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Observation of Exclusive B Decays to Final States Containing a Charmed Baryon

CLEO Collaboration

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Abstract

Using data collected in the region of the $\Upsilon(4S)$ resonance with the CLEO-II detector, we report on the first observation of exclusive decays of the B meson to final states with a charmed baryon. We have measured the branching fractions $\mathcal{B}(B^- \to \Lambda_c^+ \overline{p} \pi^-) = (0.62\ ^{+0.23}_{-0.20} \pm 0.11 \pm 0.10) \times 10^{-3}$ and $\mathcal{B}(\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^+ \pi^-) = (1.33\ ^{+0.46}_{-0.42} \pm 0.31 \pm 0.21) \times 10^{-3}$. In addition, we report upper limits for final states of the form $\overline{B} \to \Lambda_c^+ \overline{p}(n\pi)$ and $\Lambda_c^+ \overline{p}(n\pi)\pi^0$ where $(n\pi)$ denotes up to four charged pions.

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Exclusive reconstruction of B mesons to final states with a Λ_c^+ is essential in understanding the mechanisms for baryon production in B decays, which are expected to be dominated by $b \to c\overline{u}d$ transitions via internal or external W-emission [1]. These processes lead to final states of the form $\overline{B} \to \Theta_c \overline{N} X$, where $\Theta_c = \Lambda_c^+$, Σ_c or higher excitations of the ground state baryons. Inclusive production of the charmed baryon Λ_c^+ from B meson decays was first reported by ARGUS [2] and confirmed by CLEO [3]. The fraction of Λ_c^+ baryons from Σ_c^{++} and Σ_c^0 in B decays is measured to be $\sim$16 % [4]. The branching fraction for B decays to charmed baryons is $(6.4 \pm 1.1)\%$ [5]. CLEO has studied the Λ_c^+ momentum spectrum in B decays and found that two-body final states are suppressed [3]. This motivates a search for multi-body final states of $\overline{B}^0$ and B^- mesons of the form $\overline{B} \to \Lambda_c^+ \overline{p}(n\pi)$ and $\Lambda_c^+ \overline{p}(n\pi)\pi^0$, where $(n\pi)$ is up to four charged pions. These modes have previously not been observed. Throughout this study the charge conjugate process is implied.

This analysis is based on 2.39 fb^{-1} of data taken at the $\Upsilon(4S)$ resonance and 1.13 fb^{-1} of data taken at a center-of-mass energy 60 MeV less than the $\Upsilon(4S)$ resonance which is below the threshold for producing B meson pairs, hereafter referred to as continuum. Assuming equal production rates of charged and neutral B mesons, a total of 2,560,000 $\pm$ 46,000 charged and an equal number of neutral B mesons are in the data sample. The data was collected with the CLEO-II detector [6] at the Cornell Electron Storage Ring (CESR). Charged particles are tracked using three nested cylindrical wire chambers operating in a 1.5 T magnetic field. The tracking chambers are surrounded by time-of-flight (TOF) counters and an electromagnetic calorimeter which provides excellent π^0 reconstruction.

Candidate Λ_c^+ baryons are reconstructed in the modes $pK^-\pi^+$, pK_S^0 and $\Lambda\pi^+$, with $K_S^0 \to \pi^+\pi^-$ and $\Lambda \to p\pi^-$. The momentum of the Λ_c^+ is required to be less than 2.3 GeV/c, which is the kinematic limit for Λ_c^+ baryons from B meson decay. Daughter K_S^0 and Λ candidates are reconstructed from oppositely charged tracks which form a detached vertex in the plane transverse to the beam direction. The invariant mass of the K_S^0 (Λ) is required to be within 10.0 (3.0) MeV/c^2 of the known mass. Neutral pion candidates are formed from pairs of showers detected in the calorimeter which yield a $\gamma\gamma$ invariant mass within 2.5 σ of the known π^0 mass ($\sigma \sim 5$ MeV).

Particle identification is accomplished by combining the specific ionization measurements from the central drift chamber with TOF information, if available, to derive probabilities for each charged track to be consistent with the pion, kaon and proton mass hypotheses. Protons produced directly from B decay are required to have a probability for the proton hypothesis greater than 5% and a probability of less than 32% for the pion hypothesis [7]. Charged kaons and protons from Λ_c^+ decay are required to have a probability for the appropriate hypothesis greater than 5% and a probability of less than 5% for the pion hypothesis [8]. For charged pions from Λ_c^+ decay and protons from Λ decay, the probability for the respective particle hypothesis is required be greater than 0.3% [9]. No particle identification requirements are made for pion candidates from B decay in order to improve overall detection efficiency for low momentum pions and increase sensitivity to pions from the decay $\Sigma_c^{++/0} \to \Lambda_c^+\pi^\pm$. We relax the requirement for protons from B decay compared to protons from Λ_c^+ decay to increase our efficiency since the average momentum is greater for protons directly from B decay. This is necessary because the efficiency of our particle identification decreases with increasing momentum. The efficiencies of these particle identification requirements are derived from data using high purity samples of protons, kaons and pions from the decays

$\Lambda \to p\pi^-$, $D^{*+} \to D^0\pi^+$ with $D^0 \to K^-\pi^+$, and $K^0_S \to \pi^+\pi^-$, respectively.

To suppress continuum background, the normalized Fox-Wolfram second moment [10] is required to be less than 0.35. The numbers of Λ_c^+ candidates from the $\Upsilon(4S)$ data and the continuum are determined separately, where continuum data is scaled to account for the differences in luminosity and center-of-mass energies. After subtracting this contribution, the Λ_c^+ yield from B decays is 3343 ± 215.

Exclusive B decays are reconstructed by selecting Λ_c^+ candidates whose mass is within 2.5 σ of the nominal mass and forming $\Lambda_c^+\overline{p}(n\pi)$ and $\Lambda_c^+\overline{p}(n\pi)\pi^0$ combinations, where $(n\pi)$ denotes up to four charged pions. We define the beam-constrained mass as $M_B = \sqrt{E^2_{\rm beam} - (\sum_i \vec{p_i})^2}$, where $\vec{p_i}$ is the 3-momentum vector for the i^{th} daughter of the B candidate and $E_{\rm beam}$ is the beam energy. The resolution of M_B is about 2.6 MeV/c^2, a factor of five better than the resolution in invariant mass, and is dominated by the spread of the CESR beam energy.

For correctly reconstructed B mesons the measured energy, $E_{\rm meas}$, must equal the beam energy within the experimental resolution. The width of the energy difference distribution, ΔE=$E_{\rm meas}$ - $E_{\rm beam}$, is predicted by Monte Carlo to be 10 to 16 MeV, depending on the final state. We reduce the combinatorial background significantly by requiring $|\Delta E| < 25$ MeV. A further reduction in background is achieved by cutting on Θ_B, the polar angle of the B in the laboratory frame with respect to the e^+e^- axis. The distribution of $\cos\Theta_B$ is proportional to $\sin^2\Theta_B$ for $e^+e^- \to \Upsilon(4S) \to B\overline{B}$, whereas background events are distributed nearly isotropically. We require $|\cos\Theta_B| < 0.9$. If there are multiple candidates in an event with $M_B > 5.2$ GeV/c^2 for a given decay channel, the entry with the smallest absolute value of ΔE is selected.

After application of these cuts, statistically significant signals are seen in the decay modes $B^- \to \Lambda_c^+\overline{p}\pi^-$ and $\overline{B}^0 \to \Lambda_c^+\overline{p}\pi^+\pi^-$, shown in Fig. 1(a) and (c), respectively. The M_B signal distribution is fit using a Gaussian signal of width equal to 2.64 MeV/c^2 and a background function composed of a straight line with a parabolic kinematic cutoff [11]. The M_B distributions from the ΔE signal and sideband region are fit simultaneously to obtain the slope of the background function. The fits yield $12.0^{+4.4}_{-3.8}$ events for $B^- \to \Lambda_c^+\overline{p}\pi^-$, and $24.0^{+8.3}_{-7.5}$ events for $\overline{B}^0 \to \Lambda_c^+\overline{p}\pi^+\pi^-$.

The background contributions to the M_B distribution have been studied in several ways. The beam-constrained mass distribution from combinations in the ΔE sideband (defined as the region satisfying $50 < |\Delta E| < 100$ MeV) shows no enhancement in the signal region, shown in Fig. 1(b) and (d) for $B^- \to \Lambda_c^+\overline{p}\pi^-$ and $\overline{B}^0 \to \Lambda_c^+\overline{p}\pi^+\pi^-$, respectively. The background distributions are fit with the background functional form described above. Similar distributions made from the Λ_c^+ mass sidebands and continuum data show no enhancement in the signal region.

The ΔE distributions for $B^- \to \Lambda_c^+\overline{p}\pi^-$ and $\overline{B}^0 \to \Lambda_c^+\overline{p}\pi^+\pi^-$ for the M_B signal region ($\sim 2\ \sigma$), and for the M_B sideband (defined as $5.230 < M_B < 5.260 \mathrm{GeV}/c^2$), are shown in Fig. 2(a) and (b). The ΔE distribution from the M_B signal region is fit using a Gaussian whose width is fixed to the value predicted from Monte Carlo and a linear background function. After subtraction of the contribution from the M_B sideband, which also peaks due to our selection criteria, the signal yield is 10.1 ± 5.0 events for $B^- \to \Lambda_c^+\overline{p}\pi^-$ and 23.5 ± 9.3 events for $\overline{B}^0 \to \Lambda_c^+\overline{p}\pi^+\pi^-$, consistent with the yields from the fits to the M_B distributions.

The yields from the M_B distributions are quoted, however, due to their greater statistical significance.

The branching fractions are measured to be,

$$\mathcal{B}(B^- \to \Lambda_c^+ \overline{p} \pi^-) = (0.62\ ^{+0.23}_{-0.20} \pm 0.11 \pm 0.10) \times 10^{-3},$$

$$\mathcal{B}(\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^+ \pi^-) = (1.33\ ^{+0.46}_{-0.42} \pm 0.31 \pm 0.21) \times 10^{-3},$$

where the first error is statistical, the second is systematic and the third error is due to uncertainty in the Λ_c^+ branching fractions [5].

Systematic uncertainties include contributions from particle identification requirements (10%), fitting procedure (8% – 16%), charged track reconstruction (2% per track), π^0 reconstruction efficiency (5% per π^0), the number of $B\overline{B}$ events (2%), Monte Carlo statistics (2 – 4%), secondary vertex finding (1%) and the Λ_c^+ branching fractions (16%). The assumption is made that $B^- \to \Lambda_c^+ \overline{p} \pi^-$ ($\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^+ \pi^-$) proceeds via phase space decay. The reconstruction efficiency decreases by 9% (6%) if the assumption is made that 16% of the Λ_c^+ baryons come from Σ_c^{++} and Σ_c^0 [4], which is taken as a systematic error. The total systematic errors are between 13 – 23 %, depending on the decay mode.

Figure 3 shows the distributions for decay modes of the form $\overline{B}^0 \to \Lambda_c^+ \overline{p}(n\pi)\pi^0$, and Fig. 4 displays the decay modes $\overline{B}^0 \to \Lambda_c^+ \overline{p}$, $B^- \to \Lambda_c^+ \overline{p} \pi^- \pi^+ \pi^-$ and $\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^- \pi^+ \pi^- \pi^+$. In all of these B decay modes no statistically significant signals are observed and 90% C.L. upper limits are calculated and summarized in Table I. Theoretical predictions exist for a number of two-body B meson decays to charmed baryons and the limit for $\overline{B} \to \Lambda_c^+ \overline{p}$ is below theoretical predictions, which are in the range 0.04% to 0.19% [12].

In conclusion, we have made the first observation of exclusive B decays to final states including the charmed baryon Λ_c^+. The branching fractions for $B^- \to \Lambda_c^+ \overline{p} \pi^-$ and $\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^+ \pi^-$ have been measured and upper limits have been set on other decay modes.

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[8] This requirement corresponds to a 2σ consistency requirement on the selected hypothesis and a pion veto at the 2σ level.

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TABLES

TABLE I. Branching fraction results for $\overline{B} \to \Lambda_c^+ \overline{p}(n\pi)$, $\Lambda_c^+ \overline{p}(n\pi)\pi^0$ and 90% $C.L.$ upper limits.

B Mode	Events	$\mathcal{B} \times 10^3$
$\Lambda_c^+ \overline{p}$	< 2.3	< 0.21
$\Lambda_c^+ \overline{p}\pi$	$12.0\ ^{+4.4}_{-3.8}$	$0.62\ ^{+0.23}_{-0.20} \pm 0.11 \pm 0.10$
$\Lambda_c^+ \overline{p}\pi^0$	< 4.1	< 0.59
$\Lambda_c^+ \overline{p}2\pi$	$24.0\ ^{+8.3}_{-7.5}$	$1.33\ ^{+0.46}_{-0.42} \pm 0.31 \pm 0.21$
$\Lambda_c^+ \overline{p}\pi\pi^0$	< 20.6	< 3.12
$\Lambda_c^+ \overline{p}3\pi$	< 16.2	< 1.46
$\Lambda_c^+ \overline{p}2\pi\pi^0$	< 21.0	< 5.07
$\Lambda_c^+ \overline{p}4\pi$	< 13.9	< 2.74
$\Lambda_c^+ \overline{p}3\pi\pi^0$	< 28.2	< 13.4

FIGURES

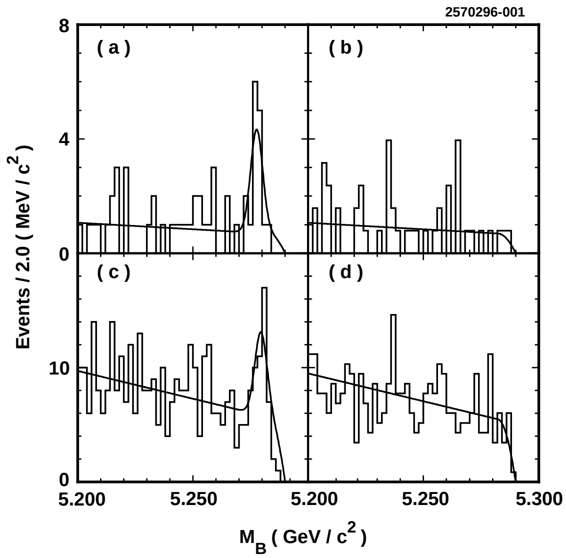

FIG. 1. M_B distribution for $B^- \to \Lambda_c^+ \overline{p} \pi^-$ from (a) ΔE signal region, (b) ΔE sideband region and M_B distribution for $\overline{B}^{\,0} \to \Lambda_c^+ \overline{p} \pi^+ \pi^-$ from (c) ΔE signal region, (d) ΔE sideband region.

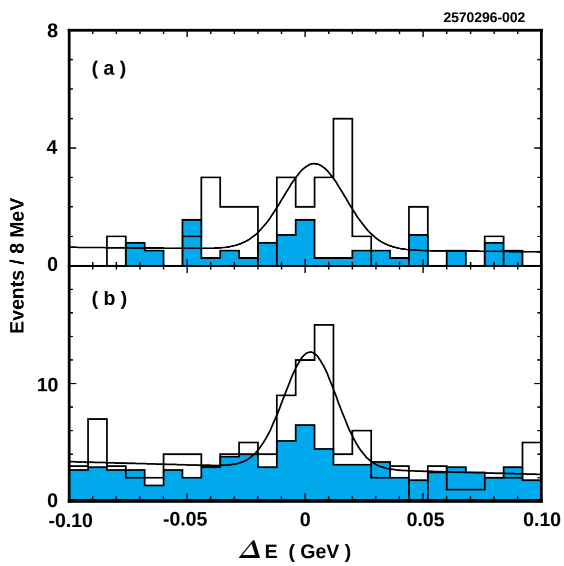

FIG. 2. ΔE distributions for (a) $B^- \to \Lambda_c^+ \overline{p} \pi^-$ and (b) $\overline{B}^{\,0} \to \Lambda_c^+ \overline{p} \pi^+ \pi^-$ from M_B signal region (histogram) and sideband region (shaded histogram).

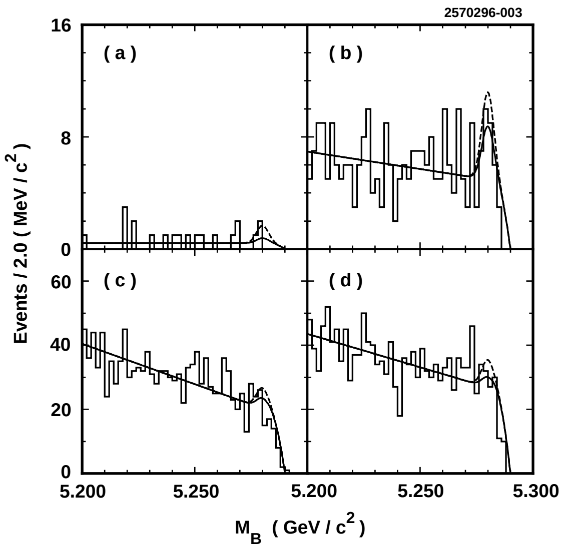

FIG. 3. M_B distribution for (a) $\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^0$, (b) $B^- \to \Lambda_c^+ \overline{p} \pi^- \pi^0$, (c) $\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^+ \pi^- \pi^0$, and (d) $B^- \to \Lambda_c^+ \overline{p} \pi^+ \pi^- \pi^- \pi^0$. The curve is the result of the fit and the dotted line is the 90% confidence level upper limit.

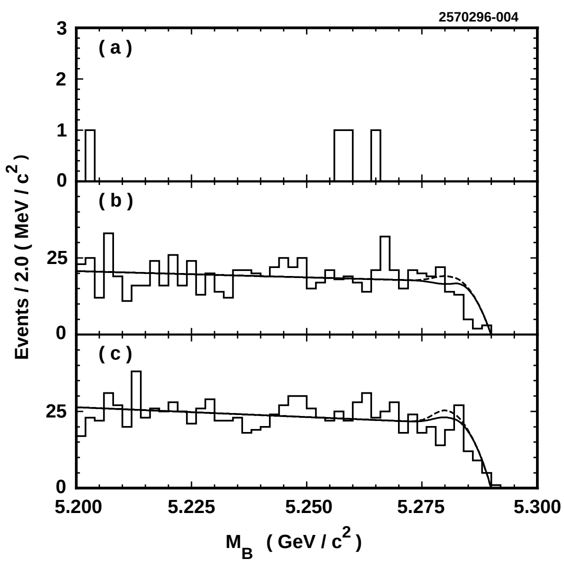

FIG. 4. M_B distribution for (a) $\overline{B}^0 \to \Lambda_c^+ \overline{p}$, (b) $B^- \to \Lambda_c^+ \overline{p} \pi^+ \pi^- \pi^-$, and (c) $\overline{B}^0 \to \Lambda_c^+ \overline{p} \pi^+ \pi^- \pi^- \pi^+$. The curve is the result of the fit and the dotted line is the 90% confidence level upper limit.