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Impact of tag-side interference on time-dependent CP asymmetry measurements using coherent $B^0\overline{B}^0$ pairs

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Interference between CKM-favored $b \to c\overline{u}d$ and doubly-CKM-suppressed $\overline{b} \to \overline{u}c\overline{d}$ amplitudes in final states used for B flavor tagging gives deviations from the standard time evolution assumed in CP-violation measurements at B factories producing coherent $B^0\overline{B}^0$ pairs. We evaluate these deviations for the standard time-dependent CP-violation measurements, the uncertainties they introduce in the measured quantities, and give suggestions for minimizing them. The uncertainty in the measured CP asymmetry for CP eigenstates is $\approx 2\%$ or less. The time-dependent analysis of $D^*\pi$, proposed for measuring $\sin(2\beta+\gamma)$, must incorporate possible tag-side interference, which could produce asymmetries as large signal asymmetry.

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I. INTRODUCTION

Measurements of time-dependent CP asymmetries in B^0 decays provide information about the irreducible phase contained in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1], which describes CP violation in the Standard Model. If a specific B decay final state has contributions from more than one amplitude and these amplitudes have different CP-violating weak phases, interference can produce a non-zero CP asymmetry. An essential ingredient in CP violation measurements in B^0 decays is flavor tagging. In this paper, we point out a subtlety of flavor tagging that has been overlooked or ignored in most recent CP violation analyses, describe the impact of this omission, and propose how to address it in some future measurements.

In the current asymmetric B-factories [2], PEP-II and KEKB, $B^0\overline{B}^0$ meson pairs are produced

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in e^+e^- interactions at the $\Upsilon(4S)$ resonance, where the pair evolves coherently in a P-wave state until one of the B mesons decays. Typically, one B decay is fully reconstructed and the flavor (whether it's a B^0 or $\overline{B}^0$) of this B, at the time of the other B's decay, is inferred from the decay products of the other B (the tag B). At the time of the tag B meson decay, the B mesons are known to be in opposite flavor states. In terms of the time difference between the two B decays, $\Delta t \equiv t_{\rm rec} - t_{\rm tag}$, the time-dependent CP asymmetry is defined as

$$A_{CP}(\Delta t) \equiv \frac{N\left(\text{tag } B^0, \Delta t\right) - N\left(\text{tag } \overline{B}^0, \Delta t\right)}{N\left(\text{tag } B^0, \Delta t\right) + N\left(\text{tag } \overline{B}^0, \Delta t\right)}\,, \tag{1}$$

where N is the number of events at Δt with a B^0 or $\overline{B}^0$ as the tag B.

Charged leptons and kaons are often used to infer the flavor of the tag B meson. The charge of a lepton from a semi-leptonic B decay has the same sign as the charge of the b quark that produced it. For example, a high-momentum e^+ (e^-) would indicate that the tag B was a B^0 ($\overline{B}^0$) at the time of its decay. Similarly, a $K^+(K^-)$ more often than not comes from a B^0 ($\overline{B}^0$). This works because the most likely b decay is $b \to c$ and the most likely c decay is $c \to s$; thus the s quark usually has the same charge as the b quark. The lepton or kaon charge does not always correctly indicate the tag-B flavor. Mistags can come from incorrect particle identification or other B decay chains that produce wrong-sign leptons or kaons. The mistag fraction must be measured in order to determine the true CP asymmetry from the measured one.

It is usually assumed that the measured CP asymmetry is entirely due to the interfering amplitudes contributing to the fully reconstructed B decay mode, and that the individual tagging states, such as $\overline{B}^0 \to D^+\pi^-$, are dominated by a single B decay amplitude. In other words, if only one B decay amplitude contributes to the tagging final state, it is safe to assume that all interference effects, such as CP violation, are due to the evolution of the fully reconstructed B. This assumption, which is valid for lepton tags, ignores the possibility of suppressed contributions to the tag-side final state with different weak phases, such as happens for non-leptonic decays.

These suppressed contributions may be important for kaon tags. For example, the $D^+\pi^-$ final state with $D^+ \to K^-\pi^+\pi^+$, which is usually associated with a $\overline{B}^0$ decay, can also be reached from a B^0 through a $\overline{b} \to \overline{u}c\overline{d}$ decay. Its amplitude is suppressed relative to the dominant $\overline{B}^0$ decay amplitude ($b \to c\overline{u}d$) by a factor of roughly $|(V_{ub}^* V_{cd})/(V_{cb} V_{ud}^*)| \approx 0.02$, and has a relative weak phase difference of γ. Both Feynman diagrams are shown in Fig. 1. The tag-side $b \to c\overline{u}d$ and

$\overline{b} \to \overline{u}c\overline{d}$ amplitudes interfere, and, through the coherent evolution of the $B^0\overline{B}^0$ pair, alter the time evolution of $A_{CP}(\Delta t)$. The subject of this paper is to investigate the consequences of this small tag-side interference in some of the standard time-dependent CP-asymmetry measurements at B factories that use coherent B decays.

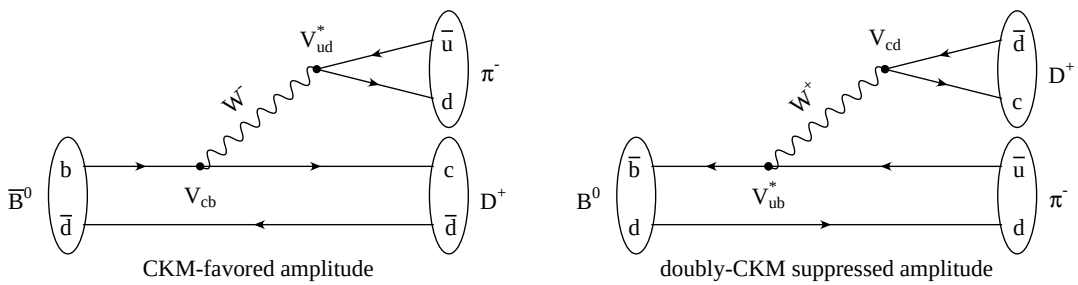

FIG. 1: The CKM-favored amplitude (left) and doubly-CKM-suppressed amplitude (right) for the final state $D^+\pi^-$. With respect to the dominant contribution, the latter is suppressed by the approximate ratio $|(V_{ub}^* V_{cd})/(V_{cb} V_{ud}^*)| \approx 0.02$ and has a relative weak phase difference of γ.

In Sections II – VI, we review the general formalism for describing the coherent evolution of the $B^0\overline{B}^0$ system, define our notation for describing the tag-side amplitude, and state the assumptions we employ in our analysis. In Section VII A, we evaluate how tag-side interference affects the mistag fraction measured from the amplitude of the time-dependent mixing (not CP) asymmetry. We find that the tag-side interference effects are *not* simply absorbed into the mistag fractions and that, to first order, the mistag fractions are unchanged by tag-side interference. In Section VII B, we evaluate the uncertainty, due to tag-side interference, in the standard mixing-induced CP asymmetry measurements – $\sin 2\beta$ from $J/\psi K_s$ and the CP asymmetry in $\pi^+\pi^-$. We find that the uncertainties are at most 5%, in the most conservative estimation, and can be limited to $< 2\%$ in most cases with reasonable assumptions. Finally, in Section VIII, we evaluate how tag-side interference affects some of the time-dependent techniques that have been proposed for measuring γ (e.g. the time-dependent analysis of $D^{*+}\pi^-$). Here, we find that tag-side interference effects can be as large as the signal asymmetry. We propose a technique for performing the analysis in a general way, which does not require assumptions about the size of tag-side interference effects and maximizes the statistical sensitivity to $(2\beta + \gamma)$. We summarize our conclusions in Section IX.

II. GENERAL COHERENT FORMALISM

In this section, we define our formalism for describing the time evolution of a pair of neutral B mesons that are coherently produced in an $\Upsilon(4S)$ decay and then subsequently decay to arbitrary final states $f_{\rm t}$ and $f_{\rm r}$ at times $t_{\rm t}$ and $t_{\rm r}$, respectively, measured in the parent B meson's rest frame. The "t" ("r") subscript refers to the tag (reconstructed) B meson or its final state. The amplitude for this process is proportional to

$$\mathcal{A} = \langle f_{\rm t}|B^0_{\rm phys}(t_{\rm t})\rangle\langle f_{\rm r}|\overline{B}^0_{\rm phys}(t_{\rm r})\rangle - \langle f_{\rm t}|\overline{B}^0_{\rm phys}(t_{\rm t})\rangle\langle f_{\rm r}|B^0_{\rm phys}(t_{\rm r})\rangle\,, \tag{2}$$

where $B^0_{\rm phys}(t)$ ($\overline{B}^0_{\rm phys}(t)$) denotes an initially-pure B^0 ($\overline{B}^0$) state after a time t. The relative minus sign between the terms reflects the antisymmetry of the P-wave $B^0\overline{B}^0$ state. Integrating over all directions for either B and the experimentally-unobservable average decay time $(t_{\rm t}+t_{\rm r})/2$, we obtain a corresponding decay rate proportional to ($\Delta t \equiv t_{\rm r} - t_{\rm t}$)

$$F(\Delta t) = e^{-\Gamma|\Delta t|}\ \left|a_+ g_+(\Delta t) + a_- g_-(\Delta t)\right|^2\ , \tag{3}$$

where Γ is the average neutral B eigenstate decay rate and we define

$$g_\pm(\Delta t) \equiv \frac{1}{2}\left(e^{-i\Delta m\Delta t/2}e^{-\Delta\Gamma\Delta t/4} \pm e^{+i\Delta m\Delta t/2}e^{+\Delta\Gamma\Delta t/4}\right) \tag{4}$$

in terms of the differences between the eigenstate masses (Δm) and decay rates ($\Delta\Gamma$).

The time-independent complex parameters $a_\pm$ in Equation (3) can be written generally as

$$a_+ = \overline{\mathcal{A}}_{\rm t}\,\mathcal{A}_{\rm r} - \mathcal{A}_{\rm t}\,\overline{\mathcal{A}}_{\rm r} \quad , \quad a_- = -\sqrt{1-z^2}\,\left(\frac{q}{p}\overline{\mathcal{A}}_{\rm t}\,\overline{\mathcal{A}}_{\rm r} - \frac{p}{q}\mathcal{A}_{\rm t}\,\mathcal{A}_{\rm r}\right) + z\,\left(\overline{\mathcal{A}}_{\rm t}\,\mathcal{A}_{\rm r} + \mathcal{A}_{\rm t}\,\overline{\mathcal{A}}_{\rm r}\right)\ , \tag{5}$$

where $\mathcal{A}_k$ ($\overline{\mathcal{A}}_k$) is the B^0 ($\overline{B}^0$) decay amplitude to f_k. The complex ratio q/p parameterizes possible CP and T violation ($|q/p| \neq 1$) in the time evolution of a neutral B state, while z, which is also complex, parametrizes possible CPT and CP violation ($z \neq 0$) in the time evolution. Note that exchanging the r and t subscripts changes the overall sign of a_+, g_-, and Δt, leaving Eq.(3) unchanged, which is required since the distinction between the B that is reconstructed and the B that is used for flavor tagging is arbitrary at this point. Explicitly, we are using the conventions

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}\,, \tag{6}$$

where M and Γ are the hermitian matrices of the effective Hamiltonian. The eigenstates of the effective Hamiltonian are defined as

$$|B_L\rangle = p\,|B^0\rangle + q\,|\overline{B}^0\rangle \tag{7}$$

$$|B_H\rangle = p\,|B^0\rangle - q\,|\overline{B}^0\rangle\,, \tag{8}$$

and $\Delta m = m_H - m_L$, which is positive by definition. If $z = 0$, as expected in the Standard Model, the two terms $a_\pm g_\pm(\Delta t)$ in Eq.(3) describe the cases where the surviving meson undergoes a net oscillation $(-)$ $B^0 \leftrightarrow \overline{B}^0$ or not $(+)$ between $t_{\rm t}$ and $t_{\rm r}$. Combining Equations (3–5) we obtain

$$F(\Delta t) = e^{-\Gamma|\Delta t|}\left[R\cosh(\Delta\Gamma\Delta t/2) + C\cos(\Delta m\Delta t) + S'\sinh(\Delta\Gamma\Delta t/2) + S\sin(\Delta m\Delta t)\right] \tag{9}$$

with coefficients which satisfy the constraint $C^2 + S^2 = R^2 - S'^2$, and are given by

$$\begin{aligned} R &\equiv \frac{1}{2}\left(|a_+|^2 + |a_-|^2\right), & S' &\equiv -\mathrm{Re}(a_+^* a_-)\,, \\ C &\equiv \frac{1}{2}\left(|a_+|^2 - |a_-|^2\right), & S &\equiv +\mathrm{Im}(a_+^* a_-)\,. \end{aligned} \tag{10}$$

In the following, we assume CPT invariance so that $z = 0$, and moreover we take $\Delta\Gamma/\Gamma \ll 1$. Thus the term S' no longer enters and $\cosh(\Delta\Gamma\Delta t/2)$ is replaced by unity. Additionally, this gives $|q/p| = 1$. The resulting time dependence, when the tagged meson is a B^0, is

$$F(\Delta t) = e^{-\Gamma|\Delta t|}\left[R + C\cos(\Delta m\Delta t) + S\sin(\Delta m\Delta t)\right]\,, \tag{11}$$

and correspondingly when the tagged meson is a $\overline{B}^0$

$$\overline{F}(\Delta t) = e^{-\Gamma|\Delta t|}\left[\overline{R} + \overline{C}\cos(\Delta m\Delta t) + \overline{S}\sin(\Delta m\Delta t)\right]. \tag{12}$$

III. CHARACTERIZATION OF THE TAGGING AMPLITUDE

The strength of the doubly-CKM-suppressed (DCS) decays can be expressed in terms of the traditional parameter [3]

$$\lambda_f = \frac{q}{p}\frac{\overline{\mathcal{A}}_f}{\mathcal{A}_f}\,. \tag{13}$$

This combination is independent of the choice of phases for the B^0 and $\overline{B}^0$ states. Suppose $|f\rangle$ is a final state that is ostensibly the result of a B^0 decay. For example, if $|f\rangle$ represents the tag B,

a K^+ would indicate that the tag B decayed as a B^0, assuming the dominant $\overline{b} \to \overline{c}u\overline{d}$ transition occurred. Then

$$\lambda_f = r_f e^{-2i\beta - i\gamma} e^{i\delta_f} \,, \tag{14}$$

where r is a real number of order 0.02 and δ_f is the strong phase difference of the $\overline{B}^0$ decay relative to that of the B^0 decay, assuming $\overline{b} \to \overline{c}u\overline{d}$ and $b \to u\overline{c}d$ transitions for the B^0 and $\overline{B}^0$ decays respectively. If, for this final state, there is only one mechanism contributing to the B^0 decay and to the $\overline{B}^0$ decay, then for the CP conjugate state $|\overline{f}\rangle$ we have

$$\lambda_f = \frac{1}{r_f} e^{-2i\beta - i\gamma} e^{-i\delta_f} \,. \tag{15}$$

We shall make the assumption of a single contributing amplitude except as noted below.

Because the DCS amplitudes are only about 2% of the allowed amplitudes, in what follows we shall drop all terms that are quadratic or higher in this suppression. In practice we combine many final states f in a single tagging category, $f \in T$. For the tagging category we then have effective values of r' and δ' defined by

$$r' e^{i\delta'} = \frac{\sum_{f\in T} \epsilon_f |\mathcal{A}_f|^2 \, r_f e^{i\delta_f}}{\sum_{f\in T} \epsilon_f |\mathcal{A}_f|^2} \,, \tag{16}$$

where ϵ_f is the relative tagging efficiency for the state f. Notice that

$$|r'| \leq \frac{\sum_{f\in T} \epsilon_f |\mathcal{A}_f|^2 |r_f|}{\sum_{f\in T} \epsilon_f |\mathcal{A}_f|^2} \,, \tag{17}$$

so there is a tendency for contributions from different tagging states to cancel, unless all contributions have nearly the same strong phase. Equation 16 holds only if terms of order r_f^2 can be ignored, as we are assuming.

IV. TIME-DEPENDENT ASYMMETRY COEFFICIENTS

In this section, we evaluate the coefficients $R(\overline{R})$, $C(\overline{C})$, and $S(\overline{S})$ of Eqns. 11(12). There are two specific cases that we will consider – the "mixing" case, where the reconstructed B meson decays in an apparent flavor eigenstate (e.g. $D^{*+}\pi^-$, normally assumed to originate from $\overline{B}^0$ decay), and the "CP" case, where the reconstructed B has decayed into a CP eigenstate. Dropping a

common factor $\mathcal{A}_{\rm t}\,\mathcal{A}_{\rm r}\,(p/q)$, we can write a_+ and a_- in terms of the λ parameters for the tag and reconstructed B mesons as

$$\begin{aligned} a_+ &= \lambda_{\rm t} - \lambda_{\rm r} \\ a_- &= 1 - \lambda_{\rm t}\,\lambda_{\rm r}\,. \end{aligned} \tag{18}$$

Quite generally then,

$$\begin{aligned} |a_+|^2 &= |\lambda_{\rm t}|^2 - 2\,{\rm Re}\,\lambda_{\rm t}\,\lambda_{\rm r}^* + |\lambda_{\rm r}|^2 \\ |a_-|^2 &= 1 - 2\,{\rm Re}\,\lambda_{\rm t}\,\lambda_{\rm r} + |\lambda_{\rm t}|^2\,|\lambda_{\rm r}|^2 \\ {\rm Im}\,a_+^* a_- &= {\rm Im}\,\lambda_{\rm r}\,(1 - |\lambda_{\rm t}|^2) \;-\; {\rm Im}\,\lambda_{\rm t}\,(1 - |\lambda_{\rm r}|^2)\,. \end{aligned} \tag{19}$$

Table I gives the coefficients for the mixing case. The only deviation from the familiar case with no DCS contributions, to first order in r and r', is the presence of a small $S(\overline{S})$ coefficient. Figure 2 shows an illustration of the time evolution for when the flavor of the two B mesons at the time of decay was opposite (unmixed) or the same (mixed). The nominal ($r = r' = 0$) case is contrasted with an example of a non-zero DCS contribution in the reconstructed B amplitude and with an example of non-zero DCS contributions to both the tag and reconstructed B amplitudes. The amplitude ratios r and r' have been enlarged by $\times 5$ with respect to the expected value (0.02) so that the DCS contributions are more clear.

Table II gives the coefficients for the CP case. All three coefficients receive corrections linear in r'. Figure 3 is an illustration of the the corrections to the time evolution for B^0 and $\overline{B}^0$ tagged CP events, also with the DCS amplitude ratio r' enlarged by $\times 5$ to make the differences more visible.

V. COMPLETELY INCLUSIVE TAGGING CATEGORIES

We can relate the effective r' and δ' to the 2×2 matrix Γ that generalizes the decay rate for the $B^0\overline{B}^0$ system. Let Γ be the class of states DX, where X represents non-charmed hadrons. Then

$$\sum_{F\in T} \frac{q}{p}\mathcal{A}_f^*\overline{\mathcal{A}}_f = \sum_{F\in T} |\mathcal{A}_f|^2\lambda_f = \sum_{F\in T} \langle B^0|\mathcal{H}|f\rangle\langle f|\mathcal{H}|B^0\rangle r' e^{-2i\beta - i\gamma + i\delta'} = \Gamma_{DX}\, r' e^{-2i\beta - i\gamma + i\delta'}\,, \tag{20}$$

where Γ_{DX} is, up to a trivial normalization, the partial width of B^0 into the class of states of the form DX. On the other hand, we can write

$$\sum_{F\in T} \frac{q}{p}\mathcal{A}_f^*\overline{\mathcal{A}}_f = \sum_{F\in T} \frac{q}{p}\langle B^0|\mathcal{H}|f\rangle\langle f|\mathcal{H}|\overline{B}^0\rangle = \frac{q}{p}\Gamma_{DX\,12}\,, \tag{21}$$

	[tag=$B^0(K^+)$, rec=B^0]	[tag=$B^0(K^+)$, rec=$\overline{B}^0$]	[tag=$\overline{B}^0(K^-)$, rec=B^0]	[tag=$\overline{B}^0(K^-)$, rec=$\overline{B}^0$]
$\lambda_{\rm t}$	$r'e^{-2i\beta-i\gamma+i\delta'}$	$r'e^{-2i\beta-i\gamma+i\delta'}$	$\frac{1}{r'}e^{-2i\beta-i\gamma-i\delta'}$	$\frac{1}{r'}e^{-2i\beta-i\gamma-i\delta'}$
$\lambda_{\rm r}$	$re^{-2i\beta-i\gamma+i\delta}$	$\frac{1}{r}e^{-2i\beta-i\gamma-i\delta}$	$re^{-2i\beta-i\gamma+i\delta}$	$\frac{1}{r}e^{-2i\beta-i\gamma-i\delta}$
$\lvert a_+\rvert^2$	0	$\frac{1}{r^2}$	$\frac{1}{r'^2}$	0
$\lvert a_-\rvert^2$	1	0	0	$\frac{1}{+r^2r'^2}$
Im $a_+^*a_-$	$-r\sin(2\beta+\gamma-\delta)$	$-\frac{r'}{r^2}\sin(2\beta+\gamma-\delta')$	$\frac{r}{r'^2}\sin(2\beta+\gamma-\delta)$	$-\frac{1}{r^2r'}\sin(2\beta+\gamma+\delta')$
	$+r'\sin(2\beta+\gamma-\delta')$	$-\frac{1}{r}\sin(2\beta+\gamma+\delta)$	$+\frac{1}{r'}\sin(2\beta+\gamma+\delta')$	$+\frac{1}{rr'^2}\sin(2\beta+\gamma+\delta)$
R	1	1	1	1
C	-1	1	1	-1
S	$-2r\sin(2\beta+\gamma-\delta)$	$-2r'\sin(2\beta+\gamma-\delta')$	$2r\sin(2\beta+\gamma-\delta)$	$-2r'\sin(2\beta+\gamma+\delta')$
	$+2r'\sin(2\beta+\gamma-\delta')$	$-2r\sin(2\beta+\gamma+\delta)$	$+2r'\sin(2\beta+\gamma+\delta')$	$+2r\sin(2\beta+\gamma+\delta)$

TABLE I: Contributions to the time dependence of tagged decays when the reconstructed decay is an apparent flavor eigenstate, with doubly-CKM-suppressed decays considered only to first order. The dependences proportional to 1 and to $\cos\Delta m\Delta t$ are unaffected. A small $\sin\Delta m\Delta t$ term is induced. Appropriate factors of r and r' have been removed to scale R to unity.

where $\Gamma_{DX\,12}$ is the contribution of states of the form DX to the off-diagonal part of the Γ matrix. So

$$r'e^{-2i\beta-i\gamma+i\delta'} = \frac{q}{p}\Gamma_{DX\ 12}/\Gamma_{DX}\,. \tag{22}$$

If tagging does not capture every state, we can think of Γ_{12} and Γ as effective quantities, limited by the partial sum over states. However, if that sum were complete, then δ' would vanish. To see this, imagine using as a basis of states f_S not the physical states that are observed but instead a basis of states that are eigenstates of the S matrix, that is a basis of states that each scatter into themselves. Because we are summing over all states in a collection connected by strong interactions, there is such a basis. Then the final state interaction phases associated with $\mathcal{A}_{f_S}$ and $\overline{\mathcal{A}}_{f_S}$ would both be $e^{i\delta_{f_S}}$. These would cancel in $\mathcal{A}^*_{f_S}\overline{\mathcal{A}}_{f_S}$. In general, because tagging is incomplete, we cannot assume that δ' vanishes.

	[tag=$B^0(K^+)$, rec=B_{CP}]	[tag=$\overline{B}^0(K^-)$, rec=B_{CP}]
$\lambda_{\rm t}$	$r' e^{-2i\beta - i\gamma + i\delta'}$	$\frac{1}{r'} e^{-2i\beta - i\gamma - i\delta'}$
$\lambda_{\rm r}$	λ_{CP}	λ_{CP}
$\lvert a_+\rvert^2$	$\lvert\lambda_{CP}\rvert^2 - 2\,\mathrm{Re}\, r' e^{-2i\beta - i\gamma + i\delta'} \lambda_{CP}^*$	$\frac{1}{r'^2} - 2\frac{1}{r'}\,\mathrm{Re}\,\lambda_{CP} e^{2i\beta + i\gamma + i\delta'}$
$\lvert a_-\rvert^2$	$1 - 2\,\mathrm{Re}\, r' e^{-2i\beta - i\gamma + i\delta'} \lambda_{CP}$	$\frac{\lvert\lambda_{CP}\rvert^2}{r'^2} - 2\frac{1}{r'}\,\mathrm{Re}\,\lambda_{CP} e^{-2i\beta - i\gamma - i\delta'}$
$\mathrm{Im}\; a_+^* a_-$	$\mathrm{Im}\,\lambda_{CP} + r'(1 - \lvert\lambda_{CP}^2\rvert)\sin(2\beta + \gamma - \delta')$	$-\frac{1}{r'^2}\,\mathrm{Im}\,\lambda_{CP} + \frac{1}{r'}(1 - \lvert\lambda_{CP}\rvert^2)\sin(2\beta + \gamma + \delta')$
R	$\frac{1+\lvert\lambda_{CP}^2\rvert}{2} - 2r'\,\mathrm{Re}\,\lambda_{CP}\cos(2\beta + \gamma - \delta')$	$\frac{1+\lvert\lambda_{CP}^2\rvert}{2} - 2r'\,\mathrm{Re}\,\lambda_{CP}\cos(2\beta + \gamma + \delta')$
C	$\frac{\lvert\lambda_{CP}^2\rvert - 1}{2} + 2r'\,\mathrm{Im}\,\lambda_{CP}\sin(2\beta + \gamma - \delta')$	$\frac{1-\lvert\lambda_{CP}^2\rvert}{2} + 2r'\,\mathrm{Im}\,\lambda_{CP}\sin(2\beta + \gamma + \delta')$
S	$\mathrm{Im}\,\lambda_{CP} + r'(1 - \lvert\lambda_{CP}^2\rvert)\sin(2\beta + \gamma - \delta')$	$-\,\mathrm{Im}\,\lambda_{CP} + r'(1 - \lvert\lambda_{CP}^2\rvert)\sin(2\beta + \gamma + \delta')$

TABLE II: Contributions to the time dependence of tagged decays when the reconstructed decay is a CP eigenstate, with doubly-CKM-suppressed decays considered only to first order. Appropriate factors of r and r' have been removed to scale R to unity in the limit in which the doubly-CKM-suppressed decays vanish.

VI. ESTIMATED SIZE OF DOUBLY-CKM SUPPRESSED AMPLITUDE

In the Introduction, we gave an estimate for the size of the DCS amplitude (r), relative to the favored amplitude, to be approximately 0.02, which is simply the ratio of the CKM elements involved $|(V_{ub}^* V_{cd})/(V_{cb} V_{ud}^*)|$. Here, we discuss the uncertainty of this estimate as well as what can be assumed, if anything, about the strong phase difference (δ) between the DCS and favored amplitudes.

We use measured charm branching fractions as a test of our simple amplitude ratio estimate. The charm decay $D^0 \to K^+\pi^-$ is doubly-CKM suppressed relative to the favored $D^0 \to K^-\pi^+$ decay. The amplitude ratio prediction, based solely on the CKM elements, gives $r \approx |(V_{cd}^* V_{us})/(V_{cs}^* V_{ud})| \approx 0.048$. The experimental value from the branching fractions [4] is 0.062 ± 0.005, which is within 25% of the ratio of CKM elements. For the singly-CKM suppressed charm decays $D^0 \to K^+K^-$ and

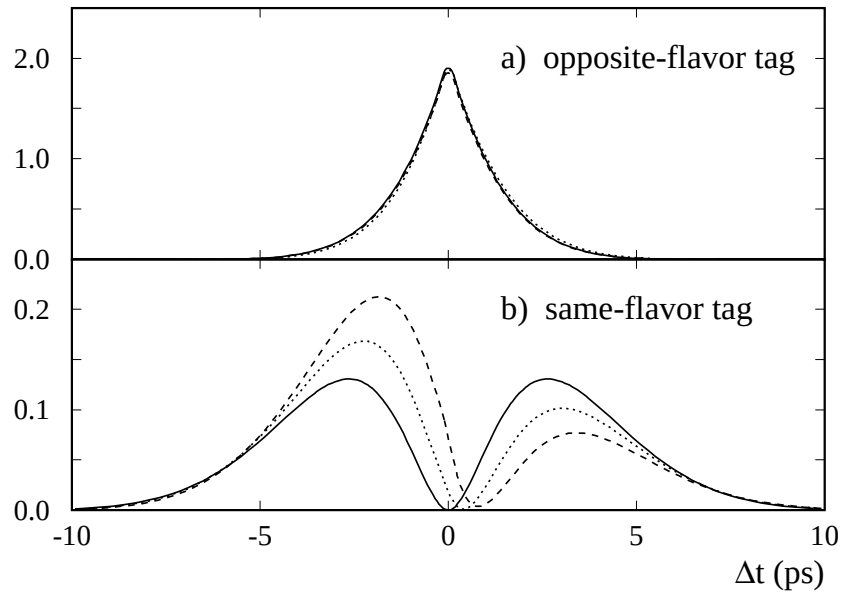

FIG. 2: Time-dependent decay distributions for the final state $D^{*-}\pi^+$, for a) a $\overline{B}^0$ tag, and b) a B^0 tag. $(2\beta+\gamma)$ is set to the value 1.86. The situation with no doubly-CKM-suppressed contribution on both the tag-side and reconstruction-side is indicated with the solid line. The dotted line has $r = 0.1$ and $\delta = 0$, but no tag-side interference. The dashed line represents the example with $r = r' = 0.1$, $\delta = 0$, and $\delta' = \pi$. In these examples, the r and r' values are $\times 5$ the expected values in order to clearly illustrate the differences with respect to the case with $r = r' = 0$.

$D^0 \to \pi^+\pi^-$ we would estimate amplitude ratios relative to the allowed amplitude of $|V_{us}/V_{ud}| \approx |V_{cd}/V_{cs}| \approx 0.23$, while the branching ratios give 0.329 ± 0.007 and 0.194 ± 0.005 for K^+K^- and $\pi^+\pi^-$, respectively.

The decay $B^0 \to D^+\pi^-$ is doubly-CKM suppressed, but this branching fraction has not been measured. We can estimate its branching fraction from the related decay mode $B^0 \to D_s^+\pi^-$, which has been observed recently [5], and has a branching fraction of $(3.2 \pm 0.9 \pm 1.0) \times 10^{-5}$. The amplitude ratio for $B^0 \to D^+\pi^-$, relative to $B^0 \to D^-\pi^+$ is estimated to be

$$r_{D\pi} \approx \sqrt{\frac{\mathcal{B}(B^0 \to D_s^+\pi^-)}{\mathcal{B}(B^0 \to D^-\pi^+)}} \left|\frac{V_{cd}}{V_{cs}}\right| \frac{f_D}{f_{D_s}} \approx 0.021 \pm 0.005 \ ,$$

where we have used $f_D/f_{D_s} = 1.11 \pm 0.01 \pm 0.01$ (from [6]) to approximate $SU(3)$ breaking effects. This is in good agreement with the naive estimate of 0.020, albeit with a large uncertainty.

There are some theoretical arguments for expecting the strong phase difference δ to be small [7],

but we know of at least one case where a non-trivial strong phase has been observed in B decay. The strong phase difference between the longitudinal and parallel polarization amplitudes of the transversity basis in $B \to J/\psi K^*(892)$ has been measured [8] to be 2.50 ± 0.22, which is about 3σ from π, in contradiction with the factorization prediction of 0 or π. The size of the effective amplitude ratio (r'), given by Equation 16, depends on the δ values of the final states included in the tagging category. As Eq. 17 shows, varying δ values between the states will tend to reduce r'.

Given the $\approx 50\%$ uncertainty on the DCS amplitude ratio r for individual final states and

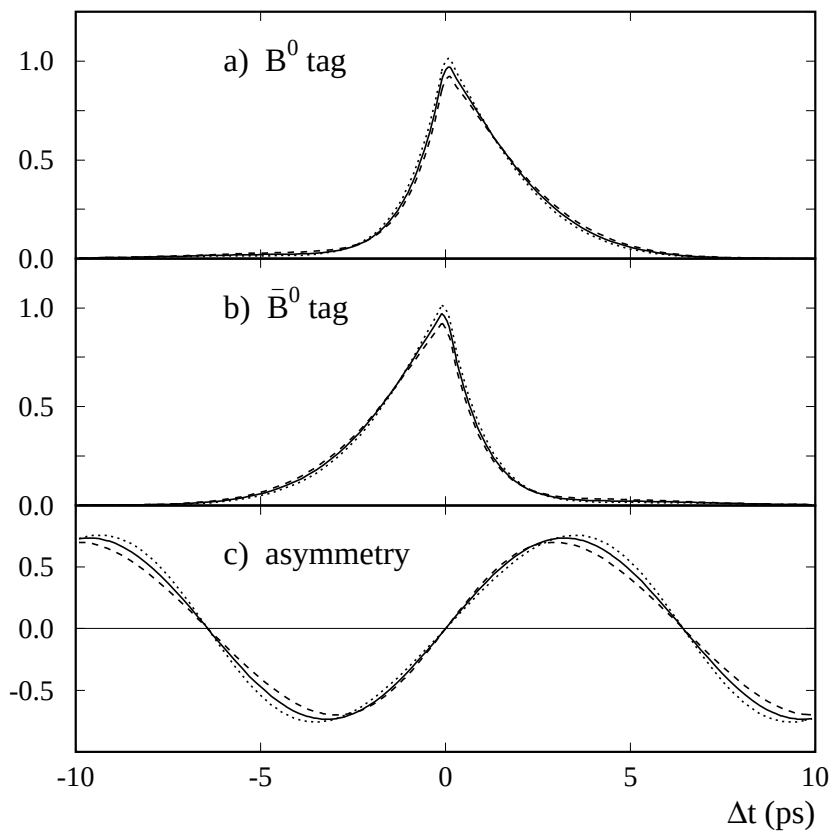

FIG. 3: Time-dependent decay distributions for the CP eigenstate $J/\psi K^0_S$, with a) a B^0 tag, and b) a $\overline{B}^0$ tag. We set $(2\beta + \gamma)$ to 1.86. The situation with no tag-side interference is indicated with the solid line. The dotted line represents the case with $r' = 0.1$ and $\delta' = 0$, and the dashed line has $r' = 0.1$, and $\delta' = \pi$. It should be noted that, adding a non-zero DCS contribution, the slope and amplitude of the time-dependent asymmetry work in opposite directions. In these examples, the r' value is $\times 5$ the expected value in order to clearly illustrate the differences with respect to the case with $r' = 0$.

the general lack of knowledge concerning strong phase differences, we conclude that the most conservative assumptions regarding the effective parameters r' and δ' would be to allow r' values from 0 (full cancellation in the sum) up to 0.04 (no cancellation with some enhancement over our 0.02 estimate) and to allow any value of δ'.

VII. UNCERTAINTIES IN FITTED ASYMMETRIES

In this section, we will discuss the uncertainties due to tag-side interference on some common time-dependent asymmetries. In addition to the assumptions that we have already made (i.e. $z = 0$, $\Delta\Gamma/\Gamma = 0$, and $|q/p| = 1$), one usually assumes that the tag-side amplitude is dominated by a single contribution, or $r' = 0$. The time dependent coefficients in Eqns. 11 and 12 simplify considerably with this assumption. For the case where the reconstructed B is a CP eigenstate, we have

$$R_{CP} = \overline{R}_{CP} \quad , \quad C_{CP} = -\overline{C}_{CP} \quad , \quad S_{CP} = -\overline{S}_{CP} \, , \tag{23}$$

which can be seen from Table II with r' set to zero. For the case where the reconstructed B is in an apparent flavor eigenstate, the coefficients in Table I with $r' = 0$ give

$$R_{\rm mix} = R_{\rm unmix} \quad , \quad C_{\rm mix} = -C_{\rm unmix} \quad , \quad S_{\rm mix} = S_{\rm unmix} = 0 \, , \tag{24}$$

where the "mix" ("unmix") subscript refers to the case where the tag and reconstructed B mesons were the same (opposite) flavor at the time of decay. In the rest of this Section, we will evaluate the bias on the fitted coefficients when fitting the data with the assumptions in Eqns. 23 or 24 of nonzero tag-side interference.

In the relations above, the R coefficients are independent of the final state configuration, so they are usually absorbed into the C and S coefficients by fitting for $\mathcal{C} \equiv (C/R)$ and $\mathcal{S} \equiv (S/R)$. A fairly reliable estimate of the fitted $\mathcal{C}$ coefficient is simply the asymmetry at $\Delta t = 0$. This would be

$$\mathcal{C}_{\rm fit} \approx \frac{C + R - \overline{C} - \overline{R}}{C + R + \overline{C} + \overline{R}} \tag{25}$$

for a CP asymmetry, or

$$\mathcal{C}_{\rm fit} \approx \frac{C_{\rm unmix} + R_{\rm unmix} - C_{\rm mix} - R_{\rm mix}}{C_{\rm unmix} + R_{\rm unmix} + C_{\rm mix} + R_{\rm mix}} \tag{26}$$

for a mixing asymmetry. A similar, but slightly less reliable, estimate for the fitted $\mathcal{S}$ coefficient in a CP asymmetry is simply the flavor-averaged $\mathcal{S}$ coefficient, or

$$\mathcal{S}_{\rm fit} \approx \frac{1}{2}\left(\frac{S}{R} - \frac{\overline{S}}{\overline{R}}\right). \tag{27}$$

Precise estimates can be derived using a simple maximum likelihood technique, where the likelihood to be maximized with respect to $\mathcal{C}_{\rm fit}$ and $\mathcal{S}_{\rm fit}$ is

$$\mathcal{L} = N \int_{-\infty}^{\infty} d\Delta t \; e^{-\Gamma|\Delta t|} \left[F(\Delta t) \; \ln F_{\rm fit}(\Delta t) + \overline{F}(\Delta t) \; \ln \overline{F}_{\rm fit}(\Delta t) \right], \tag{28}$$

with $F_{\rm fit}$ and $\overline{F}_{\rm fit}$ evaluated using the assumptions in Eq. 23. We confirmed that Eqns. 25 and 27 give reasonable estimates of $\mathcal{C}_{\rm fit}$ and $\mathcal{S}_{\rm fit}$ with unbinned maximum likelihood fits of simulated data samples.

A. Mistag calibration with flavor oscillation amplitude

As was mentioned above, the sign of the tagging kaon charge does not always give the correct flavor tag. For example, CKM-suppressed D decays, such as $D^+ \to K^+\overline{K}^0$, can produce wrong-sign kaons. Pions, incorrectly identified as kaons, can also produce wrong-sign kaons. The amplitude of any measured asymmetry using kaon tags will be reduced by a factor of $(1-2\omega)$, sometimes called the dilution factor, where ω is the fraction of tagging kaons that have the wrong sign (mistag fraction). The mistag fraction ω is usually measured from the amplitude of time-dependent flavor oscillations in a sample of reconstructed B^0 decays to flavor-specific final states [9]. The measured value of $\mathcal{C}$ will be a direct measurement of $(1-2\omega)$, which can then be used to translate measured CP asymmetry coefficients.

To first order in r and r', the R and C coefficients are the expected ones, as can be seen in Table I. The only effect is in the S coefficient, which is usually assumed to be zero in the analysis of mixing data. This means that the measured mistag fractions will be unaffected by DCS amplitude contributions, either on the tag side or the reconstructed side, since our estimator for $\mathcal{C}_{\rm fit}$ only depends on the R and C coefficients. Contrary to what one may guess, the corrections due to DCS amplitude contributions are not simply absorbed into the mistag fractions.

Using Monte Carlo pseudo-experiments, we also find that Δm_d is unaffected to the level of 0.001 ps^{-1} if allowed to float in the fit.

B. Fully reconstructed CP eigenstates

The size of CP asymmetries in B decays to CP eigenstates are in general of order one in the Standard Model. For example, CP asymmetry in $B \to J/\psi K_S^0$ (and related charmonium modes) has been measured to be $\mathcal{S}_{\rm fit} = 0.735 \pm 0.055$ [10, 11]. Any deviations due to tag-side interference (≈ 0.02) will be comparatively small (see Fig. 3), and can be treated as perturbations on the usual measurements.

In what follows, the nominal values for the fitted CP asymmetry coefficients without any tag-side interference from doubly-CKM suppressed decays are defined as

$$\mathcal{C}_0 = \frac{|\lambda_{CP}|^2 - 1}{|\lambda_{CP}|^2 + 1} \tag{29}$$

$$\mathcal{S}_0 = \frac{2{\rm Im}\lambda_{CP}}{|\lambda_{CP}|^2 + 1}. \tag{30}$$

The expected fitted coefficients, when the fit is performed with the assumptions in Eq. 23, can be found by inserting the R, C, and S values from Table II into Eqns. 25 and 27. Working to first order in r', we find

$$\begin{aligned}\mathcal{C}_{\rm fit} = \mathcal{C}_0 \big[1 \; &+ \; 2r' \cos\delta' \left\{\mathcal{G}\cos(2\beta+\gamma) - \mathcal{S}_0 \sin(2\beta+\gamma)\right\}\big] \\ &- \; 2r' \sin\delta' \left\{\mathcal{S}_0 \cos(2\beta+\gamma) + \mathcal{G}\sin(2\beta+\gamma)\right\}\end{aligned} \tag{31}$$

$$\mathcal{S}_{\rm fit} = \mathcal{S}_0 \big[1 \; + \; 2r' \cos\delta' \mathcal{G}\cos(2\beta+\gamma)\big] + 2r' \sin\delta' \mathcal{C}_0 \cos(2\beta+\gamma)\,, \tag{32}$$

where $\mathcal{G} \equiv 2{\rm Re}\lambda_{CP}/(|\lambda_{CP}|^2 + 1)$. Note that, with respect to the nominal values, there are both multiplicative and additive corrections which proportional to $\cos\delta'$ and $\sin\delta'$ respectively. In the limit of a vanishing effective tag-side strong phase difference ($\delta' \to 0$), only the multiplicative corrections remain.

For $B^0 \to J/\psi K_S^0$, the dominant tree and penguin amplitude contributions share the same weak phase. The highly suppressed u-quark penguin, which has a different relative weak phase, is typically ignored, giving the Standard Model prediction of $\lambda_{J/\psi K_S^0} = -e^{-i2\beta}$. Inserting this into Eqns. 31 and 32 gives

$$\mathcal{C}_{\rm fit}[J/\psi K_S^0] = -2r' \sin\gamma \sin\delta' \tag{33}$$

$$\mathcal{S}_{\rm fit}[J/\psi K_S^0] = \mathcal{S}_0 \left[1 - 2r' \cos\delta' \left\{\cos 2\beta \cos(2\beta+\gamma) + \mathcal{K} \sin 2\beta \sin(2\beta+\gamma)\right\}\right], \tag{34}$$

with $\mathcal{C}_0 = 0$ and $\mathcal{S}_0 = \sin 2\beta$. The last term in Eq. 34 proportional to $\mathcal{K}$ is a correction to the simple estimate given by Eq.(32). The correction $\mathcal{K}$ was derived from the more precise likelihood analysis given by Eq.(28). The value of $\mathcal{K}$ is between 0.10 and 0.35, depending on the value of $\sin 2\beta$. If we assume $\sin 2\beta = 0.74$ and allow γ to be in the range [45°,90°], then $\mathcal{K} = 0.28$ and the magnitude of the deviation of $\mathcal{S}_{\rm fit}$ away from the nominal value $\mathcal{S}_0$ is $< 0.7\, r$. The size of the deviation of $\mathcal{C}_{\rm fit}[J/\psi K^0_S]$ could be as large as $2\, r'$. These corrections to $\mathcal{S}_{\rm fit} = \mathcal{S}_0$ and $\mathcal{C}_{\rm fit} = 0$ could be as large or larger than Standard Model corrections [12].

The uncertainty estimates in the previous paragraph apply to a measurement that only uses kaon tags. In practice, all useful sources of flavor information from the tag side B are employed in order to maximize the sensitivity of the measurement. The statistical error on the measured asymmetry scales as $1/\sqrt{\sum_i Q_i}$, where each flavor tagging category contributes $Q_i = \epsilon_i(1-2\omega_i)^2$ and ϵ_i is efficiency for category i. Lepton flavor tags do not have the problem of a suppressed amplitude contribution with a different weak phase, so we assume that $r' = 0$ for lepton tags. If a measurement uses both lepton and non-lepton tags, the magnitude of the tag-side interference uncertainty will be scaled down by a factor of $Q_{\rm non-lep}/(Q_{\rm lep} + Q_{\rm non-lep})$. For example, the BaBar flavor tagging algorithm[10] has roughly $Q_{\rm lep} \approx 0.1$ and $Q_{\rm non-lep} \approx 0.2$. This gives a reduction of the tag-side interference uncertainty of about a factor of 3/2.

The CP asymmetry for $B \to \pi^+\pi^-$ is more complex. This decay has both tree and penguin amplitude contributions which are comparable in magnitude, have different weak phases, and have an experimentally unknown relative strong phase difference. Equations 31 and 32 do not become more transparent after inserting the value for $\lambda_{\pi\pi}$ given below

$$\lambda_{\pi\pi} = e^{-2i(\beta+\gamma)} \left(\frac{1 + |P/T| e^{i\delta} e^{i\gamma}}{1 + |P/T| e^{i\delta} e^{-i\gamma}} \right) , \tag{35}$$

where the t-quark penguin has been absorbed into the tree and penguin amplitudes using unitarity of the CKM matrix, as in [13]. Clearly, both the reconstructed and tag B amplitudes now depend on γ, so care must be taken in evaluating the tag-side interference uncertainty, which in general can be as large as $2\, r'$ for either the multiplicative or additive terms in Eqns. 31 and 32.

VIII. MEASUREMENT OF SIN($2\beta+\gamma$) WITH $D^{(*)}\pi$

One technique for measuring or constraining γ is to perform a time-dependent analysis of a decay mode that is known to have a non-zero DCS contribution, such as $D^{*+}\pi^-$ [14]. The time-dependent asymmetry coefficients are those given in Table I. In the usual case, tag-side interference is ignored ($r'=0$) and the amplitude of the $\sin\Delta m\Delta t$ term is $2r\sin(2\beta+\gamma\pm\delta)$, where r is the ratio of the DCS to CKM-favored amplitude contributions for the reconstructed, or non-flavor-tag, B and δ is the strong phase difference between the two amplitudes. Measuring r and $\sin(2\beta+\gamma\pm\delta)$ simultaneously is very challenging, so it is likely that r will have to be constrained from other measurements [5].

Symbol	Reco	Tag	$\sin(\Delta m\Delta t)$ coefficient
$S1$	B^0 $(D^{*-}\pi^+)$	B^0 (K^+)	$-2\,r\sin(2\beta+\gamma-\delta)+2\,r'\sin(2\beta+\gamma-\delta')$
$S2$	B^0 $(D^{*-}\pi^+)$	$\overline{B}^0$ (K^-)	$2\,r\sin(2\beta+\gamma-\delta)+2\,r'\sin(2\beta+\gamma+\delta')$
$S3$	$\overline{B}^0$ $(D^{*+}\pi^-)$	B^0 (K^+)	$-2\,r\sin(2\beta+\gamma+\delta)-2\,r'\sin(2\beta+\gamma-\delta')$
$S4$	$\overline{B}^0$ $(D^{*+}\pi^-)$	$\overline{B}^0$ (K^-)	$2\,r\sin(2\beta+\gamma+\delta)-2\,r'\sin(2\beta+\gamma+\delta')$

TABLE III: The 4 coefficients of the $\sin\Delta m\Delta t$ term in the time-dependence of $D^*\pi$. The 2nd and 3rd columns give the interpretation of the observed final state (given in parentheses) in terms of the dominant amplitude.

Since both r and r' are expected to be of the same order (≈ 0.02), it is clear that tag-side DCS interference can not be treated as a perturbation on the usual case. This effect is illustrated in Fig. 2. The time dependent analysis should be performed in a way that is general enough to accommodate $r'\approx r$ and any value of δ'.

Table III gives the $\sin\Delta m\Delta t$ coefficients, taken from Table I, for the 4 combinations of reconstructed and flavor tag B final states, where we have neglected r^2, rr', and r'^2 contributions. It is

useful to rewrite the relations for the S coefficients in the following way

$$S1 = -a + b + c \tag{36}$$

$$S2 = +a + b - c \tag{37}$$

$$S3 = -a - b - c \tag{38}$$

$$S4 = +a - b + c\,, \tag{39}$$

where the 3 variables to be determined in the time-dependent analysis are

$$a \equiv 2\,r\sin(2\beta+\gamma)\cos\delta \tag{40}$$

$$b \equiv 2\,r'\sin(2\beta+\gamma)\cos\delta' \tag{41}$$

$$c \equiv 2\cos(2\beta+\gamma)\left(r\sin\delta - r'\sin\delta'\right)\,. \tag{42}$$

This parameterization makes no assumptions about the magnitude of r' or δ', and is attractive for several reasons. First, a does not depend at all on the tag-side parameters r' and δ'. In the case where $\delta = 0$, which is favored by some [7], a is exactly what one wants to know ($\sin(2\beta + \gamma)$). Secondly, this parameterization cleanly separates the flavor-tag symmetric and antisymmetric components; the a and c coefficients are diluted by a factor of $(1 - 2\omega)$, while the b coefficient is not, since it has the same sign for tag-side B^0 and tag-side $\overline{B}^0$ events. The minimum number of independent parameters in which the S coefficients can be written is three. We recommend using the a, b, and c coefficients as the experimental parameters to be determined in the time-dependent asymmetry analysis.

The set of kaon tagging final states that yields correct tags is in general quite different from the set of final states that yields incorrect tags. This means that within a tagging category, the effective r' and δ' values for correct tags are different from those for incorrect tags. In the sum over correct and incorrect tags, the terms linear in r' that appear in the observables of the asymmetry are

$$(1-2\omega)r'e^{i\delta'} = (1-\omega)r'_c e^{i\delta'_c} - \omega r'_i e^{i\delta'_i}\,. \tag{43}$$

This equation gives effective r' and δ' parameters in terms of the mistag fraction ω, effective parameters for correct tags (r'_c and δ'_c) and incorrect tags (r'_i and δ'_i). This implies that, in order to have a completely general parameterization in the data analysis, each tagging category (kaon,

lepton, slow pion, etc.) must have different effective r' and δ' parameters, and thus different b and c parameters, due to the dependence on the mistag fraction ω. One particular case that is relevant for a kaon tag category is when $r'_i = 0$. In this case $r' = r'_c(1-\omega)/(1-2\omega)$, which means that the effective r' is enhanced by a factor of $(1-\omega)/(1-2\omega)$.

The experimental knowledge of δ depends on c, so even though the a parameter does not depend on r' and δ', one does not avoid uncertainties due to r' and δ' in the analysis. The best way to reduce this uncertainty is to take advantage of the fact that lepton tags are immune to the problem ($r' = 0$). If the fit is performed with an independent c coefficient for lepton tags, $c_{\rm lep}$ combined with the a parameter measured by all flavor tagging categories will help resolve δ and thus $(2\beta+\gamma)$.

If r' and δ' are not constrained from other measurements, one must allow for values of r' and δ' that are consistent with the measured values of b and c. Since it is possible to have a measured set of a, b, and c parameters that are consistent with $r' = 0$ when $r' \neq 0$, one must always consider all r' values between 0 and $r'_{\rm max}$ consistent with b and c, where $r'_{\rm max}$ is the largest allowed single-final-state value. This point is illustrated in Figure 4. The uncertainty on $(2\beta+\gamma)$ due to r' and δ' is maximal when a is small. In this case, the sensitivity to $(2\beta+\gamma)$ is mostly from the c coefficient and one must rely on flavor tag categories that are known to have $r' = 0$, such as lepton tags.

Using Monte Carlo pseudo-experiments, we perform a simplified study of the impact of DCS tag-side interference on a system with only two tagging categories: one for unaffected lepton tags, and the other containing kaon tags. The significance ratio of both categories is set to $Q_{\rm lep}/Q_{\rm non-lep} = 0.6$. All tests use the realistic value of 0.02 for r and r'. Each category shares the same a parameter. The lepton category constrains $c_{\rm lep}$, and the kaon category fits b and c. All fit parameters are unbiased, and conform to Gaussian distributions. Compared to the situation with no DCS contribution, having one tagging category and identical errors for its two parameters a and c, the statistical error on a is unchanged, and that on $c_{\rm lep}$ has increased by a ratio compatible with $((Q_{\rm non-lep}+Q_{\rm lep})/Q_{\rm lep})^{1/2} = 1.6$. The parameters a and b show a 20% correlation, while all other correlations are smaller than 1%.

One experimental strategy for reducing the uncertainties due to r' and δ' would be to constrain them by performing a time-dependent analysis of a flavor-specific final state that has no DCS contribution ($r = 0$), such as $D^{*+}l^-\overline{\nu}$. For such a final state, the undiluted b coefficient is the same as for $D^{*+}\pi^-$ and c now has $r = 0$. This information can be used to recover the $(2\beta+\gamma)$

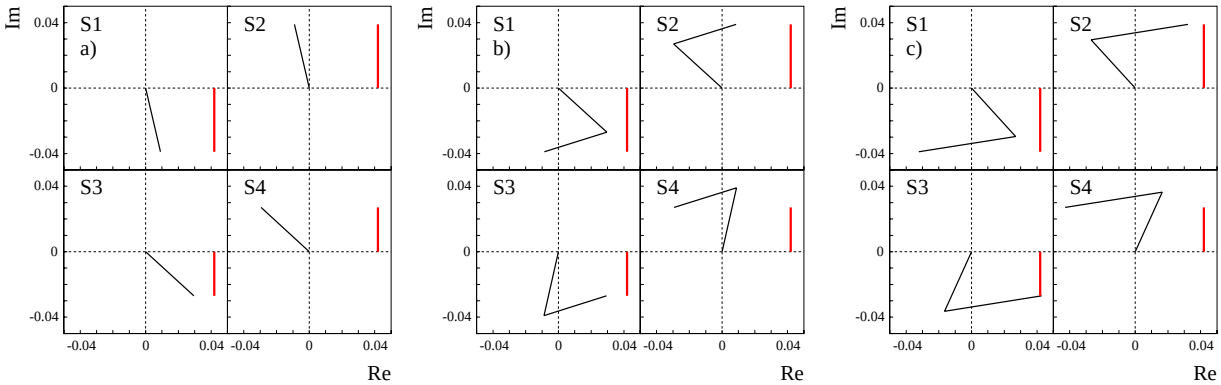

FIG. 4: An example of three amplitude configurations that all give the same set of S coefficients. For each set (a, b, or c) the S coefficients are consistent with no tag-side interference ($r' = 0$), while this is only true in the first case. Each configuration is represented by four diagrams showing the addition of the reconstructed and tag-side amplitude vectors in the complex plane. The observable coefficient (S) is the imaginary part, represented by the vertical band on the right side of each diagram. The parameters for the three configurations are: $r = 0.02$; $(2\beta+\gamma) = 2.10,\ 1.87,\ 1.73$; $r' = 0.00,\ 0.02,\ 0.03$; $\delta = 0.30,\ -0.53,\ -0.58$; and $\delta' = \mathrm{NA},\ 1.57,\ 1.57$.

sensitivity in the c coefficients in the signal sample that was lost due to the lack of knowledge of r' and δ'. Another option would be to include in the analysis events for which it was not possible to determine the flavor of the tag, so-called untagged events. From Equations 36 through 39, one can see that the untagged S coefficient for a reconstructed $D^{*-}\pi^+$ ($D^{*+}\pi^-$) is equal to $S1 + S2 = b$ ($S3 + S4 = -b$), thus untagged events provide a further constraint on b.

The measured a, b, and c coefficients for the various tagging categories and samples can be combined by forming a χ^2 using the measured parameters and the inverted covariance matrix. This assumes that the measurement uncertainties on the a, b, and c parameters are Gaussian. A constraint on $(2\beta+\gamma)$ can be derived from the χ^2 by scanning the χ^2 vs $(2\beta+\gamma)$ where for each $(2\beta+\gamma)$ value the χ^2 is minimized with respect to the unknown parameters δ, δ', and r'. If there are no external constraints on r' and δ', such as from the analysis of $D^{*+}l^-\overline{\nu}$ suggested above, the b and c parameters from non-lepton tags do not provide much information, since r' must be varied from its minimum value compatible with b to its maximum possible value (for example, see Figure 4). The non-lepton-tag b and c parameters still must be included in the fit, but they are

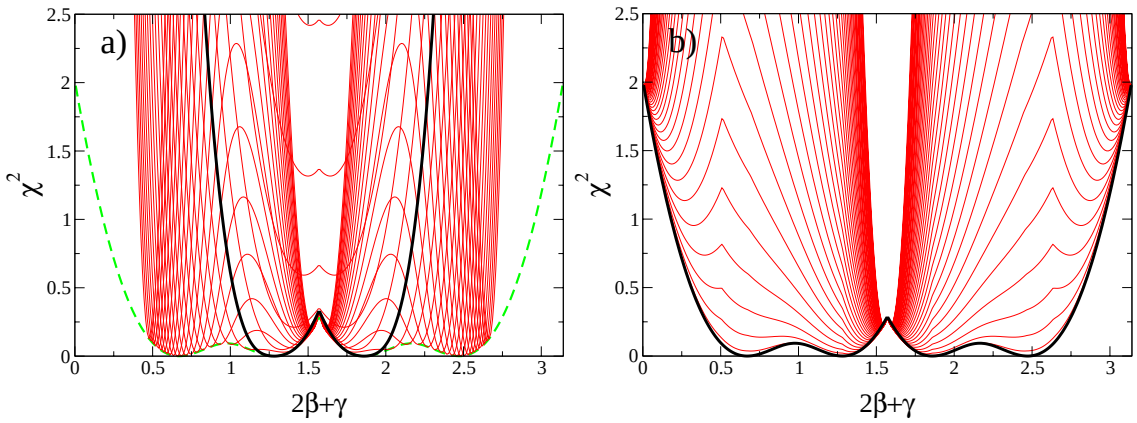

FIG. 5: Scans of χ^2 from measured a, b, and c coefficients as a function of $(2\beta+\gamma)$ illustrating two cases: one with non-zero tag-side DCS interference a) and one without tag-side interference b). The uncertainty on r' in both cases is illustrated by the collection of thin curves. The ultimate χ^2 constraint on $(2\beta+\gamma)$ is from the lowest of all of the curves for each value of $(2\beta+\gamma)$. If r' were varied to arbitrarily large values for a), this constraint would be described by the thick, dashed green curve, which is equivalent to the constraint from the $r'=0$ example, shown in b). The input values were $(2\beta+\gamma)=1.86$, $r=0.02$, and $\delta=0.9$. The tag-side parameters for a) were $r'=0.02$ and $\delta'=0.3$. The thick black curves were made with the true value of r', while the thin curves were made by scanning r' in steps of 0.001 from 0 to 0.04. The measured values were set to the correct values. The statistics of the hypothetical measurement correspond to roughly 450 fb^{-1} of B factory data from one experiment, including a constraint from $D^*l\nu$. There is a discreet ambiguity that gives exactly the same curves after adding π to the horizontal axis $(2\beta+\gamma)$.

not very useful in the χ^2 analysis.

Figure 5 shows an example of the χ^2 procedure for a hypothetical measurement where $(2\beta+\gamma)=1.86$, $r=0.02$, and $\delta=0.9$. The measured values were set to the correct values, so the χ^2 is zero at the correct and degenerate solutions. In this example, we chose to include the non-lepton b and c parameters, fix r' for the curves in Figure 5 and then vary it by its uncertainty, where each curve corresponds to a different r' value. In this case, one must use the lowest curve for each $(2\beta+\gamma)$ value. The statistical errors correspond to a measurement from $D^*\pi$ in roughly 450 fb^{-1} of B-factory data from one experiment including a constraint from $D^*l\nu$.

Three important conclusions can be drawn from Figure 5. First, comparing the $r'=0.02$ case to the $r'=0$ case, between $(2\beta+\gamma)=0.045$ and 2.7, the measurements give nearly identical

constraints on $(2\beta + \gamma)$. This means that the uncertainty on r' and δ' does not degrade the measurement. This is especially clear from the plots for $r' = 0$, where all of the red curves, which have r' set to nonzero values, give worse χ^2 values. The second important conclusion is that if r' is in fact non-zero, the constraint on $(2\beta+\gamma)$ is actually better than the case where r' is zero; the χ^2 curves for the $r' = 0.02$ case rise sharply at $(2\beta + \gamma) = 0.045$ and 2.7. If r' is non-zero, the b and c parameters in the $D^* l\nu$ sample will be non-zero. Even though r' was varied from zero to twice its true value, the non-zero b and c parameters in the $D^* l\nu$ sample provide useful information for constraining $(2\beta + \gamma)$. If r' were varied to arbitrarily large values, this information would be lost and the $(2\beta + \gamma)$ constraint would be completely equivalent to the one derived from the sample where the true r' value was zero.

Thirdly, the result for $D^*\pi$ alone, after varying r' to arbitrarily large values, is equivalent to the χ^2 curve constructed from only a and $c_{\rm lep}$. In other words, when not including the $D^* l\nu$ sample in the analysis, the b and other c parameters do not contribute to the sensitivity to $(2\beta + \gamma)$.

IX. CONCLUSIONS

Interference effects between CKM-favored $b \to c\overline{u}d$ and doubly-CKM-suppressed $\overline{b} \to \overline{u}c\overline{d}$ amplitudes in final states used for flavor tagging in coherent $B^0\overline{B}^0$ pairs from Υ(4S) decays introduce deviations from the standard time evolution assumed in CP violation measurements at the asymmetric-energy B factories. To our knowledge, the uncertainty introduced by this interference has been neglected in most B factory CP violation measurements published to date, with the exception of [10]. The uncertainties introduced in the $\sin 2\beta$ measurement in $(c\overline{c})K^0$ decay modes and the time dependent analysis of the $\pi^+\pi^-$ final state are at most of the order of 5% and can be limited to $< 2\%$ in most cases with reasonable assumptions.

In proposed measurements of $\sin(2\beta+\gamma)$ which explicitly use interference between CKM-favored and doubly-CKM-suppressed amplitude contributions in the final state that is reconstructed, such as $D^*\pi$, tag-side interference effects can be as large as the interference effects one is trying to measure. In any such analysis, the data must be analyzed in a way that is general enough to allow for tag-side interference effects. We have proposed a general framework for dealing with tag-side interference effects in $\sin(2\beta + \gamma)$ measurements. It is possible to achieve an experimental

sensitivity to $(2\beta + \gamma)$ similar to the originally proposed measurements.

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