UCB-PTH-03/03 SLAC-PUB-9650 hep-ph/0302131 February 2003

WMAPping Out Neutrinos Masses[∗](#page-0-0)

Aaron Pierce

Theoretical Physics Group Stanford Linear Accelerator Center Stanford University, Stanford, California,94309, USA

Hitoshi Murayama

Theoretical Physics Group Ernest Orlando Lawrence Berkeley National Laboratory University of California, Berkeley, California 94720, USA

Department of Physics, University of California Berkeley, California 94720, USA

Abstract

Recent data from from the Wilkinson Microwave Anisotropy Probe (WMAP) place important bounds on the neutrino sector. The precise determination of the baryon number in the universe puts a strong constraint on the number of relativistic species during Big-Bang Nucleosynthesis. WMAP data, when combined with the 2dF Galaxy Redshift Survey (2dFGRS), also directly constrain the absolute mass scale of neutrinos. These results conflict with a neutrino oscillation interpretation of the result from the Liquid Scintillator Neutrino Detector (LSND) over the entire favored mixing region. We also note that the Heidelberg–Moscow evidence for neutrinoless double beta decay is only consistent with the WMAP+2dFGRS data for the largest values of the nuclear matrix element.

[∗] The work of AP was supported by the U.S. Department of Energy under Contract DE-AC03-76SF00515. The work of H. Murayama was supported in part by the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation grant PHY-0098840

1 Introduction

Evidence for neutrino oscillation has steadily mounted over the last few years, culminating in a picture that presents a compelling argument for finite neutrino masses. The observation of a zenith-angle dependent deficit of ν_{μ} from cosmic ray showers at Super-Kamiokande [\[1\]](#page-8-0), provided strong evidence for oscillations in atmospheric neutrinos. Recent results on solar neutrinos at the Sudbury Neutrino Observatory (SNO) [\[2\]](#page-8-1) and reactor neutrinos at the Kam-LAND experiment [\[3\]](#page-8-2), have shed light on the solar neutrino problem. These experiments have provided strong evidence that the solar neutrino problem is solved by oscillations corresponding to the Large Mixing Angle solution [\[4\]](#page-8-3). Although clear oscillation data now exist in atmospheric, reactor, and solar neutrino experiments, it remains to determine the significance of the result from the Liquid Scintillator Neutrino Detector (LSND) [\[5,](#page-8-4) [6\]](#page-8-5), which claimed evidence for conversion of $\bar{\nu}_{\mu}$ to $\bar{\nu}_{e}$ with a Δm_{ν}^{2} of order 1 eV.

While these extraordinary advances in experimental neutrino physics were occurring, a concurrent revolution in experimental cosmology took place. Ushered in by the Boomerang, MAXIMA, and DASI measurements of the acoustic peaks in the Cosmic Microwave Background (CMBR) [\[7\]](#page-8-6), an era has begun wherein it is possible to make measurements of cosmological parameters with previously unimaginable precision. Most recently, the striking data [\[8\]](#page-8-7) from the Wilkinson Microwave Anisotropy Probe (WMAP) have vastly improved our knowledge of several fundamental cosmological parameters [\[9\]](#page-8-8). Because cosmology would be significantly affected by the presence of light species with masses of order 1 eV, the new WMAP data strongly constrain neutrino masses in this range. We will show this brings cosmology into sharp conflict with the LSND result in two ways.

First, WMAP determines the baryon to photon ratio very precisely. This removes an important source of uncertainty in the prediction of Big-Bang Nucleosynthesis (BBN) for the primordial abundance of 4 He. This allows for a strong limit to be placed on the number of relativistic species present at BBN. Secondly, WMAP, when combined with data from the 2 degree Field Galactic Redshift Survey (2dFGRS) [\[10\]](#page-8-9), CBI [\[11\]](#page-8-10), and ACBAR [\[12\]](#page-8-11), is able to place stringent limits on the amount that neutrinos contribute to the critical density of the universe. This results in a upper mass limit on neutrinos. These two constraints contradict the LSND result in the entire mixing region not ruled out by other experiments. The second constraint also impinges on the recent evidence, [\[13\]](#page-8-12), for neutrinoless double beta decay

from the Heidelberg-Moscow experiment.

2 The LSND Result

The LSND experiment used decays of stopped anti-muons at the LAMPF facility (Los Alamos) to look for the appearance of anti-electron-neutrinos. They reported the oscillation probability $P(\bar{\nu}_{\mu} \to \bar{\nu}_{e}) = (0.264 \pm 0.067 \pm 0.067)$ 0.045% , representing a 3.3σ signal.

If the result at the LSND experiment were a true indication of oscillations, it would have profound implications for our understanding of neutrinos. Solar and atmospheric neutrinos have already determined two neutrino mass-squared differences to be $\Delta m_{solar}^2 \sim 10^{-5} \text{ eV}^2$ and $\Delta m_{atm}^2 \sim 10^{-3} \text{ eV}^2$. However, taking into account the Bugey exclusion region [\[14\]](#page-9-0), the LSND experiment points to a mass difference (see Figure 1) $\Delta m_{LSND}^2 > 10^{-1} \text{ eV}^2$. The presence of this completely disparate mass difference necessitates the introduction of a fourth neutrino $¹$ $¹$ $¹$. Because LEP has determined the number</sup> of active neutrino species to be three, this fourth neutrino must be sterile, having extraordinarily feeble couplings to the other particles of the standard model.

The introduction of this fourth neutrino species results in principle in two characteristic types of spectra, $2 + 2$ and $3 + 1$. Two sample spectra of these types are shown in Figure 2.

However, recent results from SNO [\[2\]](#page-8-1) and Super-Kamiokande [\[17\]](#page-9-1), have indicated that the oscillations responsible for the atmospheric and solar neutrino anomaly involve transitions primarily between active neutrinos. This means that it is difficult to put the sterile part of the neutrino in either the solar or atmospheric pair in the $2 + 2$ spectrum. A recent quantitative analysis [\[18\]](#page-9-2) found this $2 + 2$ spectrum to be completely ruled out, while a 3 + 1 spectrum was allowed at the 99% confidence level. The tension for the $3 + 1$ spectrum is in large part due to the lack of a signal in shortbaseline disappearance experiments such as CDHSW[\[19\]](#page-9-3) and Bugey. Adding additional sterile neutrinos can only marginally improve this agreement. In the next two sections, we show how this allowed widow is contradicted by cosmological considerations.

¹This statement assumes CPT. If neutrinos and anti-neutrinos have different mass spectra, it may still be possible to accommodate LSND together with solar, reactor, and atmospheric neutrino data within three generations alone [\[15\]](#page-9-4).

3 Big-Bang Nucleosynthesis

By measuring the primordial abundance of ⁴He, one can place bounds on extra relativistic degrees of freedom at the time of Big Bang Nucleosynthesis (BBN) [\[20\]](#page-9-5). These bounds are usually quoted in terms of a number of effective allowed neutrino species, N_{ν}^{eff} . Additional degrees of freedom tend to increase the expansion rate of the universe, which causes neutrons to freeze out at an earlier time, at a higher abundance. This abundance translates into more primordial ⁴He for a given baryon to photon ratio, η . Therefore, knowledge of primordial ⁴He abundance along with a separate determination of η places a bound on N_{ν}^{eff} . On the other hand, for a fixed N_{ν}^{eff} , a higher η results in a higher abundance for primordial ⁴He; so, incomplete knowledge of η degrades the constraint on N_{ν}^{eff} .

In the era before precise CMBR measurements, BBN data alone were utilized to set the bound. Measurements of primordial deuterium or lithium were used for the separate determination of η . An aggressive analysis by [\[21\]](#page-9-6) cited a limit of $N_{\nu}^{\text{eff}} < 3.4$ at 2σ , and consequently found that LSND data were strongly disfavored by BBN [\[22\]](#page-9-7). However, the data for primordial light element abundances were somewhat muddled, with some measurements of lithium and deuterium preferring substantially lower values of η than others. Due to the presence of these data, a conservative bound $N_{\nu} < 4$ was often taken [\[23\]](#page-9-8). In fact, using lithium data alone, [\[24\]](#page-9-9), found that even $N_{\nu}^{eff} = 4.9$ was acceptable at the 95% confidence level.

However, after precise measurements of the CMBR, the situation has changed. The WMAP experiment has determined [\[9\]](#page-8-8) $\Omega_b h^2 = 0.224 \pm 0.001$, corresponding to an $\eta = 6.5^{+0.4}_{-0.3} \times 10^{-10}$. For the central value above, the expected ⁴He abundance, Y_p , is roughly $Y_p = 0.249 + 0.013(N_{\nu}^{eff} - 3)$. The status of primordial Helium measurements remains controversial. One helium measurement quotes a value $Y_p = 0.244 \pm 0.002$ [\[26\]](#page-9-10), while another quotes $Y_p = 0.235 \pm 0.002$ [\[25\]](#page-9-11). To deal with the discrepancy in these measurements, the Particle Data Group (PDG) assigns an additional systematic error, taking $Y_p = 0.238 \pm 0.002 \pm 0.005[27]$ $Y_p = 0.238 \pm 0.002 \pm 0.005[27]$. To be completely conservative, we will take the higher helium abundance, and assign to it the additional systematic error of the PDG, namely, we take $Y_p = 0.244 \pm 0.002 \pm 0.005$. Using the formulae of [\[28\]](#page-9-13) for ⁴He in terms of N_{ν}^{eff} and η , we find $N_{\nu}^{\text{eff}} < 3.4$ at the 95% (two-sided) confidence level, leaving no room for the extra neutrino of LSND. Using the only slightly less conservative approach of adopting the PDG central value and error, we find N_{ν}^{eff} < 3.0 at the 95% (two-sided) confidence level.

Of course, additional systematic errors in the helium abundance measurements may be found. The fact that 3 neutrinos is barely consistent at the 95% confidence level might cause some suspicion that there are unknown systematics at work. However, to get $N_{\nu}^{\text{eff}} = 4$ at the 95% level would require inflating the errors on the PDG central value dramatically, to $Y_p = 0.238 \pm 0.011.$

It is possible that an asymmetry in the electron neutrino sea could bias the rate for the processes such as $n+e^+ \rightarrow p+\bar{\nu}_e$, which could affect the value of Y_p . This would require a value of $\xi_{\nu_e} \equiv \mu_{\nu}/T = -0.05$, where μ_{ν} is the chemical potential of the degenerate neutrinos [\[29\]](#page-9-14). One might eventually hope to measure such a leptonic asymmetry in the CMBR using a satellite such as Planck [\[30\]](#page-10-0), though this may be somewhat optimistic. Current (pre-WMAP) limits yield $\xi_{\nu_e} = 0.09_{-.09}^{+0.15} {}^{(1\sigma)}_{(0.5\sigma)}$ [\[31\]](#page-10-1). CMBR constraints cannot exclude the possibility of a lepton asymmetry which could make LSND consistent with BBN. However, we will see in the next section that this scenario is disfavored due to large scale structure considerations.

Finally, one might hope to diminish the energy density contained in the sterile neutrinos using oscillations induced by matter effects in the early universe, as suggested in [\[32\]](#page-10-2). However, this mechanism requires an active neutrino to have a mass of greater than 1 eV, which is disfavored from thebounds we discuss in the next section.

4 Weighing Neutrinos with Large Scale Structure

WMAP has provided an additional constraint on LSND. As noted, for example, in [\[33\]](#page-10-3), Galactic Surveys provide a powerful tool to constrain the masses of neutrinos. Neutrinos decouple at temperatures well above those at which structure forms. They then free-stream until they become non-relativistic. This tends to smooth out structure on the smallest scales. On scales within the horizon when the neutrinos were still relativistic, the power spectrum of density fluctuations is suppressed as [\[33\]](#page-10-3):

$$
\frac{\Delta P_m}{P_m} \approx -8 \frac{\Omega_\nu}{\Omega_m} \tag{1}
$$

The 2dFGRS experiment used this fact to place a limit on the sum of neutrino masses: $\Sigma m_{\nu} < 1.8$ eV [\[10\]](#page-8-9).

Recent data from WMAP greatly improve this measurement. A key contribution is the fact that WMAP and 2dFGRS overlap in the wavenumbers probed. This allows a normalization of the 2dFGRS power spectrum from the WMAP data. The WMAP satellite also precisely determines Ω_m . Since depletion of power at small scales is sensitive to the ratio of Ω_{ν}/Ω_{m} , a more accurate determination of Ω_m leads to a better bound on the neutrino mass. The ultimate result from combining data from 2dFGRS, ACBAR, CBI, and WMAP is $\Omega_{\nu}h^2 < 0.0076$ (95 % confidence level) [\[9\]](#page-8-8). Translating this to a single active neutrino species^{[2](#page-5-0)}

$$
m_{\nu} < 0.69 \text{ eV} \ (95\% \text{ confidence level}).\tag{2}
$$

The above mass limit was placed assuming that the heavy neutrino has standard model couplings. These couplings determine when the neutrino decouples from thermal equilibrium. If the neutrino decoupled sufficiently early, it might have been substantially diluted relative to the active neutrinos. Consequently, it could contribute a relatively small amount to the critical density today. However, we do not expect this to be the case for an LSND neutrino. To see this, note that the time of decoupling of a particle is determined by balancing the rate of its interactions, Γ , against the expansion rate of the universe, H. When $\Gamma \sim H$, the particle decouples [\[34\]](#page-10-4). For ordinary neutrinos, this occurs when $G_F^2 T^5 \approx T^2 M_{pl}^{-1}$, at a temperature of 1 MeV. A sterile neutrino does not couple directly to the thermal bath. However, it can be produced through oscillations, when active neutrinos are produced, and then mix to the sterile state.

This results in an small effective coupling for the sterile neutrinos. The suppressed couplings imply an earlier decoupling. If the active fraction of the neutrino is ϵ , then the neutrino decouples when $G_F^2 T^5 \epsilon^2 \approx T^2 / M_{pl}$. However, the allowed region (see Figure 1) for LSND shows a minimum mixing angle, $\epsilon^2 \sim \sin^2 2\theta \gtrsim 10^{-3}$. This leads to a decoupling temperature of less than 10 MeV.

This temperature is less than the QCD phase transition, so that the abundance of these neutrinos will not be diluted by the entropy produced at this transition. This assures us that the limit of Eq. [\(2\)](#page-5-1) is applicable for the heavy LSND neutrino as well. Furthermore, it has been noted in [\[22\]](#page-9-7) and references therein, that MSW-type effects can serve to populate the sterile neutrino states, even after they would otherwise have gone out of equilibrium.

²This is the appropriate bound for the $3+1$ spectrum shown in Figure 2, taking a $2+2$ spectrum (or a $1 + 3$ spectrum), would only strengthen our argument.

For the analysis above to be credible, it is important to note that the time scale for oscillations corresponding to the LSND mass scale is very short. For instance, for $E_{\nu} = 1$ MeV and $\Delta m^2 = 1$ eV², the time scale is roughly one nanosecond. Therefore, we expect the sterile states to be created very quickly. Once created, they lose coherence because the active components constantly interact with the thermal bath. Once coherence is lost, the sterile neutrinos are in thermal equilibrium with the thermal bath, and the above analysis applies.

Fitting the LSND result within a two neutrino oscillation picture requires (see Figure 1) a neutrino mass greater than the square-root of smallest allowed Δm^2 . This gives $m_\nu \gtrsim 0.45$ eV. Comparing this with Eq. [\(2\)](#page-5-1), one sees that the minimum LSND result is significantly squeezed by the large scale structure measurement alone. Taking into account a full $3+1$ neutrino oscillation analysis, fully incorporating data from CDHSW and Bugey, we are forced into the small angle portion of the LSND allowed region. This means higher masses. Even at the 99% confidence level, the allowed region only includes neutrinos with masses [\[18\]](#page-9-2):

$$
m_{\nu} > 0.9 \text{ eV}.\tag{3}
$$

This conflicts with Eq. [\(2\)](#page-5-1), closing the allowed window on LSND using large scale structure measurements.

It is interesting to note that the WMAP experiment also detected a relatively early re-ionization period, $z_{reionize} \sim 20$. This implies an early generation of stars responsible for the energy of re-ionization during this period. Early star formation disfavors warm dark matter, consistent with the above statements that neutrinos make up a small fraction of the critical density.

5 Neutrinoless Double Beta Decay

The limit on the neutrino mass from the combination of WMAP and 2dF-GRS data is also interesting in the context of the neutrinoless double beta decay. The Heidelberg–Moscow experiment claimed a signal of neutrinoless double beta decay [\[13\]](#page-8-12), which would indicate that neutrinos have Majorana masses. The relevant neutrino mass for the signal is the so-called effective neutrino mass $\langle m_{\nu}\rangle_{ee} = |\sum_i m_{\nu_i} U_{ei}^2|$. The nuclear matrix elements in [\[13\]](#page-8-12) lead to the preferred range $\langle m_{\nu} \rangle_{ee} = (0.11{\text -}0.56)$ eV, while the reanalysis in [\[35\]](#page-10-5) gives 0.4–1.3 eV using a different set of nuclear matrix elements. This result does not require the presence of an additional (sterile) neutrino species, so the BBN limits need not apply. However, this high value of $\langle m_{\nu} \rangle_{ee}$ together with solar, reactor, and atmospheric neutrino data on mass splittings, require the three neutrinos to be nearly degenerate, and the WMAP+2dFGRS data would therefore require $m_{\nu_i} < 0.23$ eV. This large scale structure limit excludes the deduced range of the effective neutrino mass in [\[35\]](#page-10-5) completely. However, using the largest values of the nuclear matrix element in [\[13\]](#page-8-12), a small window is still allowed. Therefore the WMAP+2dFGRS limit severely constrains the claimed evidence for the neutrinoless double beta decay as well.

6 Conclusions

Recent precise cosmological measurements have given strong indications against the presence of an additional sterile neutrino in the range that would explain the LSND result. Bounds from BBN disfavor the presence of any additional neutrinos that do not decouple before the QCD phase transition. Large Scale Structure disfavors the presence of neutrinos with mass in the eV range. It seems as though that the only way to reconcile LSND with the cosmological data is to have CPT violation. In this case, the BBN constraint disappears, because no new light species are introduced. In addition, the large scale structure constraint is ameliorated, as only an anti-neutrino would need to be heavy, but not its CPT neutrino partner. The neutrino mixing result of LSND will be tested directly at the MiniBoone Experiment at Fermilab [\[36\]](#page-10-6).

We also note that the cosmological data do not prefer the neutrinoless double beta decay in the mass range claimed by Heidelberg–Moscow experiment, unless the nuclear matrix element is very large.

7 Acknowledgments

A. Pierce thanks L. Dixon and M. Peskin for useful and stimulating discussions that led to the idea for this work, as well as for their insightful comments on the draft. The work of A. Pierce was supported by the U.S. Department of Energy under Contract DE-AC03-76SF00515. The work of H. Murayama was supported in part by the DOE Contract DE-AC03-76SF00098 and in part by the NSF grant PHY-0098840.

References

- [1] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81**, 1562 (1998) [arXiv:hep-ex/9807003].
- [2] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **89**, 011301 (2002) [arXiv:nucl-ex/0204008].
- [3] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003) [arXiv:hep-ex/0212021].
- [4] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mi kheev and A. Y. Smi rnov, Sov. J. Nucl. Phys. **42**, 913 (1985) [Yad. Fiz. **42**, 1441 (1985)].
- [5] C. Athanassopoulos *et al.* [LSND Collaboration], Phys. Rev. Lett. **77**, 3082 (1996) [arXiv:nucl-ex/9605003].
- [6] A. Aguilar *et al.* [LSND Collaboration], Phys. Rev. D **64**, 112007 (2001) [arXiv:hep-ex/0104049].
- [7] P. de Bernardis *et al.* [Boomerang Collaboration], Nature **404**, 955 (2000) [arXiv:astro-ph/0004404]; S. Hanany *et al.*, Astrophys. J. **545**, L5 (2000) [arXiv:astro-ph/0005123]; N. W. Halverson *et al.*, Astrophys. J. **568**, 38 (2002) [arXiv:astroph/0104489].
- [8] C. L. Bennett *et al.*, arXiv:astro-ph/0302207.
- [9] D. N. Spergel *et al.*, arXiv:astro-ph/0302209.
- [10] O. Elgaroy *et al.*, Phys. Rev. Lett. **89**, 061301 (2002) [arXiv:astroph/0204152].
- [11] T. J. Pearson *et al.*, arXiv:astro-ph/0205388.
- [12] C. l. Kuo *et al.* [ACBAR collaboration], arXiv:astro-ph/0212289.
- [13] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney and I. V. Krivosheina, Mod. Phys. Lett. A **16**, 2409 (2001) [arXiv:hepph/0201231].
- [14] Y. Declais *et al.*, Nucl. Phys. B **434**, 503 (1995).
- [15] H. Murayama and T. Yanagida, Phys. Lett. B **520**, 263 (2001) [arXiv:hep-ph/0010178]; G. Barenboim, L. Borissov and J. Lykken, arXiv:hep-ph/0212116.
- [16] B. Armbruster *et al.* [KARMEN Collaboration], Phys. Rev. D **65**, 112001 (2002) [arXiv:hep-ex/0203021].
- [17] R. J. Wilkes [K2K Collaboration], arXiv:hep-ex/0212035.
- [18] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. Valle, Nucl. Phys. Proc. Suppl. **114**, 203 (2003) [arXiv:hep-ph/0209368].
- [19] F. Dydak *et al.*, Phys. Lett. B **134**, 281 (1984).
- [20] F. Hoyle and R. J. Tayler, Nature, **203**, 1108 (1964).
- [21] K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. **333**, 389 (2000) $\arXiv:astro-ph/9905320$.
- [22] K. Kainulainen and K. A. Olive, arXiv:hep-ph/0206163.
- [23] E. Lisi, S. Sarkar and F. L. Villante, Phys. Rev. D **59**, 123520 (1999) [arXiv:hep-ph/9901404].
- [24] K. A. Olive and D. Thomas, Astropart. Phys. **11**, 403 (1999) [arXiv:hepph/9811444].
- [25] Y. .I. Izotov, *el al.*, Astrophys. J. **527**, 757 (1999).
- [26] M. Peimbert, A. Peimbert and M. T. Ruiz, arXiv:astro-ph/0003154.
- [27] K. Hagiwara *et al.* [Particle Data Group Collaboration], "Review Of Particle Physics," Phys. Rev. D **66**, 010001 (2002).
- [28] R. E. Lopez and M. S. Turner, Phys. Rev. D **59**, 103502 (1999) [arXiv:astro-ph/9807279].
- [29] J. March-Russell, H. Murayama and A. Riotto, JHEP **9911**, 015 (1999) [arXiv:hep-ph/9908396].
- [30] W. H. Kinney and A. Riotto, Phys. Rev. Lett. **83**, 3366 (1999) [arXiv:hep-ph/9903459].
- [31] M. Orito, T. Kajino, G. J. Mathews and Y. Wang, Phys. Rev. D **65**, 123504 (2002) [arXiv:astro-ph/0203352].
- [32] R. Foot and R. R. Volkas, Phys. Rev. D **55**, 5147 (1997) [arXiv:hepph/9610229]; P. Di Bari, P. Lipari and M. Lusignoli, Int. J. Mod. Phys. A 15 , 2289 (2000) [arXiv:hep-ph/9907548].
- [33] W. Hu, D. J. Eisenstein and M. Tegmark, Phys. Rev. Lett. **80**, 5255 (1998) [arXiv:astro-ph/9712057].
- [34] E. W. Kolb and M. S. Turner, Redwood City, USA: Addison-Wesley (1990) 547 p. (Frontiers in physics, 69).
- [35] P. Vogel, "Limits From Neutrinoless Double-Beta Decay (Rev.)," in Particle Data Group (K. Hagiwara et al.), Phys. Rev. D **66**, 010001 (2002).
- [36] R. Stefanski [MINIBOONE Collaboration], Nucl. Phys. Proc. Suppl. **110**, 420 (2002).

Figure 1: The LSND Allowed region, with Bugey and Karmen [\[16\]](#page-9-15) exclusion regions. The constraints from the global fit [\[18\]](#page-9-2) as well as the limit from the combination of WMAP and 2dFGRS data are also shown. The contours from the global fit should, of course, would continue on to lower values of Δm^2 , but Ref. [\[18\]](#page-9-2) did not show this region.

Figure 2: Sample neutrino spectra in the light of LSND. Different permutations are also possible.