

# SUPERSYMMETRIC ROTATING BLACK HOLES AND ATTRACTORS

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## ABSTRACT

Five-dimensional stringy rotating black holes are embedded into N=2 supergravity interacting with one vector multiplet. The existence of an unbroken supersymmetry of the rotating solution is proved directly by solving the Killing spinor equations. The asymptotic enhancement of supersymmetry near the horizon in the presence of rotation is established via the calculation of the super-curvature. The area of the horizon of the rotating supersymmetric black holes is found to be  $\sqrt{Z_{\text{fix}}^3 - J^2}$ , where  $Z_{\text{fix}}$  is the extremal value of the central charge in moduli space.

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## I. INTRODUCTION

It was shown by Tod [1] that the Kerr-Newman metric in 4D reaches the supersymmetric limit when the mass is equal to the charge,  $m = |q|$ , for arbitrary value of angular momentum  $J$ . However, the extreme limit for this solution is reached at  $m^2 - |q|^2 = J^2$  and therefore the extremal Kerr-Newman solution does not have any unbroken supersymmetry. The supersymmetric solution with  $m = |q|$  for non-vanishing angular momentum  $J$  turns out to belong to the Israel-Wilson-Perjés family of metrics [2] and is known to have naked singularities [3]. Other rotating black hole solutions in 4D have been found with 1/2 and 1/4 of unbroken supersymmetry of N=4 theory but they always have naked singularities (see [4] for a review).

In contrast, 5D rotating extremal black hole solutions have been constructed which have some unbroken supersymmetry [5]. The entropy of these black holes as well as of the near extreme ones was understood via the counting of D-brane states in [5] and in [6], respectively.

Extremal supersymmetric static black hole solutions behave as attractors [7]; moduli take on fixed values at the horizon which depend only on the charges and not on the values of the moduli at infinity. What is more, the area of these black holes can be found (without a knowledge of the metric) by extremizing the value of the central charge in moduli space [8]. The extremal value  $Z_{\text{fix}}$  is then related to the area in 5D by

$$A = \frac{\pi^2}{3} Z_{\text{fix}}^{3/2}. \quad (1)$$

The precise relation is dimension-dependent; it was worked out for 4D in [8] and for 5D in [9].

In this paper, we wish to study the analogues of these questions for supersymmetric rotating black holes. The above properties for static black holes are a consequence of the fact that asymptotically close to the horizon, all the supersymmetries are unbroken [8]. We will establish that the same phenomenon occurs for rotating black holes. Using this property, we can then show that

- (a) there is still a fixed point for the moduli near the horizon, and
- (b) the relation (1) is now modified to

$$A = \frac{\pi^2}{3} \sqrt{(Z_{\text{fix}}^3 - J^2)}. \quad (2)$$

In section II, we will find the  $N = 2$  supergravity Lagrangian for which the rotating black hole of [5] (henceforth referred to as the BMPV black hole) is a solution. We will then explicitly show that the supersymmetry transformation laws are satisfied and that a Killing spinor exists. In the next section, we shall examine the integrability condition for the existence of Killing spinors. This condition is trivial at the horizon, indicating that it is satisfied for all spinors. This implies that we have full restoration of supersymmetry near the horizon. This will then imply the properties (a) and (b) above.

## II. EMBEDDING OF 5D ROTATING BLACK HOLES INTO VERY SPECIAL GEOMETRY

The general action for D=5, N=2 supergravity coupled to N=2 vector multiplets has been constructed by Günaydin, Sierra and Townsend [10]; the bosonic part is referred to as very special geometry [11]. The theory is completely defined by the prepotential

$$\mathcal{V} = \frac{1}{6} C_{IJK} X^I X^J X^K, \quad (3)$$

where the vectors are labeled by  $I = (0, i)$  where  $i = 1, \dots, n$ . This action corresponds to the compactification of 11D supergravity down to 5D on Calabi-Yau three-folds [12] with topological intersection form  $C_{IJK}$ . The bosonic part of it is

$$e^{-1} \mathcal{L} = -\frac{1}{2} R - \frac{1}{4} G_{IJ}(\phi) F_{\mu\nu}^I F^{\mu\nu J} - \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + \frac{e^{-1}}{48} \epsilon^{\mu\nu\rho\sigma\lambda} C_{IJK} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K. \quad (4)$$

The details and examples in the applications to 5D static black holes can be found in [9]. The simplest situation is pure N=2 supergravity which has  $n = 0$  and only  $C_{000}$  non-vanishing. This theory has no scalars and no vectors besides the graviphoton.

The next simplest example of (4) is the action of N=2 supergravity interacting with one vector multiplet in the form presented in [9]<sup>†</sup>

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}e^{\frac{2}{3}\phi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\phi}G_{\mu\nu}G^{\mu\nu} + \frac{1}{6}(\partial_\mu\phi)^2 - \frac{e^{-1}}{4\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}B_\lambda. \quad (5)$$

The supersymmetry transformations of the fermionic fields with vanishing fermions are

$$\begin{aligned} \delta\psi_\mu &= \nabla_\mu(\omega)\epsilon + \frac{1}{12}\left(\Gamma_\mu{}^{\rho\sigma} - 4\delta_\mu{}^\rho\Gamma^\sigma\right)\left(e^{\frac{\phi}{3}}F_{\rho\sigma} - \frac{1}{\sqrt{2}}e^{-\frac{2}{3}\phi}G_{\rho\sigma}\right)\epsilon, \\ \delta\chi &= -\frac{1}{2\sqrt{3}}\Gamma^\mu\partial_\mu\phi\epsilon + \frac{1}{4\sqrt{3}}\Gamma^{\rho\sigma}\left(e^{\frac{\phi}{3}}F_{\rho\sigma} + \sqrt{2}e^{-\frac{2}{3}\phi}G_{\rho\sigma}\right)\epsilon. \end{aligned} \quad (6)$$

The gravitational multiplet consists of the graviton  $g_{\mu\nu}$ , gravitino  $\psi_\mu$  and the graviphoton field  $(e^{\frac{\phi}{3}}F_{\rho\sigma} - \frac{1}{\sqrt{2}}e^{-\frac{2}{3}\phi}G_{\rho\sigma})$ . The vector multiplet includes a scalar field  $\phi$ , a gaugino  $\chi$  and the vector field of the vector multiplet  $(e^{\frac{\phi}{3}}F_{\rho\sigma} + \sqrt{2}e^{-\frac{2}{3}\phi}G_{\rho\sigma})$ .

Here we would like to find the rotating black hole with one half of unbroken supersymmetry in the theory of N=2 supergravity, interacting with one vector multiplet. Up to a few rescalings this is expected to be the rotating BMPV black hole [5]. The ansatz for the metric is

$$\begin{aligned} ds^2 &= \left(1 - \frac{\mu}{r^2}\right)^2 \left[ dt - \frac{4J \sin^2 \theta}{\pi(r^2 - \mu)} d\varphi + \frac{4J \cos^2 \theta}{\pi(r^2 - \mu)} d\psi \right]^2 \\ &\quad - \left(1 - \frac{\mu}{r^2}\right)^{-2} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2 + \cos^2 \theta d\psi^2). \end{aligned} \quad (7)$$

The scalar field is a constant

$$e^{2\phi} = \frac{8Q_F^2}{\pi^2 Q_H^2} = \lambda^6. \quad (8)$$

The gaugino equation for constant  $\phi$  is then reduced to:

$$\delta\chi = \frac{1}{4\sqrt{3}}\Gamma^{\rho\sigma}\left(e^{\frac{\phi}{3}}F_{\rho\sigma} + \sqrt{2}e^{-\frac{2}{3}\phi}G_{\rho\sigma}\right)\epsilon = 0. \quad (9)$$

It is satisfied by requiring that the vector field of the vector multiplet vanishes, i.e.

$$G_{\rho\sigma} = -\frac{1}{\sqrt{2}}e^\phi F_{\rho\sigma} \quad \Rightarrow \quad B = -\frac{\lambda^3}{\sqrt{2}}A. \quad (10)$$

The vector field is

$$A_t = \frac{\mu}{\lambda r^2}, \quad A_\varphi = \frac{4J \sin^2 \theta}{\pi \lambda r^2}, \quad A_\psi = -\frac{4J \cos^2 \theta}{\pi \lambda r^2}. \quad (11)$$

For the mass and charges we have

$$M_{ADM} = \frac{3\pi\mu}{4}, \quad Q_H \equiv \frac{\sqrt{2}}{4\pi^2} \int_{S^3} \star e^{-4\phi/3} G = \mu/\lambda^2, \quad Q_F \equiv \frac{\sqrt{2}}{16\pi} \int_{S^3} \star e^{2\phi/3} F = -\frac{\pi}{2\sqrt{2}}\mu\lambda. \quad (12)$$

The relation between vector fields can be inserted into the gravitino transformation:

$$\delta\psi_\mu = \nabla_\mu(\omega)\epsilon + \frac{1}{8}\left(\Gamma_\mu{}^{\rho\sigma} - 4\delta_\mu{}^\rho\Gamma^\sigma\right)e^{\frac{\phi}{3}}F_{\rho\sigma} = \nabla_\mu(\omega)\epsilon + \frac{\lambda}{8}\left(\Gamma^{\rho\sigma}\Gamma_\mu + 2\Gamma^\rho\delta_\mu{}^\sigma\right)F_{\rho\sigma}\epsilon. \quad (13)$$

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<sup>†</sup>Here, as different from [9], we will use the mostly minus metric and adapt our notation to this.

It remains to show that the rotating black hole background admits 1/2 unbroken supersymmetry, i.e. that the gravitino equation has a zero mode. It is useful at this stage to explain that our problem is actually reduced to the problem of solving for a Killing spinor in the pure N=2 theory. Indeed, we may use the relation between the vector fields in (10) as well as the fact that the scalar is a constant directly in the action (5). We get

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{3}{8}\lambda^2 F_{\mu\nu}F^{\mu\nu} + \frac{e^{-1}\lambda^3}{8}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda. \quad (14)$$

Upon rescaling the action and setting  $\tilde{F} = \lambda\frac{\sqrt{3}}{2}F$  we can rewrite the action as

$$e^{-1}\mathcal{L}_5 = -\frac{1}{4}R(\omega) - \frac{1}{4}\tilde{F}_{\mu\nu}^2 + \frac{e^{-1}}{6\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}\tilde{F}_{\mu\nu}\tilde{F}_{\rho\sigma}\tilde{A}_\lambda, \quad (15)$$

and

$$\delta\psi_\mu = \hat{\nabla}_\mu\epsilon = \nabla(\omega)\epsilon + \frac{1}{4\sqrt{3}}\left(\Gamma^{\rho\sigma}\Gamma_\mu + 2\Gamma^\rho\delta_\mu^\sigma\right)\tilde{F}_{\rho\sigma}\epsilon. \quad (16)$$

This is the bosonic action of pure N=2 supergravity theory, presented in [13]. The Killing equation (16) in the rotating black hole background has a solution:

$$\epsilon = \sqrt{1 - \frac{\mu}{r^2}} e^{\frac{1}{2}\Gamma^4\Gamma^3\theta} e^{\frac{1}{2}\Gamma^4\Gamma^1(\phi+\psi)}\epsilon_0, \quad (17)$$

where  $\epsilon_0$  is a constant spinor satisfying

$$(1 + \Gamma^0)\epsilon_0 = 0. \quad (18)$$

The existence of this solution explicitly shows that we have half of the supersymmetries unbroken in presence of rotation. Interestingly, there is no trace of the rotation in the form of the Killing spinor. It has been observed before [14] that Killing spinors in spherical coordinates may display a dependence on angles even if the geometry is spherically symmetric.

We have also found that the Killing spinor in cartesian isotropic coordinates is simply

$$\epsilon = \sqrt{1 - \frac{\mu}{r^2}}\epsilon_0$$

where the constant spinor  $\epsilon_0$  satisfies the constraint (18).

### III. ENHANCEMENT OF UNBROKEN SUPERSYMMETRY NEAR THE HORIZON

For the study of the area-entropy formula in presence of rotation  $J$  we would like to consider the near horizon geometry. For all static supersymmetric solutions near the horizon at  $r \rightarrow r_0$  and one can exhibit the  $AdS_2 \times S^3$  geometry using  $\hat{r} = (r - r_0) \rightarrow 0$  [9]

$$ds^2 = \left(\frac{2\hat{r}}{r_0}\right)^2 dt^2 - \left(\frac{2\hat{r}}{r_0}\right)^{-2} d\hat{r}^2 - r_0^2 d^2\Omega_3, \quad (19)$$

where the 3-sphere  $S^3$  is defined by

$$r_0^2 d^2\Omega_3 = r_0^2 (d\theta^2 + \sin^2\theta d\varphi^2 + \cos^2\theta d\psi^2), \quad (20)$$

and the volume of this  $S^3$ ,

$$A(J=0) = 2\pi^2 r_0^3, \quad (21)$$

gives the area of the non-rotating black hole horizon in 5D.

For the rotating solutions near the horizon at  $r^2 \rightarrow \mu \equiv r_0^2$  the metric does not split into the product space: there are non-diagonal components. Apart from this, the  $AdS_2$  part of the metric is the same but the 3-sphere is distorted:

$$ds^2 = \left(\frac{2\hat{r}}{r_0}\right)^2 dt^2 - \left(\frac{2\hat{r}}{r_0}\right)^{-2} d\hat{r}^2 - \frac{16\hat{r}J \sin^2 \theta}{r_0^3 \pi} dt d\phi + \frac{16\hat{r}J \cos^2 \theta}{r_0^3 \pi} dt d\psi - r_0^2 d^2 \Omega_3(J), \quad (22)$$

and the metric for the distorted 3-sphere is

$$r_0^2 d^2 \Omega_3(J) = r_0^2 \left( d^2 \Omega_3 - \left(\frac{4J}{r_0^3 \pi}\right)^2 (\sin^2 \theta d\varphi - \cos^2 \theta d\psi)^2 \right). \quad (23)$$

The volume of the distorted 3-sphere defines the area of the horizon of the rotating black hole

$$A(J) = 2\pi^2 \sqrt{r_0^6 - J^2}. \quad (24)$$

To clarify the relation of the radius of the distorted 3-sphere  $r_0$  to the minimum of the central charge, we need to study the supersymmetry near the horizon and find out whether it is enhanced as in the non-rotating case.

The integrability condition for the existence of the Killing spinor defines the super-curvature as

$$[\hat{\nabla}_a, \hat{\nabla}_b] \epsilon = \hat{R}_{ab} \epsilon = 0. \quad (25)$$

If this is satisfied for arbitrary  $\epsilon$ , we have fully restored supersymmetry. If a constraint on  $\epsilon$  is needed, supersymmetry is partially broken.

We will find that supersymmetry is asymptotically restored near the horizon and half broken away from the horizon.

In fact, the integrability condition takes the form

$$\hat{R}_{ab} = \frac{(r^2 - r_0^2)}{r^6} X_{ab} (1 + \Gamma^0), \quad (26)$$

where  $X_{ab}$  will be given below.

From this equation we can see that  $\hat{R}_{ab}$  approaches zero near the horizon and so the integrability condition is satisfied for arbitrary  $\epsilon$ , which implies supersymmetry is asymptotically fully restored.

Away from the horizon, when the constraint (18) is imposed, the integrability condition is also satisfied, signifying half unbroken supersymmetry, consistent with the results of the previous section.

The explicit form of  $X_{ab}$  in (26) is

$$\begin{aligned} X_{ab} &= X_{ab}^0 + \frac{4J}{\pi r} X_{ab}^1 \\ X_{ab}^0 &= r_0^2 \eta \Gamma_{ab}, \end{aligned} \quad (27)$$

where

$$\eta = \begin{cases} 1 & (a, b) = (1, 2), (1, 3) \text{ or } (1, 4) \\ 3 & (a, b) = (0, 1) \\ -1 & \text{otherwise.} \end{cases} \quad (28)$$

The nonzero terms in  $X_{ab}^1$  are

$$X_{13}^1 = -X_{24}^1 = +4 \sin \theta \Gamma^1 - 2 \cos \theta \Gamma^2 \quad (29)$$

$$X_{14}^1 = +X_{23}^1 = -4 \cos \theta \Gamma^1 - 2 \sin \theta \Gamma^2 \quad (30)$$

$$X_{12}^1 = +X_{34}^1 = +2 \cos \theta \Gamma^3 + 2 \sin \theta \Gamma^4 \quad (31)$$

with the corresponding antisymmetric parts.

The fact that the super-curvature near the horizon vanishes even in the presence of rotation allows us to extend the arguments about the universality of the area-entropy formula of static black holes to the rotating ones.

For the most general 5D rotating supersymmetric black hole we have to study both the supercurvature and the gaugino supersymmetry transformations. Using the very special geometry language we have for the theory of N=2 supergravity, interacting with  $n$  vector multiplets

$$\delta \lambda_i = -\frac{i}{2} g_{ij}(\phi) \Gamma^\mu \partial_\mu \phi^j \epsilon + \frac{1}{4} \left(\frac{3}{4}\right)^{2/3} t_{I,i} \Gamma^{\mu\nu} F_{\mu\nu}^I \epsilon. \quad (32)$$

The doubling of unbroken supersymmetry near the horizon is possible only under the condition that there is a fixed point where all  $n$  the scalars have vanishing derivatives

$$e_a{}^\mu \partial_\mu \phi^j = 0 \quad (33)$$

and simultaneously the vector fields of the vector multiplets vanish

$$t_{I,i} \Gamma^{\mu\nu} F_{\mu\nu}^I = 0 \quad \implies \quad \partial_i Z = 0. \quad (34)$$

Hence the moduli take on fixed values at the horizon, and the central charge is extremized. A particular example of this is eq. (10) which defines the fixed scalar in terms of the charges and minimizes the central charge. For a static black hole, we could then, following [8], deduce the area of the black hole to be  $A = \frac{\pi^2}{3} \sqrt{Z_{\text{fix}}^3}$ . However, as seen in equation (24), the rotating black hole has a modified area. This modifies the relation to

$$A = \frac{\pi^2}{3} \sqrt{Z_{\text{fix}}^3 - J^2}. \quad (35)$$

We have found that it is correct for all known rotating solutions, including those with scalars changing between infinity and the horizon [15,16] and may be considered as a prediction for rotating solutions which may be found later.

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#### APPENDIX

Here are some conventions, definitions and representations we used.

Greek letters denote curved space indices, roman letters denote inertial frame indices.

$$\eta_{ab} = \text{diag}(+1, -1, -1, -1, -1) \quad (36)$$

$$\Gamma^0 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \quad \Gamma^1 = \begin{bmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \quad \Gamma^2 = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \quad (37)$$

$$\Gamma^3 = \begin{bmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{bmatrix} \quad \Gamma^4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (38)$$

$$\nabla_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4} \omega_{\mu ab} \Gamma^{ab} \epsilon \quad (39)$$

$$[\hat{\nabla}_a, \hat{\nabla}_b] = [\nabla_a, \nabla_b] + \nabla_{[a} G_{b]} + [G_a, G_b], \quad (40)$$

where

$$G_a = \frac{1}{4\sqrt{3}} (\Gamma^{cd} \Gamma_a + 2\Gamma^c \delta_a^d) F_{cd}. \quad (41)$$

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