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CALCULATION OF COLLIMATOR WAKEFIELDS *

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Abstract

We present longitudinal and dipole wakefield simulations of TESLA and NLC collimator designs using a newly developed numerical algorithm, one that allows for the accurate calculation of the combination of very short bunches and very long, gently tapered structures. We demonstrate that the new algorithm is superior to the standard method of such calculations. For small taper angles our results agree with analytical formulas of Yokoya. In addition, we optimize the TESLA TTF2 "step+taper" collimator design, and compare with collimator test measurements carried out at SLAC.

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We present longitudinal and dipole wakefield simulations of TESLA and NLC collimator designs using a newly developed numerical algorithm, one that allows for the accurate calculation of the combination of very short bunches and very long, gently tapered structures. We demonstrate that the new algorithm is superior to the standard method of such calculations. For small taper angles our results agree with analytical formulas of Yokoya. In addition, we optimize the TESLA TTF2 "step+taper" collimator design, and compare with collimator test measurements carried out at SLAC.

INTRODUCTION

In future linear colliders, such as the TESLA [1] and the NLC project [2], in order to remove halo particles, the beams pass through a series of collimators before entering into collision. With the high current, low emittance beams envisioned, however, short-range transverse wakefields, generated when passing even slightly off-center through the collimator region, can spoil the projected emittance of the beams, and therefore the luminosity of the collider. Therefore, optimizing the collimator design to reduce wakefields is an important task for such projects.

A collider collimator can be described as a shallow transition from a beam pipe to a smaller aperture and then back again. Among the features making it difficult to find the wake for such structures are their finite wall conductivity, their complicated (non-cylindrically symmetric) geometry, and their long, gentle transitions. To simplify the calculation, cylindrically symmetric models are often used. In addition, for the purpose of design, the wakefields of such collimators are separated into two components, a resistive-wall component and a geometric component, where the geometric component is the wake assuming perfectly conducting walls [2].

In this report we study the geometric component of the wake of cylindrically symmetric collider collimators, using a time domain numerical method to obtain the dipole, and also the longitudinal, wakes. A main difficulty in such calculations is that—due to the short bunch length and the long, shallow tapers of the collimator—grid dispersion and errors in geometry (*e.g.* a shallow taper ends up with stair steps) can arise.

Using a recently developed numerical approach [3], one that is able to model arbitrary, cylindrically symmetric boundaries faithfully and does not suffer from longitudinal dispersion, we are able to calculate accurately the short-range wakefields of TESLA and NLC collimator designs. Our numerical results are compared to those of a standard time domain program, ABCI [4], to analytical estimates, and to measurements. Optimization of the TESLA TTF2 collimator geometry is also performed.

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CALCULATIONS

New Time-Domain Program

We will study the wakefield effects of collider collimators using the computer program ECHO. This program incorporates a newly developed finite difference, time-domain algorithm [3]. With a time step $c\Delta t = \Delta z$ (where c is the speed of light, t is time, and z is longitudinal position) allowed by the numerical stability condition, the algorithm has no longitudinal dispersion, allowing, with the use of a moving mesh, for the solution of very short bunches in very long structures. In addition, the "stair step" problem is avoided by means of the boundary approximation method of Ref. [5], a method that allows for accurate calculation in arbitrarily shaped, cylindrically symmetric structures. For longitudinal case (monopole mode) and staircase geometry approximation our scheme is reduced to the one presented in [6].

In this report we focus on perfectly conducting, cylindrically symmetric collimators of the form sketched in Fig. 1. A tube of smaller radius b and length l is connected to beam pipes of larger radius a by symmetric tapers of angle α and length L.

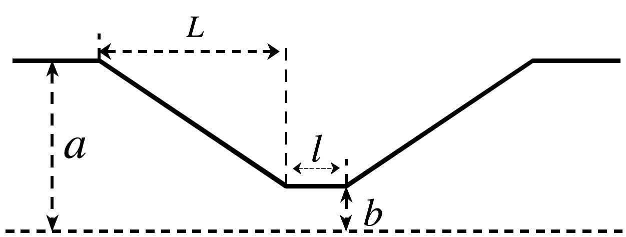

Fig.1. Collimator geometry.

For the TESLA and NLC collimators the angle α is small. K. Yokoya has shown that, if α and $\rho = \tan(\alpha)\omega b/c$ are small compared to 1 (ω is frequency), the longitudinal (monopole) and transverse (dipole) impedances of such structures are given by [7]

$$Z_{\parallel}^{0} = \Theta \frac{i\omega}{c^2} \int_{-\infty}^{\infty} (f')^2 dz, \tag{1}$$

$$Z_{\perp}^{1} = \Theta \frac{2i}{c} \int_{-\infty}^{\infty} (f'/f)^2 dz, \tag{2}$$

$f(z)$ is the beam pipe radius variation, and $\Theta = Z_0 c/4\pi$, with Z_0 the free space impedance. Note that the two impedances are purely inductive and resistive, respectively. Table 1 gives sample parameters of TESLA and NLC collimators and beams. The beams of charge Q have longitudinal distributions that are Gaussian, with rms length σ. Since the typical frequency excited ω~c/σ, we see that for TESLA $\rho = 0.03$, for NLC $\rho = 0.04$; therefore, in both cases we are in the small angle regime where Yokoya's formulas apply.

	α [mr]	a [mm]	b [mm]	l [mm]	Q [nC]	σ [mm]
TESLA	20	17.5	0.4	20	1.	0.3
NLC	20	17.5	0.2	20	1.	0.1

Table 1. Typical TESLA and NLC collimator and bunch properties.

In the time domain, Yokoya's formulas imply that the longitudinal bunch wake, $W_\parallel$, will be proportional to the derivative of the bunch shape, and the transverse bunch wake, $W_\perp$, will be proportional to the bunch shape itself. We can quantify the time domain results by the average wake ($\kappa_\parallel$ the loss factor or $\kappa_\perp$ the kick factor) and the rms of the wake, W_{rms}. Note that Yokoya's formulas imply: $\kappa_\parallel = 0$ and [for small b/a] $\kappa_\perp = 2\alpha/(\sqrt{\pi}\sigma b)$, $(W_\parallel)_{rms} \approx 0.35\alpha a/\sigma^2$, $(W_\perp)_{rms} \approx 0.44\alpha/(\sigma b)$.

Parameter Study

We begin our numerical study with the parameters in Table 1 for TESLA, and study the dependence of the results on α. Longitudinal results—$\kappa_\parallel$ and $(W_\parallel)_{rms}$—are shown in Fig. 2. The solid black curves give results obtained by ECHO with $\sigma/h = 5$ (h is the mesh size). The wake was calculated for angles down to $\alpha = 0.5^o$, in which case the total collimator length ~4 m. To test the accuracy of the numerical results, at sample points (α= 5°, 10°, 20°), the mesh was made finer, and the result changed by less than 1%. Yokoya's formula predicts $\kappa_\parallel = 0$, and $(W_\parallel)_{rms}$ as given by the dot-dashed curve in the figure (the right plot). We see that the ECHO results approach the analytical solutions for small α; at $\alpha = 10^o$ the difference is ~10%, at $\alpha = 5^o$ it is <3%. Finally, in Fig. 2, for comparison, we also display results obtained by the time-domain program, ABCI, for cases σ/h= 5, 10, 20 (gray curves); we see that for large α a much finer mesh is needed than by ECHO, and for small α the dependence is not correct.

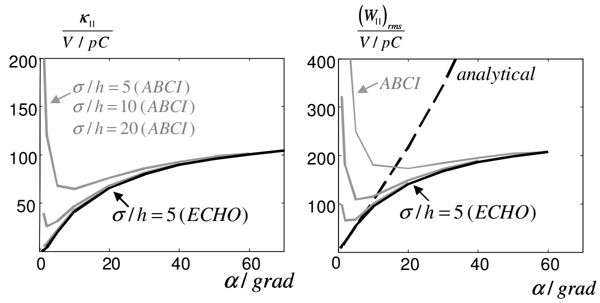

Fig. 2. Longitudinal wake dependence on collimator angle.

In Fig. 3 we plot the transverse results of ECHO—$\kappa_\perp$ and $(W_\perp)_{rms}$—and compare to the analytical results, and we see that the numerical results have the correct small α behavior.

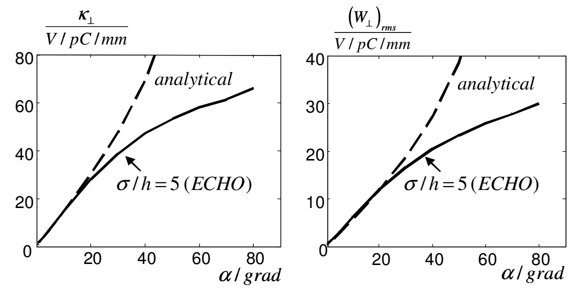

Fig. 3. Transverse wake dependence on collimator angle.

In Fig. 4 we give the wakes themselves, $W_\perp$ (left) and $W_\parallel$ (right), as o btained by ECHO, for the case α = 20 mrad and for several cases of σ/h (gray lines). We see that the numerical results approach the analytical curves (black dashes) as the mesh becomes finer.

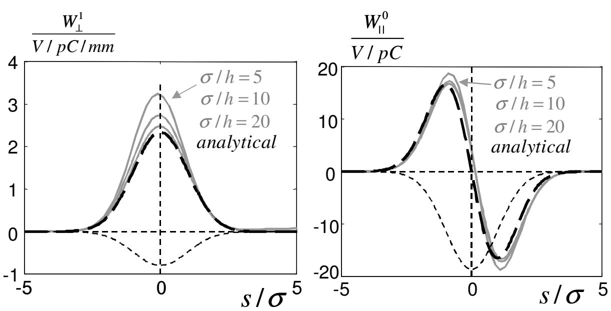

Fig. 4. Transverse (left) and longitudinal (right) wakes when α= 20 mrad.

In Table 2 we give $\kappa_\parallel$ and $\kappa_\perp$, for TESLA and NLC collimators with α = 20 mrad (near the nominal value), as obtained by ECHO. Also given are some results of ABCI (in parentheses) and the analytical asymptotic values. We clearly see that, for a given mesh, ECHO is much more accurate than ABCI. We note also that the absolute error for ECHO remains nearly unchanged as the length of the collimator is increased, unlike a program like ABCI, which requires an ever increasingly dense mesh.

σ/h	$\kappa_\parallel$ [V/pC]		$\kappa_\perp$ [V/pC/mm]	
	TESLA	NLC	TESLA	NLC
5	1.46	33.6	2.24 (135)	12.8
10	1.42	**32.9**	1.91 (44.3)	11.2
20	**1.42**	-	1.74 (13.4)	10.6
Analytical	0		**1.65**	**10**

Table 2: Dependence of $\kappa_{\|}$ and $\kappa_{\perp}$ on mesh density when α = 20 mrad, as obtained by ECHO. ABCI results are given in parentheses.

Collimator Shape Optimization

We have seen that reducing the taper angle decreases the wakefield effect. However, to obtain a significant effect, the types of collimator that we have studied so far may need to be meters long. An alternative solution is to consider a type of collimator that we call a "step+taper" collimator [8]. An example is shown in Fig. 5 where the parameters correspond to the TESLA TTF2 collimator [9]. We perform ECHO calculations for a very short bunch, $\sigma = 0.05$ mm.

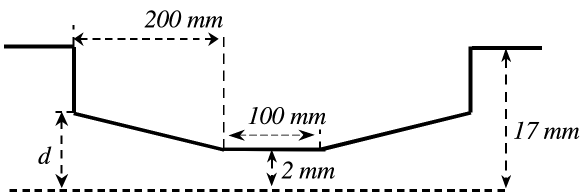

Fig. 5. Geometry of the TESLA TTF2 "step+taper" collimator.

Fig. 6 displays κ and W_{rms}, both longitudinal and transverse, as functions of collimator parameter *d*. As can be seen all functions have a minimum; the overall minimum can be taken to be $d \approx$ 4.5 mm.

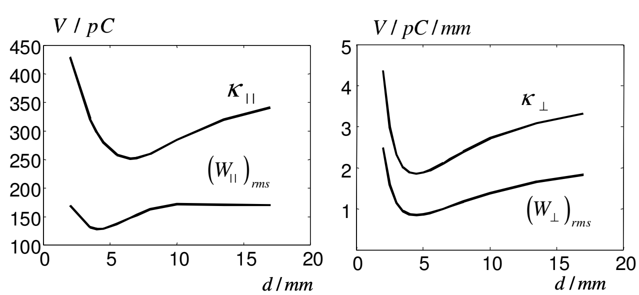

Fig. 6. Collimator geometry optimization.

Comparison with Measurement

Finally, we compare ECHO numerical results with experimental data. At SLAC dedicated test chambers with collimators were constructed, installed in the SLAC linac, and then the transverse wakefield effect was measured with beam [10]. One chamber has a square collimator that we will assume has a wakefield that is similar to that of a round chamber. The parameters are: α= 335 mrad, *a*= 19 mm, *b* = 1.9 mm, *l* = 0 mm, and *L* = 51 mm. (Note that the taper angle α is much larger than will be the case in future colliders.) The measured, simulated, and analytical small angle asymptotic results are given in the Table 3. The ECHO results were checked by refining the mesh, and were found to be accurate to better than 1%. Note that $\rho = 0.55(1.02)$ for the case $\sigma = 1.2(0.65)$ mm, and it is therefore not surprising, in the latter case, that simulated and analytical results are in disagreement. Measurement and simulation agree well for the former case, but significantly disagree in the latter, shorter bunch case, a disagreement that is not understood.

σ [mm]	Measured	Simulated	Analytical
1.2	1.2 ± 0.1	1.268	1.34
0.65	1.4 ± 0.1	1.908	2.48

Table 3. SLAC collimator measurement comparison. Given are kick factors, $\kappa_{\perp}$, in units of V/pC/mm.

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