SLAC-PUB-9920
SU-ITP-96-17
hep-th/9604193
May 5, 2003

BRANE-ANTI-BRANE DEMOCRACY

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ABSTRACT

We suggest a duality invariant formula for the entropy and temperature of non-extreme black holes in supersymmetric string theory. The entropy is given in terms of the duality invariant parameter of the deviation from extremality and 56 SU(8) covariant central charges. It interpolates between the entropies of Schwarzschild solution and extremal solutions with various amount of unbroken supersymmetries and therefore serves for classification of black holes in supersymmetric string theories. We introduce the second auxiliary 56 via E(7) symmetric constraint. The symmetric and antisymmetric combinations of these two multiplets are related via moduli to the corresponding two fundamental representations of E(7): brane and anti-brane "numbers." Using the CPT as well as C symmetry of the entropy formula and duality one can explain the mysterious simplicity of the non-extreme black hole area formula in terms of branes and anti-branes.

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Work supported in part by Department of Energy Contract DE-AC03-76SF00515
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1 Introduction

The concept of p-brane democracy was introduced by Townsend [1]. The basic idea is that the non-perturbative d=4 string theory treats various p-branes on equal footing. U-duality [2], which is related to a discrete version of E(7) symmetry of N=8 supergravity [3], removes the distinction between $p = 1$ wrapping modes of the string and other p-branes with $p > 1$ of $d = 10$ theory and their wrapping modes. U-duality manifests itself in particular in the symmetry of the entropy of extreme black holes under $E(7; \mathbf{Z})$ [4]. As a result of 1/8 of unbroken supersymmetry one can prove [5] that the entropy depends only on the conserved charges which transform as one 56-dimensional fundamental multiplet of E(7). The entropy is therefore given by the unique quartic invariant of E(7) constructed from one fundamental multiplet of charges. The independence of the moduli at fixed values of conserved charges follows directly from unbroken supersymmetry. An analogous picture was found in $d = 5$ where the corresponding entropy is given by a triple invariant of $E(6)$ constructed from one 27-dimensional fundamental representation of E(6) [6, 5]. It is worth mentioning that in each case the unique expression for the entropy is protected by the supersymmetric non-renormalization theorem from quantum corrections.

In view of these facts it looked rather puzzling that the entropy formulas for non-extreme black holes in $d = 5$ [7] and in $d = 4$ [8] have been recently suggested in a form of triple and quartic invariants respectively based in each case on two fundamental representations of E(6) (E(7)) in $d = 5$ ($d = 4$). The first 27 (56) in d=5 (d=4) corresponds to the number of solitons and the second one to the number of anti-solitons.

The purpose of this letter is first to obtain the duality invariant entropy and temperature formulas for non-extreme black holes of N=8 supergravity in d=4. We will present a simple formula for the product of the entropy and temperature ST in terms of the eigenvalues of the supersymmetry charges. The formula for the entropy will involve only the mass of the black hole and the central charge matrix and will generalize the previously known formulas of this kind for N=4 supergravity. There is nothing puzzling about this formula since there is only one $N \times N$ antisymmetric complex central charge matrix Z_{AB} with $A, B = 1, \ldots, N$ in the theory and no obvious room for branes and anti-branes. The central charge is invariant under E(7) and transforms covariantly under the global $SU(8)$ symmetry.

Starting from this formula we will introduce the second $SU(8)$ covariant charge Y_{AB} as a function of the parameter of the deviation of the theory from extremality r_0 and of the central charge Z_{AB}. The total procedure is very much in spirit of Dirac's treatment of constrained systems: originally one has more variables then necessary to describe the physical states. The symmetries of the theory have enough room to be manifest when all of these variables are present. The variables are constrained, however. If one would like to describe the system only in terms of unconstrained variables one has to solve the constraints and break some of the symmetries.

Our auxiliary variable Y_{AB} will serve the following purpose: whereas the central charge Z_{AB} will be represented as an antisymmetric function of branes and anti-branes, which changes its sign when branes are changed into anti-branes, the auxiliary variable Y_{AB} will be constructed as a symmetric function of branes and anti-branes.

Starting with our two 56 of SU(8) which have a very clear interpretation via an E(7) symmetric constraint between Z's and Y's, we will then rewrite our entropy formula in terms of a symmetric function of 56 "brane-numbers" and 56 "anti-brane-numbers" as in [7, 8] using the quartic invariant of E(7) applied to two multiplets. In this way we will confirm the entropy formulas suggested in [7, 8] and explain the origin of the two multiplets in a covariant manner.

Moreover, our way of derivation of the formulas of the non-extreme black holes will suggest the new interpretation which will go under the name "brane-anti-brane democracy" and will state not only the equal rights of various p-branes but also the equal rights of states with positive and negative charges. This can be understood as follows. For the non-extreme configurations the integer charges $(p^I, q_I),\ I = 1, \ldots, 28$, will be given by the antisymmetric combinations of branes and anti-branes

$$p^I = m^I - \bar{m}^I \ , \qquad q_I = n_I - \bar{n}_I \ . \tag{1}$$

This means that the charges can be positive or negative, depending on whether $n > \bar{n}$ and $m > \bar{m}$ or vice versa. The non-extreme entropy will depend on (m^I, n_I) and on $(\bar{m}^I, \bar{n}_I)$ in some symmetric way, which reflects the degeneracy of the non-extreme black holes entropy on the sign of charges.

For the extreme solutions again the charges can be positive or negative in each of the 28 groups, the entropy being degenerate in this signs. Note that the brane (m^I, n_I) and anti-brane numbers $(\bar{m}^I, \bar{n}_I)$ can only be non-negative numbers in the extreme limit. The charges are given by the number of branes

$$(p^I)_{\rm extr} = (|p^I|)_{\rm extr} = m^I \ , \qquad (q_I)_{\rm extr} = (|q_I|)_{\rm extr} = n_I \ , \tag{2}$$

for those values of I for which the charges are positive and by the number of anti-branes

$$(p^I)_{\rm extr} = (-|p^I|)_{\rm extr} = -\bar{m}^I \ , \qquad (q_I)_{\rm extr} = (-|q_I|)_{\rm extr} = -\bar{n}_I \ , \tag{3}$$

for those values of I for which the charges are negative. Therefore the entropy for all solutions with all possible values of charges in all gauge groups in extreme limit will depend only on absolute values of all charges $(|p^I|, |q_I|)$. Since we will consider even non-extreme black holes in the context of supersymmetric theories, the CPT as well as C transformation will be defined in terms of the transformations of the supersymmetry generators. The CPT transformation on supersymmetry charges acts as follows: $Q \rightarrow iQ^*$. This results in $P_\mu \rightarrow P_\mu$ and $Z \rightarrow -Z^*$. This is realized in terms of charges as $p \rightarrow p$ and $q \rightarrow -q$. Therefore CPT on branes and anti-branes act as follows:

$$n \Longleftrightarrow \bar{n} \ , \qquad m \Longleftrightarrow m \ , \qquad \bar{m} \Longleftrightarrow \bar{m} \ . \tag{4}$$

The charge conjugation C acts on supersymmetry generators as follows: $Q \rightarrow iQ$ and $Q^* \rightarrow -iQ^*$, which results in $P_\mu \rightarrow P_\mu,\ Z \rightarrow -Z,\ Z^* \rightarrow -Z^*$. This leads to

$$n \Longleftrightarrow \bar{n} \ , \qquad m \Longleftrightarrow \bar{m} \ . \tag{5}$$

2 Supersymmetry algebra, temperature and entropy of non-extreme black holes

There exists a simple relation [9] between the product of temperature T and entropy S of black holes in supersymmetric theories and the parameter of the deviation of the theory from extremality r_0:

$$2\pi S\,T = r_0 \equiv \frac{r_+ - r_-}{2}\ . \tag{6}$$

This parameter r_0 defines the distance between the event horizon r_+ and the inner horizon r_- of the non-extreme black holes.

We will derive the universal formula for $2\pi S\,T = r_0$ in terms of supersymmetry charges by using the symplectic form of the supersymmetry algebra [10]. The d=4 N-extended supersymmetry algebra is most conveniently described for massive states at rest in terms of $2N$-component spinors. In doublet form they are given by

$$Q^a_\alpha = \begin{pmatrix} Q_{\alpha A} \\ \\ Q^{*\alpha A} \end{pmatrix}, \qquad \begin{array}{l} Q^a_\alpha = Q_{\alpha A} \quad \text{for} \quad a = 1,\dots,N, \qquad \alpha,\beta = 1,2\ , \\ \\ Q^a_\alpha = Q^{*\alpha A} = \epsilon^{\alpha\beta} Q^{*A}_\beta \quad \text{for} \quad a = N+1,\dots,2N\ . \end{array} \tag{7}$$

These spinors satisfy a symplectic reality condition $Q^{*a}_\alpha = \epsilon^{\alpha\beta}\Omega_{ab}Q^b_\beta$ with

$$\epsilon^{\alpha\beta} = -\epsilon_{\alpha\beta} = (i\sigma_2)_{\alpha\beta}, \qquad \Omega^{ab} = -\Omega_{ab} = \begin{pmatrix} 0 & \mathrm{I} \\ -\mathrm{I} & 0 \end{pmatrix}\ , \tag{8}$$

and the supersymmetry algebra in a symplectic form is

$$\{Q^a_\alpha, Q^b_\beta\} = \epsilon_{\alpha\beta} \begin{pmatrix} Z & M\mathrm{I} \\ \\ -M\mathrm{I} & Z^* \end{pmatrix} \equiv \epsilon_{\alpha\beta}\ \Lambda^{ab}\ . \tag{9}$$

The $2N \times 2N$ matrix Λ^{ab} is written in terms of $N \times N$ numerical antisymmetric complex matrix $Z_{AB},, Z^{*AB}$ and the mass. The numbers Z_{AB} are the eigenvalues of the central charge operators in a given supermultiplet. Any complex antisymmetric matrix Z_{AB} can be brought to the normal form using some $U(N)$ rotation [11]. For example, for $N = 4$ and for $N = 8$ respectively we get

$$\tilde{Z}_{AB} = i\sigma_2 \begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix}, \qquad \tilde{Z}_{AB} = i\sigma_2 \begin{pmatrix} z_1 & 0 & 0 & 0 \\ 0 & z_2 & 0 & 0 \\ 0 & 0 & z_3 & 0 \\ 0 & 0 & 0 & z_4 \end{pmatrix}, \tag{10}$$

where z_i are non-negative real numbers, $i = 1, \dots N/2$. We used the following notation: $\tilde{Z}_{12} = -\tilde{Z}_{21} = z_1, \dots, \tilde{Z}_{78} = -\tilde{Z}_{87} = z_4$.

In terms of the mass of the black hole and the central charge the product of the entropy and temperature for N=4 case can be written as

$$(2\pi S\,T)^2 = r_0^2 \equiv \frac{(M^2 - |z_1|^2)(M^2 - |z_2|^2)}{M^2} \geq 0\ , \tag{11}$$

and for N=8 case as

$$(2\pi S\,T)^2 = r_0^2 \equiv \frac{(M^2-|z_1|^2)(M^2-|z_2|^2)(M^2-|z_3|^2)(M^2-|z_4|^2)}{M^6} \geq 0\ . \tag{12}$$

The product of temperature and entropy then has a simple representations in terms of the eigenvalues of the supersymmetry generators.

$$(2\pi S\,T)^2 = r_0^2 = \frac{det^{1/2}\{Q_\alpha^c, Q_\gamma^b\}}{[\frac{1}{4N}\Omega_{ac}\epsilon^{\beta\gamma}\{Q_\alpha^c, Q_\gamma^b\}]^{N-2}}\ . \tag{13}$$

This formula is completely symmetric in terms of 32 supersymmetry generators of N=1, d=11 supersymmetry in case we study N=8, d=4 theory and symmetric in 16 supersymmetry generators of N=1, d=10 theory when we study N=4, d=4 theory.

As long as $2\pi S\,T = r_0 \neq 0$ the black holes are non-extreme. The extreme ones have $r_0 = 0$, which means that some supersymmetry charges have to have zero eigenvalues. Those with the non-vanishing area of the horizon and entropy have vanishing temperature.

$$r_0 = 0\ , \qquad T = 0\ , \qquad S \neq 0\ . \tag{14}$$

The extreme ones with the vanishing area of the horizon $S = A/4\pi = 0$ usually do not have a well defined temperature since the horizon is singular.

The analysis performed in this section shows that one can describe the geometry starting with the simplest solutions related to the normal form of the central charge matrix Z_{AB} and derive the formulas of the type (11), (12). The result can then be generalized to the form in which it is no longer necessary to assume that the central charge matrix is diagonal.

3 Thermodynamics of black holes in N=2,4,8 supergravity

In this section we will use the known results for the non-extreme black holes in N=4 supergravity [9] and represent them in the form suitable for generalization to N=8 theory. The entropy given by 1/4 of the area was found in [9] to be equal to

$$S = \pi(r_0 + M + \Sigma)(r_0 + M - \Sigma)\ , \tag{15}$$

where Σ is the dilaton charge of the black hole. Using the fact that

$$\Sigma = -\frac{z_1 z_2}{M}$$

one can show that the entropy is equal to

$$S_{N=4} = \pi \left(r_0 + \sqrt{r_0^2 + (z_1 + z_2)^2} \right) \left(r_0 + \sqrt{r_0^2 + (z_1 - z_2)^2} \right) \ . \tag{16}$$

It is easy to get the entropy formula in pure the N=2 supergravity by setting $z_2 = 0$

$$S_{N=2} = \pi \left(r_0 + \sqrt{r_0^2 + (z_1)^2} \right)^2 \ . \tag{17}$$

The generalization of N=4 formula to the N=8 theory is straightforward

$$\begin{aligned} S_{N=8} = \pi & \left(r_0 + \sqrt{r_0^2 + (z_1 + z_2 + z_3 + z_4)^2} \right)^{1/2} \left(r_0 + \sqrt{r_0^2 + (z_1 + z_2 - z_3 - z_4)^2} \right)^{1/2} \\ & \left(r_0 + \sqrt{r_0^2 + (z_1 - z_2 + z_3 - z_4)^2} \right)^{1/2} \left(r_0 + \sqrt{r_0^2 + (z_1 - z_2 - z_3 + z_4)^2} \right)^{1/2} \ . \end{aligned} \tag{18}$$

This expression agrees with the non-extreme black hole solution with four-charges [12] upon identification of central charges of N=8 theory performed in [4]. There is one more important criterion for the validity of this formula: at the extreme limit $r_0 = 0$ the mass of the solution becomes equal to the largest of the eigenvalues, for example, $z_1 \equiv Z$, which depends on the conserved charges and on moduli:

$$M_{\rm extr} = |Z\left((p,q), \phi_\infty\right)| \ . \tag{19}$$

According to universality of the supersymmetric attractors [5], near the horizon all central charges besides the largest one $|Z|$ have to vanish, i.e. $z_2 = z_3 = z_4 = 0$, and any of these expressions for N=2,4,8 has to reduce to

$$S_{\rm extr} = \pi |Z\left((p,q), \phi_{\rm h}[(p,q)]\right)| \ , \tag{20}$$

so that the entropy depends only on conserved charges (56 in N=8, 12 in N=4, 2 in N=2). The central charge Z_{AB} enters the local supersymmetry transformations rules, since it is a charge of the graviphoton.

To switch from the SU(8) basis to the SO(8) basis in the general case of N=8 theory when neither Z_{AB} nor ζ_{ij} are in the normal form, we will use

$$\zeta_{ij} = \frac{1}{2\sqrt{2}} Z_{AB} (\gamma^{ij})_A{}^B \tag{21}$$

with the inverse relation

$$Z_{AB} = \frac{1}{4\sqrt{2}} \zeta_{ij} (\gamma^{ij})_A{}^B \ , \tag{22}$$

where the matrices $(\gamma^{ij})_A{}^B$, $i,j = 1,\ldots,8$, form the algebra of $SO(8)$. In the normal frame they corresponds to linear combinations

$$\begin{aligned} \zeta_1 \equiv \zeta_{12} &= \frac{1}{\sqrt{2}}(z_1 + z_2 + z_3 + z_4) \ , \qquad \zeta_2 \equiv \zeta_{34} = \frac{1}{\sqrt{2}}(z_1 + z_2 - z_3 - z_4) \ , \\ \zeta_3 \equiv \zeta_{56} &= \frac{1}{\sqrt{2}}(z_1 - z_2 + z_3 - z_4) \ , \qquad \zeta_4 \equiv \zeta_{78} = \frac{1}{\sqrt{2}}(z_1 - z_2 + z_3 - z_4) \ . \end{aligned} \tag{23}$$

The entropy formula can now be rewritten as follows:

$$S = \pi(\det)^{1/4}\left\{r_0\delta_{ik} + \sqrt{(r_0)^2\delta_{ik} - 2\zeta_{ij}\zeta^*_{jk}}\right\} \ . \tag{24}$$

Here we can also use the diagonal basis, and consider the case where only $\zeta_1 = \zeta_2$ and $\zeta_3 = \zeta_4$ are not vanishing (they still can be complex) and we get

$$S = \prod_{i=1}^{4}\left\{r_0 + \sqrt{(r_0)^2 + 2\zeta_i\zeta^*_i}\right\}^{1/2} \ . \tag{25}$$

This formula describes in particular the entropy of the non-extreme $U(1)\times U(1)$ axion-dilaton black holes in which both gauge groups have electric as well as magnetic charges [9].

Now we can introduce the auxiliary $SU(8)$ multiplet (Y_{AB}, Y^{*AB}), which is a function of r_0 and (Z_{AB}, Z^{*AB}). First, let us again switch to the $SO(8)$ basis

$$Y_{AB} = \frac{1}{4\sqrt{2}}\Upsilon_{ij}(\gamma^{ij})_A{}^B \ , \qquad \Upsilon_{ij} = \frac{1}{2\sqrt{2}}Y_{AB}(\gamma^{ij})_A{}^B \ . \tag{26}$$

The definition of our new multiplet Y is the following:

$$(r_0)^2\delta_{ik} - 2\zeta_{ij}\zeta^*_{jk} \equiv -2\Upsilon_{ij}\Upsilon^*_{jk} \ . \tag{27}$$

Thus we have a non-linear relation (27) between the parameter of deviation of extremality, central charges of the theory (Z_{AB}, Z^{*AB}) and new auxiliary $SU(8)$ multiplet (Y_{AB}, Y^{*AB}). The expression for the entropy which was given only in terms of r_0 and central charges can be rewritten using the auxiliary multiplet Υ as

$$S = \pi(\det)^{1/4}\left\{r_0\delta_{ik} + \sqrt{-2\Upsilon_{ij}\Upsilon^*_{jk}}\right\} \ . \tag{28}$$

It will be useful from now to use the basis with 28 real electric and 28 real magnetic charges

$$\zeta_{ij} = Q_{ij} + iP^{ij} \ , \qquad \Upsilon_{ij} = Q'_{ij} + iP'^{ij} \ , \tag{29}$$

$$\zeta^{*ij} = Q_{ij} - iP^{ij} \ , \qquad \Upsilon^{*ij} = Q'_{ij} - iP'^{ij} \ . \tag{30}$$

The charges $A \equiv (P^{ij}, Q_{ij})$ are related to the conserved charges $a \equiv (p^{ij}, q_{ij})$ via moduli $A = Va$. For our auxiliary multiplet $A' \equiv (P'^{ij}, Q'_{ij})$ we will assume the same relation $A' = Va'$ with $a' \equiv (p'^{ij}, q'_{ij})$, it is consistent with the constraint (27). One more step is to define the symmetric and antisymmetric combinations of the original and the auxiliary multiplets:

$$P'^{ij} + P^{ij} = M^{ij} \ , \qquad Q'_{ij} + Q_{ij} = N_{ij} \ , \tag{31}$$
$$P'^{ij} - P^{ij} = \bar{M}^{ij} \ , \qquad Q'_{ij} - Q_{ij} = \bar{N}_{ij} \ . \tag{32}$$

Our constraint (27) simplifies to

$$(r_0)^2\delta_{ik} = -2(M^{ij}\bar{M}^{jk} + N_{ij}\bar{N}_{jk}) \ . \tag{33}$$

One can exclude r_0 completely, using the constraint (33), and rewrite the entropy formula entirely in terms of these two 56s of $SU(8)$ (M,N) and $(\bar{M},\bar{N})$ as follows:

$$S = 2\pi(\det)^{1/4}\left\{2\sqrt{M^{ij}\bar{M}^{jk}+N_{ij}\bar{N}_{jk}}+\sqrt{(M+\bar{M})^{ij}(M+\bar{M})^{jk}+(N+\bar{N})_{ij}(N+\bar{N}_{jk})}\right\} . \tag{34}$$

To rewrite this formula in terms of 56 branes and 56 anti-branes we have to review some elements of N=8 theory [3]. We consider the theory in the symmetric gauge with fixed local $SU(8)$ symmetry. In this gauge the scalars are taken to be the coordinates of the coset space $\frac{E_7}{SU(8)}$. The matrix V describing the scalars forms an element of E(7) before the local gauge fixing and has 133 entries, but when unitarity constraint $V = V^\dagger$ is imposed, it depends only on 35 complex scalars. In this gauge the hidden symmetry acts on the charges as well as on moduli as follows

$$a = \begin{pmatrix} p \\ q \end{pmatrix} , \qquad a \longrightarrow Ea , \qquad V \longrightarrow hVE^{-1} , \qquad A = Va = \begin{pmatrix} P \\ Q \end{pmatrix} \longrightarrow hA . \tag{35}$$

Here E is an element of $E(7)$ and h is the global $SU(8)$ which is the residual symmetry after gauge fixing, which preserves the gauge. It follows that the global symmetry of $A = Va$ is the $SU(8)$ symmetry. The same transformation rules apply to the the auxiliary multiplet of E(7) , a' and the corresponding $SU(8)$ partner of it with E(7) blind, which transforms only under $SU(8)$, A'.

$$a' = \begin{pmatrix} p'^I \\ q'_I \end{pmatrix} , \qquad a \longrightarrow Ea' , \qquad V \longrightarrow hVE^{-1} , \qquad A' = Va' = \begin{pmatrix} P' \\ Q' \end{pmatrix} \longrightarrow hA' . \tag{36}$$

As already explained in the Introduction, we will build a symmetric and antisymmetric combination of our two E(7) multiplets and call them branes and anti-branes, respectively.

$$a = \begin{pmatrix} p^{ij} \\ q_{ij} \end{pmatrix} = \begin{pmatrix} (m-\bar{m})^{ij} \\ (n-\bar{n})_{ij} \end{pmatrix} , \qquad a' = \begin{pmatrix} p'^{ij} \\ q'_{ij} \end{pmatrix} = \begin{pmatrix} (m+\bar{m})^{ij} \\ (n+\bar{n})_{ij} \end{pmatrix} . \tag{37}$$

The symmetric and antisymmetric $SU(8)$ partners of these two E(7) multiplets are

$$A' + A = \begin{pmatrix} M^{ij} \\ N'_{ij} \end{pmatrix} , \qquad A' - A = \begin{pmatrix} \bar{M}^{ij} \\ \bar{N}'_{ij} \end{pmatrix} . \tag{38}$$

The moduli matrix $V = V^\dagger$ in the symmetric gauge is a function of the matrix $y_{ij,kl}$ which defines the inhomogeneous coordinates of $\frac{E_7}{SU(8)}$. E(7) acts on the 70 coordinates $y_{ij,kl}$ by fractional transformation

$$y' = \frac{B + yD}{A + yC} . \tag{39}$$

and A, B, C, D are 28 by 28 constant matrices defined in [3].

Having introduced all these objects, which in addition to charges contain an auxiliary multiplet a', we will proceed from the other side and use the entropy formulas (24), (28). The question is whether our entropy (34)

$$S\left\{(M,N),(\bar{M},\bar{N})\right\}$$

can be proved to depend only on (m,n) and $(\bar{m},\bar{n})$ and not on moduli V. Equivalently one may try to show that the 70 moduli are functions of (m,n) and $(\bar{m},\bar{n})$. By using only the E(7) duality symmetry of the theory or the $SU(8)$ subgroup of it we cannot prove that the entropy does not depend on moduli. However, fortunately, the symmetry of the entropy is larger: the original formula (24) for the entropy in terms of central charges in addition to an $SU(8)$ symmetry of ζ_{ij}, ζ^{*ij} has a CPT symmetry under $\zeta_{ij} \longrightarrow -\zeta^{*ij}$, C symmetry under $\zeta_{ij} \longrightarrow -\zeta_{ij}$ as well as a U(1) symmetry $\zeta_{ij} \longrightarrow -e^{i\alpha}\zeta^{ij}$. Therefore we can use the global $U(8)$ symmetry of the entropy.

The entropy for the most general solution can be therefore reduced to the expression which it has for the simplest solution in the normal frame.

$$S = 2\pi(\det)^{1/4}\left\{r_0\delta_{ik} + \sqrt{-2\Upsilon_{ij}\Upsilon^*_{jk}}\right\} = 2\pi(\det)^{1/4}\left\{r_0\delta_{ik} + \sqrt{-2\tilde{\Upsilon}_{ij}\tilde{\Upsilon}^*_{jk}}\right\} . \tag{40}$$

This can be further presented as

$$S = 2\pi(\det)^{1/4}\left\{2\sqrt{N_{ij}\bar{N}_{jk}} + \sqrt{+(N+\bar{N})_{ij}(N+\bar{N})_{jk}}\right\} , \tag{41}$$

which is equal to

$$S = 2\pi(\sqrt{N_{12}} + \sqrt{\bar{N}_{12}})(\sqrt{N_{34}} + \sqrt{\bar{N}_{34}})(\sqrt{N_{56}} + \sqrt{\bar{N}_{56}})(\sqrt{N_{78}} + \sqrt{\bar{N}_{78}}) . \tag{42}$$

It has been found in [8] that 3 moduli of the normal frame solution are functions of 8 branes and anti-branes. Therefore it was possible to present an entropy in a form where it depends only on $\tilde{n}, \tilde{\bar{n}}$ which are the normal form representative of the E(7) multiplets (m,n), $(\bar{m},\bar{n})$.

$$S = 2\pi(\sqrt{n_{12}} + \sqrt{\bar{n}_{12}})(\sqrt{n_{34}} + \sqrt{\bar{n}_{34}})(\sqrt{n_{56}} + \sqrt{\bar{n}_{56}})(\sqrt{n_{78}} + \sqrt{\bar{n}_{78}}) . \tag{43}$$

It was suggested in [8], following the related observation in d=5 theory in [7], to consider the entropy formula, generalizing the normal frame solution to an E(7) invariant form.

$$S = 2\pi\sum_{i,j,k,l}\sqrt{T_{\hat{A}\hat{B}\hat{C}\hat{D}}f_i^{\hat{A}}f_j^{\hat{B}}f_k^{\hat{C}}f_l^{\hat{D}}} , \qquad i,j,k,l = 1,2 , \qquad \hat{A},\hat{B},\hat{C},\hat{D} = 1,\ldots,56 , \tag{44}$$

where $T_{\hat{A}\hat{B}\hat{C}\hat{D}}$ is the quartic invariant considered in [4], and $f_1^{\hat{A}} = (m^{ij}, n_{ij})$, $f_2^{\hat{A}} = (\bar{m}^{ij}, \bar{n}_{ij})$. The mysterious part of this formula was the appearance of two full E(7) multiplets, and the interpretation of this multiplets since the classical solution depends only 28 vector fields. Now we are in a position to prove this formula for the entropy of non-extreme black holes and explain the origin of the second multiplet.

We start with our U(8) symmetric formula (34) which depends on two SU(8) multiplets and is invariant under E(7). We could rewrite this formula as a function of the moduli and the "brane-numbers" and "anti-brane numbers" $(m,n,\bar{m},\bar{n})$ i.e.

$$S\left\{(M,N),(\bar{M},\bar{N})\right\} = \hat{S}\left\{(m,n),(\bar{m},\bar{n}),y_{ij,kl}\right\} . \tag{45}$$

However, we see from equation (43) that the entropy in the normal frame can be written as a function of the "brane-numbers" and "anti-brane-numbers" alone. This shows that in the normal frame, all the moduli appearing in the function $\hat{S}$ are determined as functions of $(n, \bar{n})$.

$$S_{\rm normal}\left\{(0,\tilde{n}),(0,\tilde{\bar{n}})\right\} = \hat{S}_{\rm normal}\left\{(0,\tilde{n}),(0,\tilde{\bar{n}}),\tilde{y}_{ij,kl}(\tilde{n},\tilde{\bar{n}})\right\} \ . \tag{46}$$

The E(7) symmetric generalization of the left hand side of this equation due to the uniqueness of the quartic invariant [3] is given by the formula (44). For this to be consistent with the right hand side of eq. (46) moduli in this formula have to transform under E(7) via it dependence on $\tilde{n}, \tilde{\bar{n}}$. Therefore the transformed value of the moduli according to eq. (39) will provide the generic expression for the 70 moduli as functions of all branes and anti-branes $y_{ij,kl}\left((m,n),(\bar{m},\bar{n})\right)$.

Thus we conclude that entropy of the the non-extreme black holes is duality invariant and has a nice symmetric C-invariant form given in eq. (44) as conjectured in [8]. Under C-conjugation

$$f_1 \Longleftrightarrow f_2 \ . \tag{47}$$

Thus the entropy can indeed be written entirely in terms of branes and anti-branes, which however are not independent but satisfy an E(7) symmetric constraint (33), which allows to express the moduli in terms of branes and anti-branes:

$$(r_0)^2\delta_{ik} = -2(M^{ij}\bar{M}^{jk} + N_{ij}\bar{N}_{jk}) = -2\left((\bar{a}^\dagger V^\dagger)^{ij}(Va)^{jk} + (\bar{a}^\dagger V^\dagger)_{ij}(Va)_{jk}\right) \ . \tag{48}$$

In the normal frame this set of constraints becomes

$$r_0^2 = 2N_{12}\bar{N}_{12} = 2N_{34}\bar{N}_{34} = 2N_{56}\bar{N}_{56} = 2N_{78}\bar{N}_{78} \ . \tag{49}$$

Together with diagonal form of relations $A = Va,\ A' = Va'$

$$N_{12} = {V_{12}}^{12}n_{12}\ , \qquad N_{34} = {V_{34}}^{34}n_{34}\ , \qquad N_{56} = {V_{56}}^{56}n_{56}\ , \qquad N_{78} = {V_{78}}^{78}n_{78}\ , \tag{50}$$

$$\bar{N}_{12} = {V_{12}}^{12}\bar{n}_{12}\ , \qquad \bar{N}_{34} = {V_{34}}^{34}\bar{n}_{34}\ , \qquad \bar{N}_{56} = {V_{56}}^{56}\bar{n}_{56}\ , \qquad \bar{N}_{78} = {V_{78}}^{78}\bar{n}_{78}\ , \tag{51}$$

this leads to

$$(V_1)^2 n_1\bar{n}_1 = (V_2)^2 n_2\bar{n}_2 = (V_3)^2 n_3\bar{n}_3 = (V_4)^2 n_4\bar{n}_4\ , \tag{52}$$

where as before we have simplified the notation, e.g. $n_{12} \equiv n_1, {V_{12}}^{12} \equiv V_1$. Thus we have derived from our U(8) covariant constraint the fact established in [8] that three values of moduli,

$$\left(\frac{V_2}{V_1}\right)^2 = \frac{n_1\bar{n}_1}{n_2\bar{n}_2}\ , \qquad \left(\frac{V_3}{V_2}\right)^2 = \frac{n_2\bar{n}_2}{n_3\bar{n}_3}\ , \qquad \left(\frac{V_4}{V_3}\right)^2 = \frac{n_3\bar{n}_3}{n_4\bar{n}_4}\ , \tag{53}$$

are the functions of 4 branes and 4 anti-branes. This seemed earlier to be a mysterious property of the solution. We have established this property by introducing into the theory an auxiliary SU(8) multiplet constrained to the original one in E(7) symmetric way. This has allowed to realize the degeneracy of the entropy formula in the signs of the charges in each of 28 gauge groups in a

[3] One could also have a symplectic quadratic invariant of E(7) but it would not have a correct supersymmetric limit and will not match the normal frame solution

manifest way. Thus the entropy formula (44) given in terms of two E(7) multiplets by Cartan's quartic invariant, when the 2 multiplets are constrained as in eq. (48), is the entropy formula for non-extreme black holes in N=8 supergravity.

In conclusion, we have found entropy formulas for N=8 supergravity black holes in terms of central charges (18), (24) which interpolate between Schwarzschild solution with all central charges vanishing $Z_{AB} = 0$, Reissner-Nordström solutions with one non-vanishing skew eigenvalue of the central charge and various other non-extreme solutions. It simultaneously includes those with 1/8, 1/4 and 1/2 of unbroken supersymmetry, depending on how many of the supersymmetric positivity bounds are saturated, i.e. whether $M = z_1 > z_2, z_3, z_4$, or $M = z_1 = z_2 > z_3, z_4$, or $M = z_1 = z_2 = z_3 = z_4$.

This formula, together with the expression for the product of the temperature and entropy in terms of supersymmetry charges (13), presents the classification of extreme as well as non-extreme black holes in supersymmetric theories. This kind of classification was suggested before in the limited context of pure N=4 supergravity, when only z_1, z_2 where available [9].

We have also rederived this entropy formula in terms of conserved charges of the theory and moduli and, finally, in terms of branes and anti-branes. We gave an explanation for the appearance of branes and anti-branes in the description of these black holes as a set of constrained multiplets which enable us to realize the full symmetries of the theory.

4 Acknowledgements

Stimulating discussions with B. Kol, A. Linde and L. Susskind are gratefully acknowledged. The work of R.K. is supported by the NSF grant PHY-9219345. The work of A.R is supported in part by the Department of Energy under contract no. DE-AC03-76SF00515.

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