fermions for small values of m_Q we must have a second-order phase transition as m_Q is increased. It is not clear how the theory behaves on the other side of this phase transition. In the $\not{R}$ case obviously only a $SU(N_f) \times U(1)_B$ symmetry remains, with no massless fermions. This situation is schematically described as follows

$\not{R}$	R	
$\Downarrow$	$\nearrow$	$\searrow$
$SU(N_f) \times U(1)_B$	$SU(N_f) \times U(1)_B$	$SU(N_f) \times U(1)_B \times U(1)_R$
massive fermions	massive fermions	massless fermions

6 Problems of Approximate Supersymmetry on the Lattice

(i) Can the phenomena we have discussed in this work be seen in lattice gauge theory simulations? In general, gauge theories on the lattice cannot be made supersymmetric at the fundamental level. We expect that lattice simulations of these theories will also contain small dimension 4 perturbations which violate supersymmetry. Our analysis has been based on the assumption that, if the phenomena discussed by Seiberg survive perturbations which are relevant in the infrared, they should also survive small marginal perturbations.

(ii) Can one reach the continuum limit in such lattice simulations? Typically in lattice gauge theory simulations with scalar fields, there is no continuum limit; instead, one finds a first order phase transition as a function of the scalar field mass parameter [14]. Renormalization effects in a gauge theory can induce an unstable potential for a scalar field coupled to the gauge bosons, leading to a 'fluctuation-induced first-order phase transition'. How can this be avoided?

(iii) To analyze this question, consider the renormalization group equations for an approximately supersymmetric gauge theory. Viewed as a conventional renormalizable gauge theory, SQCD has three coupling constants, the gauge coupling g, the quark-squark-gluino coupling g_λ, and the four-scalar coupling g_D. The scalar potential has the specific form

$$V = \frac{g_D^2}{2}\left[Q^\dagger \tau^A Q - \overline{Q}\tau^A \overline{Q}^\dagger\right]^2 , \tag{6.21}$$

where τ^A is an $SU(N_c)$ matrix. If we relax the constraint of supersymmetry, there are four possible invariants under the symmetries of the problem, including the continuous global symmetries and parity $Q \leftrightarrow \overline{Q}^\dagger$. The most general linear combination of these invariants can be generated by the renormalization group flow.

We can view the effects of renormalization relatively simply, however, by restricting our attention to the vicinity of the surface given by the three couplings g, g_λ, and g_D. In this surface, the beta functions of the three couplings are given by

$$
\begin{aligned}
\beta_g &= -\frac{1}{(4\pi)^2}[3N_c - N_f]g^3 \\
\beta_{g_\lambda} &= -\frac{1}{(4\pi)^2}\left[g_\lambda g^2(3N_c + 3C_2(N_c)) - g_\lambda^3(3C_2(N_c) + N_f)\right] \\
\beta_{g_D^2} &= -\frac{1}{(4\pi)^2}\left[4g_\lambda^4 N_c + 2g_D^4(N_c - N_f - 2C_2(N_c)) + 12g_D^2 g^2 C_2(N_c) - 8g_D^2 g_\lambda^2 C_2(N_c)\right] ,
\end{aligned}
$$

where $C_2(N_c) = (N_c^2 - 1)/2N_c$. These three functions all reduce to the standard SQCD beta function on the supersymmetric subspace; for $g^2 = g_\lambda^2 = g_D^2$, $\beta_g = \beta_{g_\lambda} = \beta_{g_D^2}/2g$. Note that, for $N_c \sim N_f$ and the three couplings in reasonable ratio, all three couplings are asymptotically free.

The potential instability to a first order phase transition arises because a new structure in the potential is induced by the renormalization group flow. To lowest order, the form of the potential induced is

$$V_E = \frac{g_E^2}{2}\left[Q^\dagger\{\tau^A, \tau^B\}Q + \overline{Q}\{\tau^A, \tau^B\}\overline{Q}^\dagger\right]^2 . \tag{6.22}$$

On the surface $g_E^2 = 0$, the beta function for g_E^2 is

$$\beta_{g_E^2} = -\frac{1}{(4\pi)^2}\left[4g_\lambda^4 - 3g^4 - g_D^4\right]. \tag{6.23}$$

This equation implies that, if one leaves out the gluinos, g_E^2 becomes negative in the infrared, leading to a fluctuation-induced first-order phase transition. According to (6.23), this effect is removed if the lattice simulation includes gluinos, and if the gluino coupling g_λ is large enough.

With this provision to avoid possible first-order phase transitions, we expect that lattice simulations with an approximately supersymmetric action can reach the continuum limit and test our predictions for softly broken supersymmetric QCD.

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