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Electron cloud at high beam currents *

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Abstract

The density and the wake fields of the e-cloud are quite different at low and high beam currents. The wake fields are derived and applied to the upgraded PEP-II B-factory.

1 Introduction: Example

There are plans for upgrading the PEP-II B-factory to higher luminosity [1]. This could be achieved, mostly, by increasing the beam currents up to 10-20 Amp. Table I presents four possible scenarios of upgrading the PEP-II B-factory. Many potential problems hinder with the plans, the most obvious of them are related to the RF and the synchrotron radiation (SR) heat loading. Here I would like to consider only adverse effects of the beam interaction with the electron cloud.

The present wisdom predicts that the density of the cloud is defined by the condition of neutrality

$$< n >= \frac{N_b}{\pi s_b b^2}. \quad (1)$$

Therefore, the interaction with the cloud and, particularly, the tune shift

$$\Delta Q_\beta = \frac{2\pi r_e R^2 n_e}{\gamma Q_\beta} \quad (2)$$

grow proportional to the beam current. The variation of the tune along the bunch is of the same order. For the nominal PEP-II parameters, Table I (1st column), $\Delta Q_\beta = 0.052$ and is unacceptably large for higher currents.

I would like to argue that such a prediction might be wrong and the path to the high currents, at least from the point of view of e-cloud effects, is not hopeless.

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Table 1: Parameters for upgraded PEP-II LER

Parameter	(I)	(II)	(III)	(IV)
n_b	750	1658	3400	3492
I_{beam}, Amp	1.750	4.0/1.4	10.0	18.0
I_{bunch}/mA	2.33	2.41	2.94	5.15
σ_z	1.1 9	0.8	0.5	0.13
$\alpha, 10^{-3}$	1.23	1.23	2.41	2.41
$\delta_0, 10^{-4}$	7.7	7.7	7.7	7.7
$N_b\, 10^{-11}$	1.07	1.1	1.35	2.36

2 Relevant parameters

There are two groups of electrons in the cloud: primary photo-electrons generated by the SR photons and secondary electrons generated by the beam induced multi-pactoring. Electrons in the first group generated at the beam pipe wall with the radius b interact with the parent bunch and accelerated (by a short bunch) to the velocity $v/c = 2N_b r_e/b$, where r_e is the classical electron radius and N_b is the bunch population. Electrons in the second group, generally, miss the parent bunch and move from the beam pipe wall with the velocity $v/c = \sqrt{2E_0/mc^2}$ until the next bunch arrives. The velocity is defined by the average energy $E_0 \simeq 5$ eV of the secondary electrons and, at high N_b, is smaller than velocity of the first group.

The process of the cloud formation depends, respectively, on two parameters:

$$\kappa = \frac{2N_b r_e s_b}{b^2} \tag{3}$$

$$\zeta = \frac{s_b}{b}\sqrt{\frac{2E_0}{mc^2}} \tag{4}$$

These parameters are the distance (in units of b) passed by electrons of each group before the next bunch arrives.

At low currents, $\kappa << 1$, electron interacts with many bunches before it reaches the opposite wall. In the opposite extreme case, $\kappa > 2$, all electrons go wall-to-wall in one bunch spacing.

The transition to the second regime can be expected, therefore, for $\kappa \simeq 1$ where the cloud is quite different than it is at low currents. For $\kappa > 1$ and $\zeta < 1$, secondary electrons are confined within the layer $\zeta < (r/b) < 1$ at the wall and are wiped out of the region $0 < (r/b) < \zeta$ close to the beam by each passing bunch. This makes the range of parameters ($\kappa > 1$ and $2 - \kappa < \zeta < 1$) quite desirable to suppress the adverse effects of the e-cloud on the beam dynamics.

The initial energy of the electron and the space-charge force neglected above do not change substantially this statement. The case of high κ is considered here for the upgrades of the PEP-II B-factory.

The heat load to the wall increases with beam current but dependence on the current is different in low and high current regimes. The energy of an electron thrown to the wall by the passing bunch $E_w \simeq (mc^2/2)(2N_b r_e/b)^2$ and, therefore, the heat load of a bunch is proportional to N_b^3 at low currents, but only N_b^2 at high currents because the cloud density at saturation may be independent on the beam current.

(It may be worth noting also that at the very large currents, the energy of electrons hitting the wall is so large that secondary electron yield (SEY) Y rolls off and multi-pactoring at such high currents is always suppressed. This happens at $\kappa > \zeta\sqrt{E/E_0}$, where $E \simeq 2$ keV, $\kappa \simeq 10$. We will not consider that extreme case).

3 Density of the e-cloud at high-beam currents

The e-cloud density at low currents is given by the condition of neutrality. It means that the sum averaged in time of the fields of the beam and of the space-charge is zero at the wall.

The condition of neutrality implies that secondary electrons remain in the cloud for a time long enough to affect the secondary electrons generated by the following bunches. In other words, the condition of neutrality and the quasi-steady equilibrium distribution of the e-cloud are justified only for small κ.

It is not the case at the high currents. In this case, all primary photo-electrons disappear just in one pass. The secondary electrons are produced with low energy $E_0 \simeq$ 5eV and are locked up at the wall. The density of the secondary electrons grows until the space-charge potential of the secondary electrons is lower than E_0,

$$U \simeq \pi e^2 b^2 [1 - (1-\zeta)^2] n_0 \propto E_0. \tag{5}$$

This is a very moderate density $n_0 \simeq 2.8\,10^6\ cm^{-3}$.

The radius of the Larmor circles in the arcs may be changed by the kick from a passing bunch provided the bunch is short, $\omega_L \sigma_B / c << 1$ where $\omega_L = eH/mc$. Otherwise, there is the adiabatic invariant $L = m\omega_L r^2$ and the energy $E = L\omega$ of the Larmor motion is preserved. It means that electrons in the arcs are accumulated and may define the beam stability at the high bunch currents.

4 Simulations

Simple simulations were carried out for a round beam pipe $b = 4.5$ cm assuming that particles move only radially. Space charge was included. A bunch and the distance between bunches $s_b = 275$ cm were sliced and interaction with each slice was described as a kick. There was no source of particles except initial fill and multipactoring: particle crossing the wall with a low energy was killed and one with the energy $E > 40$ eV was replaced with $\eta = 1.45$ new electrons randomly distributed over the energy range 5 ± 2 eV. The four currents considered in simulations correspond to parameters $\zeta = 0.27$ and $\kappa = 0.22$, 0.94, 1.54 and 2.63, respectively. These cases are noted below as (a), (b), (c), and (d), respectively. Results of the simulations are shown in Figs. 1,2,3.

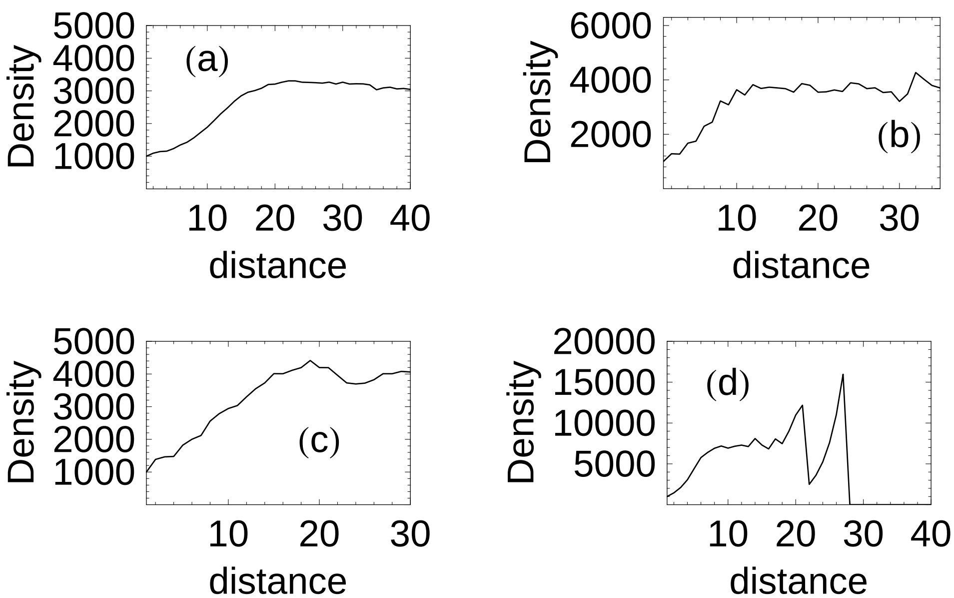

Figure 1: Total number of particles vs time (in units of bunch spacing). $\zeta = 0.27$ and $I_{beam} = 0.5\,A$, $2.15\,A$, $3.5\,A$ and $6.0\,A$ for (a),(b),(c), and (d), respectively.

The results of the simulations are consistent with the qualitative argument given above:

1. The density increases with the current and goes to saturation but, at the highest current, drops to zero. This can be expected when the average density exceeds the lock-up threshold.

2. The snap-shot of the cloud distribution substantially varies in time between bunches at high currents and has only small modulation at low current.

3. Although the average density increases with current, the variation of the density at the beam line in time is substantially different for different beam currents: it is about a constant in the case (a), it is maximum at the each other bunch in the case (b), and, at the high current, the bunch sees almost zero density cloud as it can be expected for $\kappa > 2$. I think that the situation (b) can explain why luminosity of each other bunch drops in the PEP-II [2].

5 Wakes and tune shifts at high currents

The wake field of the electron cloud at low currents is defined by electrons oscillating in the vicinity (3-5) $\sigma_\perp$ of the beam. Such electrons pass the memory of the offset of the

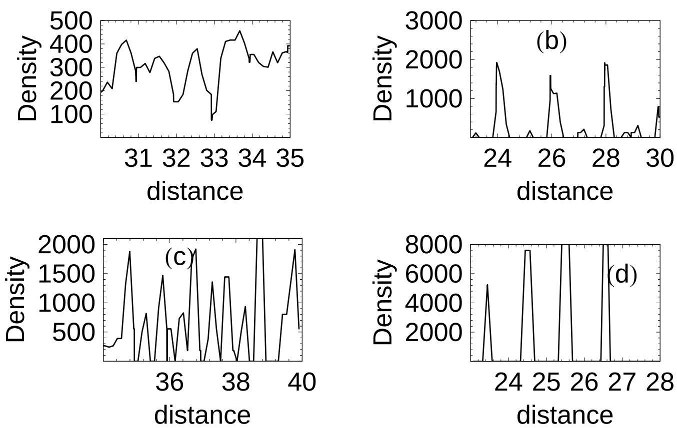

Figure 2: *Density at the beam line for the four beam currents vs time (in units of the passing bunch number). In the case (b), the density goes to zero for each other bunch. In the case (d), all bunches see minimum density.*

previous bunch to the following bunches.

The integrated single-bunch wake for a long bunch can be approximated[3], see Fig. 4, by the wake of a single mode with frequency $\mu\Omega_0$,

$$W_{eff}(z) = W_{eff} 2\pi R \frac{2n_e}{(1+\sigma_y/\sigma_x)\lambda_b}(\frac{\Omega_0}{c})\sin(\mu\xi)e^{-\frac{\mu\xi}{2Q}}. \tag{6}$$

Here, n_e is the cloud density, $\lambda_b = N_b/(\sigma_z\sqrt{2\pi})$ is the bunch linear density, Ω_0 is the linear frequency of the vertical electron oscillations, $(\Omega_0/c)^2 = 2\lambda_b/(\sigma_y(\sigma_x+\sigma_y))$ and $\xi = \Omega_{bunch} z/c$. Numeric calculations [4] which take into account the frequency spread of the electrons of the e-cloud, defined parameters $W_{eff} = 1.2$, $\mu = 0.9$, $Q = 5$ which are with good accuracy independent on the rms size of the cloud.

Additional effect is given by possible asymmetry of the cloud due to primary photo-electrons or ante-chamber. For an estimate, the field of an anti-symmetric cloud with the cloud centroid at a and the linear density dN/ds can be described as a field of a thread with the linear charge density dN/ds displaced by a from the axes of the round beam pipe. The horizontal component of the m-th azimuthal harmonic of the field of the thread is

$$E_x^{(m)} = \frac{2e}{a}\frac{dN}{ds}(\frac{r}{a})^{m-1}[1-(\frac{a}{b})^{2m}]\cos[(m-1)\phi]. \tag{7}$$

The $m = 1$ harmonic gives the steady-state horizontal force and changes the equilibrium energy of the beam by $\Delta E/E = eE_x^{(1)}\rho/E$, where ρ is the bend radius. Effect is very small. For example, let us consider the jet of the primary photo-electrons with the linear density

$$\frac{dN}{ds} = Y_{e\gamma}\frac{5\alpha_0\gamma}{2\sqrt{3}\rho}N_b\frac{L_{arcs}}{2\pi R}, \tag{8}$$

where $Y_{e\gamma} \simeq 0.1$ is number of electrons per SR photon, $\alpha_0 = 1/137$, and $L_{arcs} = 2\pi\rho$ is the total length of the bends. Let us assume that the primary photo-electrons get the kick $v/c = 2N_b r_e/b$ from the parent bunch and move to the radius $a = b - (2N_b r_e/b)s_b$ to the moment when the next bunch arrives. Taking the bunch population $N_b = 10^{11}$, the bunch spacing $s_b = 250$ cm, $\rho = 13.5$ m and $b = 4.5$ cm, we get $dN/ds = 1.8\,10^7\ 1/cm$, $a = 1.36$ cm, and $\Delta E = 3.5$ eV for 2.2 km PEP-II LER.

Effect of the asymmetry due to the ante-chamber at low beam currents depends on the parameter $\omega_{pl}s_b/c = \sqrt{2\kappa}$, where ω_{pl} is the plasma frequency $\omega_{pl}/c = \sqrt{4\pi n_0 r_e}$. Hence, at low currents $\kappa < 1$, any asymmetry of the cloud density generated by a bunch is preserved to the next bunch but hardly is larger than the effect of the asymmetry of the photo-electrons estimated above.

The mechanism of the bunch interaction through the e-cloud is different at high currents and is defined by azimuthal asymmetry of the distribution of the secondary electrons due to bunch transverse offset. The bunch with the offset x gives the asymmetric kick $(v/c)_{\pm} = 2N_b r_e/(b \pm x)$ to the electrons in the cloud. They reach the wall and produce secondary electrons at the different moments $t_{\pm}$. The secondary electrons propagating toward the following bunch are at the different distances $a_{\pm} = b - \zeta(1 - ct_{\pm}/b)$ from the beam line when the bunch arrives. The interaction with the bunch is given by the field $E_x(a_-) - E_x(a_+)$ of the $m = 1$ harmonic, see Eq. (7). Expanding the field over x, the result can be described as the transverse bunch-to-bunch wake $W_{\perp}$. For small $\zeta << 1$, the integrated wake is

$$W_{\perp} = 2\pi R \frac{8}{b^2} \frac{1}{N_b} \frac{dN}{ds} \frac{\zeta}{\kappa}, \tag{9}$$

where $a = b(1 - \zeta)$. For $n_0 = (dN/ds)/(\pi b^2) \simeq 10^6\ cm^{-3}$, $N_b = 10^{11}$, $2\pi R = 2.2$ km, and $b = 2.5$ cm, we get $\zeta = 0.4$, $\kappa = 2.25$, and $W_{\perp} = 11\ V/pC/cm$.

The azimuthal harmonic $m = 2$ of the e-cloud distribution gives the tune shift

$$\Delta Q_{x,y} = \mp \frac{r_e R^2}{\gamma Q_{x,y} b^2} (\frac{dN}{ds})[(\frac{b}{a})^2 - (\frac{a}{b})^2]. \tag{10}$$

For the same parameters n_0, N_b and b as above, we get $dQ/dI_{beam} = 4.5\,10^{-3}\ 1/Amp$.

It is worth noting that the effect of the jets of the primary photo-electrons on the beam varies along the bunch due to the changing distance from the jet to the beam line. This may cause variation of the tune shift and orbit distortion along the bunch.

6 Head-tail instability

The wake generated by the interaction with the cloud leads to the head-tail instability [3]. A peculiar feature of the e-cloud wake that it depends on I_{bunch} due to the electron frequency dependence. The Satoh-Chin's formalism [5] can be used, in principal, to define the threshold of instability. The stability is defined by the eigen values of a matrix which has to be, as usual, replaced by a matrix of a finite rang. Simulations with a low order matrix show a certain threshold of the head-tail instability. However, the bunch again become stable at higher currents. This reduction of the growth rate may be a result of a large number of electron oscillations per bunch length $\Omega_{bunch}\sigma_l/c >> 1$ at large N_b. At the present time, it is not clear whether such an explanation is correct until the numeric results are checked with the matrices of higher rang (of the order of $(\Omega_{bunch}\sigma_l/c)2$).

7 Conclusion

The present theory predicts that the e-cloud becomes more dangerous at high currents. The situation might be not hopeless. The condition of neutrality predicting the growth of the e-density with current might be replaced by the lock-up condition independent of current. The distribution of electrons in the cloud changes and, at the high currents, becomes hollow. In particular, the density at the beam line which defines beam stability decreases. The head-tail instability is stabilized at high currents due to high electron frequencies.

These prediction and, in particular, the adverse effect of density fluctuations, could be verified with existing codes.

References

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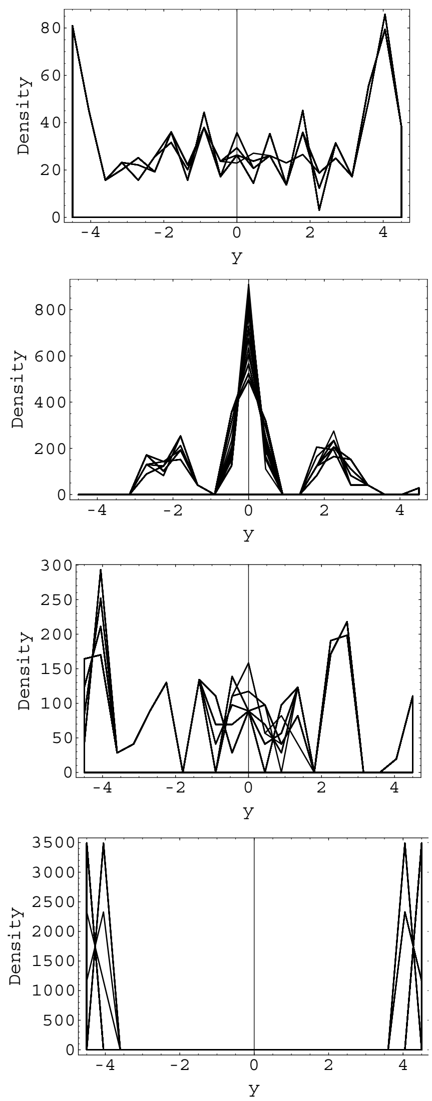

Figure 3: *Snap shots of the e-cloud distribution along the beam pipe diameter. Current increases from the top to bottom: 0.5, 2.5, 3.5 and 6 Amp, respectively.*

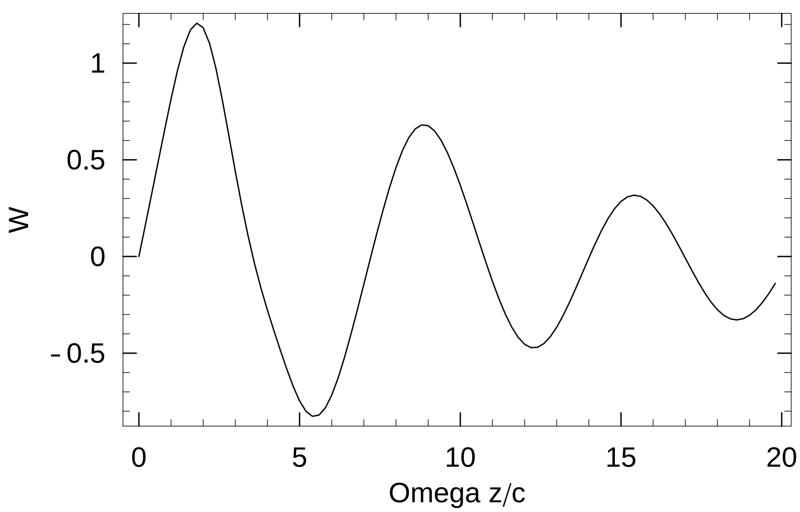

Figure 4: *Effective wake* $W_{eff}(\zeta, 0)$ *of the cloud as function of* $\zeta = \Omega_0 z/c$.