SLAC-PUB-9665
hep-th/0303138
March 2003

# Chiral rings, Mirror Symmetry and the Fate of Localized Tachyons

Sang-Jin Sin

*Stanford Linear Accelerator Center, Stanford University, Stanford CA 94305* *

*and*

*Department of Physics, Hanyang University, Seoul, 133-791, Korea* †

March 14, 2003

**Abstract**

We study the localized tachyon condensation of non-supersymmetric orbifold backgrounds in their mirror Landau-Ginzburg picture. We first show that the R-charges of chiral primaries increase under the process of condensing the tachyon in the same chiral ring. Then, utilizing the existence of four copies of (2,2) worldsheet supersymmetry, we show that the minimal tachyon mass in twisted sectors increases in CFT and type 0 string and it plays the role of the c-function of the twisted sectors. We also study the GSO projection in detail and show that type II decays to only to type II while type 0 can mix with type 0 and II under the RG-flow.

*Work supported partially by the department of Energy under contract number DE-AC03-76SF005515.

†Permanent address

# 1 Introduction

The study of open string tachyon condensation[1] has led to many interesting consequences including classification of the D-brane charge by K-theory. While the closed string tachyon condensation involve the change of the background spacetime and much more difficult, if we consider the case where tachyons can be localized at the singularity, one may expect the maximal analogy with the open string case. Along this direction, the study of localized tachyon condensation was considered first in [2] using the brane probe and renormalization group flow and by many others[3, 4, 5, 6, 7, 8]. The basic picture is that tachyon condensation induces cascade of decays of the orbifolds to less singular ones until the spacetime supersymmetry is restored. Therefore the localized tachyon condensation has geometric description as the resolution of the spacetime singularities.

Soon after, Vafa[3] considered the problem in the Landau-Ginzburg (LG) formulation using the Mirror symmetry and confirmed the result of [2]. In [4], Harvey, Kutasov, Martinec and Moore studied the same problem using the RG flow as deformation of chiral ring and in term of toric geometry. In both papers, the worldsheet N=2 supersymmetry was utilized in essential ways.

The tachyon condensation process can be regarded as a RG-flow, along which there is a decreasing quantity, c-function, [9] for unitary conformal field theories. However, under the the localized tachyon condensation in non-compact space, $c$ is constant[2, 4] since it measure the bulk degree of freedom. Therefore it would be very interesting to have a quantity which has a property of monotonicity along the RG-flow like the c-function of Zamolodchikov . Along this line, the authors of [4] suggested a quantity, $g_{cl}$, which is a closed string analogue of the ground state degeneracy in open string theory[10]. On the other hand, Dabholkar and Vafa[5] suggested the maximal R-charge of Ramond sector (see [16, 17] for ealier study on this quantity), as a c-function of the twisted sector describing the localized tachyon condensation. Although both suggestions have well motivated physical intuition, the prediction of two quantity are slightly different[7]. The prediction of $g_{cl}$ is also not compatible with [2] as pointed out in [4]. In [7], it was suggested that the lowest twisted tachyon mass increases along RG flow. Using the spectral flow and CTP invariance of the Ramond sector and the mass and the R-charge for chiral primaries, one can easily see that the proposal of [7] is equivalent to the GSO projected version of the one given by [5].

The monotonically increasing property of R-charge is related to a theorem in singularity theory called semi-continuity of spectrum[12] in singularity theory, which was conjectured by Arnold [11] and proved later by Varchenko[13] and Steenbrink[14]. These mathematical result can be applied[16] to the Landau-Ginzburg theory with the help of non-renormalization property of $N = 2$ supersymmetric world sheet theory. In our case, the LG theory that is mirror to the orbifold $\mathbb{C}^r/\mathbb{Z}_n$ is not an ordinary LG theory but an orbifolded LG [15] model and hence the theorem can not be applied directly. Although it is easy to see the monotonicity of R-charge for $\mathbb{C}/Z_n$ case, it is not trivial for $\mathbb{C}^2/Z_n$ case. Therefore the proposal of [5] is conjecture rather than a theorem even at the on shell (CFT) level. The main goal of this paper is to prove this conjecture, i.e, *the lowest tachyon mass or equivalently the minimal R-charge in orbifold CFT and type 0 theories increases when we compare those in UV and IR fixed points.* The behavior of R-charge in the intermediate stage is very interesting but it is out of the scope. Interestingly, our method applies only to the orbifolded Landau-Ginzburg thoery and does not apply to the generic LG theories. In this sense, our method is complementary to the method used in mathematical literature.

We use the mirror LG picture of Vafa and the existence of the two copies of (2,2) worldsheet supersymmetries. We need several preparation to achieve the goal. In CFT of $\mathbb{C}^2/Z_n$, there are $2^2$ extended chiral ring structures according to the choice of complex structures of each $\mathbb{C}$ factor. We call them as *cc*, *ca*, *ac* and *aa* rings. For string spectrum, we need to put spectrum of all 4 sectors together. When we consider the behavior of *cc* ring elements under the condensation of a tachyon in *cc* ring, we can establish an explicit mapping between spectrum of initial and final orbifold conformal field theories. We will be able to show that individual R-charge of tachyons increases under the process. This is possible since we have control over the RG-process due to the world sheet (2,2) supersymmetry off the criticality, which povide the non-renormalization theorem. However, we have to deal with other cases as well: what happen to the R-charges of operators in *ca* or other rings when a tachyon operator in *cc* ring condensate? The answer is that we lose control, since we lose all supersymmetry off the criticality hence we do not have non-renormalization theorem.

What saves us from this difficulty is the presence of the enhenced $2^r$ copies of (2,2) worldsheet SUSY in orbifold CFT's. This is because its presence allows us to choose the supersymmetry generators $G^{\pm}_{-\frac{1}{2}}$ and complex structure such that the condensing tachyon belongs to $cc$-ring. We can then determine the generators of the daughter theories. Since we know that the final products of the decay are again orbifold theories [2, 3, 4], knowing the fate of the $cc$-ring element is enough to establish the fate of entire spectrum. We will be able to establish linear mappings for each of 4 chiral rings, some of which do not necessarily describe tachyon condensation process of individual R-charges. They just connect between the spectrum of mother and daughter theories. We can also show that the linear mapping has the property such that the R-charges of their images are bigger than the R-charges of the originals. The mere existence of such mappings will enable us to show our main goal: the minimal charge increases under TC.

In order to discuss the string theory, we need one more element to discuss: the GSO projection. The GSO projection in the context of orbifold theory is quite non-trivial. For example, the orbifold thoeries $\mathbb{C}^2/\mathbb{Z}_{n(1,-1)}$ and $\mathbb{C}^2/\mathbb{Z}_{n(1,n-1)}$ are identical in CFT level, but they have completely different spectra after GSO projection. We will also examine whether there is a GSO projected version of the theorem discussed above. We will illustrate that in some examples of type II theory, even in the case of condensing marginal operator, the minimal charge still increases. This is a behavior not expected from a c-function. In fact, The c-theorem is a property of CFT and GSO is imposed by hand in a way that has nothing to do with the dynamics of CFT. Therefore it is likely that the c-theorem does not hold for type II string theory in general. We also show that type II string should decay into type II only, while type 0 string can decay into either type 0 and type II.

The rest of the paper goes as follows: In section 2, we will summarize the basic elements of mirror Landau-Ginzburg theory of orbifold CFT. In section 3, explicit construction of four chiral ring elements and corresponding monomials (i.e, their mirror representations) will be constructed. After constructing a standard chiral rings of (2,2) SUSY of $\mathbb{C}^1/Z_n$ and $\mathbb{C}^2/Z_n$ in section 3.1 and 3.2 respectively, we will show in section 3.3 that any tachyon constructed from the mode of worldsheet fermion can be considered as an BPS state, i,e, an chiral ring element by considering all 4 copies of (2,2) worldsheet SUSY. In section 4, we consider the behavior of individual R-charges of a chiral ring when tachyon in the same ring condenses. In section 4.1, we first establish a linear map that connect the initial and final state. then in section 4.2, we complete the prescription of Vafa[3] by determining the generator of the orbifold theories of the daughter theory when given chiral ring element condensates. In section 4.3, using all the preliminaries established in section 3 and 4, we can prove our main statement. In section 5.1, we will review the relation between the R-charge and tachyon mass to show that the property of minimal charge in NS sector will be identical to that of the maximal charge in Ramond sector In section 5.2, our main statement described above is proved. The result so far is at the level of CFT and type 0 string theory where no tachyon spectrum is projected out. In section 6, we address the question on the behavior of the minimal R-charge after the GSO projection. In section 6.1, we discuss the chiral GSO projection in the orbifold CFT, using the relation of partition functions of Green-Schwarz formalism and those in the NSR formalism. We also discuss in type 0 and II in orbifold theory in detail. In section 6.2, we discuss the GSO projected version of the theorem proved in section 5. In section 7, we discuss the transition between the type 0 and type II. we conclude with discussions.

# 2 Landau-Ginzburg as Mirror pairs of Orbifolds

The notion of mirror symmetry in Calabi-Yau manifolds is T-duality on torus fibration whose fibers are supersymmetric 3-torus [18]. In [19], the mirror symmetry is derived by by applying the T-duality to the 2 dimensional gauge theory that flows to non-linear sigma model in IR. The T-duality turns the non-linear sigma model to Landau-Ginzburg model where a superpotential is generated by the vortex gas of high energy gauge system. The analysis is made precise in [20] and the method can be applied to the case whose target space given by a toric manifold, which is a torus fibration over a manifold. The basic reason why orbifold can be discussed in terms of Landau Ginzburg model is because the former can be thought as a limit of non-linear sigma model. In following subsection we give a brief summary of Vafa's work [3] on localized tachyon condensation by applying above ideas to non-compact target space.

## 2.1 Mirror symmetry and Orbifolds

The orbifold $\mathbb{C}^r/Z_n$ is defined by the $\mathbb{Z}_n$ action given by equivalence relation

$$(X_1, ..., X_r) \sim (\omega^{k_1} X_1, ..., \omega^{k_r} X_r), \quad \omega = e^{2\pi i/n}. \tag{2.1}$$

We call $(k_1, \cdots, k_r)$ as the generator of the $\mathbb{Z}_n$ action.

In gauged linear sigma model(GLSM) [19], the $U(1)$ gauge symmetry acts on charged fields $(X_0, X_1, ..., X_r)$ of charges $(-n, k_1, ..., k_r)$ by

$$(X_0, X_1, ..., X_r) \mapsto (X_0 e^{-in\theta}, X_1 e^{ik_1\theta}, ..., X_r e^{ik_r\theta}). \tag{2.2}$$

The geometry of vacuum manifold of GLSM is described by the D-term constraints

$$-n|X_0|^2 + \sum_i k_i |X_i|^2 = t. \tag{2.3}$$

Notice that in $t \to -\infty$ limit, $X_0$ should take large vacuum expectation value. Then the U(1) is broken to $\mathbb{Z}_n$ acting on $X_i$'s precisely as eq.(2.1) and the $X_i$'s are massless fields. Hence we get orbifold as $t \to -\infty$ limit of GLSM. On the other hand, in the $t \to \infty$ limit, the target space corresponds to the $O(-n)$ bundle over the weighted projected space $WP_{k_1,...,k_r}$ defined by

$$(X_1, ..., X_r) \sim (\lambda^{k_1} X_1, ..., \lambda^{k_r} X_r), \quad \lambda \neq 0 \tag{2.4}$$

where at least one of the $X_i$ is non-zero. $X_0$ direction corresponds to the non-compact fiber of this bundle. Here $t$ plays role of size of the $WP_{k_1,...,k_r}$.

By dualizing this GLSM, we get a LG model with a superpotential

$$W = \sum_{i=0}^{r} \exp(-Y_i), \tag{2.5}$$

where twisted chiral fields $Y_i$ are periodic $Y_i \sim Y_i + 2\pi i$ and related to $X_i$ by

$$Re[Y_i] = |X_i|^2. \tag{2.6}$$

By introducing the variable

$$u_i := e^{-Y_i/n}, \tag{2.7}$$

the D-term constraint is expressed as

$$e^{-Y_0} = e^{t/n} \prod_i u^{k_i}. \tag{2.8}$$

The periodicity of $Y_i$ imposes the identification :

$$u_i \sim e^{2\pi i/n} u_i \tag{2.9}$$

, namely we need to mod out each $u_i$ by $\mathbb{Z}_n$. The periodicity of $Y_0$ requires the right hand side of (2.8) to be invariant under this $Z_n$ phase multiplication. Therefore the group we have to mod out is $(\mathbb{Z}_n)^{n-1}$ rather than $(\mathbb{Z}_n)^n$. One summarize this result symbolically

$$[W = \sum_{i=1}^{r} u_i^n + e^{t/n} \prod_i u^{k_i}]//(\mathbb{Z}_n)^{r-1}. \tag{2.10}$$

Therefore the mirror of the orbifold ($t \to -\infty$ limit) is the orbifolded LG model:

$$[W = \sum_{i=1}^{r} u_i^n]//(\mathbb{Z}_n)^{r-1}. \tag{2.11}$$

The information about $k_i$ is hidden in the constraint of the $\mathbb{Z}_n$ action: it should preserve the monomial $T = \prod_{i=1}^r u^{k_i}$. The ground states of $n-1$ twisted sectors are N=2 chiral primaries and give twisted fields. The first twisted field is identified with $T = \prod_i u_i^{k_i}$ and $T^l$ is twisted fields of $l$-th sector. Since the R-charge of $u_i$ is $1/n$, the R-charge of $T$ is $R[T] = \sum_i k_i/n$. Since it is chiral primary, the conformal dimension is given by $\Delta_T = \frac{1}{2} R[T]$. The generic deformation by all twist fields is given by

$$[W = \sum_i u_i^n + \sum_{l=1}^{n-1} t_l T^l]//(\mathbb{Z}_n)^{r-1}, \tag{2.12}$$

for some complex parameters $t_l$ representing the strength of the condensation of $T^l$. In order to make the dimension of $T^l$ lowest possible, we should replace $T^l$ by $\prod_i u_i^{n\{lk_i/n\}}$, where $\{x\}$ is the fractional part of $x$.

The GSO projection is given by $W \to -W$. For odd $n$, one can use $u_i \to -u_i$ for G-parity transoformation. For even $n$, one can use $u_i^n \to -u_i^n$, and $u_1^{k_1} u_2^{k_2} \to -u_1^{k_1} u_2^{k_2}$. Finally the RG flow correspond to $W \to \Lambda^{-1} W$; due to the non-renormalization of F-term, it is simply given by the scaling dimension of $\int d^2x d^2\theta$. Under the scaling $u_i \to \Lambda^{1/n} u_i$, $t$ should run by

$$t(\Lambda) = t + (\sum_i k_i - n) \log \Lambda. \tag{2.13}$$

## 2.2 Local ring of LG v.s Chiral ring of orbifold CFT

One important aspect of the mirror of the orbifold is that it is not a just a Landau-Ginzburg theory but the orbifolded version of it as it was denoted by in eq.(2.10). Due to this, the chiral ring structure of the theory is very different from that of LG model. For example, the dimension of the local ring of the super potential $W = \sum_i^r u_i^n$ is $(n-1)^r$, while the dimension of the chiral ring of the $\mathbb{C}^r/\mathbb{Z}_n$, the Witten index of the orbifold $\mathrm{Tr}(-1)^F$, is always $n-1$, regardless of $r$. For example, for $\mathbb{C}^2/\mathbb{Z}_n$ case the monomial basis of local ring of superpotential $W = u_1^n + u_2^n$ is

$$\{u_1^{p_1}u_2^{p_2}|(p_1,p_2) = (n\{jk_1/n\}, n\{jk_2/n\}), j = 0,1,...,n-2\}, \tag{2.14}$$

for some $(k_1,k_2)$, which we call as generator.

Another distinguished feature of LG theory as a mirror of the NLSM is in the counting of the U(1) charge of the local ring. In usual LG model, the monomial $u_1p_1u_2^{p_2}$ gives the weight vector $(p_1,p_2)$. [1] This is not true in this case due to the unusual kinetic term in terms of $u_1, u_2$, the variable that gives polynomial super-potential. Namely, the identification $u_i = e^{-Y_i/n}$ leads us to the kinetic term $(\partial Y_i)^2 = (\partial u_i/u_i)^2$, which is large in IR limit $u \to 0$. The potential term $V = |\nabla_u W|^2$. Since we need to measure the weight of the superpotential with respect to the kinetic term, we need to shift $(p_1,p_2)$ to $(p_1+1,p_2+1)$, to measure the weight and charge correctly. This may be seen more clearly by rewriting the bosonic action as

$$S = \int_\Sigma (\sum_i (|\partial u_i|^2 - |u_i \frac{\partial W}{\partial u_i}|^2)/u_i^2. \tag{2.15}$$

As a consequence, the identity operator has a weight vector $(1,1)$ and there is no monomial having the weight vector $(0,0)$ representing the vacuum of untwisted vector. This standard basis is very awkward to use due to the mismatch of the power and the weight. Furthermore, in this local ring basis (2.14) , we are in lack of monomials which is necessary to describe the some of the twisted sectors: it does not have any monomial whose weight vector is $(p,0)$. To overcome these difficulty, it is proper to consider the ideal generated by $u_i\nabla_{u_i}W = \nabla_{Y_i}W$ rather

[1] The weight here is integer normalized one, i.e, weight multiplied by $n$.

than that generated by $\nabla_{u_i}$. Then the ideal is given by

$$I = \{u_1^n f_1 + u_2^n f_2 | f_1, f_2 \text{ are arbitrary holomorphic polynomial}\}, \tag{2.16}$$

and our local ring is identified by $u_i^n \equiv 0$ instead of usual $u_i^{n-1} \equiv 0$.

Summarizing, the local ring of the mirror LG model of the orbifold is given by

$$\mathcal{R} = \mathbb{C}[u_1, u_2]/I[u_i \nabla_{u_i} W], \tag{2.17}$$

and the monomial basis of our orbifolded LG theory is given by

$$\{u_1^{p_1} u_2^{p_2} | (p_1, p_2) = (n\{jk_1/n\}, n\{jk_2/n\}), j = 1, ..., n-1\}, \tag{2.18}$$

Notice that comparing with eq. (2.14), the range of $j$ is shifted by 1. There is no shift in measuring the weight so that $u_1^{p_1} u_2^{p_2}$ has weight $(p_1, p_2)$ and charge $(p_1/n, p_2/n)$, which is natural and desired. Including $j = 0$ is natural in this construction and it corresponds to the untwisted sector.

# 3 Chiral Rings of the Orbifolds

Here we construct chiral rings of orbifolds[21] in terms of modes, which in turn will allows us to construct the chiral ring in terms of monomials of LG model. First we work out $\mathbb{C}^1/\mathbb{Z}_n$ for simplicity. For $\mathbb{C}^2/\mathbb{Z}_n$, the existence of the 4 copies of (2,2) worldsheet SUSY will enables to prove that for any worldsheet fermion generated tachyon can be constructed as a BPS state, i.e, a member of a chiral ring.

## 3.1 $\mathbb{C}^1/\mathbb{Z}_n$

The Energy momentum tensor of the NSR string on the cone $\mathbb{C}^1/\mathbb{Z}_n$ is

$$T = -\partial X \partial X^* + \frac{1}{2}\psi^* \partial \psi + \frac{1}{2}\psi \partial \psi^*, \tag{3.1}$$

where $X = X^1 + iX^2,\ X^* = X^1 - iX^2$ and $\psi$ and $\psi^*$ are Weyl fermions which are conjugate to each other with respect to the target space complex structure. All the fields appearing here describe worldsheet left movers. We denote the corresponding worldsheet complex conjugate by bared fields: $\bar{X}, \bar{X}^*, \bar{\psi}, \bar{\psi}^*$. The $N = 2$ world sheet SCFT algebra is generated by $T$, $G^+ = \psi^* \partial X$, $G^- = \psi \partial X^*$ and $J = \psi^* \psi$. The orbifold symmetry group

$$\mathbb{Z}_n = \{g^l | l = 0, 1, 2, \cdots, n-1, \text{with } \; g^n = 1\} \tag{3.2}$$

act on the fields in NS sector by

$$\begin{aligned} g \cdot X(\sigma + 2\pi, \tau) &= e^{2\pi i k/N} g \cdot X(\sigma, \tau), \\ g \cdot X^*(\sigma + 2\pi, \tau) &= e^{-2\pi i k/N} g \cdot X^*(\sigma, \tau), \\ g \cdot \psi(\sigma + 2\pi, \tau) &= -e^{2\pi i k/N} g \cdot \psi(\sigma, \tau), \\ g \cdot \psi^*(\sigma + 2\pi, \tau) &= -e^{-2\pi i k/N} g \cdot \psi^*(\sigma, \tau). \end{aligned} \tag{3.3}$$

The mode expansion of the the fields in the conformal plane are given by

$$\begin{aligned} \partial X(z) &= \; \textstyle\sum_{n \in \mathbb{Z}} \alpha_{n+a}/z^{n+1+a}, \\ \partial X^*(z) &= \; \textstyle\sum_{n \in \mathbb{Z}} \alpha^*_{n-a}/z^{n+1-a}, \\ \psi(z) &= \; \textstyle\sum_{r \in \mathbb{Z}+\frac{1}{2}} \psi_{r+a}/z^{r+\frac{1}{2}+a}, \\ \psi^*(z) &= \; \textstyle\sum_{r \in \mathbb{Z}+\frac{1}{2}} \psi^*_{r-a}/z^{r+\frac{1}{2}-a}, \end{aligned} \tag{3.4}$$

where $a = k/N$. The quantization conditions are:

$$[\alpha_{n+a}, \alpha^*_{-m-a}] = \delta_{m,n}, \;\; \{\psi_{r+a}, \psi^*_{-s-a}\} = \delta_{r,s}. \tag{3.5}$$

Hence the conjugate variables are given by

$$\begin{aligned} \alpha^\dagger_{n+a} = \alpha^*_{-n-a}, \quad (\alpha^*_{n-a})^\dagger = \alpha_{-n+a}, \\ \psi^\dagger_{r+a} = \psi^*_{-r-a}, \quad (\psi^*_{r-a})^\dagger = \psi_{-r+a}. \end{aligned} \tag{3.6}$$

The vacuum is defined as a state that is annihilated by all positive modes. Notice that as $a$ grows greater than $\frac{1}{2}$, $\psi_{-\frac{1}{2}+a}$ ($\psi^*_{\frac{1}{2}-a}$)changes from a creation(annihilation) to an annihilation (creation) operator. The (left mode) hamiltonian of the orbifolded complex plane is

$$H_L = \frac{1}{2} \sum \left[\alpha^*_{-n-a}\alpha_{n+a} + \alpha_{-n+a}\alpha^*_{n-a}\right] + \frac{1}{2} \sum_{r \in \mathbb{Z}+\frac{1}{2}} \left[(r+a)\psi^*_{-r-a}\psi_{r+a} + (r-a)\psi_{-r+a}\psi^*_{n-a}\right]. \tag{3.7}$$

The contribution of the left modes to the zero energy is

$$E_0^L = \frac{1}{2}\sum_{n=0}^{\infty}(n+a) + \frac{1}{2}\sum_{n=1}^{\infty}(n-a) - \frac{1}{2}\sum_{n=0}^{\infty}(n+\frac{1}{2}+a) - \frac{1}{2}\sum_{n=0}^{\infty}(\frac{1}{2}+n-a). \tag{3.8}$$

If we define

$$f(a) = \sum_{n=0}^{\infty}(n+a) = 1/24 - (a-1/2)^2/2, \tag{3.9}$$

then $f(a) = f(1-a)$ and $f(a+1/2) = f(-a+1/2)$ so that the above sum gives $E_0^L = a/2 - 1/8$. Embedding the cone to the string theory to make the target space $\mathbb{C}/\mathbb{Z}_n \times R^{7,1}$, we need to add the zero energy fluctuation of the 6 transverse flat space, $6 \times (-1/24)(1+1/2) = -3/8$ to the zero point energy, which finally become

$$E_0^L = \frac{1}{2}(a-1), \quad for \;\; 0 < a < \frac{1}{2}. \tag{3.10}$$

If $1/2 < a < 1$, then $(a-\frac{1}{2})\psi^*_{\frac{1}{2}-a}\psi_{-\frac{1}{2}+a}$ should be added to the normal ordered Hamiltonian while $(\frac{1}{2}-a)\psi_{-\frac{1}{2}+a}\psi^*_{\frac{1}{2}-a}$ should be removed from it. Therefore the zero point energy should be modified to be

$$E_0^L = \frac{1}{2}(a-1) - \frac{1}{2}\left[-\left(\frac{1}{2}-a\right) + \left(a-\frac{1}{2}\right)\right] = -\frac{1}{2}a, \quad for \;\; \frac{1}{2} < a < 1. \tag{3.11}$$

From this we can identify the weight and charge of twisted ground states using

$$E_0^L = \Delta - 1/2, \quad \text{and} \quad q = \pm 2\Delta, \tag{3.12}$$

where we take + if the the ground state is a chiral state and − for if it is anti-chiral state.

We now construct next level chiral and anti-chiral primary states by applying the creation operator.

$$\begin{aligned} G^+_{-\frac{1}{2}} &= \sum \psi^*_{-n-\frac{1}{2}-a}\alpha_{n+a} = \psi^*_{\frac{1}{2}-a}\alpha_{-1+a} + \cdots, \\ G^-_{-\frac{1}{2}} &= \sum \psi_{-n-\frac{1}{2}+a}\alpha^*_{n-a} = \psi_{-\frac{1}{2}+a}\alpha^*_{-a} + \cdots. \end{aligned} \tag{3.13}$$

Notice that for $0 < a < \frac{1}{2}$, $\psi^*_{\frac{1}{2}-a}|0>= 0$, hence

$$G^+_{-\frac{1}{2}}|0>= 0 \tag{3.14}$$

so that $|0>$ is a chiral state. It has a weight $a/2$ and R-charge $a$, so that the local ring element of LG theory corresponding to $|0>_a$ can be identified as $u^k$:

$$|0>_a \sim u^k. \tag{3.15}$$

The first excited state is $\psi_{a-\frac{1}{2}}|0>$ which is annihilated by $G^-_{-\frac{1}{2}}$:

$$G^-_{-\frac{1}{2}}\psi_{a-\frac{1}{2}}|0>= 0. \tag{3.16}$$

Therefore it is an anti-chiral state. Its weight is $\frac{1}{2}(1-a)$ and charge is $a-1$, hence it corresponds to $\bar{u}^{n-k}$:

$$\psi_{a-\frac{1}{2}}|0>\sim \bar{u}^{n-k}. \tag{3.17}$$

For $\frac{1}{2} < a < 1$, $\psi_{a-\frac{1}{2}}|0>= 0$, hence $G^-_{-\frac{1}{2}}|0>= 0$ so that $|0>$ is a anti-chiral state. It has weight $\frac{1}{2}(1-a)$ and charge $a-1$, hence the corresponding local ring element is $\bar{u}^{n-k}$. The first excited state is $\psi^*_{\frac{1}{2}-a}|0>$ which is annihilated by $G^+_{-\frac{1}{2}}$ therefore it is a chiral state. Its weight is $a/2$ and the charge is $a$ hence the corresponding local ring element is again $u^k$. Using the weight and charge relation for chiral and anti-chiral states, we see that $\psi^*$ has $+1$ charge and $\psi$ has $-1$ charge.

So far we have worked out the first twisted sector for arbitrary generator $k$. For the $j$-th twisted sector, we can easily extend the above identifications by observing that $a$ is the fractional part of $jk/n$;

$$a = \{jk/n\}. \tag{3.18}$$

The result is that for all chiral operators, the local ring elements are given by $u^{n\{jk/n\}}$ and for the anti-chiral operators they are given by $\bar{u}^{n-n\{jk/n\}}$. In both cases $j$ runs from 1 to $n-1$ for twisted sectors. It is worthwhile to observe that

$$n(1 - \{jk/n\}) = n\{j(n-k)/n\}, \tag{3.19}$$

so that the generator of the anti-chiral ring is $\bar{u}^{n-k}$, while that of chiral ring is $u^k$. Since it is the building block for the results in higher dimensional theories, we tabulate the above results in table 1.

| Region | vacuum | annihilator | creators |
|---|---|---|---|
| $0 < a < \frac{1}{2}$ | $\lvert 0 >\sim u^{na}$ | $\psi^*_{\frac{1}{2}-a}$ | $\psi_{a-\frac{1}{2}}\lvert 0 >\sim \bar{u}^{n-na}$ |
| $\frac{1}{2} < a < 1$ | $\lvert 0 >\sim \bar{u}^{n-na}$ | $\psi_{a-\frac{1}{2}}$ | $\psi^*_{\frac{1}{2}-a}\lvert 0 >\sim u^{na}$ |

Table 1: Twisted vacuum and first excited states. The chirality is equal to the holomorphic structure of the target space, i.e, chiral(anti-chiral) states correspond to monomial of $u$ ($\bar{u}$).

What about the case $a = -k/n < 0$? The answer can be read off from what we already have by noticing that above structure is periodic in $a$ with period 1, because we should shift the mode if $a$ is bigger than 1. The effect of $a \to -a$ amounts to exchanging the role of $\psi$ and $\psi^*$. Therefore in this case, the local ring elements of LG dual are given by $u^{n-n\{jk/n\}}$ and for the anti-chiral operators they are given by $\bar{u}^{n\{jk/n\}}$.

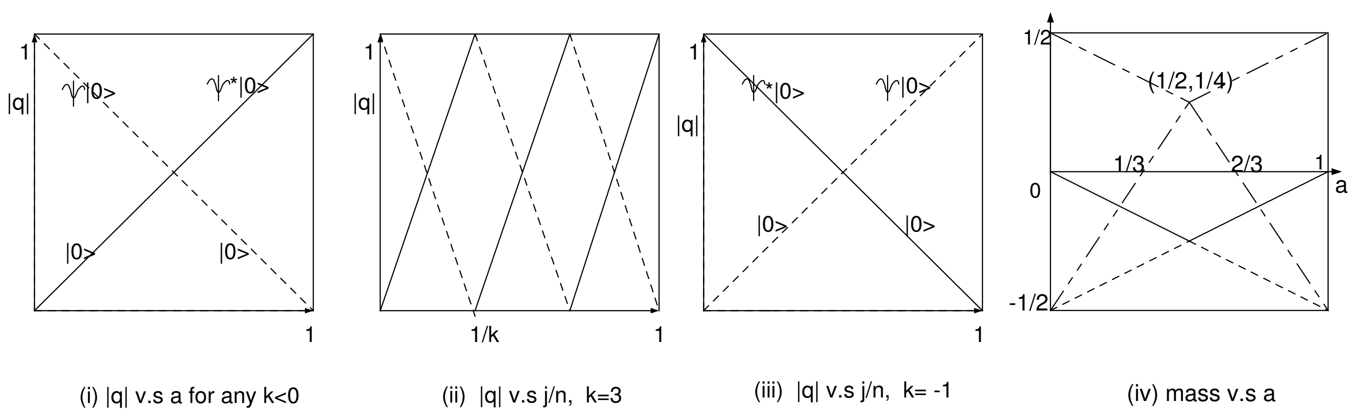


Figure 1: Spectrum versus twists in $\mathbb{C}^1/\mathbb{Z}_n$: (i) $2\Delta = |q|$ v.s $a = \{jk/n\}$ for any $k > 0$. The states on solid lines are chiral while those on the dotted lines are anti-chiral. (ii) $|q|$ v.s $j/n$ for k=3, (iii) $|q|$ v.s $j/n$ for $k = -1$. For $k < 0$, the role of chiral and anti-chiral states are interchanged. (iv) All possible Tachyom mass v.s $a$. Dotted lines are for the (twisted) vacuum, solid lines are for worldsheet fermion excitations, the rests are for scalar excitations. Notice that the lowest tachyon mass is always generated by worldsheet fermion.

The first three graphs in Fig.1 show the weight versus twist $a$ for the various cases. The charge can be read off by the $q = \pm 2\Delta$ rule. We are interested in $2\Delta$ since left and right moving parts contribute the same to the masses of the states represented by these polynomials. The last figure in Fig.1 is mass spectrum $\frac{1}{4}\alpha' M^2 = E_0^L$ as function of the twist $a$ for *all* possible tachyons including the scalar excitations:

$$\begin{aligned}
\alpha^*_{-a}|0\rangle: \ E_0^L &= \ (3a-1)/2, \ \text{for} \quad 0<a<\frac{1}{2} \\
&= \quad a/2, \quad \text{for} \quad \frac{1}{2}<a<1 \\
\alpha_{-(1-a)}|0\rangle: \ E_0^L &= \ (1-a)/, \ \text{for} \quad 0<a<\frac{1}{2} \\
&= \ (2-3a)/2, \ \text{for} \quad \frac{1}{2}<a<1.
\end{aligned} \tag{3.20}$$

These scalar excitations $\alpha^*_{-a}|0\rangle, \alpha_{-(1-a)}|0\rangle$ are tachyons if $0<a<1/3$, $2/3<a<1$ respectively. They can not be characterized as a chiral or anti-chiral states. Furthermore it never gives the lowest tachyon mass, hence we will not pay much attention afterward.

## 3.2 $\mathbb{C}^2/\mathbb{Z}_n$

Now we extend the result of previous section to $\mathbb{C}^2/\mathbb{Z}_n$ case, which is our main interests. We introduce two sets of (bosonic and fermionic) complex fields $X^{(1)}, \psi^{(1)}, \psi^{*(1)}; X^{(2)}, X^{*(2)}, \psi^{(2)}, \psi^{*(2)}$ and specify how the orbifold group $\mathbb{Z}_n$ is acting on each set of fields. The group action is the same as before except that $\mathbb{Z}_n$ can act on first and second set of fields with different generators $k_1$ and $k_2$. For example, in the first twisted sector,

$$\begin{aligned}
g \cdot X^{(1)}(\sigma+2\pi,\tau) &= e^{2\pi i k_1/N} g \cdot X^{(1)}(\sigma,\tau), \\
g \cdot X^{(2)}(\sigma+2\pi,\tau) &= e^{2\pi i k_2/N} g \cdot X^{(2)}(\sigma,\tau).
\end{aligned} \tag{3.21}$$

Since three parameter $n, k_1, k_2$ fix an $\mathbb{C}^2/\mathbb{Z}_n$ orbifold theory completely, we use notation $n(k_1,k_2)$ to denote it.

Let $a_i = k_i/n$ as before. For $0<a_i<\frac{1}{2}$, the zero energy fluctuation can be calculated as

$$E_0^L = \left(\frac{1}{2}a_1 - \frac{1}{8}\right) + \left(\frac{1}{2}a_2 - \frac{1}{8}\right) - \frac{1}{24}\left(1+\frac{1}{2}\right) \times 4 = \frac{1}{2}\left(a_1+a_2-1\right). \tag{3.22}$$

Therefore the weight of twisted vacuum is given by

$$\Delta_0 = \frac{1}{2}(a_1+a_2). \tag{3.23}$$

| $(a_1\text{-}\frac{1}{2}, a_2\text{-}\frac{1}{2})$ | $b$ | $b^\dagger$ | chiral state | anti-chiral state | neither |
|---|---|---|---|---|---|
| $--$ | $\psi_1^*, \psi_2^*$ | $\psi_1, \psi_2$ | $\|0>$ $\sim u_1^{na_1} u_2^{na_2}$ | $\psi_1\psi_2\|0>$ $\sim \bar{u}_1^{n-na_1} \bar{u}_2^{n-na_2}$ | $\psi_1\|0>\sim \bar{u}_1^{n-na_1} u_2^{na_2}$ $\psi_2\|0>\sim {u_1}^{na_1} \bar{u}_2^{n-na_2}$ |
| $-+$ | $\psi_1^*, \psi_2$ | $\psi_1, \psi_2^*$ | $\psi_2^*\|0>$ $\sim u_1^{na_1} u_2^{na_2}$ | $\psi_1\|0>$ $\sim \bar{u}_1^{n-na_1} \bar{u}_2^{n-na_2}$ | $\|0>\sim {u_1}^{na_1} \bar{u}_2^{n-na_2}$ $\psi_1\psi_2^*\|0>\sim \bar{u}_1^{n-na_1} u_2^{na_2}$ |
| $+-$ | $\psi_1, \psi_2^*$ | $\psi_1^*, \psi_2$ | $\psi_1^*\|0>$ $\sim u_1^{na_1} u_2^{na_2}$ | $\psi_2\|0>$ $\sim \bar{u}_1^{n-na_1} \bar{u}_2^{n-na_2}$ | $\|0>\sim \bar{u}_1^{n-na_1} u_2^{na_2}$ $\psi_1^*\psi_2\|0>\sim {u_1}^{na_1} \bar{u}_2^{n-na_2}$ |
| $++$ | $\psi_1, \psi_2$ | $\psi_1^*, \psi_2^*$ | $\psi_1^*\psi_2^*\|0>$ $\sim u_1^{na_1} u_2^{na_2}$ | $\|0>$ $\sim \bar{u}_1^{n-na_1} \bar{u}_2^{n-na_2}$ | $\psi_1^*\|0>\sim {u_1}^{na_1} \bar{u}_2^{n-na_2}$ $\psi_2^*\|0>\sim \bar{u}_1^{n-na_1} u_2^{na_2}$ |

Table 2: Oscillator and monomial representations of chiral and anti-chiral rings. $+-$ means $(a_1 - \frac{1}{2}) > 0, (a_2 - \frac{1}{2}) < 0$.

We define

$$G^+ = G_1^+ + G_2^+, \tag{3.24}$$

where $G_i^+ = \psi^{*(i)}\partial X^{(i)}$. For abbreviation, we use following notations;

$$\psi_i := \psi^{(i)}_{a_i - \frac{1}{2}} \quad \text{and} \quad \psi_i^* := \psi^{*(i)}_{\frac{1}{2}-a_i}. \tag{3.25}$$

Then for $a_1 < \frac{1}{2}, a_2 < \frac{1}{2}$, we have $\psi_1^*|0>= 0$ and $\psi_2^*|0>= 0$, which gives $G^+_{-\frac{1}{2}}|0>= 0$ so that the twisted vacuum is a chiral state, whose associated local ring element is identified:

$$|0>\sim u_1^{n\{jk_1/n\}} u_2^{n\{jk_2/n\}}. \tag{3.26}$$

Both $\psi_1, \psi_2$ are creation operators and $G^-_{-\frac{1}{2}}\psi_1\psi_2|0>= 0$, so that $\psi_1\psi_2|0>$ is an anti-chiral state. By considering weight and charge, corresponding monomial is found to be

$$\psi_1\psi_2|0>\sim \bar{u}_1^{n-n\{jk_1/n\}} \bar{u}_2^{n-n\{jk_2/n\}}. \tag{3.27}$$

So far, $\psi_i^*|0>$'s are neither chiral($c$) nor anti-chiral($a$). One can work out other three cases in similar fashion. We summarize the result in the Table 2.

Notice that (anti-)chiral states in different parameter ranges have different oscillator representations but have the same polynomial expressions as local ring elements.

When some of $a_i < 0$, one can get the similar result by exchanging the role of $\psi$ and $\psi^*$, and $u_i$ and $\bar{u}_i$. As a result, for the factor with the negative twist $a_i = -\{jk_i/n\}$, we need to

| $(a_1,a_2)$ | $c$-ring | $2\Delta$ | $a$-ring | $2\Delta$ |
|---|---|---|---|---|
| $++$ | $u_1^{na_1}u_2^{na_2}$ | $a_1+a_2$ | $\bar{u}_1^{n(1-a_1)}\bar{u}_2^{n(1-a_2)}$ | $2-a_1-a_2$ |
| $+-$ | $u_1^{na_1}u_2^{n(1-\lvert a_2\rvert)}$ | $a_1-\lvert a_2\rvert+1$ | $\bar{u}_1^{n(1-a_1)}\bar{u}_2^{na_2}$ | $1-a_1+\lvert a_2\rvert$ |
| $-+$ | $u_1^{n(1-\lvert a_1\rvert)}u_2^{na_2}$ | $1-\lvert a_1\rvert+a_2$ | $\bar{u}_1^{na_1}\bar{u}_2^{n(1-a_2)}$ | $\lvert a_1\rvert-a_2+1$ |
| $--$ | $u_1^{n(1-\lvert a_1\rvert)}u_2^{n(1-\lvert a_2\rvert)}$ | $2-\lvert a_1\rvert-\lvert a_2\rvert$ | $\bar{u}_1^{na_1}\bar{u}_2^{na_2}$ | $\lvert a_1\rvert+\lvert a_2\rvert$ |

Table 3: Monomial basis of chiral and anti-chiral rings and their weights when some of $a_i$ is negative. $+-$ means $a_1>0, a_2<0$. The R-charges can be read off by the rule $q=\pm 2\Delta$.

use $u_i^{n-n\{jk_i/n\}}$ for the chiral states and $\bar{u}_i^{n\{jk_i/n\}}$ for the anti-chiral states, while for the factor with the positive twist $\{jk_i/n\}$, we need to use $u_i^{n\{jk_i/n\}}$ for the chiral states and $\bar{u}_i^{n\{jk_i/n\}}$ for the anti-chiral states. For example: if only $a_2$ is negative, the chiral states are associated with $u_1^{n\{jk_1/n\}}u_2^{n-n\{jk_2/n\}}$, while the anti-chiral states are associated with $\bar{u}_1^{n-n\{jk_1/n\}}\bar{u}_2^{n\{jk_2/n\}}$. We summarize the result in the table 3.

### 3.3 Chiral rings with Enhenced (2,2) SUSY

In studying the tachyon condensation, characterizing a state as a chiral or anti-chiral state gives an extremely powerful result since we can utilize the (2,2) worldsheet supersymmetry. If all the tachyon spectrum are chiral or anti-chiral, the analysis of the tachyon condensation could be much easier. However, in reality it is not the case. For example, when $a_2<\frac{1}{2}<a_1$, $\psi_1^*|0>\sim u_1^{na_1}u_2^{na_2}$ and $\psi_2|0>\sim \bar{u}_1^{n(1-a_1)}\bar{u}_2^{n(1-a_1)}$ are chiral and anti-chiral state respectively, while $|0>$ and $\psi_1^*\psi_2|0>$ are neither of them.[2] This issue is particularly relevant in case the

[2] One may argue that we have not considered the left-right combination and it might be such that left-right combination non BPS tachyon might be projected out. however, Examining the low temperature behavior of the partition function[7], we can easily see that the string theory does contains a tachyon with $\frac{1}{4}\alpha' M^2=-\frac{1}{2}|a_1-a_2|$ as well as $\frac{1}{4}\alpha' M^2=-\frac{1}{2}|a_1+a_2-1|$. In fact, since we are looking for lowest tachyonic spectrum which comes from (NS,NS) sector the level matching condition requires that $\Delta_L=\Delta_R$ and we do not get $-|a_1-a_2|$ from the (chiral,chiral) or (anti-chiral,anti-chiral) states. That is, those spectrum with mass of the form $\frac{1}{4}\alpha' M^2 = -\frac{1}{2}|a_1-a_2|$ is in fact not a SUSY state according to our definition of (2,2) SUSY. For the level matching between left NS and right R sectors, we need to consider the modular invariance that leads to GSO projection $n(E_{NS}-E_R)=0 \mod 1$ [23]. Even in the case we combine left chiral and right anti-chiral, we do not get the spectrum of type $\frac{1}{4}\alpha' M^2=-\frac{1}{2}|a_1-a_2|$.

lowest mass in the given twisted sector is neither chiral nor anti-chiral.[3]

However, we will see that one can improve the situation by recognizing that there are enhenced SUSY in orbifold thoeries. We will show that all twisted sector tachyons generated by world sheet fermions can be considered as a chiral states by redefining the generator of the supersymmetry algebra. For this, let's define $L^{(i)}, G^{(i)\pm}, J^{(i)}$ as the generators of $N=2$ superconformal algebra in $i$-th complex plane. Usually we define $L = L^{(1)} + L^{(2)}$, $J = J^{(1)} + J^{(2)}$ and $G^+ = G^{(1)+} + G^{(2)+}$ and the last was used above to identify the chiralities. However, it is a simple matter to check that we can also define the $N=2$ superconformal algebra by defining $G^+ = G^{(1)+} + G^{(2)-}$ with corresponding change in $J = J^{(1)} - J^{(2)}$ but the same $L = L^{(1)} + L^{(2)}$. We call this (+-) choice of $G^+$ as $G^+_{ca}$, while we call the previous (++) choice as $G^+_{cc}$. The fact that we need to change the sign of $J^{(2)}$ means that we need to count the $U(1)$ charge of $u_2, \bar{u}_2$ as $-1, +1$ respectively while $u_1, \bar{u}_1$ as $1, -1$ as before. The choice of $G^+$ corresponds to the target space complex structure. This phenomena is due to the special geometry of target space in which each complex plane have independent complex structure so that to define a complex structure of the whole target space, we need to specify one in each complex plane.

Since $J \sim \psi^* \psi$ and $G^+ \sim \psi^* \partial X$ and $G^- \sim \psi \partial X^*$, the above change of generator construction corresponds to the change in the complex structure in the target space, i.e, interchanging stared field and un-stared fields with the notion of positivity of charge also changed: $\psi^*$ has $-1$ charge and $\psi$ has $+1$ charge, which is opposite to the previous case.

Since the chirality is defined by this new choice of $G^+$, we now have different classification of tachyon states: for example, in $a_2 < \frac{1}{2} < a_1 < 1$ case, $\psi_1^* \psi_2 |0> \sim u_1^{na_1} \bar{u}_2^{n(1-a_2)}$ and $|0> \sim \bar{u}_1^{n(1-a_1)} u_2^{na_2}$ are chiral and anti-chiral state respectively. Notice that they were neither *chiral* nor *anti−chiral* under $G^+_{cc}$. On the other hand, $\psi_1^* |0> \sim u_1^{na_1} u_2^{na_2}$ and $\psi_2 |0> \sim \bar{u}_1^{n(1-a_1)} \bar{u}_2^{n(1-a_1)}$ are neither chiral nor anti-chiral in the new definition of $G^+$. Similarly, we can classify other parameter zones. The result can be conceptually summarized as follows: for $G_{cc}$, $G_{ca}$, $G_{ac}$, $G_{aa}$ the monomial basis of local chiral ring is generated by $u_1^{k_1} u_2^{k_2}$, $u_1^{k_1} \bar{u}_2^{n-k_2}$, $\bar{u}_1^{n-k_1} u_2^{k_2}$ and $\bar{u}_1^{n-k_1} \bar{u}_2^{n-k_2}$ respectively, while the anti-chiral ring is generated by $\bar{u}_1^{n-k_1} \bar{u}_2^{n-k_2}$, $\bar{u}_1^{n-k_1} u_2^{k_2}$, $u_1^{k_1} \bar{u}_2^{n-k_2}$, $u_1^{k_1} u_2^{k_2}$ respectively. Notice that anti-chiral ring of $G_{cc}$ is chiral ring of $G_{aa}$, while anti-chiral ring of $G_{ca}$

[3]One example is the 10(1,3) theory.

| $G$ | $c$-ring | $2\Delta$ | $a$-ring | $2\Delta$ |
|---|---|---|---|---|
| $G_{cc}$ | $u_1^{na_1} u_2^{na_2}$ | $a_1 + a_2$ | $\bar{u}_1^{n(1-a_1)} \bar{u}_2^{n(1-a_2)}$ | $2 - a_1 - a_2$ |
| $G_{ca}$ | $u_1^{na_1} \bar{u}_2^{n(1-a_2)}$ | $a_1 - a_2 + 1$ | $\bar{u}_1^{n(1-a_1)} {u_2}^{na_2}$ | $1 - a_1 + a_2$ |
| $G_{ac}$ | $\bar{u}_1^{n(1-a_1)} u_2^{na_2}$ | $1 - a_1 + a_2$ | ${u_1}^{na_1} \bar{u}_2^{n(1-a_2)}$ | $a_1 - a_2 + 1$ |
| $G_{aa}$ | $\bar{u}_1^{n(1-a_1)} \bar{u}_2^{n(1-a_2)}$ | $2 - a_1 - a_2$ | ${u_1}^{na_1} {u_2}^{na_2}$ | $a_1 + a_2$ |

Table 4: Monomial basis of chiral and anti-chiral rings and their weights for various choices of target space complex structures.

is chiral ring of $G_{ac}$. Therefore we may consider only chiral ring of each complex structure. We call the chiral ring of $G_{cc}$ complex structure as $cc$-ring. We define $ca$-ring, $ac$-ring and $aa$-ring similarly.

It is convenient to consider the weight of a state as sum of contribution from each complex plane. For example, the weight of $u_1^{na_1} u_2^{na_2}$ can be considered as sum of $a_1$ from $u_1$ and $a_2$ from $u_2$. $(a_1, a_2)$ form a point in the weight space. As we vary $j$ in $a_i = \{jk_i/n\}$, the trajectory of the point in weight space will give us a parametric plot in the plane. In the figure 2, we draw for weight points of $cc$ and $aa$ rings in the first figure and those of $ca$ and $ac$ rings in the second figure of figure 2. In order to compare these spectrum with $a_1$ and/or $a_2$ negative cases, we

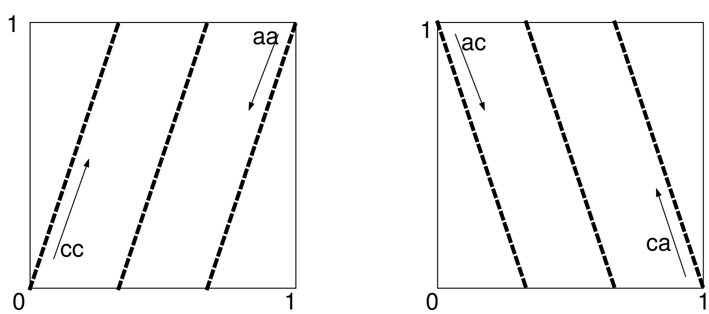


Figure 2: Weight points of $cc, aa$ and $ac, ca$ rings in weight space. x- and y-axis represent $2\Delta_1(j)$ and $2\Delta_2(j)$. Arrows represent the direction and starting point of corresponding ring as $j$ increases from 1 to n-1. Plot is drawn for $k_1 = 1,\ k_2 = 3$.

work out the weight of the states in table 4. By comparing the two table 3 and table 4, it is clear that the spectrum of $ca$-ring of $n(k_1, k_2)$ theory is equal to the $cc$-chiral ring of $n(k_1, -k_2)$ theory. So the change in complex structure $u_i \to \bar{u}_i$ is equivalent to the change in generator

| $(a_1\text{-}\frac{1}{2}, a_2\text{-}\frac{1}{2})$ | $cc$ | $ca$ | $ac$ | $aa$ |
|---|---|---|---|---|
| $--$ | $\vert 0>$ | $\psi_2\vert 0>$ | $\psi_1\vert 0>$ | $\psi_1\psi_2\vert 0>$ |
| $-+$ | $\psi_2^*\vert 0>$ | $\vert 0>$ | $\psi_1\psi_2^*\vert 0>$ | $\psi_1\vert 0>$ |
| $+-$ | $\psi_1^*\vert 0>$ | $\psi_1^*\psi_2\vert 0>$ | $\vert 0>$ | $\psi_2\vert 0>$ |
| $++$ | $\psi_1^*\psi_2^*\vert 0>$ | $\psi_1^*\vert 0>$ | $\psi_2^*\vert 0>$ | $\vert 0>$ |

Table 5: For a given twisted sector, any tachyon generated by worldsheet fermions is an element of one of the 4 possible chiral rings.

$k_i \to -k_i$ keeping the complex structure fixed. For string theory, we have to consider all four different complex structures. That is, we may consider 4 sets of spectra generated by $(k_1, k_2)$, $(-k_1, k_2)$, $(k_1, -k_2)$ and $(-k_1, -k_2)$ all together.

Summarizing, we have shown that any of the lowest tachyon spectrum generated by the worldsheet fermions, can be considered as chiral state by choosing a worldsheet SUSY generator appropriately; any of them belongs to one of 4 classes: *cc*-, *ca*-, *ac*-, *aa*- ring depending on the choice of complex structure of $\mathbb{C}^2$. This is explicit in the Table 5 We emphasize that these chiral rings does not co-exist at the same time. For example, when *cc*-ring is active ( chosen), then *aa*-ring exist as its anti-chiral ring and other two are not chiral or anti-chiral ring. But for our purpose, for any tachyon state, there is a choice of complex structure in which the given state is a chiral primary. For example, if a tachyon in *ca* ring is condensed, the spectrum of entire *ca*-ring is well controlled by the worldsheet supersymmetries generated by $G_1^+, G_2^-$. As a consequence, we will be able to calculate the fate of the those controlled spectrum. This is powerful since if we know that initial and final thoeries are described by an orbifold theories [2, 3, 4], knowing those a few spectrum completely fixes entire tower of the string spectrum in the final theory. The same phenomena arise for all $\mathbb{C}^r/\mathbb{Z}_n$. Any worldsheet fermion generated tachyon state is a chiral primary by properly choosing the target space complex structure among $2^r$ possibilities defined by the $\sum_{i=1}^r G_i^\pm$. There are $2^r$ $(2,2)$ world sheet supersymmetries instead of one. [4]

We end this section with a few comments.

[4]In fact this happens for any tensor product of N=2 SCFT's.

- The notion of enhenced symmetry already appeared in literature implicitly. For example in [4, 28], the notion of $cc, ca$ ring is discussed and the chiral ring elements was described in terms of bosonization.

- The weight space is a lattice in torus of size $n \times n$. The identification of weights by modulo $n$ corresponds to shifting string modes. However, periodicity of the generator $(k_1, k_2)$ is $2n$ and $(k_1, k_2)$ and $(k_1, k_2 + n)$ do not generate the same theory in general. We choose the standard range of $k_i$ between $-n+1$ to $n-1$. This is because the GSO projection depends not only the R-charge vector $(\{jk_1/n\}, \{jk_2/n\})$ but also the G-parity number $G = [jk_1/n] + [jk_2/n]$. We will comeback to this when we discuss the GSO projection.

- When $n$ and $k_i$ are not relatively prime, we have a chiral primary whose R-charge vector is $(p/n, 0)$. We call this as the reducible case and eliminate from our interests. This is a spectrum that is not completely localized at the tip of the orbifold. Sometimes, even in the case we started from non-reducible theory, a tachyon condensation leads us to the reducible case.

# 4 The fate of orbifolds under localized tachyon condensation

For $\mathbb{C}^2/\mathbb{Z}_{n(k_1,k_2)}$ case, if one consider the condensation of tachyon in the $l$-th twisted sector that corresponds to chiral ring element $u_1^{p_1} u_2^{p_2}$, with $p_1 = n\{lk_1/n\}$ and $p_2 = n\{lk_2/n\}$, the theory is given by the super potential

$$[W = u_1^n + u_2^n + e^{t/n} u_1^{p_1} u_2^{p_2}]//Z_n. \tag{4.1}$$

In [3], Vafa showed that, as a consequence of the tachyon condensation, the final point of the process is sum of two orbifold theories: One located at north and the other at the south poles of blown up $P^2$ singularity of the orbifold in the limit where the radius of the sphere is infinite. Schematically, we represent this transition by

$$\mathbb{C}^2/\mathbb{Z}_{n(k_1,k_2)} \to \mathbb{C}^2/\mathbb{Z}_{p_1(*,*)} \oplus \mathbb{C}^2/\mathbb{Z}_{p_2(*,*)}, \tag{4.2}$$

with yet unknown generators for the final theories. We first determine these generators, thereby specify the final theories completely. To do this we need to know how the spectrums of chiral primaries are transformed under the tachyon condensation. For this we will utilize the fact we established last section: any tachyon generated by a worldsheet fermion can be represented as an element of a chiral ring which is one of *cc*, *ca*, *ac* and *aa* rings. When a tachyon in, say, *cc*-ring condenses, R-charges of other elements in *cc*-ring is controlled by the BPS relation. Given any operator in the *cc*-ring of initial theory, we will be able to calculate precise final value of the weight or R-charge of that operator. In our case, the initial and final theories are both orbifold theories [2, 3, 4]. Once this is accepted, we can determine the generator of the final theory hence determine entire spectrum of the final theory by considering just one chiral ring.

## 4.1 Tachyon Spectrum under the localized tachyon condensation

Consider $u_2 \sim 0$ and $u_2^n \sim e^{t/n} u_1^{p_1} u_2^{p_2}$ region, which should be described by [5]

$$[W \sim u_1^n + e^{t/n} u_1^{p_1} u_2^{p_2}]//Z_n. \tag{4.3}$$

By introducing the new variables $v_1 = u_1^{n/p_2}$ and $v_2 = e^{t/np_2} u_1^{p_1/p_2} u_2$. The single valuedness of $v_i$ induces the $Z_n$ but single valuedness of $u_1^n$ and $u_1^{p_1} u_2^{p_2}$ implies that $v_1, v_2$ are orbifolded by $Z_{p_2}$. By substitution, we can express $u_1^{q_1} u_2^{q_2}$ in terms of $v_1, v_2$:

$$u_1^{q_1} u_2^{q_2} = v_1^{Q_1} v_2^{Q_2}, \tag{4.4}$$

where

$$(Q_1, Q_2) = (-p \times q/n, q_2), \tag{4.5}$$

with $p \times q = (p_1 q_2 - p_2 q_1)$. Notice that map $T_p^- : (q_1, q_2) \mapsto (Q_1, Q_2)$ is linear map acting on the integrally normalized weight space and can be described by a matrix

$$T_p^- = \begin{pmatrix} p_2/n & -p_1/n \\ 0 & 1 \end{pmatrix}. \tag{4.6}$$

[5]This section is strongly influenced by an unpublished work of Allan Adams on toric variety.

It is working near $u_2 \sim 0$. It maps $(n, 0) \to (p_2, 0)$ and $(p_1, p_2) \to (0, p_2)$. It corresponds to $u_1^n \to v_1^{p_2}$ and $u_1^{p_1} u_2^{p_2} \to v_2^{p_2}$. In integrally normalized weight space, the volume of triangle $\triangle AOB$ is $np_2/2$, while that of its image is $p_2^2/2$ and the ratio is correctly encoded in $\det T_p^- = p_2/n$.

One should notice that $Q_1, Q_2$ are not integers in general. However, when both $p$ and $q$ are weight vectors of elements of orbifold chiral ring, generated by $(k_1, k_2)$, they are integers. This is because if $p = (n\{lk_1/n\}, n\{lk_2/n\}), q = (n\{jk_1/n\}, n\{jk_1/n\})$, $s := p \times q/n$, then

$$s = n\{lk_1/n\}\{jk_2/n\} - n\{lk_2/n\}\{jk_1/n\} \in \mathbb{Z} \tag{4.7}$$

for any integers $n, k, l, j$. For $k_1 = 1$, $s = -l[jk_2/n] + j[lk_2/n]$. Especially interesting case will be $q = k = (1, k_2)$, in which case, we have $s = [lk_2/n] = (lk_2 - p_2)/n$. Geometrically, $s$ is proportional to the area spanned by two vectors $p$ and $q$. Therefore it is zero if $p$ and $q$ are parallel.

Another interesting quantity is the R-charge. The R-charges are determined by the marginality condition. In the original theory, $u_i$ has R-charge $1/n$ since $u_i^n$ has R-charge 1. We express this as $R[u_i^n] = 1$. Therefore $R[u_1^{p_1} u_2^{p_2}] = (p_1 + p_2)/n$. The diagonal in charge space is the line connecting A$(1, 0)$ and B$(0, 1)$. Any operator whose R charge is on this diagonal corresponds to the marginal operator. The points below the diagonal correspond to the relevant operators and those above it correspond to the irrelevant operators. When a tachyon, P, is fully condensed, the marginal line is changed from diagonal line AB to line AP or BP. AP gives down-theory and BP gives the up-theory. $\Delta_+$ is the cone spanned by $\vec{OB}$ and $\vec{OP}$, and similarly $\Delta_-$ is the cone spanned by $\vec{OA}$ and $\vec{OP}$.

Let P be the point $(p_1/n, p_2/n)$ in charge space that corresponds to a chiral primary that is undergoing condensation, and Q be any charge point $(q_1/n, q_2/n)$ and A, B now corresponds to $(1, 0)$ and $(0, 1)$. One can work out the action of $T_p^-$ from other point of view. If P represent the chiral primary of $l$-th twisted sector, $(p_1/n, p_2/n) := (\{lk_1/n\}, \{lk_1/n\})$. Near $u_2 \sim 0$ region, the marginality condition is changed to $R[u_1^{p_1} u_2^{p_2}] = 1, R[u_1^n] = 1$. In terms of new variable $R[v_i^{p_2}] = 1$. The linear transformation

$$\tilde{T}_p^- : (q_1/n, q_2/n) \to (Q_1/p_2, Q_2/p_2), \tag{4.8}$$

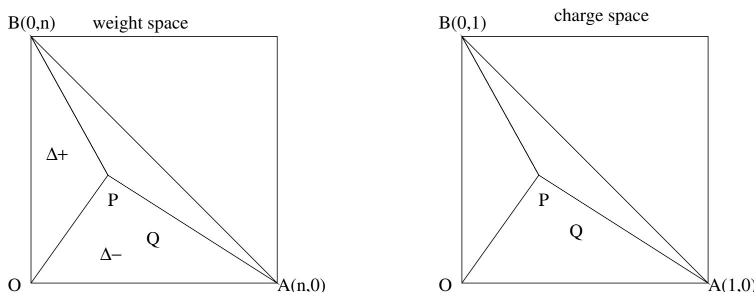


Figure 3: Integrally normalized weight/charge space(left) can be considered as the space of power of local ring elements. It is defined as a two dimensional torus with size n. In true weight/charge space(right), $u_1^n$ and $u_2^n$ is located at A(1,0) and B(0,1) respectively.

can be determined by its action on P and (1,0). Once $\tilde{T}_p^-$ is decided, we get $T_p^-$ from the relation, $\tilde{T}_p^- = \frac{n}{p_2}T_p^-$. The result of course agrees with the one given by eq.(4.6). Under this mapping, the lower triangle $\triangle POA$ in figure 3 in charge space is mapped to the entire $\triangle BOA$, which defines one of theory in the final stage of the tachyon condensation. We call it down-theory. [6]

Similarly, by considering $u_1 \sim 0$ region, we get the mapping $\tilde{T}_p^+$ that maps the upper triangle $\triangle BOP$ to $\triangle BOA$. By the relation $T_p^+ = (p_1/n)\tilde{T}_p^+$ we can obtain the mapping in weight space:

$$T_p^+ q = \begin{pmatrix} 1 & 0 \\ -p_2/n & p_1/n \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} q_1 \\ p \times q/n \end{pmatrix} \tag{4.9}$$

Notice that $T_p^+$ leaves all the vertical lines in weight space fixed while $T_p^-$ leaves horizontal lines invariant. [7]

Now we ask: given an operator with $q = (q_1, q_2)$, should we map with $T_p^+$ or $T_p^-$? The answer is that we should use the map that gives smaller R-charge. The difference of the R-charge after

[6]Conversely, if we require that $\tilde{T}_p^-$ maps $\triangle POA$ to $\triangle BOA$, $\tilde{T}_p^-$ is completely determined. The mapping $T^-$ in the integrally normalized weight space is induced by $T^- = (p_2/n)\tilde{T}^-$. The normalization is dictated from the condition that $T$ maps from integer vectors to integer vectors. Finally $T_p^-(n, 0) = (p_2, 0)$ and $T_p^-(p_1, p_2) = (0, p_2)$ so that the identification $u_1^n = v^{p_2}$, $u_1^{p_1} u_2^{p_2} = v^{p_2}$ is dictated.

[7]It maps $(0, n) \to (0, p_1)$ and $(p_1, p_2) \to (p_1, 0)$, i.e, $u_2^n \to v_2^{p_1}$ and $u_1^{p_1} u_2^{p_2} \to v_1^{p_1}$. In weight space, the volume of triangle $\triangle AOB$ is $np_1/2$, while that of its image is $p_1^2/2$ and the ratio is correctly encoded in $\det T_p^+ = p_1/n < 1$. On the charge space, however, $\tilde{T}_p^+ = n/p_1 \cdot T_p^+$ has determinant $n/p_1 > 1$ indicating that it expand the volume of charge space. Similar statement can be made for $\tilde{T}_p^-$. It is precisely this aspect that is responsible for the monotonically increasing property of R-charge under the tachyon condensation, as will show later.

the mapping is given by

$$\delta := R[T_p^+ q] - R[T_p^- q] = \frac{p \times q}{n p_1 p_2}(p_1 + p_2 - n) \quad \begin{array}{ll} < 0 & \text{if } q \in \Delta_+ \\ > 0 & \text{if } q \in \Delta_-, \end{array} \tag{4.10}$$

where $\Delta_+$ is the cone spanned by $\vec{OB}$ and $\vec{OP}$, and similarly $\Delta_-$ is the cone spanned by $\vec{OA}$ and $\vec{OP}$. Notice that we are condensing relevant operator $p$ so that $p_1 + p_2 < n$. The line $BP$ is mapped to the marginal line of a final theory, the up-theory, and the line $AP$ is mapped to that of down-theory. Therefore the emerging picture is following: The parallelogram $OBDP$ spanned by $\vec{OB}$ and $\vec{OP}$ is mapped to the up-theory whose weight space size is $p_1$. Similarly, the parallelogram $OPEA$ spanned by $\vec{OP}$ and $\vec{OA}$ is mapped to the down-theory whose weight space size is $p_2$. See figure 2. From eq. (4.7), it is easy to see that chiral ring elements of Mother theory are mapped to chiral ring elements of the daughter theories, under the condensation of a chiral ring element. Any operator $q'$ outside these two parallelograms can be parallel translated to inside one of above two parallelograms by the vector $\vec{OP}$ a few times if necessary. In daughter theories, if $q' \in \Delta_+$, then $T_p^+ q'$ can be translated horizontally by $p_1$ a few times to a point in the up-theory. Similarly, if $q' \in \Delta_-$, then $T_p^- q'$ can be translated vertically by $p_1$ a few times to a point in the up-theory.

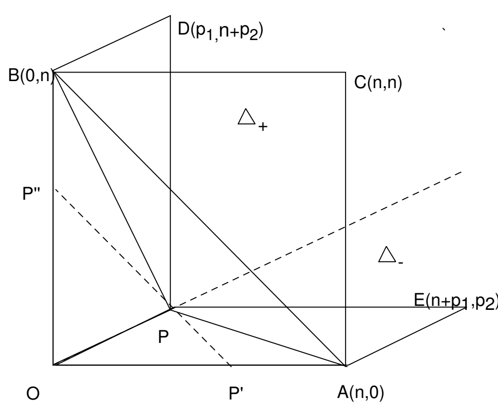


Figure 4: Under the condensation of operator P, the parallelogram $OBDP$ is mapped to the up-theory and $OPEA$ is mapped to the down-theory. Translation parallel to OP is mapped to horizontal in up theory and vertical in down theory.

## 4.2 Fate of orbifolds under localized tachyon condensation

With these preparation, we can answer to our original question: what are the generators of final theories? We noticed that there are two theories in the final stage. These two theories are described by the difference of the marginal lines in the weight space: extension of $BP$ or that of $AP$. We call the former as the up-theory, describing $u_1 \sim 0$ region, and the latter as down-theory, describing the $u_2 \sim 0$ region. In terms of the charge space, up-theory is obtained by mapping $\tilde{T}_p^+ : \triangle BOP \mapsto \triangle BOA$ and down-theory is obtained by mapping $\tilde{T}_p^- : \triangle BOP \mapsto \triangle BOA$.

The up-theory is a orbifold $\mathbb{C}^2/\mathbb{Z}_{p_1}$ and the down theory is another orbifold $\mathbb{C}^2/\mathbb{Z}_{p_2}$. Let $k = (k_1, k_2)$ be the generator of the original theory. Then the generator of the up-theory is given by $T_p^+(k) = (k_1, p \times k/n)$ and that of the $T_p^-(k) = (-p \times k/n, k_2)$. Since $(k_1, k_2) \sim (-k_1, -k_2)$ as a generator, one can also use $T_p^-(-k) = (p \times k/n, -k_2)$ instead of $T_p^-(k)$. Therefore we can describe the process of condensation of tachyon with charge $p = (p_1, p_2)$ as follows:

$$\mathbb{C}^2/\mathbb{Z}_{n(k_1,k_2)} \longrightarrow \mathbb{C}^2/\mathbb{Z}_{p_1(k_1,p\times k/n)} \oplus \mathbb{C}^2/\mathbb{Z}_{p_2(-p\times k/n,k_2)}. \tag{4.11}$$

To simplify the notation, we use $n(k_1, k_2)$ for $\mathbb{C}^2/\mathbb{Z}_{k_1,k_2}$ and $s = p \times k/n$. Then,

$$n(k_1, k_2) \underset{(p_1,p_2)}{\longrightarrow} p_1(k_1, s) \oplus p_2(-s, k_2). \tag{4.12}$$

Especially interesting cases are those when one of $k_i = 1$.

$$\begin{aligned} n(1, k_2) &\underset{(p_1,p_2)}{\longrightarrow} p_1(1, s) \oplus p_2(-s, k_2), \text{ if } k_1 = 1 \\ n(k_1, 1) &\underset{(p_1,p_2)}{\longrightarrow} p_1(k_1, s) \oplus p_2(-s, 1), \text{ if } k_2 = 1. \end{aligned} \tag{4.13}$$

In order to check the validity of our method, we work out examples that contains all of examples studied in APS and HKMM, where some of $k_1 = 1$ case is considered. We also use the abbreviation $n(k_2) := n(1, k_2)$.

1. $2l(-1) \underset{(l,l)}{\longrightarrow} l(-1) \oplus l(-1)$, with $s = -1$. APS example 5.2
2. $2l(3) \underset{(l,l)}{\longrightarrow} l(1) \oplus l(-3)$, with $s = 1$. APS Ex.5.3
3. $5(3) \underset{(2,1)}{\longrightarrow} 2(1) \oplus \mathbb{C}^2$, with $s = 1$. A generic tachyon condensation. APS Ex.5.4

4. $n(1) \xrightarrow[(p,p)]{} p(1,0) \oplus p(0,1)$: all charges are on the diagonal $q_1 = q_2$ line, so $s = 0$. This is two copies of $\mathbb{C}^1/\mathbb{Z}_p \times \mathbb{C}$.

5. $n(-1) \xrightarrow[(l,n-l)]{} l(-1) \oplus n - l(-1)$: all charges are on the marginal line $q_1 + q_2 = n$. $s = -1$.

6. $n(-3) \xrightarrow[(j,-3j)]{} j(-\alpha) \oplus \alpha n - 3j(\alpha, -3)$, where $\alpha = [3j/n] + 1$. Notice $p = (j, -3j) \equiv (j, \alpha n - 3j)$, so that $s = -\alpha$. $\alpha = 1$ case is the example 4.3.3 of HKMM.

Now, what about the generic case where neither $k_1$ nor $k_2$ is equal to 1? We first discuss the non-reducible cases where $\{lk_i/n\} \neq 0$ for any $l = 1, ..., n-1$. This is the case if $k_i$ and $n$ are relatively prime. Then we can choose a new generator $(1, k)$ such that

$$\{j(1,k)|j = 1, ..., n-1\} = \{l(k_1, k_2)|l = 1, ..., n-1\}, \tag{4.14}$$

because we can find $k$ such that for any given $l$, $lk_1 = j \bmod n$ and $lk_2 = jk \bmod n$ for some $j$. In fact $k$ is given by

$$k \equiv k_2/k_1 \bmod n. \tag{4.15}$$

Therefore *generic case is isomorphic to* $n(1,k)$ *type.*[8] For example, 11(2,3) is identical to 11(1,7) and also to 11(8,1), since $3/2 \equiv 7,\ 2/3 \equiv 8 \bmod 11$.

Some times we meet situation where $s = 0$, where we need more care. For example, if we condensate the generator $(1, k)$ itself, eq.(4.13) predict that

$$n(1,k) \to 1(1,0) \oplus k(0,k). \tag{4.16}$$

For the first element 1(1,0), it is right since the upper triangle does not contain any tachyon operator, however, for the second element , this can not be true since we have non-trivial operator in the lower triangle. $s = 0$ is caused since $p$ and $(1, k)$ is parallel. So we need to choose a generator of the lower triangle other than $(1, k)$. Assuming $k$ and $n$ are relatively prime, $k$ has multiplicative inverse modulo $n$, which we denote by $k^{-1}$. We also introduce $s' = p \times (k^{-1}, 1)/n$. Then we have $n(1,k) = n(k^{-1}, 1)$. Now the image of the new generator under $T_p^-$ is $(-s', 1)$. It is easy to show that $ks' = s - ap_2$ where $a$ is defined by $k^{-1}k = na + 1$.

[8]So far we proved this fact in the conformal filed theory level before GSO projection.

Therefore $p_2(-s,k) = p_2(-s',1)$ if $s$ is not 0. So we get

$$n(1,k) \xrightarrow[(p_1,p_2)]{} p_1(1,s) \oplus p_2(-s',1). \tag{4.17}$$

The equations (4.13), (4.17) are the main formula of this section. When one of $s, s'$ is 0 and the other is not, we use the non-zero one. For example, when the condensing operator is of the form $j(k^{-1},1)$, $s' = 0$ and it is better to use $p_2(-s,k)$ for the exactly same reason as we use $p_2(-s',1)$ when $s = 0$. When $ss' \neq 0$ two are equivalent in conformal field theory level. [9] We give a few example.

- If we condensate an operator with $p = j(1,k)$, its band number $G := [j/n] + [jk/n] = 0$ and $s = 0$. However, $s' = j(1,k) \wedge (k^{-1},1) = -aj \neq 0$ unless $k = 1$ ( or, $a = 0$). The transition is described as

  $$n(1,k) \xrightarrow[j(1,k)]{} j(1,0) \oplus jk(ja,1). \tag{4.18}$$

  More explicitly, for $p = (2,6)$ in 11(1,3), $j = 2$, $s = 0$, $k = 3$, $k^{-1} = 4$, $4 \cdot 3 = 11 \cdot 1 + 1$ hence $a = 1$ and $s' = -2$ so that

  $$11(1,3) \xrightarrow[(2,6)]{} 2(1,0) \oplus 6(2,1). \tag{4.19}$$

  Notice that $6(2,1)$ contains an operator (0,3) so that this is a reducible orbifold. Even in the case we start with irreducible orbifold, we can get reducible orbifold as a result of tachyon condensation. This happen if and only if there is an operator sitting on the line which connect (0,0) and the condensing one, $p$.

  For later use, we tabulate all possible tachyon condensation processes for model $11(1,3)$ and $10(1,3)$ in table 6 and table7. In tables, we should consider only the process by relevant operators, namely those with $n - (p_1 + p_2) > 0$, otherwise it is a process by an irrelevant operator which disappears in the infrared limit.

[9] For string theory level, two prescriptions are different if $s$ and $s'$ does not have the same G-parity (even or odd-ness). we need to use the one that has the same parity as that of k. This will be discussed further in later section.

| $j$ | $(p_1, p_2)$ | $G = [3j/11]$ | $n - (p_1 + p_2)$ | process |
|---|---|---|---|---|
| 1 | (1,3) | 0 | 7 | $11(1,3) \mapsto 1(1,0) \oplus 3(1,1)$ |
| 2 | (2,6) | 0 | 3 | $11(1,3) \mapsto 2(1,0) \oplus 6(2,1)$ |
| 3 | (3,9) | 0 | -1 | irrelevant process |
| 4 | (4,1) | 1 | 6 | $11(1,3) \mapsto 4(1,1) \oplus 1(0,1)$ |
| 5 | (5,4) | 1 | 2 | $11(1,3) \mapsto 5(1,1) \oplus 4(1,1)$ |
| 6 | (6,7) | 1 | -2 | irrelevant process |
| 7 | (7,10) | 1 | -6 | irrelevant process |
| 8 | (8,2) | 2 | 1 | $11(1,3) \mapsto 8(1,2) \oplus 2(0,1)$ |
| 9 | (9,5) | 2 | -3 | irrelevant process |
| 10 | (10,8) | 2 | -7 | irrelevant process |

Table 6: All possible tachyon condensation process in 11(1,3) model

| j | $(p_1, p_2)$ | $G = [3j/10]$ | $n - (p_1 + p_2)$ | process |
|---|---|---|---|---|
| 1 | (1,3) | 0 | 6 | $10(1,3) \mapsto 1(1,0) \oplus 3(0,1)$ |
| 2 | (2,6) | 0 | 2 | $10(1,3) \mapsto 2(1,0) \oplus 6(0,1)$ |
| 3 | (3,9) | 0 | -2 | irrelevant process |
| 4 | (4,2) | 1 | 4 | $10(1,3) \mapsto 4(1,1) \oplus 2(1,1)$ |
| 5 | (5,5) | 1 | 0 | $10(1,3) \mapsto 5(1,1) \oplus 5(1,2)$ |
| 6 | (6,8) | 1 | -4 | irrelevant process |
| 7 | (7,1) | 2 | 2 | $10(1,3) \mapsto 7(1,2) \oplus 1(0,1)$ |
| 8 | (8,4) | 2 | -2 | irrelevant process |
| 9 | (9,7) | 2 | -6 | irrelevant process |

Table 7: All possible localized tachyon condensation in model $10(1,3)$

Notice that (5,5) is a marginal deformation.

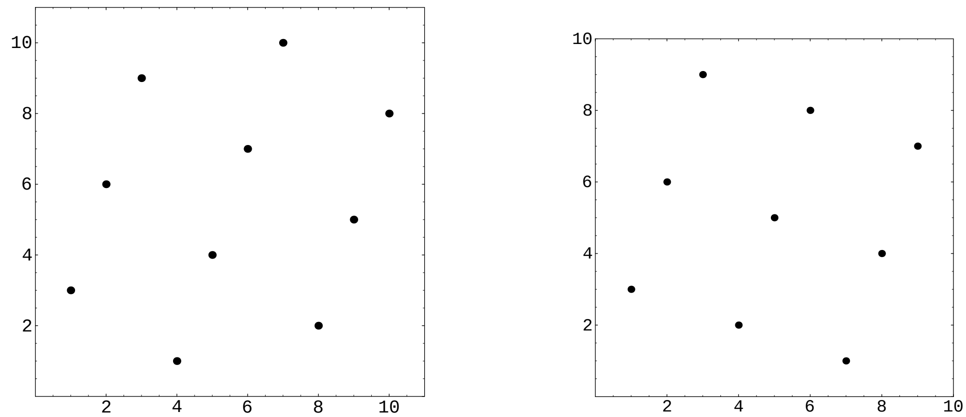


Figure 5: Charges of 11(1,3) (left) and 10(1,3) (right) in Weight space.

# 5 A semi-c-theorem for twisted sector

We start our discussion with the precise relation between the R-charges of Ramond sector and the tachyon masses orbifold theories.

## 5.1 R-charge and Tachyon mass

In [6], the tachyon potential is argued to be the same as the deficit angle, since the orbifold cone can be regarded as a consequence of the 8-brane, which is a point source from the transverse 2 dimensional point of view. More explicitly, we have equation of motion

$$g^{\mu\nu}R_{\mu\nu} = \delta^2(x)V(T). \tag{5.1}$$

By integrating out both side in 2 dimension, we get $V_0 = 2\pi(1-\frac{1}{n})$. In this context, the statement that tachyon potential decrease is nothing but that the minimal $\Delta_{min} = 1/2n$ increase. In fact not only the minimal weight of the entire twisted sector, but also the minimal weight ($l/2n$) in any ($l$-th) twisted sector decreases. So that all of them can play the role of the 'c-function'.

We start with (tachyon) mass formula in terms of the conformal weight in the NS-sector:

$$\frac{1}{4}\alpha' M^2 = \Delta - \frac{1}{2}. \tag{5.2}$$

For $\mathbb{C}^1/\mathbb{Z}_n$ model, $\Delta_{min} = 1/2n$ so that $\alpha' M^2_{min} = -2(1-1/n)$ is proportional to the deficit angle of the cone. The maximal R-charge and the minimal tachyon mass can be related. Let's imbed the orbifold into 8 dimensional transverse target space of lightcone string theory. Then the transverse spacetime is $\mathbb{C}^r/\mathbb{Z}_n \times R^{8-2r}$. Since the ground states of the twisted sectors are chiral or anti-chiral primary,

$$q = \pm 2\Delta. \tag{5.3}$$

On the other hand, we can relate the charges of the NS sector to that of the Ramond sector by the spectral flow:

$$q_R = q_{NS} - \hat{c}/2. \tag{5.4}$$

Then,

$$\frac{1}{4}\alpha' M^2 = \frac{1}{2}(q_R + \frac{\hat{c}}{2}) - \frac{1}{2}. \tag{5.5}$$

Therefore

$$\alpha' M^2_{min} = Q^5_{min} + \hat{c} - 2, \tag{5.6}$$

where 2 in $Q^5_{min} = 2q_{R,min}$ comes from summing left and right R-charges. Using the CPT symmetry on the Ramond sector, we have $q_R^{min} = -q_R^{max}$. Therefore above statement can also be written as

$$\max\left|\alpha' M^2\right| = Q^5_{max} + 2 - \hat{c}. \tag{5.7}$$

One should notice that $\hat{c} = r$ in above formula and the mass and charge are the same for $\mathbb{C}^2/\mathbb{Z}_n$ models, while they are different for $\mathbb{C}^1/\mathbb{Z}_n$ and $\mathbb{C}^3/\mathbb{Z}_n$.

Applying above result to $\mathbb{C}^1/\mathbb{Z}_n$, with $\hat{c} = 1$, $q_R^{max} = \frac{n-1}{n} - \frac{1}{2} = \frac{1}{2} - \frac{1}{n}$, which is *not* proportional to the deficit angle. This is puzzling[5] since for any N=2 SCFT,

$$0 =< \Phi|\{G^+_{-3/2}, G^-_{3/2}\}|\Phi >=< \Phi|(2L_0 - 3J_0 + 2\hat{c})|\Phi > \tag{5.8}$$

together with $\Delta = q/2$ for any chiral primary, lead us a general statement

$$\hat{c} = q_{NS}^{max} = 2q_R^{max}. \tag{5.9}$$

The puzzle came from using $\hat{c} = 1$ and $q_{NS}^{max} = 1 - 1/n$ at the same time. In fact, without including untwisted sector, $\hat{c} = 1$ is not reached. This consideration lead us to define the central charge for twisted sector by maximal R-charge of chiral primaries in the twisted sector, namely,

$$c_t := q_{NS}^{max}. \tag{5.10}$$

[10] As we will show later, $c_t$ still has a property of c-function: If an orbifold goes to another orbifold, the $c_t$ of the IR is smaller than that of UV. In fact, it is precisely this twisted sector which is described by the chiral ring of mirror LG model, as observed in [5] in the context of $\mathbb{C}^1/Z_n$ case.

## 5.2 Chiral rings and R-charge under the tachyon condensation

One of a great interest in tachyon condensation is how the spectrum flows under the RG-flow. In lack of of control of off-shell theory, it is in general difficult question to address. However, spectrum of UV and IR theories are readily available since both are conformal field theory. Since any worldsheet fermion generated tachyon can be thought as a chiral ring element for some choice of supersymmetry, we can assume, without loss of generality, that the condensing tachyon with weight $p = (p_1, p_2)$ is an element of $cc$-ring. Then consider other element in the same $cc$-ring whose weight is $q = (q_1, q_2)$. For definiteness, let's say $q \in \Delta_-$. The R-charge of it is $R_q = (q_1 + q_2)/n$. Now after the condensation of $p = (p_1, p_2)$, $q$ is moved to $q' = T_p^-(q)$, whose R-charge is

$$(q_2 - p \times q/n)/p_2. \tag{5.11}$$

If $p$ represents a tachyonic (massless) state, it must be below (on) the diagonal. Namely,

$$p_1 + p_2 \leq n. \tag{5.12}$$

Therefore The difference in the R-charge between before and after the process is given by

$$R_{q'} - R_q = (n - p_1 - p_2) q_2 / n p_2 \geq 0. \tag{5.13}$$

The same inequality holds for $q \in \Delta_+$. For $p, q \in ac$ ring, we can apply the same argument by replacing

$$p \to \bar{p} = (p_1, n - p_2), \quad q \to \bar{q} = (q_1, n - q_2). \tag{5.14}$$

[10] A side remark: If we blindly use $c_t$ in the spectral flow of the charge, i.e, $q_R = q_{NS} - \hat{c}_t/2$, then the maximal charge of the Ramond state is also proportional to the deficit angle and the puzzle is gone. It is not clear yet, if this can be justified.

Therefore we arrive at the result: *The R-charge of a relevant chiral primary operator increases under condensation of tachyon in the same ring.*

The eq. (5.13) also shows that under the condensation of marginal operator, there is no change in R-charge of any operator. Due to the mass-charge relation discussed before, we can make the same statements for the tachyon mass. The above statement shows that any of the spectrum is a candidate of the c-function of the twisted sector. However, this statement does not exclude the possibility of level crossing. That is, the ordering of the R-charge can be changed during the process.

What happen to the R-charges of operators in *ca* ring when a tachyon operator in *cc* ring condensate? The answer is that we lose control, since we lose the world sheet (2,2) super symmetry off the criticality and we lose non-renormalization theorem. In fact if one naively apply $T_p^{\pm}$ to the *ca*-ring elements we get non-integer power of $u_i$'s. Similarly, when we condense a *ca* ring element, we lose control over the *cc* ring spectrum.

However, when an element in *ca* ring turn on, we have control over other *ca* ring elements instead. It is holomorphic and protected by worldsheet SUSY $G^+_{ca}$. Since we have choice of selecting complex structure in each plane independently, we can choose any combination of complex structure to define the holomorphic co-ordinate of $\mathbb{C}^2$. We can call $u_1, \bar{u}_2$ as the holomorphic co-ordinates just as we can call $u_1, u_2$ as a holomorphic coordinate. As far as other combination does not enter in the theory, things are protected by the worldsheet supersymmetry.

Now let $q_0$ denote the a state of minimal R-charge, namely,

$$R[q_0] \leq R[q], \quad \text{for} \quad \text{all} \;\; q. \tag{5.15}$$

We want to compare the minimal charge of the initial charge and that of the final state. Let $q'$ be a minimal charge of a final theory. There are two theories in the final states and one choose any of it, say up-theory. Then $q'$ should come from a $q \in \Delta_+$ such that $q' = T_p^+(q)$. Due to the monotonicity of R-charge, we have $R[q'] > R[q]$. On the other hand, $R[q]$ can not be smaller than $R[q_0]$, by definition of $q_0$. Therefore we have inequality

$$R[q^{initial}_{min}] < R[q^{final}_{min}]. \tag{5.16}$$

The same inequality holds for the down-theory as well.

Some of the relevant operators, which are precisely those in the triangle $\triangle BPA$, will be pushed out to irrelevant operator after P is condensed. One may worry about the converse possibility that some irrelevant operators of the initial theories flow to the relevant operator. Following lemma tells us that it does not happen.

*Lemma :* Relevant chiral primary states of final theory comes only from the relevant ones in the initial theory.

*Proof:* Let $q'/p_2$ be the charge of a relevant operator in the down-theory and $q$ be its pre-image in the original theory, i.e, $q' = T_p^-(q)$. Our question is whether $q_1' + q_2' < p_2$ implies $q_1 + q_2 < n$ or not. This can be answered simply by calculating the inverse of $T_p^-$.

$$q = (T_p^-)^{-1}(q') = \frac{n}{p_2}\begin{pmatrix} 1 & p_1/n \\ 0 & p_2/n \end{pmatrix}\begin{pmatrix} q_1' \\ q_2' \end{pmatrix} = \begin{pmatrix} (nq_1' + p_1 q_2')/p_2 \\ q_2' \end{pmatrix}. \tag{5.17}$$

Now,

$$q_1 + q_2 = (nq_1' + q_2'(p_1 + p_2))/p_2 \le n(q_1' + q_2')/p_2 < n. \tag{5.18}$$

Following is an easy consequence.: *Minimal R-charge of the cc (ca) ring increases under condensation of tachyon in cc (ca)-ring. More explicitly,*

$$\min_{l=1}^{n-1} \left(\{lk_1/n\} + \{lk_2/n\}\right) \tag{5.19}$$

*increases under tachyon condensation.*

In string theory, we need to consider both rings together. Therefore we are interested in the behaviour of the R-charge which is smallest in the union of *cc* ring and *ca*-ring. To do this, we reconsider the problem of the fate of *ca*-ring under the condensation of tachyon in *cc*-ring. Although we do not have any control over the flow of the *ca* ring spectrum, we know what is the final theory and its total set of the spectrum. We ask whether any tachyon mass of the final theory can be considered as an image of some mapping with the property of R-charge increasing. To do this we want to show that there is a map that takes the some of chiral ring of the mother theory to *ca* or *ac* ring of the daughter theories. Notice that, in general, the *ca* ring of $n(k_1, k_2)$ is *cc* ring of $n(k_1, -k_2)$ and the daughter theory has structure $p_1(k_1, p \times k/n) \oplus p_2(-p \times k/n, k_2)$. First, the *ca* ring of the daughter theory $p_1(k_1, p \times k/n)$ is *cc*-ring of $p_1(k_1, -p \times k/n)$ which is expected to be the image of the $cc$ ring of $n(k_1, -k_2)$ under some mapping $F_p^+$, which is not necessarily associated with physical process. It turns out that $F_p^+$ can be chosen as $T^+_{(p_1,-p_2)}$:

$$F_p^+(\bar{q}) := T_{p'}^+(\bar{q}) = (q_1, p_1 - p \times q/n), \tag{5.20}$$

where $\bar{q} = (q_1, n - q_2) \in ca$ ring and $p' = (p_1, -p_2)$. One can check that

$$R[F_p^+(\bar{q})] > R[\bar{q}] \;\; if \;\; \bar{q} \in ca \; ring. \tag{5.21}$$

Similarly, the $ac$ ring of the daughter theory $p_2(-p \times k/n, k_2)$ is $cc$ ring of $p_2(p \times k/n, k_2)$, which can be considered as the image of the $cc$ ring of $n(-k_1, k_2)$ by the map $F_p^-$ defined by

$$F_p^-(\tilde{q}) := T_{-p'}^-(n - q_1, q_2)) = (p_2 + p \times q/n, q_2), \tag{5.22}$$

where $\tilde{q} = (n - q_1, q_2) \in ac$ ring. It can be also shown that

$$R[F_p^-(\tilde{q})] > R[\tilde{q}] \;\; if \;\; \tilde{q} \in ac \; ring. \tag{5.23}$$

Now let $q_0'$ be the tachyon with lowest mass in the daughter theory. Let it belong to the up-theory. Then we can without loss of generality assume that it belongs to $ca$ ring due to the equivalence $ca$ and $ac$ ring in their spectrum. (If it belongs to $cc$ ring, we have shown already what we want to show.) Then $q' = F_p^+(\bar{q})$ for some $\bar{q}$, which has bigger charge than the minimal charge of initial theory. Using the property of eq. (5.21),

$$R[q'] \geq R[q] \geq R[q_0], \tag{5.24}$$

as desired. Similarly, if $q'$ belongs to down theory, we can assume that it belongs to the $ac$-ring of the down theory. Then Then $q' = F_p^-(\tilde{q})$ for some $\tilde{q}$, which has bigger charge than the minimal charge of initial theory $q_0$. Using the property of eq. (5.23),

$$R[q'] \geq R[\tilde{q}] \geq R[q_0], \tag{5.25}$$

as desired.

Therefore we proved following: In the conformal field theory of orbifolds theories, the minimal R charge of the final theory is bigger than that of the initial theory under the condensation of any tachyon generated by a world sheet fermion;

$$\min_{l=1}^{n-1}[\min\left(\{lk_1/n\} + \{lk_2/n\} - 1, \{lk_1/n\} - \{lk_2/n\}\right)]. \tag{5.26}$$

increases when we compare its value in the initial and final theories.

Again a few remarks are in order:

- These theorems are world sheet fact. The same statement conjectured in [7] is the GSO projected version for which we need to take into account the GSO projection. However, tachyons in NS-NS sector is not projected out by the type 0 GSO projection, so the above conclusion is true in type 0 string theory level. For type II we will discuss in detail in next subsection.

- There are two independent theories in the final stage of tachyon condensation. Each theory will have its own minimal charges. We should take the smaller of the two, since the minimal mass of final theory is the minimal over all final spectra. Namely, the minimal R-charges of two theories in final stage are not to be added to compare with the initial one, contrary to the treatment of $g_{cl}$ in [4].

- This in fact is equivalent to a conjecture stated by Dabholkar and Vafa in [5]. More precisely, the R-charge here is that in NS sector. The R-charge of Ramond sector is related to that of NS sector by spectral flow. $q_R = q_{NS} - \hat{c}/2$, where $\hat{c} = 2$. Since there are CPT invariance in Ramond sector, the statement that minimal charge increases is equivalent to statement that maximal charge decreases. [11]

# 6 Chiral GSO projection and the c-theorem

What we have computed so far is the spectrum (R-charge) and their fate in the tachyon condensations in conformal field theory level. To understand the string mass spectrum, it is necessary

[11] In fact one can prove this directly: For notational conveniency, we consider only for the $cc$-ring under the condensation of $cc$-ring elements. Namely, we want to show that $\max_{l=1}^{n-1}(\{lk_1/n\} + \{lk_2/n\} - 1)$ decreases. Let $q_0$ be the minimal R-charge operator of initial theory, and $q$, say in $\Delta_+$, is the operator whose image $q' = T_p^+(q)$ gives the minimal R-charge of the final theory. Then the maximal R-charge of the final theory is provided by $(p_1, p_1) - q'$, which is the image of $p + \vec{OB} - q$. The maximal R-charge of initial theory is given by $(n, n) - q_0$. So our goal is to show $R[(n, n) - q_0] > R[(p_1, p_1) - q']$, which comes from $R[q_0] < R[q']$ proved above.

to consider GSO projection.

## 6.1 GSO projection and Type II v.s Type 0 in Orbifold

By considering the low temperature limit of orbifold partition functions[22, 23, 24], one can prove that the GSO projection is acting on chiral rings in the following manner [7].

**Theorem** Let $q_l := (n\{k_1 l/n\}, n\{k_2 l/n\})$ in the $cc$ ring of $n(k_1, k_2)$ theory, and $G_q = [lk_1/n] + [lk_2/n]$. If $G_q$ is odd the chiral GSO projection keeps $q$ in $cc$ ring, project out $\bar{q}$ in $ca$ ring. If $G_q$ is even, it keeps $\bar{q}$ in $ca$ ring, project out $q$ in $cc$ ring. [12]

In earlier section, we proved that for the orbifold $n(k_1, k_2)$, the R-charge of $cc, aa, ca, ac$ rings are given by

$$\{lk_1/n\} + \{lk_2/n\},\ 2 - \{lk_1/n\} - \{lk_2/n\},\ \{lk_1/n\} + 1 - \{lk_2/n\},\ 1 - \{lk_1/n\} + \{lk_2/n\}, \quad (6.1)$$

respectively. In considering monotonicity of minimal R-charge in type II string picture, one should worry about two possibilities that endanger the statement. The first is that some tachyonic state that was projected out in the initial theory flows to a state of the final theory that survive the chiral GSO spectrum. The second is that the minimal state which survive in the mother theory flows to the state which is projected out. If one of these happen to the minimal R-charge, it is not a decreasing quantity in the type II string theory.

Under the change of $k_i \to n - k_i$ together with $l \to n - l$, $\{lk_i/n\}$ is invariant, since $(n - k_i)(n - l)/n = n - l - k_i + lk_i/n$. This implies that conformal field theory of $n(k_1, k_2)$ theory is equivalent to that of $n(n - k_1, n - k_2)$: because $cc$ and $aa$ rings have the same set of operators, and so do the $ca$ and $ac$ rings. What about the $G$-parity? Following result answer this question.

$$[(n-l)(n-k_1)/n] + [(n-l)(n-k_2)/n] \equiv [lk_1/n] + [lk_2/n] + k_1 + k_2 \mod 2. \quad (6.2)$$

This means that, for $G$ even (odd), operators with given value of R-charge appear in $cc$ and $aa$ rings, or $ca$ and $ac$ rings, with the same (opposite) G-parity.

[12] Following seems to hold: $G_q \equiv k \times q/n$ mod 2. This can be easily shown for the canonical representation $n(1, k)$.

The action on of GSO on 4 chiral rings depends on whether $k_1 + k_2$ is even or odd. For $k_1 + k_2$ even, if a R-charge in $cc$ ($ca$) ring is projected out, the same value of R-charge is also projected out from $aa$ ($ac$) ring as well.

On the contrary, for $k_1 + k_2$ odd, $cc$ and $aa$ rings are complementary to each other in GSO projection: If a R-charge is projected out from one ring, it is not projected out from the other. Similarly, $ca$ and $ac$ rings are complementary in GSO. Therefore as string theory spectrum, no R-charge is projected out by the chiral GSO projection for odd $G$. Only multiplicity of the spectrum is reduced by half under the GSO projection. This is the character of the type 0 theory. We will discuss when we get type 0 and type II in detail below.

We now consider the GSO projection and orbifold action to generalize the argument of [2]. First we consider $\mathbb{C}^1/\mathbb{Z}_n$. Let $g$ be the orbifold action acting on complex plane;

$$g = e^{2\pi ikJ/n}, \quad k = -n+1, \cdots, n-2, n-1 \tag{6.3}$$

where $J$ is the rotation generator in the complex plane that is orbifolded.

$$g^n = (-1)^{kF_s}, \tag{6.4}$$

where $F_s$ means spacetime fermion number and we used $J = 1/2$ for the spacetime fermion. Hence if $k$ is even, then $g^n = 1$ and $g$ is a good generator of $\mathbb{Z}_n$ action. On the other hand, if k is odd, $g^n = (-1)^{F_s} \neq 1$, and $g$ is not a generator of $\mathbb{Z}_n$ action. In fact $g$ is the generator of $\mathbb{Z}_{2n}$ action. The $\mathbb{Z}_{2n}$ projection operator $P$ project out all spacetime fermion, since

$$P = \frac{1}{2n}\sum_{l=0}^{2n-1} g^l = \frac{1}{2}(1 + (-1)^{F_s}) \cdot \sum_{l=0}^{n-1} g^l/n. \tag{6.5}$$

The consequence is type 0 string where there is no spacetime fermion. More precisely, the bulk fermion in untwisted sector is cancelled by those of n-th twisted sector.

In order to get type II string for $k$ odd case, we need to change the projection operator by

$$g' = e^{2\pi ikJ/n}(-1)^{-2\pi iJ}, \tag{6.6}$$

so that $g'^n = (-1)^{(k-n)F_s}$. Therefore we need to require $n$ be odd integer. The net result is as follow: when $k$ is odd, instead of change $g \to g'$, we can change generator of the orbifold action from $k$ to $k-n$ in eq.(6.3). One should notice that the new generator $k'=k-n$ is even. Therefore, we can summarize : *if the generator $k$ is even, the theory is type II, otherwise it is type 0.*

Now we consider $\mathbb{C}^2/\mathbb{Z}_n$. The twist operator is

$$g = \exp(2\pi i(k_1 J_1 + k_2 J_2)/n), \tag{6.7}$$

$g^n = (-1)^{F_s(k_1+k_2)} = (-1)^{F_s}$. Therefore $g$ define a type II theory for $k_1+k_2$ even, and a type 0 theory for $k_1+k_2$ odd. In order to get a type II theory for $k_1+k_2 = odd$, the twist operator should be modified to

$$g' = \exp(2\pi i(k_1 J_1 + k_2 J_2)/n)(-1)^{F_s} \tag{6.8}$$

Since $g'^n = (-1)^{(k_1+k_2-n)F_s}$, we need odd $n$ to get $g^n = 1$ in the case $k_1+k_2$ is odd. The net result is that to get the type II theory, we can shift the generator

$$(k_1,k_2) \to (k_1-n,k_2) \;\; or \;\; (k_1,k_2) \to (k_1,k_2-n), \tag{6.9}$$

instead of changing the twist operator $g \to g'$. This works only if $n$ is odd. Notice that in the shifted generator, $k_1'+k_2' = k_1+k_2-n$ =even. [13] Since we can choose $k_1 = 1$ without loss of generality, we may fix our convention such that for even $k$ ($1+k = odd$), we need to change $k \to k-n$ so that we have to consider $n(1,k-n)$ instead of $n(1,k)$. We can summarize: *if $k_1+k_2$ is even, the theory is type II, otherwise it is type 0.* From now on we will assume that the twist operator is standard one given by $g$ and that parity of $k_1+k_2$ determine whether the given orbifold is type 0 or type II.

**Examples:**

1. $n(1,1)$: $G = [j/n]+[j/n] = 0$, hence all *cc*-ring and *aa*-ring elements are projected out. All *ca* and *ac* ring elements survive under GSO.
2. $n(1,-1)$: $G = [j/n]+[-j/n] = 0+[-1+(n-j)/n] = -1$, hence all *cc*-ring and *aa*-ring elements survive under GSO. All *ca* and *ac* ring elements are projected out.

[13] It is easy to show that these two choices as well as other possibility $(k_1,k_2) \to (k_1(1+n), k_2(1+n))$ coming from $g = \exp(2\pi i(k_1 J_1 + k_2 J_2)/n)(-1)^{F_s(k_1+k_2)}$, defines the same GSO projected spectrum.

3. $n(1,n-1)$: $G=[j(n-1)/n]=[j-j/n]=j-1$. Hence, alternating. Surviving elements are $j=1$: (1,1); $j=2$: (2,n-2); $j=3$: (3,3); etc.

4. $n(1,1-n)$: $G=[j(1-n)/n]=[-j+j/n]=-j$: Alternating projection. Surviving elements are $j=1$: (1,1); $j=2$: (2,n-2); $j=3$: (3,3); etc.

From the examples above, it is quite obvious that the set of surviving spectrum of $n(1,k)$ and that of $n(1,-k)$ is identical. The reason is because the *ca* ring of $n(1,k)$ is the same as the *cc* ring of $n(1,-k)$ and this relation is true even at the GSO projection. One can see this by simply calculating the $G$ parity of *cc* ring of each theory. For $n(1,k)$, $G=[jk/n]$ and for $n(1,k)$, $G=[-jk/n]=-[jk/n]-1$. They differ by one as desired. Therefore two theories are isomorphic as string theories.

On the other hand, $n(1,k)$ and $n(1,k-n)$ have the same spectrum before GSO projection, but they are very different after GSO projection.

## 6.2 GSO projection and semi-c-theorem

As we discussed before, when $k_1+k_2$ is odd, half of the states are projected out but the set of spectrum is not projected out. Only multiplicity of each R-charge is halfed. Therefore if $k_1+k_2$ is odd, the semi-c-theorem is compatible with chiral GSO projection, namely,

**Theorem** In type 0 string theory, $c_t$ or the minimal twisted tachyon mass of non-supersymmetric orbifold $\mathbb{C}^2/Z_{n(k_1,k_2)}$ in string theory, increases under tachyon condensation.

What happen if $k_1+k_2$ is even? In this case, some R-charges are in fact projected out. Therefore the validity of the semi-c-theorem depends on whether or not the minimal charge is projected out by GSO projection. If it is not projected out, then above theorem hold. We will get some idea from the details of examples given below.

Example 1. 11(1,3) theory with $5(1,3)=(5,4)$ condensation.
To see the general feature of string spectrum, we study the case where $n,k_1,k_2$ are relatively prime. We already tabulated all possible tachyon condensation process of 11(1,3) theory in table 6. Here we give a weight diagrams of mother and daughter theories where charges of *cc*

and $ca$ elements are put together. For simplicity we give the diagram for just one process

$$11(1,3) \underset{(5,4)}{\longrightarrow} 5(1,1) \oplus 4(1,1). \tag{6.10}$$

In this example, the final products after GSO projection are two separated supersymmetric orbifolds. All $cc$ elements are projected out while all $ca$ elements are marginal. See figure 6. The second and third diagrams in figure 6 is calculated by $T_p^+$ and $T_p^-$ for $cc$ ring and by $\bar{T}_p^+$ and $\bar{T}_p^-$ for $ca$ ring. The result is entirely consistent with the expectation as the diagram of 5(1,1) and 4(1,1) theories.

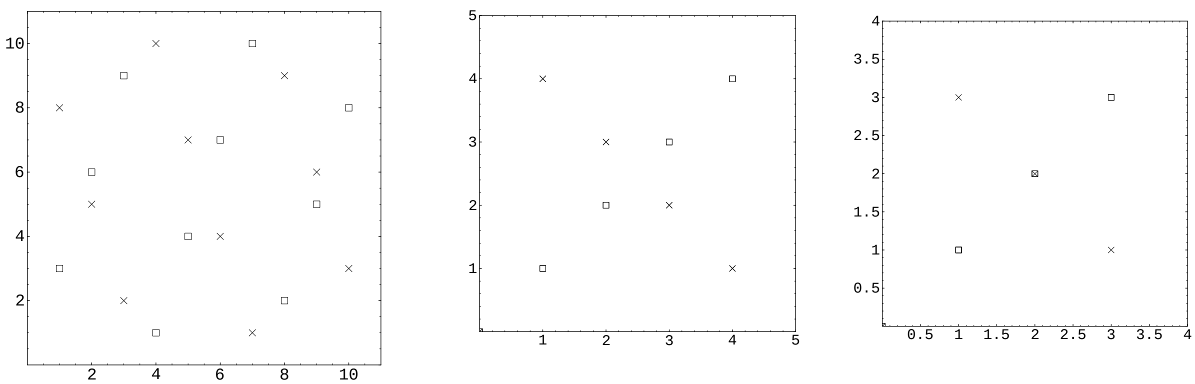


Figure 6: $11(1,3) \underset{(5,4)}{\longrightarrow} 5(1,1) \oplus 4(1,1)$. All charges of $cc$(boxes) and $ca$(crosses) elements are put together. The first big box is for the mother theory and two small boxes are for daughter theories. After GSO projection, both pieces of the final theory are supersymmetric.

Example 2. 11(1,3) Model with $8(1,3) = (8,2)$ condensation: A failure of counter example. One might try to find a counter example of $c_t$ theorem in

$$11(1,3) \underset{(8,2)}{\longrightarrow} 8(1,2) \oplus 2(1,-3). \tag{6.11}$$

In type 0 theory (with further twist by the fermion mumber), it provide the example where we get a reducible variety after the tachyon condensation. With standard twist operator, 11(1,3) is type II while 8(1,2) is a type 0 theory. Furthermore the lowest possible tachyon before GSO in the mother theory is given by $(1,3)$ which are in $cc$ ring and $G = 0$, hence it is projected out. The lowest surviving R-charge is given by (4,1) in $cc$ and $(3,2)$ in $ca$. On the other hand, the lowest R-charge is given by (1,2) in $aa$ of 8(1,2). ( as a $aa$ element (1,2)=7(-1,-2) and G=-3 hence survive. However, as the $cc$ element (1,2) is projected out.) Important fact is that this process is possible in CFT but not allowed in type II theory simply because $(8, 2)$ is projected out. (As a $cc$ element, it has $G = 2$.) Otherwise, it would give an explicit counter example for the semi-c-theorem in type II theory. This suggest that it might be true that GSO projection is compatible with semi-c-theorem. The proof of the compatibility theorem would contains two lemmas:

1. The image of GSO positive spectrum of mother theory is also GSO positive.
2. The image of minimal charge is minimal in the daughter theory. That is, the level crossing does not happen for the minimal charge.

While we have not proved it due to the complexity of the GSO, it is also nontrivial to find a counter example for this. However, we find some subtlety in next example.

Example 3.

$$10(1, 3) \underset{(5,5)}{\longrightarrow} 5(1, 1) \oplus 5(1, -3). \tag{6.12}$$

This example is particular case of a class studied in APS as well as in HKMM and disputed between the two papers. We study this in detail since it reveal much of the subtlety of GSO projection. The minimal charge of $10(1, 3)$ occur in $j = 1$ of $cc$ ring. Its value is $R = 4/10$ and it is projected out since $G = 0$ is even. However, $R = 4/10$ occur also in $ca$ rings at $3(1, -3) \equiv (3, 1)$ with $G = 0$ hence GSO surviving. See figure 7. Therefore the minimal charge of $10(1, 3)$ is not projected out due to the degeneracy in R-charge of $cc$ and $ca$. the image of $(1, 3)$ under $T_p^+$ in the final theory is $(1, 1)$ which belongs to up-theory $5(1, 1)$. Both ($cc$) and ($aa$) rings have $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$ as their elements. Before, GSO projection, the minimal R-charge of mother and daughter theories are the same, $R = (1 + 3)/10 = (1 + 1)/5$, which is what we expect from the experience of a c-theorem. However, none of these ($cc$) and ($aa$) ring elements survive under the chiral GSO (All have $G = 0$).

On the other hand, the $ca$+$ac$ ring of 5(1,1) has $\{(1, 4), (2, 3), (3, 2), (4, 1)\}$, and all of them survive. Notice that all surviving are marginal operators. It is a supersymmetric model. We should not, however, conclude that the minimal tachyon mass increased yet, since there is another daughter theory, $5(1, -3)$. Its minimal charge before GSO is $3/5$, which is already bigger than the minimal charge of the mother theory, $4/10$. Therefore the GSO projected spectra implies that the minimal charge increases under the condensation of *marginal operator*. However, it is contradictory to our expectation for a c-theorem. This is an aspect of subtlety of the c-theorem imposed by the chiral GSO projection.

Therefore, though the twisted sector c-theorem is proved and consistent with our expectation in CFT level and type 0 theory, it is not consistent with our expectation for type II theory. Perhaps there is no c-theorem in string theory level. In fact, while the c-theorem is fact of CFT, not at the level of GSO projected string theory. Nevertheless we could still ask whether $c_t$ is non-decreasing for type II as well. We hope to come back to this question in future.

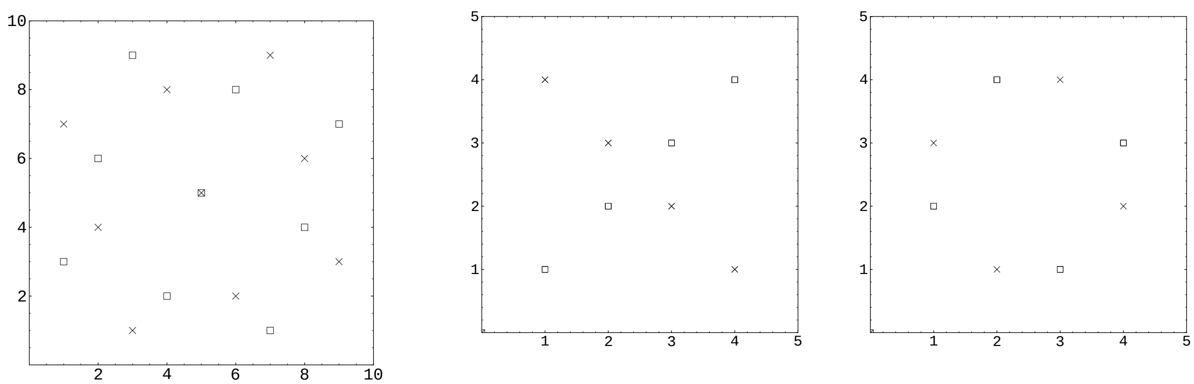

Figure 7: $10(1,3) \xrightarrow[(5,5)]{} 5(1,1) \oplus 5(1,-3)$. All charges of $cc$ and $ca$ elements are put together. After GSO projection, one piece of the final theory is supersymmetric and the other one is still tachyonic.

# 7 Transition between type 0 and type II by tachyon condensation

Now we can answer following more important question. Is the decay product of type II string theory is a type II? or Transition from type II to type 0 allowed? We can answer this question by our earlier results. First, one can represent any type II theory as a $n(1,k)$ with odd integer. $k$. For $s = p \times k/n \neq 0$, we can use

$$n(1,k) \xrightarrow[(p_1,p_2)]{} p_1(1,s) \oplus p_2(-s,k), \tag{7.1}$$

for the decay. Here one should notice that $s$ is equal to the band number of $p$: $s = G_p = [jk/n]$ if $p = j(1,k)$ mod $n$. For $p$ to survive the GSO projection of $cc$ ring, $s$ must be an odd integer. [14] Since both $1+s$ and $k-s$ are even integers, both of the daughter theories are type II theories. *Therefore type II theories can decay only type II theories.*

Can we find examples of type 0 theories to decay to a type II? One can give a similar analysis given above. In this case, $k$ is even but $s_p$ can be both even or odd since no operators are projected out. In either case we get one daughter theory type 0 and the other one type II as a product of decay of type 0 mother theory. For explicitness, we workout the decays of $11(1,2)$ in the table (8) and one can see that $11(1,2) \mapsto 7(1,1) \oplus 3(1,-2)$ is the candidate. [15] The $cc$ ring elements of 11(1,2) decay to the $cc$ ring elements of 7(1,1) and all projected out by the final theory GSO, while $ca$ ring elements of 11(1,2) decay into the SUSY spectrum of final theories. One should remember that in this example the bulk tachyon of the original theory is not from untwisted sector but from the n-th twisted sector operator (11,11). Under the tachyon condensation by $p = (7,3)$ operator, (11,11) is mapped to (11,4)≡(4,4) by $T_p^+$ and to (-4,11)≡(2,2) by $T_p^-$, either of them are irrelevant operator of final theories. The bulk tachyon spectrum is "lost" in the process of tachyon condensation to become a massive state. These final states are in fact projected out by the GSO of the final theories.

One may ask whether a super-symmetric orbifold can produce a type 0 theories by turning on a marginal operator? Fortunately this does not happen, as we can see shortly. SUSY case is either $k_1 = -k_2$ with generator $(1,-1)$ or $k_1 = k_2$ with generator $(1,1)$. For $n(1,-1)$, the $ca$ or $ac$ rings are completely projected out from the initial theory hence nothing in the final theory. $cc$ ring elements decay to diagonal elements as one can see directly or from the transition rule,

$$n(1,-1) \to n-m(1,1) \oplus m(1,1). \tag{7.2}$$

Therefore by condensing a marginal operator, we get only SUSY theories. For $n(1,1)$ theory, we get the same story by interchanging the role of $cc$ and $ca$.

What is the physical interpretation of this phenomena? The simplest interpretation is that

[14]Therefore for type II transition, there is no need to use $n(1,k) \xrightarrow[(p_1,p_2)]{} p_1(1,s) \oplus p_2(-s',1)$.

[15]if we use $p_2(-s',1)$ instead of $p_2(s,-k)$, we would get $3(1,1)$ for the second factor. It would incorrectly indicates that a type 0 theory can decay to two type II theories.

| $j$ | $(p_1, p_2)$ | G | $n-(p_1+p_2)$ | process |
|---|---|---|---|---|
| 1 | (1,2) | 0 | 8 | $11(1,2) \mapsto 1(1,0) \oplus 2(1,1)$ |
| 2 | (2,4) | 0 | 5 | $11(1,2) \mapsto 2(1,0) \oplus 4(2,1)$ |
| 3 | (3,6) | 0 | 2 | $11(1,2) \mapsto 3(1,0) \oplus 6(3,1)$ |
| 4 | (4,8) | 0 | -1 | irrelevant process |
| 5 | (5,10) | 0 | -4 | irrelevant process |
| 6 | (6,1) | 1 | 4 | $11(1,2) \mapsto 6(1,1) \oplus 1(0,1)$ |
| 7 | (7,3) | 1 | 1 | $11(1,2) \mapsto 7(1,1) \oplus 3(1,-2)$ |
| 8 | (8,5) | 1 | -2 | irrelevant process |
| 9 | (9,7) | 1 | -5 | irrelevant process |
| 10 | (10,9) | 1 | -8 | irrelevant process |

Table 8: All possible LTC process of 11(1,2)

the dynamics of the string theory is given by that of CFT and the separate notion of type II and type 0 is not preserved under the tachyon condensation. Similar result were obtained in [4] by somewhat different reasoning.

# 8 Conclusion

In this paper, we have studied the localized condensation in non-supersymmetric orbifold using the (2,2) world sheet SUSY in mirror LG picture. We study the localized tachyon condensation in Mirror Landau-Ginzburg picture as well as the toric geometry picture of non-supersymmetric orbifold backgrounds. Due to the two copies of (2,2) worldsheet supersymmetry, any worldsheet fermion generated tachyon can be considerred as a BPS state. Utilizing this fact, we show that the R-charge of chiral primaries increases under the process of localized tachyon condensation. The minimal tachyon mass in twisted sectors increases in CFT and type 0 string and plays the role of the c-function in the twisted sectors. We also study the GSO projection in detail and show that type II decay to only to type II while type 0 can decay to the mixture of type 0 and II mix. By working out how the individual chiral primaries are mapped under the tachyon condensation, we have proved that R-charges of chiral primaries increase under tachyon condensation. We studied the GSO projection and found that in many aspects, the separate notion of type II and type 0 is not preserved under the tachyon condensation.

We now discuss the limitation and related future works. First of all, our work is confined to orbifold fixed points before and after the tachyon condensation. It would be interesting to work out the detail of the off-shell. One may ask what is the geometry for the finite condensation co-efficient of LG in terms of gauged linear sigma model? A work related to this question has appeared [28]. Another related work is [26], where the Bondi energy [25] as a c-function was discussed based on the earlier work by Tseytlin [27].

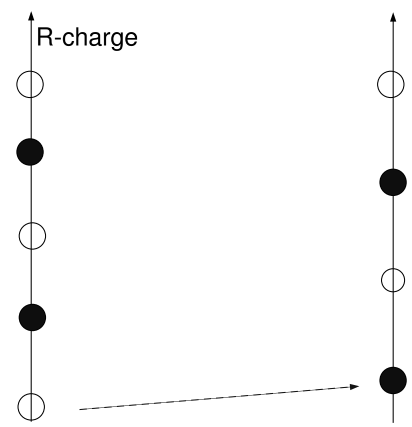


Figure 8: The black dots denote the GSO even and white dots denote GSO odd (projected out) spectrum. It is plausible that the minimal charge of mother theory is projected out by the GSO and that of the daughter theory is kept. However, in all example we considered, such possibility is forbidden due to GSO projection, a surprising phenomena.

Secondly, our work is mostly about CFT and type 0 theory rather than type II theory. For type II theory, there is only one way by which the theorem can be broken, namely, if the minimal charge of mother theory is projected out by the GSO and the minimal charge of the daughter theory is kept, then it may happen that the minimal charge of the daughter theory is smaller than that of the mother theory. It is very plausible that such possibility happens. See figure 8. Surprisingly, however, in all example we considered, the tachyon condensation that cause such possibility is forbidden due to GSO projection (ironically) as illustrated in the example 2 in section 6 by the theory 11(1,3) with (8,2). We have neither proved nor disproved the theorem of non-decreasing property of the minimal R-charge due to the complicated nature of GSO projection acting on the spectrum. We have shown, however, even in the case the theorem works for type II theory, it does not work in the way that would be expected from c-function behavior, since the marginal deformation still increases the minimal R-charge for some case as shown in the example 3 of section 6.2. We wish to come back to this issue in later publication.

Finally we mention that the basic lemma proven in section 4 can be treated in Toric geometry without using the mirror picture. In the Appendix A, we show how it can be done using the toric diagram by proving the equivalence of toric diagram and integer normalized weight diagram of Landau-Ginzburg picture.

## Appendix: A. Toric and Weight diagrams

Our goal here is to show the equivalence of tachyon transition in LG picture,

$$n(1,k) \xrightarrow[(p_1,p_2)]{} p_1(1,s) \oplus p_2(-s',1), \tag{8.1}$$

with $s = p \wedge (1,k)/n$, $s' = p \wedge (k^{-1},1)/n$ and that in toric picture

$$n(k) \xrightarrow[(n',-k')]{} n'(k') \oplus n''(k''), \tag{8.2}$$

where

$$n'' = kn' - nk' \text{ and } -k'' = cn' - dk' \tag{8.3}$$

with integer $c,d$ satisfying $cn - dk = 1$. [16] Notice that it is assumed that $k,n$ is relatively prime.

The data of weight diagram of LG model can be related to that of toric geometry by a linear map $U : LG \to Toric$ and its inverse $U^{-1}$:

$$U = \begin{pmatrix} 1 & 0 \\ -k/n & 1/n \end{pmatrix}, \quad U^{-1} = \begin{pmatrix} 1 & 0 \\ k & n \end{pmatrix}. \tag{8.4}$$

The weight $(p_1,p_2)$ of the condensing tachyon is related to the corresponding toric data $n'(k')$ by

$$\binom{p_1}{p_2} = U^{-1}\begin{pmatrix} n' \\ -k' \end{pmatrix} = \begin{pmatrix} n' \\ kn' - nk' \end{pmatrix}, \tag{8.5}$$

which gives $p_1, p_2$:

$$p_1 = n', \quad p_2 = kn' - nk', \tag{8.6}$$

from which $s$ can calculated in terms of toric data:

$$s = p \wedge (1,k)/n = (n', kn' - nk') \wedge (1,k)/n = k'. \tag{8.7}$$

Now, since $p_1(1,s)$ is trivially equal to $n'(k')$, we only need to show the equivalence of $p_2(-s',1)$ with $n''(k'')$. The question is whether $k'' \equiv -s' \mod p_2$ or equivalently,

$$(cn' - dk') \equiv \ (p_1 - k^{-1}p_2)/n \mod p_2 \tag{8.8}$$

[16] If $(c,d)$ is a solution of this equation, $(c + k'm, d + n'm)$ is also a solution. The result is the $(n''. - k'') \to (n'', -k''+n''m)$ which is just an $SL_2Z$ transformation $\begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix}$ which corresponds to a holomorphic coordinate transformation of a toric variety.

is true or not. Multiplying both sides by $k$, $(cn' - dk')k \equiv\ (kp_1 - k^{-1}kp_2)/n \text{mod } p_2$. Using $cn - dk = 1$, $s = (kp_1 - p_2)/n$ and $k^{-1}k = 1 + an$, left hand side is equal to $k'$ and right hand side is $s - ap_2$. From $s = k'$, we now have proved eq.(8.8). Now $-kk'' = ks' \text{ mod } p_2$ implies $k'' \equiv -s' \text{ mod } p_2$, provided $k$ and $p_2$ are relatively prime to each other, completing the proof of our desired result.

Remark: It is interesting to observe that for a general chiral ring element $q = (j, n\{jk/n\})$, $Uq = (j, k \times q/n) = \tilde{T}_k^+(q/n) = (j, -[jk/n])$, which means formally, $U$ coincide with tachyon condensation mapping for generator condensation. This fact directly generalizes to the general $(k_1, k_2)$.

## Appendix B. Compatibility of GSO projection and $n(k_1, k_2) \equiv n(1, k)$

The conformal field theory spectrum of $n(k_1, k_2)$ are the same with that of $n(1, k)$ as well as with $n(k^{-1}, 1)$ for $k = k_2/k_1 \text{ mod } n$. It is an important to know what happen to if we take into account the GSO projection. According to the quantum symmetry of orbifold theory[29], one can map from first twisted sector to $l$-th twisted sector for arbitrary $l$ in the 1-1 fashion. For type II theory, we need one more requirement: $k$ or $k^{-1}$ must be odd. Otherwise, we can not preserve the type II condition $k_1 + k_2$=odd integer. To convince ourselves, we study a few concrete examples.

- **11(1,3), 11(4,1), 11(5,4);**

First of all, 11(1,3) is a type II ($k_1 + k_2$ =even), while the other two theories are of type 0 ($k_1 + k_2$ =odd). In type 0 theories, no spectrum is projected out, hence we can say 11(4,1) and 11(5,4) are equivalent string theories without further consideration.

- **11(1,3), 11(4,1), 11(8,2);**

11(4,1) is type 0 and the other two are type II theories. So we should compare 11(1,3) and 11(8,2) in detail to see the equivalence. We first work out charge of all elements of each ring of each theory with their $G$ values in one triplet $(q_1, q_2, G)$. Since we already know that for $k_1 + k_2$ =even case, $cc(ca)$ and $aa(ac)$ are equivalent, we only have to consider $cc$ and $ca$ only.

We list of operators in $(q_1, q_2, G)$ format;

$$(q_1, q_2, G) \equiv (q_1', q_2', G') \quad \text{if} \;\; q_1 = q_1', q_2 = q_2', G \equiv G' \bmod 2.$$

First, for 11(1,3)

cc ring:

$\{(1,3,0),(2,6,0),(3,9,0),(4,1,1),(5,4,1),(6,7,1),(7,10,1),(8,2,2),(9,5,2),(10,8,2)\}$

ca ring:

$\{(1,8,0),(2,5,0),(3,2,0),(4,10,1),(5,7,1),(6,4,1),(7,1,1),(8,9,2),(9,6,2),(10,3,2)\}$

Now, for 11(8,2)

cc ring:

$\{(8,2,0),(5,4,1),(2,6,2),(10,8,2),(7,10,3),(4,1,5),(1,3,6),(9,5,6),(6,7,7),(3,9,8)\}$

ca ring:

$\{(8,9,0),(5,7,1),(2,5,2),(10,3,2),(7,1,3),(4,10,5),(1,8,6),(9,6,6),(6,4,7),(3,2,8)\}$

We now need to project out even-$G$ operators from $cc$ and $aa$ rings and odd-$G$ operators from $ca$ and $ac$ rings. Surviving operators are listed below in $(q_1, q_2, G)$ format;

11(1,3)

cc ring: $\{(4,1,1),(5,4,1),(6,7,1),(7,10,1)\}$

ca ring: $\{(1,8,0),(2,5,0),(3,2,0),(8,9,2),(9,6,2),(10,3,2)\}$

11(8,2)

cc ring: $\{(5,4,1),(7,10,3),(4,1,5),(6,7,7)\}$

ca ring: $\{(8,9,0),(2,5,2),(10,3,2),(1,8,6),(9,6,6),(3,2,8)\}$

Therefore, two theories are identical.

• **10(1,3) and 10(7,1);** One more example

Before GSO projection; 10(1,3)

cc ring:

$\{(1,3,0),(2,6,0),(3,9,0),(4,2,1),(5,5,1),(6,8,1),(7,1,2),(8,4,2),(9,7,2)\}$

ca ring:

$\{(1,7,0),(2,4,0),(3,1,0),(4,8,1),(5,5,1),(6,2,1),(7,9,2),(8,6,2),(9,3,2)\}$

10(7,1)

cc ring:

$\{(7,1,0),(4,2,1),(1,3,2),(8,4,2),(5,5,3),(2,6,4),(9,7,4),(6,8,5),(3,9,6)\}$

ca ring:

$\{(7,9,0),(4,8,1),(1,7,2),(8,6,2),(5,5,3),(2,4,4),(9,3,4),(6,2,5),(3,1,6)\}$

After GSO projection;

10(1,3)

cc ring: $\{(4,2,1),(5,5,1),(6,8,1)\}$

ca ring: $\{(1,7,0),(2,4,0),(3,1,0),(7,9,2),(8,6,2),(9,3,2)\}$

10(7,1)

cc ring: $\{(4,2,1),(5,5,3),(6,8,5)\}$

ca ring: $\{(7,9,0),(1,7,2),(8,6,2),(2,4,4),(9,3,4),(3,1,6)\}$

Once again, they are equivalent by comparison.

## Acknowledgement

I would like to thank Lance Dixon, Michael Gutperle, Shamit Kachru, Amir Kashani-Poor, Matthias Klein and M. Peskin for helpful discussions. I'd like to give special thank to Allan Adams for his collaboration in the initial stage of the work as well as many stimulating discussions, and to Eva Silverstein for her support during the author's stay at SLAC as well as her interests and discussions on the work. This work is partially supported by the Korea Research Foundation Grant (KRF-2002-013-D00030).

# References


[1] A. Sen, "Non-BPS states and branes in string theory," arXiv:hep-th/9904207, and references citing it; For review, see
A. Sen, "Stable non-BPS states in string theory," JHEP **9806**, 007 (1998) [arXiv:hep-th/9803194], and references therein.

[2] A. Adams, J. Polchinski and E. Silverstein, "Don't panic! Closed string tachyons in ALE space-times," JHEP **0110**, 029 (2001) [arXiv:hep-th/0108075].

[3] C. Vafa, "Mirror symmetry and closed string tachyon condensation," arXiv:hep-th/0111051.

[4] J. A. Harvey, D. Kutasov, E. J. Martinec and G. Moore, "Localized tachyons and RG flows," arXiv:hep-th/0111154.

[5] A. Dabholkar and C. Vafa, "tt* geometry and closed string tachyon potential," arXiv:hep-th/0111155.

[6] A. Dabholkar, "Tachyon condensation and black hole entropy," Phys. Rev. Lett. **88**, 091301 (2002) [arXiv:hep-th/0111004].

[7] Sang-Jin Sin, "Tachyon mass, c-function and Counting localized degrees of freedom," Nucl. Phys. B637 (2002)395, arXiv:hep-th/0202097.

[8] Y. Michishita and P. Yi, "D-brane probe and closed string tachyons," Phys. Rev. D **65**, 086006 (2002) [arXiv:hep-th/0111199],
Soonkeon Nam and Sang-Jin Sin, "Condensation of Localized Tachyons and Spacetime Supersymmetry", arXiv:hep-th/0201132;
R. Rabadan and J. Simon, "M-theory lift of brane-antibrane systems and localised closed string tachyons," JHEP **0205**, 045 (2002) [arXiv:hep-th/0203243];
A. M. Uranga, "Localized instabilities at conifolds," arXiv:hep-th/0204079;
T. Sarkar, "Brane probes, toric geometry, and closed string tachyons," Nucl. Phys. B **648**, 497 (2003) [arXiv:hep-th/0206109]. T. Suyama, "Closed string tachyons and RG flows,"

JHEP **0210**, 051 (2002) [arXiv:hep-th/0210054];

J. L. Barbon and E. Rabinovici, "Remarks on black hole instabilities and closed string tachyons," arXiv:hep-th/0211212;

Y. H. He, "Closed string tachyons, non-supersymmetric orbifolds and generalised McKay correspondence," arXiv:hep-th/0301162.

[9] A. B. Zamolodchikov, "'Irreversibility' Of The Flux Of The Renormalization Group In A 2-D Field Theory," JETP Lett. **43**, 730(1986) [Pisma Zh. Eksp. Teor. Fiz. **43**, 565 (1986)].

[10] I. Affleck and A. W. Ludwig, "Universal Noninteger 'Ground State Degeneracy' In Critical Quantum Systems," Phys. Rev. Lett. **67**, 161 (1991).

[11] V.I.Arnol'd, "On some problems in singularity theory", In *Geometry and Analysis*, Papers dedicated to V.K.Patodi, 1-9(1981).

[12] V.I. Arnol'd, Sinularity theory (Selected papers), Cambridge University Press 1981, V.I. Arnol'd(Ed), Dynamical systems VI, Springer.

[13] A.N. Varchenko, "Semi-continuity of the spectrum and an upper bound for the number of singular points of a projective hypersurface", Soviet. math., Dolk.27, 735-739 (1983)

[14] J.H.M.Steenbrink, "Semicontinuity of the singularity spectrum". Invent. Math. 79, 557-566.(1985)

[15] C. Vafa, "String Vacua And Orbifoldized L-G Models," Mod. Phys. Lett. A **4**, 1169 (1989);

K. A. Intriligator and C. Vafa, "Landau-Ginzburg Orbifolds," Nucl. Phys. B **339**, 95 (1990).

[16] S. Cecotti and C. Vafa, "Topological antitopological fusion," Nucl. Phys. B **367**, 359(1991).

[17] S. Cecotti, P. Fendley, K. A. Intriligator and C. Vafa, "A New supersymmetric index," Nucl. Phys. B **386**, 405 (1992) [arXiv:hep-th/9204102].

[18] A. Strominger, S. T. Yau and E. Zaslow, "Mirror symmetry is T-duality," Nucl. Phys. B **479**, 243 (1996) [arXiv:hep-th/9606040].

[19] E. Witten, "Phases of N = 2 theories in two dimensions," Nucl. Phys. B **403**, 159 (1993) [arXiv:hep-th/9301042].

[20] K. Hori and C. Vafa, "Mirror symmetry," arXiv:hep-th/0002222.

[21] L. J. Dixon, D. Friedan, E. J. Martinec and S. H. Shenker, "The Conformal Field Theory Of Orbifolds," Nucl. Phys. B **282**, 13 (1987).

[22] A. Dabholkar, "Strings on a cone and black hole entropy," Nucl. Phys. B **439**, 650 (1995) [arXiv:hep-th/9408098].

[23] D. A. Lowe and A. Strominger, "Strings near a Rindler or black hole horizon," Phys. Rev. D **51**, 1793(1995) [arXiv:hep-th/9410215].

[24] T. Takayanagi and T. Uesugi, "Orbifolds as Melvin geometry," arXiv:hep-th/0110099.

[25] H.Bondi, M. Van der Burg and A. Metzner, "Gravitational waves in general relativity VII. Waves from axi-symmetric isolated systems"' Proc. Roy. Soc. London, Ser. A269,21(1962).

[26] M. Gutperle, M. Headrick, S. Minwalla and V. Schomerus, "Space-time energy decreases under world-sheet RG flow," JHEP **0301**, 073 (2003) [arXiv:hep-th/0211063].

[27] A. A. Tseytlin, "Conditions of Weyl invariance of two-dimensional sigma model from equations of stationarity of central charge action," Phys. Lett. B **194**, 63 (1987).

[28] E. J. Martinec and G. Moore, "On decay of K-theory," arXiv:hep-th/0212059;
E. J. Martinec, "Defects, decay, and dissipated states," arXiv:hep-th/0210231.

[29] C. Vafa, "Quantum Symmetries Of String Vacua," Mod. Phys. Lett. A **4**, 1615 (1989).