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A World Average for $B \to X_s\gamma$.

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Abstract

We combined the measurements of several different experiments, taking into account the correlated errors, to find $\mathcal{B}(B \to X_s\gamma) = 3.34 \pm 0.38 \times 10^{-4}$.

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1 Introduction

Table 1 gives all the measurements made of $\mathcal{B}(B \to X_s\gamma)$ made to date. In this note we attempt to construct a world average. We neglect the first CLEO result since this is superceded by the later result. The CLEO,BELLE and *BABAR*measurements explicitly factor out the model dependence which has been computed in each case using the model of Kagan and Neubert. The model dependence occurs because it is only possible to experimentally measure the spectrum above a threshold $E > E_0$. It is then necessary to correct for the missing part of the spectrum which necessarily involves making a theoretical assumption. In the model of Kagan and Neubert the spectrum is parameterized by the b-quark mass and they recommend $m_b = 4.80 \pm 0.15\,\mathrm{GeV}$ with the spectrum extrapolated down to $E_0 = 250\,\mathrm{MeV}$. The CLEO and BaBar experiment use $m_b = 4.80 \pm 0.15\,\mathrm{GeV}$ while BELLE uses $m_b = 4.75 \pm 0.10\,\mathrm{GeV}$. Ideally we would need to "normalize" the three results to a common value of the b-quark mass and error. However for the present we assume that the changes resulting from this normalization are small compared to the present errors and combine the results as stated. BaBar,BELLE and CLEO extrapolate to $E_0 = 250\,\mathrm{MeV}$.

The ALEPH measurement includes the theory error in the systematic. However this error (the Fermi momentum systematic) is quoted in their paper as being 0.028×10^{-4} compared to the statistical and systematic errors of $0.80, 0.72 \times 10^{-4}$ and so is negligible. However the measurement is for $b \to s\gamma$ in $Z \to b\bar{b}$ which is expected to have a different branching ratio than at the $\Upsilon(4S)$ of the other measurements. For comparitive purposes we scale the ALEPH result by 0.934 which is the ratio of the expectations for the $\Upsilon(4S)$ and $Z \to b\bar{b}$ processes quoted in their paper, to get $2.95 \pm 0.75 \pm 0.67 \times 10^{-4}$.

Experiment	Reference	$\mathcal{B}(B \to X_s\gamma) \times 10^{-4}$
CLEO 95	[1]	$2.32 \pm 0.57(stat) \pm 0.35(sys)$
ALEPH 98	[2]	$3.11 \pm 0.80(stat) \pm 0.72(sys)$
BELLE 01	[3]	$3.36 \pm 0.53(stat) \pm 0.42(sys)^{+0.50}_{-0.54}(th)$
CLEO 01	[4]	$3.21 \pm 0.43(stat) \pm 0.27(sys)^{+0.18}_{-0.10}(th)$
BaBar 02a	[5]	$4.3 \pm 0.5(stat) \pm 0.80(sys) \pm 1.3(th)$
BaBar 02a	[6]	$3.88 \pm 0.36(stat) \pm 0.37(sys)^{+0.43}_{-0.23}(th)$

Table 1: Previously-measured values of $\mathcal{B}(B \to X_s\gamma)$.

2 Combined Branching Ratios

2.1 General Formalism for combining errors

We illustrate analytically the technique for combining measurements with correlated systematics by combining two measurements. To extend this to the combination of several measurements we use a simple program that implements this numerically.

Consider that we make two measurements x_1 and x_2. If we combine these two measurements in some way with a function $f(x_1, x_2)$ then the error σ_f on the combined measurement is given by:

$$\sigma_f^2 = D^T V D \tag{1}$$

where V is the covariance matrix

$$V_{ij} = cov(x_i, x_j) =< x_i x_j > - < x_i >< x_j >$$

and

$$D = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix}$$

Each measurement has three types of errors, a statistical error σ, a systematic that is unique to the particular measurement κ, and a systematic that is common to the two measurements S. The measurement of x can be considered to be the sum of three terms $x_i = x_i^\sigma + x_i^\kappa + x_i^S$ each with independent errors σ_i,κ_i and S_i respectively then

$$\begin{aligned} cov(x_1, x_1) &= < x_1 x_1 > - < x_1 >< x_1 >= \sigma_1^2 + \kappa_1^2 + S_1^2 \\ cov(x_1, x_2) &= cov(x_1^S, x_2^S) = S_1 S_2 \end{aligned}$$

where we have used the fact that the individual errors are independenet and so the covariance between the different components vanishes e.g $cov(x_i^\sigma, x_i^\kappa) = 0$ and the common systematic is a completely correlated error. The covariance matrix is then

$$V = \begin{pmatrix} \sigma_1^2 + \kappa_1^2 + S1^2 & S_1 S_2 \\ S_2 S_1 & \sigma_2^2 + \kappa_2^2 + S_2^2 \end{pmatrix}$$

3 Combining using a Weighted Average

The appropriate weighted averaging procedure is derived as follows. Consider the combination of any two measurements x_1, x_2 with error matrix

$$V = \begin{pmatrix} \sigma_1^2 + \kappa_1^2 + S_1^2 & S_1 S_2 \\ S_1 S_2 & \sigma_2^2 + \kappa_2^2 + S_2^2 \end{pmatrix}$$

To find the best estimate for the quantity x we minimise the χ^2 [7].

$$\chi^2 = \vec{x}^T V^{-1} \vec{x} \tag{2}$$

where

$$\vec{x} = (x_1 - x, x_2 - x)$$

Inserting the form for V above (det V =$|V|$).

$$\chi^2 = \frac{1}{|V|} \left[(x_1 - x)^2 (\sigma_2^2 + \kappa_2^2 + s_2^2) + (x_2 - x)^2 (\sigma_1^2 + +\kappa_1^2 + s_1^2) + -2(x_1 - x)(x_2 - x) s_1 s_2 \right]$$

$$\frac{1}{2}|V| \frac{d\chi^2}{dx} = (x_1 - x)(\sigma_2^2 + +\kappa_2^2 s_2^2) + (x_2 - x)^2 (\sigma_1^2 + \kappa_1^2 + s_1^2) - (x_1 - x) s_1 s_2 - (x_2 - x) s_1 s_2 = 0$$

Solving for x

$$x = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$$

where

$$w_1 = \frac{1}{\sigma_1^2 + \kappa_1^2 + S_1^2 - S_1 S_2} \tag{3}$$

$$w_2 = \frac{1}{\sigma_2^2 + \kappa_2^2 + S_2^2 - S_1 S_2}$$

So to combine the measurements we use

$$B_{combined} = \frac{\Sigma_i w_i B_i}{\Sigma_i w_i}$$

where B_i is the branching fraction of the individual mode and w_i is the weight. Note that it is possible to have a negative weight in equation refeqn:weight. This can occur if one of the measurements has a much larger correlated error. The error on the averaged branching ratio is given by equation 1.

This has been computed analytically for the two variable case. In the case of N variables it is easier to compute numerically. The minimization criteria for the χ^2 results in a set of linear equations which can be solved by matrix inversion.

4 Average of $\mathcal{B}(B \to X_s\gamma)$ Measurements

We assume that the statistical and systematic errors are uncorrelated and that the theory error is completely correlated. The two babar measurements have correlated systematic errors, for instance in the photon efficiency, but these correlations are small and can be neglected. The CLEO, BELLE and BaBar theory errors all come from the extropolation of the photon spectrum (or the dual hadronic mass spectrum) into the unmeasured region using the same theoretical model and so the assumption of complete correlation is reasonable. We average the five measurements given in the table below. We symmetrize the errors where they are asymmetric by averaging the high and low values.

Experiment	Index	Reference	$\mathcal{B}(B \to X_s\gamma) \times 10^{-4}$	Weight
ALEPH 98	1	[2]	$2.95 \pm 0.75(stat) \pm 0.67(sys)$	0.99
BELLE 01	2	[3]	$3.36 \pm 0.53(stat) \pm 0.42(sys)^{+0.50}_{-0.54}(th)$	0.93
CLEO 01	3	[4]	$3.21 \pm 0.43(stat) \pm 0.27(sys)^{+0.18}_{-0.10}(th)$	3.28
BaBar 02a	4	[5]	$4.3 \pm 0.5(stat) \pm 0.80(sys) \pm 1.3(th)$	-0.49
BaBar 02a	5	[6]	$3.88 \pm 0.36(stat) \pm 0.37(sys)^{+0.43}_{-0.23}(th)$	2.39

Table 2: Previously-measured values of $\mathcal{B}(B \to X_s\gamma)$.

The covariance matrix with row and column index as in table 2 is given by

$$V = \begin{pmatrix} 1.011 & 0 & 0 & 0 & 0 \\ 0 & 0.7277 & 0.0728 & 0.676 & 0.1716 \\ 0 & 0.0728 & 0.2774 & 0.182 & 0.0462 \\ 0 & 0.676 & 0.182 & 2.58 & 0.429 \\ 0 & 0.1716 & 0.0462 & 0.429 & 0.3754 \end{pmatrix}$$

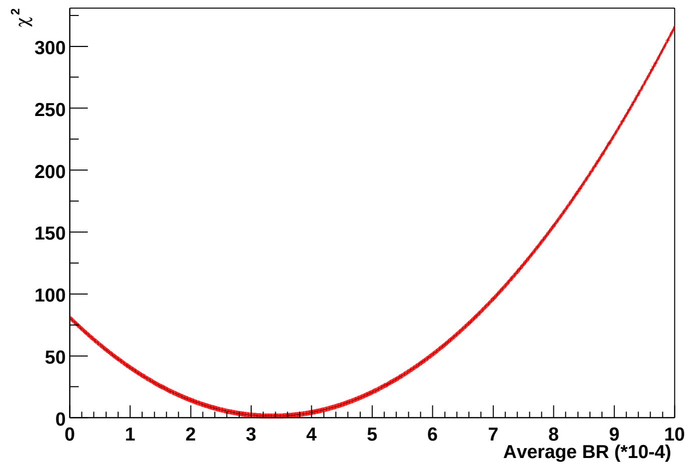

Figure 1: The Mimimal χ^2

Figure 1 shows the behavior of the χ^2 from equation 2 around the minima. The resultant world average is:

$$\mathcal{B}(B \to X_s\gamma) = 3.34 \pm 0.38 \times 10^{-4}$$

References

[1] CLEO Collaboration, M.S. Alam *et al.*, Phys. Rev. Lett. **74**, 2885 (1995).

[2] ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. B **429**, 169 (1998).

[3] BELLE Collaboration, K. Abe *et al.*, Phys. Lett. B **511**, 151 (2001).

[4] CLEO Collaboration, S. Chen *et al.*, Phys. Rev. Lett. **87**, 251807 (2001).

[5] *BABAR*Collaboration, B. Aubert *et al.*, hep-ex/0207074 2002 .

[6] *BABAR*Collaboration, B. Aubert *et al.*, hep-ex/0207076 2002 .

[7] "Probability and Statistics in Particle Physics"
Frodessen, Skjeggestad and Tofte
See Chapter 10 "Least Squares Method" eq 10.20.