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Stringy Resolutions of Null Singularities

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We study string theory in supersymmetric time-dependent backgrounds. In the framework of general relativity, supersymmetry for spacetimes without flux implies the existence of a covariantly constant null vector, and a relatively simple form of the metric. As a result, the local nature of any such spacetime can be easily understood. We show that we can view any such geometry as a sequence of solutions to lower-dimensional Euclidean gravity. If we choose the lower-dimensional solutions to degenerate at some light-cone time, we obtain null singularities, which may be thought of as generalizations of the parabolic orbifold singularity. We find that in string theory, many such null singularities get repaired by α'-corrections - in particular, by worldsheet instantons. As a consequence, the resulting string theory solutions do not suffer from any instability. Even though the CFT description of these solutions is not always valid, they can still be well understood after taking the effects of light D-branes into account; the breakdown of the worldsheet conformal field theory is purely gauge-theoretic, not involving strong gravitational effects.

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1. Introduction

One of the most interesting features of string theory is its ability to describe certain singular spacetimes, whose description in general relativity inevitably breaks down. We have learned much about the rich subject of static singularities and their resolutions (for a review see *e.g.* [1-4]). Little is known, however, about time-dependent singularities.

Recently, there has been an interesting attempt [5] to understand the null singularity of the parabolic orbifold of Minkowski space [6], where the orbifold group is generated by a parabolic element of the Lorentz group. Even though the parabolic orbifold is a limit of a well-behaved string background [7-9], namely the null-brane [10, 11], in the singular limit the backreaction of (almost) any particle on the geometry becomes large, and perturbation theory breaks down [12,7-9]. (Recently, there has been a lot of interest in time-dependent backgrounds in general [13-44]. For other related work see *e.g.* [45-51].)

The parabolic orbifold can be thought of as a circle fibration over a nine-dimensional Minkowski space, with the fiber shrinking along a null direction to a zero size. In this paper, we will be interested in more general cases, with a smaller (but nonzero) amount of supersymmetry, where more general fibers shrink along a null direction in a slightly more general way. To achieve this in a solution of general relativity or string theory, we will have to consider also more general base spaces – instead of the Minkowski space, the fibrations will be over plane waves.

Some of the solutions constructed in this way will not have a much better behavior in string theory than the parabolic orbifold. We will see, however, that decreasing the amount of supersymmetry has rather dramatic consequences – there are many distinct null singularities of general relativity which are perfectly well-behaved within the framework of string theory! In other words, the ill-behaved singularities are exceptions, rather than generic cases.

The paper is organized as follows. In section 2, we review some basic facts about the parabolic orbifold singularity. In section 3, we discuss the properties of general purely geometric solutions to supergravity which have a covariantly constant spinor (and therefore also a covariantly constant vector). In section 4, we consider a special case of these solutions, namely supersymmetric fibrations over plane waves in general relativity. In section 5, we provide a string theory description of these fibrations. In section 6, we resolve null singularities. In conclusions, we conclude. In appendix A, we setup the coordinate system used in section 3, and we prove some of its important properties. In appendix B, we discuss the stability issues.

2. A Warm-up Example: The Parabolic Orbifold Singularity

Before get to the case of more general null singularities, let us briefly review the properties of one of the most simple null singularities – the parabolic orbifold singularity. Although we do not know how to resolve it in string theory, it will provide a useful intuition for a class of null singularities which do have a non-singular behavior in string theory.

2.1. The Parabolic Orbifold of Minkowski Space

This orbifold can be obtained from a three-dimensional Minkowski space $\mathbf{R}^{1,2}$ (cross $\mathbf{R}^7$ in case we want to consider superstring theory) by modding out by a group isomorphic to $\mathbb{Z}$ and generated by a parabolic element of $SO(1,2)$:

$$g_0 = e^{i\beta J}, \quad J \equiv \frac{1}{\sqrt{2}}\, J^{01} + \frac{1}{\sqrt{2}}\, J^{12}. \tag{2.1}$$

In terms of the coordinates

$$\mathrm{x}^+ = \frac{\mathrm{x}^0 + \mathrm{x}^1}{\sqrt{2}}, \quad \mathrm{x}^- = \frac{\mathrm{x}^0 - \mathrm{x}^1}{\sqrt{2}}, \quad \mathrm{x} = \mathrm{x}^2, \tag{2.2}$$

the generator (2.1) acts as

$$\begin{pmatrix} \mathrm{x}^+ \\ \mathrm{x}^- \\ \mathrm{x} \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} \mathrm{x}^+ \\ \mathrm{x}^- + \beta \mathrm{x} + \frac{1}{2}\beta^2 \mathrm{x}^+ \\ \mathrm{x} + \beta \mathrm{x}^+ \end{pmatrix} \tag{2.3}$$

If we introduce a new set of coordinates

$$\begin{aligned} u &= \mathrm{x}^+ \\ v &= \mathrm{x}^- - \frac{\mathrm{x}^2}{2\mathrm{x}^+} \\ x &= \frac{\mathrm{x}}{\mathrm{x}^+}, \end{aligned} \tag{2.4}$$

the orbifold identifications (2.1) become very simple:

$$(u, v, x) \sim (u, v, x + \beta), \tag{2.5}$$

and the metric can be written in a the following form

$$ds^2 = -2\ dudv + u^2 dx^2. \tag{2.6}$$

Strictly speaking, the definition of v and x in (2.4) is sensible only for non-zero u. The slice of $u = 0$ corresponds to a null singularity, close to which the full orbifold spacetime is not even Hausdorff.

If we interpret the coordinate u as the light-cone time, we can view the region of negative u as a light-cone-time evolution of a shrinking circle. Its circumference is given by βu, and in particular, for $u = 0$ the circle degenerates to a zero size.[1]

This singularity does not seem to be better-behaved in string theory than in general relativity. It has been argued that adding just a single particle (with non-zero p^u) into the orbifold causes so large backreaction that the approximation of small perturbations around the background geometry fails, and in particular, the string perturbation theory is invalid. One can see this effect also directly from the singular behavior of the string scattering amplitudes.

One important feature of the geometry (2.6) is that in a certain sense, it represents an infinite distance in the moduli space of circles, traversed in a finite light-cone time. (This follows from the fact that the zero size circle is infinitely far from any other point in the moduli space of the S^1, whether or not we include the α' corrections of string theory). For this reason it will be interesting to consider more general null singularities, where the circle is replaced by some different internal space whose zero volume limit is not an infinite distance from the rest of the moduli space.

3. General Properties Of Null Geometries With A Constant Spinor

In this section, we will be interested in the properties of general solutions to ten-dimensional supergravity which have at least one conserved Majorana-Weyl spinor, assuming that there are no fluxes and that the dilaton is constant. We will choose the background metric in the 10d 'string' frame to be the same as in the 10d Einstein frame. This is possible because we will keep the dilaton fixed. (The condition of a constant dilaton can be relaxed quite easily. We will not do so in order to keep the discussion relatively simple.)

[1] It is also possible to consider more general null orbifolds which can be interpreted as d-dimensional tori shrinking along u. These orbifolds can be obtained by modding out the Minkowski space by $\mathbb{Z}^d$ generated by exponentials of $J^{0i} + J^{1i}$ for $i = 1..d$, in the rectangular case.

The geometries of this type will necessarily admit a covariantly constant null vector[2], which can be seen as follows. With the simplifying assumptions above, unbroken supersymmetry implies [1,52] that there exists some number of covariantly constant Majorana-Weyl spinors $\eta^{(I)}$, *i.e.* spinors satisfying

$$\Gamma\eta^{(I)} = +\eta^{(I)} \qquad \eta^{(I)*} = C^*\eta^{(I)}, \tag{3.1}$$

where Γ and C are the 10D chirality and charge conjugation matrices, respectively, and the star denotes complex conjugation. Now, we can construct covariantly constant vector fields as linear combinations of $\bar{\eta}^{(I)}\Gamma^\mu\eta^{(J)}$, which will be symmetric in I and J, since the matrices $(C^*\Gamma^0\Gamma^\mu)_{\alpha\beta}$ are symmetric in the spinor indices α, β. The number of independent vector fields obtained in this way will be non-zero[3], since for example for the I-th spinor, $\eta^{\dagger(I)}\Gamma^0\Gamma^\mu\eta^{(I)} = 0$ would imply $\eta^{\dagger(I)}\Gamma^0\Gamma^0\eta^{(I)} = \eta^{\dagger(I)}\eta^{(I)} = 0$, and consequently $\eta^{(I)} = 0$. Furthermore we can show that every vector $l^{\mu(I)} \equiv \bar{\eta}^{(I)}\Gamma^0\Gamma^\mu\eta^{(I)}$ is light-like: the quantity

$$(C^*\Gamma^0\Gamma^\mu)_{\alpha\beta}(C^*\Gamma^0\Gamma_\mu)_{\gamma\delta} + \text{permutations of } \alpha, \beta, \gamma, \delta \tag{3.2}$$

vanishes[4], and contracting this into $\eta^{(I)}_\alpha\eta^{(I)}_\beta\eta^{(I)}_\gamma\eta^{(I)}_\delta$ shows that $l^{\mu(I)}l^{(I)}_\mu = 0$. We will be, of course, most interested in the case when none of the linear combinations of $l^{\mu(I)}$ is time-like. As a result, the covariantly constant null vector l^μ will be unique up to a constant rescaling.

There are several important properties of these spacetimes, which we prove (and refine) in appendix A. First of all, we can define a certain special coordinate u with the property that the slices of constant u are light-like surfaces. Locally in u, *i.e.* for some range $(u_\mathrm{a}, u_\mathrm{b})$, the geometry can be written as a fibration over a null geodesic with affine

[2] This means that they will belong to the family of plane-fronted waves with parallel rays (pp-waves), because pp-waves are *defined* to be spacetimes with a covariantly constant null vector. Note also that if a vector is covariant constant, it is also a Killing vector.

[3] This is a consequence of working in a spacetime of Lorentzian signature. For instance in the case of six-dimensional Euclidean supergravity on a generic Calabi-Yau three-fold, having a covariantly constant spinor does not imply the existence of any Killing vectors.

[4] See for example pg. 246 of [52]

parameter u. Now, if we start at any non-singular fiber, then locally in u, we can always find coordinates v and x^i, $i = 1..8$, such that the metric becomes[5]

$$ds^2_{10} = -2\,du\,dv + h_{ij}(u, x^k)\,dx^i\,dx^j. \tag{3.3}$$

Here, x^i parameterize a space which can be either compact or non-compact. In these coordinates the constant vector l^μ can be written as $l^u = 0,\ l^v = 1,\ l^i = 0$. The metric (3.3) does not depend on v, which is a consequence of l^μ being a Killing vector.

Note that the metric of the parabolic orbifold (2.6) is precisely of the form (3.3), with x^i parameterizing $S^1 \times \mathbf{R}^7$ and $h_{11} = u^2$, $h_{ij} = \delta_{ij}$ for $i, j = 2..8$. (In (2.6), the directions 2..8 were suppressed.)

3.1. Slide-Show Interpretation of the Spacetimes

The metric (3.3) has very simple transformation properties under boosts. If we perform a coordinate change

$$u = u_0 + \Omega(u' - u_0), \quad v = \Omega^{-1} v', \tag{3.4}$$

it becomes

$$ds^2_{10} = -2\,du'\,dv' + h_{ij}(u_0 + \Omega u' - \Omega u_0, x^k)\,dx^i\,dx^j. \tag{3.5}$$

This is just a manifestation of the Doppler effect for gravitational waves. We see that by choosing Ω to be small, we can make the metric arbitrarily slowly varying. In other words, where is no scale associated with the u-dependence of the metric. In the strict limit $\Omega \to 0$, we obtain

$$ds^2_{10} = -2\,du'\,dv' + h_{ij}(u_0, x^k)\,dx^i\,dx^j. \tag{3.6}$$

If the original metric satisfied Einstein's equations, then the frozen metric (3.6) must also satisfy them. The spacetime (3.6) has the form of a direct product of a two-dimensional Minkowski space, and an eight-dimensional space parameterized by x^i. This implies that $h_{ij}(u_0, x^k)$ must be a solution of 8d Euclidean gravity for any fixed u_0. (It is clear that if

[5] If the fibers are topologically non-trivial, we might need to use more patches of coordinates $x^i_{(a)}$, labelled by an index a. In order to keep the notation simple, we will not always explicitly mention this fact. The coordinate v will be, however, always globally well-defined. We will assume that v is non-compact, because for a compact v, there would be closed causal curves through every point.

the original solution (3.3) was supersymmetric, then also these 8d solutions must admit a covariantly constant spinor.)

As a result, the spacetime (3.3) can be interpreted as a series of slices of constant u, where each slice is a direct product of a line, parameterized by v, and a solution to eight-dimensional Euclidean gravity. The precise metric on these slices will vary with the light-cone time u.

It is natural to ask whether any path in the space of (supersymmetric) solutions of 8d Euclidean gravity gives rise to a solution to 10d Einstein's equations in this way. We will see in the following sections (in a slightly less general context) that this is not the case, and that there is one additional condition on such path that has to be satisfied.

4. Supersymmetric Fibrations Over Plane Waves

The null singularities we will be eventually most interested in correspond to some compact $\tilde{d}$-dimensional spaces $\mathcal{M}$ (with metric h_{ab}) collapsing to a zero size at some light-cone time u_s. For simplicity, we will concentrate on the case where (3.3) takes the form

$$ds_{10}^2 = -2\,du\,dv + a^2(u)\,dy^\alpha\,dy^\alpha + h_{ab}(u, x^c)\,dx^a\,dx^b. \tag{4.1}$$

In other words, we will split the x^i coordinates into y^α, $\alpha = 1..(d-2)$, which will be coordinates on a flat $(d-2)$-dimensional plane with a u-dependent scale factor, and x^a, $a = 1...\tilde{d}$, parameterizing the compact space $\mathcal{M}$. Here $d = 10 - \tilde{d}$. The section of the spacetime spanned by u, v, and y^α has the most simple form possible in this context – it is a d-dimensional plane wave. By the argument from the previous section, we see that at any fixed u_0, the compact manifold $\mathcal{M}$ has to be of special holonomy.

4.1. Kaluza-Klein reduction

Now we can perform a Kaluza-Klein decomposition of the metric (4.1) in order to obtain a lower-dimensional description. The spacetime (4.1) can be viewed as an $\mathcal{M}$-fibration over a plane wave. Since the fiber $\mathcal{M}$ is everywhere orthogonal to the base, the KK gauge fields will be zero, and the lower-dimensional metric in the 'string' frame will be simply equal to the metric on the base,

$$ds_{d,s}^2 = -2\,du\,dv + a^2(u)\,dy^\alpha\,dy^\alpha. \tag{4.2}$$

This is a plane wave metric in the usual Rosen coordinates. There will be also some number of d-dimensional effective scalars $\phi^A(u)$, $A = 1...n$, corresponding to the moduli of $\mathcal{M}$. In particular, there will be a certain function ω of the scalars describing the total volume of $\mathcal{M}$, $V = V_0\,\exp(\omega)$.

If we want to express the metric (4.2) in the d-dimensional Einstein frame, we have to perform a Weyl rescaling.

$$ds^2_{d,\mathrm{E}} = \left(\frac{V(u)}{V_0}\right)^{2/(d-2)} \cdot\ ds^2_{d,\mathrm{s}} = \exp\left(\frac{2\omega(u)}{d-2}\right)\ ds^2_{d,\mathrm{s}}. \tag{4.3}$$

When $V(u) < V_0$ the distances look shorter in the Einstein frame than in the 'string' frame. By a simple reparameterization of u and a redefinition of a, we can put the metric (4.3) into the Rosen form even in the Einstein frame

$$ds^2_{d,\mathrm{E}} = -2\,d\tilde{u}\,dv + \tilde{a}^2(\tilde{u})\,dy^\alpha\,dy^\alpha. \tag{4.4}$$

More explicitly

$$\tilde{u} = \int \left(\frac{V(u)}{V_0}\right)^{2/(d-2)} du, \quad \tilde{a}(\tilde{u}) = \left(\frac{V(u)}{V_0}\right)^{1/(d-2)} a(u). \tag{4.5}$$

4.2. Equations of motion

The equations of motion for the ten-dimensional *background* (4.1) are equivalent to the d-dimensional Einstein's equations for the metric (4.3), (4.4)

$$R^{(E)}_{\mu\nu} - \tfrac{1}{2}R^{(E)}g^{(E)}_{\mu\nu} = 8\pi T^{(E)}_{\mu\nu}. \tag{4.6}$$

The energy-momentum tensor is sourced by the minimally coupled scalars ϕ^A, and can be expressed as

$$T^{(E)}_{\mu\nu} = \mathcal{G}_{AB}\,\partial_\mu\phi^A\partial_\nu\phi^B - \tfrac{1}{2}g^{(E)}_{\mu\nu}\,\mathcal{G}_{AB}\,\partial_\sigma\phi^A\partial^\sigma\phi^B, \tag{4.7}$$

where $\mathcal{G}_{AB}(\phi^C)$ is the metric on the moduli space of $\mathcal{M}$. In our case, the scalars ϕ^C depend only on $\tilde{u}$. As a result, the second term in (4.7) vanishes, and the only non-zero component of the energy-momentum tensor is

$$T^{(E)}_{\tilde{u}\tilde{u}} = \mathcal{G}_{AB}(\phi^C)\,\dot{\phi}^A\,\dot{\phi}^B, \tag{4.8}$$

where the dots denote differentiation with respect to $\tilde{u}$. The Einstein tensor on the left-hand side of (4.6) also takes a simple form, the only non-zero component being

$$G^{(E)}_{\tilde{u}\tilde{u}} = R^{(E)}_{\tilde{u}\tilde{u}} = (d-2)\, \frac{\ddot{\tilde{a}}}{\tilde{a}} \ . \tag{4.9}$$

We see that in our case, Einstein's equations for the metric (4.1) reduce just to a single equation,

$$8\pi \mathcal{G}_{AB}(\phi^C)\, \dot{\phi}^A\, \dot{\phi}^B = (d-2)\, \frac{\ddot{\tilde{a}}}{\tilde{a}} \ . \tag{4.10}$$

Given a path $\phi^A(\tilde{u})$ in moduli space, one can always solve locally in $\tilde{u}$ for the scale factor $a(\tilde{u})$.

The equations of motion for the scalars are trivial: Assuming that the scalars depend just on $\tilde{u}$, they will be automatically satisfied.

4.3. Brinkmann coordinates for the plane wave

For completeness, let us mention that there is also another useful set of coordinates for the plane-wave part of (4.1), known as the Brinkmann coordinates [53]. By a redefinition of v and y^α, which is well known in the theory of plane waves [54], we can put the metric (4.1) into the following form

$$ds^2_{10} = -2\, du\, d\mathrm{v} + b(u)\, \mathrm{y}^\alpha \mathrm{y}^\alpha\, du^2 + \, d\mathrm{y}^\alpha\, d\mathrm{y}^\alpha + h_{ab}(u, x^c)\, dx^a\, dx^b. \tag{4.11}$$

Similarly, the d-dimensional metric (4.4) can be written as

$$ds^2_{d,\mathrm{E}} = -2\, d\tilde{u}\, d\mathrm{v} + \tilde{b}(\tilde{u})\, \tilde{\mathrm{y}}^\alpha \tilde{\mathrm{y}}^\alpha\, d\tilde{u}^2 + \, d\tilde{\mathrm{y}}^\alpha\, d\tilde{\mathrm{y}}^\alpha. \tag{4.12}$$

The equation of motion (4.10) then becomes

$$8\pi \mathcal{G}_{AB}(\phi^C)\, \dot{\phi}^A\, \dot{\phi}^B = (d-2)\, \tilde{b}. \tag{4.13}$$

The advantage of the Brinkmann coordinates is that they can cover all of the plane wave without ever degenerating.

4.4. Supersymmetry

Any path in the moduli space of $\mathcal{M}$ leads to a supersymmetric gravitational solution of the type (4.1), (4.11), as long as we satisfy Einstein's equations (4.10), (4.13) by an appropriate choice of $\tilde{a}$ or $\tilde{b}$. The fact that the solution will be supersymmetric can be seen from the d-dimensional description. As we said, in d-dimensions, the metric will be of the plane wave form, and the only other non-zero fields will be the effective scalars ϕ^A. It is known that every plane wave admits a covariantly constant spinor. More explicitly, its existence can be seen as follows.

Let us work in the globally well-behaved Brinkmann coordinates (4.12). If we choose the vielbein to be $e^{(\tilde{u})} = d\tilde{u}$, $e^{(\mathrm{v})} = d\mathrm{v} - \frac{1}{2}\tilde{b}(\tilde{u})\, \tilde{\mathrm{y}}^\alpha \tilde{\mathrm{y}}^\alpha\, d\tilde{u}$, $e^{(\alpha)} = d\tilde{\mathrm{y}}^\alpha$, we can express the covariant derivatives for the spinors as $\nabla_{\tilde{u}} = \partial_{\tilde{u}} - \frac{1}{2}\tilde{b}(\tilde{u})\, \tilde{\mathrm{y}}^\alpha\, \Gamma_{\mathrm{v}}\Gamma_\alpha, \nabla_{\mathrm{v}} = \partial_{\mathrm{v}}, \nabla_\alpha = \partial_\alpha$. Clearly, any spinor ϵ which is constant for this choice of vielbein and which satisfies $\Gamma_{\mathrm{v}}\epsilon = 0$ will be also covariantly constant, $\nabla_\mu \epsilon = 0$.

Now it is not hard to see that the supersymmetry generated by any such spinor is preserved by the background we consider. The supersymmetric variation of the gravitino obviously vanishes because it is proportional to the covariant derivative of the spinor, $\nabla_\mu\epsilon$. Similarly, the variations of spin-$\frac{1}{2}$ fermions vanish, because the only possible non-trivial terms all contain $\Gamma^\mu \partial_\mu \phi^A(\tilde{u})\, \epsilon$, which is zero due to $\Gamma_{\mathrm{v}}\epsilon = 0$.

4.5. Conclusion

Any supersymmetric spacetime of the form (4.1), or (4.11), can be thought of as a path in the moduli space of a special-holonomy manifold $\mathcal{M}$, which is parameterized by u (resp. $\tilde{u}$). Conversely, if we choose any such path, we can always construct a gravitational solution of the form (4.1), or (4.11), because the only non-trivial Einstein equation (4.10), (4.13) can be easily solved by a suitable choice of $\tilde{a}(\tilde{u})$, or $\tilde{b}(\tilde{u})$. Any solution constructed in this way will be supersymmetric.

5. Stringy Description Of Supersymmetric Fibrations Over Plane Waves

To obtain a string theory description of the fibrations over plane waves studied in the previous section, one might try to write down a non-linear sigma model based on the target space metric (4.1), (4.11), and then correct it order by order in α' to obtain a conformal field theory. However, since we are interested in cases in which the size of the fiber becomes of order the string length and where the non-linear sigma model perturbation theory breaks down, we would not get too far in this way.

Let us first discuss string theory counterparts of non-singular solutions to general relativity (4.1), (4.11), postponing the discussion of the singular cases until section 7. To get a handle on the string theory description of the spacetimes (4.1), (4.11), we will use their transformation properties under boosts. As discussed in section 3, by performing a large enough boost, we can make the u-dependence (resp. $\tilde{u}$-dependence) of the solutions arbitrarily slow. Alternatively, we can simply *start* with a slowly varying spacetime. For such slowly varying geometry, the low-energy effective description of the spacetime will be perfectly valid.

This reduces the problem of finding the string theory counterpart of (4.1), (4.11) to the well-studied problem of finding moduli space metrics for ordinary string theory compactifications. The lower-dimensional metric will be given by the same expressions as before (4.4), (4.12). Also the stringy equations of motion will look very similar to (4.10), (4.13), namely

$$8\pi \mathcal{G}^{(str)}_{AB}(\phi^C)\,\dot{\phi}^A\,\dot{\phi}^B = (d-2)\,\frac{\ddot{\tilde{a}}}{\tilde{a}}, \tag{5.1}$$

or

$$8\pi \mathcal{G}^{(str)}_{AB}(\phi^C)\,\dot{\phi}^A\,\dot{\phi}^B = (d-2)\,\tilde{b}, \tag{5.2}$$

with the difference that now we have to use the α'-corrected moduli space metric for string theory compactified on $\mathcal{M}$. (If we wish to work at non-zero string coupling, we should also include the g_s-corrections.)

Since we have boosted the original spacetime to make it very slowly varying, it is clear that if there are any α'-corrections to the equations of motion besides those already included in the moduli space metric $\mathcal{G}^{(str)}_{AB}$, they must be rather small. It is interesting to note that actually any such corrections which are perturbative in α' vanish identically for our solution. This is because perturbative corrections to the equations of motion correspond to some higher-derivative terms added to the Einstein equations (4.6). They have to be generally covariant, and in our case, they have to be made from the lower-dimensional metric and the scalars. Since the metric (4.4) and the scalars ϕ^A depend only on $\tilde{u}$, constructing a non-vanishing tensor with two free indices and more than two derivatives requires contracting at least two $\tilde{u}$-indices. Such a contraction, however, makes any tensor vanish, because the corresponding metric coefficient is zero. We see that the symmetries of the problem, and in particular the absence of any scale associated with the

$\tilde{u}$-dependence of the solution, forbid any further perturbative α'-corrections to (5.1). This argument generalizes that of [46] to the case where the low-energy effective dynamics of string theory are those of gravity coupled to scalar fields.

5.1. CFT description

Even in static cases it is hard to get an explicit lagrangian for the worldsheet CFT when the curvature of the target space becomes of order the string scale. To analyze such compactifications, one has to rely on some less direct methods. It is clear that in general, we will not be able to write down explicitly the CFT action providing a string theory description of the spacetimes (4.1), (4.11) when the fiber becomes small. We can, however, at least write down its general form. Let us denote $\mathcal{L}[\psi^K;\phi^A]$ the worldsheet lagrangian which corresponds to the space $\mathcal{M}$ at some fixed values ϕ^A of its moduli and which functionally depends on some worldsheet fields ψ^K. Now, the worldsheet action describing the spacetimes of interest can be written schematically as

$$S=-\frac{1}{4\pi\alpha'}\int d^2\sigma\,\sqrt{-\gamma}\,\left(-2\,\partial_a u\,\partial^a \mathrm{v}+b(u)\,\mathrm{y}^\alpha \mathrm{y}^\alpha\,\partial_a u\,\partial^a u+\,\partial_a \mathrm{y}^\alpha\,\partial^a \mathrm{y}^\alpha\,+\mathcal{L}[\psi^K;\phi^A(u)]\right), \tag{5.3}$$

where we have ignored all the fermions not contained in $\mathcal{L}$. We should stress that here, ϕ^A are not independent worldsheet fields, but merely some functionals of u related to $b(u)$ by (5.2). If we were powerful enough, this CFT would allow us, in principle, to compute string scattering amplitudes beyond the low-energy field theory approximation.

6. Generalizations To Geometries With Non-Trivial Potentials And Fluxes

Most of what we said about fibrations over plane waves has a straightforward generalizations to the cases where the fiber $\mathcal{M}$ carries some non-trivial potentials and fluxes (with the exception of the worldsheet point of view, since Ramond-Ramond fields or non-trivial dilaton are usually problematic for string perturbation theory in general). All we have to do is to replace the moduli space of metric $\mathcal{G}_{AB}$ on $\mathcal{M}$ by an appropriate moduli space of the desired string theory compactifications. It is not clear to us, however, whether there is also any simple generalization of the statement that in supergravity, any purely geometric solution with null supersymmetry takes locally the form (3.3).

7. Stringy Resolutions Of Null Singularities

We have seen that in general relativity the local structure of spacetimes of the form (4.1), (4.11) can be understood in terms of paths in the moduli space of the compact manifold $\mathcal{M}$. If we choose the path to reach (in finite $\tilde{u}$) the boundary of the classical moduli space where the $\mathcal{M}$ shrinks to a zero size, the spacetime will have a null singularity which can be thought of as a generalization of the parabolic orbifold singularity. At this point, general relativity certainly breaks down. Moreover, it seems that (almost) any particle added to the spacetime would make the singularity spacelike, essentially because a finite amount energy would be focused into an infinitely small region.

It seems that string theory is too weak to change anything substantial in this kind of story. In the case of the parabolic orbifold of Minkowski space, we have seen very well how string theory loses its fight against the null singularity! Unless we choose a slightly different (non-singular) classical solution to begin with, we do not know, at the present time, how to deal with such spacetime.

This is all true, but the reason why this happened is that we were really harsh. We constrained the string theory by such a large amount of supersymmetry that it could not protect itself by using one of its most powerful weapons – the worldsheet instantons! If we decide to reduce the amount of supersymmetry, it will have extremely dramatic consequences.

7.1. General considerations

We will consider null singularities which arise when the fiber $\mathcal{M}$ shrinks to a zero size in finite $\tilde{u}$. We do not have to assume that the ten-dimensional dilaton is necessarily constant, and also, we may allow the space $\mathcal{M}$ to carry non-trivial potentials or fluxes.

In general relativity, the moduli space distance to a configuration of a vanishing volume is always infinite. String theory offers more possibilities (assuming that we have a reasonable definition of the volume even for $\mathcal{M}$ of order the string length):

In some cases, usually with a large amount of supersymmetry, there are no important corrections to the moduli space metric and the distance to the zero volume configuration remains infinite. If we want to cover this infinite distance in a finite light-cone time $\tilde{u}$, there will be no justification for the low-energy approximation we have been using. Moreover, the zero volume limit does not lie inside of the moduli space, but rather on its boundary. It is not clear whether it is makes any sense to ask what should happen once we reach this boundary. We do not know, at present, how to study such singularities in a controllable way.

On the other hand in more generic cases, the zero-volume limit is either just a finite distance in the quantum-corrected moduli space([55], [56]), or it does not exist at all. For the corresponding string theory solutions, the lower-dimensional description we have been using so far is perfectly valid. (At certain points of the moduli space we might be forced to include more fields into the lower-dimensional description.) For this reason, there will be no instability similar to that of the parabolic orbifold which was studied in detail in [9]. After all, the system we are considering is just a string theory compactification moving arbitrarily slowly in its moduli space.

Rather than continuing this general discussion, let us now focus on a more specific context.

7.2. Calabi-Yau three-fold fibrations over plane waves

If we compactify type IIA string theory on a Calabi-Yau three-fold, we obtain in four dimensions an $\mathcal{N} = 2$ effective field theory which contains one gravity multiplet (with no scalars), $h^{1,1}$ vector multiplets (each containing two real scalars, for example the overall volume modulus), and $h^{2,1} + 1$ hypermultiplets (each containing four real scalars, for example the dilaton). The moduli space of the whole theory exactly factorizes into the vector multiplet moduli space and the hypermultiplets moduli space (up to discrete quotients), and for this reason we can consider these two spaces separately.

In particular, we will consider motions only in the vector multiplet moduli space, since it is the vector multiplets which control the Kähler parameters (including the overall volume) of the Calabi-Yau manifolds. The vector multiplet moduli space metric in type IIA receives no g_s-corrections, and in principle, it could be determined from the classical contribution and the contributions of the worldsheet instantons at zero string coupling (there are no perturbative α'-corrections). In practice, it much more convenient to use the mirror map between $\mathcal{M}$ and a different Calabi-Yau manifold $\mathcal{W}$ in type IIB, because in type IIB, the vector multiplet moduli space metric does not receive any α' or g_s-corrections at all.

The overall quantum volume of the Calabi-Yau $\mathcal{M}$ may be defined to be equal to the mass of the D6-brane wrapping $\mathcal{M}$ [57,58]. Of course, for a large Calabi-Yau, this definition coincides with the classical definition of the volume. For a typical Calabi-Yau (or maybe in all cases), there is a finite-distance point in the quantum-corrected moduli space where the D6-branes become massless. This means that we can construct fibrations over plane waves where the fiber $\mathcal{M}$ literally shrinks to a zero size at some $u = u_s$. Because the zero-volume point is a finite distance in the moduli space, we will not lose any control over the solution. The lower dimensional effective description will still be perfectly valid, it will just contain new light degrees of freedom coming from the D6-branes.

7.3. An Example: A Shrinking Quintic

The quintic hypersurface in $\mathbf{CP}^4$ (denoted $P_4(5)$) is given in projective coordinates by the equation

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \tag{7.1}$$

(or by one of its possible deformations by other fifth order monomials). We can choose the metric on $P_4(5)$ to be Ricci-flat, and $P_4(5)$ becomes a Calabi-Yau manifold. The Hodge numbers which give rise to its moduli are $h^{1,1} = 1$ and $h^{2,1} = 101$. This means that in type IIA, the moduli space of vector multiplets (which control the Kähler parameters of the quintic) will have complex dimension one. In other words, there will be just one vector multiplet.

The vector multiplet moduli space and its metric have been completely determined. Schematically, it is depicted in fig. 1. Note that there are three interesting points: the infinite volume limit, the Landau-Ginzburg orbifold point (*i.e.* the Gepner point), and the zero-volume point $\mathcal{P}_0$, where D6-branes wrapping the whole quintic become massless.

The metric is finite everywhere except in the vicinity of $\mathcal{P}_0$, where it has a logarithmic divergence caused by the light D6-branes. Let us see more explicitly whether we can reach the zero-volume point $\mathcal{P}_0$ in a finite light-cone time $\tilde{u}$ without causing large curvatures in four dimensions, *i.e.* with having $\tilde{b}$ in (4.12) bounded by some finite value. The only non-trivial equation of motion is (4.13), or in our case

$$\tilde{b} = 4\pi \mathcal{G}_{AB}(\phi^C)\, \dot{\phi}^A\, \dot{\phi}^B, \quad A, B, C = 1, 2. \tag{7.2}$$

We can choose ϕ^1 to be the 'Kähler form' J of the quintic, and ϕ^2 can be the 'B-field period, B, as in fig. 1. Let us also define $\rho = J - J_0$. If we choose B to be constant along the path in the moduli space, the equation (7.2) becomes

$$\tilde{b} = 4\pi \mathcal{G}_{11}\, \dot{\rho}^2. \tag{7.3}$$

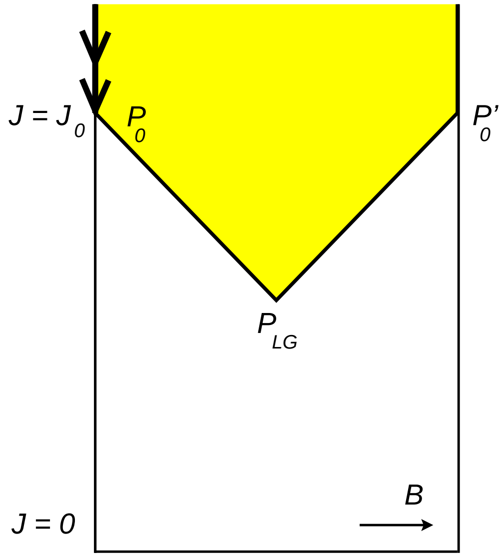

Fig. 1: The shaded region in this figure represents schematically the quantum vector multiplet moduli space of the quintic. The semi-infinite line going upwards from $\mathcal{P}_0$ should be identified with a similar line originating from $\mathcal{P}'_0$. Similarily, $\mathcal{P}_0$-$\mathcal{P}_{LG}$ is to be identified with $\mathcal{P}'_0$-$\mathcal{P}_{LG}$. There are three important points in the moduli space: The zero-volume point $\mathcal{P}_0$, the Landau-Ginzburg orbifold point $\mathcal{P}_{LG}$, and the infinite volume limit $J = \infty$. Going to the zero-volume point, for instance along the path indicated by the arrows, corresponds to a perfectly well-behaved string theory solution. Without α'-corrections, however, we would obtain a null singularity of general relativity if we tried to go to zero-volume.

Near the zero-volume point the metric has a logarithmic behavior, and we may write

$$\tilde{b}\; d\tilde{u}^2 \propto \log \rho \; d\rho^2. \tag{7.4}$$

Because the integral of $(\log \rho)^{1/2}$ from zero to any finite positive ρ is finite, we see that indeed, it is possible to reach the zero-volume point within a finite interval of the light-cone time and with the four-dimensional curvature being small.

8. Conclusions

We have seen that understanding null singularities in string theory does not always pose a much harder problem than understanding static string compactifications. In particular, we have seen that many null singularities have a perfectly non-singular description within the framework of string theory at weak coupling.

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Appendix A. Setting Up The Coordinate System

In the appendix A, we will set up the coordinate system used in section 3 and prove some of its important properties along the way.

A.1. The non-singular case

In the following, we will make two assumptions: (1) We will assume that the spacetime is a connected manifold of Lorentzian signature which admits a covariantly constant null vector. (Which means that it is a pp-wave, a plane-fronted wave with parallel rays.) This is true, in particular, in the case of null-supersymmetric supergravity solutions without flux and with a constant dilaton. (2) We will assume there are no closed causal curves. The only reason we need this assumption is to make sure that the null isometry $\mathcal{I}$ defined below is non-compact. If we wanted to accept also compact null isometries, we could relax this condition.

Let us denote the covariantly constant null vector l^μ. (In principle, there might be more such vectors which would be linearly independent, but we will use only one of them, denoted l^μ, in all of our considerations.) The covariantly constant vector field l^μ may be used to define a scalar field u at any point P in the spacetime as

$$u(P) = \int_{c(P,P_0)} l_\mu \, ds^\mu, \tag{A.1}$$

where $c(P, P_0)$ is a path connecting the point P to some fixed reference point P_0, and ds^μ is a line element along the path. Because l^μ is covariantly constant, the integral does not depend on the particular choice of $c(P, P_0)$, and the scalar u is well-defined.[6] As a result, the spacetime will be foliated by slices $\mathcal{S}_u$ of constant u.

The hypersurfaces $\mathcal{S}_u$ have the following property: Any geodesic which is tangent to $\mathcal{S}_u$ at one point lies entirely inside that particular $\mathcal{S}_u$. This is a simple consequence of the fact that the change of u with the affine parameter λ of the geodesic can be written as

$$\frac{du}{d\lambda} = l_\mu \, \frac{ds^\mu}{d\lambda} \,. \tag{A.2}$$

The scalar product of a covariantly constant vector with a tangent vector of a geodesic does not change under parallel transport along that geodesic. As a result, if $du/d\lambda$ vanishes at one point, it will be zero at any other point of the geodesic.

The scalar u is globally well-defined and we will use it as a coordinate. In addition, we would like to define coordinates v and x^i.

Start at an arbitrary surface $\mathcal{S}_{u_0}$, corresponding to $u = u_0$. The null Killing vector l^μ is, by definition (A.1), tangent to any surface $\mathcal{S}_u$, and in particular, it generates a non-compact continuous isometry $\mathcal{I}$ which takes $\mathcal{S}_{u_0}$ to itself. (It is non-compact because we have assumed that there are no closed causal curves.) Since l^μ is non-trivial and covariantly constant, it is everywhere non-vanishing. This means that the isometry $\mathcal{I}$ acts freely. As a result, there exists a smooth cross-section Σ_{u_0} of $\mathcal{S}_{u_0}$ such that (1) no point on Σ_{u_0} is the image of any other point of Σ_{u_0} by a non-trivial isometry action $\mathcal{I}$, and (2) Σ_{u_0} together with its images by $\mathcal{I}$ covers the whole $\mathcal{S}_{u_0}$. (Clearly, Σ_{u_0} will be homeomorphic to the coset space $\mathcal{S}_{u_0} \,/\, I$.)

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[6] Strictly speaking, this is true only if $\int_\gamma l_\mu \, ds^\mu$ vanishes for all one-cycles γ in the spacetime. This will be the case in all examples we are interested in, and we can simply assume that this requirement is satisfied. Nevertheless, if $\int_\gamma l_\mu \, ds^\mu$ does not vanish for some one-cycle γ, it just means that u is a multi-valued scalar in the spacetime. (An example of such spacetime would be a gravitational wave propagating in the S^1-direction in, say, $\mathbf{R}^{1,8} \times S^1$.) In this case, we can always go to the covering space, where u is single valued. Since all the statements we make in this appendix and in section 3 are only for some restricted range of $u \in (u_\mathrm{a}, u_\mathrm{b})$, it will not be important whether u is single-valued or multi-valued. Roughly speaking, this is because we can choose u_a and u_b such that the one-cycle γ (for which $\int_\gamma l_\mu \, ds^\mu \neq 0$) intersects $u = u_\mathrm{a}$ and $u = u_\mathrm{b}$.

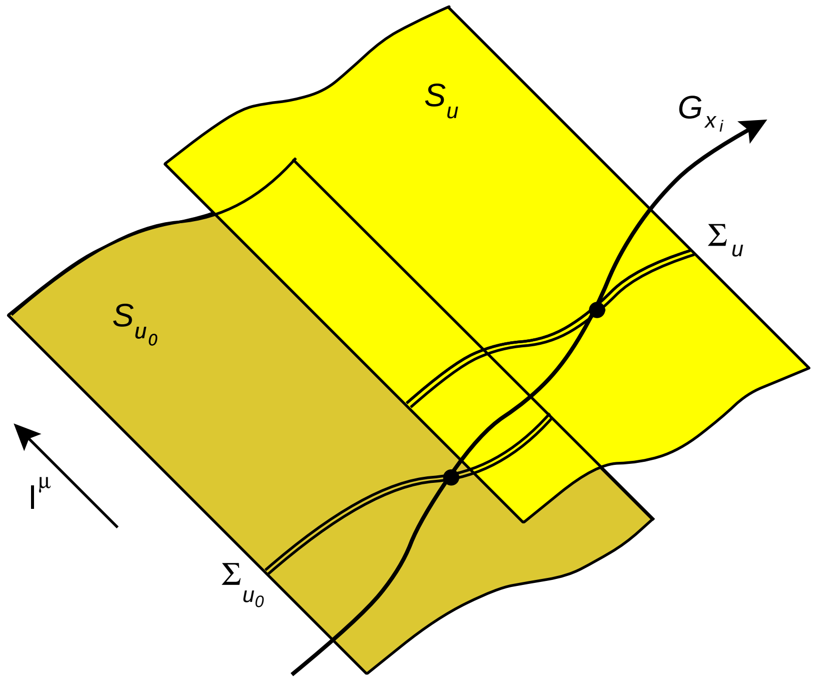

Fig. 2: Setting up a coordinate system is not an easy task if your tools are curved.

We can set up an arbitrary coordinate system on Σ_{u_0} with coordinates x^i.[7] Now, we will try to extend these coordinates also to some cross-sections Σ_u of other hypersurfaces $\mathcal{S}_u$. At every point of Σ_{u_0}, we will find a null direction normal to Σ_{u_0} and independent of l^μ. These directions define a congruence of null geodesics $\mathcal{G}_{x^i}$ which will necessarily intersect all the surfaces $\mathcal{S}_u$. We define the coordinates x^i along any geodesic to be equal to the value of x^i at the intersection of the geodesic with Σ_{u_0}. This definition will be sensible only in some range $(u_\mathrm{a}, u_\mathrm{b})$ of u, $u_\mathrm{a} < u_0 < u_\mathrm{b}$, because the null congruences will almost inevitably have some caustics. The fact that Σ_{u_0} is smooth guaranties that $u_\mathrm{a} < u_0$ and $u_0 < u_\mathrm{b}$. ¿From now on we will restrict our attention to the range $(u_\mathrm{a}, u_\mathrm{b})$.

So far we have defined u globally and x^i on one cross-section Σ_u of every $\mathcal{S}_u$, $u \in (u_\mathrm{a}, u_\mathrm{b})$. Now we will extend the definition of x^i to any spacetime point with $u \in (u_\mathrm{a}, u_\mathrm{b})$ in a simple way manner using the null isometry $\mathcal{I}$ generated by l^μ. At every point of every surface Σ_u, $u \in (u_\mathrm{a}, u_\mathrm{b})$, we construct a null geodesic $\tilde{\mathcal{G}}_{u,x^i}$ in the l^μ-direction, and define

[7] If Σ_{u_0} is topologically non-trivial, we might need to use more patches of coordinates $x^i_{(a)}$, labelled by an index a. We will not express this fact explicitly, in order not to obscure the notation even more.

x^i to be constant along each of these geodesics. We will use the same geodesics also in the following step.

Having specified u and x^i for every point of interest, *i.e.* at every point with $u \in (u_\mathrm{a}, u_\mathrm{b})$, the only other coordinate to be defined is v. We can choose v to be equal to zero at all the cross-sections Σ_u constructed previously. Clearly, the direction in which only v will increase is along the geodesics $\tilde{\mathcal{G}}_{u,x^i}$. Let us calibrate the affine parameter λ_{u,x^i} of each $\tilde{\mathcal{G}}_{u,x^i}$ in such a way that the tangent vector $ds^\mu/d\lambda_{u,x^i}$ equals l^μ at any Σ_u. Now define v along any geodesic $\tilde{\mathcal{G}}_{u,x^i}$ to be equal to the corresponding value of the affine parameter λ_{u,x^i}. This guarantees that the isometries $\mathcal{I}$ generated by l^μ will be realized as constant shifts of v without changing u and x^i. In other words, dv will be a Killing vector.

In the coordinate system we have just constructed, there are important simplifications in the metric. If we write its general form as

$$ds^2 = g_{uu}\, du^2 + g_{vv}\, dv^2 + 2\, g_{uv}\, du\, dv + 2\, g_{ui}\, du\, dx^i + 2\, g_{vi}\, dv\, dx^i + g_{ij}\, dx^i\, dx^j, \qquad \text{(A.3)}$$

we notice the following properties:

- None of the metric coefficients depends on v, because ∂_v is by construction a Killing vector.
- The coefficient g_{uu} vanishes, because ∂_u is a vector tangent to some null geodesic which is an $\mathcal{I}$-translation of one of the null geodesics $\mathcal{G}_{x^i}$.
- The coefficient g_{vv} vanishes, because ∂_v is a vector tangent to some null geodesic $\tilde{\mathcal{G}}_{u,x^i}$.
- The coefficient g_{uv} is equal to -1, by the definition of u (A.1) and the definition of v.
- The coefficient g_{vi} vanishes, because any vector (in our case ∂_{x^i}) tangent to a surface of constant u is perpendicular to $\partial_v \sim l^\mu$.
- The last statement will show now is that g_{ui} vanishes. The geodesic equation for $\mathcal{G}_{x^i}$ can be written as

$$g_{\mu\nu}\frac{d^2 s^\nu}{d\lambda^2} + g_{\mu\nu}\Gamma^\nu_{\rho\sigma}\frac{ds^\rho}{d\lambda}\frac{ds^\sigma}{d\lambda} = 0. \qquad \text{(A.4)}$$

The only coordinate that varies along $\mathcal{G}_{x^i}$ is u, so the equation becomes

$$g_{\mu u}\frac{d^2 u}{d\lambda^2} + g_{\mu\nu}\Gamma^\nu_{uu}\left(\frac{du}{d\lambda}\right)^2 = 0. \qquad \text{(A.5)}$$

Because g_{uu} vanishes, the Christoffel symbol simplifies, and we get

$$g_{\mu u}\frac{d^2u}{d\lambda^2} + g_{\mu u,u}\left(\frac{du}{d\lambda}\right)^2 = 0. \tag{A.6}$$

To determine $d^2u/d\lambda^2$, we can set $\mu = v$ in (A.6),

$$g_{vu}\frac{d^2u}{d\lambda^2} + g_{vu,u}\left(\frac{du}{d\lambda}\right)^2 = 0. \tag{A.7}$$

Since g_{vu} is a constant, $d^2u/d\lambda^2 = 0$ must vanish. This means that u is a good affine parameter for the geodesic $\mathcal{G}_{x^i}$, as could have been anticipated. Returning back to (A.6), we see that

$$g_{\mu u,u} = 0 \tag{A.8}$$

for any μ. In particular, we can take $\mu = i$.

The geodesic $\mathcal{G}_{x^i}$ was constructed in such a way that at $u = u_0$ it is perpendicular to Σ_{u_0}, which means that g_{iu} vanishes at Σ_{u_0}. Equation (A.8) then implies that g_{iu} vanishes at any Σ_u. It is now trivial to extend this result to the whole spacetime between $u = u_\mathrm{a}$ and $u = u_\mathrm{b}$, since the coordinates x^i have been defined by $\mathcal{I}$-translations of the coordinates x^i at various Σ_u.

We have just shown that in our coordinate system, the metric in the region between $u = u_\mathrm{a}$ and $u = u_\mathrm{b}$ takes the form

$$ds^2 = -2\,du\,dv + h_{ij}(u, x^k)\,dx^i\,dx^j, \tag{A.9}$$

which was the goal of this section.

A.2. The singular cases

Even if the spacetime is singular, having a covariantly constant vector implies that we can define u in the same way as in the previous section. This means that the spacetime will still be foliated by surfaces $\mathcal{S}_u$ of constant u. Now there are two possibilities.

(1) If there exists a family of non-singular $\mathcal{S}_u$ which degenerates at some u_s, then we can apply the results of the previous section to the non-singular region, and generally, we can study the properties of this singularity by looking at the path in the space of solutions of 8d Euclidean gravity in the spirit of section 4. There are however two pathological cases which cannot be understood in this way. One of them is an orbifold singularity which

corresponds to a $\mathbb{Z}_2$ action reflecting the u-direction, and which introduces closed causal curves. The other one is the case where there are infinitely many conjugate points near the singularity, which implies that in no open interval (u_0, u_s) touching the singularity we can use one set of coordinates leading to (A.9) everywhere. Heuristically, this corresponds to a gravitational wave with an unbounded frequency.

(2) Even if every $\mathcal{S}_u$ is singular, there should be some family of S_u where the singular loci are at least codimension one in S_u, since otherwise we would not even know how to define the spacetime. If is quite possible that there is a suitable generalization of the arguments from the previous section which can be applied to this case as well. This would go, however, beyond the scope of the present paper.

Appendix B. Stability

From a certain point of view, the stringy resolutions of null singularities discussed in section 7 are just plane waves with some number of scalars varying in the same light-like directions. For this reason, it is obvious that they are stable (for a recent discussion of the stability of plane waves see [37,38].)

It is, however, quite interesting to see intuitively why effects considered in a great detail in [9] do not pose a problem here. In the case of shrinking Calabi-Yau manifolds, we cannot use any arguments based on ten-dimensional supergravity when the size of the Calabi-Yau becomes of order the string length. However, we can still ask whether there is any instability related to the evolution of the spacetime before the Calabi-Yau shrinks to a string size.

The closest simple analog of such spacetimes would be an S^1-fibration over a nine-dimensional plane wave, which for $u \equiv \mathrm{x}^+$ smaller than some x_c^+ looks exactly as the parabolic orbifold, but where the after x_c^+ the circle stopes shrinking and expands again. Such spacetimes have been constructed in [8].

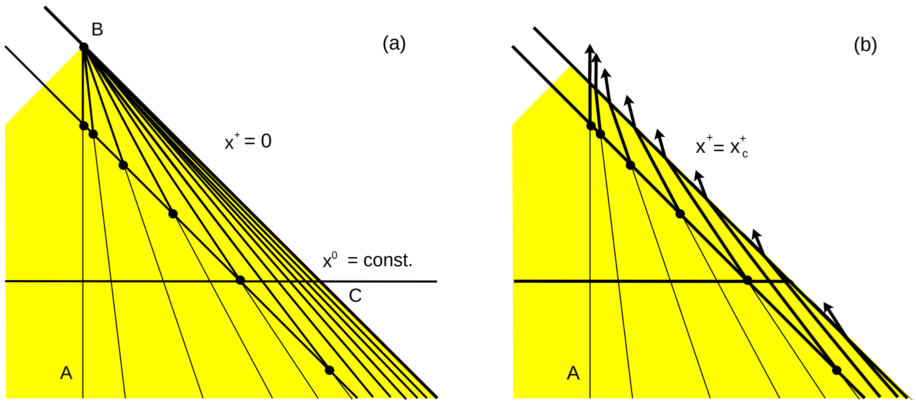

Fig. 3: (a) A schematic picture of the parabolic orbifold, showing only coordinates x^+ and x^-. Close to the singularity, images of any particle become infinitely dense. (b) If one cuts off the spacetime at some finite $x_c^+ < 0$ and replaces it with a plane wave where the circle expands again, the images never come too close to each other, and the resulting spacetime is stable. The part of the geometry with $\mathrm{x}^+ > \mathrm{x}_c^+$, not shown in this figure, has a non-zero curvature.

To see intuitively why these spacetimes are stable but the parabolic orbifold is not, consider the situation in fig. 3. If there is a massive particle in the parabolic orbifold (a) which starts at some spacetime point A and reaches the singularity at point B, then there will be also infinitely many images of the same particle which are all in the past light cone of B, and which reach the singularity at point B. At any slice of constant time $\mathrm{x}^0 = \mathrm{const.}$, there will be a finite density of images everywhere except close to the singularity at a point denoted C. It is precisely the infinite concentration of images at C which causes a large backreaction, and as a consequence, an instability of the singularity itself.

If one cuts off the singularity (b) at some light-cone-time $x_c^+ < 0$ and replaces it with a plane wave where the circle expands again, as in [8], the image particles never come too close to each other, and the particle density is finite everywhere. This is in accord with the usual intuition that a compactification on a finite-size circle should be stable even when the size of the circle varies with time. A spacetime of this type would be stable even if we replaced the S^1 with, say T^6. There is no need of a large number of non-compact directions, unlike in the case of the null-brane considered in [7-9]. This can be seen by a simple analysis of the Kaluza-Klein modes in this kind of geometry. Kaluza-Klein excitations will always have finite energy and finite energy density, and provided that the string coupling constant is not too large, their scattering can be studied perturbatively. Of course, there will be also scattering processes which produce a finite-size black holes.

This is however the same situation as in flat space, and it cannot be considered to be an instability of the spacetime itself.

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