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Two-loop corrections to $gg \to \gamma\gamma$

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An overview of the calculation of the two-loop helicity amplitudes for scattering of two gluons into two photons is presented. These matrix elements enter into the recent improved calculation of the QCD background to Higgs boson decay into a pair of photons, which is the preferred search mode at the LHC for the case of a light Higgs boson.

Recent years have seen an enormous improvement in our ability to calculate two-loop amplitudes with more than a single kinematic invariant. The initial calculations of this type were for four-point scattering in maximally supersymmetric theories [1] and special helicity configurations in QCD [2]. In this talk we will focus on the two-loop amplitudes for gluon fusion into two photons [3]. This calculation is among the more general two-loop processes [4, 5, 6, 7, 8, 9, 10, 11] that have become doable, thanks to the development of rather general two-loop integration techniques [12]. Further summaries of these developments may be found in a number of talks at this conference [13].

Gluon fusion into two photons is phenomenologically interesting because the preferred mode for discovering a light Higgs boson ($M_H < 140$ GeV) at the LHC is through its decay into two photons. There are large backgrounds to this decay coming from radiation from either partons or hadrons [14, 15]. Although the gluon fusion contributions to the background are formally of higher order in the perturbative expansion they are greatly enhanced by the large gluon distribution at small x at the LHC.

The amplitudes described here were calculated in the helicity formalism, using the 't Hooft-Veltman dimensional regularization scheme [16]. Helicity methods have a long history, having been used rather productively at tree and one-loop levels (see *e.g.* refs. [17, 18]). At two loops the first of the $2 \to 2$ amplitudes were also evaluated using helicity states [1, 2]. In the $gg \to \gamma\gamma$ process under consideration here, the tree amplitudes vanish and the one-loop amplitudes give the leading order contributions. Thus the next-to-leading order contributions to gluon fusion require an interference of two-loop amplitudes with one-loop amplitudes. Instead of evaluating this interference directly, we again make use of helicity.

To generate the loop momentum integrals, we did not use Feynman diagrams, but instead used a unitarity-based technique [19, 20]. This technique exploits a duality between loop and phase space integrals and has already been employed in a number of two-loop calculations [1, 2, 10]. This duality was also used very recently to calculate the exact NNLO contribution to the total cross section for Higgs boson production at the LHC [21].

In performing the loop momentum integrals some minor extensions of the general reduction algorithms developed for four-point massless integrals [12] were needed. Any four-dimensional polarization vector can be expanded in terms of the three independent momenta in the problem

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plus a dual vector. This means that dot products of polarization vectors with loop momenta can be re-expressed in terms of dot products of loop momenta with external momenta and also the dual vector. Integrals where no dual vector appears can be reduced using the previously constructed reduction algorithms. This leaves integrals containing dual vectors dotted into loop momenta to be evaluated; as discussed in ref. [8] such integrals tend to be simpler to evaluate. An alternative method for dealing with helicity at two loops was recently given in ref. [9].

We take the quarks in the loops to be massless since the mass of the Higgs boson, and therefore the energy scale of the experiments for which this calculation is relevant, is well above the mass of all quarks other than the top. Moreover, the top quark can be ignored since the aforementioned energy scale is well below the $2m_t = 350$ GeV threshold.

The renormalized $gg \to \gamma\gamma$ amplitude may be expanded as

$$\mathcal{M}_{gg\to\gamma\gamma} \;=\; 4\pi\alpha \left[\frac{\alpha_s(\mu^2)}{2\pi} \mathcal{M}^{(1)}_{gg\to\gamma\gamma} + \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \mathcal{M}^{(2)}_{gg\to\gamma\gamma} + \cdots \right],$$

where $\mathcal{M}^{(L)}_{gg\to\gamma\gamma}$ is the Lth loop contribution, $\alpha_s(\mu^2)$ is the $\overline{\rm MS}$ running QCD coupling and α is the QED fine structure constant.

We use Catani's 'Magic Formula' [22] for two-loop infrared divergences to organize our results. This formula was obtained for general QCD amplitudes, but with minor modifications it also applies to mixed QED and QCD amplitudes. In cases where the tree-level amplitude vanishes (as happens for $gg \to \gamma\gamma$), Catani's formula reduces to,

$$\mathcal{M}^{(2)}_{gg\to\gamma\gamma} = I^{(1)}_{gg\to\gamma\gamma}(\epsilon)\mathcal{M}^{(1)}_{gg\to\gamma\gamma} + \mathcal{M}^{(2)\rm fin}_{gg\to\gamma\gamma}\,, \qquad (1)$$

where $I^{(1)}(\epsilon)$ contains the infrared singularities, and $\mathcal{M}^{(2)\rm fin}$ is a finite remainder. In our case the color factors can only be proportional to $\delta^{a_1a_2}$, so $I^{(1)}(\epsilon)$ is relatively simple:

$$I^{(1)}_{gg\to\gamma\gamma}(\epsilon,\mu^2;\{p\}) = -N\frac{e^{-\epsilon\psi(1)}}{\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon^2} + \left(\frac{11}{6} - \frac{1}{3}\frac{N_f}{N}\right)\frac{1}{\epsilon}\right]\left(\frac{\mu^2}{-s}\right)^{\epsilon},$$

where $N = 3$ in QCD and N_f is the number of light flavors. In this formula the infrared divergences are encoded as poles in the dimensional regularization parameter $\epsilon = (4 - D)/2$.

The one-loop amplitudes were given in ref. [3] through their relation to the one-loop four-gluon amplitudes. We can write

$$\mathcal{M}^{(1)}_{gg\to\gamma\gamma} = 2\,\delta^{a_1a_2}\left(\sum_{i=1}^{N_f} Q_i^2\right) M^{(1)}\,,$$

where Q_i is the electric charge of the ith quark and $M^{(1)}$ is equal to the sum of non-cyclic permutations of the fermionic contributions to the four-gluon primitive amplitudes [20].

It is convenient to extract overall spinor phases from each helicity amplitude,

$$M^{(1)}(1^{\lambda_1},2^{\lambda_2},3^{\lambda_3},4^{\lambda_4}) = S_{\lambda_1\lambda_2\lambda_3\lambda_4} M^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4},$$

where

$$S_{++++} = i\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\,, \quad S_{--++} = i\frac{\langle 1\,2\rangle\,[3\,4]}{[1\,2]\,\langle 3\,4\rangle}\,,$$
$$S_{-+++} = i\frac{\langle 1\,2\rangle\,\langle 1\,4\rangle\,[2\,4]}{\langle 3\,4\rangle\,\langle 2\,3\rangle\,\langle 2\,4\rangle}\,.$$

Through $\mathcal{O}(\epsilon^0)$ the amplitudes simplify greatly and the $M^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ reduce to,

$$\begin{aligned}
M^{(1)}_{++++} &= 1 + \mathcal{O}(\epsilon)\,,\\
M^{(1)}_{-+++} &= M^{(1)}_{+-++} = M^{(1)}_{++-+} = M^{(1)}_{+++-}\\
&= 1 + \mathcal{O}(\epsilon),\\
M^{(1)}_{--++} &= -\frac{1}{2}\frac{t^2+u^2}{s^2}\left[\ln^2\left(\frac{t}{u}\right) + \pi^2\right]\\
&\quad - \frac{t-u}{s}\ln\left(\frac{t}{u}\right) - 1 + \mathcal{O}(\epsilon)\,,\\
M^{(1)}_{-+-+} &= -\frac{1}{2}\frac{t^2+s^2}{u^2}\ln^2\left(-\frac{t}{s}\right)\\
&\quad - i\pi\left[\frac{t^2+s^2}{u^2}\ln\left(-\frac{t}{s}\right) + \frac{t-s}{u}\right]\\
&\quad - \frac{t-s}{u}\ln\left(-\frac{t}{s}\right) - 1 + \mathcal{O}(\epsilon)\,,\\
M^{(1)}_{+--+}(s,t,u) &= M^{(1)}_{-+-+}(s,u,t)\,.
\end{aligned}$$

where we are using an "all-outgoing" convention for the momentum (p_i) and helicity (λ_i) labeling. The Mandelstam variables are $s = (p_1 + p_2)^2$, $t = (p_1 + p_4)^2$, and $u = (p_1 + p_3)^2$.

We consider both QCD corrections with internal gluon lines and QED corrections with internal photons. For the QCD corrections, the dependence of the finite remainder in eq. (1) on quark charges, N, N_f and the renormalization scale μ, may be extracted as,

$$\mathcal{M}^{(2)\text{fin}}_{gg\to\gamma\gamma} = 2\,\delta^{a_1 a_2}\left(\sum_{j=1}^{N_f} Q_j^2\right) S_{\lambda_1\lambda_2\lambda_3\lambda_4} \times \left[\frac{11N - 2N_f}{6}\Big(\ln(\mu^2/s) + i\pi\Big) M^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4} + N F^{\text{L}}_{\lambda_1\lambda_2\lambda_3\lambda_4} - \frac{1}{N} F^{\text{SL}}_{\lambda_1\lambda_2\lambda_3\lambda_4}\right]. \quad (2)$$

The two-loop renormalized QED corrections are a little simpler, since in this case the amplitudes are free of infrared divergences,

$$\mathcal{M}^{(2)\text{QED}}_{gg\to\gamma\gamma} = 4\,\delta^{a_1 a_2}\left(\sum_{j=1}^{N_f} Q_j^4\right) \times S_{\lambda_1\lambda_2\lambda_3\lambda_4} F^{\text{SL}}_{\lambda_1\lambda_2\lambda_3\lambda_4}\,. \quad (3)$$

We quote our results in the physical s-channel ($s > 0$; t, $u < 0$). In order to reduce the size of the expressions we define

$$x = \frac{t}{s}, \quad y = \frac{u}{s}, \quad X = \ln(-x), \quad Y = \ln(-y),$$
$$\tilde{X} = X + i\pi, \quad \tilde{Y} = Y + i\pi,$$
$$\Xi = \tilde{X}^2 + \pi^2, \quad \Upsilon = \tilde{Y}^2 + \pi^2,$$
$$Z_\pm = X \pm Y, \quad \tilde{Z} = (X - Y)^2 + \pi^2,$$
$$A_n^\pm = \text{Li}_n(-x) \pm \zeta_n, \quad B = \text{Li}_2(-x) - \frac{\pi^2}{6},$$
$$C_n^\pm(x,y) = \text{Li}_n(-x) \pm \text{Li}_n(-y).$$

The explicit forms for the $F^{\text{L}}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ appearing in eq. (2) are

$$F^{\text{L}}_{++++} = \frac{1}{2}\,,$$

$$F^{\text{L}}_{-+++} = \frac{1}{8}\left[-(1 - xy)\tilde{Z} + 2\left(\frac{9}{y} - 10x\right)\tilde{X} + \left(2 + 4\frac{x}{y^2} - 5\frac{x^2}{y^2}\right)\Xi\right] + \Big\{t \leftrightarrow u\Big\}\,,$$

$$F^{\text{L}}_{++-+} = \frac{1}{8}\left[\left(2 + 6\frac{x}{y^2} - 3\frac{x^2}{y^2}\right)\Xi - (x - y)^2\tilde{Z} + 2\left(\frac{9}{y} - 8x\right)\tilde{X}\right] + \Big\{t \leftrightarrow u\Big\}\,,$$

$$\begin{aligned}
F^{\text{L}}_{--++} = {} & -(x^2 + y^2)\Big[4\text{Li}_4(-x) + \frac{1}{48}Z_+^4 \\
& + (\tilde{Y} - 3\tilde{X})\text{Li}_3(-x) + \Xi\,\text{Li}_2(-x) \\
& + i\frac{\pi}{12}Z_+^3 + i\frac{\pi^3}{2}X - \frac{\pi^2}{12}X^2 - \frac{109}{720}\pi^4\Big] \\
& + \frac{1}{2}x(1 - 3y)\Big[\text{Li}_3(-x/y) - Z_-\text{Li}_2(-x/y) \\
& - \zeta_3 + \frac{1}{2}Y\tilde{Z}\Big] + \frac{1}{8}\left(14(x - y) - \frac{8}{y} + \frac{9}{y^2}\right)\Xi \\
& + \frac{1}{16}(38xy - 13)\tilde{Z} - \frac{\pi^2}{12} - \frac{9}{4}\left(\frac{1}{y} + 2x\right)\tilde{X} \\
& + \frac{1}{4}x^2\Big[Z_-^3 + 3\tilde{Y}\tilde{Z}\Big] + \frac{1}{4} + \Big\{t \leftrightarrow u\Big\}\,,
\end{aligned}$$

$$\begin{aligned}
F^{\text{L}}_{-+-+} = {} & -2\frac{x^2 + 1}{y^2}\Big[A_4^- - \frac{1}{2}\tilde{X}A_3^- - \frac{1}{48}X^4 \\
& + \frac{\pi^2}{6}\left(B - \frac{1}{2}X^2\right) + \frac{1}{24}\tilde{X}^2\Xi\Big] + \frac{4}{9}\pi^2\frac{x}{y} \\
& + 2\frac{3(1 - x)^2 - 2}{y^2}\Big[C_4^-(x,y) + \text{Li}_4(-x/y) \\
& - \tilde{Y}A_3^- + \frac{\pi^2}{6}\left(\text{Li}_2(-x) + \frac{1}{2}Y^2\right) + \frac{1}{24}Y^4 \\
& - \frac{1}{6}XY^3 - \frac{7}{360}\pi^4\Big] - \frac{2}{3}\left(8 - x + 30\frac{x}{y}\right)\Big[\\
& \text{Li}_3(-y) - \zeta_3 - \tilde{Y}\left(\text{Li}_2(-y) - \frac{\pi^2}{6}\right) - \frac{1}{2}X\Upsilon\Big] \\
& + \frac{1}{6}\left(4y + 27 + \frac{42}{y} + \frac{4}{y^2}\right)\Big[i\frac{\pi}{2}X^2 - \tilde{X}B \\
& + A_3^- - \pi^2 X\Big] + \frac{1}{12}\left(3 - \frac{2}{y} - 12\frac{x}{y^2}\right)\tilde{X}\Xi \\
& + 2\left(1 + \frac{2}{y}\right)\left(\zeta_3 - \frac{\pi^2}{6}\tilde{Y}\right) + \frac{1}{24}\Big(y^2 - 24y \\
& + 44 - 8\frac{x^3}{y}\Big)\tilde{Z} - \frac{1}{24}\left(15 - 14\frac{x}{y} - 48\frac{x}{y^2}\right)\Xi \\
& + \frac{1}{24}\left(8\frac{x}{y} + 60 - 24\frac{y}{x} + 27\frac{y^2}{x^2}\right)\Upsilon - \frac{1}{3}y\tilde{X}\Upsilon
\end{aligned}$$

$$+\frac{1}{12}(2x^2-54x-27y^2)\biggl(\frac{1}{y}\tilde{X}+\frac{1}{x}\tilde{Y}\biggr).$$

Similarly, the subleading color contributions in eqs. (2) and (3) are,

$$F^{\rm SL}_{++++}=-\frac{3}{2},$$

$$F^{\rm SL}_{-+++}=\frac{1}{8}\biggl[\frac{x^2+1}{y^2}\Xi+\frac{1}{2}(x^2+y^2)\tilde{Z} -4\biggl(\frac{1}{y}-x\biggr)\tilde{X}\biggr]+\Bigl\{t\leftrightarrow u\Bigr\},$$

$$F^{\rm SL}_{++-+}=F^{\rm SL}_{+-++}=F^{\rm SL}_{+++-}=F^{\rm SL}_{-+++},$$

$$\begin{aligned}F^{\rm SL}_{--++}=&-\frac{1}{4}-2x^2\biggl[C_4^+(x,y)-\tilde{X}C_3^+(x,y)\\&+\frac{1}{12}X^4-\frac{1}{3}X^3Y+\frac{\pi^2}{12}XY-\frac{4}{90}\pi^4\\&+i\frac{\pi}{6}X\Bigl(X^2-3XY+\pi^2\Bigr)\biggr]+\frac{\pi^2}{12}\\&-(x-y)\Bigl(\mathrm{Li}_4(-x/y)-\frac{\pi^2}{6}\mathrm{Li}_2(-x)\Bigr)\\&-x\biggl[2\mathrm{Li}_3(-x)-\mathrm{Li}_3(-x/y)-3\zeta_3\\&+Z_-(\mathrm{Li}_2(-x/y)+X^2)-2\tilde{X}\mathrm{Li}_2(-x)\\&+\frac{1}{12}(5Z_-+18i\pi)\tilde{Z}-i\pi(Y^2+\pi^2)\\&-\frac{2}{3}X(X^2+\pi^2)\biggr]-\frac{1}{8}(2xy+3)\tilde{Z}\\&+\frac{1-2x^2}{4y^2}\Xi+\biggl(\frac{1}{2y}+x\biggr)\tilde{X}+\Bigl\{t\leftrightarrow u\Bigr\},\end{aligned}$$

$$\begin{aligned}F^{\rm SL}_{-+-+}=&-\frac{1}{2}-2\frac{x^2+1}{y^2}\biggl[C_4^-(x/y,y)+\frac{7}{360}\pi^4\\&+\frac{1}{24}(X^4+2i\pi X^3-4XY^3+Y^4+2\pi^2Y^2)\\&+\frac{1}{2}(\tilde{X}-2\tilde{Y})A_3^-\biggr]-2\frac{x-1}{y}\biggl[A_4^--\frac{1}{2}\tilde{X}A_3^-\\&+\frac{\pi^2}{6}\Bigl(B-\frac{1}{2}X^2\Bigr)-\frac{1}{48}X^4\biggr]-\frac{2-y^2}{4x^2}\Upsilon\\&+\biggl(2\frac{x}{y}-1\biggr)\biggl[A_3^+-\tilde{X}\mathrm{Li}_2(-x)-\frac{1}{6}X^3\\&-\frac{\pi^2}{3}Z_+\biggr]+\frac{\pi^2}{6}+2\biggl(2\frac{x}{y}+1\biggr)\biggl[\mathrm{Li}_3(-y)\\&+\tilde{Y}\mathrm{Li}_2(-x)-\zeta_3+\frac{1}{4}X(2Y^2+\pi^2)\\&-\frac{1}{8}X^2(X+3i\pi)\biggr]-\frac{1}{4}(2x^2-y^2)\tilde{Z}\\&-\frac{1}{4}\Bigl(3+2\frac{x}{y^2}\Bigr)\Xi+\frac{1}{2}(2x+y^2)\biggl[\frac{1}{y}\tilde{X}+\frac{1}{x}\tilde{Y}\biggr],\end{aligned}$$

$$F^{\rm SL}_{+--+}(s,t,u)=F^{\rm SL}_{-+-+}(s,u,t).$$

The reliability of these results was ensured by performing a series of checks described in ref. [3].

A companion talk in these proceedings [23] describes the application of the amplitudes presented here to obtain [24] an improved prediction for the QCD background to Higgs production at the LHC, when the Higgs decays into two photons.

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