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THE SPIN STRUCTURE FUNCTION g_2

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Abstract. We have measured the spin structure functions g_2^p and g_2^d over the kinematic range $0.02 \leq x \leq 0.8$ and $0.7 \leq Q^2 \leq 20$ GeV^2 by scattering 29.1 and 32.3 GeV longitudinally polarized electrons from transversely polarized NH_3 and ^{6}LiD targets. Our measured g_2 approximately follows the twist-2 Wandzura-Wilczek calculation. The twist-3 reduced matrix elements d_2^p and d_2^n are less than two standard deviations from zero. The data are inconsistent with the Burkhardt-Cottingham sum rule. The Efremov-Leader-Teryaev integral is consistent with zero within our measured kinematic range.

The deep-inelastic spin structure functions of the nucleons, $g_1(x,Q^2)$ and $g_2(x,Q^2)$, depend on the spin distribution of the partons and their correlations. The function g_1 can be primarily understood in terms of the quark parton model (QPM) and perturbative QCD with higher twist terms at low Q^2. The function g_2 is of particular interest since it has contributions from quark-gluon correlations and other higher twist terms at leading order in Q^2 which cannot be described perturbatively. By interpreting g_2 using the operator product expansion (OPE) [1, 2], it is possible to study contributions to the nucleon spin structure beyond the simple QPM.

The structure function g_2 can be written [3]:

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \overline{g_2}(x,Q^2)$$

where

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y} dy,$$

$$\overline{g_2}(x,Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left(\frac{m}{M} h_T(y,Q^2) + \xi(y,Q^2)\right)\frac{dy}{y},$$

x is the Bjorken scaling variable and Q^2 is the absolute value of the virtual photon four-momentum squared. The twist-2 term g_2^{WW} was derived by Wandzura and Wilczek [4] and depends only on g_1. The function $h_T(x,Q^2)$ is an additional twist-2 contribution [3, 5] that depends on the transverse polarization density. The h_T contribution to $\overline{g_2}$ is suppressed by the ratio of the quark to nucleon masses m/M [5] and its effect is thus small for up and down quarks. The twist-3 part (ξ) comes from quark-gluon correlations. Low-precision measurements of g_2 exist for the proton and deuteron [6, 7, 8], as well as for the neutron [9, 10]. Here, we report new, precise measurements of g_2 for the proton and deuteron.

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Electron beams with energies of 29.1 and 32.3 GeV and longitudinal polarization $P_b = (83.2 \pm 3.0)\%$ struck approximately transversely polarized NH_3 [12] (average polarization $< P_t >= 0.70)$ or ^{6}LiD $(< P_t >= 0.22)$ targets. The beam helicity was randomly chosen pulse by pulse. Scattered electrons were detected in three independent spectrometers centered at 2.75°, 5.5°, and 10.5°. The two small-angle spectrometers were the same as in SLAC E155 [11], while the large-angle spectrometer had additional hodoscopes and a more efficient pre-radiator shower counter. Further information on the experimental apparatus can be found in references [11, 12, 13]. The approximately equal amounts of data taken with the two beam energies and opposites signs of target polarization gave consistent results.

The measured asymmetry, $\tilde{A_\perp}$, differs from $A_\perp$ because the target polarizations were not exactly perpendicular to the beam line. We determined $\tilde{A_\perp}$ using:

$$\tilde{A_\perp} = \frac{1}{f_{RC}} \left[\frac{C_1}{f P_t} \left(\frac{A_{raw}}{P_b} - A_{EW} \right) + C_2 \frac{\sigma_p}{\sigma_d} \tilde{A^p_\perp} \right] + A_{RC}$$

where A_{raw} is the measured counting rate asymmetry from the two beam helicities, including small corrections for pion and charge symmetric backgrounds, dead-time and tracking efficiency, and A_{EW} is the electroweak asymmetry. The target dilution factor, f, is the fraction of free polarizable protons (≈ 0.13) or deuterons (≈ 0.18). C_1 and C_2 are nuclear corrections. The quantities f_{RC} and A_{RC} are radiative corrections determined using a method similar to E143 [12]. The detailed results for $\tilde{A_\perp}$ are shown in Ref. [14]. The multiplicative uncertainties due to target and beam polarization and dilution factor combined are 5.1% (proton) and 6.2% (deuteron). are small compared to the statistical errors. We determined $g_2(x, Q^2)$ from $\tilde{A_\perp}$ (dominant contribution) and the previously measured g_1.

The data cover the kinematic range $0.02 \leq x \leq 0.8$ and $0.7 \leq Q^2 \leq 20$ GeV^2 with an average Q^2 of 5 GeV^2. Tables of the complete results are in Ref. [14]. Figure 1 (left) shows the values of xg_2 as a function of Q^2 for several values of x along with results from E143 [12] and E155 [8]. The systematic error on xg_2 is much smaller than the statistical error. The former includes the systematic errors on $\tilde{A_\perp}$, the 5% normalization uncertainty of g_1, the 2% uncertainty of F_2, and the systematic errors of R. The data approximately follow the Q^2 dependence of g_2^{WW} (solid curve), although for the proton, the data points are slightly lower than g_2^{WW} at low and intermediate x, and higher at high x. The predictions of Stratmann [15] are closer to the data.

We obtained values at the average Q^2 for each x bin by using the Q^2-dependence of g_2^{WW}. Figure 1 (right) show the averaged xg_2 of this experiment. The figure also has xg_2^{WW} calculated using our parameterization of g_1. The combined new data for p disagree with g_2^{WW} with a χ^2/dof of 3.1 for 10 degrees of freedom. For d the new data agree with g_2^{WW} with a χ^2/dof of 1.2 for 10 dof. The data for g_2^p are inconsistent with zero (χ^2/dof=15.5) while g_2^d differs from zero only at $x \sim 0.4$. Also shown in Fig. 1 (right) is the bag model calculation of Stratmann [15] which is in good agreement with the data, chiral soliton models calculations [16, 17] which are too negative at $x \sim 0.4$, and the bag model calculation of Song [5] which is in clear disagreement with the data.

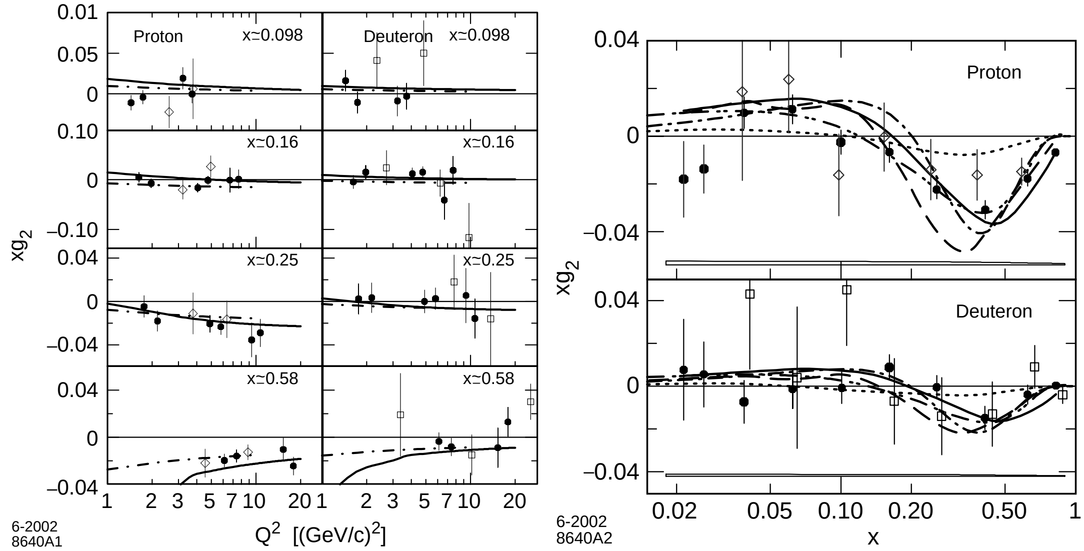

FIGURE 1. LEFT) xg_2^p and xg_2^d as a function of Q^2 for selected values of x from this experiment (solid), E143 [12] (open diamond) and E155 [8] (open square). Errors are statistical, the systematic errors are small. The curves show xg_2^{WW} (solid) and the bag model of Stratmann [15] (dash-dot).
RIGHT) The Q^2-averaged structure function $x\mathrm{g}_2$ from this experiment (solid circle), E143 [7] (open diamond) and E155 [8] (open square). The errors are statistical; systematic errors are shown as the width of the bar at the bottom. Also shown is our twist-2 g_2^{WW} at the average Q^2 of this experiment at each value of x (solid line), the bag model calculations of Stratmann [15] (dash-dot-dot) and Song [5] (dot) and the chiral soliton models of Weigel and Gamberg [16] (dash dot) and Wakamatsu [17] (dash)

The OPE allows us to write the hadronic matrix element in deep-inelastic scattering in terms of a series of renormalized operators of increasing twist [1, 2]. The moments of g_1 and g_2 for even $n \geq 2$ at fixed Q^2 can be related to twist-3 reduced matrix elements, d_n, and higher-twist terms which are suppressed by powers of $1/Q$. Neglecting quark mass terms:

$$d_n = 2\frac{n+1}{n}\int_0^1 dx\, x^n \overline{g_2}(x,Q^2).$$

The matrix element d_n measures deviations of g_2 from the twist-2 g_2^{WW} term. Note that some authors [2, 18] define d_n with an additional factor of two. We calculated d_2 with the assumption that $\overline{g_2}$ is independent of Q^2 in the measured region. This is not unreasonable since d_2 depends only logarithmically on Q^2 [1]. The part of the integral for x below the measured region was assumed to be zero because of the x^2 suppression. For $x \geq 0.8$ we used $\overline{g_2} \propto (1-x)^m$ where m=2 or 3, normalized to the data for $x \geq 0.5$. Because $\overline{g_2}$ is small at high x, the contribution was negligible for both cases. We obtained values of $d_2^p = 0.0025 \pm 0.0016 \pm 0.0010$ and $d_2^d = 0.0054 \pm 0.0023 \pm 0.0005$ at an average Q^2 of 5 GeV^2. We combined these results with those from SLAC experiments on the neutron (E142 [9] and E154 [10]) and proton and deuteron (E143 [12] and E155 [8])

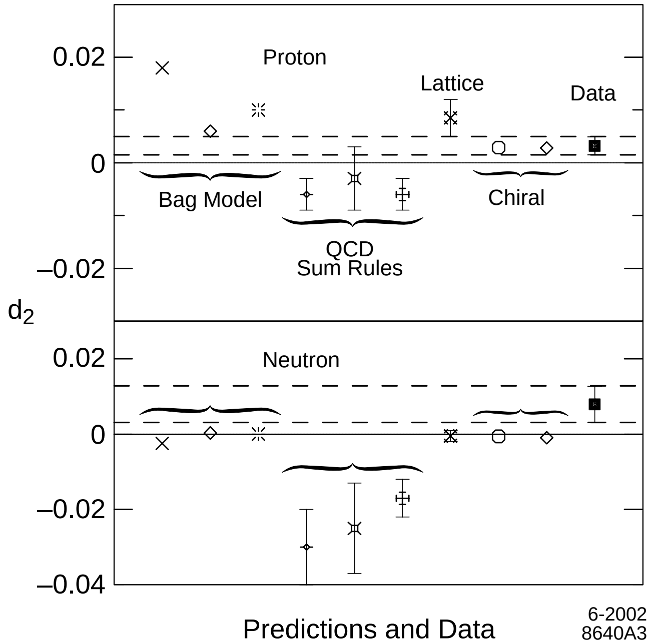

FIGURE 2. The twist-3 matrix element d_2 for the proton and neutron from the combined data from this and other SLAC experiments (E142 [9], E143 [12], E154 [10] and E155 [8] (DATA). The region between the dashed lines indicates the experimental errors. Also shown are theoretical model values from left to right: bag models [5, 15, 19], QCD Sum Rules [20, 21, 22], Lattice QCD [18] and chiral soliton models [16, 17].

to obtained average values d_2^p =0.0032±0.0017 and d_2^n =0.0079±0.0048. These are consistent with zero (no twist-3) to within two standard deviations. The values of the 2^{nd} moments alone are: $\int_0^1 dx\ x^2 g_2(x,Q^2)$=−0.0072± 0.0005±0.0003 (p) and −0.0019± 0.0007±0.0001 (d).

Figure 2 shows the experimental values of d_2^p and d_2^n plotted along with theoretical models from left to right: bag models (Song [5], Stratmann [15], and Ji [19]); sum rules (Stein [20], BBK [21], Ehrnsperger [22]); chiral soliton models [16, 17]; and lattice QCD calculations ($Q^2 = 5$ GeV^2, $\beta = 6.4$) [18]. The lattice and chiral calculations are in good agreement with the proton data and two standard deviations below the neutron data. The sum rule calculations are significantly lower than the data. The Non Singlet combination, $3 \cdot (d_2^p - d_2^n) = -0.0141 \pm 0.0170$ is consistent with an instanton vacuum calculation of ~ 0.001 [23].

The Burkhardt-Cottingham (BC) sum rule [24] for g_2 at large Q^2, $\int_0^1 g_2(x)dx = 0$, was derived from virtual Compton scattering dispersion relations. It does not follow from the OPE since $n = 0$. Its validity depends on the lack of singularities for g_2 at $x = 0$, and a dramatic rise of g_2 at low x could invalidate the sum rule. We evaluated the BC integral in the measured region of $0.02 \le x \le 0.8$ at $Q^2 = 5$ GeV^2. The results for the proton and deuteron are −0.044±0.008±0.003 and −0.008±0.012±0.002 respectively. Averaging with the E143 and E155 results which cover a slightly more restrictive x range gives −0.042±0.008 and −0.006±0.011. This does not represent a conclusive test of the sum rule because the behavior of g_2 as $x \to 0$ is not known. However, if we assume that $g_2 = g_2^{WW}$ for $x < 0.02$, and use the relation $\int_0^x g_2^{WW}(y)dy = x\left[g_2^{WW}(x) + g_1(x)\right]$, there is an additional contribution of 0.020 (p) and 0.004 (d). This leaves a $\sim 2.8\sigma$ deviation from zero for the proton.

The Efremov-Leader-Teryaev (ELT) sum rule [25] involves the valence quark contributions to g_1 and g_2: $\int_0^1 x[g_1^V(x) + 2g_2^V(x)]dx = 0$. If the sea quarks are the same in protons and neutron this becomes $\int_0^1 x[g_1^p(x) + 2g_2^p(x) - g_1^n(x) - 2g_2^n(x)]dx = 0$. We evaluated this ELT integral in the measured region using the fit to g_1. The result at $Q^2 = 5$ GeV^2 is $-0.013\pm$ 0.008$\pm$0.002 , which is consistent with the expected value of zero. Including the data of E143 [12] and E155 [8] leads to $-0.011\pm$ 0.008. The extrapolation to x=0 is not known, but is suppressed by a factor of x. The values of the 1^{st} moments at $Q^2 = 5$ GeV^2 are: $\int_0^1 dx\, xg_2(x,Q^2)$=$-0.0157 \pm 0.0012 \pm 0.0005$ (p) and $-0.0037 \pm 0.0016 \pm 0.0002$ (d).

In summary, our results for g_2 follow approximately the twist-2 g_2^{WW} shape, but deviate significantly at some values of x. The twist-3 matrix elements d_2 are less than two standard deviations from zero. The data over the measured range are inconsistent with the BC sum rule and consistent with the ELT integral.

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