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BLACK HOLES, HAWKING RADIATION AND THE INFORMATION PARADOX[1]

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This talk is about results obtained by Kirill Melnikov and myself pertaining to the canonical quantization of a massless scalar field in the presence of a Schwarzschild black hole. After a brief summary of what we did and how we reproduce the familiar Hawking temperature and energy flux, I focus attention on how our discussion differs from other treatments. In particular I show that we can define a system which fakes an equilibrium thermodynamic object whose entropy is given by the $A/4$ (where A is the area of the black hole horizon), but for which the assignment of a classical entropy to the system is incorrect. Finally I briefly discuss a discretized version of the theory which seems to indicate that things work in a surprising way near $r = 0$.

1 Introduction

In this talk I will discuss results obtained by Kirill Melnikov and myself[1] pertaining to the canonical quantization of a massless scalar field theory in the presence of a Schwarzschild black hole. The main difference of this work from earlier work is that we study the future evolution of a well defined state of a quantum field theory defined on a given space-like hypersurface as a function of time and show that we can obtain explicit expressions for Hawking radiation[2] without computing things at null infinity.

In particular, we show that for a large black hole, the usual formula for Hawking radiation is obtained well before one is forced to deal with the evaporation of the hole and within a Hamiltonian framework, which explicitly preserves unitarity. We actually derive a variant of the usual Hawking result: namely, if one starts from any reasonable state and waits long enough, then an Unruh thermometer[3] located at a fixed Schwarzshild r will measure a

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temperature inversely proportional to the mass of the black hole. We also show there is a uniquely defined finite energy flux through any sphere of large but finite Schwarzschild r and any large but finite time, t, and that the size of this flux agrees with the Hawking result.

Since we derive the usual Hawking results for the apparent temperature of a black hole it is fair to ask what we have to say that is new. To our minds the most important difference is that our derivations are done within an explicitly unitary framework and we derive all results for observers located at a finite distance from the black hole and for large but finite times after the initial state begins to evolve. This differs significantly from derivations which compute transitions from past null infinity to future null infinity. Furthermore, this difference allows us to construct a variation of the original problem which shows that an observer could mistakenly conclude he is dealing with an equilibrium system with a Bekenstein entropy[4] well before any question of black hole evaporation or information loss becomes relevant. Needless to say, this leads to a difference in interpretation which I will discuss at the appropriate time.

The physical picture which emerges from our analysis is that, consistent with the general theorem which says that the Scwarzschild metric does not admit any global timelike Killing vector field, our globally defined Hamiltonian is perforce time dependent. It is this time dependence which provides the mechanism which generates the Hawking radiation. Moreover, since this time dependence continues forever, we see that the Hawking radiation is not to be viewed as an equilibrium phenomenon, but rather as the steady state behavior which one might expect in such a system. There is no finite time at which the concepts of equilibrium thermodynamics apply.

2 The Plan

I begin by spending a few moments showing you how the foliation of Schwarzschild space-time is done and then briefly discuss the quantization of the scalar field theory. After that I briefly show how the Heisenberg equations of motion are used to study the time evolution of the system and how these are approximately solved in the Schwarzschild background metric. With these preliminaries out of the way I focus on variants of the problem which address physics issues which come up. First, I address the question of how to choose an initial state. After this I exhibit a variant of the problem which shows how an outside observer can conclude that he is studying an equilibrium system with a well defined temperature and entropy, even though nothing could be further from the truth. Finally, I conclude by discussing the much more interesting question of what is happening near the real singularity at $r = 0$. The ideas in this part of my talk are much more speculative, but the results are thought provoking.

3 Preliminaries

I begin by considering a massless scalar field theory with Lagrangian density

$$\mathcal{L} = \sqrt{-g}\,[g^{\mu\nu}\,\partial_\mu\phi(x)\,\partial_\nu\phi(x)] \tag{1}$$

in the background of a Schwarzschild black hole of mass M. In the usual Schwarzschild coordinates the metric $g_{\mu\nu}$ takes the familiar form

$$ds^2 = -(1-\frac{2M}{r})\,dt^2 + (1-\frac{2M}{r})^{-1}\,dr^2 + r^2 d\Omega^2, \tag{2}$$

where we have set Newton's constant, G, to one. As is well known, the apparent metric singularity at $r = 2M$ is a coordinate artifact and, as such, does not pose a problem. The true issue for canonical quantization is that we need to define a family of spacelike slices which foliate the spacetime in order to define initial data and form the Hamiltonian. Inspection of Eq.(2) shows that surfaces of constant Schwarzschild time change from spacelike to timelike at the horizon ($r = 2M$) and so they do not fulfill our requirements.

In order to exhibit a satisfactory family of spacelike slices it is convenient to introduce dimensionless versions of r and t by rescaling $r \rightarrow 2Mr$ and $t \rightarrow 2Mt$. Using these variables we introduce the Kruskal coordinates X and Y by the equations:

$$XY = (r-1)\,e^r, \qquad \frac{X}{|Y|} = e^{ts}. \tag{3}$$

In these coordinates the Schwarzschild metric takes the form

$$ds^2 = \frac{32\,e^{|-r|}dX\,dY}{r} + r^2 d\Omega^2. \tag{4}$$

Eq.(3) tells us that fixed Schwarzschild r is a hyperbola in the X, Y-plane, as shown in Fig.1, and that a surface of fixed Schwarzschild time corresponds to a straight line $X = |Y|\,e^{ts}$ (such lines are not shown in Fig.1).

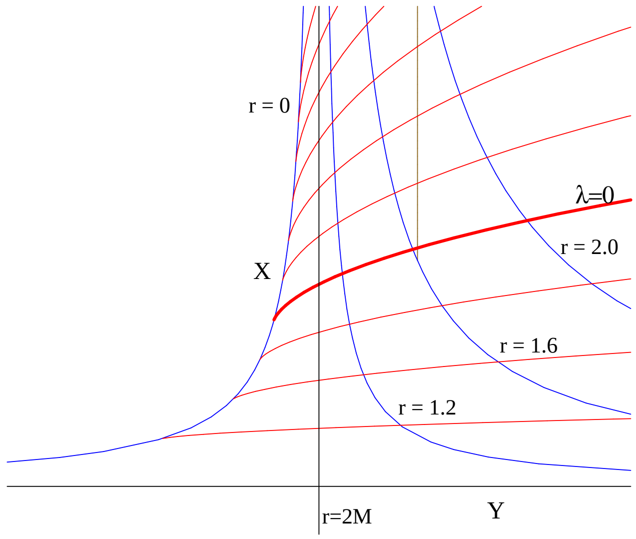

Figure 1: Space-time foliation overlaid on Kruskal coordinates.

Next we introduce Painlevé coordinates, which are derived from Schwarzschild coordinates by making an r-dependent shift in Schwarzschild time, i.e.,

$$t = \lambda - 2\sqrt{r} - \ln\left(\left|\frac{\sqrt{r}-1}{\sqrt{r}+1}\right|\right). \tag{5}$$

The spacelike surfaces we wish to define correspond to surfaces of fixed Painlevé time. They are the almost horizontal curves shown in Fig.1. It is obvious these surfaces are everywhere spacelike since, in Painlevé coordinates, the Schwarzschild metric takes the form

$$ds^2 = -\left(1 - \frac{1}{r}\right) d\lambda^2 + \frac{2\,d\lambda\,dr}{\sqrt{r}} + dr^2 + r^2 d\Omega^2. \tag{6}$$

To carry out the canonical quantization of the scalar field theory it is convenient to make one more change of variables. This leads to Lemaître coordinates, which are related to Painlevé λ and r by

$$r(\lambda, r_{sch}) = \left(r_{sch}^{3/2} - \frac{3}{2}\lambda\right)^{2/3} = \left(\frac{3}{2}(\eta - \lambda)\right)^{2/3}. \tag{7}$$

In Lemaître coordinates the metric takes the form

$$ds^2 = -d\lambda^2 + \frac{1}{r(\lambda,\eta)} d\eta^2 + r(\lambda,\eta)^2 d\Omega^2. \tag{8}$$

The Lemaître form of the metric has the property that it is manifestly free of coordinate singularities at $r = 1$, has no cross terms in $d\lambda$ and dr, and allows a completely straightforward canonical quantization procedure.

4 Canonical Quantization

The Lagrangian for the massless scalar field in Lemaître coordinates is rotationally invariant, so we can study the theory one angular momentum mode at a time. Expanding the field $\phi(\lambda, \eta, \theta, \phi)$ in spherical harmonics in θ and ϕ and restricting attention to the $L = 0$ mode we find the Lemaître coordinate form of the $L = 0$ scalar field Lagrangian to be

$$\mathcal{L} = \sqrt{-g}\,\frac{1}{2}\left[(\partial_\lambda \phi_0\,(\lambda,\eta))^2 - r\,(\partial_\eta \phi_0\,(\lambda,\eta))^2\right] \tag{9}$$

where the determinant $\sqrt{-g}$ is

$$\sqrt{-g} = r^{3/2} = \frac{3}{2}(\eta - \lambda). \tag{10}$$

From this we see that the momentum conjugate to the field is

$$\pi_0(\lambda,\eta) = \frac{3\,(\eta-\lambda)}{2}\,\partial_\lambda \phi_0(\lambda,\eta), \tag{11}$$

and the canonical Hamiltonian is

$$H(\lambda) = \frac{1}{2}\int_\lambda^\infty d\eta\,\left(\frac{2\,\pi_0(\lambda,\eta)^2}{3(\eta-\lambda)} + \frac{3}{2} r\,(\eta-\lambda)(\partial_\eta \phi_0(\lambda,\eta))^2\right). \tag{12}$$

The commutation relations for ϕ_0 and π_0 are

$$[\pi_0(\lambda,\eta), \phi_0(\lambda,\eta')] = -i\,\delta(\eta-\eta'). \tag{13}$$

It follows from Eq.12 and the canonical commutation relations that the Heisenberg equation of motion for the field is

$$\partial_\lambda\,[(\eta-\lambda)\partial_\lambda\phi_0] - \partial_\eta\,[(\eta-\lambda)\,r\,\partial_\eta\phi_0] = 0. \tag{14}$$

Before discussing the solution of the Heisenberg equations of motion I want to emphasize that it is simple to find all of the eigenstates of $H(0)$, because it is just a free field Hamiltonian in disguise. To see this change variables back to Schwarzschild r, using $\eta = (2/3)r^{3/2}$ and rescale the fields by

$$\pi_0(r) = \sqrt{r}\,\pi_1(r), \quad \phi_0(r) = \frac{\phi_1(r)}{r}. \tag{15}$$

This converts Eq.(12) to

$$H(0) = \frac{1}{2}\int_0^\infty dr\,\left(\pi_1(r)^2 + r^2(\partial_r\frac{\phi_1}{r})^2\right), \tag{16}$$

which is the Hamiltonian of the $L = 0$ mode of a free massless field in flat space. To construct the eigenstates of this Hamiltonian simply expand the fields in terms of annihilation and creation operators,

$$\phi_1(r) = \int_0^\infty \frac{d\omega}{\sqrt{\pi\,\omega}}\sin(\omega r)\left(a_\omega^\dagger + a_\omega\right), \quad \pi_1(r) = i\int_0^\infty d\omega\,\sqrt{\frac{\omega}{\pi}}\sin(\omega r)\left(a_\omega^\dagger - a_\omega\right), \tag{17}$$

and define the vacuum state $|0\rangle$ to be the state that is annihilated by all of the a_ω's.

5 Geometric Optics Approximation

To see how the initial state evolves it is best to use the Heisenberg representation and solve for the Heisenberg fields at later times as a function of the fields defined on the initial surface of quantization. To do this we use a geometric optics approximation.

To briefly describe this geometric optics approximation let us study Eq.(14) in Painlevé coordinates (λ, r), since these coordinates are non-singular and the dependence of the solutions on λ and r factorizes. The WKB approach to this problem is to assume a solution of the form $\phi_0 = r^{-1}e^{i\omega\lambda}f_\omega(r)$ and substitute this ansatz into the field equation. In this way one obtains that, for large ω, $f_\omega(r)$ can be written as

$$\ln f_\omega(r) \;=\; i\omega S_{1,2}(r) + \mathcal{O}(\omega^{-1}),\;\; S_{1,2}(r) = \pm r - 2\sqrt{r} \pm \ln((\sqrt{r}\pm 1)^2). \tag{18}$$

We now observe that these solutions are constant along incoming or outgoing null geodesics where an incoming null-geodesic starting at the point x_1 at time $\lambda = 0$ is a curve $r(\lambda)$ such that

$$S_1(x_1) = \lambda + S_1(r(\lambda)), \tag{19}$$

and similarly, an outgoing geodesic starting at x_2 at $\lambda = 0$, is a curve $r(\lambda)$ such that

$$S_2(x_2) = \lambda + S_2(r(\lambda)), \tag{20}$$

where $S_{1,2}$ are as defined in Eq.(18).

It is simple to convert this observation into an ansatz for the solution to the S-wave field equation by imitating the general form of the solution of the same sort of problem in flat space; i.e. we say that for general (λ, r)

$$\phi_0(\lambda, r) = \frac{1}{r}\left(\tilde{\phi}_1(\lambda + S_1(r)) + \tilde{\phi}_2(\lambda + S_2(r))\right), \tag{21}$$

and the functions $\tilde{\phi}_{1,2}(S_{1,2}(r)) = f_{1,2}(r)$ are to be determined from the boundary conditions

$$\phi_0(0, r) = \frac{\phi_1(r)}{r}, \quad \partial_\lambda\, \phi(\lambda, r)|_{\lambda=0} = \sqrt{r}\pi_1(r), \tag{22}$$

where $\phi_1(r)$ and $\pi_1(r)$ are the rescaled operators we introduced to quantize the theory on the initial surface $\lambda = 0$.

Substituting Eq.(21) into Eq.(22) we obtain

$$f_{1,2}(x) = \frac{1}{2}\int_0^x \mathrm{d}\xi \left[\phi_1'(\xi) \pm \pi_1(\xi) \mp \frac{\phi_1(\xi)}{\xi^{3/2}}\right], \tag{23}$$

where $\phi_1' = \mathrm{d}\phi_1/\mathrm{d}\xi$ and $S_{1,2}(x_{1,2}) = \lambda + S_{1,2}(r)$. Given that the field ϕ_1 and its momentum π_1 are expressed through the creation and annihilation operators defined at $\lambda = 0$, we can compute any Green's function of the field ϕ_0 at any later time.

I will refer you to our paper[1] to see how the calculation for the response of the Unruh thermometer and outgoing flux are carried out. The important point of these calculations are that the effect gets its contributions from null geodesics which leave the initial surface of quantization from points just outside, but exponentially close to, the horizon.

6 The Infalling Mirror

Having described the general framework, a question comes to mind; namely, "Is there a problem because our initial state is defined to be a single coherent quantum state both inside and outside the horizon?". The issue is that there is no physical mechanism for preparing such a system, since there is no way degrees of freedom inside the horizon communicate with those outside. To address this issue and show it is a non-problem we considered a variant of the original problem in which we imagine a black hole which, up to some Lemaître time λ, is surrounded by a perfectly reflecting mirror of radius R_0. This is technically implemented by assuming the field vanishes inside and on the spherical surface R_0 at large times in the past. In this way we guarantee that the physical state which we start from when we quantize the theory is completely outside of the horizon.

Next, at some finite time t we assume the mirror starts to fall into the black hole along one of the Lemaître time-lines. We then carry out the computation for an Unruh thermometer, or outgoing flux, at large times in the future and at large distances from the black hole. The result is, of course, unchanged. Details do differ, however. Now the null geodesics which arrive at the measuring apparatus at late times do not originate from a point on the initial surface of quantization at a point exponentially close to the horizon (since there is no field inside the reflecting sphere). Rather, they come from geodesics which begin life as infalling null geodesics which are then reflected from the infalling sphere just before it crosses the horizon. In this way this problem behaves in much the same way as the analysis of a black hole which is assembled by infalling radiation.

This variant of the problem actually establishes two facts: first, that there is no real problem associated with starting from a coherent quantum state defined to be inside and outside the horizon; second, it shows that the point on the initial surface of quantization which corresponds to the place from which the Hawking radiation arises depends upon when one chooses to let go of the mirror. Clearly, in such a situation, modifying the initial state so as to suppress the Hawking radiation is a very unphysical thing to do.

7 The Two Mirror Problem – Bekenstein Entropy

The next item I wish to discuss is a modification of the original problem in which we place one mirror close to the black hole and another at a large distance from the black hole. In this case, so long as both mirrors remain static the theory has a well defined, time-independent Hamiltonian and a unique ground state. So long as both mirrors remain static an observer outside the larger mirror will, by dropping a test charge, be able to measure the mass of the black hole but nothing else. If he sticks an Unruh thermometer through a small hole in the surface he will measure zero temperature.

Now, if we allow the inner mirror to collapse, as in the previous section, then simple modification of the preceding analysis leads to the following results: first, since all Hawking radiation is reflected from the outer mirror, the rate of evaporation of the black hole is proportional to $1/M^5$; second, at times which are long, but short with respect to the time needed for significant evaporation of the black hole, the outside observer who sticks an Unruh thermometer through a small hole in the mirror will measure a temperature proportional to $1/8\pi M$; third, since no radiation leaves the outer sphere the outside observer always measures the same total mass, M. Thus, at times long after the inner mirror collapses the outside observer thinks he is dealing with a body of energy M and temperature $T_H = 1/8\pi M$. Following Bekenstein he would say that he is dealing with an equilibrium thermodynamic system (since he sees nothing changing) for which

$$dU = dM = TdS = dS/8\pi M \tag{24}$$

from which it follows that $S = A_{BH}/4\pi$, where A_{BH} is the area of the horizon of the black hole. However, he would reach this conclusion for a system which is always in a pure quantum state. What is wrong with this analysis?

The answer is clear if we look inside the static mirror. In that case we realize that we are not dealing with an equilibrium system at all. Rather we have a system with a time dependent Hamiltonian and the temperature we see is the result of steady-state and not thermal behavior. Nevertheless, from the point of view of the outside observer there is no way to know this fact.

8 What Is Going On At $r = 0$?

I must begin by emphasizing what should have been clear from the rest of this paper, that none of what I have said addresses issues associated with quantum gravity. Our work focuses on what is happening if one studies field theory in a classical gravitational background. Having said this, there is an issue which we discuss in our paper which overlaps with this question; namely, what is going on near the real singularity at $r = 0$. Since, except for the case of a two-dimensional black hole, the geometric optics approximation doesn't work in this region we took a different approach to analyze the problem; namely, we introduced a lattice in the Lemaître coordinate η. This lattice is peculiar in that it does nothing to regulate the ultraviolet behavior of the field theory at large distances but it does make things better behaved inside the horizon near

$r = 0$. Adopting the point of view that too close to $r = 0$ quantum gravity becomes important we discuss a system for which the singularity is excised and a surface put at some $r = \epsilon$. We find, consistent with what happens in the case of the two-dimensional black hole, that this space-like surface must be included when one integrates to get the Hamiltonian at times later than the time assigned to the surface of quantization. When we do this and solve for the eigenstates of the Hamiltonian at different times we are surprised to find that states which one would have thought to be totally contained on the surface $r = \epsilon$ stretch through the horizon. It would take too much space to discuss this here, but clearly this result raises many questions which deserve further study.

9 Conclusions

To my mind there are three issues raised in this analysis. First, despite what has been said by many, the behavior of a massless quantum field in the background of a Schwarzschild black hole seems to be unitary and no important issues arise, except for the behavior of the theory near $r = 0$ where we expect issues of quantum gravity to significantly modify any semi-classical analysis. Thus, since no obstruction exists to treating this system according to the usual rules of quantum mechanics I do not believe any analysis of the semi-classical system will shed light upon the question of what the true quantum completion of gravity should be. Second, the discussion of the two mirror example raises a serious question of interpretation of the Bekenstein's discussion of black hole entropy for the case of a Schwarzschild black hole. This discussion, which closely parallels Bekenstein's original argument, shows that for the case of the Schwarzschild black hole one can construct a steady-state system which, from the point of view of an external observer, mimics the behavior of a system in thermodynamic equilibrium without being one. Finally, the discussion of the discretized system raises the intriguing possibility that due to the mixing of low-energy states, there is something non-trivial to understand about how the quantum system behaves with respect to information stored in the black hole.

[1] ON UNITARY EVOLUTION OF A MASSLESS SCALAR FIELD IN A SCHWARZSCHILD BACKGROUND: HAWKING RADIATION AND THE INFORMATION PARADOX. By Kirill Melnikov, Marvin Wein-

stein (SLAC). SLAC-PUB-9224, May 2002. 45pp. e-Print Archive: hep-th/0205223

[2] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); J.B. Hartle and S.W. Hawking, Phys. Rev. **D13**, 2188 (1976).

[3] W.G. Unruh, Phys. Rev. **D14**, 870 (1976).

[4] J. D. Bekensten, Phys. Rev. **D7**, 2333 (1973), J. D. Bekenstein, Phys. Rev. **D9**, 3292 (1974).